Unit 11: Exponents and Polynomials

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Lesson 1: Integer Exponents

Topic 1: Exponential Notation

Learning Objectives

- Evaluate expressions containing exponents.
- Evaluate exponential notations with exponents of 0 and 1.
- Write an exponential expression involving negative exponents with positive exponents.

Topic 2: Simplify by using the Product, Quotient and Power Rules

Learning Objectives

- Use the product rule to multiply exponential expressions with like bases.
- Use the power rule to raise powers to powers.
- Use the quotient rule to divide exponential expressions with like bases.
- Simplify expressions using a combination of the properties.

Topic 3: Products and Quotients Raised to Powers

Learning Objectives

- Raise a product to a power.
- Raise a quotient to a power.
- Simplify expressions using a combination of the properties.

Topic 4: Scientific Notation

Learning Objectives

- Convert between scientific and decimal notation.
- Multiply and divide using scientific notation.
- Solve application problems.

Lesson 2: Polynomials with Single Variables

Topic 1: Introduction to Single Variable Polynomials

Learning Objectives

- Identify the terms, the coefficients and the exponents of a polynomial.
- Evaluate a polynomial for given values of the variable.
- Simplify polynomials by collecting like terms.

Topic 2: Adding and Subtracting Polynomials

Learning Objectives

- Add polynomials.
- Find the opposite of a polynomial.
- Subtract polynomials.

Topic 3: Multiplying Polynomials

Learning Objectives

- Multiply monomials.
- Multiply monomials times polynomials.
- Multiply two binomials.
- Multiply any two polynomials.

Topic 4: Multiplying Special Cases

Learning Objectives

- Square a binomial.
- Multiply the sum and difference of the same two terms.

Topic 5: Dividing by a Monomial

Learning Objectives

- Divide a monomial by a monomial.
- Divide a polynomial by a monomial.

Topic 6: Dividing by Binomials and Polynomials

Learning Objectives

- Divide a polynomial by a binomial.
- Divide a polynomial by another polynomial.

Lesson 3: Polynomials with Several Variables

Topic 1: Simplifying and Evaluating Polynomials with More Than One Term

Learning Objectives

- Evaluate a polynomial for given values of each variable.
- Simplify polynomials by collecting like terms.

Topic 2: Operations with Polynomials

Learning Objectives

- Add polynomials with more than one variable.
- Subtract polynomials with more than one variable.
- Multiply polynomials with more than one variable.
- Divide polynomials with more than one variable.

Unit 11: Exponents and Polynomials

Instructor Notes

The Mathematics of Exponents and Polynomials

Exponents are explored in-depth in this unit, from positive, negative and zero exponents to the product, quotient, and power rules. In addition, scientific notation is explained and illustrated with real-life applications.

Students also learn how to work with polynomials, regardless of how many terms (and variables) they contain. Basic terminology, and the addition, subtraction, multiplication, and division of polynomials are covered, as are techniques for simplifying and evaluating polynomials.

When this unit is completed, students will have gained the ability to work fluently with exponents and polynomials—critical skills for those who move on to higher-level mathematics classes.

Teaching Tips: Challenges and Approaches

Exponents are not new to students—they should be familiar with what an exponent is and understand that it represents repeated multiplication. However, polynomials are a new idea. The combination of exponents and polynomials is a lot for students to absorb and there are many stumbling blocks.

Common Mistakes

Exponent and polynomial problems are dangerous grounds even for students who understand the basic ideas and techniques. When problems involve so many steps, terms, and rules, it's easy to overlook a variable, misplace a decimal, or forget parentheses. These bookkeeping errors are especially frustrating for students—they work hard to understand the concepts, only to find they still can't seem to get the right answers. That can give them distaste for algebra that's hard to overcome. Emphasize to the students to be especially mindful of the following:

- Signs. Students often lose track of negative signs when applying the distributive property or when solving a subtraction problem by adding the opposite. They should double check these manipulations before proceeding.

- Powers. When they first start working with exponents, many students will have difficulty deciding just which parts of a term the power applies to. They'll think $5x^3$ means $5^3 \bullet x^3$

or conversely that $(5x)^3$ equals $5 \bullet x^3$. Give them practice problems until the rules are ingrained before moving on.

- Like terms. Like terms must contain the same exact variables raised to the same exact power. Students will often think it's enough if just the variables are the same. Or they won't realize that order doesn't matter, and monomials such as $4x^2y$ and $-5yx^2$ are like terms. Explain carefully what constitutes a like term with each major topic.

Show your Work

Students are required to learn a lot of rules in this unit. They'll understand them more easily and remember them better if they see how these rules are derived.

For example, the idea that any non-zero number or variable raised to the 0 power equals 1 is counter-intuitive—most students will assume it has to be 0 because any number multiplied by 0 is 0.

The best thing you can do is to show them something similar to this:

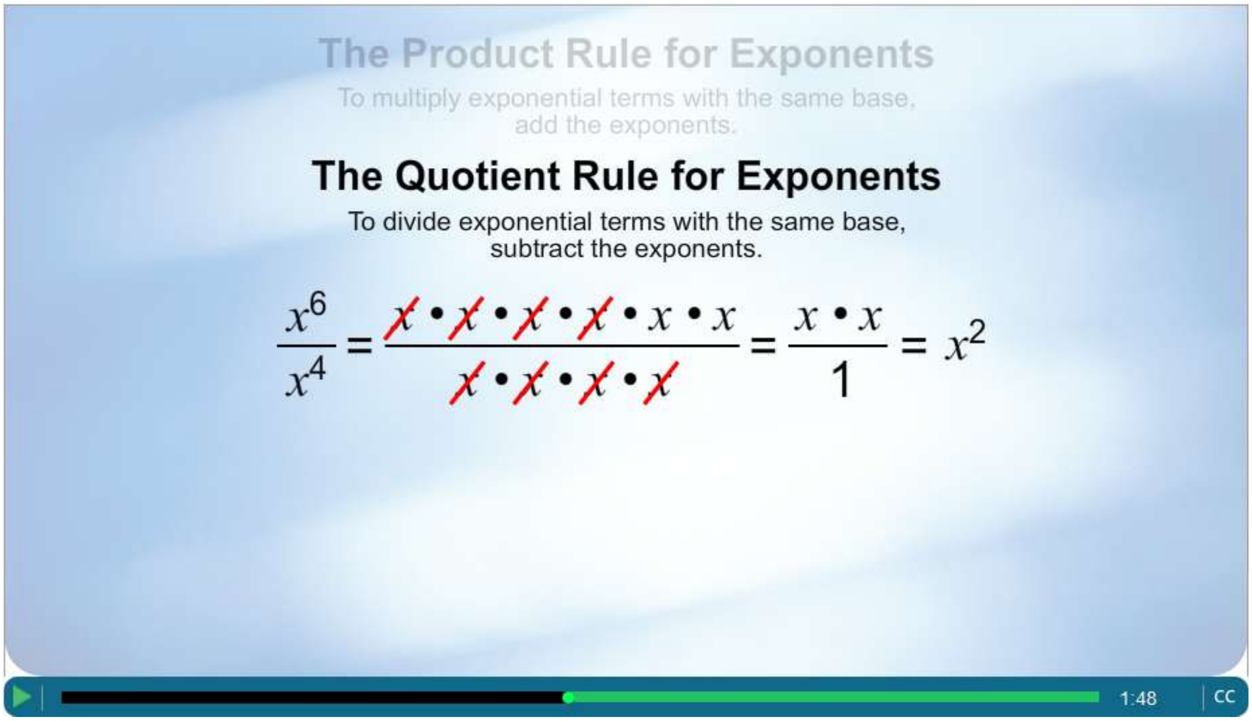
Exponential Form	Expanded Form	Value
10^5	$10 \bullet 10 \bullet 10 \bullet 10 \bullet 10$	100,000
10^4	$10 \bullet 10 \bullet 10 \bullet 10$	10,000
10^3	$10 \bullet 10 \bullet 10$	1,000
10^2	$10 \bullet 10$	100
10^1	10	10
10^0	1	1
10^{-1}	$\frac{1}{10^1}$	$\frac{1}{10}$
10^{-2}	$\frac{1}{10^2}$	$\frac{1}{100}$

[From Lesson 1, Topic 1, Topic Text]

Start by writing down the chart above from 10^5 to 10^1. Point out that as the exponent decreases by 1, the expanded form and the value are divided by 10. Ask what 10^0 must be based on the ongoing pattern. After a little thought, students should see that it equals 10 divided by 10, which is 1, not 0.

Once this is understood, continue on to introduce the concept of negative exponents—another stumbling block for students. They may think that a negative exponent means that the answer has to be negative. This table should help them see that it is not the case.

In the same way, demonstrate why quotient, product, and power rules work (and make calculations much easier!) by writing out some problems the long way:



[From Lesson 1, Topic 2, Presentation]

This teaches students two important lessons: they can simplify an expression with exponents much faster and more accurately using the rules, and if they forget the rules, they can still get the right answer by taking the long way.

With more advanced or intuitive students, you may want to give them several similar problems to work and see if they can recognize a pattern and figure out the rules themselves. For example, after introducing the multiplication of two binomials, have them try a number of problems like $(x + 5)(x – 5)$. Then ask your students to come up with a general rule for multiplying two binomials that differ only in the operation.

Scientific Notation

Scientific notation may be new to students, but this is an important topic for them to understand if they are going to study higher levels of mathematics or science. It's also a rare opportunity in this unit to bring in real-world situations to show the usefulness of exponents:

[From Lesson 1, Topic 4, Presentation]

A chart like the one that follows and some individual and group practice will quickly accustom students to recognizing numbers that are in scientific notation format.

Number	Scientific Notation?	Explanation
1.85×10^{-2}	yes	$1 \le 1.85 < 10$ -2 is an integer
$1.083 \times 10^{\frac{1}{2}}$	no	$\frac{1}{2}$ is not an integer
0.82×10^{14}	no	0.82 is not ≥ 1
10×10^{3}	no	10 is not < 10

[From Lesson 1, Topic 4, Presentation]

As students begin converting numbers in and out of scientific notation, encourage them to draw loops or marks on the paper to help them track the movement of the decimal point. Later, it's also useful to go over how a calculator gives an answer in scientific notation—students may overlook the 'e' in the display and think the co-efficient is the complete answer.

While operations on these numbers are simple for students once they understand the rules of exponents, at first they may not realize that their answers may no longer be in scientific notation and might have to be rewritten.

Operations on Polynomials

Evaluating polynomials can get complicated quickly, but there are several problem-solving techniques that make the task easier and more accurate.

Addition and Subtraction

Point out to students that addition and subtraction of polynomials is very similar to what they learned for simplifying expressions. Use the commutative and associative properties to regroup the parts of a polynomial and then combine like terms. Encourage them to change subtraction, including subtraction of a second polynomial, to addition of the opposite. You may wish to illustrate both the across and the column method of addition (and subtraction if they insist). Students will vary in which method they prefer.

Multiplication

Multiplication of polynomials is a little more difficult, especially when the polynomials that are being multiplied are not monomials.

FOIL – first, outer, inner, last – is a traditional method that most students learn to multiply two binomials together. While there is nothing inherently wrong with this approach, it does have limitations. This mnemonic only works when multiplying two binomials, and when students who have grown comfortable with it are confronted by a more challenging situation like $(x + y + 2)(3 – 2x)$, they'll often struggle. As a result, it may be more productive to teach a more general rule from the beginning, such as "When multiplying two polynomials, multiply everything in the first set of parentheses to everything in the second set of parentheses".

If students struggle to keep track of all of the terms, suggest they multiply vertically by putting one polynomial on top of the other. This tends to line up the like terms and makes it easy to get the final answer, as illustrated below:

Example					
Problem	**$(3x + 6)(5x^2 + 3x + 10)$**				
			$3x$	**$+ 6$**	Set up the problem in a vertical form, and begin by multiplying $3x + 6$ by $+ 10$. Place the products underneath, as shown.
	x	$5x^2$	$+ 3x$	$+ 10$	
			$+ 30x$	**$+ 60$**	
			$3x$	**$+ 6$**	Now multiply $3x + 6$ by $+ 3x$. Notice that $(6)(3x) = 18x$; since this term is like $30x$, place it directly beneath it.
	x	$5x^2$	$+ 3x$	$+ 10$	
			$+ 30x$	$+ 60$	
		$+ 9x^2$	**$+ 18x$**		
			$3x$	**$+ 6$**	Finally, multiply $3x + 6$ by $5x^2$. Notice that $30x^2$ is placed underneath $9x^2$.
	x	$5x^2$	$+ 3x$	$+ 10$	
			$+ 30x$	$+ 60$	
		$+ 9x^2$	$+ 18x$		
	$+ 15x^3$	**$+ 30x^2$**			
			$3x$	$+ 6$	Now add like terms.
	x	$5x^2$	$+ 3x$	$+ 10$	
			$+ 30x$	$+ 60$	
		$+ 9x^2$	$+ 18x$		
	$+ 15x^3$	$+ 30x^2$			
	$+ 15x^3$	$+ 39x^2$	$+ 48x$	$+ 60$	
Answer	$15x^3 + 39x^2 + 48x + 60$				

[From Lesson 2, Topic 3, Topic Text]

(Note: It's easier to put the polynomial with the most terms on top when using the vertical method.)

Division

Intermediate students will need to divide polynomials by other polynomials. This is probably the most difficult part of this unit. Students will invent many creative ways to get answers, but unfortunately most of them will be incorrect. Tell them that division of polynomials by other polynomials is essentially the same long division that they already know how to do. Then show them this is true by working a long division problem with numbers only side by side and step by step with a trinomial divided by a binomial problem.

One common error will be that your students will forget to subtract entire expressions—they will subtract just the first term. Another problem is that sometimes students don't know when to stop. Tell them that it is okay to stop when the degree of the remainder is less than the degree of the divisor.

Because there are so many places to go wrong, be especially insistent that students always check their answers for any long division problem involving polynomials.

Keep in Mind

This unit covers both beginning and intermediate algebra topics. More difficult examples and problems are included for intermediate learners, but in most cases these can be also used to challenge beginners. However, material on dividing polynomials by polynomials and on polynomials with several variables are best left to intermediate students only.

Additional Resources

In all mathematics, the best way to really learn new skills and ideas is repetition. Problem solving is woven into every aspect of this course—each topic includes warm-up, practice, and review problems for students to solve on their own. The presentations, worked examples, and topic texts demonstrate how to tackle even more problems. But practice makes perfect, and some students will benefit from additional work.

A good site for reviewing the rules of exponents is http://www.ltcconline.net/greenl/java/BasicAlgebra/ExponentRules/ExponentRules.html. Practice with converting between scientific and decimal notation as well as performing operations using scientific notation can be found at http://janus.astro.umd.edu/cgi-bin/astro/scinote.pl.

Practice with multiplying polynomials can be found at http://www.mathsnet.net/algebra/e12.html (you can get additional problems on this site by clicking on “more on this topic”).

Summary

After completing this unit, students will be able to simplify exponential expressions by using exponent rules, and convert numbers in and out of scientific notation. They will know how to add, subtract, multiply and divide polynomials. This unit sets the stage for the further study of algebra.

Unit 11: Exponents and Polynomials

Instructor Overview
Tutor Simulation: How Big Is Big?

Purpose

This simulation allows students to demonstrate their ability use exponents and polynomials. Students will be asked to apply what they have learned to solve a problem involving:

- Solving formulas
- Writing and modifying formulas
- Using exponents
- Simplifying and solving polynomials
- Adding and subtracting polynomials
- Multiplying and dividing polynomials
- Using scientific notation

Problem

Students are presented with the following problem:

> How big is big? You will be analyzing three-dimensional shapes by exploring what happens when their dimensions change. You will use your knowledge of exponents and polynomials to look at a few different shapes and see how their volume changes as their sides or diameters change.

Recommendations

Tutor simulations are designed to give students a chance to assess their understanding of unit material in a personal, risk-free situation. Before directing students to the simulation,

- Make sure they have completed all other unit material.
- Explain the mechanics of tutor simulations.
    - Students will be given a problem and then guided through its solution by a video tutor;
    - After each answer is chosen, students should wait for tutor feedback before continuing;
    - After the simulation is completed, students will be given an assessment of their efforts. If areas of concern are found, the students should review unit materials or seek help from their instructor.
- Emphasize that this is an exploration, not an exam.

Developmental Math – An Open Curriculum
Instructor Guide

Unit 11 – Puzzle

Unit 11: Exponents and Polynomials

Instructor Overview
Puzzle: Polynomial Poke

Objectives

Polynomial Poke challenges students' familiarity with polynomial nomenclature. To play the game successfully, they must be able to distinguish between cubic, quadratic, and linear terms, and recognize monomials, binomials, and trinomials.

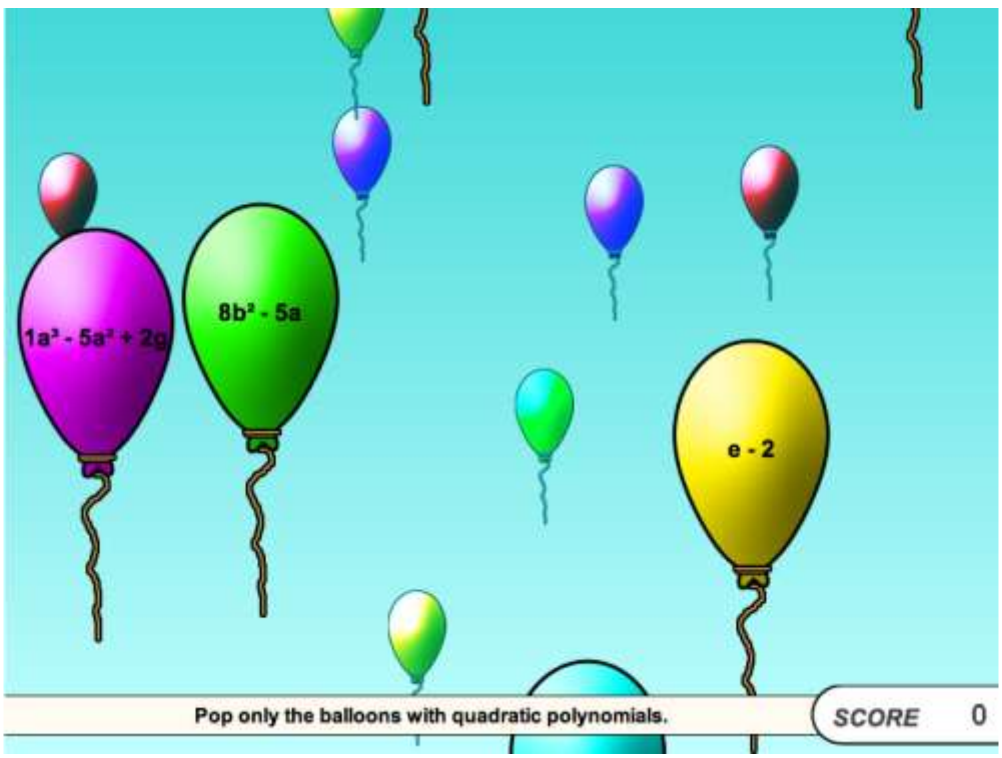


Figure 1. *Polynomial Poke* asks players to pop balloons that contain specified types of polynomials.

Description

There are three levels in this puzzle, which each consisting of 10 groups of floating balloons containing polynomials. In the first level, learners are challenged to pop balloons in order of degree of monomials, from cubic to quadratic to linear. In the second level, players must pop balloons depending on the number of terms in their polynomials. In the third level, players are asked to pop only those balloons that contain a specified degree of polynomial. Players earn points for correct answers, and lose points for popping balloons out of sequence.

The puzzle is designed for the single player, but it could be played in a classroom with students identifying the order or the degree and calling out which balloon to pop.

Unit 11: Exponents and Polynomials

Instructor Overview
Project: Looking for Patterns

Student Instructions

Introduction

The main business of science is to uncover patterns in nature. Mathematics is a tool that is of great importance to scientists as they complete this task.

Task

In this project you play the part of an amateur scientist seeking to uncover patterns in various physical situations. You will collect and analyze some data using your knowledge of scientific notation and arithmetic involving monomials and binomials.

Instructions

Work with at least one other person to complete the following exercises. Solve each problem in order and save your work along the way, as you will create a presentation on one of the four parts to be given to your class.

- **First Problem – Data Collection:** In this experiment, you will examine the relationship between the length of a pendulum arm and the amount of time it takes for the pendulum to swing back and forth. To complete this, you will need some string, tape, a stopwatch or some other timing device, and a large metal washer or heavy ring. *[Note: We did not own a stopwatch, so we used the online stopwatch at http://www.online-stopwatch.com.]*

    - Tie the washer or ring onto one end of the string and tape the top of the string to a wall. Then, choose an initial angle from which to let the ring go and mark that with an "X". See the Figure 1 below.

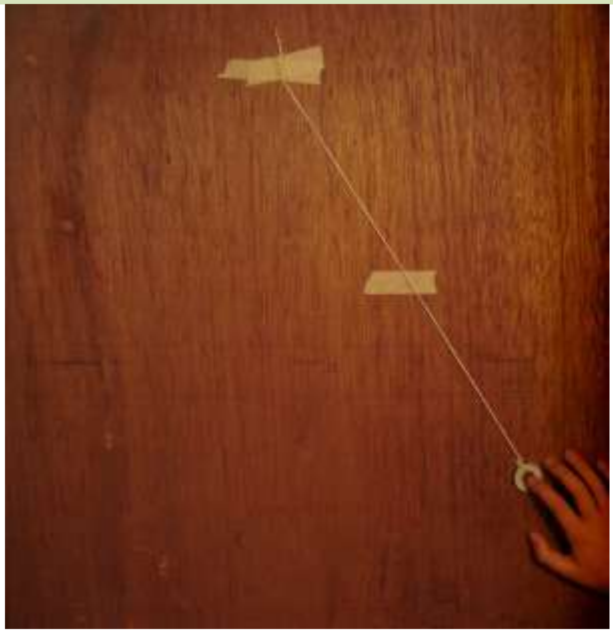


Figure 1: *This figure shows the string taped to the wall at the top with a washer attached to the end. The other piece of tape has a red X on it marking the angle at which we will release the pendulum each time. It should be high enough so that it will work for the short, medium, and long pieces of string.*

- You will vary the length of string in this experiment, and for each length, you will measure the *period*, or the length of time it takes for the pendulum to complete one back and forth motion. If you try this once, you will notice that it is difficult to measure one period since the pendulum moves so quickly. To help us, we will time how long it takes for the pendulum to complete *five* (5) cycles and then divide this total time by five to get the period.

- Choose a "long" length of string (for the next two parts, you will cut this string to make it shorter), and write that length in the space provided above the charge below. Measure the period 12 times, cross out the high and the low and calculate the average of the numbers that are left.

Trial #1: LONG STRING

Length of String = ___________ cm *Average Period* = ___________

seconds

Trial No.	1	2	3	4	5	6	7	8	9	10	11	12
Time of 1 Period												

- Cut your string so that it is now a "medium" length, and write that length in the space provided above the charge below. Measure the period 12 times, cross out the high and the low and calculate the average of the numbers that are left.

Trial #2: MEDIUM STRING

Length of String = ____________ cm

Average Period = ___________ seconds

Trial No.	1	2	3	4	5	6	7	8	9	10	11	12
Time of 1 Period												

- Finally, cut the string so that it is "short," and write that length in the space provided above the charge below. Measure the period 12 times, cross out the high and the low and calculate the average of the numbers that are left.

Trial #3: SHORT STRING

Length of String = ____________ cm

Average Period = ___________ seconds

Trial No.	1	2	3	4	5	6	7	8	9	10	11	12
Time of 1 Period												

- **Second Problem – Finding the Pattern:** We will now try to find a pattern in this data.
  - As with many events in physics, the dividing two quantities in this case should give a constant. Complete the chart below to determine the pattern in this case. Circle the relationship in the top row of the table that shows the pattern. Round your answers to

the nearest hundredths.

	Period (in seconds)	*Length of String (in cm)*	$\frac{\text{Length}}{\text{Period}}$	$\frac{\sqrt{\text{Length}}}{\text{Period}}$	$\frac{\text{Length}^2}{\text{Period}}$
Trial #1					
Trial #2					
Trial #3					

- There are many other situations in which this type of pattern arises. One of the most famous comes from an astronomer named Johannes Kepler. He carefully studied astronomical data collected without the aid of a telescope by Tyco Brahe and noticed several patterns, which are today called "Kepler's Three Laws of Planetary Motion."

Johannes Kepler

1571-1630

Tyco Brahe

1546-1601

The table below records data on the period of some planets in our solar system as well as their average distance from the center of the Sun. Convert each of the numbers into scientific notation. Since the numbers are large, this will make our calculations easier. Remember to round each of your numbers to three significant digits (that is, round a number such as 5.376429×10^{23} to 5.38×10^{23}).

	Period (in days)	*Period in Scientific Notation*	*Distance from Sun (in meters)*	*Distance from Sun in Scientific Notation*
Mercury	88.00		58,000,000,000	
Venus	224.70		108,000,000,000	

Earth	365.25		149,000,000,000	
Jupiter	4331.86		778,000,000,000	
Neptune	60,193.20		4,490,000,000,000	

- Finally, complete the chart below to determine the pattern in this case. Circle the relationship in the top row of the table that shows the pattern.

	Period (in days)	***Distance from Sun (in meters)***	$\frac{\text{Period}^2}{\text{Distance}}$	$\frac{\text{Period}}{\text{Distance}^3}$	$\frac{\text{Period}^2}{\text{Distance}^3}$
Mercury					
Venus					
Earth					
Jupiter					
Neptune					

- **Third Problem – Finding Units:** One very important task of the scientist is to keep track of units. This helps catch arithmetic errors and gives the proper meaning to different quantities. For example, if we were to calculate the speed of an object, we could divide the distance traveled (in miles) by the time elapsed (in hours) to get $\frac{\text{distance}}{\text{time}} \to \frac{\text{miles}}{\text{hour}}$, or simply "miles per hour."

  Here, we ask you to calculate the units of the constants you computed above as well as some other famous constants from science. Do not be thrown off that some of these constants have long names or that the units are unfamiliar to you. In the end, you are just performing algebra with letters that have exponents on them!

  - Use this method to determine the units on the constant you computed for the pendulum.

  - Use this method to determine the units on the constant you computed Kepler's Law. Use *d* to stand for "days" and *m* to stand for "meters."

- Finally, we will introduce you to some other famous constants and ask you to determine the units on them:
  - In the study of fluid flow through a tube (such as blood flow through an artery), Poisueille's Law describes the relationship between the flow rate Q of a fluid (with units $m^3 \cdot s^{-1}$), the radius R of the tube (with units m), the length of the tube (with units m), and the pressure P of the fluid in the tube (with units $N \cdot m^{-2}$). This law states that the following quantity will be constant: $\frac{P \cdot R^4}{L \cdot Q}$. Calculate the units on this constant (which is known as *Dynamic Viscosity*).

  - In geology, Darcy's Law describes the relationship between the flow rate Q of fluid through porous rock (with units $cm^3 \cdot s^{-1}$), the cross-sectional area of the rock A (with units m^2), the pressure on the fluid P (with units $N \cdot m^{-2}$), the length that the fluid must flow L (with units m), and the thickness (viscosity) of the fluid u (with units $N \cdot s \cdot m^{-2}$). This law states that the following quantity will be constant: $\frac{Q \cdot u \cdot L}{A \cdot P}$. Calculate the units on this constant (which is known as the *Permeability of the Rock*).

  - In chemistry, the Ideal Gas Law describes a gas in a closed container with pressure P (with units $N \cdot m^{-2}$), volume V (with units m^3), amount of gas n (with units mol), and temperature T (with units K). This law states that the following quantity will be constant for any gas in a closed container: $\frac{P \cdot V}{n \cdot T}$. Calculate the units on this constant (which is known as the *Ideal Gas Constant*). *[Hint: even though the unit* mol *has 3 letters in it, you treat it just as if it were one whole letter. You need not know the meaning of all of these units to do the calculation, but just FYI, N is a unit of force called a Newton and K is a unit of temperature called a Kelvin, and mol measures how many molecules of gas are present.]*

- **Fourth Problem – Long-Term Patterns:** One very different kind of pattern involves two polynomials that are divided. Such expressions are difficult to deal with and so many scientists prefer to approximate them with simpler expressions, especially for large values of *x*. Below are three such expressions. For each: (1) use long division to rewrite the expression as $\text{quotient} + \frac{\text{remainder}}{\text{divisor}}$, (2) complete the chart, and (3) write an expression which approximates the original for large values of *x*. *[Hint: To help you see the patterns, we suggest that you keep as many decimal places as possible in your answers.]*

Expression #1

Expression: $\frac{5x^2+15x+11}{x^2+3x+2}$		**Rewritten:** ____________ + ____________	
Value of x	Value of expression	Value of quotient	Value of $\frac{\text{remainder}}{\text{divisor}}$
10			
20			
50			
100			
1,000			

Expression #2

Expression: $\frac{5x^3 + 15x^2 + 10x + 1}{x^2 + 3x + 2}$		**Rewritten:** __________ + __________	
Value of x	Value of expression	Value of quotient	Value of $\frac{\text{remainder}}{\text{divisor}}$
10			
20			
50			
100			
1,000			

Expression #3

Expression: $\frac{5x^4+15x^3+10x^2+1}{x^2+3x+2}$		**Rewritten:** ______ + ______	
Value of x	Value of expression	Value of quotient	Value of $\frac{\text{remainder}}{\text{divisor}}$
10			
20			
50			
100			
1,000			

Collaboration

Get together with another group to compare your answers to each of the four problems. Discuss any differences in your answers and come to conclusions everyone agrees upon.

Conclusions

Lastly, prepare your final presentation for the part assigned to you by your instructor. Be sure to clearly explain your reasoning at each stage. Then, present your solution to the class.

Instructor Notes

Assignment Procedures

This project contains several different types of problems in order to give students practice in simplifying monomials, using scientific notation, and dividing by monomials and binomials. However, the third and fourth problems do not depend on the first two problems or on one another. Therefore, this project can be easily tailored by assigning only those problems corresponding to those skills you would like to reinforce from the section of material.

Problem 1

We would like to note a few points of physics.

1. The length of time it takes for the pendulum to complete one back and forth motion does not depend on the initial angle at which the pendulum begins (or the mass on the end of the pendulum), but for simplicity we have students begin the pendulum at the same angle each time. It would be an interesting extension of the activity if you were to ask them to also change the initial angle of the pendulum and re-examine the data to see if this makes a difference.
2. The relationship we are having the students explore here is valid only for "small" angles. What constitutes "small" can depend on many factors, but we have had good success as long as the initial angle has been smaller than 45 degrees.

Answers will of course vary but our data is given below.

Trial #1: LONG STRING

Length of String = ___58.5_____ cm

Average Period = ____7.5____ seconds

Trial No.	1	2	3	4	5	6	7	8	9	10	11	12
Time of 1 Period	7.3	7.6	7.5	~~7.7~~	7.6	7.5	~~7.2~~	7.5	7.4	7.6	7.3	7.6

Trial #2: MEDIUM STRING

Length of String = ___31.0_____ cm

Average Period = ___5.7_____ seconds

Trial No.	1	2	3	4	5	6	7	8	9	10	11	12
Time of 1 Period	5.9	~~6.1~~	5.8	5.8	5.6	5.9	5.3	5.3	5.9	~~5.2~~	5.9	5.9

Trial #3: SHORT STRING

Length of String = ___14.0______ cm

Average Period = _____4.0______ seconds

Trial No.	1	2	3	4	5	6	7	8	9	10	11	12
Time of 1 Period	~~4.4~~	4.2	3.6	4.3	4.3	~~3.5~~	3.8	3.7	3.8	4.4	4.3	3.8

Problem 2

Although the original data will vary and the actual numbers in each of the three right columns will depend on many factors in the experiment, the numbers in the middle column should be clearly similar when compared to the numbers in the other two columns. Our calculations are shown below.

	Period (in seconds)	***Length of String (in cm)***	$\frac{\text{Length}}{\text{Period}}$	$\frac{\sqrt{\text{Length}}}{\text{Period}}$	$\frac{\text{Length}^2}{\text{Period}}$
Trial #1	7.5	58.5	7.80	1.02	456.30
Trial #2	5.7	31.0	5.44	0.98	168.60
Trial #3	4.0	14.0	3.50	0.94	49.00

	Period (in days)	***Period in Scientific Notation***	***Distance from Sun (in meters)***	***Distance from Sun in Scientific Notation***
Mercury	88.00	8.80×10^{1}	58,000,000,000	5.80×10^{10}
Venus	224.70	2.25×10^{2}	108,000,000,000	1.08×10^{11}
Earth	365.25	3.65×10^{2}	149,000,000,000	1.49×10^{11}
Jupiter	4331.86	4.33×10^{3}	778,000,000,000	7.78×10^{11}
Neptune	60,193.20	6.02×10^{4}	4,490,000,000,000	4.49×10^{12}

	Period (in days)	***Distance from Sun (in meters)***	$\frac{\text{Period}^2}{\text{Distance}}$	$\frac{\text{Period}}{\text{Distance}^3}$	$\frac{\text{Period}^2}{\text{Distance}^3}$
Mercury	8.80×10^{1}	5.80×10^{10}	1.34×10^{-7}	4.51×10^{-31}	3.97×10^{-29}
Venus	2.25×10^{2}	1.08×10^{11}	4.69×10^{-7}	1.79×10^{-31}	4.02×10^{-29}
Earth	3.65×10^{2}	1.49×10^{11}	8.94×10^{-7}	1.10×10^{-31}	4.03×10^{-29}
Jupiter	4.33×10^{3}	7.78×10^{11}	2.41×10^{-5}	9.19×10^{-33}	3.98×10^{-29}
Neptune	6.02×10^{4}	4.49×10^{12}	8.07×10^{-4}	6.65×10^{-34}	4.00×10^{-29}

Problem 3

The units will be:

$$\frac{\sqrt{\text{Length}}}{\text{Period}} \rightarrow \frac{\sqrt{\text{cm}}}{\text{s}}$$

$$\frac{\text{Period}^2}{\text{Length}^3} \rightarrow \frac{\text{days}^2}{\text{meters}^3} = \frac{\text{d}^2}{\text{m}^3} \text{ or } \text{d}^2 \cdot \text{m}^{-3}$$

$$\frac{\left(N\cdot m^{-2}\right)\cdot m^4}{m\cdot\left(m^3\cdot s^{-1}\right)}=\frac{N\cdot m^2}{m^4\cdot s^{-1}}=\frac{N\cdot s}{m^2}\text{ or }N\cdot s\cdot m^{-2}$$

$$\frac{\left(m^3\cdot s^{-1}\right)\cdot\left(N\cdot s\cdot m^{-2}\right)\cdot m}{m^2\cdot\left(N\cdot m^{-2}\right)}=\frac{N\cdot m^2}{N}=m^2$$

$$\frac{\left(N\cdot m^{-2}\right)\cdot m^3}{mol\cdot K}=\frac{N\cdot m}{mol\cdot K}\text{ or }N\cdot m\cdot mol^{-1}\cdot K^{-1}$$

Problem 4
The answers are in the table below. For large values of *x*, this expression is approximate equal to 5.

Expression #1

Expression: $\frac{5x^2+15x+11}{x^2+3x+2}$		**Rewritten:** $5+\frac{1}{x^2+3x+2}$	
Value of x	Value of expression	Value of quotient	Value of $\frac{\text{remainder}}{\text{divisor}}$
10	5.007575758	5	0.007575758
20	5.002164502	5	0.002164502
50	5.000377074	5	0.000377074
100	5.000097068	5	0.000097068
1,000	5.000000997	5	0.000000997

For large values of *x*, this expression is approximate equal to *5x*.

Expression #2

Expression: $\frac{5x^3+15x^2+10x+1}{x^2+3x+2}$		**Rewritten:** $5x+\frac{1}{x^2+3x+2}$	
Value of x	Value of expression	Value of quotient	Value of $\frac{\text{remainder}}{\text{divisor}}$
10	50.00757576	50	0.007575758
20	100.002164502	100	0.002164502
50	250.000377074	250	0.000377074
100	500.000097068	500	0.000097068
1,000	5,000.00000010	5,000	0.00000010

For large values of *x*, this expression is approximate equal to $5x^2$.

Expression #3

Expression: $\frac{5x^4+15x^3+10x^2+1}{x^2+3x+2}$		**Rewritten:** $5x^2+\frac{1}{x^2+3x+2}$	
Value of x	Value of expression	Value of quotient	Value of $\frac{\text{remainder}}{\text{divisor}}$
10	500.00757576	500	0.00757576
20	2,000.0021645	2,000	0.0021645
50	12,500.000377	12,500	0.000377
100	50,000.0000971	50,000	0.0000971
1,000	5,000,000.0000001	5,000,000	0.0000001

Recommendations

- Have students work in teams to encourage brainstorming and cooperative learning.
- Assign a specific timeline for completion of the project that includes milestone dates.
- Provide students feedback as they complete each milestone.
- Ensure that each member of student groups has a specific job.

Technology Integration

This project provides abundant opportunities for technology integration, and gives students the chance to research and collaborate using online technology. The students' instructions list several websites that provide information on numbering systems, game design, and graphics.

The following are other examples of free Internet resources that can be used to support this project:

http://www.moodle.org

An Open Source Course Management System (CMS), also known as a Learning Management System (LMS) or a Virtual Learning Environment (VLE). Moodle has become very popular among educators around the world as a tool for creating online dynamic websites for their students.

http://www.wikispaces.com/site/for/teachers or http://pbworks.com/content/edu+overview

Allows you to create a secure online Wiki workspace in about 60 seconds. Encourage classroom participation with interactive Wiki pages that students can view and edit from any computer. Share class resources and completed student work.

http://www.docs.google.com

Allows a student to collaborate in real-time from any computer. Google Docs provides free access and storage for word processing, spreadsheets, presentations, and surveys. This is ideal for group projects.

http://why.openoffice.org/

The leading open-source office software suite for word processing, spreadsheets, presentations, graphics, databases and more. It can read and write files from other common office software packages like Microsoft Word or Excel and MacWorks. It can be downloaded and used completely free of charge for any purpose.

Rubric

Score	Content	Presentation/Communication
4	• The solution shows a deep understanding of the problem including the ability to identify the appropriate mathematical concepts and the information necessary for its solution. • The solution completely addresses all mathematical components presented in the task. • The solution puts to use the underlying mathematical concepts upon which the task is designed and applies procedures accurately to correctly solve the problem and verify the results. • Mathematically relevant observations and/or connections are made.	• There is a clear, effective explanation detailing how the problem is solved. All of the steps are included so that the reader does not need to infer how and why decisions were made. • Mathematical representation is actively used as a means of communicating ideas related to the solution of the problem. • There is precise and appropriate use of mathematical terminology and notation. • Your project is professional looking with graphics and effective use of color.
3	• The solution shows that the student has a broad understanding of the problem and the major concepts necessary for its solution. • The solution addresses all of the mathematical components presented in the task. • The student uses a strategy that includes mathematical procedures and some mathematical reasoning that leads to a solution of the problem. • Most parts of the project are correct with only minor mathematical errors.	• There is a clear explanation. • There is appropriate use of accurate mathematical representation. • There is effective use of mathematical terminology and notation. • Your project is neat with graphics and effective use of color.
2	• The solution is not complete indicating that parts of the problem are not understood. • The solution addresses some, but not all of the mathematical components presented in the task. • The student uses a strategy that is partially useful, and demonstrates some evidence of mathematical reasoning. • Some parts of the project may be correct, but major errors are noted and the student could not completely carry out mathematical procedures.	• Your project is hard to follow because the material is presented in a manner that jumps around between unconnected topics. • There is some use of appropriate mathematical representation. • There is some use of mathematical terminology and notation appropriate to the problem. • Your project contains low quality graphics and colors that do not add interest to the project.
1	• There is no solution, or the solution has no relationship to the task. • No evidence of a strategy, procedure, or mathematical reasoning and/or uses a strategy that does not help solve the problem.	• There is no explanation of the solution, the explanation cannot be understood or it is unrelated to the problem. • There is no use or inappropriate use of mathematical representations (e.g.

	• The solution addresses none of the mathematical components presented in the task. • There were so many errors in mathematical procedures that the problem could not be solved.	figures, diagrams, graphs, tables, etc.). • There is no use, or mostly inappropriate use, of mathematical terminology and notation. • Your project is missing graphics and uses little to no color.

Unit 11: Exponents and Polynomials

Common Core Standards

Unit 11, Lesson 1, Topic 1: Exponential Notation

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.8.EE.**	Expressions and Equations
CATEGORY / CLUSTER		Work with radicals and integer exponents.
STANDARD	**8.EE.1.**	Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 3^2 x 3^-5 = 3^-3 = 1/3^3 = 1/27. (SBAC Summative Assessment Target: 1.02)

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.N.**	Number and Quantity
CATEGORY / CLUSTER	**N-RN.**	The Real Number System
STANDARD		Extend the properties of exponents to rational exponents.
EXPECTATION	**N-RN.2.**	Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Unit 11, Lesson 1, Topic 2: Simplify by using the Product, Quotient and Power Rules

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.8.EE.**	Expressions and Equations
CATEGORY / CLUSTER		Work with radicals and integer exponents.
STANDARD	**8.EE.1.**	Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 3^2 x 3^-5 = 3^-3 = 1/3^3 = 1/27. (SBAC Summative Assessment Target: 1.02)

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.N.**	Number and Quantity

CATEGORY / CLUSTER	N-RN.	The Real Number System
STANDARD		Extend the properties of exponents to rational exponents.
EXPECTATION	N-RN.2.	Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Unit 11, Lesson 1, Topic 3: Products and Quotients Raised to Powers

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	CC.8.EE.	Expressions and Equations
CATEGORY / CLUSTER		Work with radicals and integer exponents.
STANDARD	8.EE.1.	Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 3^2 x 3^-5 = 3^-3 = 1/3^3 = 1/27. (SBAC Summative Assessment Target: 1.02)

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	CC.N.	Number and Quantity
CATEGORY / CLUSTER	N-RN.	The Real Number System
STANDARD		Extend the properties of exponents to rational exponents.
EXPECTATION	N-RN.2.	Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Unit 11, Lesson 1, Topic 4: Scientific Notation

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	CC.8.EE.	Expressions and Equations
CATEGORY / CLUSTER		Work with radicals and integer exponents.
STANDARD	8.EE.3.	Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger. (SBAC Summative Assessment Target: 1.02)
STANDARD	8.EE.4.	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific

		notation that has been generated by technology. (SBAC Summative Assessment Target: 1.04)

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Unit 11, Lesson 2, Topic 1: Introduction to Single Variable Polynomials

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Interpret the structure of expressions.
EXPECTATION	**A-SSE.1.**	Interpret expressions that represent a quantity in terms of its context.
GRADE EXPECTATION	**A-SSE.1(a)**	Interpret parts of an expression, such as terms, factors, and coefficients.

Unit 11, Lesson 2, Topic 2: Adding and Subtracting Polynomials

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Interpret the structure of expressions.
EXPECTATION	**A-SSE.1.**	Interpret expressions that represent a quantity in terms of its context.
GRADE EXPECTATION	**A-SSE.1(a)**	Interpret parts of an expression, such as terms, factors, and coefficients.
STRAND / DOMAIN	**CC.A.**	Algebra

CATEGORY / CLUSTER	A-APR.	Arithmetic with Polynomials and Rational Functions
STANDARD		Perform arithmetic operations on polynomials.
EXPECTATION	A-APR.1.	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Unit 11, Lesson 2, Topic 3: Multiplying Polynomials

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	CC.A.	Algebra
CATEGORY / CLUSTER	A-SSE.	Seeing Structure in Expressions
STANDARD		Interpret the structure of expressions.
EXPECTATION	A-SSE.1.	Interpret expressions that represent a quantity in terms of its context.
GRADE EXPECTATION	A-SSE.1(a)	Interpret parts of an expression, such as terms, factors, and coefficients.
STRAND / DOMAIN	CC.A.	Algebra
CATEGORY / CLUSTER	A-APR.	Arithmetic with Polynomials and Rational Functions
STANDARD		Perform arithmetic operations on polynomials.
EXPECTATION	A-APR.1.	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Unit 11, Lesson 2, Topic 4: Multiplying Special Cases

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	CC.A.	Algebra
CATEGORY / CLUSTER	A-APR.	Arithmetic with Polynomials and Rational Functions
STANDARD		Perform arithmetic operations on polynomials.

EXPECTATION	A-APR.1.	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
STRAND / DOMAIN	CC.A.	Algebra
CATEGORY / CLUSTER	A-APR.	Arithmetic with Polynomials and Rational Functions
STANDARD		Use polynomial identities to solve problems.
EXPECTATION	A-APR.5.	(+) Know and apply the Binomial Theorem for the expansion of (x + y)^n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Unit 11, Lesson 2, Topic 5: Dividing by a Monomial

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.

Unit 11, Lesson 2, Topic 6: Dividing by Binomials and Polynomials

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	CC.A.	Algebra
CATEGORY / CLUSTER	A-SSE.	Seeing Structure in Expressions
STANDARD		Interpret the structure of expressions.
EXPECTATION	A-SSE.1.	Interpret expressions that represent a quantity in terms of its context.
GRADE EXPECTATION	A-SSE.1(a)	Interpret parts of an expression, such as terms, factors, and coefficients.

Unit 11, Lesson 3, Topic 1: Simplifying and Evaluating Polynomials with More than One Term

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Interpret the structure of expressions.
EXPECTATION	**A-SSE.1.**	Interpret expressions that represent a quantity in terms of its context.
GRADE EXPECTATION	**A-SSE.1(a)**	Interpret parts of an expression, such as terms, factors, and coefficients.

Unit 11, Lesson 3, Topic 2: Operations with Polynomials

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Interpret the structure of expressions.
EXPECTATION	**A-SSE.1.**	Interpret expressions that represent a quantity in terms of its context.
GRADE EXPECTATION	**A-SSE.1(a)**	Interpret parts of an expression, such as terms, factors, and coefficients.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-APR.**	Arithmetic with Polynomials and Rational Functions
STANDARD		Perform arithmetic operations on polynomials.
EXPECTATION	**A-APR.1.**	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.