

Time Rate of Consolidation Settlement

- We know how to evaluate total settlement of primary consolidation S_c which will take place in a certain clay layer.
- However this settlement usually takes place over time, much longer than the time of construction.
- One question one might ask is in how much **time** that magnitude of settlement will take place. Also might be interested in knowing the value of S_c for a **given** time, or the **time** required for a **certain** magnitude of settlement.
- In certain situations, engineers may need to know the followings information:
 1. The amount of settlement $S_c(t)$ ~ at a specific time, t , before the end of consolidation, or
 2. The time, t , required for a specific settlement amount, before the end of consolidation.

How to get to know the rate of consolidation?

- From the spring analogy we can see that S_c is directly related to how much water has squeezed out of the soil voids.
- How much water has squeezed out and thus the change in void ratio e is in turn directly proportional to the amount of excess p.w.p. that has dissipated.
- Therefore, the rate of settlement is directly related to the rate of excess p.w.p. dissipation.
- What we need is a **governing equation** that predict the change in p.w.p. with time and hence e , at any point in **TIME** and **SPACE** in the consolidation clay layer.
- In other words, we need something to tell us how we get from the moment the load is entirely carried by the water to the point the load is completely supported by the soil.
- It is the **THEORY OF CONSOLIDATION** which tells us that.

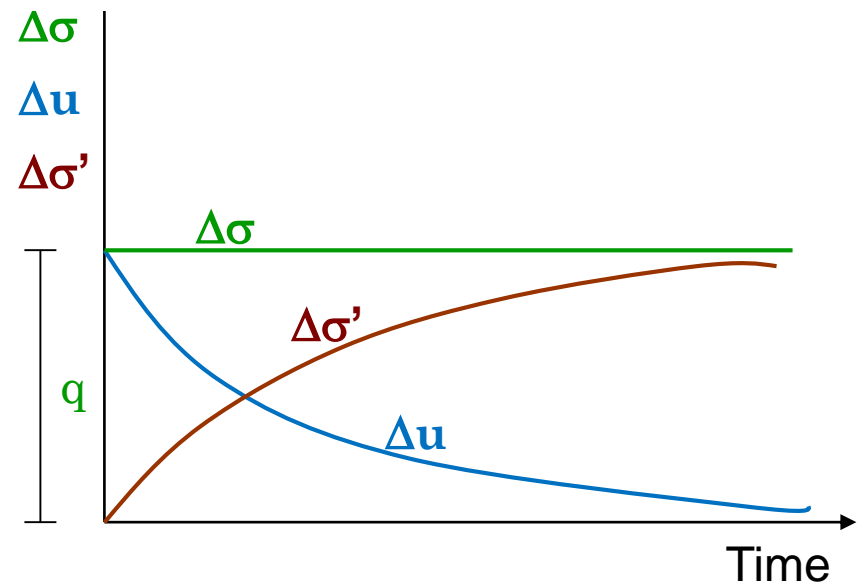
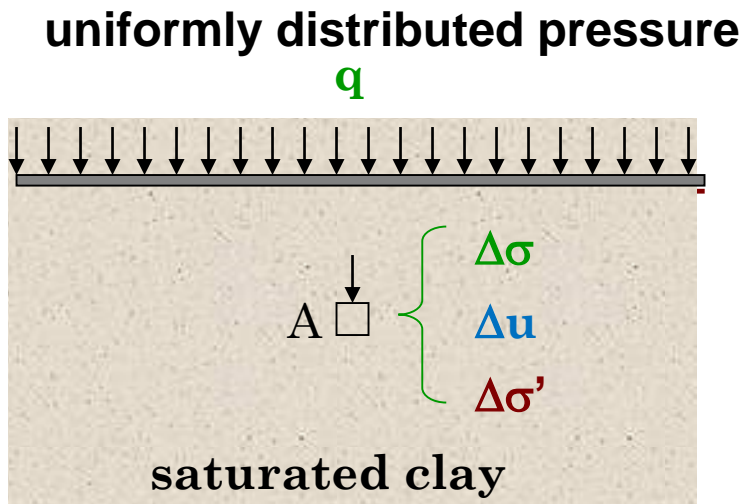
Spring Analogy

- $\Delta\sigma$, the increase in total stress remains the same during consolidation, while effective stress $\Delta\sigma'$ **increases**.
- Δu the excess pore-water pressure **decreases** (due to drainage) transferring the load from water to the soil.

Excess pore pressure (Δu)

is the difference between the current pore pressure (u) and the steady state pore pressure (u_o).

$$\Delta u = u - u_o$$



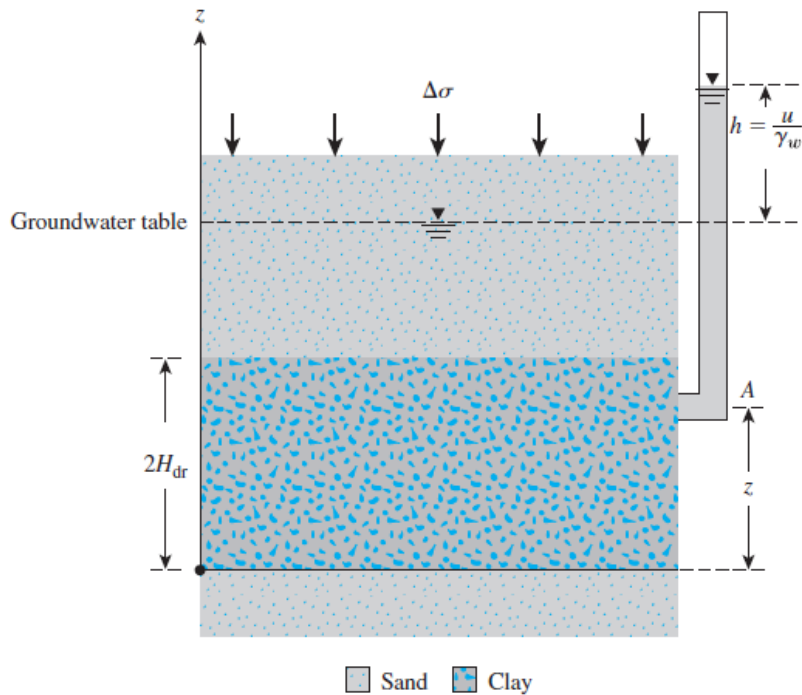
1-D Theory of Consolidation

- ❑ **Terzaghi** developed a theory based on the assumption that an increment of load **immediately** is transferred to the pore water to create **excess** pore water pressure (p.w.p).
- ❑ Then as the pore water squeezed out, the excess p.w.p. relaxes gradually transferring the load to effective stress.
- ❑ He assumed that all drainage of excess pore water is **vertical** toward one or two horizontal drainage faces. This is described as **ONE-DIMENSIONAL CONSOLIDATION**.
- ❑ **3-D** consolidation theory is now available but more cumbersome.
- ❑ However **1-D** theory is useful and still the one used in practice, and it tends to overpredict settlement.

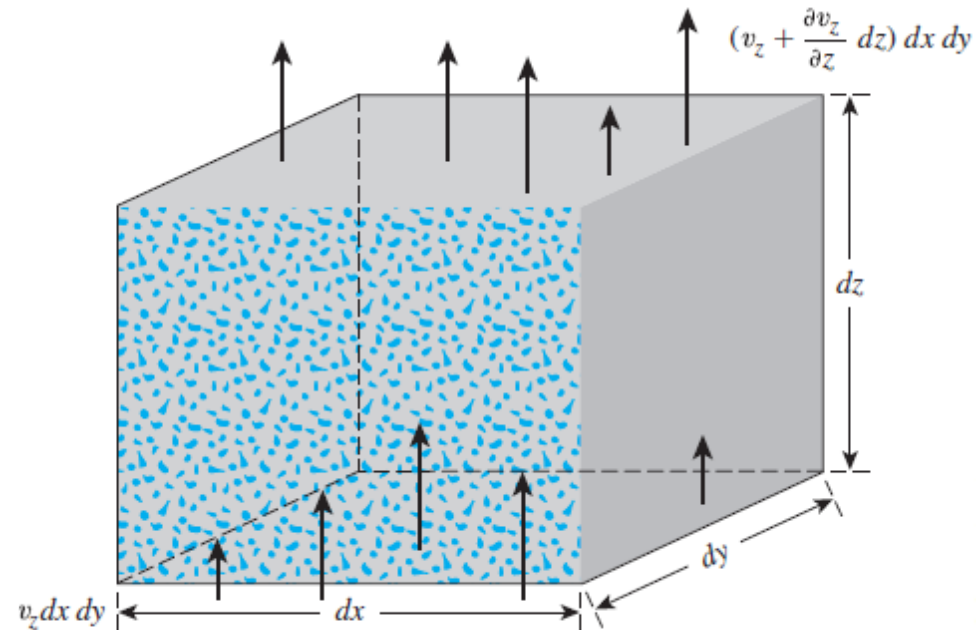
ASSUMPTIONS

- ✓ The soil is homogeneous.
- ✓ The soil is fully saturated.
- ✓ The solid particles and water are incompressible.
- ✓ Compression and flow are 1-D (vertical).
- ✓ Darcy's law is valid at all hydraulic gradients.
- ✓ The coefficient of permeability and the coefficient of volume change remain **constant** throughout the process.
- ✓ Strains are small.

Mathematical Derivation



(a)



(b)

Rate of outflow of water - Rate of inflow of water = Rate of Volume Change

$$\left(v_z + \frac{\partial v_z}{\partial z} dz \right) dx dy - v_z dx dy = \frac{\partial V}{\partial t}$$

$$\frac{\partial v_z}{\partial z} dx dy dz = \frac{\partial V}{\partial t} \quad (1)$$

where V = volume of the soil element
 v_z = velocity of flow in z direction

Mathematical Derivation

$$v_z = ki = -k \frac{\partial h}{\partial z} = -\frac{k}{\gamma_w} \frac{\partial u}{\partial z}$$

$$\frac{\partial v_z}{\partial z} = \frac{\partial}{\partial z} \left(-\frac{k}{\gamma_w} \frac{\partial u}{\partial z} \right) = -\frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2}$$

$$-\frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2} dxdydz = \frac{\partial V}{\partial t} \quad (2)$$

$$\frac{\partial V}{\partial t} = \frac{\partial(V_s + eV_s)}{\partial t} = \frac{\partial V_s}{\partial t} + V_s \frac{\partial e}{\partial t} + e \frac{\partial V_s}{\partial t}$$

$$\frac{\partial V_s}{\partial t} = 0$$

$$\frac{\partial V}{\partial t} = V_s \frac{\partial e}{\partial t}$$

$$V_s = \frac{V}{1 + e_o} = \frac{dxdydz}{1 + e_o}$$

$$\frac{\partial V}{\partial t} = \frac{dxdydz}{1 + e_o} \frac{\partial e}{\partial t} \quad (3)$$

The one-dimensional consolidation equation derived by Terzaghi

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

From (2) and (3)

$$-\frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2} = \frac{1}{1 + e_o} \frac{\partial e}{\partial t}$$

$$\partial e = a_v \partial(\nabla \sigma') = -a_v \partial u$$

$$-\frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2} = -\frac{a_v}{1 + e_o} \frac{\partial u}{\partial t} = -m_v \frac{\partial u}{\partial t}$$

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

$$c_v = \frac{k}{\gamma_w m_v} = \frac{k}{\gamma_w \left(\frac{a_v}{1 + e_o} \right)}$$

Solution of Terzaghi's 1-D consolidation equation

Terzaghi's equation is a linear **partial** differential equation in one dependent variable. It can be solved by one of various methods with the following boundary conditions:

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

$$z = 0, \quad u = 0$$

$$z = 2H_{dr}, \quad u = 0$$

$$t = 0, \quad u = u_o$$

The solution yields



$$u = \sum_{m=0}^{m=\infty} \left[\frac{2u_o}{M} \sin \left(\frac{Mz}{H_{dr}} \right) \right] e^{-M^2 T_v} \quad (*)$$

Where

u = excess pore water pressure

u_o = initial pore water pressure

$M = \pi/2 (2m+1)$ m = an integer

z = depth

H_{dr} = maximum drainage path

$$T_v = \frac{c_v t}{H_{dr}^2} = \text{time factor}$$

Remarks

- ❑ The theory relates three variables:
 - Excess pore water pressure u
 - The depth z below the top of the clay layer
 - The time t from the moment of application of load

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

Or it gives u at any depth z at any time t

- ❑ The solution was for doubly drained stratum.
- ❑ Finding degree of consolidation for single drainage is exactly the same procedure as for double drainage case except here $H_d =$ the entire depth of the drainage layer when substituting in equations or when using the figure of isochrones.
- ❑ Eq. (*) represents the relationship between time, depth, p.w.p for constant initial pore water pressure u_0 .
- ❑ If we know the coefficient of consolidation C_v and the initial p.w.p. distribution along with the layer thickness and boundary conditions, we can find the value of u at any depth z at any time t .

Degree of Consolidation

- The progress of consolidation after sometime **t** and at any depth **z** in the consolidating layer can be related to the **void ratio** at that time and the final **change** in void ratio.
- This relationship is called the DEGREE or PERCENT of CONSOLIDATION or CONSOLIDATION RATIO.
- Because consolidation progress by the dissipation of excess pore water pressure, the degree of consolidation at a distance **z** at any time **t** is given by:

$$U_z = \frac{\text{excess pore water pressure dissipated}}{\text{initial excess pore water pressure}}$$

$$U_z = \frac{u_o - u_z}{u_o} = 1 - \frac{u_z}{u_o} \quad \dots\dots(**)$$

u_z = excess pore water pressure at time t .

u_o = initial excess pore water pressure

Degree of Consolidation

Substituting the expression for excess pore water pressure, i.e.

$$u = \sum_{m=0}^{m=\infty} \left[\frac{2u_o}{M} \sin \left(\frac{Mz}{H_{dr}} \right) \right] e^{-M^2 T_v}$$

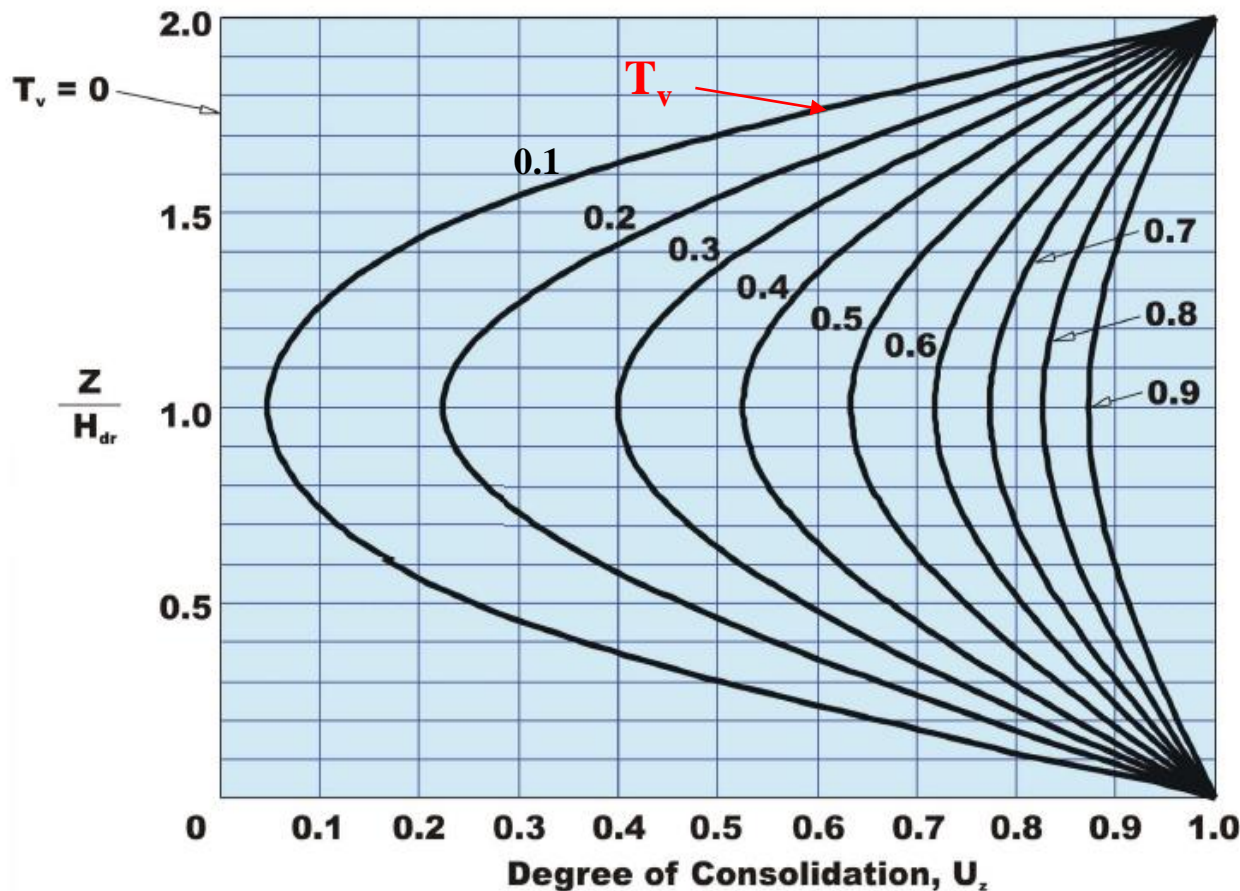
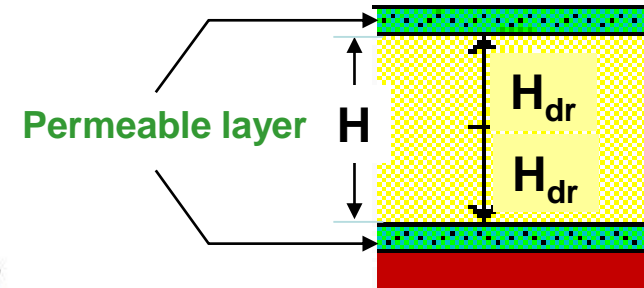
into Eq. (**) yields

$$U_z = 1 - \sum_{m=0}^{m=\infty} \left[\frac{2}{M} \sin \left(\frac{Mz}{H_{dr}} \right) \right] e^{-M^2 T_v} \quad \dots\dots (***)$$

- The above equation can be used to find the degree of consolidation at depth **z** at a given time **t**.
- At any given time excess pore water pressure **u_z** varies with **depth**, and hence the degree of consolidation **U_z** also varies.
- If we have a situation of **one-way drainage** Eq. (***) is still be valid, however the length of the drainage path is equal to the **total** thickness of the clay layer.

Degree of Consolidation

$$U_z = 1 - \sum_{m=0}^{\infty} \left[\frac{2}{M} \sin \left(\frac{Mz}{H_{dr}} \right) \right] e^{-M^2 T_v}$$

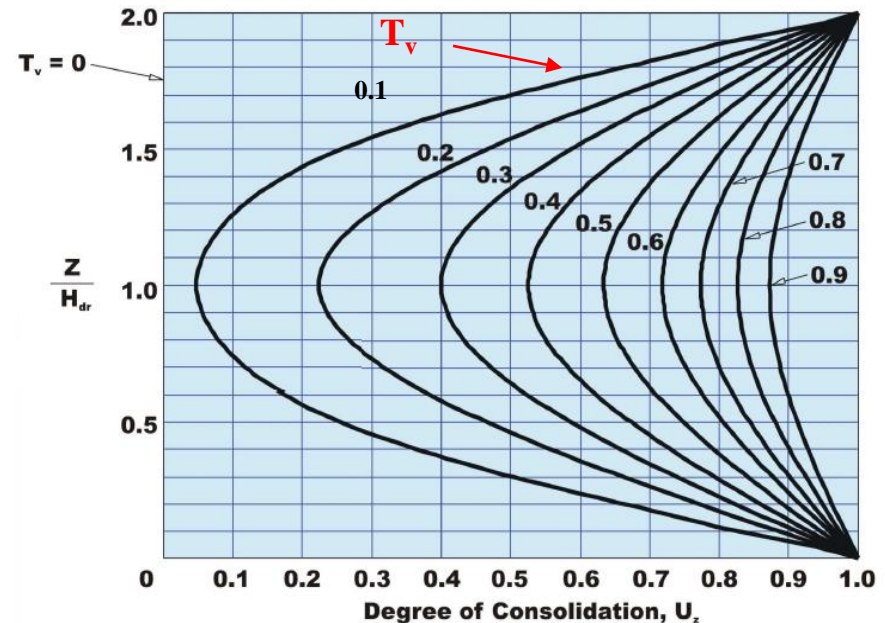


$$T_v = \frac{c_v t}{H_{dr}^2} = \text{time factor}$$

Variation of U_z with T_v and Z/H_{dr}

Remarks

- From this figure it is possible to find the amount or **degree of consolidation** (and therefore u and σ') for any real time after the start of loading and at any point in the consolidating layer.

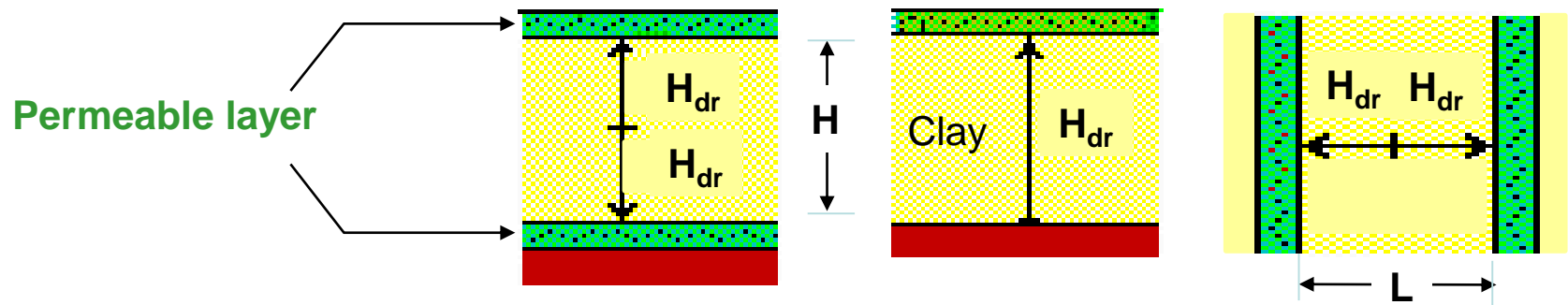


Variation of U_z with T_v and z/H_{dr}

- All you need to know is the C_v for the particular soil deposit, the total thickness of the layer, and boundary drainage conditions.
- These curves are called **isochrones** because they are lines of equal **times**.
- With the advent of digital computer the value of U_z can be readily evaluated directly from the equation without resorting to chart.

Length of the drainage path, H_{dr}

- During consolidation water escapes from the soil to the surface or to a permeable sub-surface layer above or below (where $\Delta u = 0$).
- The rate of consolidation depends on the longest path taken by a drop of water. The length of this longest path is the **drainage path length**, H_{dr}



- **Typical cases are:**
 - An open layer, a permeable layer both above and below ($H_{dr} = H/2$)
 - A half-closed layer, a permeable layer either above or below ($H_{dr} = H$)
 - Vertical sand drains, horizontal drainage ($H_{dr} = L/2$)

Degree of Consolidation

$$U_z = \frac{\text{excess pore water pressure dissipated}}{\text{initial excess pore water pressure}}$$

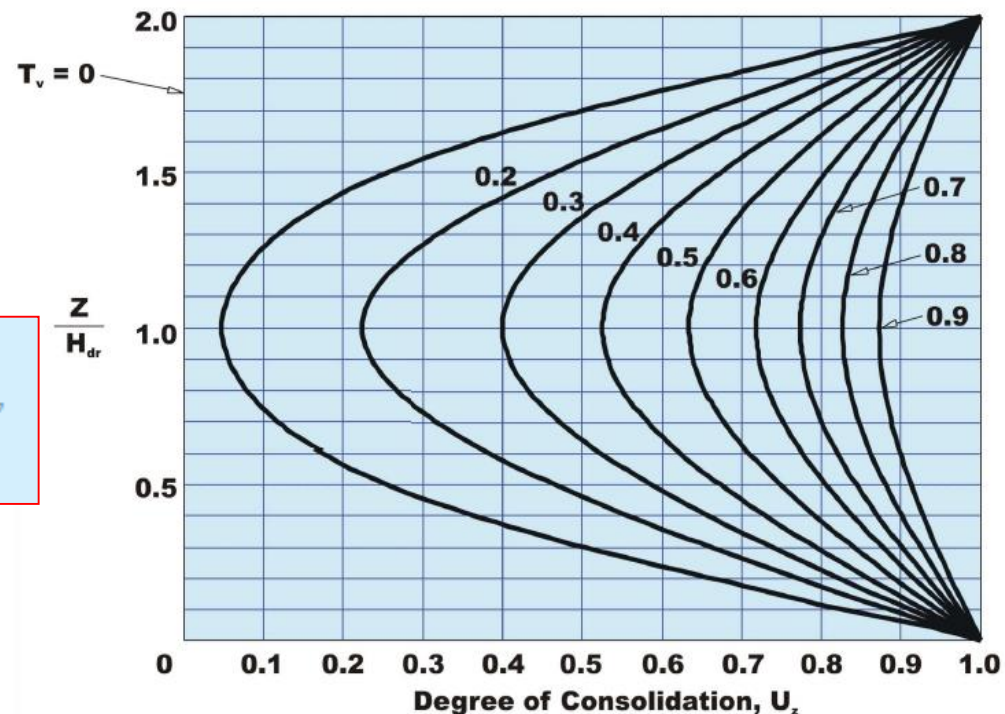
$$U_z = \frac{u_o - u_z}{u_o} = 1 - \frac{u_z}{u_o}$$

$$u = \sum_{m=0}^{\infty} \left[\frac{2u_o}{M} \sin \left(\frac{Mz}{H_{dr}} \right) \right] e^{-M^2 T_v}$$

$$T_v = \frac{c_v t}{H_{dr}^2} = \text{time factor}$$

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

$$U_z = 1 - \sum_{n=0}^{\infty} \left[\frac{2}{M} \sin \left(\frac{Mz}{H_{dr}} \right) \right] e^{-M^2 T_v}$$



Example

A 12 m thick clay layer is **doubly** drained (This means that a very pervious layer compared to the clay exists on top of and under the 12 m clay layer. The coefficient of consolidation $C_v = 8.0 \times 10^{-8} \text{ m}^2/\text{s}$.)

Required:

Find the degree or percent consolidation for the clay 5 yr after loading at depths of 3, 6, 9, and 12 m.

Solution:

First, compute the time factor.

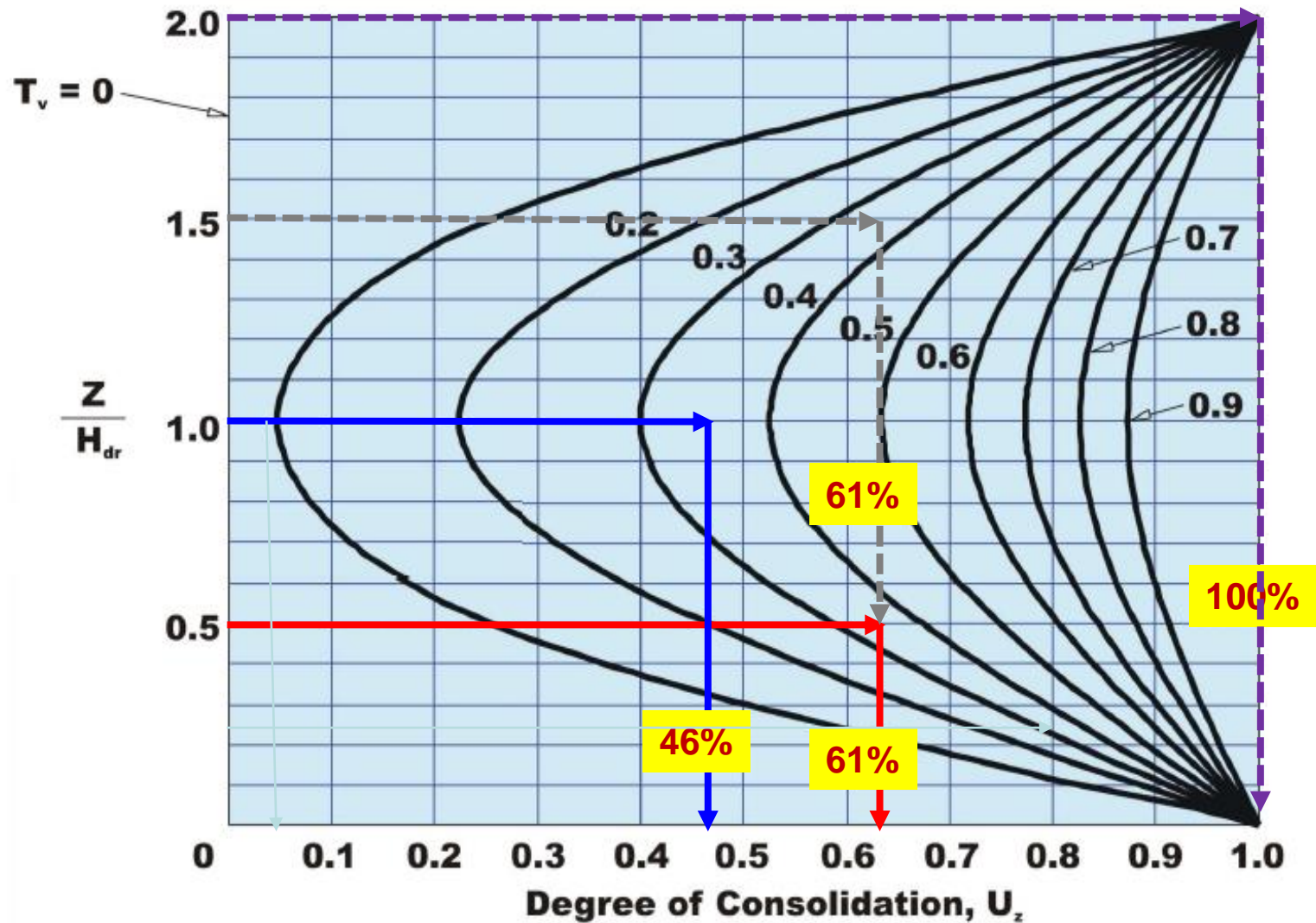
$$T = \frac{c_v t}{H_{dr}^2}$$
$$= \frac{8.0 \times 10^{-8} \text{ m}^2/\text{s} (3.1536 \times 10^7 \text{ s/yr}) (5 \text{ yr})}{(6)^2} = 0.35$$

Note that $2H = 12 \text{ m}$ and $H_{dr} = 6 \text{ m}$ since there is double drainage.

Next, from Fig. 9.3 we obtain (by interpolation) for $T = 0.35$:

At $z = 3 \text{ m}$,	$z/H = 0.50$,	$U_z = 61\%$
At $z = 6 \text{ m}$,	$z/H = 1.0$,	$U_z = 46\%$
At $z = 9 \text{ m}$,	$z/H = 1.50$,	$U_z = 61\%$
At $z = 12 \text{ m}$,	$z/H = 2.0$,	$U_z = 100\%$

At $z = 3$ m, $z/H = 0.50$,
 At $z = 6$ m, $z/H = 1.0$,
 At $z = 9$ m, $z/H = 1.50$,
 At $z = 12$ m, $z/H = 2.0$,



Average Degree of Consolidation

- In most cases, we are not interested in how much a given point in a layer has consolidated.
- Of more practical interest is the *average degree or percent consolidation* of the entire layer.
- This value, denoted by U or U_{av} , is a measure of how much the entire layer has consolidated and thus it can be directly related to the *total settlement* of the layer at a given *time* after loading.
- Note that U can be expressed as either a *decimal* or a *percentage*.
- To obtain the average degree of consolidation over the entire layer corresponding to a given *time factor* we have to find the area under the T_v curve.

Average Degree of Consolidation

The average degree of consolidation for the entire depth of clay layer is,

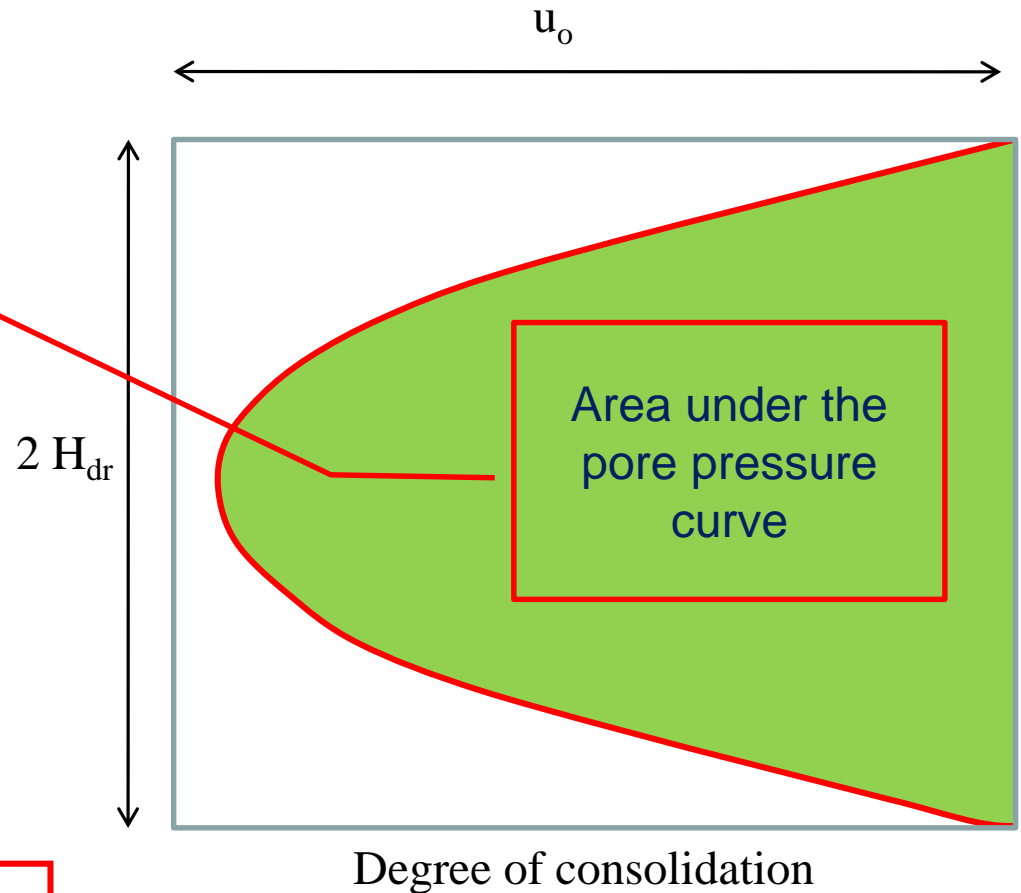
$$U = 1 - \frac{\left(\frac{1}{2H_{dr}} \int_0^{2H_{dr}} u_z dz \right)}{u_o} \dots (1)$$

Substituting the expression of u_z given by

$$u = \sum_{m=0}^{\infty} \left[\frac{2u_o}{M} \sin \left(\frac{Mz}{H_{dr}} \right) \right] e^{-M^2 T_v}$$

Into Eq. (1) and integrating, yields

$$U = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} e^{-M^2 T_v}$$



Summary

- Because consolidation progress by the dissipation of excess pore water pressure, the degree of consolidation at a distance **z** at any time **t** is given by:

$$U_z = \frac{\text{excess pore water pressure dissipated}}{\text{initial excess pore water pressure}}$$



$$U_z = \frac{u_o - u_z}{u_o} = 1 - \frac{u_z}{u_o}$$



$$U = 1 - \frac{\left(\frac{1}{2H_{dr}} \right)^2 \int_0^{2H_{dr}} u_z dz}{u_o}$$



$$U = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} e^{-M^2 T_v}$$



$$u = \sum_{m=0}^{\infty} \left[\frac{2u_o}{M} \sin \left(\frac{Mz}{H_{dr}} \right) \right] e^{-M^2 T_v}$$



$$U_z = 1 - \sum_{m=0}^{\infty} \left[\frac{2}{M} \sin \left(\frac{Mz}{H_{dr}} \right) \right] e^{-M^2 T_v}$$

Degree of consolidation

Average Degree of consolidation

Average Degree of Consolidation

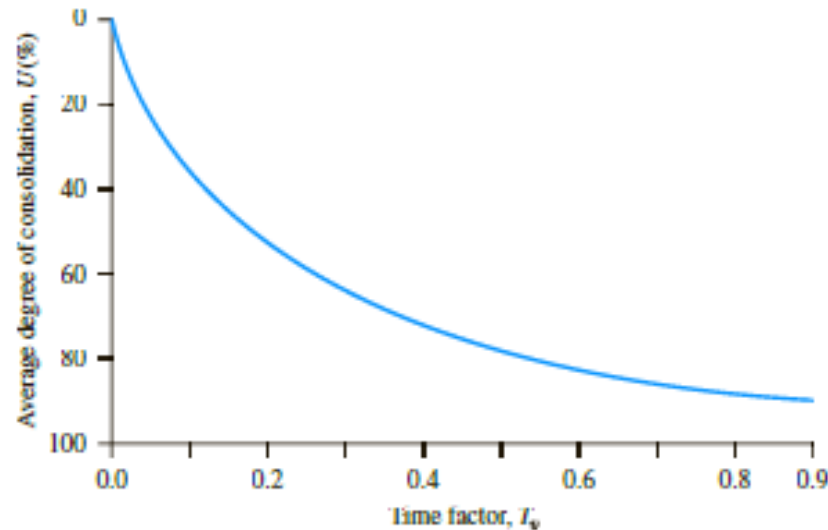


Fig. 11.30 Variation of U with T_v

$$U = \frac{S_{c(t)}}{S_c}$$



$$S_{c(t)} = U(t)S_c$$

$S_{c(t)}$ = Settlement at any time, t

S_c = Ultimate primary consolidation settlement of the layer.

Average Degree of Consolidation

Table 11.7 Variation of T_v with U

U (%)	T_v	U (%)	T_v	U (%)	T_v	U (%)	T_v
0	0	26	0.0531	52	0.212	78	0.529
1	0.00008	27	0.0572	53	0.221	79	0.547
2	0.0003	28	0.0615	54	0.230	80	0.567
3	0.00071	29	0.0660	55	0.239	81	0.588
4	0.00126	30	0.0707	56	0.248	82	0.610
5	0.00196	31	0.0754	57	0.257	83	0.633
6	0.00283	32	0.0803	58	0.267	84	0.658
7	0.00385	33	0.0855	59	0.276	85	0.684
8	0.00502	34	0.0907	60	0.286	86	0.712
9	0.00636	35	0.0962	61	0.297	87	0.742
10	0.00785	36	0.102	62	0.307	88	0.774
11	0.0095	37	0.107	63	0.318	89	0.809
12	0.0113	38	0.113	64	0.329	90	0.848
13	0.0133	39	0.119	65	0.340	91	0.891
14	0.0154	40	0.126	66	0.352	92	0.938
15	0.0177	41	0.132	67	0.364	93	0.993
16	0.0201	42	0.138	68	0.377	94	1.055
17	0.0227	43	0.145	69	0.390	95	1.129
18	0.0254	44	0.152	70	0.403	96	1.219
19	0.0283	45	0.159	71	0.417	97	1.336
20	0.0314	46	0.166	72	0.431	98	1.500
21	0.0346	47	0.173	73	0.446	99	1.781
22	0.0380	48	0.181	74	0.461	100	∞
23	0.0415	49	0.188	75	0.477		
24	0.0452	50	0.197	76	0.493		
25	0.0491	51	0.204	77	0.511		

Approximate relationships for U vs. T_v

- Many correlations of variation of U with T_v have been proposed.
- **Terzaghi** proposed the followings:

For $U = 0$ to 60%

$$T_v = \frac{\pi}{4} \left(\frac{U\%}{100} \right)^2 \quad \text{or} \quad U = \sqrt{\frac{4T_v}{\pi}}$$

For $U > 60\%$

$$T_v = 1.781 - 0.933 \log(100 - U\%)$$

or

$$U = 100 - 10^{-\frac{T_v - 1.781}{0.933}}$$

Approximate relationships for U vs. T_v

$$\frac{U\%}{100} = \frac{(4T_v/\pi)^{0.5}}{[1 + (4T_v/\pi)^{2.8}]^{0.179}}$$

or:

$$T_v = \frac{(\pi/4)(U\%/100)^2}{[1 - (U\%/100)^{5.6}]^{0.357}}$$

$$S_{c(t)} = U_{(t)} S_c$$

Note

These equations can be applied for all ranges of U value with small errors .

Error in T_v of less than **1%** for $0\% < U < 90\%$ and less than **3%** for $90\% < U < 100\%$.

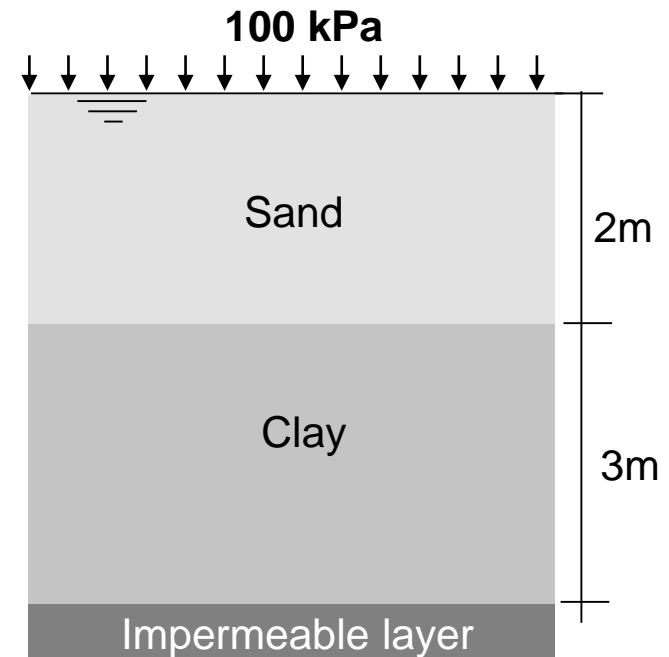
Example

A soil profile consists of a **sand** layer **2 m** thick, whose top is the ground surface, and a **clay** layer **3 m** thick with an **impermeable** boundary located at its base. The water table is at the ground surface. A widespread load of **100 kPa** is applied at the ground surface.

(i) What is the excess water pressure, Δu corresponding to:

- **$t = 0$** (i.e. immediately after applying the load)
- **$t = \infty$** (very long time after applying the load)

(ii) Determine the time required to reach **50%** consolidation if you know that $C_v = 6.5 \text{ m}^2/\text{year}$.



Solution

(i) Immediately after applying the load, the degree of consolidation $U_z = 0\%$ and the pore water would carry the entire load:

at $t = 0 \rightarrow \Delta u_0 = \Delta \sigma = 100 \text{ kPa}$

On contrary, after very long time, the degree of consolidation $U = 100\%$ and the clay particles would carry the load completely:

at $t = \infty \rightarrow \Delta u_{\infty} = 0$

(ii) The time required to achieve 50% consolidation can be calculated from the equation:

$$T_v = \frac{C_v t}{H_{dr}^2} \rightarrow$$

- c_v = coefficient of consolidation (given) = $6.5 \text{ m}^2/\text{year}$
- H_{dr} = the drainage path length = height of clay = 3m (because the water drain away from the sand layer only)
- T_v = is the time factor for $U=50\%$, and can approximately be calculated from:

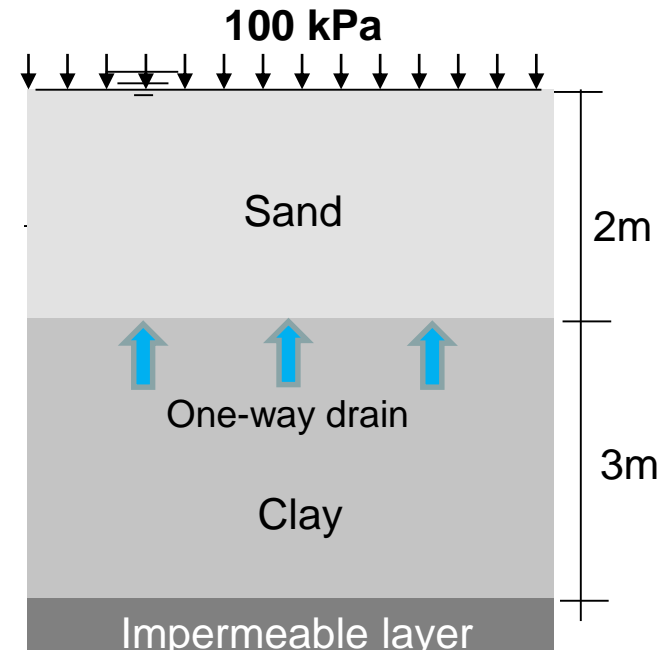
$$T_v = \frac{\pi}{4} \left(\frac{U\%}{100} \right)^2 \approx 0.197$$

Can also be obtained from the theoretical relationship or graph

Substitution of these values in the above equation of t :

$t \approx 0.27 \text{ year}$

$$t = H_{dr}^2 \cdot T_v / c_v$$



Example

An **open** layer of clay **4 m** thick is subjected to loading that increases the average effective vertical stress from **185 kPa** to **310 kPa**. Assuming **$m_v = 0.00025 \text{ m}^2/\text{kN}$** , **$C_v = 0.75 \text{ m}^2/\text{year}$** , determine:

- i. The ultimate consolidation settlement
- ii. The settlement at the end of 1 year,
- iii. The time in days for 50% consolidation,
- iv. The time in days for 25 mm of settlement to occur.

Solution

(i) The consolidation settlement for a layer of thickness H can be represented by the coefficient of volume compressibility m_v defined by:

$$\begin{aligned} S_c &= m_v H \Delta \sigma'_z \\ &= 0.00025 * 4 * 125 = 0.125\text{m} = \underline{\underline{125\text{mm}}}. \end{aligned}$$

Example

(ii) The procedure for calculation of the settlement at a specific time includes:

• Calculate time factor: $T_v = \frac{C_v t}{H_{dr}^2} = \dots\dots\dots = \underline{0.1875}$

• Calculate average degree of consolidation

$$U_t = \dots\dots\dots = \underline{0.49}$$

• Calculate the consolidation settlement at the specific time (t) from:

$$S_t = U_t \cdot S_c = \dots\dots\dots = \underline{61 \text{ mm}}$$

(iii) For 50% consolidation $T_v = 0.197$, therefore from $T_v = \frac{C_v t}{H_{dr}^2}$

$$\dots\dots\dots \rightarrow t = 1.05 \text{ year} = \underline{384 \text{ days}}$$

(vi) For $S_t = 25 \text{ mm}$ $U_t = 0.20$, therefore

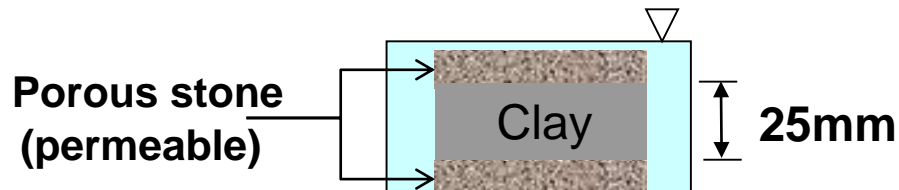
$$\dots\dots\dots \rightarrow t = 0.1675 \text{ year} = \underline{61 \text{ days}}$$

Example

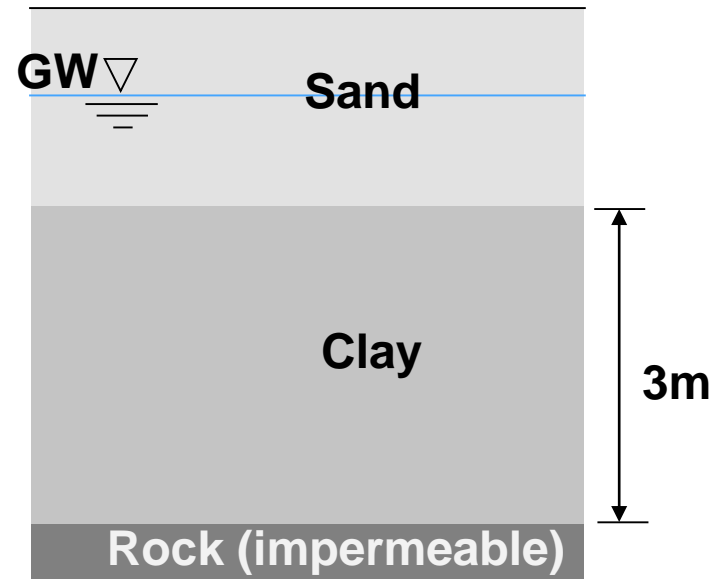
The time required for **50%** consolidation of a **25-mm-thick** clay layer (drained at **both** top and bottom) in the laboratory is **2 min. 20 sec.**

- (i) How long (in **days**) will it take for a **3-m-thick** clay layer of the same clay in the field under the same pressure increment to reach **50%** consolidation? In the field, there is a **rock** layer at the bottom of the clay.

Laboratory



Field



- (ii) How long (in days) will it take in the field for **30%** primary consolidation to occur? Assuming:

$$T_v = \frac{\pi}{4} \left(\frac{U\%}{100} \right)^2$$

Solution

(i) As the clay in lab and field reached the same consolidation degree ($U=50\%$), Thus, *The time factor in the lab test = The time factor for the field*

Approach I:

$$T_{50} = \frac{c_v t_{\text{lab}}}{H_{\text{dr}(\text{lab})}^2} = \frac{c_v t_{\text{field}}}{H_{\text{dr}(\text{field})}^2}$$

or

$$\frac{t_{\text{lab}}}{H_{\text{dr}(\text{lab})}^2} = \frac{t_{\text{field}}}{H_{\text{dr}(\text{field})}^2}$$

$$\frac{140 \text{ sec}}{\left(\frac{12.5 \text{ mm}}{1000 \text{ m}} \right)^2} = \frac{t_{\text{field}}}{(3)^2}$$

$$t_{\text{field}} = 8,064,000 \text{ sec} = 93.33 \text{ days}$$

Approach II:

From Lab.

At $U=50\%$ > $T_v = 0.197$

From $T_v = C_v t / H_d^2$ > $C_v = 2.2 \times 10^{-7} \text{ m}^2/\text{S}$

In the field

$$0.197 = \frac{2.2 \times 10^{-7} * t}{(3)^2}$$

$t = 93.3 \text{ days}$

(ii)

$$T_v = \frac{\pi}{4} \left(\frac{U\%}{100} \right)^2$$

$$T_v = \frac{3.14 \times (0.3)^2}{4} = 0.071$$

$$T_v = \frac{C_v * t}{H_d^2}$$

$$0.071 = \frac{2.2 \times 10^{-7} * t}{(3)^2}$$

$t = 33.5 \text{ days}$

Example 11.13

Example 11.13

The time required for 50% consolidation of a 25-mm-thick clay layer (drained at both top and bottom) in the laboratory is 3 min 15 sec. How long (in days) will it take for a 3-m-thick clay layer of the same clay in the field under the same pressure increment to reach 50% consolidation? In the field, the clay layer is drained at the top only.

Solution

$$T_{50} = \frac{c_v t_{\text{lab}}}{H_{\text{dr(lab)}}^2} = \frac{c_v t_{\text{field}}}{H_{\text{dr(field)}}^2}$$

or

$$\begin{aligned} \frac{t_{\text{lab}}}{H_{\text{dr(lab)}}^2} &= \frac{t_{\text{field}}}{H_{\text{dr(field)}}^2} \\ \frac{195 \text{ sec}}{\left(\frac{0.025 \text{ m}}{2}\right)^2} &= \frac{t_{\text{field}}}{(3 \text{ m})^2} \\ t_{\text{field}} &= 11,232,000 \text{ sec} = 130 \text{ days} \end{aligned}$$

Example 11.14

Example 11.14

Refer to Example 11.13. How long (in days) will it take in the field for 70% primary consolidation to occur?

Solution

$$T_v = \frac{c_v t}{H_{dr}^2}$$

From Table 11.7, for $U = 50\%$, $T_v = 0.197$ and for $U = 70\%$, $T_v = 0.403$. So,

$$\frac{T_{v_{50}}}{T_{v_{70}}} = \frac{\left(\frac{c_v t_{50}}{H_{dr}^2} \right)}{\left(\frac{c_v t_{70}}{H_{dr}^2} \right)}$$

Hence,

$$\frac{0.197}{0.403} = \frac{t_{50}}{t_{70}} = \frac{130 \text{ days}}{t_{70}}; t_{70} \approx 266 \text{ days}$$

Example 11.15

Example 11.15

A 3-m-thick layer (double drainage) of saturated clay under a surcharge loading underwent 90% primary consolidation in 75 days. Find the coefficient of consolidation of clay for the pressure range.

Solution

$$T_{90} = \frac{c_v T_{90}}{H_{dr}^2}$$

Because the clay layer has two-way drainage, $H_{dr} = 3 \text{ m}/2 = 1.5 \text{ m}$. Also, $T_{90} = 0.848$ (see Table 11.7). So,

$$0.848 = \frac{c_v (75 \times 24 \times 60 \times 60)}{(1.5 \times 100)^2}$$

$$c_v = \frac{0.848 \times 2.25 \times 10^4}{75 \times 24 \times 60 \times 60} = 0.00294 \text{ cm}^2/\text{sec}$$

Example 11.16

Example 11.16

For a normally consolidated clay,

- $\sigma'_O = 200 \text{ kN/m}^2$
- $\sigma'_O + \Delta\sigma = 400 \text{ kN/m}^2$
- $e = e_O = 1.22$
- $e = 0.98$

The hydraulic conductivity, k , of the clay for the loading range is $0.61 \times 10^{-4} \text{ m/day}$. How long (in days) will it take for a 4-m-thick clay layer (drained on one side) in the field to reach 60% consolidation?

Solution

The coefficient of volume compressibility is

$$m_v = \frac{a_v}{1 + e_{av}} = \frac{\left(\frac{\Delta e}{\Delta \sigma'} \right)}{1 + e_{av}}$$

$$\Delta e = 1.22 - 0.98 = 0.24$$

$$\Delta \sigma' = 400 - 200 = 200 \text{ kN/m}^2$$

$$e_{av} = \frac{1.22 + 0.98}{2} = 1.1$$

So,

$$m_v = \frac{\frac{0.24}{200}}{1 + 1.1} = 0.00057 \text{ m}^2/\text{kN}$$

$$c_v = \frac{k}{m_v \gamma_w} = \frac{0.61 \times 10^{-4} \text{ m/day}}{(0.00057 \text{ m}^2/\text{kN})(9.81 \text{ kN/m}^3)} = 0.0109 \text{ m}^2/\text{day}$$

$$T_{60} = \frac{c_v t_{60}}{H_{dr}^2}$$

$$t_{60} = \frac{T_{60} H_{dr}^2}{c_v}$$

From Table 11.7, for $U = 60\%$, $T_{60} = 0.286$, so

$$t_{60} = \frac{(0.286)(4)^2}{0.0109} = 419.8 \text{ days}$$

Example 11.17

Example 11.17

For a laboratory consolidation test on a soil specimen (drained on both sides), the following results were obtained.

- Thickness of the clay specimen = 25 mm
- $\sigma'_1 = 50 \text{ kN/m}^2$
- $\sigma'_2 = 120 \text{ kN/m}^2$
- $e_1 = 0.92$
- $e_2 = 0.78$
- Time for 50% consolidation = 2.5 min

Determine the hydraulic conductivity of the clay for the loading range.

Solution

$$m_v = \frac{u_v}{1 + e_{av}} = \frac{\left(\frac{\Delta e}{\Delta \sigma'}\right)}{1 + e_{av}}$$

$$= \frac{\frac{0.92 - 0.78}{120 - 50}}{1 + \frac{0.92 + 0.78}{2}} = 0.00108 \text{ m}^2/\text{kN}$$

$$c_v = \frac{T_{50} H_{dr}^2}{t_{50}}$$

From Table 11.7, for $U = 50\%$, $T_v = 0.197$, so

$$c_v = \frac{(0.197) \left(\frac{0.025 \text{ m}}{2}\right)^2}{2.5 \text{ min}} = 1.23 \times 10^{-5} \text{ m}^2/\text{min}$$

$$\begin{aligned} k &= c_v m_v \gamma_w = (1.23 \times 10^{-5})(0.00108)(9.81) \\ &= 1.303 \times 10^{-7} \text{ m/min} \end{aligned}$$

Example

For a normally consolidated laboratory clay specimen drained on **both** sides, the following are given:

- $\sigma'_0 = 150 \text{ kN/m}^2$, $e_0 = 1.1$
 - $\sigma'_0 + \Delta\sigma' = 300 \text{ kN/m}^2$, $e = 0.9$
 - Thickness of clay specimen = 25 mm
 - Time for 50% consolidation = 2 min
-
- i. For the clay specimen and the given loading range, determine the hydraulic conductivity (also called coefficient of permeability, **k**) estimated in: m/min.
 - ii. How long (in days) will it take for a **3 m** clay layer in the field (drained on **one** side) to reach **60%** consolidation?

Solution

- i. The hydraulic conductivity (coefficient of permeability, k) can be calculated from:

$$c_v = \frac{k}{\gamma_w m_v} = \frac{k}{\gamma_w \left(\frac{a_v}{1 + e_o} \right)} \longrightarrow k = c_v m_v \gamma_w$$

$$m_v = \frac{\Delta e}{\Delta \sigma' (1 + e_o)}$$

$$m_v = \Delta e / (1 + e_o) / \Delta \sigma' = \mathbf{0.00063} \quad \text{m}^2/\text{kN}$$

for $U=50\%$, T_v can be calculated from:

$$T_v = \frac{\pi}{4} \left(\frac{U\%}{100} \right)^2 \longrightarrow T_{50} \approx \dots \mathbf{0.197}$$

$$T_v = \frac{c_v t}{H_{dr}^2} \longrightarrow c_v = H_{dr}^2 \cdot T_v / t = (\mathbf{0.0125})^2 \times \mathbf{0.197} / 2 = \mathbf{0.000015} \quad \text{m}^2/\text{min}$$

$$k = c_v m_v \gamma_w = 0.000015 \times 0.00063 \times 9.81 = 9.27 \times 10^{-8} \text{ m/min}$$

Solution

ii. Time factor relation with time:

$$T_{60} = \frac{c_v t_{60}}{H_{dr}^2} \longrightarrow t_{60} = \frac{T_{60} H_{dr}^2}{c_v}$$

Because the clay layer has one-way drainage, $H_{dr} = 3$ m

for $U=60\%$, T_{60} can be calculated from:

$$T_v = \frac{\pi}{4} \left(\frac{U\%}{100} \right)^2 \longrightarrow T_{60} \approx 0.285$$

$$t_{60} = H_{dr}^2 \cdot T_v / c_v = (3)^2 \times 0.286 / (0.000015) \dots = 171600 \text{ min} \\ = 119.16 \text{ days}$$

Determination of coefficient of consolidation (C_v)

- In the calculation of time rate of settlement, the coefficient of consolidation C_v is required.

$$T_v = \frac{C_v t}{H_{dr}^2}$$

- C_v is determined from the results of **one-dimensional consolidation test**.
- For a given load increment on a specimen, two graphical methods are commonly used for determining C_v from laboratory one-dimensional consolidation tests.
 - **Logarithm-of-time method** - by Casagrande and Fadum (1940),
 - **Square-root-of-time method** - by Taylor (1942).
- The procedure involves plotting thickness changes (i.e. settlement) against a suitable function of time (either **log(time)** or **√time**) and then fitting to this the **theoretical T_v : U_t** curve.
- The procedure for determining C_v allows us to separate the **SECONDARY COMPRESSION** from the **PRIMARY CONSOLIDATION**.
- The procedures are based on the similarity between the shapes of the **theoretical** and **experimental** curves when plotted versus the square root of T_v and t .

Logarithm-of-time method (Casagrande's method)

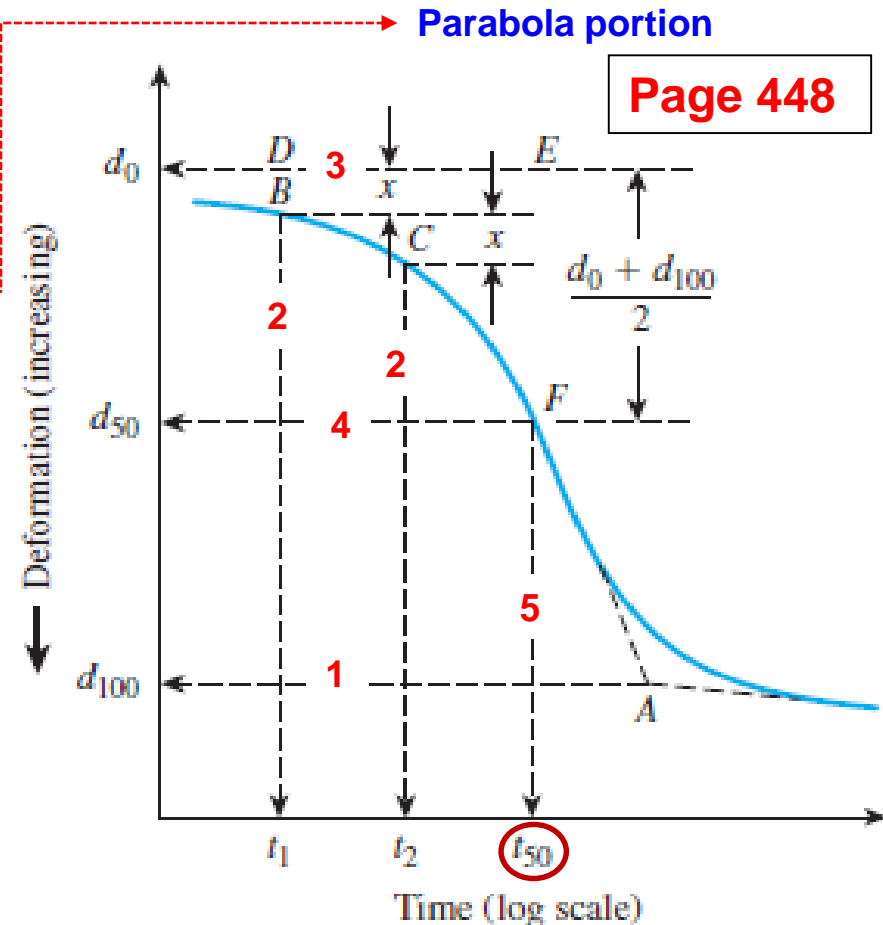
1. Extend the straight line portion of primary and secondary consolidation curve to intersect at A. A is d_{100} , the deformation at the end of consolidation
2. Select times t_1 and t_2 on the curve such that $t_2 = 4t_1$. Let the difference is equal to x
3. Draw a horizontal line (DE) such that the vertical distance BD is equal to x . The deformation of DE is equal to d_0 .
4. The ordinate of point F represents the deformation at 50% primary consolidation, and its abscissa represents t_{50}

$$c_v = \frac{0.197 H_{dr}^2}{t_{50}}$$

Note: This is only for the case of constant or linear u_0 .

where H_{dr} = average longest drainage path during consolidation.

For specimens drained at both top and bottom, H_{dr} equals one-half the average height of the specimen during consolidation. For specimens drained on only one side, H_{dr} equals the average height of the specimen during consolidation.



EXAMPLE 11.19

Example 11.19

During a laboratory consolidation test, the time and dial gauge readings obtained from an increase of pressure on the specimen from 50 kN/m² to 100 kN/m² are given here.

Time (min)	Dial gauge reading (cm × 10 ⁴)	Time (min)	Dial gauge reading (cm × 10 ⁴)
0	3975	16.0	4572
0.1	4082	30.0	4737
0.25	4102	60.0	4923
0.5	4128	120.0	5080
1.0	4166	240.0	5207
2.0	4224	480.0	5283
4.0	4298	960.0	5334
8.0	4420	1440.0	5364

Using the logarithm-of-time method, determine c_v . The average height of the specimen during consolidation was 2.24 cm, and it was drained at the top and bottom.

Solution

The semilogarithmic plot of dial reading versus time is shown in Figure 11.38. For this, $t_1 = 0.1$ min, $t_2 = 0.4$ min have been used to determine d_0 . Following the procedure outlined in Figure 11.34, $t_{50} \approx 19$ min. From Eq. (11.72),

$$c_v = \frac{0.197 H_{dr}^2}{t_{50}} = \frac{0.197 \left(\frac{2.24}{2} \right)^2}{19} = 0.013 \text{ cm}^2/\text{min} = 2.17 \times 10^{-4} \text{ cm}^2/\text{sec}$$

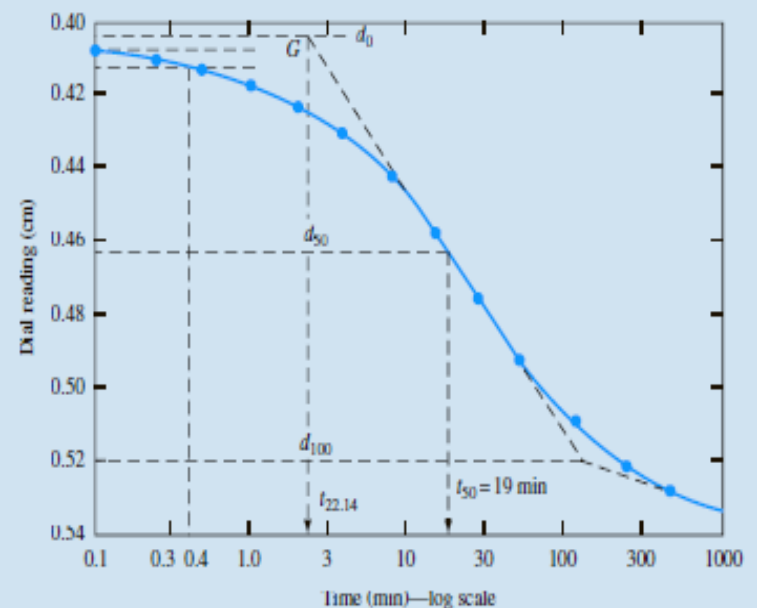


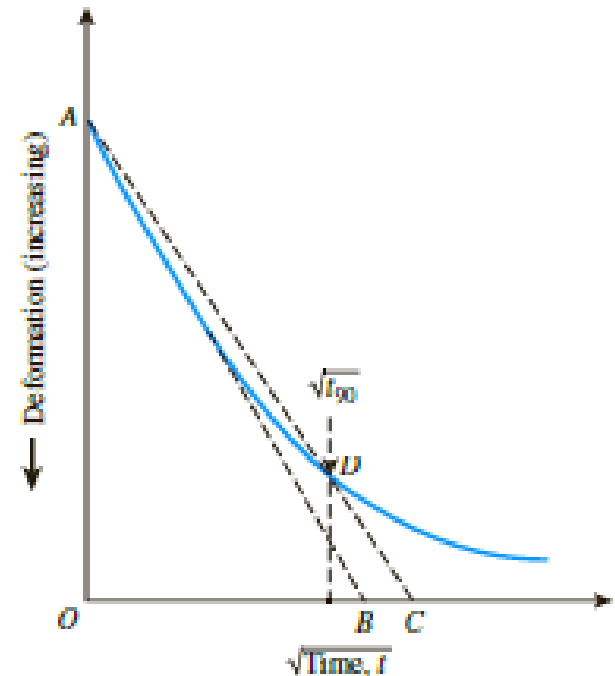
Figure 11.38

Square-root-of-time method (Taylor's method)

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1. Draw the line **AB** through the early portion of the curve
2. Draw the line **AC** such that **OC = 1.15 AB**. Find the point of intersection of line **AC** with the curve (point **D**).
3. The abscissa of **D** gives the square root of time for **90%** consolidation.
4. The coefficient of consolidation is therefore:

$$C_v = \frac{T_{90} H_{dr}^2}{t_{90}} = \frac{0.848 H_{dr}^2}{t_{90}}$$



Remarks

- ❑ For samples drained at top and bottom, H_d equals **one-half** of the AVERGAE height of sample during consolidation. For samples drained only on one side, H_d equals the average height of sample during consolidation.
- ❑ The curves of actual deformation dial readings versus real time for a given load increment often have very similar shapes to the **theoretical U-T_v** curves.
- ❑ We take advantage of this observation to determine the C_v by so-called “curve fitting methods” developed by Casagrande and Taylor.
- ❑ These empirical procedures were developed to fit approximately the observed laboratory test data to the Terzaghi’s theory of consolidation.
- ❑ Often C_v as obtained by the **square time** method is slightly **greater** than C_v by the **log t** fitting method.
- ❑ C_v is determined for a **specific** load increment. It is different from load increment to another.
- ❑ Taylor’s method is more useful primarily when the **100** percent consolidation point cannot be estimated from a semi-logarithmic plot of the laboratory time-settlement data.

EXAMPLE 11.22

Example 11.22

Calculate the settlement of the 10-ft-thick clay layer (Figure 11.40) that will result from the load carried by a 5-ft-square foundation. The clay is normally consolidated. Use the weighted average method [Eq. (11.76)] to calculate the average increase of effective pressure in the clay layer.

Solution

For normally consolidated clay, from Eq. (11.35),

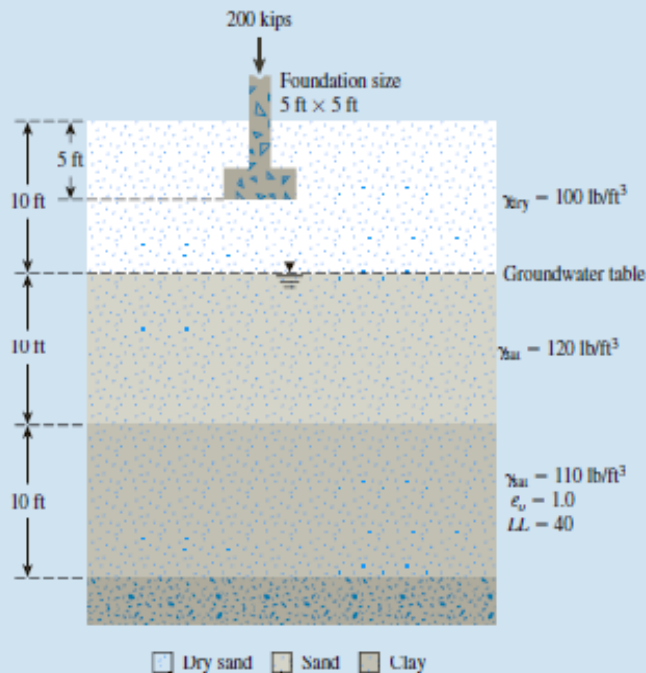


Figure 11.40

$$S_c = \frac{C_c H}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o}$$

where

$$C_c = 0.009(LL - 10) = 0.009(40 - 10) = 0.27$$

$$H = 10 \times 12 = 120 \text{ in.}$$

$$e_o = 1.0$$

$$\begin{aligned} \sigma'_o &= 10 \text{ ft} \times \gamma_{\text{dry(sand)}} + 10 \text{ ft} [\gamma_{\text{sat(sand)}} - 62.4] + \frac{10}{2} [\gamma_{\text{sat(clay)}} - 62.4] \\ &= 10 \times 100 + 10(120 - 62.4) + 5(110 - 62.4) \\ &= 1814 \text{ lb/ft}^2 \end{aligned}$$

From Eq. (11.76),

$$\Delta\sigma'_{av} = \frac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6}$$

$\Delta\sigma'_t$, $\Delta\sigma'_m$, and $\Delta\sigma'_b$ below the center of the foundation can be obtained from Eq. (10.36).

Now we can prepare the following table (Note: $L/B = 5/5 = 1$):

m_1	z (ft)	$b = B/2$ (ft)	$n_1 = z/b$	q (kip/ft ²)	I_4	$\Delta\sigma' = qI_4$ (kip/ft ²)
1	15	2.5	6	$\frac{200}{5 \times 5} = 8$	0.051	$0.408 = \Delta\sigma'_t$
1	20	2.5	8	8	0.029	$0.232 = \Delta\sigma'_m$
1	25	2.5	10	8	0.019	$0.152 = \Delta\sigma'_b$

So,

$$\Delta\sigma'_{av} = \frac{0.408 + (4)(0.232) + 0.152}{6} = 0.248 \text{ kip/ft}^2 = 248 \text{ lb/ft}^2$$

Hence,

$$S_c = \frac{(0.27)(120)}{1 + 1} \log \frac{1814 + 248}{1814} \approx 0.9 \text{ in.}$$

The end