## Unit 13: Two-Way Tables

## PREREQUISITES

Students should know how to compute percentages.

## ADDITIONAL TOPIC COVERAGE

Additional coverage on two-way tables can be found in The Basic Practice of Statistics, Chapter 6, Two-Way Tables.

## ACTIVITY DESCRIPTION

Following the activity description, there is a questionnaire with three questions. Feel free to use this questionnaire or to use a modified version. If your class consists entirely of students of one class (freshman, sophomore, junior, senior), you might replace question 1 with the following question on gender:

## What is your gender? Male Female.

If you decide to add questions, give students an opportunity to create the questions. Questions 2 and 3 are directly related to the video. In these questions students are asked to rate the physical beauty of their campus (or school) and their current level of happiness. Regardless of how you modify the questionnaire, be sure to include these two questions.

After the class data have been collected, they should be entered into an Excel or statistical software spreadsheet and distributed to the class. Sample data are available if you decide not to collect data from your class. The sample answers to the activity are based on these data. Once students have collected the data, they can complete the activity. This activity can be done either individually or in groups.

The categorical data collected from the Happiness Survey is ordinal data. In other words, there is an inherent order in the categories. When making tables or graphic displays using software, students will need to impose that order (unless the categories' alphabetical order is the same as the inherent order). So, for all tables and graphic displays that involve the variable Physical Beauty, check that the categories Bad, OK, and Good appear in that order (or the reverse order) and not in alphabetical order Bad, Good, and OK.

## HAPPINESS SURVEY

Circle your answers to the following questions:

What is your class year?
Fr So Jr Sr

Rate the physical beauty of your campus (or school):
Bad OK Good

Rate your level of happiness today:
Unhappy So-so Happy

| Happiness | Physical Beauty | Class |
| :---: | :---: | :---: |
| Happy | Good | Jr |
| Happy | Good | Jr |
| Unhappy | Good | Jr |
| Unhappy | Good | Sr |
| Happy | OK | Jr |
| Happy | Good | Sr |
| Happy | Good | Sr |
| Unhappy | Bad | Sr |
| Happy | Good | Jr |
| Happy | Good | Jr |
| So So | Good | Sr |
| Happy | OK | Jr |
| Happy | OK | Sr |
| Happy | OK | Sr |
| Happy | Good | Sr |
| So So | Good | Sr |
| Happy | OK | Sr |
| Happy | OK | Sr |
| Unhappy | Good | Sr |
| Happy | OK | Sr |
| Happy | OK | Jr |
| Happy | Bad | Sr |
| Happy | Good | Sr |

Table T1. Sample Data collected from a college introductory statistics class.

## THE VIDEO SOLUTIONS

1. Sample answers: race, gender, car color.
2. Somerville included a Happiness Survey.
3. The row variable was Happiness - the values of happiness were used to label the rows of the table. The column variable was Physical Beauty and its values were used to label the columns of the table.
4. The percentage of respondents rating Somerville's physical beauty as Bad went down as the level of happiness went up.

## UNIT ACTIVITY SOLUTIONS

1. Sample answer (based on sample data):

|  |  | Physical Beauty |  |  | Total |
| :---: | :--- | :---: | :---: | :---: | :---: |
|  |  | Bad | OK | Good |  |
| Happiness | Happy | 1 | 8 | 8 | 17 |
|  | So So | 0 | 0 | 2 | 2 |
|  | Unhappy | 1 | 0 | 3 | 4 |
| Total |  | 2 | 8 | 13 | 23 |

2. Sample answer: $17 / 23 \times 100 \%=73.9 \%$
3. Sample answer: $13 / 23 \times 100 \%=56.5 \%$
4. a.

| $\%$ | Physical Beauty |  |  | Total |  |
| :---: | :--- | :---: | :---: | :---: | :---: |
|  |  |  | Bad |  |  | OK |
| Happin Happiness | Good |  |  |  |  |  |
|  | Happy | $5.90 \%$ | $47.10 \%$ | $47.10 \%$ | $100 \%$ |
|  | So So | $0.0 \%$ | $0.0 \%$ | $100.00 \%$ | $100 \%$ |
|  | Unhappy | $25.00 \%$ | $0.0 \%$ | $75.00 \%$ | $100 \%$ |

b. Sample answer: Percent of happy students who rated campus physical beauty as Good: $8 / 17 \times 100 \%=47.1 \%$. Percent of unhappy students who rated campus physical beauty as Good: $3 / 4 \times 100 \%=75.0 \%$. A higher percentage of the unhappy students rated campus beauty as Good. However, there were only four students who rated themselves as unhappy.
5. a. Sample answer: Students' bar charts should show three distributions, one for each level of physical beauty. (See chart on next page...)

b. Sample answer: Fifty percent of the students who rated the physical beauty of the campus as bad were happy and fifty percent were unhappy. A higher percentage of students who rated campus physical beauty as Good were happy (61.5\%) compared to those who rated campus physical beauty as bad. All eight of the students who rated campus physical beauty as OK were happy.
6. Sample answer. The class consisted only of juniors and seniors; $65.2 \%$ were seniors and $34.8 \%$ were juniors. The two-way tables below show the breakdown of Class with Happiness and Physical Beauty.

| Count |  | Happiness |  |  | Total |
| :--- | :--- | :---: | :---: | :---: | :---: |
|  |  | Happy | So So | Unhappy |  |
| Class | Junior | 7 | 0 | 1 | 8 |
|  | Senior | 10 | 2 | 3 | 15 |
| Total |  | 17 | 2 | 4 | 23 |


| Count |  | Physical Beauty |  |  | Total |
| :--- | :--- | :---: | :---: | :---: | :---: |
|  |  | OK | Good |  |  |
| Class | Junior | 0 | 3 | 5 | 8 |
|  | Senior | 2 | 5 | 8 | 15 |
| Total |  | 2 | 8 | 13 | 23 |

The juniors were happier than the seniors; $87.5 \%$ of the juniors rated themselves as happy compared to only $66.7 \%$ of the seniors. Below is a two-way table showing the distribution of Happiness conditioned on Class.

| $\%$ |  | Happiness |  |  | within Class |
| :--- | :--- | :---: | :---: | :---: | :---: |
|  |  | Happy | So So | Unhappy |  |
| Class | Junior | $87.50 \%$ | $0.0 \%$ | $12.50 \%$ | $100.00 \%$ |
|  | Senior | $66.70 \%$ | $13.30 \%$ | $20.00 \%$ | $100.00 \%$ |

The graphic display below shows the distribution of Physical Beauty for each level of Class.


Notice that a higher percentage of juniors rated the campus physical beauty as Good compared to seniors; $62.5 \%$ of the juniors rated campus physical beauty as Good compared to $53.3 \%$ of the seniors. Notice also that none of the juniors rated campus physical beauty as Bad compared to $13.3 \%$ of the seniors. So, it appears that juniors have a "rosier" outlook than the seniors, both in terms of their happiness but also in how they see the beauty of their surroundings.

## EXERCISE SOLUTIONS

1. a.

|  |  | Intelligence |  |  | Total |
| :--- | :--- | :---: | :---: | :---: | :---: |
|  |  | Below <br> Average | Average | Above <br> Average |  |
| Gender | Female | 437 | 2243 | 4072 | 6752 |
|  | Male | 456 | 1643 | 4593 | 6692 |
| Total |  | 893 | 3886 | 8665 | 13444 |

b. Male: $6692 / 13444 \times 100 \%=49.8 \%$; Female: $6752 / 13444 \times 100 \%=50.2 \%$
c. Above Average: $8665 / 13444 \times 100 \%=64.5 \%$. A sizable majority felt that their intelligence was above average compared to others their age.
2. a.

|  |  | Intelligence |  |  |  |
| :---: | :--- | :---: | :---: | :---: | :---: |
|  |  | Below Average | Average | Above Average | Total |
| Gender | Female | $437 / 6752 \times 100 \%$ | $2243 / 6752 \times 100 \%$ | $4072 / 6752 \times 100 \%$ |  |
|  |  | $6.47 \%$ | 0.3322 | $60.31 \%$ | $100 \%$ |
|  | Male | $456 / 6692 \times 100 \%$ | $1643 / 6692 \times 100 \%$ | $4593 / 6692 \times 100 \%$ |  |
|  | $6.81 \%$ | $24.60 \%$ | $68.60 \%$ | $100 \%$ |  |

b.

c. Sample answer: The percentages of male and female students who rated their intelligence as below average compared to others their age were about the same, $6.5 \%$ for females and $6.8 \%$ for males. A higher percentage of female students rated their intelligence as average compared to males, $33.2 \%$ for females and only $24.6 \%$ for males. A higher percentage of male students rated themselves as having above average intelligence than female students, 68.6\% for males compared to only $60.3 \%$ for females. However, it should be noted a majority of both male and female 12th graders responded that they had above average intelligence compared to others their age.
3. a. Notice that the total in the table below is $100.2 \%$. This is due to rounding the percentages.

|  |  | Political Preference |  |  |  |  |  |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Rep | Ind | Dem | Oth | No Pref <br> /Hvnt Decid | Total |
| Gender |  | Female | 9.6 | 5.5 | 12.4 | 0.7 | 22.1 |
|  |  | 12.3 | 6.6 | 10.1 | 1.4 | 19.5 | 49.9 |
| Total |  | 21.9 | 12.1 | 22.5 | 2.1 | 41.6 | 100.2 |

b.

|  |  | Political Preference |  |  |  |  |  |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Rep | Ind | Dem | Oth | No Pref/ <br> Hvnt Decid |  |
| Gender | Female | 19.2 | 10.9 | 24.6 | 1.3 | 44 |  |
|  | Male | 24.6 | 13.2 | 20.2 | 2.8 | 39.2 |  |

C.

|  |  | Political Preference |  |  |  |  |  |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Rep | Ind | Dem | Oth | No Pref/ <br> Hvnt Decid |  |
| Gender | Female | 44 | 45.4 | 55.1 | 32.7 | 53.1 |  |
|  | Male | 56 | 54.6 | 44.9 | 67.3 | 46.1 |  |

4. a. Percentage of respondents who were female and Democrats: $12.4 \%$ Percentage of respondents who were males and Independents: 6.6\%
b. Males were more likely to identify themselves as Republican than females; $24.6 \%$ of the males responded Republican compared to only $19.2 \%$ of the females.
c. Republicans were more likely to be male; $56 \%$ of the students who responded they were Republicans were male and only $44.0 \%$ were female.

In this question, the condition is Republican; there were 2895 students in this category. Then these 2895 students were broken down by gender. In (b) students are first broken down by gender; there were 6637 females and 6583 males. Then we see how many of each gender were Republican.
d. Sample answer: A higher percentage of male students (24.6\%) responded Republican compared to females (19.2\%). A higher percentage of female students (24.6\%) responded Democrat compared to males. It is interesting to note that the same percentage of females responded Democrat as males who responded Republican. A considerably higher percentage of females (44.0\%) reported having no preference or were undecided compared to males $(39.1 \%)$. A very low percentage of either gender responded "Other" for political preference.


## REVIEW QUESTIONS SOLUTIONS

1. a.

|  |  | Smoking |  |  |  |  | Total |  |  |  |  |  |  |  |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Never | Once or <br> twice | Occ/ <br> not reg | Reg <br> in past | Reg now |  |  |  |  |  |  |  |  |
| Gender | Female | 4244 | 1228 | 649 | 253 | 428 | 6802 |  |  |  |  |  |  |  |
|  | Male | 3957 | 1182 | 793 | 337 | 543 | 6812 |  |  |  |  |  |  |  |
|  | Total |  |  |  |  |  |  |  |  | 8201 | 2410 | 1442 | 590 | 971 | 13614 |

b. Females who never smoked: $4244 / 6802 \times 100 \%=62.4 \%$

Males who never smoked: $3957 / 6812 \times 100 \%=58.1 \%$
A higher percentage of the female respondents (62.4\%) did not smoke compared to the male respondents (58.1\%).
C.

2. a. 13,627
b. Never smoked: $8229 / 13627 \times 100 \%=60.4 \%$

Smoked at least once: $100 \%-60.4 \%=39.6 \%$
c. $3465 / 13627 \times 100 \%=25.4 \%$
3. a. $3465 / 4751 \times 100 \%=72.9 \%$
b. $(926+46) / 2221 \times 100 \%=43.8 \%$
4. a. $(489+168) / 967 \times 100 \%=67.9 \%$
b. $(330+126) / 591 \times 100 \%=77.2 \%$
c. $(3792+3465) / 8229 \times 100 \%=88.2 \%$
5. a. Grade D: sum = 99\%; Grade C-, C, or C+ : sum = 100\%;

Grade B-, B, or B+: sum = $100 \%$; Grade A- or A: sum $=99 \%$. The failure to sum to $100 \%$ is due to round-off error.
b. Sample answer: Here are the most striking patterns showing a relationship between grades and alcohol consumption. As grades increase, the percentage of students who had not consumed alcohol increases. As grades increase, the percentage of students who had consumed alcohol 6 or more times in the last 30 days decreases.

