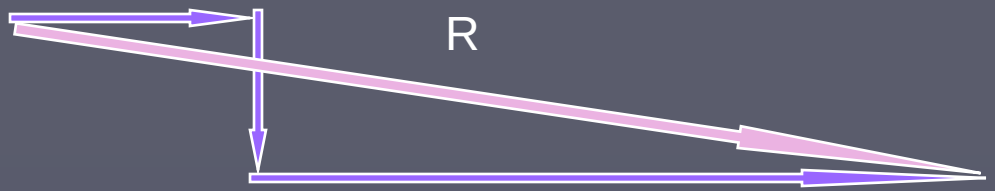
Vector Addition

Graphical and Algebraic Representations

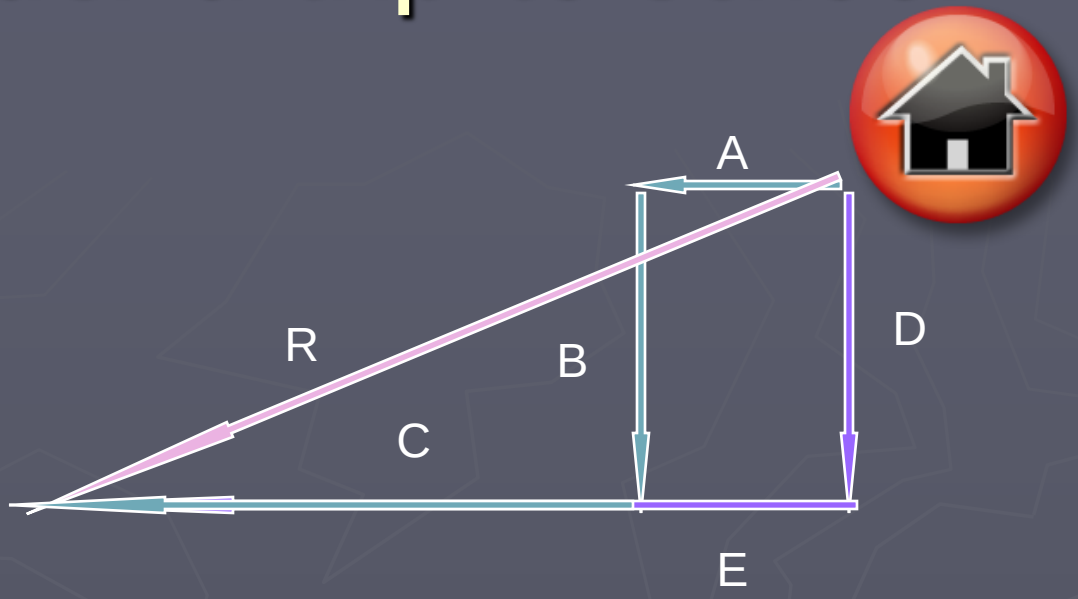
Review

- Vectors are arrows drawn to represent magnitude AND direction of a concept.
- Vectors can be combined to represent a total (Resultant) **graphically**.
- **Algebra** required includes Pythagorean theorem and trig ratios.
- Draw arrows proportionally according to quantity of a measurement.
- Vectors can have positive and negative directions, but not magnitudes.



- VECTORS CAN BE MOVED (as long as orientation remains constant.)
- **Resultant**- a vector that is equal to the sum of two or more vectors.
- Use the Tail-to-Head method of combining vectors.
- The magnitude of the resultant can be found by measuring R with a ruler and applying the length to a **scale** you have established.

Consider a trip to school



- If you compare the displacement for route ABC to route DE, you will find the resultant R is the same.

Resultant Magnitude

- If two vectors are at right angles to each other, the magnitude of the resultant can be found by : $R^2 = A^2 + B^2$

- If the vectors are at some angle other than 90º, then you can use the Law of Cosines:

  $R^2 = A^2 + B^2 - 2AB\cos\theta$

Your turn to practice

Find the magnitude of the sum of a 15-km displacement and a 25-km displacement when the angle between them is 135º.

Given:

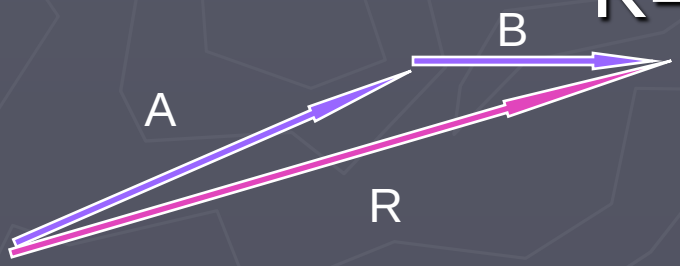
A=25km

B=15km

Θ=135º

Unknown:

R=?



Solution

- Strategy:

Use the Law of Cosines

- Calculations:

$R^2 = A^2 + B^2 - 2AB\cos\theta$

$R^2 = (25\text{km})^2+(15\text{km})^2-2(25\text{km})(15\text{km})(\cos135^\circ)$

$R^2 = 625\text{km}^2 + 225\text{km}^2 - 750\text{km}^2(\cos135^\circ)$

$R^2 = 1380\ \text{km}^2$

$R = \sqrt{(1380\ \text{km}^2)}$

$R = 37\ \text{km}$

Your turn to Practice

Open your textbook to pg. 67.

Follow the format given to you in the preceding example (Sketch, Label given and unknowns, SHOW YOUR WORK, Solve)

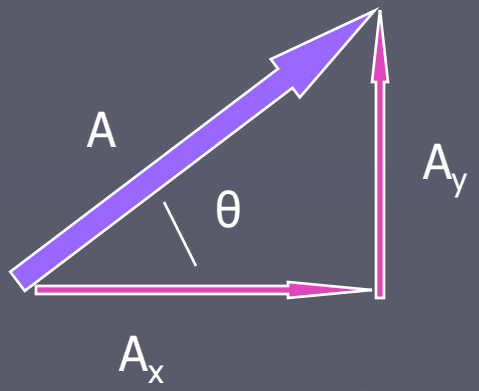
Answer the following practice questions:

#1, #2, #3

Read page 68. Complete questions 5-10 pg 71.

Components of Vectors

- Choose a coordinate system to help define the direction of your vectors. (For earth surface motions, usually the x-axis points east and the y-axis points north; for motion through the air, typically the x-axis is horizontal and the POSITIVE y-axis is vertical (upward).
- Once on a grid, the direction of a vector is the angle it makes with the x-axis measured counterclockwise.



Vector A is broken up into two COMPONENT vectors.

$A = A_x + A_y$

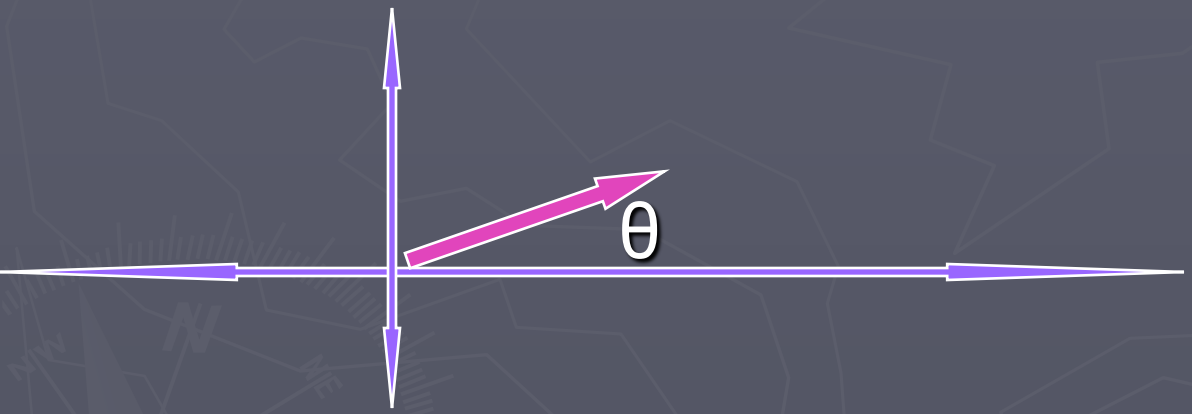
The process of breaking a vector into its components is called **vector resolution**.

Since $\cos\theta$ = adjacent/hypot. = A_x/A so $A_x = A\cos\theta$

Since $\sin\theta$ = opposite/hypot. = A_y/A so $A_y = A\sin\theta$

A Practice Problem

A bus travels 23.0 km on a straight road that is 30º north of east. What are the east and north components of its displacement?



- Define your coordinates for position.
- (Use a system where the x-axis is east)
- Sketch the vector measuring the angle θ counterclockwise from the x-axis.

Calculate and Solve

Given:

$A = 23.0$ km

$\Theta = 30^o$

Unknown:

$A_x = ?$

$A_y = ?$

Calculations:

$A_x = A \cos\theta$

$A_x = (23.0 \text{ km}) \cos\theta$

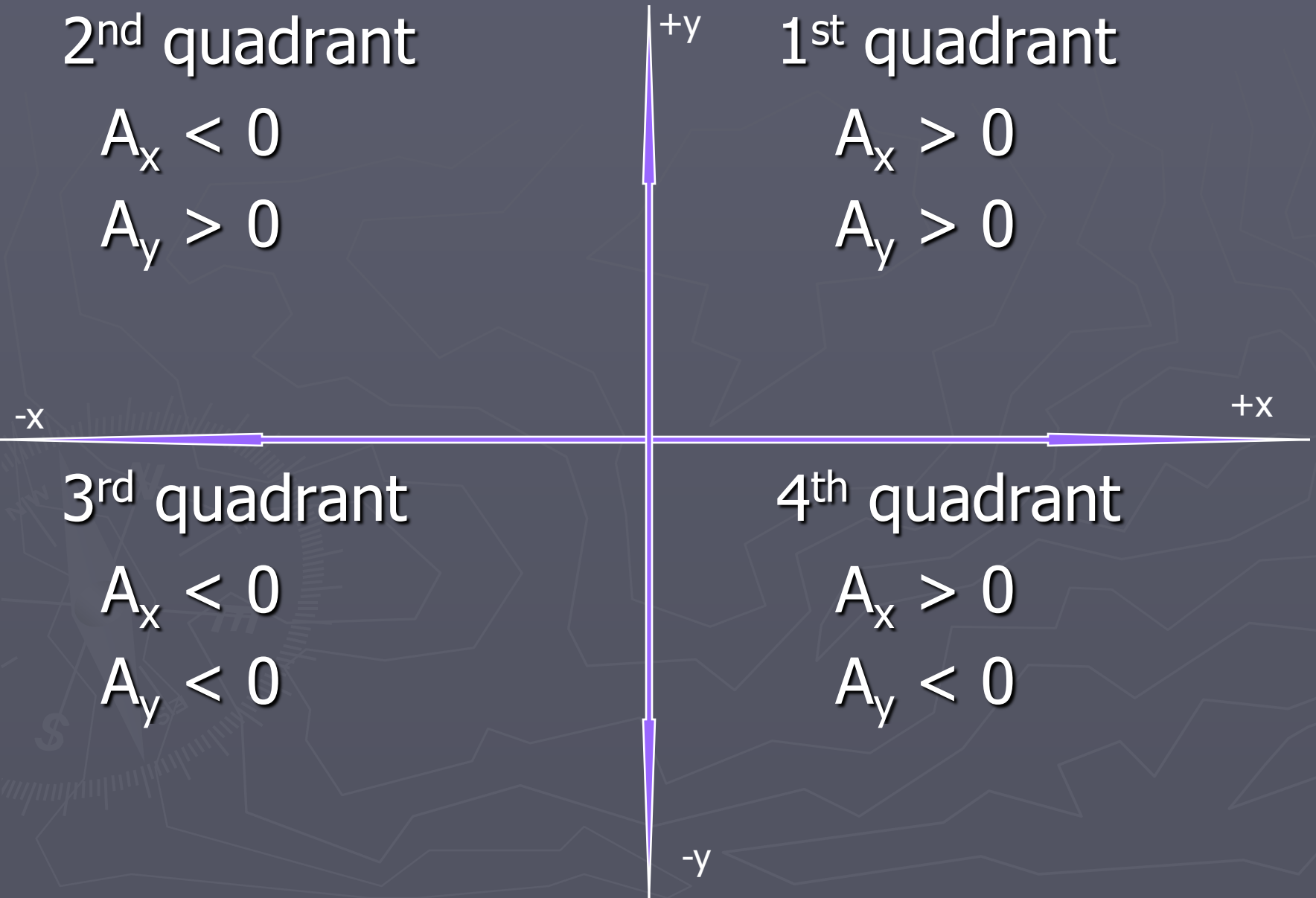
$A_x = +19.9$ km

$A_y = A \sin\theta$

$A_y = (23.0 \text{ km}) \sin\theta$

$A_y = +11.5$ km

Signs of Components



2nd quadrant

$A_x < 0$

$A_y > 0$

1st quadrant

$A_x > 0$

$A_y > 0$

3rd quadrant

$A_x < 0$

$A_y < 0$

4th quadrant

$A_x > 0$

$A_y < 0$

Algebraic addition of vectors

- Two or more vectors can be added by first resolving them into their x- and y-components.
- X components are added to make an x resultant: $R_x = A_x + B_x + C_x + \ldots$
- Y components are added to make a y resultant: $R_y = A_y + B_y + C_y + \ldots$
- Because R_x and R_y are at 90º they can be added by: $R^2 = R_x^2 + R_y^2$

Now find the angle

- To find the angle of the resultant vector, remember the tan of the angle the vector makes with the x-axis is

$$\tan \theta = \frac{R_y}{R_x}$$

- You can find the angle by using $\tan^{-1}$ on your calculator.

$$\theta = \tan^{-1}\left(\frac{Ry}{Rx}\right)$$

Your turn to Practice

- Open your book to page 74.
- Follow the format examples given previously.
- SHOW ALL WORK
- Sketch your vectors or components.
- Do problems #11, #12, #13, and #14

Practice Continued

- Turn to page 76 in your book
- Follow the format shown previously
- All vectors need rulers and PROTRACTORS
- Solve questions # 15, #16, #17, and #18