Vedic Math Presentation

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practice and master mental math – the Vedic way

Vedic Math

Veda means *Knowledge*

Vedic Maths over 2000 years old
Rediscovered in 20th century by Bharati Krishna

Comprised of Sutras and sub-Sutras
which are aphoristic formulas

A system of Mental Mathematics

Recommended Reference Book
Vedic Mathematics – Teacher's Manual - Elementary Level
Kenneth R. Williams
ISBN: 81-208-2774-0

Vedic Math

Completing the Whole

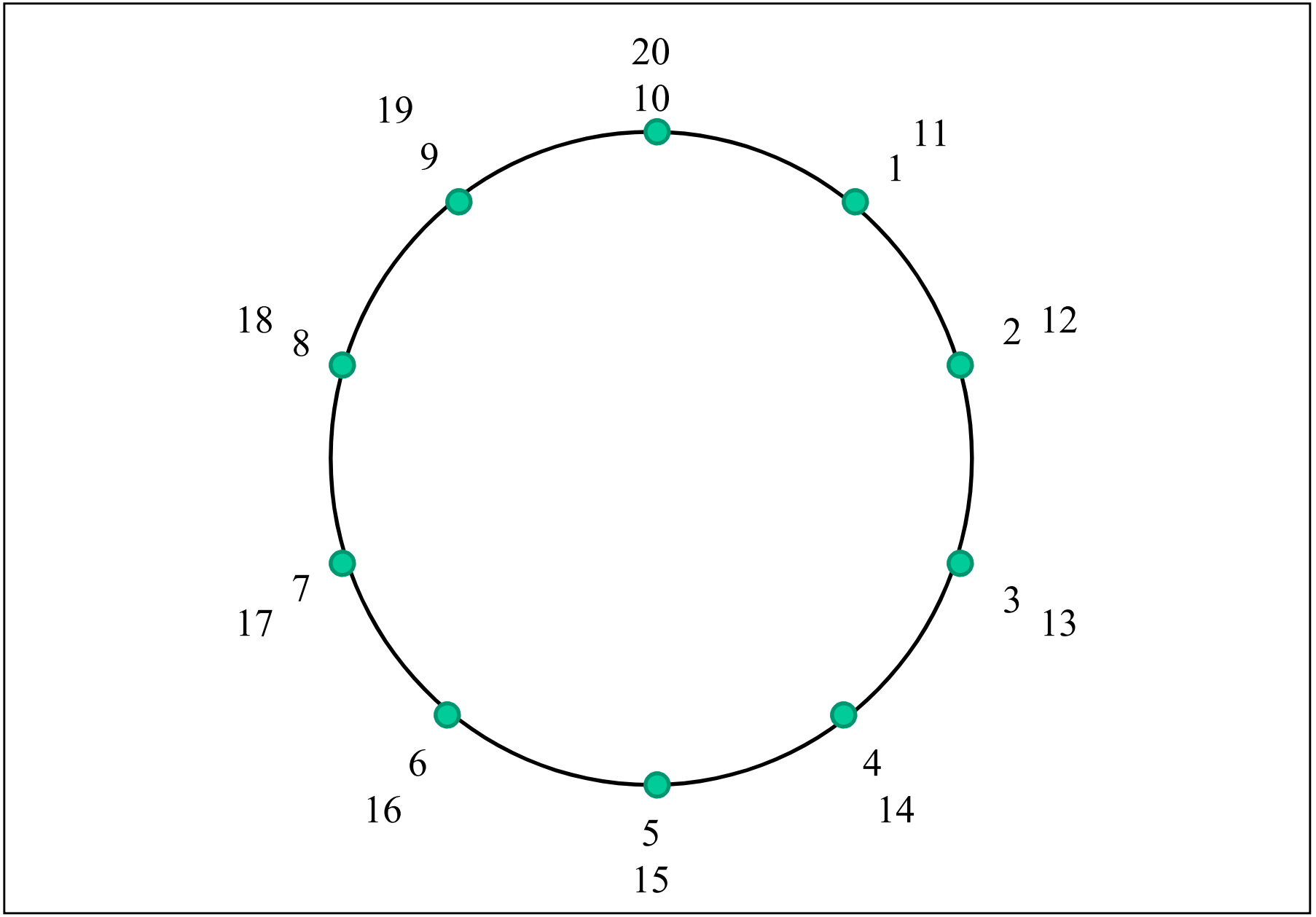
lessons

The Ten Point Circle
Using Subtraction to Simply Addition
Using Addition to Simplify Subtraction
Simplifying Addition by Groups of 10

sutras

By Completion or Non-Completion
By the Deficiency

The Ten Point Circle

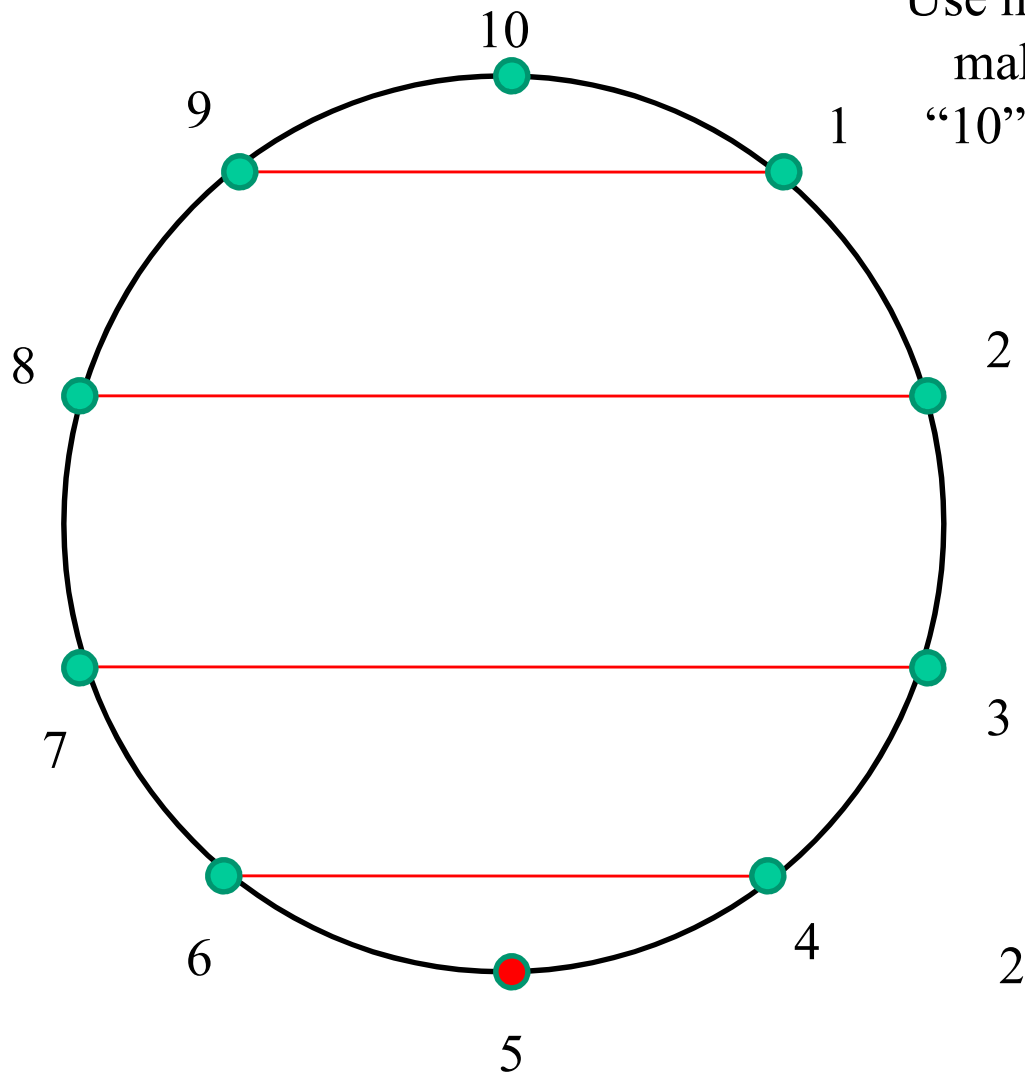


The Ten Point Circle

By Completion or Non-Completion

five number pairs

1 + 9
2 + 8
3 + 7
4 + 6
5 + 5



Use number pairs to make groups of "10" when adding numbers

example:
24 + 26 = 50

Below a Multiple of Ten

By the Deficiency

View a number as to how close it is to the next multiple of ten

49 is close to 50 and is 1 short
38 is close to 40 and is 2 short

It's easy to add zeros!

Make use of this!

$$59 + 4 = 63$$

59 is close to 60 and is 1 below it
So, 59 + 4 is 1 below 60 + 4
$59 + 4 = 60 + 4 - 1 = 64 - 1 = 63$

Practice This Process Mentally!

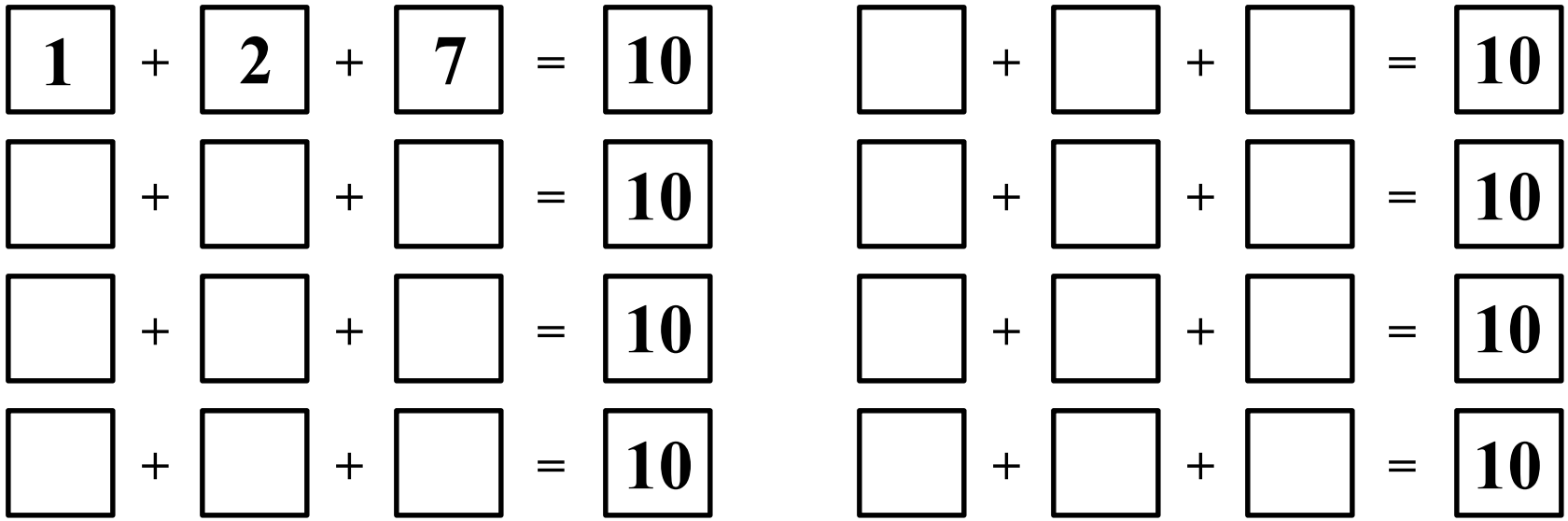
$$38 + 24 = 62$$

38 is close to 40 and is 2 below it
So, 38 + 24 is 2 below 40 + 24
$38 + 24 = 40 + 24 - 2 = 64 - 2 = 62$

Sum to Ten

The Ten Point Circle illustrates the pairs of numbers whose sum is 10

There are eight unique groups of three numbers that sum to 10
1 + 2 + 7 = 10 is an example



Can you find the other seven groups of three numbers summing to 10?

Adding a List of Numbers

By Completion or Non-Completion

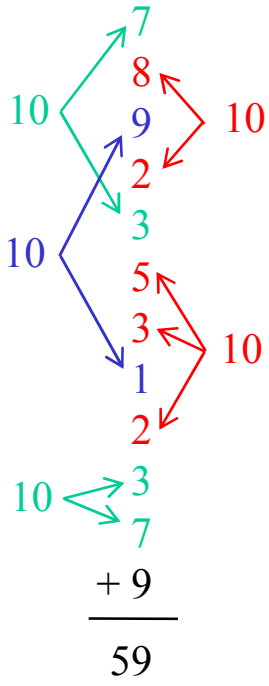
Look for number pairs that make a multiple of 10

$$7 + 6 + 3 + 4$$

The list can be sequentially added as follows
7+6 is 13,
13+3 is 16
And 16+4 is 20

OR
You could look for number pairs that make multiples of 10
7+3 is 10,
6+4 is 10
10+10 is 20

$$48 + 16 + 61 + 32$$



Subtracting Near a Base

By Completion or Non-Completion

When subtracting a number close to a multiple of 10,
Just subtract from the multiple of 10 and correct the answer accordingly

$$53 - 29$$

29 is close to 30, just 1 lower,
So subtract 30 from 53 making 23
Then add 1 to make 24

$$53 - 29 = 53 - (30 - 1) = 53 - 30 + 1 = 23 + 1 = 24$$

mentally

$\rightarrow (53 - 30) + 1$

$\downarrow$

$(23) + 1$

$\downarrow$

24

This process can be done **mentally**

$$45 - 18$$

$$45 - 18 = 45 - (20 - 2) = 45 - 20 + 2 = 25 + 2 = 27$$

Vedic Math

Doubling and Halving

lessons

Mentally Multiplying and Dividing by 2
Mentally Multiplying and Dividing by 4 and 8
Mentally Multiplying and Dividing by 5, 50 and 25
Using Number Relationships to Simplify a Problem

sutras

Proportionately

Doubling

Proportionately

Adding a number to itself is called ***Doubling***

$$\begin{array}{rl} & 23 \\ + & 23 \\ \hline & 46 \end{array}$$

2 + 2 → ← *3 + 3*

Mentally, we can double each column and then combine results

$$\begin{array}{rl} & 58 \\ + & 58 \\ \hline & 116 \end{array}$$

← *double 50 and add double 8*

← *100 + 16 = 116*

Grouping columns may simply the problem

$$\begin{array}{rl} & 263 \\ + & 263 \\ \hline & 526 \end{array}$$

← *double 260 and add double 3*

← *520 + 6 = 526*

Practice Doubling Mentally

Doubling

Proportionately

Practice how to approach a problem

$$\begin{array}{r} 736 \\ +\ 736 \\ \hline 1472 \end{array}$$

Mentally, the problem can be "broken" into two problems

$$736 = 700 + 36$$

double each of these and combine

$$1400 + 72 = \mathbf{1472}$$

Practice Doubling Mentally

Multiplying by 4 and 8

Proportionately

Doubling can be used to multiply by 4

just double the number twice

$$35 \times 4 = (35 \times 2) \times 2$$

so,

$$35 \times 4 = 70 \times 2 = 140$$

similarly,

$$163 \times 4 = 326 \times 2 = 652$$

Doubling can be used to multiply by 8

just double the number three times

$$35 \times 8 = ((35 \times 2) \times 2) \times 2$$

so,

$$35 \times 8 = 70 \times 4 = 140 \times 2 = 280$$

similarly,

$$163 \times 8 = 326 \times 4 = 652 \times 2 = 1304$$

Halving

Proportionately

Halving is the opposite of Doubling

*Half of **42** is **21***
just half each column

*Half of **56** is **28***
just half 50 and half 6 then add

Mentally, the problem can be "broken" into two problems

736 = 700 + 36

half each of these and combine
*350 + 18 = **368***

Practice Halving Mentally

Dividing by 4 and 8

Proportionately

Halving can be used to divide by 4

just half the number twice

$$72 \div 4 = (72 \div 2) \div 2$$

so,

$$72 \div 4 = 36 \div 2 = 18$$

similarly,

$$164 \div 4 = 82 \div 2 = 41$$

Halving can be used to divide by 8

just half the number three times

$$72 \div 8 = ((72 \div 2) \div 2) \div 2$$

so,

$$72 \div 8 = (36 \div 2) \div 2 = 18 \div 2 = 9$$

similarly,

$$504 \div 8 = 252 \div 4 = 126 \div 4 = 63$$

Multiplying by 5, 50 and 25

Proportionately

Multiply by 5 by multiplying by 10 and halving the result

$$26 \times 5 = (26 \times 10) \div 2 = 260 \div 2 = 130$$

It's easy to multiply by 10 and 100! Make use of this!

Multiply by 50 by multiplying by 100 and half the result

$$43 \times 50 = (43 \times 100) \div 2 = 4300 \div 2 = 2150$$

Multiply by 25 by multiplying by 100 and half the result twice

$$68 \times 25 = (68 \times 100) \div 4 = 6800 \div 4 = 3400 \div 2 = 1700$$

Dividing by 5, 50 and 25

Proportionately

Divide by 5 by doubling and dividing the result by 10

$$320 \div 5 = (2 \times 320) \div 10 = 640 \div 10 = 64$$

It's easy to divide by 10 and 100! Make use of this!

Divide by 50 by doubling and dividing the result by 100

$$850 \div 50 = (850 \times 2) \div 100 = 1700 \div 100 = 17$$

Divide by 25 by doubling twice and dividing the result by 100

$$325 \div 25 = (325 \times 4) \div 100 = 1300 \div 100 = 13$$

Proportionately

We know certain number facts well, such as 8 x 7 = 56

But given the problem 16 x 7,
we may use long multiplication,
Instead, ***proportionately*** *allows us to use*
our number facts along with halving and doubling

16 x 7 = 2 x (8 x 7) = 2 x (56) = 112

all of which can be done ***mentally***!

similarly,
18 x 14= (2x9) x (2x7) = 4 x (9x7) = 4 x 63 = 2 x 126 = 252

Vedic Math

Digit Sums

lessons

Definition of Digit Sum
Nine Point Circle
"Casting Out" 9's
Checking with Digit Sums

sutras

When the Samuccaya is the Same it is Zero

Digit Sums

A ***Digit Sum*** is the sum of all of the digits of a number
and is found by adding all of the digits of a number

The Digit Sum of 35 is 3+5 = 8

The Digit Sum of 142 is 1+4+2 = 7

If the sum of the digits is greater than 9, then
sum the digits of the result again until the result is less than 10

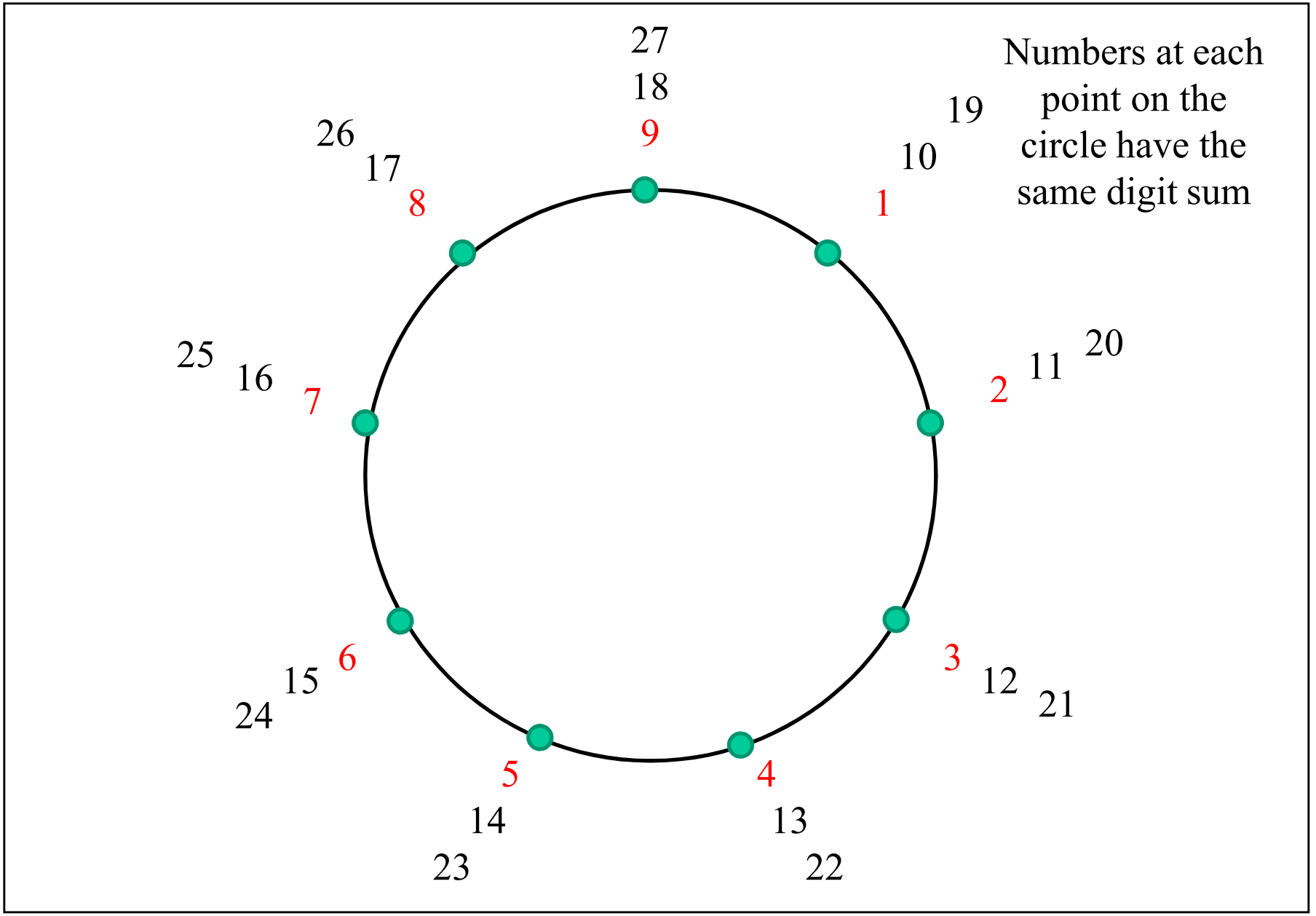
>9, so sum the digits again

The Digit Sum of 57 is 5+7 = 12 → 1+2 = 3
So the Digit Sum of 57 is 3

The Digit Sum of 687 is 6+8+7 = 21 → 2+1 = 3
So the Digit Sum of 687 is 3

Keep finding the Digit Sum of the result until it's less than 10
0 and 9 are equivalent!

Nine Point Circle



Casting Out 9's

When finding the Digit Sum of a number,
9's can be "cast out"

The Digit Sum of 94993 is 4+3 = 7
"cast out" the 9's

When finding the Digit Sum of a number,
Group of numbers that sum to 9 can be "cast out"

The Digit Sum of 549673 is 7
"cast out" the 5+4, 9 and 6+3, leaving just 7

By Casting Out 9's,
Finding a Digit Sum can be done more quickly
and mentally!

Checking with Digit Sums

Both Addition and Multiplication preserve Digit Sums

$$\begin{array}{rcl} 21 & \rightarrow & 3 \\ +\,14 & \rightarrow & 5 \\ \hline 35 & \rightarrow & 8 \end{array} \qquad 3+5 = 8$$

$$\begin{array}{rcl} 26 & \rightarrow & 8 \\ +\,35 & \rightarrow & 8 \\ \hline 61 & \rightarrow & 7 \end{array} \qquad 8+8 = 16 \rightarrow 8$$

If the sum of the Digit Sums does NOT equal
The Digit Sum of the sum, then there's a problem!

$$\begin{array}{rcl} 21 & \rightarrow & 3 \\ \times\,14 & \rightarrow & 5 \\ \hline 294 & \rightarrow & 6 \end{array} \qquad 3\times5 = 15 \rightarrow 6$$

$$\begin{array}{rcl} 26 & \rightarrow & 8 \\ \times\,35 & \rightarrow & 8 \\ \hline 910 & \rightarrow & 1 \end{array} \qquad 8\times8 = 64 \rightarrow 1$$

If the product of the Digit Sums does NOT equal
The Digit Sum of the product, then there's a problem!

Digit Sums of Perfect Squares

All Perfect Squares
end in 1, 4, 5, 6, 9 or 0
and
digit sums are 1, 4, 7 or 9

4539
ends in 9
digit sum is 3
*Therefore, **4539** is **not** a perfect square*

5776
ends in 6
digit sum is 7
*Therefore, **5776 may** be a perfect square*

Vedic Math

Left to Right

Vedic Math

All from 9 and the last from 10

lessons

Subtracting Number from 10^n
Applications with Money

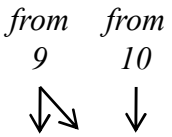
sutras

All From 9 and the Last From 10

All From 9 and the Last From 10

When subtracting a number from a power of 10
Subtract *all* digits *from 9 and last from 10*

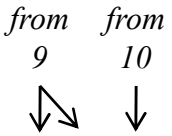
$$\begin{array}{r} 1\,0\,0\,0 \\ -\,2\,7\,6 \\ \hline 7\,2\,4 \end{array}$$



$$2\,7\,6 \quad \downarrow\downarrow\downarrow \quad 7\,2\,4$$

If the number ends in zero,
use the last non-zero number as the last number

$$\begin{array}{r} 1\,0\,0\,0\,0 \\ -\,4\,2\,5\,0 \\ \hline 5\,7\,5\,0 \end{array}$$

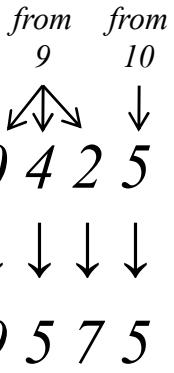


$$4\,2\,5\,0 \quad \downarrow\downarrow\downarrow\downarrow \quad 5\,7\,5\,0$$

All From 9 and the Last From 10

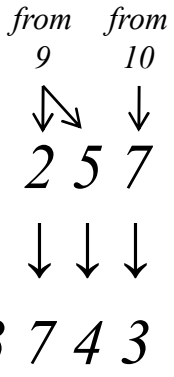
If the number is less digits, then append zeros to the start

$$\begin{array}{r} 10000 \\ -\,425 \\ \hline 9575 \end{array}$$



When subtracting from a multiple of a power of 10,
Just decrement the first digit by 1, then subtract remaining digits

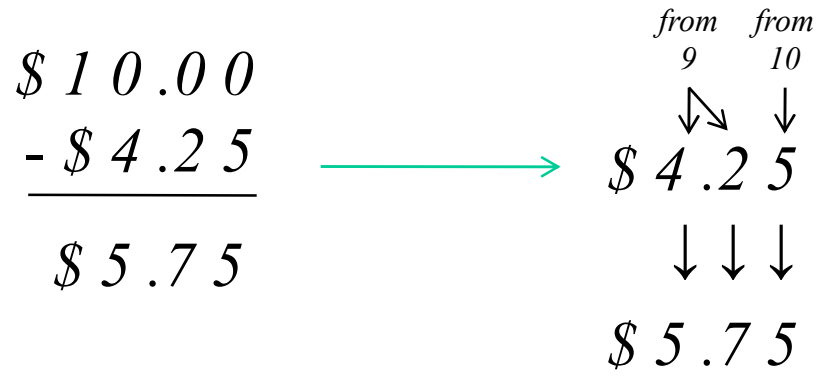
$$\begin{array}{r} 4000 \\ -\,257 \\ \hline 3743 \end{array}$$



Money

All from 9 and the Last from 10

A great application of "All from 9 and last from 10" is money. Change can be calculated by applying this sutra mentally!



It is often the case the payment is made with bills only, these are multiples of "100"

THINK MENTALLY!

Vedic Math

Number Splitting

lessons

Splitting Number to Simplify Problem

sutras

Proportionately

Number Splitting

Quick mental calculations can be performed more easily if the the numbers are **"split"** into more manageable parts

This sum can look somewhat daunting

$$\begin{array}{r} 3642 \\ +\ 2439 \\ \hline \end{array}$$

"split" into two more manageable sums

$$\begin{array}{r|l} 36 & 42 \\ +\ 24 & 39 \\ \hline 60 & 81 \end{array}$$

The "split" allows one to add "36+24" and "42+39", both of which can be done ***mentally***

Think about where to place the "split" line.
It's often best to avoid number "carries" over the line

$$\begin{array}{r} 342 \\ +\ 587 \\ \hline \end{array}$$

$$\begin{array}{r|l} 3 & 42 \\ +\ 5 & 87 \\ \hline 9 & 29 \end{array}$$

carry → *1*

a carry of "1" over the line is required

$$\begin{array}{r|l} 34 & 2 \\ +\ 58 & 7 \\ \hline 92 & 9 \end{array}$$

No carry is required

Number Splitting

The same can be done for **subtraction** also!

$$\begin{array}{r} 3642 \\ -\ 2439 \\ \hline \end{array} \longrightarrow \begin{array}{r|l} 36 & 42 \\ -\ 24 & 39 \\ \hline 12 & 03 \end{array}$$

The "split" allows one to subtract "36-24" and "42-39", both of which can be done ***mentally***

The same can be done for **multiplication** also!

$$\begin{array}{r} 263 \\ \times\ 2 \\ \hline \end{array} \longrightarrow \begin{array}{r|r} 26 & 3 \\ \times\ 2 & \times\ 2 \\ \hline 52 & 6 \end{array} \longrightarrow 526$$

The same can be done for **division** also!

$$\begin{array}{r} 6234 \\ \div\ 2 \\ \hline \end{array} \longrightarrow \begin{array}{r|r} 62 & 34 \\ \div\ 2 & \div\ 2 \\ \hline 31 & 17 \end{array} \longrightarrow 3117$$

Number Splitting

The “split” may require more “parts”

$$\begin{array}{r} 30155 \\ \div\, 5 \\ \hline \end{array} \longrightarrow \begin{array}{r|l|l} 30 & 15 & 5 \\ \div\, 5 & \div\, 5 & \div\, 5 \\ \hline 6 & 03 & 1 \end{array} \longrightarrow 631$$

$$\begin{array}{r} 244506 \\ \div\, 3 \\ \hline \end{array} \longrightarrow \begin{array}{r|l|l} 24 & 45 & 06 \\ \div\, 3 & \div\, 3 & \div\, 3 \\ \hline 8 & 15 & 2 \end{array} \longrightarrow 81502$$

Vedic Math

Base Multiplication

lessons

Multiplying Numbers Just Above or Below 10^n
Using Number Relationships to Simplify Problems
Multiplying Numbers Near Different Bases
Squaring Numbers Near a Base

sutras

Vertically and Crosswise
Proportionately

Multiplying Numbers Just Above 10

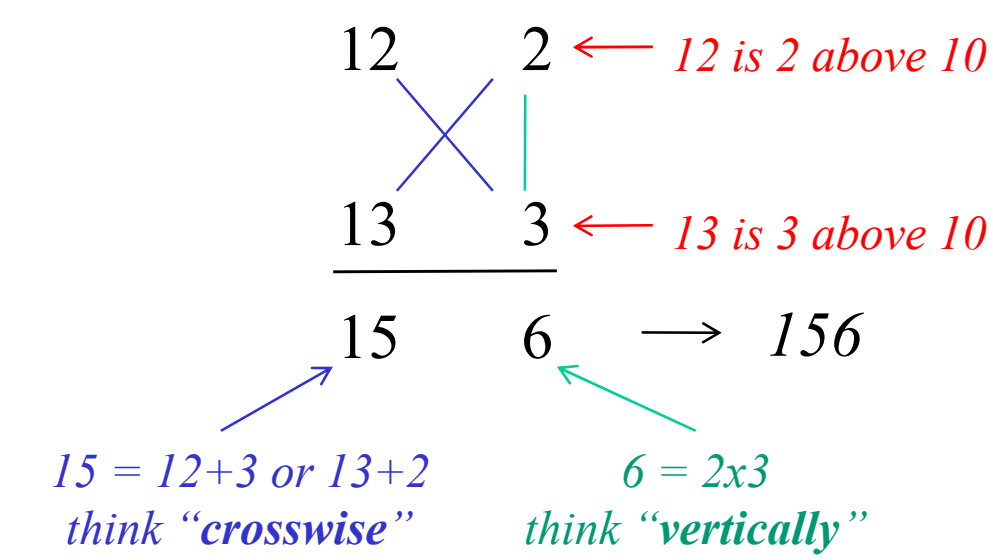
Vertically and Crosswise

Traditionally, the multiplication of 2-digit numbers requires
Four single digit multiplies and series of summations
to combine the results

Traditional Method

$$\begin{array}{r} 12 \\ \times\ 13 \\ \hline 36 \\ 12 \\ \hline 156 \end{array}$$

Vedic Method



The Vedic Method requires an addition (*crosswise*), a single digit Multiplication (*vertically*) and possibly a carry

Multiplying Numbers Just Above 10

Vertically and Crosswise

A carry may be required when combining the *crosswise* and *vertical* results

Traditional Method

$$\begin{array}{r} 16 \\ \times\ 13 \\ \hline 48 \\ 16 \\ \hline 208 \end{array}$$

Vedic Method

16 6 ← *16 is 6 above 10*

13 3 ← *13 is 3 above 10*

19 18 → *208*

↑

carry "1" and add to 19

Multiplying Numbers Just Below 100

Vertically and Crosswise

Traditionally, the multiplication of 2-digit numbers requires
Four single digit multiplies and series of summations
to combine the results

Traditional Method

$$\begin{array}{r} 98 \\ \times\ 88 \\ \hline 784 \\ 784 \\ \hline 8624 \end{array}$$

Vedic Method

98 2 ← *98 is 2 below 100*

88 12 ← *88 is 12 below 100*

86 24 → *8624*

86 = 98–12 or 88-2
*think "**crosswise**"*

24 = 2x12
*think "**vertically**"*

The Vedic Method requires a subtraction (*crosswise*), and
a single digit multiplication (*vertically*)

Multiplying Numbers Just Above 100

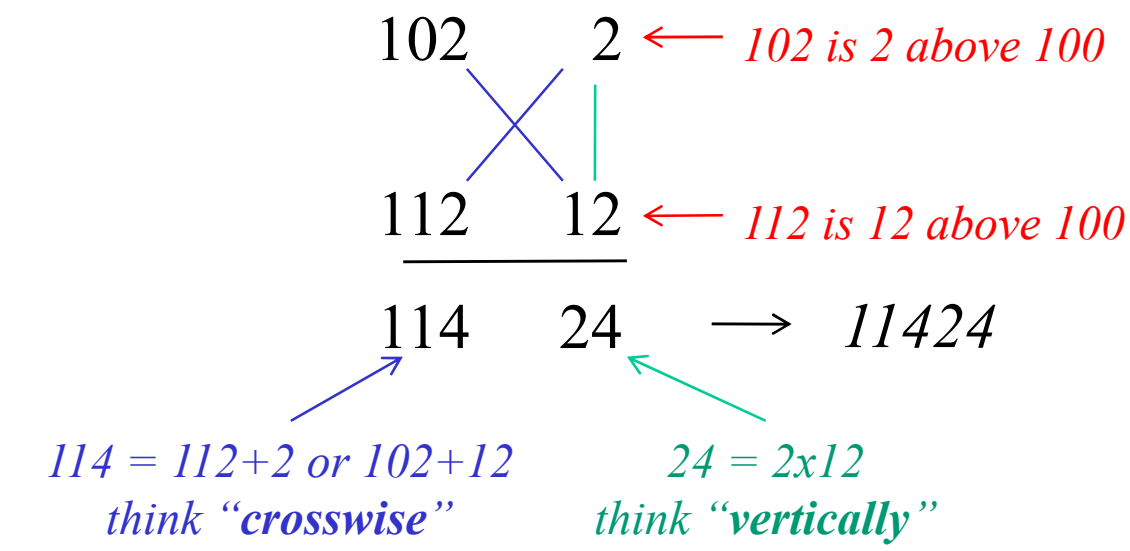
Vertically and Crosswise

Traditionally, the multiplication of 3-digit numbers requires
Six single digit multiplies and series of summations
to combine the results

Traditional Method

$$\begin{array}{r} 102 \\ \times\ 112 \\ \hline 204 \\ 102 \\ 102 \\ \hline 11424 \end{array}$$

Vedic Method



The Vedic Method requires an addition (*crosswise*), and
a multiplication (*vertically*)

Multiplying Numbers on Either Side of 100

Vertically and Crosswise

Multiplying Numbers Close to 100 but on either side

Traditional Method

$$\begin{array}{r} 112 \\ \times\ 98 \\ \hline 896 \\ 1008 \\ \hline 10976 \end{array}$$

Vedic Method

112 +12 ← *112 is 12 above 100*

98 -2 ← *98 is 2 below100*

110 -24 = *11000 – 24 - 10976*

110 = 112-2 or 98+12

-24 = -2x12

All from 9 and the Last from 10

So, what's 103 x 96? *9900 – 12 = 9888*

Multiplying Larger Numbers

Vertically and Crosswise

Traditionally, the multiplication of larger numbers requires numerous single digit multiplies and series of summations to combine the results

Traditional Method

```
  672
x 998
-----
 5376
6048
6048
-----
670656
```

Vedic Method

672 328 ← *672 is 328 below 1000*

"all from 9, the last from 10"

998 2 ← *998 is 2 below 1000*

670 656 → *670656*

670 = 672-2
*think "**crosswise**"*

656 = 2x328
*think "**vertically**"*

The Vedic Method requires an addition (*crosswise*), and a multiplication (*vertically*)

Proportionately

Often problems can be simplified in order to be performed mentally more easily

Traditional Method

$$\begin{array}{r} 212 \\ \times\ 104 \\ \hline 848 \\ 212 \\ \hline 22048 \end{array}$$

Vedic Method

$212 \times 104 \longrightarrow 2 \times (106 \times 104)$ *rewrite problem*

$2 \times (106 \times 104) = 2 \times 11024$ *vertically/ crosswise*

$= 22048$ *doubling*

another example

46 x 192 can be rewritten as 92 x 96 by *doubling* and *halving*

$46 \times 192 = 92 \times 96 = 8832$ *vertically/ crosswise*

Multiplying Numbers Near Different Bases

Vertically and Crosswise

Multiplying numbers near different bases
uses the same techniques

94 x 9997

9997 3 ← *9997 is 3 below 10000*

94 6 ← *94 is 6 below 100*

9397 18 ⟶ *939718*

*9397 = 9997-**6x100***
*think "**crosswise**"*

18 = 3x6
*think "**vertically**"*

10006 x 1002

10006 6 ← *10006 is 6 above 10000*

1002 2 ← *1002 is 2 above 1000*

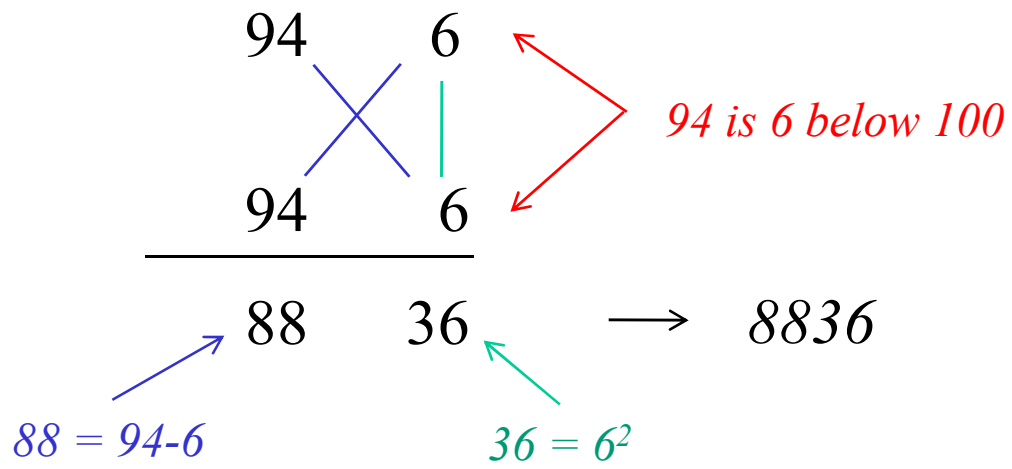
*10026 = 10006+**20***
*think "**crosswise**"* ⟶ 10026 12 ⟶ *10026012*

Squaring Numbers Near a Base

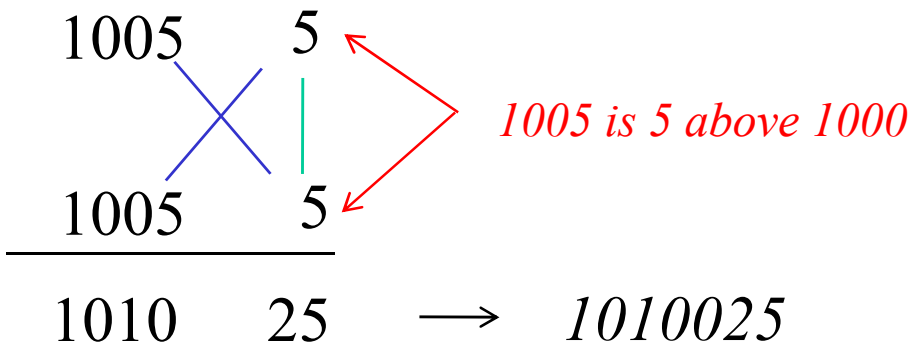
Vertically and Crosswise

To square numbers near a base, just apply the same techniques

94^2



1005^2



Vedic Math

Bar Numbers

lessons

Define Bar Numbers
Bar Number Arithmetic
Using Bar Numbers

sutras

All From 9 and the Last From 10

Bar Numbers

29 is close to the number 30
Let's rewrite 29 as $3\bar{1}$

$3\bar{1}$ means 30 – 1 or 29

Bar numbers are analogous to time
The time 5:45 can be
"45 minutes past 5"
OR
"15 minutes before 6"

$5\bar{2}$ means 50 – 2 or 48

$6\bar{3}$ means 60 – 3 or 57

$4\bar{1}2$ means 400 – 10 + 2 = 392

$4\bar{13}$ means 400 – 13 = 387

Note the distinction between
$4\bar{1}2 = 392$
and
$4\bar{12} = 388$

Subtraction using Bar Numbers

$$\begin{array}{r} 4\ 3\ 5 \\ -\ 2\ 7\ 6 \\ \hline 1\ 5\ 9 \end{array}$$

Traditionally, subtraction is performed on columns right to left "borrowing" from the next left column when necessary

However, subtracting each column independently gives the following:

$$4 - 2 = 2$$
$$3 - 7 = -4$$
$$5 - 6 = -1$$

Negative numbers can be replaced with their bar number equivalent , so

$$4 - 2 = 2$$
$$3 - 7 = \overline{4}$$
$$5 - 6 = \overline{1}$$

So,

$$\begin{array}{r} 4\ 3\ 5 \\ -\ 2\ 7\ 6 \\ \hline 2\ \overline{4}\ \overline{1} \end{array} = 2\overline{41} = 159$$

Arithmetic of Bar Numbers

$$\begin{array}{rcl} 2\ 9 & \rightarrow & 3\ \bar{1} \\ +\ 4\ 8 & \rightarrow & 5\ \bar{2} \\ \hline 7\ 7 & & \end{array}$$

The original problem can be rewritten three different ways using bar numbers

$$\begin{array}{r} 3\ \bar{1} \\ +\ 4\ 8 \\ \hline 7\ 7 \end{array} \qquad \begin{array}{r} 2\ 9 \\ +\ 5\ \bar{2} \\ \hline 7\ 7 \end{array} \qquad \begin{array}{r} 3\ \bar{1} \\ +\ 5\ \bar{2} \\ \hline 8\ \bar{3} \end{array} = 7\ 7$$

7 = 8-1 *7 = 9-2* *3 = 2+1*

In each case, the result is the same, 77!

Arithmetic of Bar Numbers

Addition:

$$\begin{array}{r} 2\ 8 \\ +\ 4\ 3 \\ \hline 7\ 1 \end{array} \quad \rightarrow \quad \begin{array}{r} 3\ \bar{1} \\ +\ 4\ 2 \\ \hline 7\ 1 \end{array} \quad \leftarrow \quad 1 = 2+\bar{1} = 2-1$$

Subtraction:

$$\begin{array}{r} 6\ 3 \\ -\ 3\ 7 \\ \hline 2\ 6 \end{array} \quad \rightarrow \quad \begin{array}{r} 6\ 3 \\ -\ 4\ \bar{3} \\ \hline 2\ 6 \end{array} \quad \leftarrow \quad 6 = 3-\bar{3} = 3+3$$

Multiplication:

$$\begin{array}{r} 2\ 8 \\ x\ \ 3 \\ \hline 8\ 4 \end{array} \quad \rightarrow \quad \begin{array}{r} 3\ \bar{2} \\ x\ \ 3 \\ \hline 9\ \bar{6} = 84 \end{array} \quad \swarrow \quad \bar{6} = \bar{2}x3$$

Division:

$$\begin{array}{r} 8\ 7 \\ \div\ \ 3 \\ \hline 7\ 1 \end{array} \quad \rightarrow \quad \begin{array}{r} 9\ \bar{3} \\ \div\ \ 3 \\ \hline 3\ \bar{1} = 29 \end{array} \quad \swarrow \quad \bar{1} = \bar{3}\div 3$$

Vedic Math

Special Multiplication

lessons

Multiply by 11
Multiply ab x *ac where b+c=10*
Multiply ba x *ca where b+c=10*
Multiply by 99, 999, etc
Using the Average to Multiply

sutras

Vertically and Crosswise
By One More than The One Before
By One Less than The One Before
The First by the First, The Last by the Last
Specific General

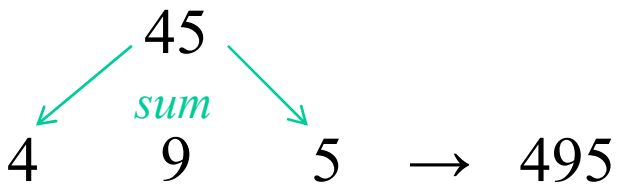
Multiplying by 11

Multiplying by 11 can be performed easily

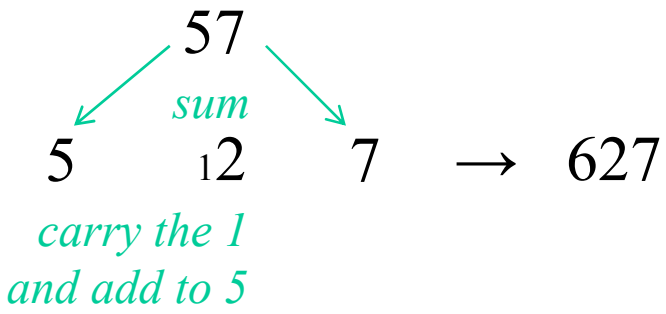
Traditional Method

$$\begin{array}{r} 45 \\ \times\ 11 \\ \hline 45 \\ 45 \\ \hline 495 \end{array}$$

Vedic Method



Carries may be required when the sum exceeds 9

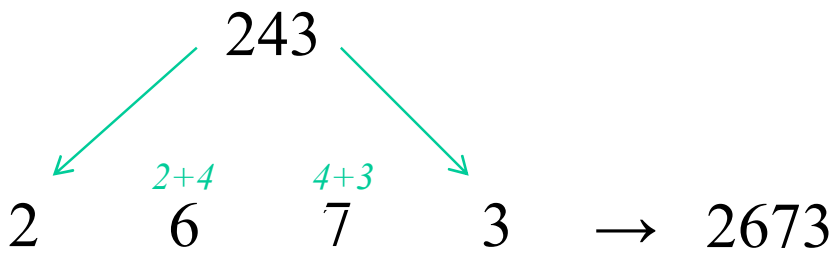


Multiplying by 11

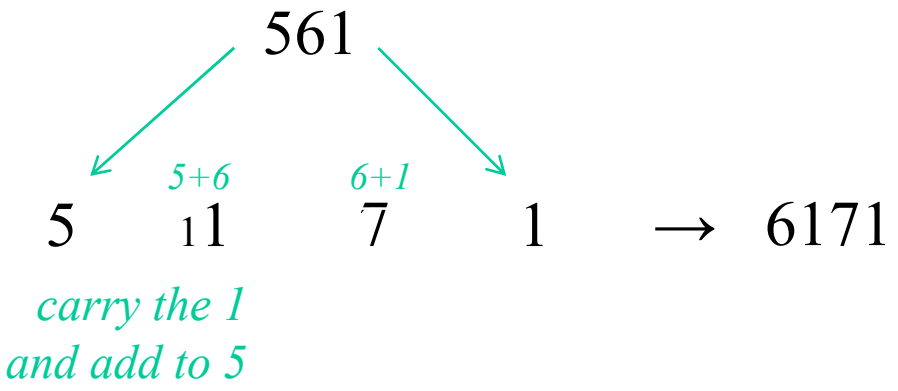
Traditional Method

$$\begin{array}{r} 243 \\ \times\ 11 \\ \hline 243 \\ 243 \\ \hline 2673 \end{array}$$

Vedic Method



Carries may be required when a center sum exceeds 9



Special Multiplication

By One More Than The One Before

When 2-digit numbers start with the same digit
and their last digits sum to 10,
The product of the two numbers can be easily computed

both numbers start with 3

32 x 38

the sum of the last digits is 10

$$\begin{array}{r} 3\ 2 \\ \times\ 3\ 8 \\ \hline 12\ 16 \end{array} \longrightarrow 1216$$

12 = 3x4 *16 = 2x8*

"By one more than the one before"

As the numbers get further from 35, their product gets further from $35^2 = 1225$ by a factor of the difference squared

35 x 35 = ***1225*** = 1225 – 0
34 x 36 = ***1224*** = 1225 – ***1*** = 1225 - $\boldsymbol{1}^2$
33 x 37 = ***1221*** = 1225 – ***4*** = 1225 – $\boldsymbol{2}^2$
32 x 38 = ***1216*** = 1225 – ***9*** = 1225 – $\boldsymbol{3}^2$
31 x 39 = ***1209*** = 1225 – ***16*** = 1225 - $\boldsymbol{4}^2$

Special Multiplication

The First by the First
The Last by the Last

When 2-digit numbers end with the same digit
and their first digits sum to 10,
The product of the two numbers can be easily computed

both numbers End with 4

24 x 84

the sum of the first digits is 10

2 4
x 8 4

20 16 ⟶ *2016*

20 = 2x8 + 4 *16 = 4x4*

3 6
x 7 6

27 36 ⟶ *2736*

6 2
x 4 2

26 04 ⟶ *2604*

Special Multiplication

By One Less Than The One Before

When multiplying by a number with all digits equal to "9", The product of the two numbers can be easily computed in two parts

$$\begin{array}{r} 32 \\ \times\ 99 \\ \hline \end{array}$$

31 68 ⟶ *3168*

31 = 32-1
"By one less than the one before"

68 = 100-32
"All from 9, the last from 10"

$$\begin{array}{r} 568 \\ \times\ 999 \\ \hline \end{array}$$

567 432 ⟶ *567432*

$$\begin{array}{r} 1285 \\ \times\ 9999 \\ \hline \end{array}$$

1284 8715 ⟶ *12848715*

Special Multiplication

Specific
General

When multiplying numbers, the average can sometimes be used to determine their product

31×29

their average is 30

square this and subtract 1 to determine the product

$$31 \times 29 = 30^2 - 1^2 = 900 - 1 = 899$$

30 = (31+29)/2

1 = (31-30)²

$$38 \times 42 = 40^2 - 2^2$$
$$= 1600 - 4 = 1596$$ *"all from 9, the last from 10"*

$$47 \times 53 = 50^2 - 3^2$$
$$= 2500 - 9 = 2491$$ *"all from 9, the last from 10"*

Vedic Math

General Multiplication

lessons

Multiply 2-Digit Numbers
Multiply 3-Digit Numbers

sutras

Vertically and Crosswise

Multiplying 2-Digit Numbers

Vertically and Crosswise

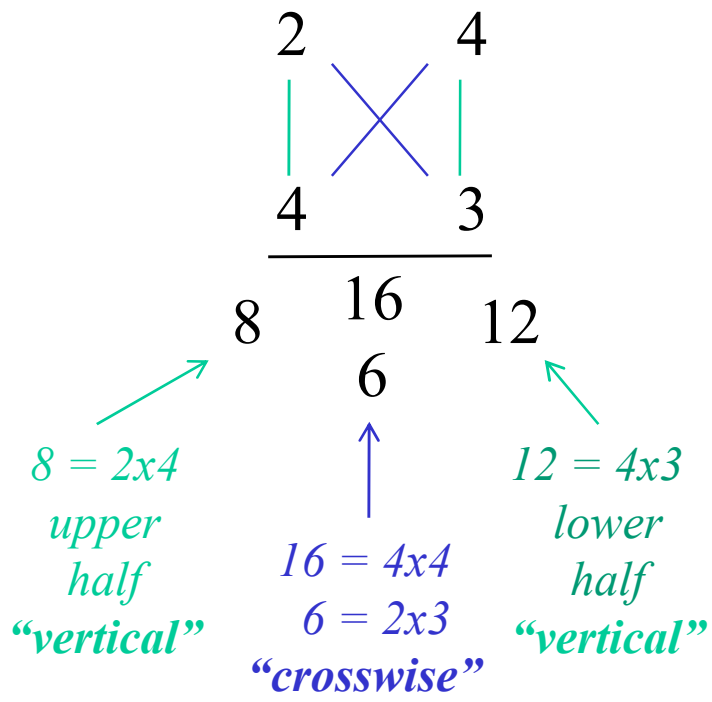
Traditional Method

$$\begin{array}{r} 24 \\ \times\ 43 \\ \hline 72 \\ 96 \\ \hline 1032 \end{array}$$

Intermediate Method

$$\begin{array}{rl} 24 & \\ \times\ 43 & \\ \hline 12 & =3\times4 \\ 60 & =3\times20 \\ 160 & =40\times4 \\ 800 & =40\times20 \\ \hline 1032 & \end{array}$$

Vedic Method



$$812 + 220 = 1032$$

$$46 \times 52 = 2012 + 380 = 2392$$

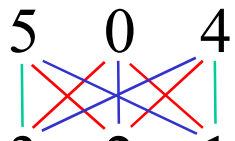
Multiplying 3-Digit Numbers

Find 504 x 321

Traditional Method

$$\begin{array}{r} 504 \\ \times\ 321 \\ \hline 504 \\ 1008 \\ 1512 \\ \hline 161784 \end{array}$$

Vedic Method



15, 10, 17, 8, 4

161784

Mentally, we think 15; 160; 1617; 16178; 161784

Find 123 x 321

3, 8, 14, 8, 3

39483

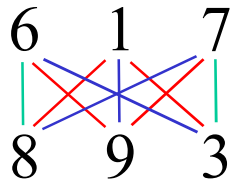
Mentally, we think 3; 38; 394; 3948; 39483

Multiplying 3-Digit Numbers

Vertically and Crosswise

Find 617 x 893

Vedic
Method



48, 62, 83, 66, 21

550981

Mentally, we think 48;542; 5503; 55096; 550981

Squaring

lessons

Squaring Numbers Ending in 5
Squaring Numbers Near 50
General Squaring using the Duplex of a Number

sutras

By One More Than the One Before
Proportionately

Squaring Numbers that End in 5

By One More Than the One Before

$35^2 \rightarrow 3 \bullet 4 \,/\, 25 \rightarrow 12 \,/\, 25 \rightarrow$ ***1225***

The result is comprised of two "parts"
The bottom "part" is always 25
The top "part" is n(n+1)

$75^2 \rightarrow 7 \bullet 8 \,/\, 25 \rightarrow 56 \,/\, 25 \rightarrow$ ***5625***

$45^2 \rightarrow 4 \bullet 5 \,/\, 25 \rightarrow 20 \,/\, 25 \rightarrow$ ***2025***

$95^2 \rightarrow 9 \bullet 10 \,/\, 25 \rightarrow 90 \,/\, 25 \rightarrow$ ***9025***

$115^2 \rightarrow 11 \bullet 12 \,/\, 25 \rightarrow 132 \,/\, 25 \rightarrow$ ***13225***

Squaring Numbers Near 50

$54^2 \rightarrow 5^2+4 \,/\, 4^2 \rightarrow 29 \,/\, 16 \rightarrow$ ***2916***

For numbers greater than 50

The result is comprised of two "parts"
The bottom "part" is always the bottom digit squared
The top "part" is the top digit squared plus the bottom digit

$48^2 \rightarrow 5^2-2 \,/\, 2^2 \rightarrow$ ***23*** $/\, 4 \rightarrow$ ***2304***

For numbers less than 50

The result is comprised of two "parts"
The bottom "part" is always the bar of the bottom digit squared
The top "part" is the top digit squared minus
the bar of the bottom digit

$53^2 \rightarrow 25+3 \,/\, 9 \rightarrow 28 \,/\, 9 \rightarrow$ ***2809***

$46^2 \rightarrow 25-4 \,/\, 16 \rightarrow 21 \,/\, 16 \rightarrow$ ***2116***

General Squaring

The ***Duplex, D,*** of a number

1 digit – $D(n) = n^2$

e.g. D(5) = 25

2 digits – D(n) = twice the product of the digits

e.g. D(26) = 2(2)(6) = 24

3 digits – D(n) = twice the product of the outer digits + the square of the middle digit

e.g. $D(137) = 2(1)(7) + 3^2 = 14 + 9 = 23$

The square of a number is the "total" of its Duplexes

$34^2 = 1156$

D(3) = 9, D(34) = 24, D(4) = 16

9, 24, 16 → 1156

$56^2 = 3136$

D(5) = 25, D(56) = 60, D(6) = 36

25, 60, 36 → 3136

General Squaring

$47^2 = 1156$

D(4) = 16, D(47) = 56, D(4) = 49

16, 56, 49 → 2209

16 49
5 6
22 09

Number Splitting

123^2 - *Split 123 into two parts 12 / 3*

D(12) = 144, D(123) = 72, D(3) = 9

144, 72, 9 → 15129

412^2 - *Split 412 into two parts 4 / 12*

D(4) = 16, D(412) = 96, D(12) = 144

16, 96, 144 → 169744

General Squaring (3 and 4-digit numbers)

$341^2 = 116281$

D(3) = 9, D(34) = 24, D(341) = 22, D(41) = 8, D(1) = 1

9, 24, 22, 8, 1 → 116281

$263^2 =$

D(2) = 4, D(26) = 24, D(263) = 48, D(63) = 36, D(3) = 9

4, 24, 48, 36, 9 → 69169

$4332^2 =$

D(4) = 16, D(43) = 24, D(433) = 33, D(4332) = 34,
D(332) = 21, D(32) = 12, D(2) = 4

16, 24, 33, 34, 21, 12, 4 → 18766224

Vedic Math

Special Division

lessons

Division by 9
Division Below and Above 10^n

sutras

Need to Determine This

Division by 9

$32 \div 9$ 9) 3 2

3 r5 *where 5 = 3+2*

$52 \div 9$ 9) 5 2

5 r7 *where 7 = 5+2*

$75 \div 9$ 9) 7 5

7 r12 *where 12 = 7+5* *remainder > 9*

= 8 r3

$3102 \div 9$ 9) 3 1 0 2

3 4 4 r6

$312 \div 9$ 9) 3 1 2

3 4 r6

When dividing by 9,
*The **remainder** is always the*
***digit sum** of the original number*

Division by 9 with Carries

$3172 \div 9$ $\quad 9\,)\,3\ 17\ 2$

$3\ 4\ {}_{1}1\ \text{r}13 \;=\; 351\ \text{r}13 \;=\; 352\ \text{r}4$

$5555 \div 9$ $\quad 9\,)\,5\ 5\ 5$

$5\ {}_{1}0\ \text{r}15 \;=\; 61\ \text{r}6$

Short Cut

$3172 \div 9$ $\quad 9\,)\,3\ 1\ 7\ 2$ *Check to see if next sum is >9, if so add 1*

$3\ 5\ 2\ \text{r}4$

$6153 \div 9$ $\quad 9\,)\,6\ 1\ 5\ 3$

$6\ 8\ 3\ \text{r}6$

Division Below a Base Number

235 ÷ 88

88) 2 | 3 5
1 2 | 2 4
2 | 5 9

1. *Drop 2 (the first digit)*
2. *Create 12 = $\overline{88}$*
3. *Multiple 12 by 2 = 24*
4. *Add 35 and24*

211 ÷ 75

76) 2 | 1 1
2 5 | 5 0
2 | 6 1

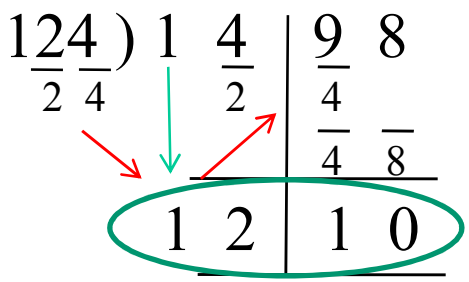
1121123 ÷ 8989

8989) 1 1 2 | 1 1 2 3
1 0 1 1 | 1 0 | 1 1
2 | 0 2 2
4 | 0 4 4

1 2 4 6 4 8 7

Division Above a Base Number

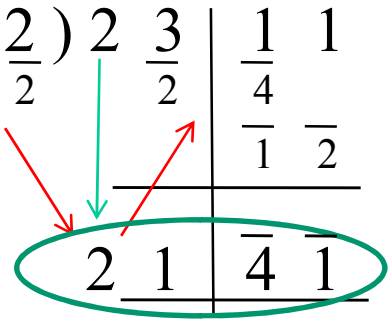
1498 ÷ 124



1. *Drop 1 (the first digit)*
2. *Create $\overline{24}$*
3. *Multiply $\overline{24}$ by 1 = $\overline{24}$*
4. *Add 4 + $\overline{2}$ = 2*
5. *Multiply $\overline{24}$ by 2 = $\overline{48}$*
6. *Add columns*

12 r10

2311 ÷ 112



= 20 r71 *where 71 = 112-41*

General Division

lessons

General Division
Decimalizing the Remainder

sutras

General Division

308 ÷ 51

	5	1	3	0	8
					0
				6	2

= 6 r2

1. *Create flag, 1, from divisor 2nd digit*
2. *5 goes into 30 6 times with r0*
3. *08 – 1x6 = 2*

234 ÷ 54

	5	4	2	3	4
					3
				4	18

= 4 r18

1. *Create flag, 4, from divisor 2nd digit*
2. *5 goes into 23 4 times with r3*
3. *34 – 4x4 = 18*

503 ÷ 72

	7	2	5	0	3
					1
				7	$\overline{1}$

= 6 r71

1. *Create flag, 2, from divisor 2nd digit*
2. *7 goes into 50 7 times with r1*
3. *13 – 2x7 = $\overline{1}$ = 72 – 1 = 71*

General Division

$19902 \div 62$

6	2	1	9	9	0	2
				1		
		3				

1. *Create flag, 2, from divisor 2nd digit*
2. *6 goes into 19 3 times with r1*

6	2	1	9	9	0	2
				1	1	0
		3	2	1	0	

1. *Compute 19 – 2x3 = 13*
2. *6 goes into 13 2 times with r1*

6	2	1	9	9	0	2
				1	1	0
		3	2	1	0	

1. *Compute 10 – 2x2 = 6*
2. *6 goes into 6 1 times with r0*

6	2	1	9	9	0	2
				1	1	0
		3	2	1	0	

1. *Compute 02 – 2x1 = 0 (remainder)*

$= 321\ r0$

General Division

$92054 \div 63$

6	3	9	2	0	5	4
			3			
		1				

1. *Create flag, 3, from divisor 2nd digit*
2. *6 goes into 9 1 times with r3*

6	3	9	2	0	5	4
			3	5		0
		1	4			

1. *Compute 32 – 3x1 = 29*
2. *6 goes into 29 4 times with r5*

6	3	9	2	0	5	4
			3	5	2	
		1	4	6		

1. *Compute 50 – 3x4 = 38*
2. *6 goes into 38 6 times with r2*

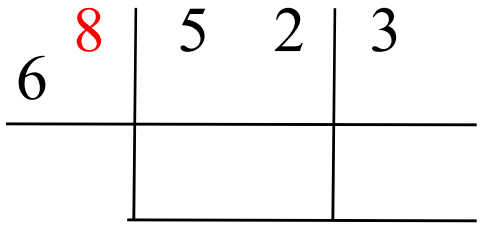
6	3	9	2	0	5	4
			3	5	2	1
		1	4	6	1	11

1. *Compute 25 – 3x6 = 7*
2. *6 goes into 7 1 time with r1*
3. *Compute 14 – 3x1 = 11 remainder*

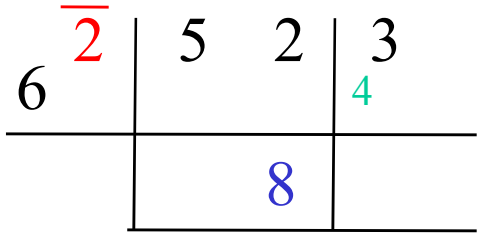
$= 1461\ r11$

General Division

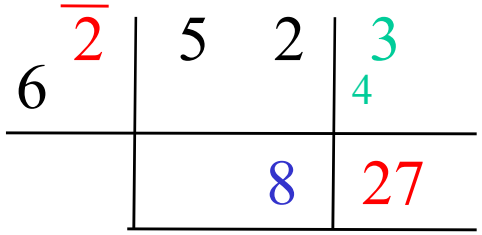
$543 \div 68$



1. *Create flag, 8, from divisor 2nd digit*



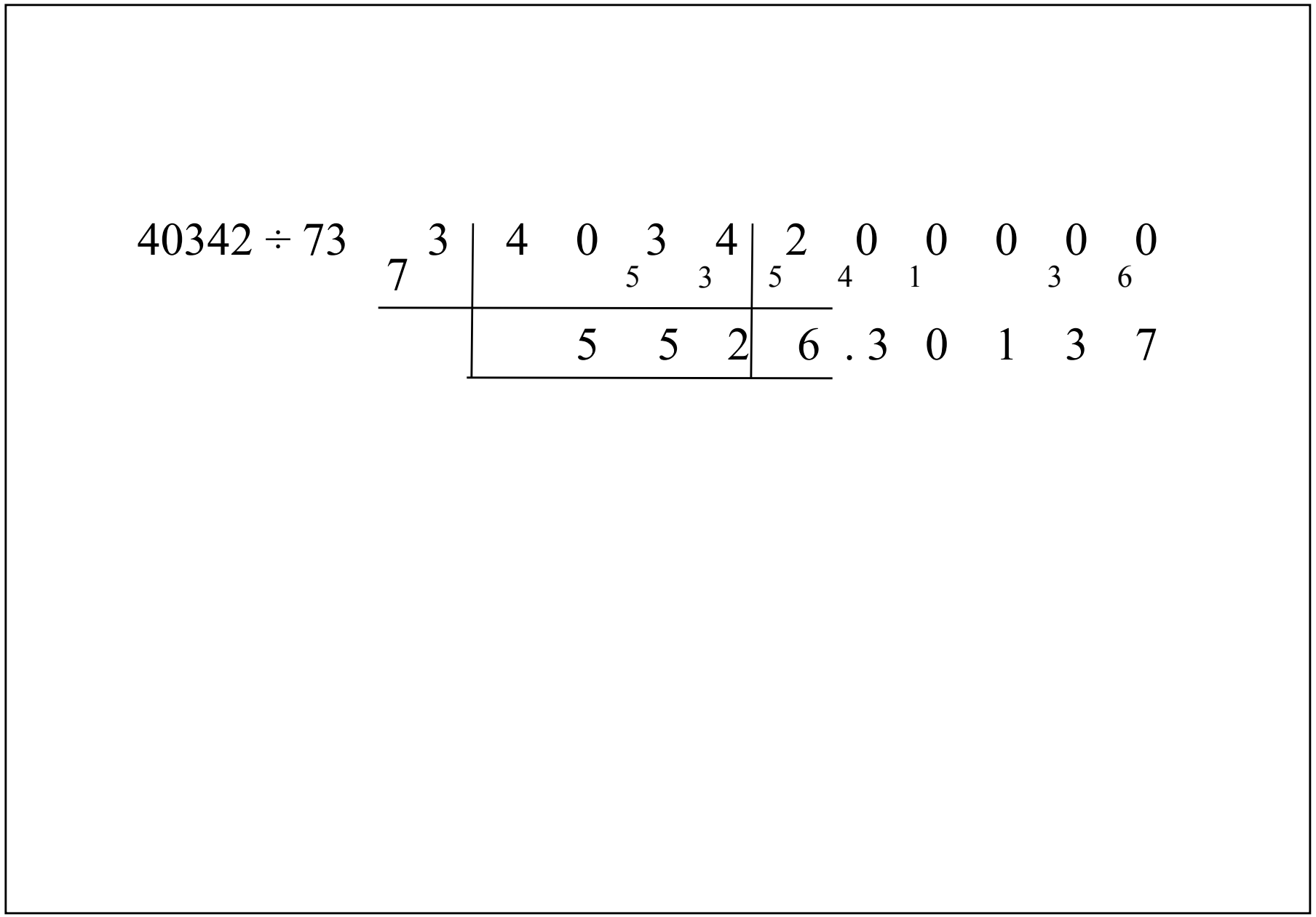
1. *Change flag to a bar number 2*
2. *6 goes into 52 8 times with r4*



1. *Compute 43 - 2x8 = 27 remainder*

$= 8\ r27$

Decimalizing the Reminder



Questions and Comments