

# Textbook of Mechanical Vibrations

### **SECOND EDITION**

#### Rao V. Dukkipati

Professor of Mechanical Engineering Fairfield University, Fairfield (Connecticut)

J. Srinivas

Associate Professor Department of Mechanical Engineering National Institute of Technology Rourkela



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#### TEXTBOOK OF MECHANICAL VIBRATIONS, Second Edition Rao V. Dukkipati and J. Srinivas

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Our parents and beloved family

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## Preface

In this second edition, while retaining the approach and organization of the first edition, the following new features and improvements have been incorporated.

- A new chapter on *nonlinear vibrations* is introduced, emphasizing the importance of nonlinear oscillations and their analysis approaches for predicting the stability of the system. The concept of limit cycles and bifurcation analysis as well as prediction of chaotic oscillating conditions are explained in a lucid style.
- One appendix on *transverse vibrations of beams* summarizing the mode shape functions for important boundary conditions has been introduced.

Most typographical errors have also been corrected.

It is for the information of the readers that Professor R.V. Dukkipati passed away on 13 February, 2010.

J. Srinivas

### **Preface to the First Edition**

The purpose of this book is to impart a basic understanding, both physically and mathematically, of the principles of vibrations and their applications. In this book, an attempt is made to present in simple and systematic manner, the techniques that can be applied to the study of mechanical vibrations. Special emphasis is placed on analytical developments and computational solutions. Being intended as a first course on the theory of mechanical vibrations, the concepts have been presented in simple terms and the solution procedures have been explained in detail.

The book is comprehensive and self-contained. It assumes no previous knowledge of dynamics and vibrations of the reader and will serve to fulfill the needs of the following categories of readers:

- Senior undergraduate and postgraduate students taking the course on mechanical vibrations
- Design and research engineers who need to select and develop mathematical models for analytical and design purposes
- Practising engineers and managers who want to learn the basic principles and concepts involved in mechanical vibration analysis and apply these to their own work place concerns.

Besides, the book may be adopted for a short professional course on the subject it presents. Since the book addresses a wide range of audience, the level of mathematics has been intentionally kept low. All the principles presented are illustrated by numerous worked-out examples. The book strikes an optimum balance between theory and practice.

In the organization and presentation of the material, utmost care has been taken to introduce the various concepts and their application to problems in a gradual step-by-step manner. The subject of mechanical vibrations deals with the methods and means for formulating mathematical models of physical systems and evolving their solutions. Both these aspects: the tools for formulating the mathematical equations and also the methods of solving them have been emphasized.

Each chapter in this book consists of concise but thorough statement of the fundamental theory; principles and methods, followed by a number of selected illustrative worked examples. A number of sample unsolved exercise problems have also been included so as to enable the earnest student to reinforce and extend the comprehension of the concepts. Answers to selected problems are given towards the end of the book. The numerous reference titles listed in the bibliography at the end of the book serve as guidelines for further study and research.

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