

**Unit Name:** Unit 1: Extending the Number System

**Lesson Plan Number & Title:** Lesson 7: Operations with Complex Numbers

**Grade Level:** High School Math II

**Lesson Overview:**

Students will develop methods for simplifying and calculating complex number operations based upon  $i^2 = -1$ . A deeper understanding of the applications of complex numbers in calculating electrical impedance is addressed in the lesson.

**Focus/Driving Question:**

How are algebraic properties applied to complex numbers?

**West Virginia College- and Career-Readiness Standards:**

M.2HS.5

Use the relation  $i^2 = -1$  and the commutative, associative and distributive properties to add, subtract and multiply complex numbers. Instructional Note: Limit to multiplications that involve  $i^2$  as the highest power of  $i$ .

**Manage the Lesson:**

**Step 1:** Establish student understanding by connecting the learning to previously taught lessons through the [Classifying Complex Numbers](http://alex.state.al.us/uploads/11364/Classifying%20Complex%20Numbers.ppt) PowerPoint

(<http://alex.state.al.us/uploads/11364/Classifying%20Complex%20Numbers.ppt>). With this instructional strategy, students are introduced to complex numbers and how they relate to the number system. It permits questioning of student understanding and guides them to the standard form of a complex number.

**Step 2:** Develop and connect the lesson vocabulary in viewing the following video Adding and Subtracting Complex Numbers - [http://www.youtube.com/watch?v=xSwrgNfS\\_LY](http://www.youtube.com/watch?v=xSwrgNfS_LY). Ask students to create their own definitions with corresponding examples for addition to the word wall. Include examples and definitions of the commutative, associative and distributive properties.

**Step 3:** Build upon student knowledge from previous activities by student investigation with the [Graphic Organizer-Adding, Subtracting and Multiplying](#). Have students work individually, in pairs or small groups to summarize in their own words the process for adding, subtracting, and multiplying polynomials. Upon completion of the organizer, students should be able to create an example for each type and find each answer. Then, using a whole group discussion format, everyone shares their responses.

**Step 4:** Students will demonstrate their knowledge through the incorporation of student practice utilizing a variety of materials. The combination of materials listed can be adapted to your students learning styles and abilities. For example, breaking the assignment into shorter tasks can guide your instruction and provide informal assessment on student mastery. Not all of these materials may be needed for student mastery of the objectives. All of the materials listed below may not be needed for student mastery of the lesson objective. When planning lesson implementation, select the materials most appropriate for your student's needs.

*Instructional Video*

Multiplying Complex Numbers - <http://www.youtube.com/watch?v=cTqZYEtQhA> (multiplication of complex numbers explanation)

*Instructional Activities*

TI Classroom Activities: Complex Numbers -

<https://education.ti.com/en/timathnspired/us/detail?id=6FD90593B6FF446CB9BE76C9AF380ECE&sa=291B0ACD31104D178C0EA77ABC7FB53A&t=81A3FC9ACCC24ABAA3BF97F1A038AFA9> (develop the

rules for complex number addition, subtraction and multiplication)

TI Classroom Activity: Complex Number Addition -

<https://education.ti.com/en/us/activity/detail?id=07EF321269B64BB398EABD1C0E0D9061> (complex number addition developed on the graphing calculator, includes handout)

Operations with Complex Numbers Worksheet -

<http://www.kutasoftware.com/FreeWorksheets/Alg2Worksheets/Operations%20with%20Complex%20Numbers.pdf> (printable on arithmetic operations with complex numbers)

Multiplying Complex Numbers Worksheet - <http://www.mathworksheetsgo.com/sheets/algebra-2/complex-numbers/multiply-complex-numbers-worksheet.php> (Multiplying complex numbers printable with an extension lesson on complex conjugates (use the extension lesson if your students are advanced and you want to challenge their reasoning))

**Rally the Table Activity** This activity is designed to be used after the students have a basic understanding of how to add, subtract, multiply and divide complex numbers. It encourages multiple representations reasoning by students

**Applications of Complex Numbers** examples of applications of complex numbers followed by several problems-may be used individually, in pairs, small group or whole group instruction-includes solutions **Complex Number Lesson** (<http://alex.state.al.us/uploads/11364/Complex%20Number%20Lesson.ppt>)

PowerPoint for instruction on complex number operations

**Operations with Complex Numbers WS** printable page of complex number operations

**Create Problems** printable where students create complex arithmetic problems and provide solutions in a pair and share activity may be adapted to multiplication as well

**Showdown (radical and complex)** small group activity and competition in which students demonstrate their learning of complex numbers and operations, reviews radicals as well.

*Computer Practice*

Cyclic Nature of the Powers of  $i$  -

<http://www.regentsprep.org/Regents/math/algtrig/ATO6/powerlesson.htm> (Instructional website and online practice)

Practice with Powers of  $i$  - <http://www.regentsprep.org/Regents/math/algtrig/ATO6/powerprac.htm>

Practice with Multiplication and Dividing Complex Numbers -

<http://www.regentsprep.org/Regents/math/algtrig/ATO6/multprac.htm> (online practice for multiplication of complex numbers)

SAS Curriculum Pathways Simplifying Complex Number Expressions -

<http://www.sascurriculumpathways.com/portal/> (Quick Launch #1425 - practice tool and online quiz)

*Puzzles and Games*

Complex Adding Maze - <http://www.regentsprep.org/Regents/math/algtrig/ATO6/complexmaze.htm> (online game for addition and subtraction of complex numbers)

Complex Number Bingo - <http://www.regentsprep.org/Regents/math/algtrig/ATO6/ComplexRes.htm>

*Teacher Information*

Does Anyone Ever Really Use Complex Numbers? -

<http://www.regentsprep.org/Regents/math/algtrig/ATO6/electricalresouce.htm> (This resource can provide additional insight on the Applications Activity in the Instructional Activities)

**Step 5:** Ask students to complete the following problem as a think-pair-share with the **Complex Numbers Exit Slip**. The activity is designed to extend their reasoning regarding complex numbers. Some students may need support on this activity. Before handing out the slips, you may want to remind them about squaring a number and the inverse operations used in solving an equation. In summation, connect the problem to imaginary numbers and their related values.

**Step 6:** Reflect with your students regarding the lesson's focus question "How are algebraic properties applied to complex numbers?" Ask your students to write a response demonstrating addition, subtraction

and multiplication of complex numbers. In whole group discussion, compare the student response and use the opportunity to assess/or correct student misunderstandings.

**Academic Vocabulary Development:**

*Impedance* – opposition to flow of alternating current

Review real number, imaginary number and complex number from the Word Wall and prior lessons.

Develop the algebraic properties of commutative, associative and distributive in terms of integers. Ask students if they can think of examples using imaginary numbers. How might the properties be applicable to their examples? Add the properties and the corresponding examples to the Word Wall.

**Launch/Introduction:**

Establish student understanding in Step One of Manage the Process by connecting learning to prior lessons through the [Classifying Complex Numbers](#) power point. It provides opportunities for student questioning on how complex numbers relate to the real numbers and the number system.

**Investigate/Explore:**

Students read and explore with the [Graphic Organizer-Adding, Subtracting and Multiplying](#). Instructors can have students work individually, in pairs or small groups to summarize in their own words the process for adding, subtracting, and multiplying polynomials. Upon completion of the organizer, students should be able to create an example for each type and find each answer. Then, with teacher facilitation of whole group discussion, everyone shares their responses.

The purpose of the lesson is the development of a complex number and its basic operations of addition, subtraction, and multiplication. The launch activity encourages flexible instructional strategies for initiating the use of standard form and classifying complex numbers through a power point and whole group discussion. The development of vocabulary and algebraic properties related to complex numbers can be guided by the instructor through the student created examples on the word wall. The instructor can choose to do this portion (create an example) as individual, small group or whole group instruction depending on student abilities or necessary modifications. The process of instructional practice by students can be flexible instruction by the assignment of handouts, games, or online computer practice according to instructional resources and differentiation determined by the instructor. Assessment of the lesson is a continual part of the instructional process, whether informal or formal and should guide the instructor in the directing of instruction. Both students and instructors are encouraged to reflect upon the lesson and knowledge gained with the focus question in terms of knowledge gained regarding the use of algebraic properties and operations of complex numbers.

**Summarize/Debrief:**

Have students complete the [Complex Numbers Exit Slip](#) as a think-pair-share activity. The intent of the experience is to extend their reasoning regarding complex numbers. Some students may need support on this activity. Before handing out the slips, you may want to remind them about squaring a number and the inverse operations used in solving an equation. Remind students to write down any brainstorming ideas they may have on the paper. In summation, connect the problem to imaginary numbers and their related values as part of a whole group discussion.

**Materials:**

Graphing Calculator, Word Wall Materials (construction paper, markers), foldable (white copy paper or notebook paper, markers or colored pencils, scissors), optional-computers

[Classifying Complex Numbers](#) PowerPoint

[Graphic Organizer-Adding, Subtracting and Multiplying](#)

[Rally the Table Activity](#)

[Applications of Complex Numbers](#)  
[Complex Number Lesson](#)  
[Operations with Complex Numbers WS](#)  
[Create Problems](#)  
[Showdown \(radical and complex\)](#)  
[Complex Numbers Exit Slip](#)

**Websites:**

<http://alex.state.al.us/uploads/11364/Classifying%20Complex%20Numbers.ppt>  
<http://alex.state.al.us/uploads/11364/Complex%20Number%20Lesson.ppt>  
[http://www.youtube.com/watch?v=xSwrgNfS\\_LY](http://www.youtube.com/watch?v=xSwrgNfS_LY)  
<http://www.youtube.com/watch?v=cTqZYEtTQhA>  
<https://education.ti.com/en/timathnspired/us/detail?id=6FD90593B6FF446CB9BE76C9AF380ECE&sa=291B0ACD31104D178C0EA77ABC7FB53A&t=81A3FC9ACCC24ABAA3BF97F1A038AFA9>  
<https://education.ti.com/en/us/activity/detail?id=07EF321269B64BB398EABD1C0E0D9061>  
<http://www.regentsprep.org/Regents/math/algtrig/ATO6/powerlesson.htm>  
<http://www.regentsprep.org/Regents/math/algtrig/ATO6/powerprac.htm>  
<http://www.regentsprep.org/Regents/math/algtrig/ATO6/multprac.htm>  
<http://www.regentsprep.org/Regents/math/algtrig/ATO6/complexmaze.htm>  
<http://www.regentsprep.org/Regents/math/algtrig/ATO6/electricalresouce.htm>  
<http://www.kutasoftware.com/FreeWorksheets/Alg2Worksheets/Operations%20with%20Complex%20Numbers.pdf>  
<http://www.mathworksheetsgo.com/sheets/algebra-2/complex-numbers/multiply-complex-numbers-worksheet.php>  
<http://www.sascurriculumpathways.com/portal/>  
<http://www.regentsprep.org/Regents/math/algtrig/ATO6/ComplexRes.htm>

**Career Connection:**

Engineering and Technical Cluster and Science and Natural Resources Cluster use complex and imaginary numbers when calculating electrical impedance.

**Lesson Reflection:**

Reflect with your students regarding the lesson's focus question "How are algebraic properties applied to complex numbers?" Ask your students to write a response demonstrating addition, subtraction and multiplication of complex numbers. In whole group discussion, compare the student response and use the opportunity to assess/or correct student misunderstandings. The teacher could reflect on how the lesson went, what parts went well or what parts need to be revised.

# Classifying Complex Numbers

downloaded from

<http://alex.state.al.us/uploads/11364/Classifying%20Complex%20Numbers.ppt>

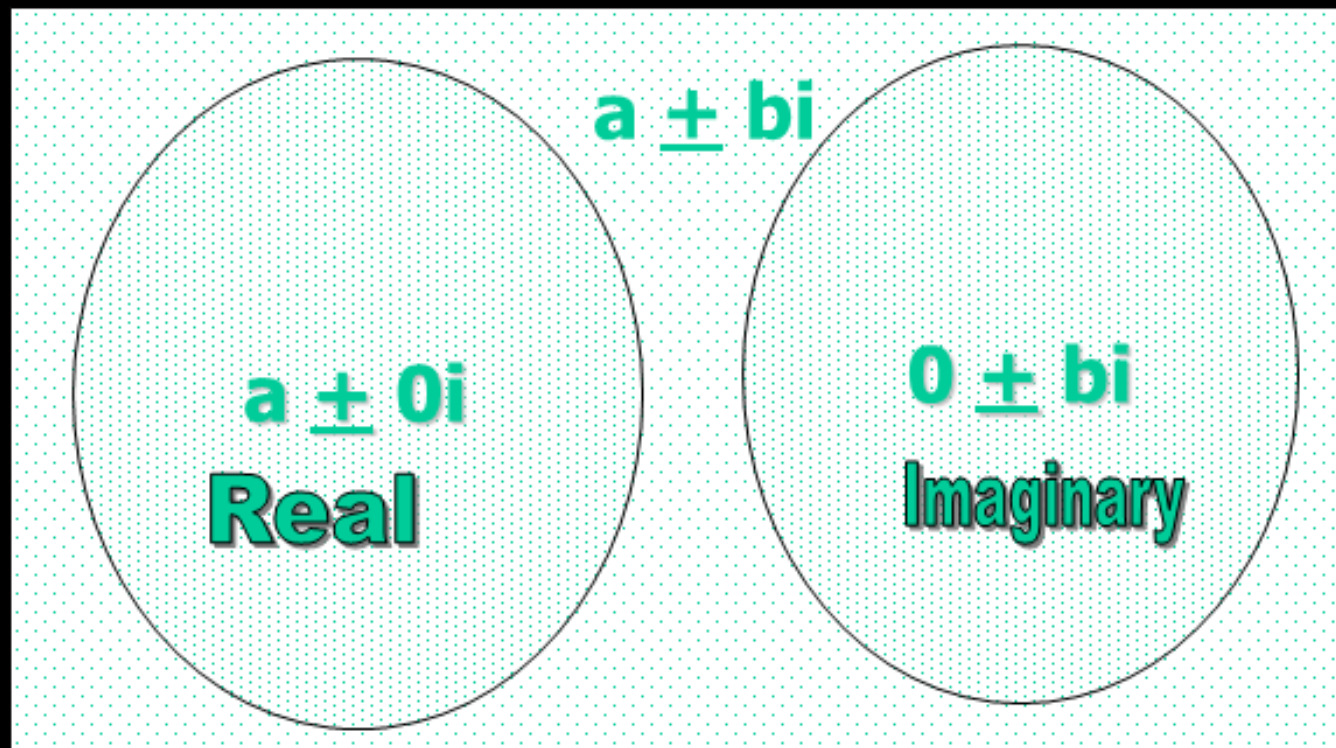
## Instructions

Click to select an answer.

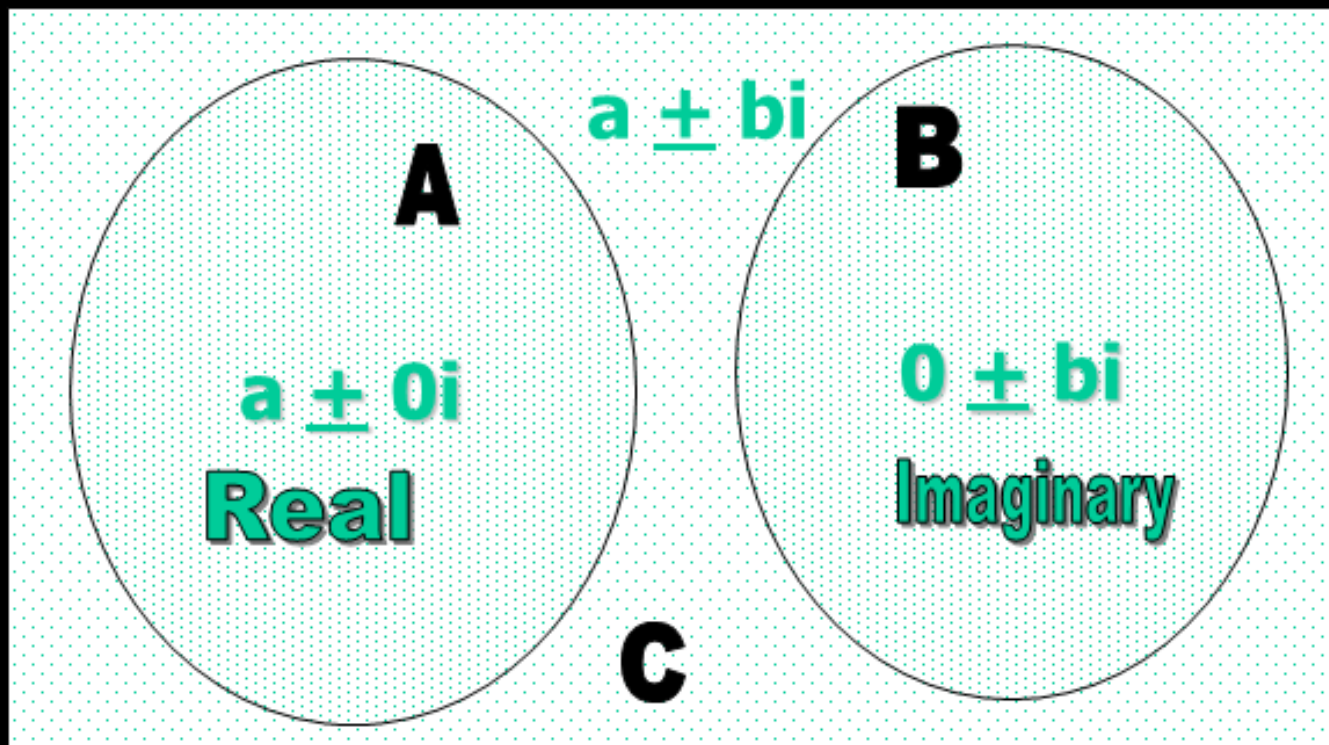
Otherwise use the spacebar to advance.

## COMPLEX NUMBERS

all numbers of the form  $a \pm bi$ ,  
where  $a$  and  $b$  are real numbers

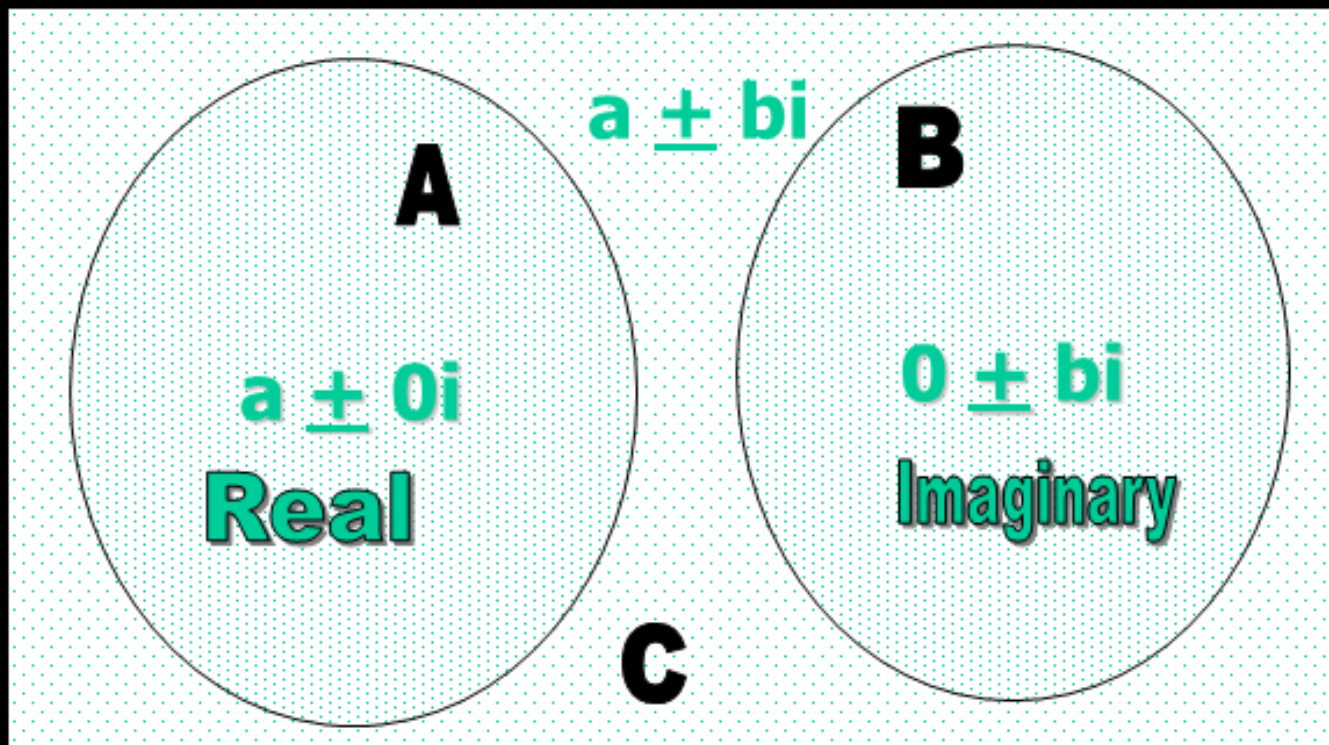


Classify each number shown  
as belonging in set A, B or C  
by clicking in the set NOT on the letter





$$-7 + 3i$$

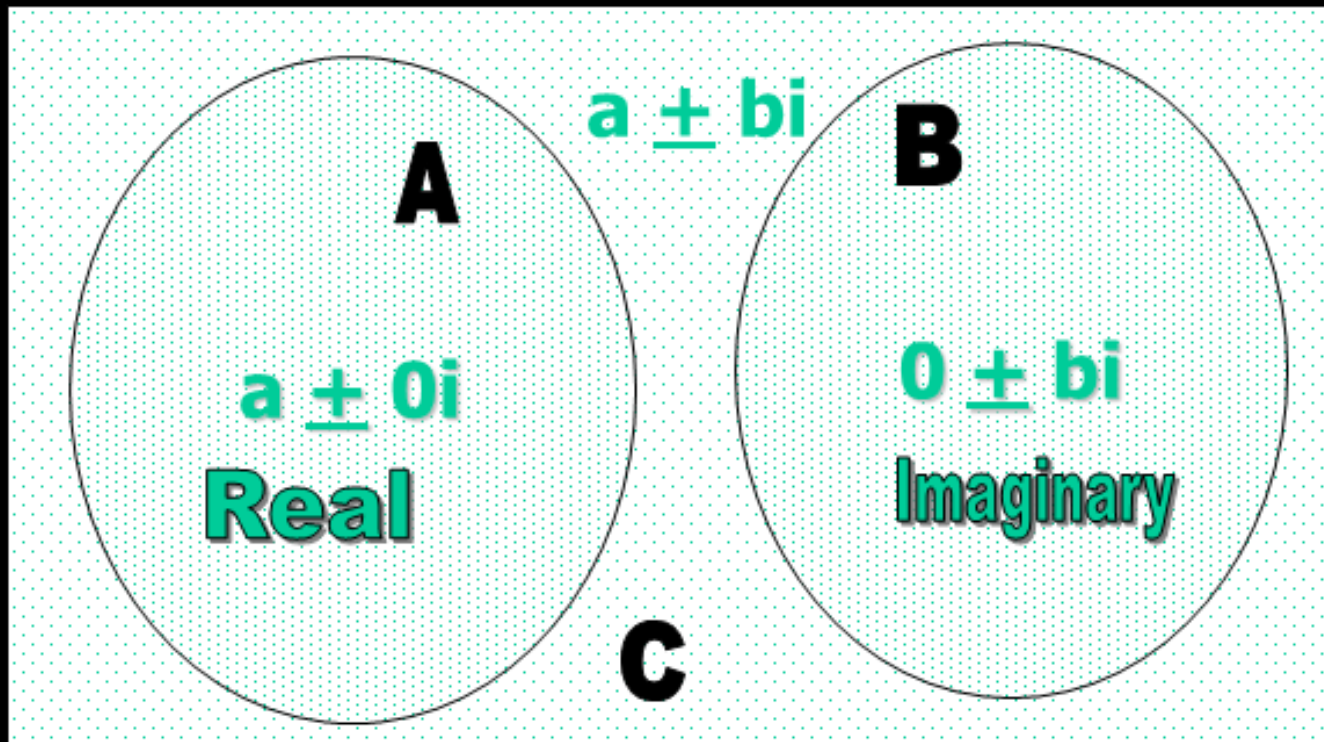




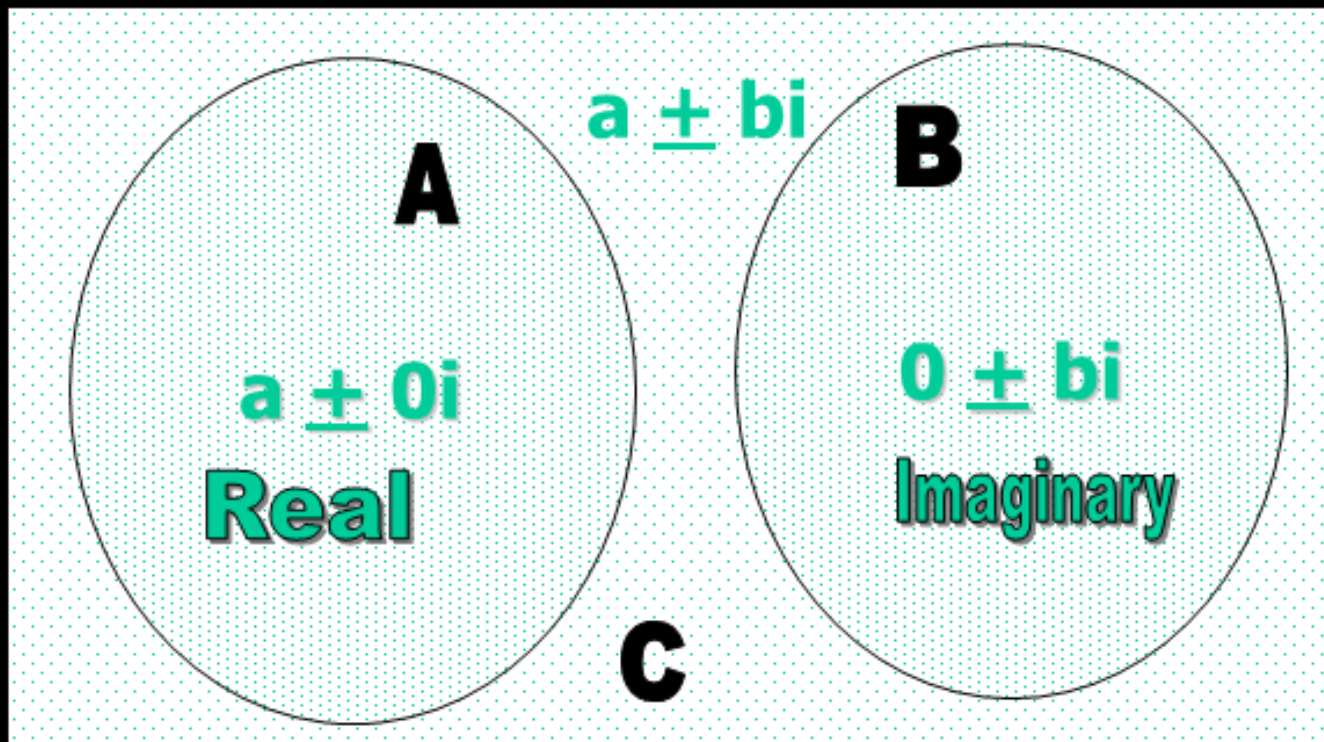
**Incorrect.**

**Set A is for strictly real numbers like 14, -9,  $\frac{3}{4}$ .**

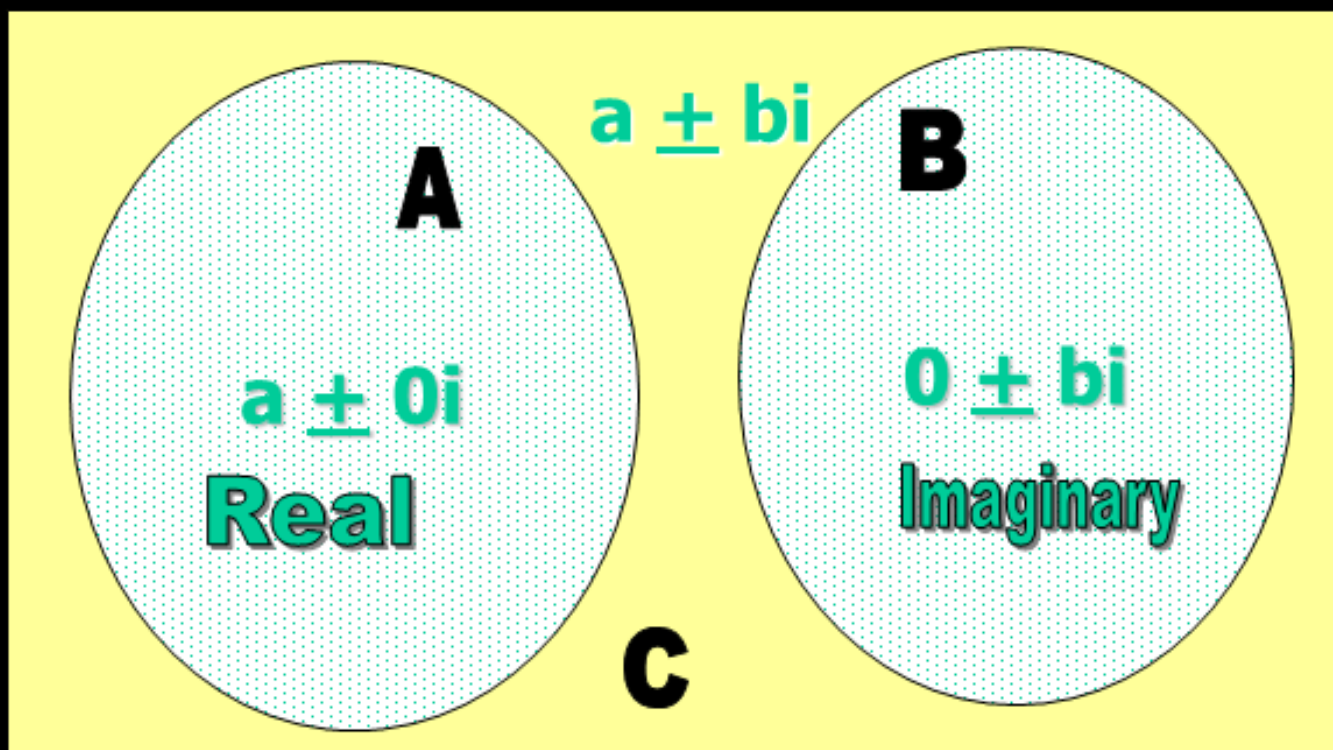
**Press spacebar.**



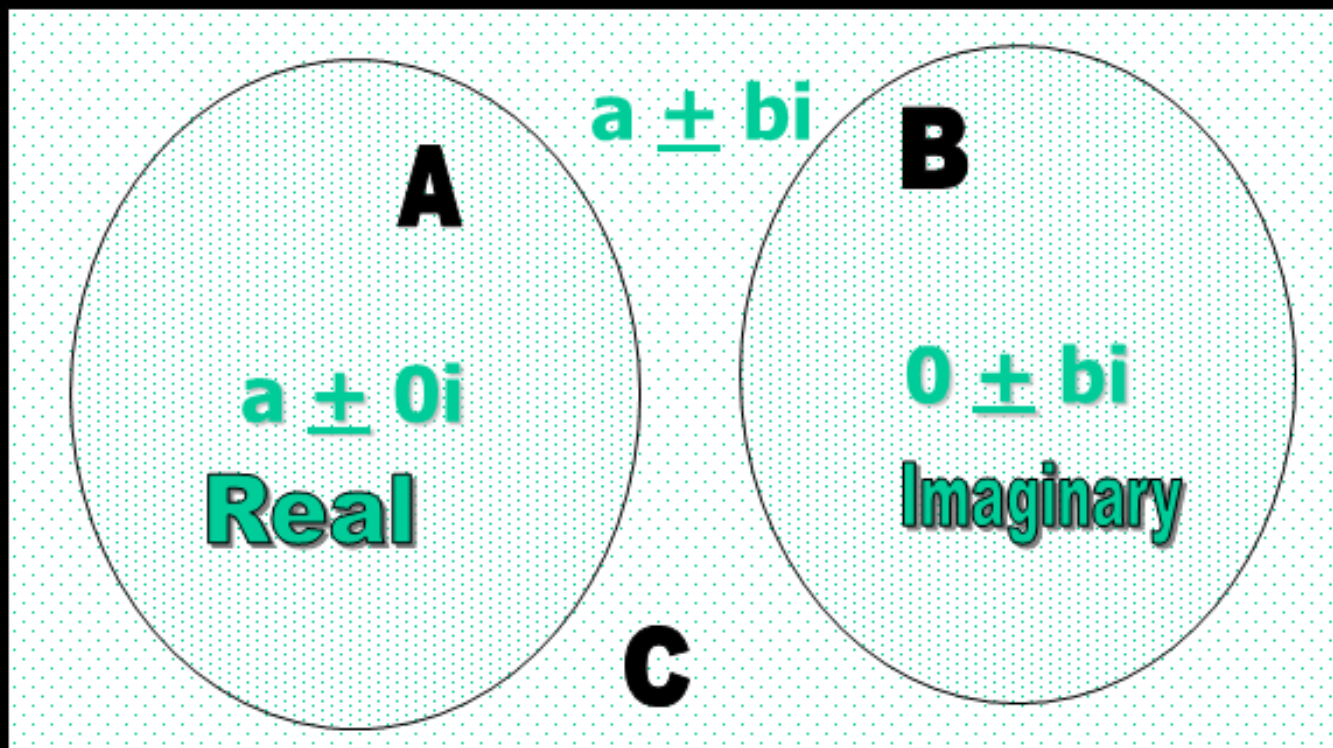
**Incorrect.**  
**Set B is for strictly imaginary numbers**  
**like  $-7i$ ,  $11i$ , or  $\frac{1}{2}i$ . Press spacebar.**



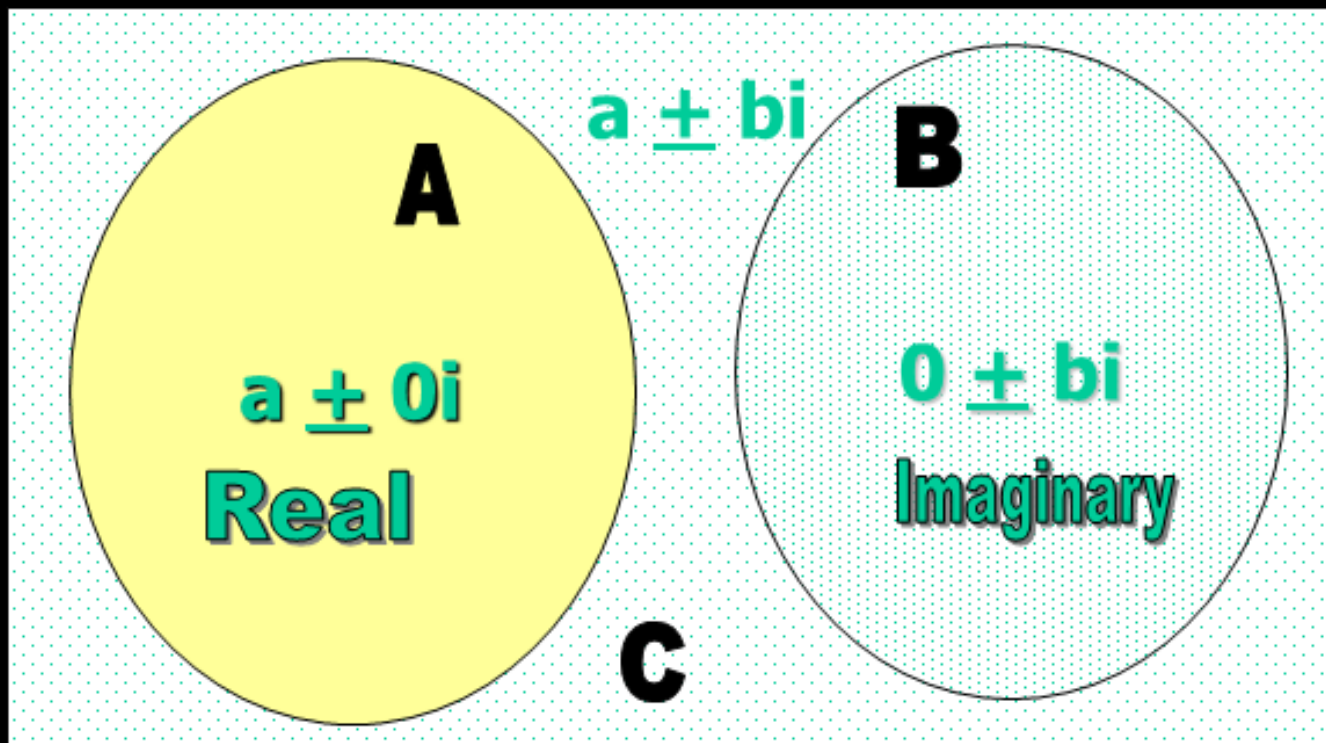
**CORRECT!**  
**Press spacebar twice.**



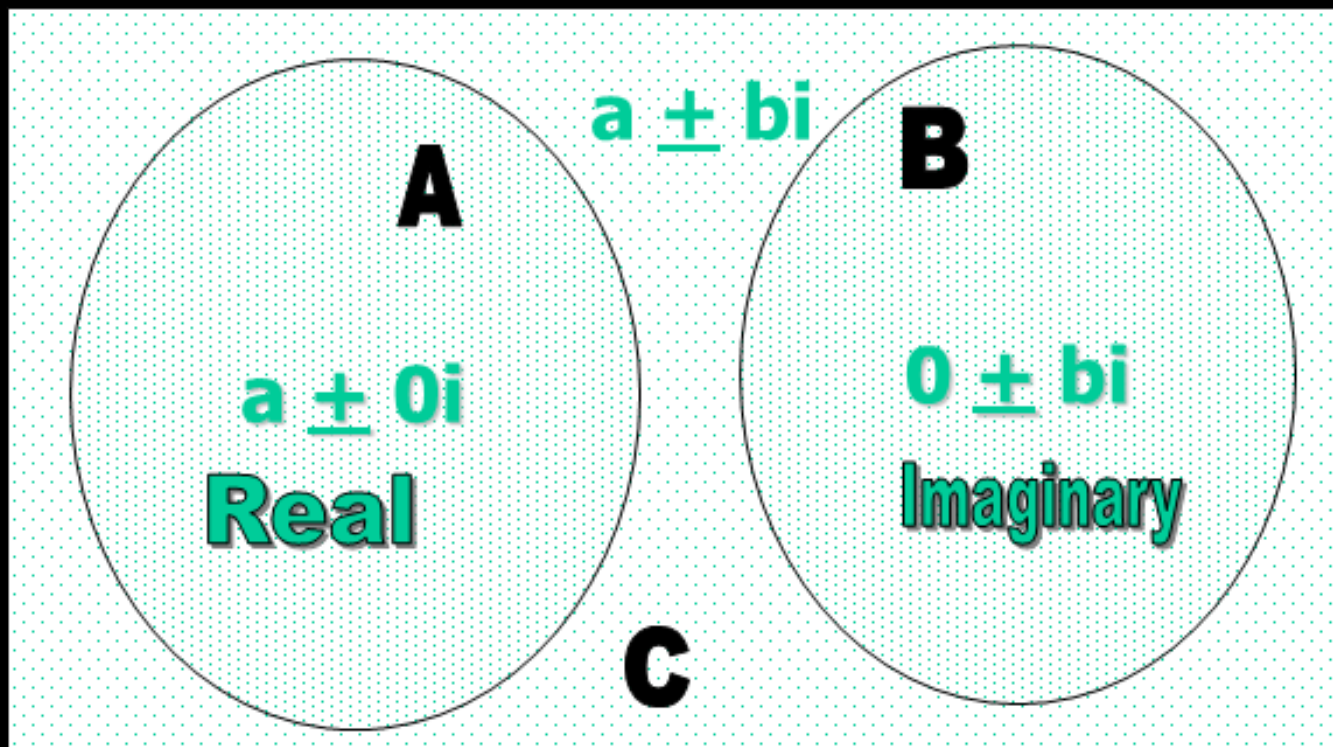
**-7**



**CORRECT!**  
**Press spacebar twice.**

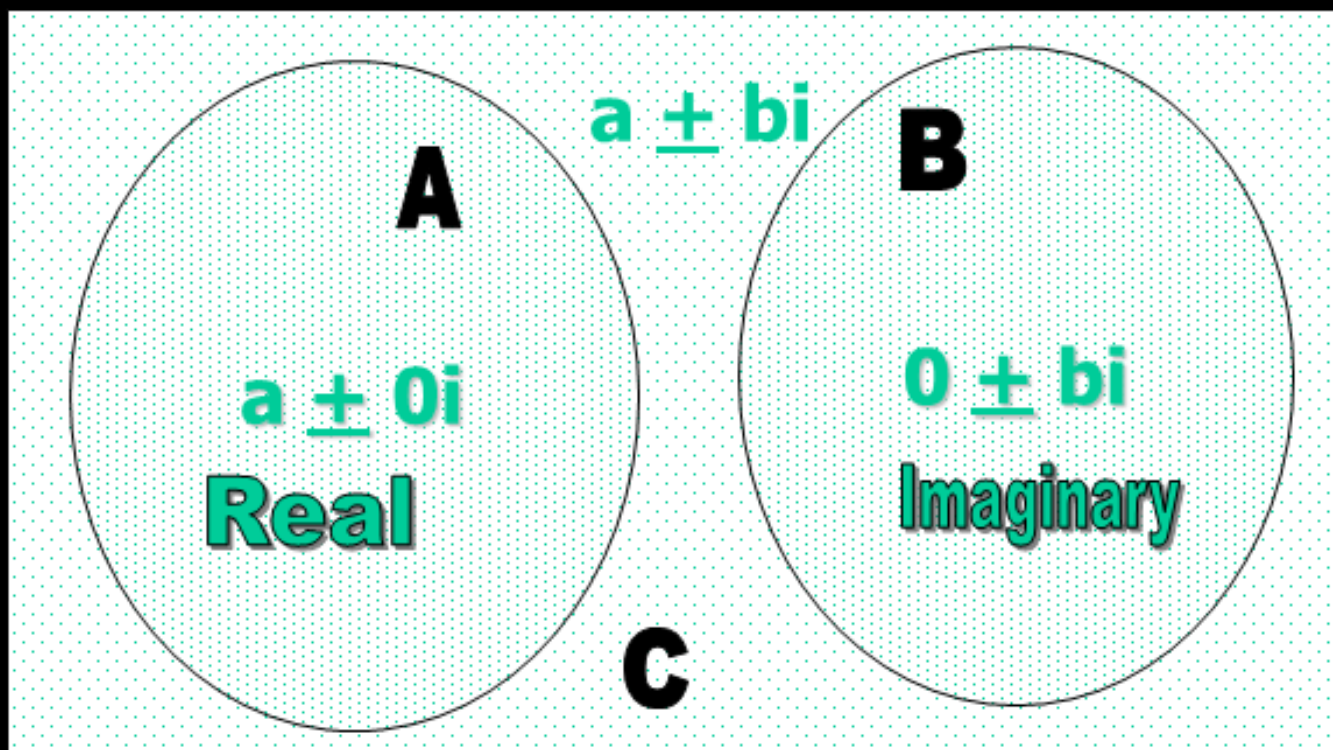


**Incorrect.**  
**Set B is for strictly imaginary numbers**  
**like  $-7i$ ,  $11i$ , or  $\frac{1}{2}i$ . Press spacebar.**



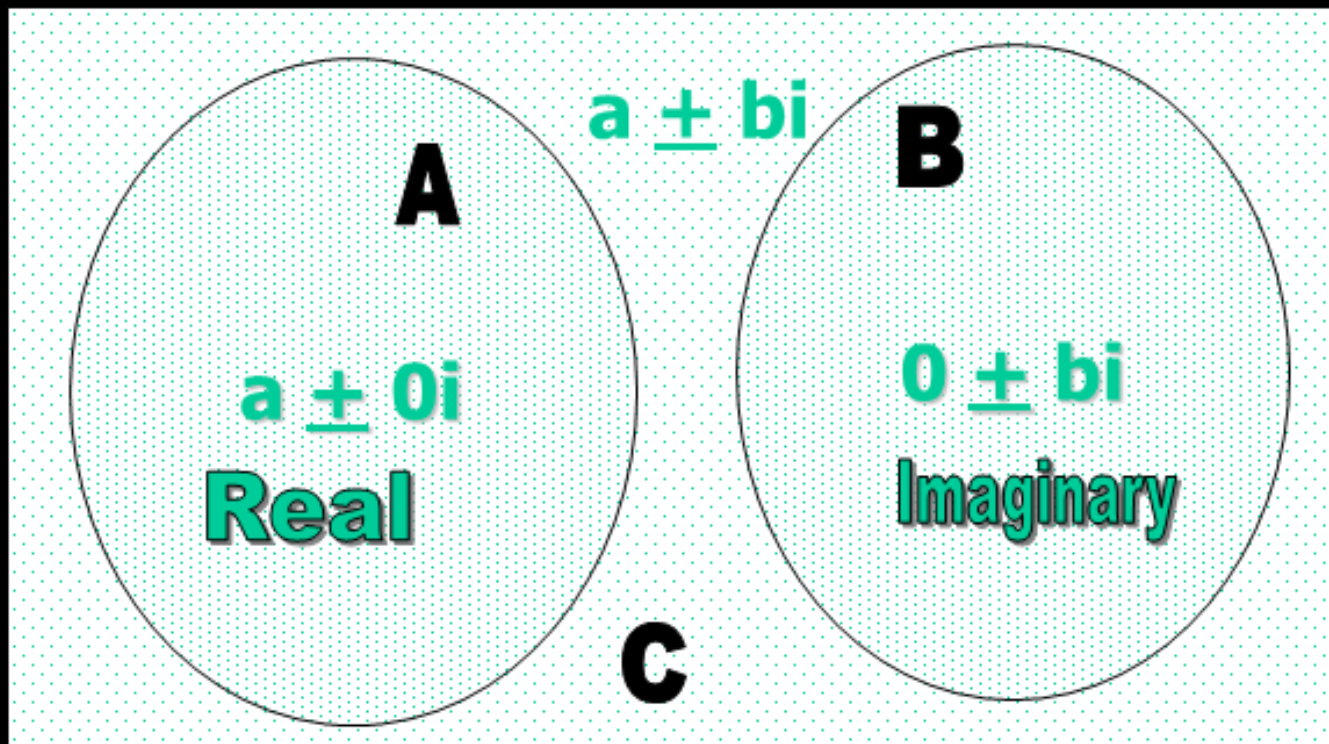
**Incorrect.**

**Set C is for strictly complex numbers  
like  $4 - 3i$ ,  $\frac{1}{2} + i$ , or  $-3 + 6i$ . Press spacebar.**

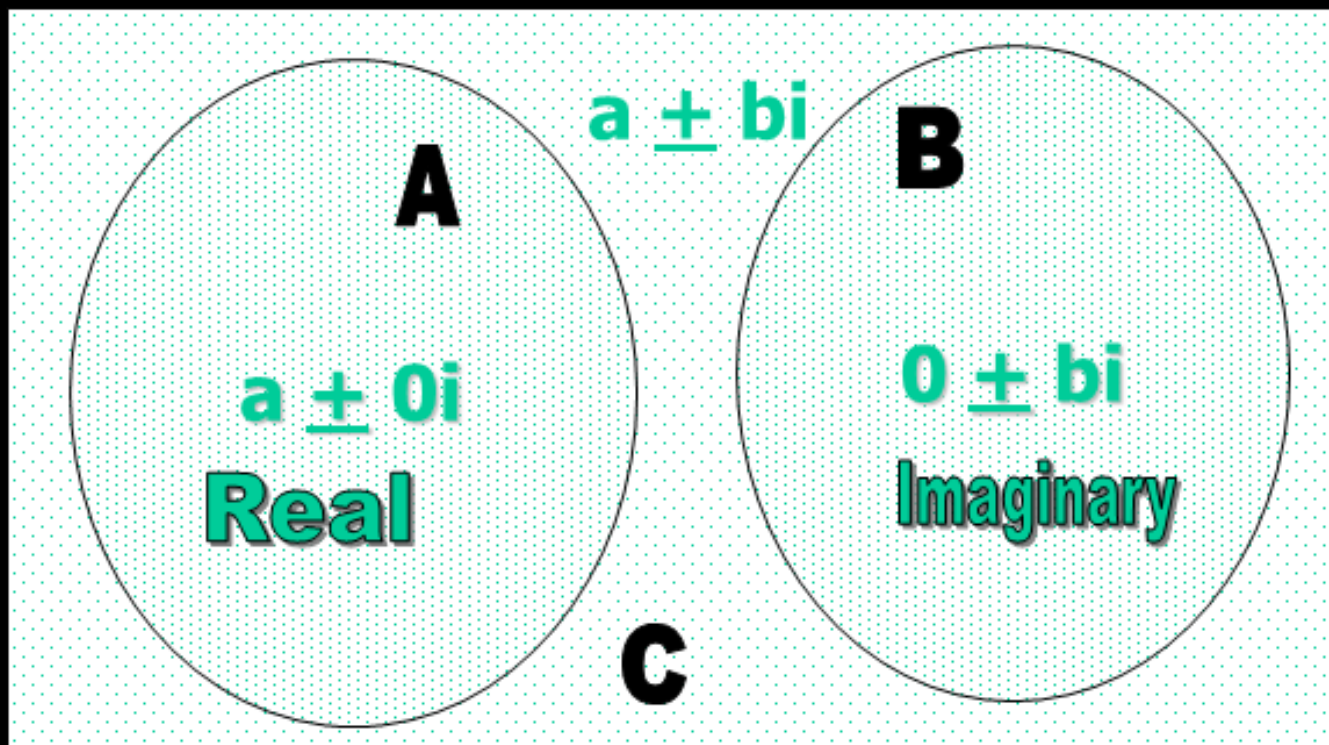




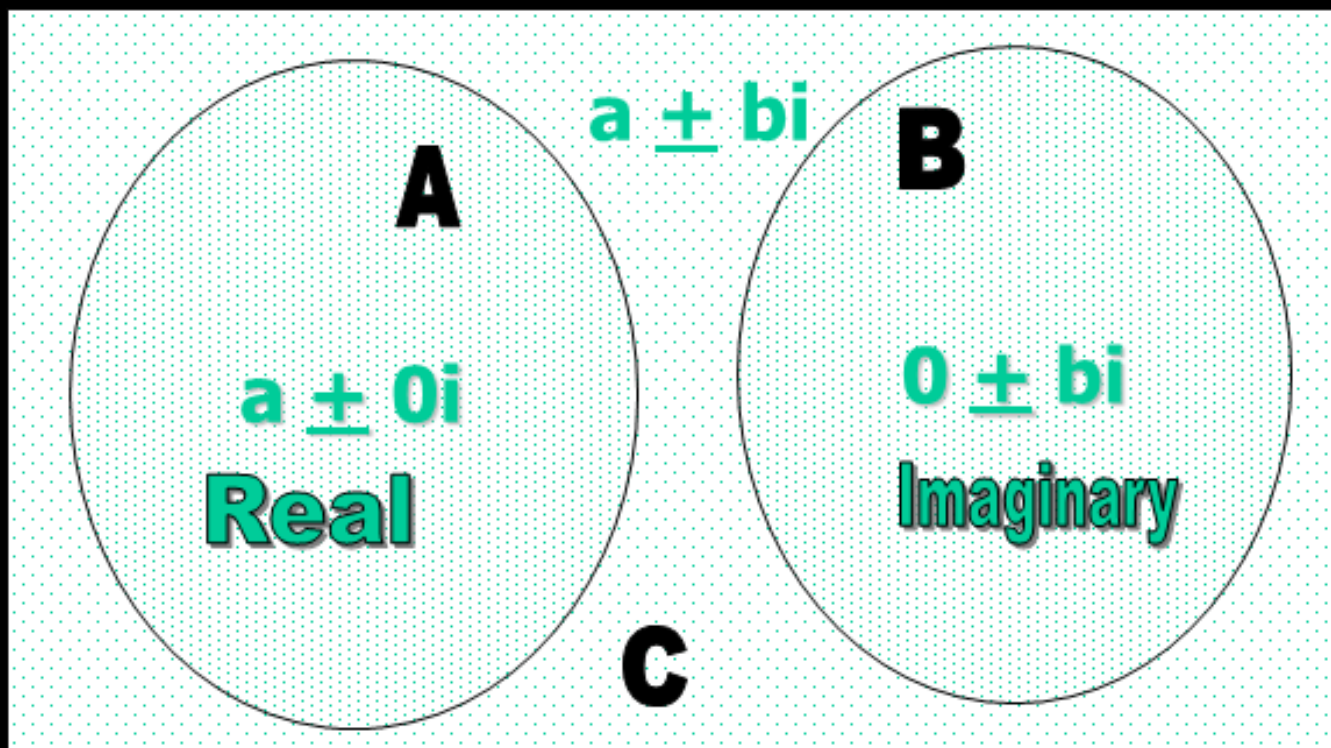
$$12 - 3i$$



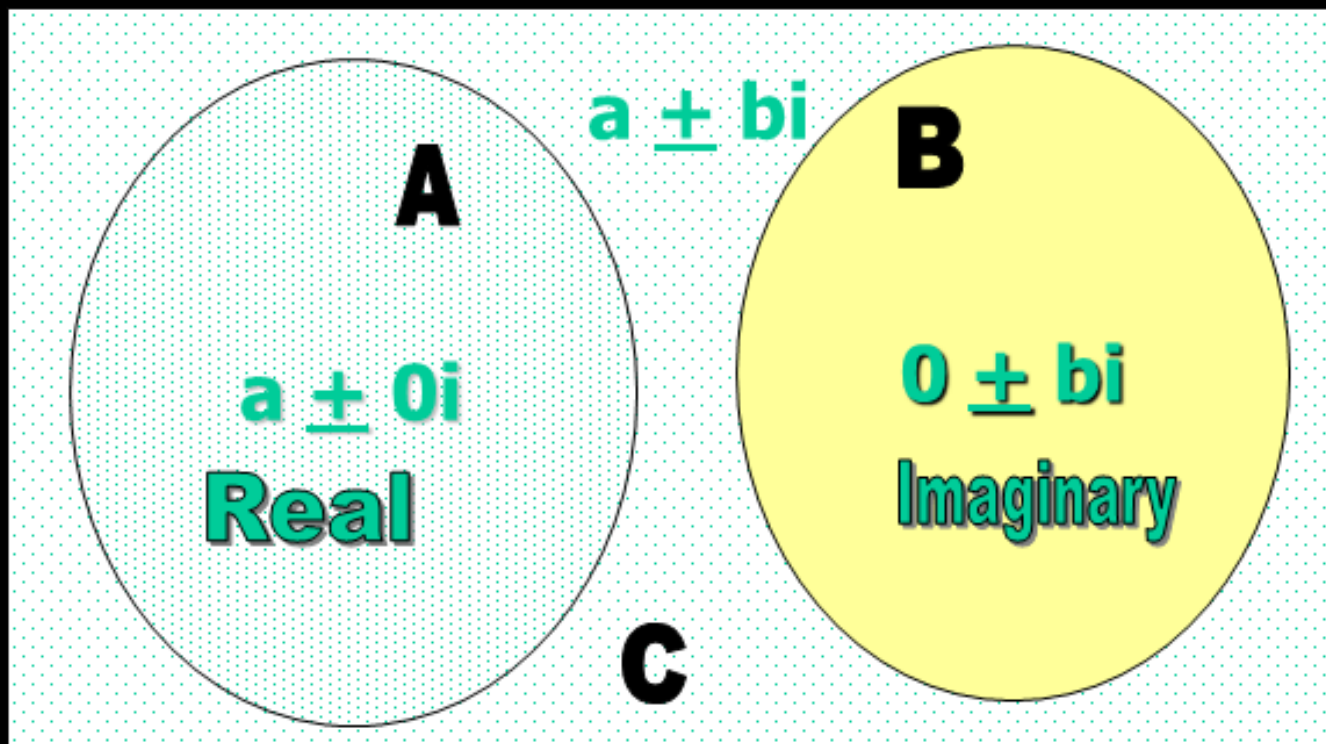
4



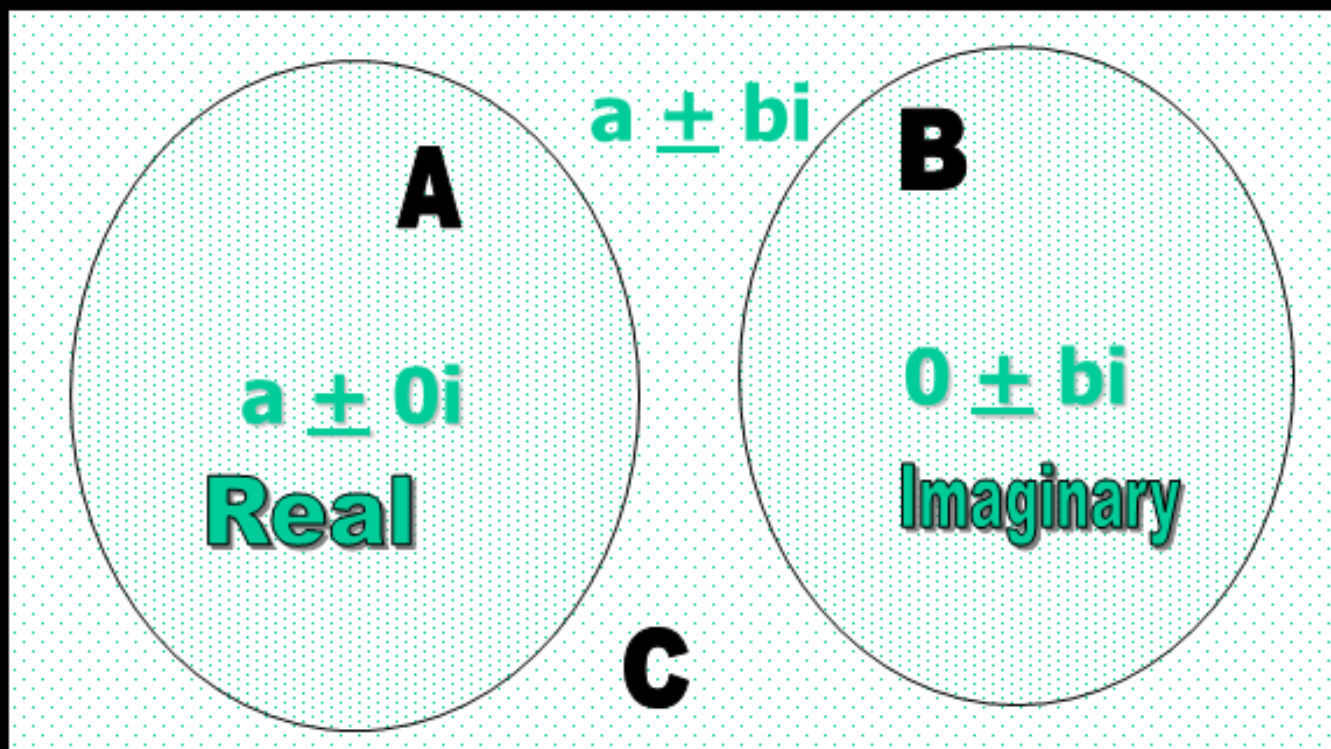
**-5i**



**CORRECT!**  
**Press spacebar twice.**

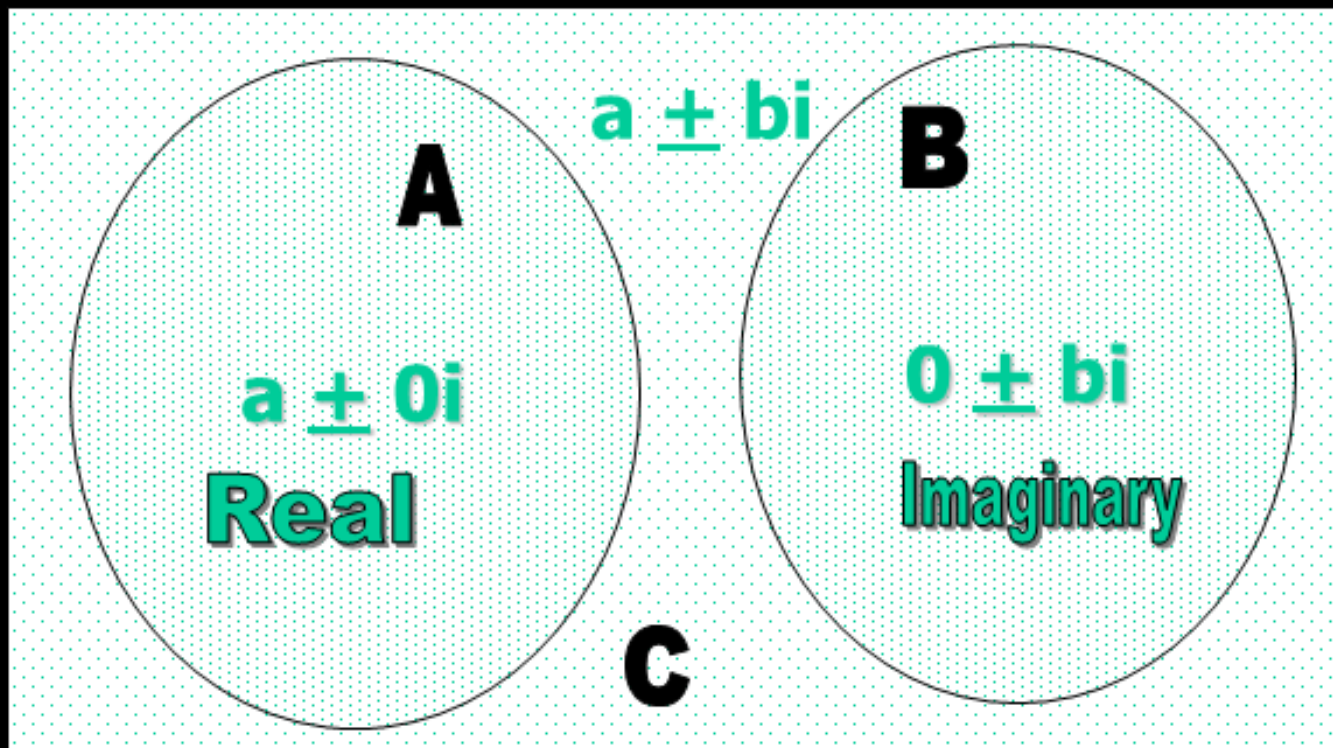


**Incorrect.**  
**Set A is for strictly real numbers**  
**like  $-7$ ,  $11$ , or  $\frac{1}{2}$ . Press spacebar.**

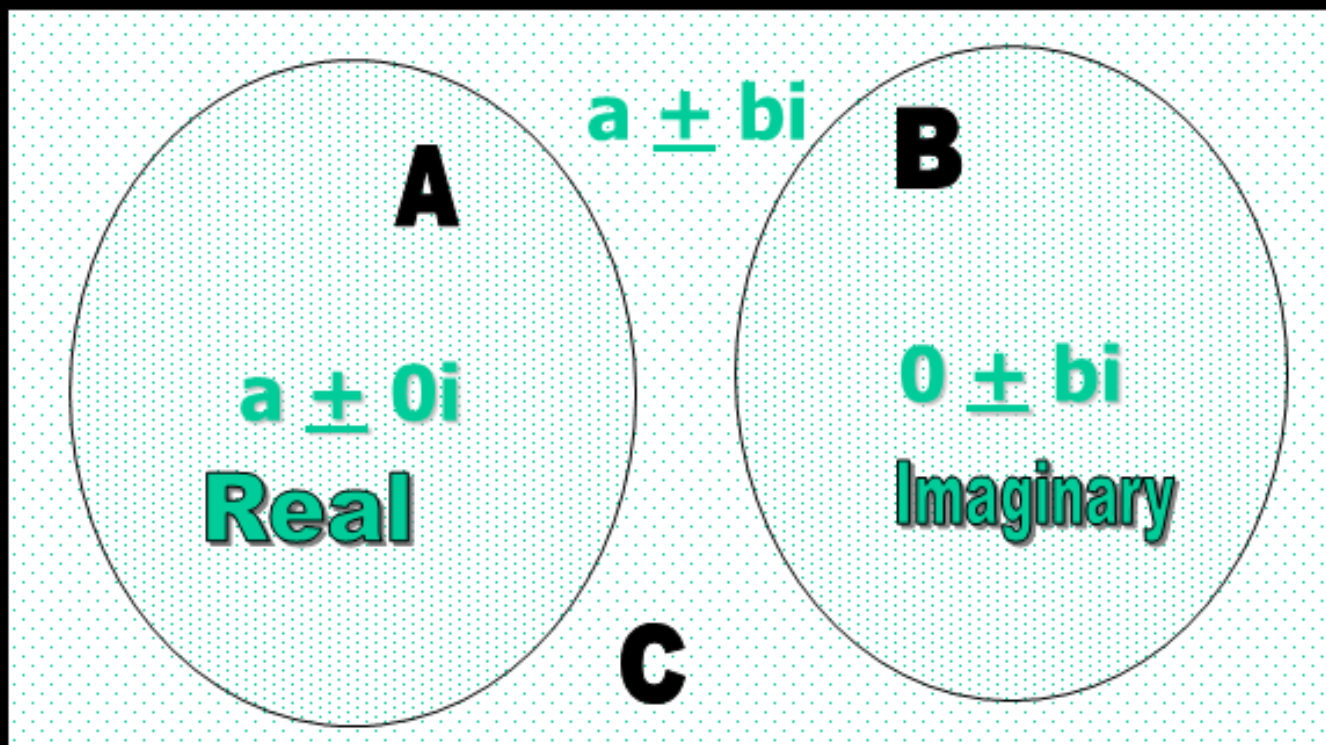


**Incorrect.**

**Set C is for strictly complex numbers  
like  $4 - 3i$ ,  $\frac{1}{2} + i$ , or  $-3 + 6i$ . Press spacebar.**

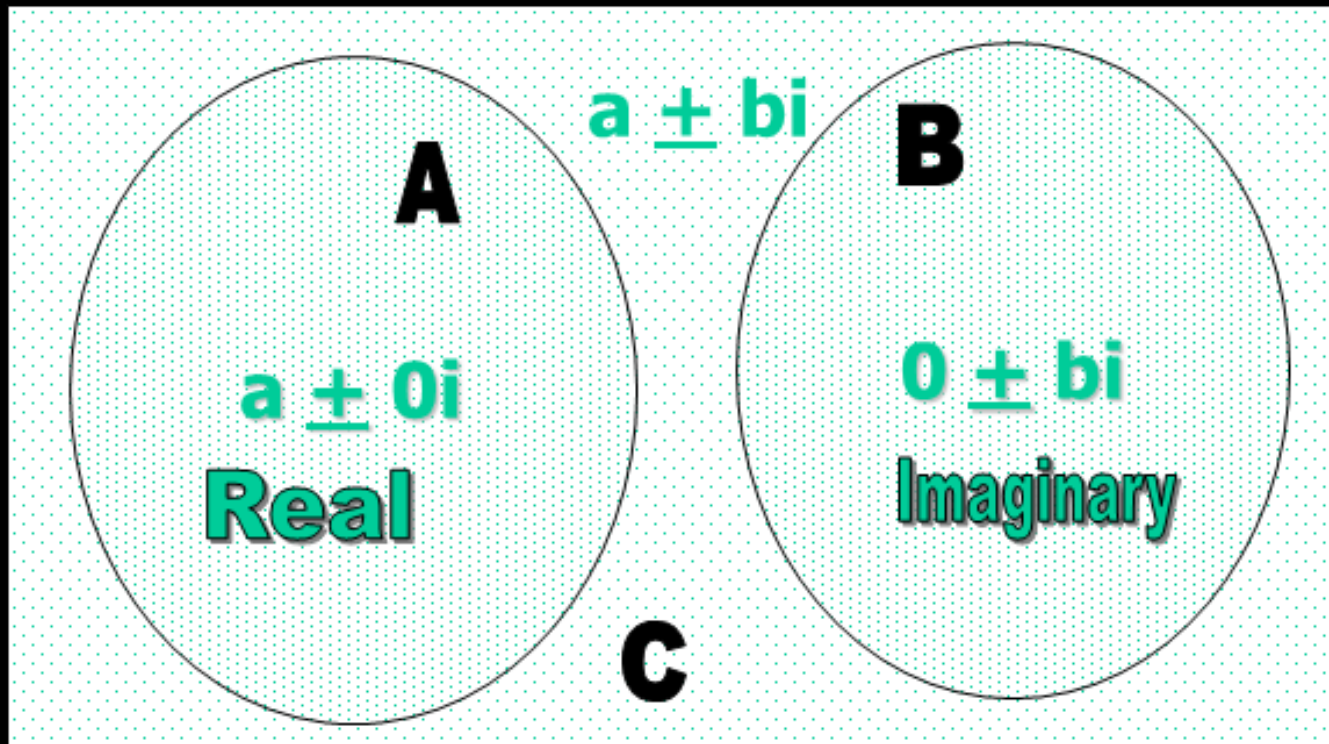


13

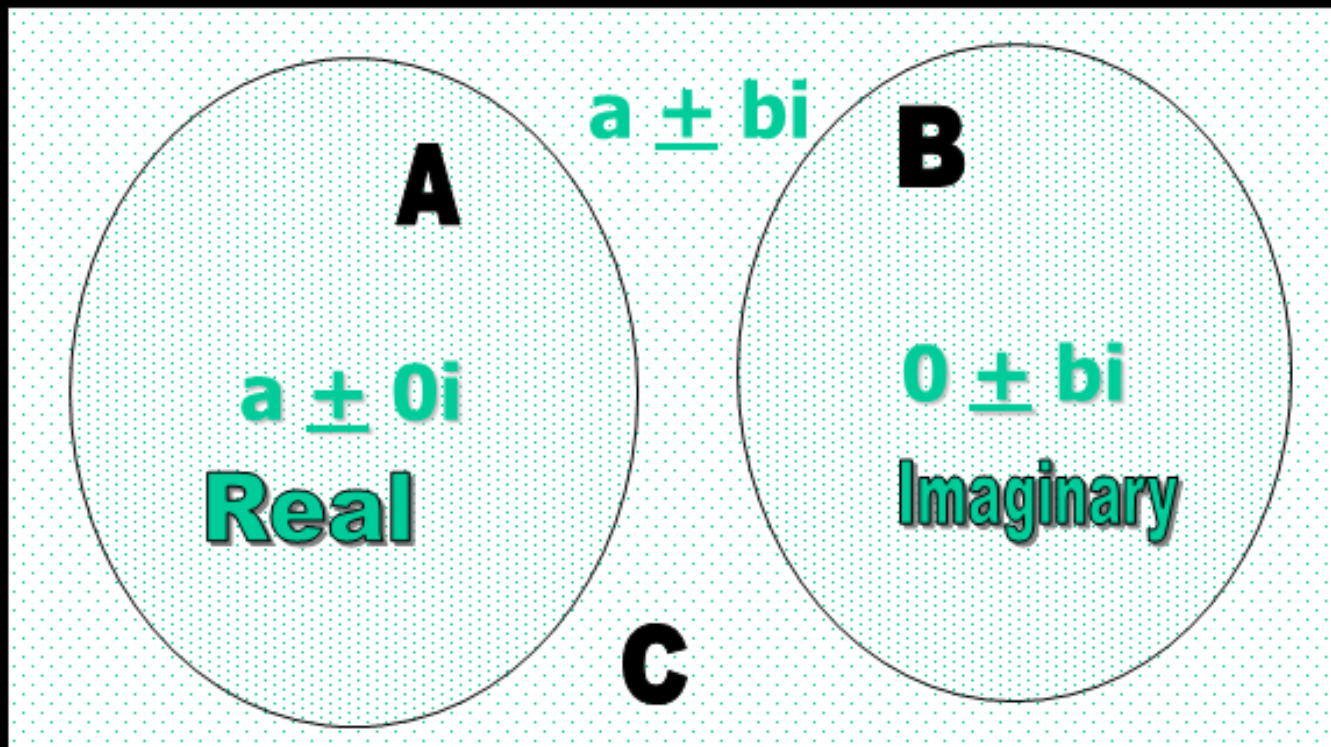




**2i**



$$1 + \left(\frac{2}{3}\right)i$$



# Classifying Complex Numbers

Session is over.

# Graphic Organizer – Adding, Subtracting and Multiplying

Adapted from an activity by Emily and Katie via <http://www.ilovemath.org>

## Guidelines for Adding, Subtracting, and Multiplying Complex Numbers

### Adding

Parentheses do not change the problem. Combine like terms (real with real and imaginary with imaginary) – by, combining coefficients.

- Examples -

$$(7 + 6i) + (8 - 3i) =$$

$$(-5 - 2i) + (-8i) =$$

### Subtracting

All signs for each term must be switched in the set of parentheses that follow the subtraction sign. Then follow the rules for adding complex numbers.

- Examples -

$$(12 + i) - (-4 + 5i) =$$

$$(2) - (8 - 7i) =$$

### Multiplying

When multiplying two complex numbers, you are just using the distributive property multiple times. Simplify the resulting expression. Remember

$$i^2 = -1$$

- Examples -

$$(3 + 5i)(6 - 9i) =$$

$$(6 - 3i)(6 + 3i) =$$

On a sheet of notebook paper, summarize in your own words the process for adding, subtracting, and multiplying polynomials. Create an example for each type and find each answer.

## Rally The Table Activity

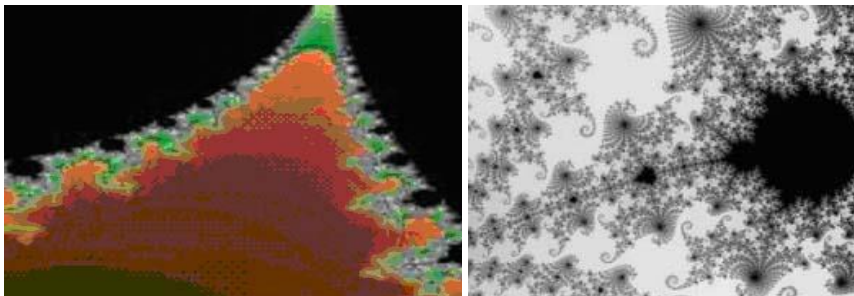
This activity is designed to be used after the students have a basic understanding of how to add, subtract, multiply and divide complex numbers. The structure of this activity will provide students with developing multiple representations about operations on complex numbers.

- Display a simple complex number, such as  $3+2i$ .
- Ask the class to provide an equivalent representation of the complex expression provided.  
Examples include:  $(4+i)-(1-i)$ ,  $\frac{9+6i}{3}$ ,  $\frac{1}{2}(6+4i)$ , etc.
- After the class has an understanding of the task, break the class up into groups of 3 – 4 students and complete the following Rally Table activity.
- Rally Table
  - Tell each group to get out one sheet of paper for all of the group members to use.
  - Tell the groups that the person whose birthday is closest to today will be the first person to complete the task.
  - Display a complex number.
  - Instruct the person who has the piece of paper to write an equivalent representation for this number on the paper and then pass the paper to the person to the right.
  - The next person then writes a different equivalent form of the number on the paper and then passes the paper to their right.
  - This process continues for 3 minutes.
- After time expires, each group will then share their answers in a “round – robin” format. For each correct response the group will get one point. The group with the most points at the end will be the “champ!”

## Application of Complex Numbers

Where are complex numbers used?

In real life, complex numbers are used by engineers and physicists to measure electrical currents, to analyze stresses in structures such as bridges and buildings, and to study the flow of liquids. Engineers who design speakers use complex numbers. Engineers who design and test the strength of bridges use complex numbers. Scientists who do experiments on ways to make energy using fuel cells, batteries, and solar cells use complex numbers. Complex numbers are also used for generating fractals, which are geometric objects created by making a repeating pattern.



Electrical current measurements are important to people who want to use electricity in a creative way. If you want to redesign your stereo to make it play the bass louder, or to add components like a turn-table, you might want to make some electrical measurements to make sure the components are compatible with each other. Electrical current is used as a direct current (DC) or alternating current (AC). When working with AC, complex numbers are needed.

Electrical current equation  $V = I \cdot Z$

V is voltage, I is current, Z is impedance

$$Z = V / I$$

We can also separate the current and voltage using complex notation

$$Z = V + Ii$$

Definitions to Know:

Voltage – the difference in electrical charge between two points in a circuit, units are volts

Current – a flow of electric charge, units are amperes or amps

Impedance – the opposition to current flow in AC circuits. In DC voltages, the term resistance is used. Impedance is simply the measure of how the flow of electrons is resisted. Units are ohms.

Adapted from: [http://tip.columbia.edu/index.php?option=com\\_content&task=view&id=86&Itemid=57](http://tip.columbia.edu/index.php?option=com_content&task=view&id=86&Itemid=57)

Name \_\_\_\_\_

Problems

1) The impedance in one part of a circuit is  $4 + 12i$  ohms. The impedance in another part of the circuit is  $3 - 7i$  ohms. What is the total impedance in the circuit?

2) The voltage in a circuit is  $45 + 10i$  volts. The impedance is  $3 + 4i$  ohms. What is the current?

$$V =$$

$$Z =$$

$$V = I \cdot Z$$

$$I = V / Z$$

3) The current in a circuit is  $8 + 3i$  amps. The impedance is  $1 - 4i$  ohms. What is the voltage?

$$I =$$

$$Z =$$

$$V = I \cdot Z$$

$$V =$$



Teacher Notes:

Students should be able to perform basic operations (addition, subtraction, division, and multiplication) with complex numbers in the form of  $a + bi$  by hand and by using the graphing calculator to check their answers.

Problems Solution Key

$$1) I_T = I_1 + I_2 = (4 + 12i) + (3 - 7i) \\ I_T = 7 + 5i \text{ ohms.}$$

$$2) V = 45 + 10i \text{ volts} \\ Z = 3 + 4i \text{ ohms} \\ V = I \cdot Z \\ I = V / Z = (45 + 10i) / (3 + 4i) \\ I = \frac{(45 + 10i)}{(3 + 4i)} \times \frac{(3 - 4i)}{(3 - 4i)} \\ I = \frac{(135 - 180i + 30i - 40i^2)}{(9 - 12i + 12i - 16i^2)} \\ I = \frac{(135 - 180i + 30i + 40)}{(9 + 16)} \\ I = \frac{175 - 150i}{25}$$

$$I = 7 - 6i \text{ amps}$$

$$3) I = 8 + 3i \text{ amps} \\ Z = 1 - 4i \text{ ohms} \\ V = I \cdot Z = I \cdot Z \\ V = (8 + 3i) \cdot (1 - 4i) \\ V = (8 - 32i + 3i - 12i^2) = (8 - 32i + 3i + 12) \\ V = 20 - 29i \text{ volts}$$

# The complex numbers

Downloaded from [http://alex.state.al.us/lesson\\_view.php?id=11364](http://alex.state.al.us/lesson_view.php?id=11364)

**To make many of the rules of mathematics apply universally we need to enlarge our number field.**

If we desire that every integer has an inverse element, we accept the existence of rational numbers.

If we desire every polynomial equation to have root(s) equal in number to its highest variable power, we must extend the real number field  $\mathcal{R}$  to a larger field  $\mathcal{C}$  of complex numbers.

## Where was '*i*' hiding?

- You may remember being told that you can't take the square root of a negative number. That's because you had no numbers that, when squared, were negative. Squaring a negative number always gives you a positive. So you couldn't very well square-root a negative and expect to come up with anything sensible.

## Where was '*i*' hiding?

- Now, however, you can take the square root of a negative number, but it involves using a new number to do it. At one time, nobody believed that any "real world" use would be found for this new number, other than easing the computations involved in solving certain equations, so the new number was viewed as being a '*pretend number*' invented for convenience sake.



## A complex number

A complex number is an ordered pair of real numbers  $(a,b)$ . We call 'a' the real part and 'b' the imaginary part of the complex number. We write that new number as  $a \pm bi$ . The ' $\pm$ ' is used to indicate the sign of the imaginary number part. The real number part represented by 'a' which can be either positive or negative.

Examples :

$$2 - 4i$$

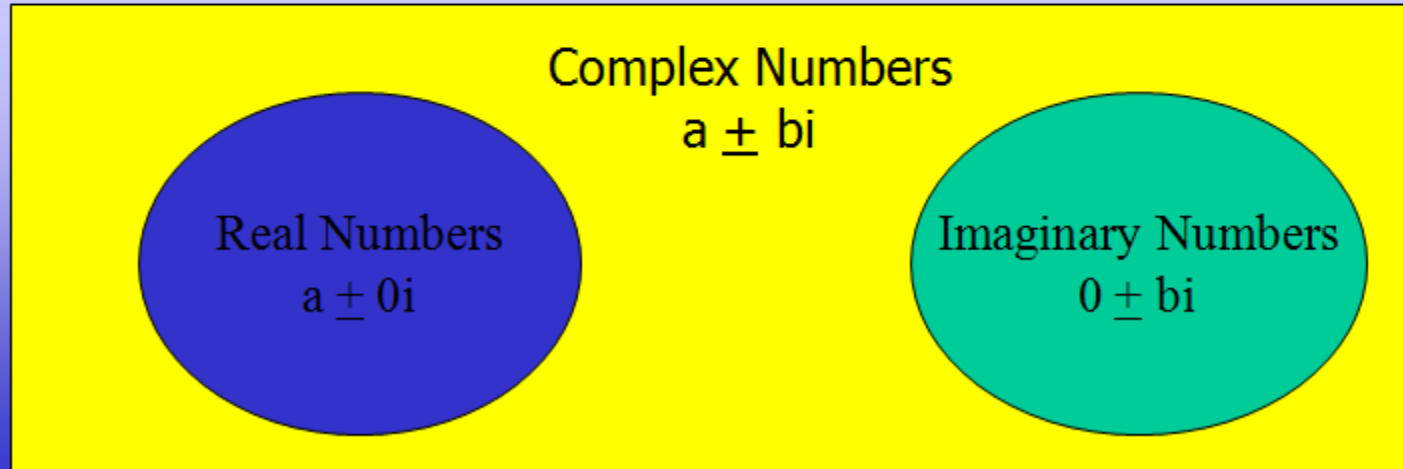
$$-3 + 5i$$

$$-5 + \frac{3}{4}i$$

These are examples of numbers that we say are strictly complex.

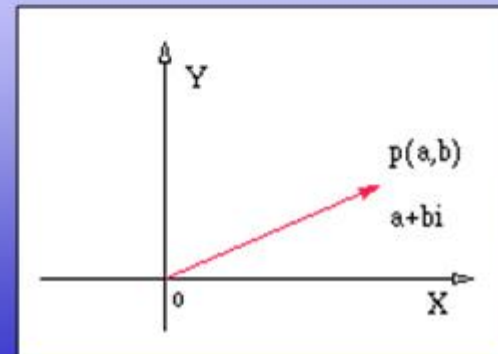
# VENN DIAGRAM Representation

- Since all number belong to the Complex number field,  $\mathcal{C}$ , all number can be classified as complex. The Real number field,  $\mathcal{R}$ , and the imaginary numbers,  $i$ , are subsets of this field as illustrated below.



## Graphical representation of a complex number

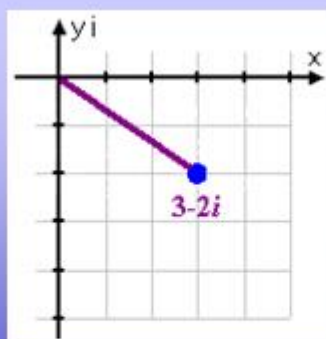
- A complex number has a representation in a plane. Simply take the x-axis as the real numbers and an y-axis as the imaginary numbers. Thus, giving the complex number  $a + bi$  the representation as point P with coordinates  $(a,b)$ .





# Graphing a Complex Number

- Therefore, complex numbers can be represented by a two dimensional graph.
- Here we see the graph of the complex number  $3 - 2i$ .



## Operations with Complex Numbers

**Simplify.**

1)  $i + 6i$

2)  $3 + 4 + 6i$

3)  $3i + i$

4)  $-8i - 7i$

5)  $-1 - 8i - 4 - i$

6)  $7 + i + 4 + 4$

7)  $-3 + 6i - (-5 - 3i) - 8i$

8)  $3 + 3i + 8 - 2i - 7$

9)  $4i(-2 - 8i)$

10)  $5i \cdot -i$

11)  $5i \cdot i \cdot -2i$

12)  $-4i \cdot 5i$

13)  $(-2 - i)(4 + i)$

14)  $(7 - 6i)(-8 + 3i)$

15)  $7i \cdot 3i(-8 - 6i)$

16)  $(4 - 5i)(4 + i)$

17)  $(2 - 4i)(-6 + 4i)$

18)  $(-3 + 2i)(-6 - 8i)$

19)  $(8 - 6i)(-4 - 4i)$

20)  $(1 - 7i)^2$

21)  $6(-7 + 6i)(-4 + 2i)$

22)  $(-2 - 2i)(-4 - 3i)(7 + 8i)$

23)  $5i + 7i \cdot i$

24)  $(6i)^3$

25)  $6i \cdot -4i + 8$

26)  $-6(4 - 6i)$

27)  $(8 - 3i)^2$

28)  $3 + 7i - 3i - 4$

29)  $-3i \cdot 6i - 3(-7 + 6i)$

30)  $-6i(8 - 6i)(-8 - 8i)$

**Critical thinking questions:**

31) How are the following problems different?

Simplify:  $(2 + x)(3 - 2x)$

Simplify:  $(2 + i)(3 - 2i)$

32) How are the following problems different?

Simplify:  $2 + x - (3 - 2x)$

Simplify:  $2 + i - (3 - 2i)$

## Operations with Complex Numbers Key

Simplify.

1)  $i + 6i$

$7i$

2)  $3 + 4 + 6i$

$7 + 6i$

3)  $3i + i$

$4i$

4)  $-8i - 7i$

$-15i$

5)  $-1 - 8i - 4 - i$

$-5 - 9i$

6)  $7 + i + 4 + i$

$11 + 2i$

7)  $-3 + 6i - (-5 - 3i) - 8i$

$2 + i$

8)  $3 + 3i + 8 - 2i - 7$

$4 + i$

9)  $4i(-2 - 8i)$

$32 - 8i$

10)  $5i \cdot -i$

$5$

11)  $5i \cdot i \cdot -2i$

$10i$

12)  $-4i \cdot 5i$

$20$

13)  $(-2 - i)(4 + i)$

$-7 - 6i$

14)  $(7 - 6i)(-8 + 3i)$

$-38 + 69i$

15)  $7i \cdot 3i(-8 - 6i)$

$168 + 126i$

16)  $(4 - 5i)(4 + i)$

$21 - 16i$

17)  $(2 - 4i)(-6 + 4i)$

$4 + 32i$

18)  $(-3 + 2i)(-6 - 8i)$

$34 + 12i$

19)  $(8 - 6i)(-4 - 4i)$

$-56 - 8i$

20)  $(1 - 7i)^2$

$-48 - 14i$

21)  $6(-7 + 6i)(-4 + 2i)$

$96 - 228i$

22)  $(-2 - 2i)(-4 - 3i)(7 + 8i)$

$-98 + 114i$

23)  $5i + 7i \cdot i$

$-7 + 5i$

24)  $(6i)^3$

$-216i$

25)  $6i \cdot -4i + 8$

$32$

26)  $-6(4 - 6i)$

$-24 + 36i$

27)  $(8 - 3i)^2$

$55 - 48i$

28)  $3 + 7i - 3i - 4$

$-1 + 4i$

29)  $-3i \cdot 6i - 3(-7 + 6i)$

$39 - 18i$

30)  $-6i(8 - 6i)(-8 - 8i)$

$-96 + 672i$

**Critical thinking questions:**

31) How are the following problems different?

Simplify:  $(2 + x)(3 - 2x)$

Simplify:  $(2 + i)(3 - 2i)$

$i^2 = -1$  so it leads to a few more steps

32) How are the following problems different?

Simplify:  $2 + x - (3 - 2x)$

Simplify:  $2 + i - (3 - 2i)$

There is no difference.

Name \_\_\_\_\_

**Create Problems**  
(Complex numbers)

Instructions: Create 6 problems involving the addition and subtraction of complex numbers (3 of each type). After you create your problems, solve them on a separate piece of paper. After you have solved your problems share them with another student in the room. When they are finished, correct their paper.

1)

2)

3)

4)

5)

6)

### Showdown Setup:

- Each group selects a team captain who will set the pace of work.
- Each student needs a handful of answer slips (there are 16 problems in total).
- Each group needs a team answer sheet.

### Process:

- 1) The captain shows problem 1. All team members work individually and silently on the problem. Write answers on answer slips, turning them upside-down when done (or totally stuck).
- 2) When the captain sees all slips are upside-down, he/she calls "Showdown". Group members show and compare their answers, explain their work, and come up with a group answer. The captain writes the group answer on the team answer sheet.
- 3) The captain turns the problem card over and compares the given answer to the team's answer. If the answers are different, discuss. *At this point*, you can ask me for help if you can't figure out how to get the answer.
- 4) Move on to the next problem and repeat.

**Question 1**

Without using a calculator, give a decimal approximation of  $\sqrt{24}$ , rounded to the tenths place.

**Question 2**

Simplify  $\sqrt{\frac{28}{3}}$

**Question 3**

Simplify:  $2(4\sqrt{2} - 6) - 3\sqrt{50} + 1$

**Question 4**

Simplify:  $\left(2 - \frac{3}{5}i\right)(20i)$

**Question 5**

Simplify:  $i\left(3 - \frac{1}{2}i\right)$

**Question 6**

Write in the form **a + bi**:  $7 + \sqrt{-9}$

**Question 7**

If  $(4 + bi)(4 - bi) = 80$ , find the value of  $b$ .

**Question 8**

Simplify  $2i\sqrt{-64}$



<p><b>Answer 2</b></p> $\sqrt{\frac{28}{3}} = \frac{\sqrt{28}}{\sqrt{3}} = \frac{2\sqrt{7}}{\sqrt{3}} = \frac{2\sqrt{21}}{3}$	<p><b>Answer 1</b></p> $\sqrt{24} \approx 4.9$
<p><b>Answer 4</b></p> $\begin{aligned} \left(2 - \frac{3}{5}i\right)(20i) \\ = 40i - 12i^2 \\ = 40i + 12 \\ = 12 + 40i \end{aligned}$	<p><b>Answer 3</b></p> $\begin{aligned} 2(4\sqrt{2} - 6) - 3\sqrt{50} + 1 \\ = 8\sqrt{2} - 12 - 3 \cdot 5\sqrt{2} + 1 \\ = -7\sqrt{2} - 11 \end{aligned}$
<p><b>Answer 6</b></p> $7 + 3i$	<p><b>Answer 5</b></p> $\begin{aligned} i\left(3 - \frac{1}{2}i\right) \\ 3i - \frac{1}{2}i^2 \\ 3i - \frac{1}{2}(-1) \\ 3i + \frac{1}{2} \\ \frac{1}{2} + 3i \end{aligned}$
<p><b>Answer 8</b></p> $\begin{aligned} 2i \times 6i \\ 12i^2 \\ -12 \end{aligned}$	<p><b>Answer 7</b></p> $\begin{aligned} (4 + bi)(4 - bi) &= 16 - b^2i^2 \\ &= 16 + b^2 = 80 \\ b^2 &= 64 \\ b &= 8 \end{aligned}$

**Question 9**

Evaluate  $i^{243}$

**Question 10**

Simplify  $(i - i^2)^2$

**Question 11**

Simplify  $\sqrt{-9} + 6$

**Question 12**

Evaluate  $i^{23}$

**Question 13**

$(7 + 2\sqrt{-9}) + (6 + 3\sqrt{-36})$

**Question 14**

Write in the form of  $a + bi$

$5i$

**Question 15**

Evaluate  $i^{15}$

**Question 16**

Simplify  $(10 - 3i) - (8 + 3i)$

**Answer 10**

$$\begin{aligned} &= (i - i^2)(i - i^2) \\ &= i^2 - 2i^3 + i^4 \\ &= (-1) - 2(-i) + 1 \\ &= 2i \end{aligned}$$

**Answer 9**

$$\begin{aligned} 243 \div 4 &= 60R3, \\ i^{243} &= i^3 = -i \end{aligned}$$

**Answer 12**

$$\begin{aligned} i^3 \times i^{20} \\ i^3 \times 1 \\ i^3 \end{aligned}$$

**Answer 11**

$$\begin{aligned} 3i + 6 \\ 6 + 3i \end{aligned}$$

**Answer 14**

$$0 + 5i$$

**Answer 13**

$$\begin{aligned} (7 + 6i) + (6 + 18i) \\ (7 + 6) + (6i + 18i) \\ 13 + 24i \end{aligned}$$

**Answer 16**

$$2 - 6i$$

**Answer 15**

$$i^3$$

## Showdown Answer Sheet

**Team Captain:** \_\_\_\_\_

**Other Members:** \_\_\_\_\_

**1)**

**9)**

**2)**

**10)**

**3)**

**11)**

**4)**

**12)**

**5)**

**13)**

**6)**

**14)**

**7)**

**15)**

**8)**

**16)**

Homework

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## More Review Problems

*Do all scratch work on binder paper and staple it to the handout. Write the answers on this sheet.*

- 1) Simplify each expression completely. Make sure to rationalize denominators and to write complex numbers in the form  $a + bi$ .

a)  $(2i^2 - 3i)^2$

b)  $\sqrt{\frac{3}{5} + \frac{3}{25}}$

c)  $\sqrt{-25} - 8i^{23}$

d)  $\frac{-1}{2 - \sqrt{3}}$

e)  $(2i)(3i)(4i)(5i)$

f)  $\sqrt{\left(\frac{2}{3} - \frac{1}{4}i\right)\left(\frac{2}{3} + \frac{1}{4}i\right)}$

g)  $(2 + 5i) \div (2 + 7i)$

h)  $(-\sqrt{147} + 2) - 5(1 - 5\sqrt{3})$

## Complex Number Exit Slip

In the space below, use what you have learned in this lesson to provide the two solutions to  $x^2 + 1 = 0$ .

Consider the steps needed to solve an equation and isolate "x". What inverse operations need to be applied in obtaining your solution? Predict what think will happen in generating a solution. Discuss with a partner and then share your ideas with the class. Record your ideas on the paper.

My first solution:

My second solution: