Synthesis of PID tuning for a new parallel cascade control structure

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Abstract: Most of the published papers on parallel cascade control strategies present improvement of dynamic performance of stable processes. In this paper, a new parallel cascade control scheme is proposed for controlling stable and unstable processes with time delay. The two main features of the proposed scheme are: the primary process output completely tracks the primary setpoint and the servo response decouples the regulatory response in the nominal system. The proposed structure has only two controllers. The inner loop controller is designed based on IMC approach. The outer loop controller is a PID controller in series with lead/lag filter which is designed based on the desired complementary sensitivity function. Significant improvement in the load disturbance rejection performances are obtained when compared to some recent methods in the literature. Simulation results show the superiority and usefulness of the proposed control method over the existing ones.

Keywords: Parallel cascade control, unstable process, time delay, load disturbance, IMC

1. INTRODUCTION

In process industries, parallel cascade control (which was first proposed by Luyben (1973)) is widely used to reduce the effects of possible disturbances and to improve the dynamic performance of the closed-loop system. A parallel cascade system is one in which both the manipulated variable and the disturbance affect the primary and the secondary output through the parallel actions while in a series cascade both actions on the primary output take place through secondary one. The conventional parallel cascade control structure proposed by Luyben (1973) is shown in Fig. 1. The parallel cascade control structure consists of two loops: primary (outer) loop and secondary (inner) loop. In parallel cascade control, the secondary loop dynamics should be much faster than the primary loop because the disturbances entering in to the secondary loop should be rejected immediately so that it reduces steady state error in the primary loop. The design of parallel cascade control for regulatory response and a method for selection of secondary measurement under different disturbances were addressed by Yu (1988) and Shen and Yu (1990), respectively. Semino and Brambilla (1996) used a conventional feedback controller in the secondary loop and an IMC controller in the primary loop. Lee et al. (2006) proposed an analytical method of PID controller design for parallel cascade control taking into account the interaction between primary and secondary control loops. The design of parallel cascade control systems has attracted relatively less attention despite the clear benefits of the parallel cascade control and its wide range applications in process industries. If a long time delay exists in the outer loop, the cascade control may not give satisfactory closed-loop responses to setpoint changes. To overcome



Fig. 1. Conventional parallel cascade control structure

this problem, many researchers (Kaya (2001), Kaya and Atherton (2008), Uma et al. (2009), Padhan and Majhi (2012)) use a dead time compensator scheme in the outer loop of the series cascade control system.

Recently, Rao et al. (2009) proposed a parallel cascade control structure (consists of three controllers and a filter) in which they have used a dead time compensator in the outer loop. The structure proposed by Lee et al. (2006) consists of two controllers and two filters. Till date, most of the published papers on parallel cascade control strategies are for control of stable processes. In this paper, further results are presented for a new parallel cascade control structure and controller design for controlling unstable process with time delay. For easy in plant operation, a simple structure with less number of controllers is desirable. This paper shows how effective control can be achieved for both long time delay and unstable processes with only two controllers. IFAC Conference on Advances in PID Control PID'12 Brescia (Italy), March 28-30, 2012

For clear interpretation, the proposed cascade control structure has been addressed in section 2. The controller design procedures are explained in section 3. The simulation results are provided in section 4 followed by the conclusions in the end.

2. A NEW PARALLEL CASCADE CONTROL STRUCTURE

The proposed parallel cascade control structure (shown in Fig. 2) has two controllers, namely, G_{c1} and G_{c2} . G_{c2} in the inner loop stabilizes the process by rejecting the disturbances entering the inner loop. Unlike the conventional parallel cascade control structure, the proposed structure uses the outer loop controller G_{c1} in the feedback path. Although, G_{c1} is primarily meant for load disturbance rejection, it also takes part in stabilizing the unstable process in the outer loop. $G_{p1} = \tilde{G}_{p1}e^{-\theta_1 s}$ and $G_{p2} = \tilde{G}_{p2}e^{-\theta_2 s}$ are the transfer functions of the primary and secondary processes respectively. $G_{m1} = \tilde{G}_{m1}e^{-\theta_{m1}s}$ and $G_{m2} = \tilde{G}_{m2}e^{-\theta_{m2}s}$ are the transfer functions of the primary and secondary process models respectively. G_{d1} and G_{d2} are the transfer functions of the disturbances for primary and secondary loops respectively. The overall outer loop process transfer function is

and

$$G_m = \tilde{G}_m e^{-\theta_m s} \tag{2}$$

is the transfer function of the overall outer loop process model. The closed-loop transfer function relating the primary process response (y_1) to the reference (r_1) can be written as

 $G_p = \tilde{G}_p e^{-\theta_p s} = G_{c2} G_{p1} = G_{c2} \tilde{G}_{p1} e^{-\theta_1 s}$

$$\frac{y_1}{r_1} = \frac{G_{c2}G_{p1}\left(1 + G_{c1}\tilde{G}_m e^{-\theta_m s}\right)}{\tilde{G}_m\left(1 + G_{c1}G_{c2}G_{p1} + G_{c2}G_{p2} - G_{c2}G_{m2}\right)}$$
(3)

where $\tilde{G}_m e^{-\theta_m s} = G_m$ is the transfer function model of the overall process dynamics. Similarly, the closed-loop transfer function relating the primary process output (y_1) to the disturbance input d is given by

$$\frac{y_1}{d} = \frac{G_{d1}\left(1 + G_{c2}G_{p2} - G_{c2}G_{m2}\right) - G_{d2}G_{c2}G_{p1}}{1 + G_{c1}G_{c2}G_{p1} + G_{c2}G_{p2} - G_{c2}G_{m2}}$$
(4)

Based on the assumption that the model used perfectly matches the process dynamics, (3) and (4) reduce to

$$\frac{y_1}{r_1} = e^{-\theta_m s} \tag{5}$$

and

$$\frac{y_1}{d} = \frac{G_{d1} - G_{d2}G_{c2}G_{p1}}{1 + G_{c1}G_{c2}G_{p1}} \tag{6}$$

respectively. It concludes from (5) that the primary process output follows the setpoint input and the closed-loop system is always stable under the nominal condition. Also, from (5) and (6), it is evident that the new structure decouples the servo response from the regulatory response for the nominal system. (Note: In order to avoid complexity the transfer functions $G_{c1}(s)$, $G_{c2}(s)$, $G_{p1}(s)$...etc have been represented as G_{c1} , G_{c2} , G_{p1} ...etc respectively, throughout the paper)



Fig. 2. Proposed parallel cascade control structure

2.1 Process models

(1)

In practice, in the case of parallel cascade control, the inner loop process has no or a negligible time delay while the outer loop process has a large time delay compared to the inner loop. Generally, in the industrial applications, the dynamics of secondary process is stable and that of the primary process is stable or unstable. Therefore, the inner loop process transfer function is assumed to be a first order plus time delay(FOPTD):

$$G_{p2} = \frac{k_2 e^{-\theta_2 s}}{\tau_2 s + 1} \tag{7}$$

The outer loop process transfer functions are assumed in the following form

$$G_{p1} = \frac{k_1 e^{-\theta_1 s}}{\tau_1 s + 1}$$
(8)

for a FOPTD process and

$$G_{p1} = \frac{k_1 e^{-\theta_1 s}}{\tau_1 s - 1}$$
(9)

for an unstable first order plus time delay (UFOPTD) process.

3. CONTROLLER DESIGN PROCEDURES

The design methods for the controllers $(G_{c1} \text{ and } G_{c2})$ are explained in this section in details.

3.1 Design of the inner loop controller G_{c2}

The secondary loop controller is designed based on internal model control (IMC) principles (Morari and Zafiriou (1989)). The design procedure given in Padhan and Majhi (2011) is reproduced below for the controller G_{c2} . The inner loop is referred to as IMC since the plant model G_{m2} appears in the control structure. The inner loop process model G_{m2} is given by

$$G_{m2} = \frac{k_{m2}e^{-\theta_{m2}s}}{\tau_{m2}s + 1} \tag{10}$$

According to the IMC controller design, decomposing the process model into two parts results in

$$G_{m2}(s) = G_{m2M}(s)G_{m2A}(s)$$
 (11)

where G_{m2M} contains the invertible portion of the model and G_{m2A} contains all the non-invertible portion. The invertible portions are the part of the model with stable poles. The non-invertible portions are the portion of model with right half plane zeros and time delays.

The ideal IMC controller is the inverse of the invertible portion of the process model i.e.

$$G_{c2i} = G_{m2M}^{-1} \tag{12}$$

In order to make the IMC controller proper, it is necessary to introduce a low-pass filter $(f_{c2} = \frac{1}{(\lambda_2 s + 1)^n})$ with a steady state gain of 1. Where λ_2 is the filter time constant and the index *n* should be selected to make the IMC controller realizable.

Now, the secondary loop controller G_{c2} is given by

$$G_{c2} = G_{c2i} f_{c2} = \frac{\tau_{m2} s + 1}{k_{m2} \left(\lambda_2 s + 1\right)}$$
(13)

where λ_2 is the adjustable tuning parameter. The response speed is determined by the parameter λ_2 . In order to achieve good control performance, the inner loop should be faster than the outer loop. The smaller the value of λ_2 the better the performance of the parallel cascade control system. The value of λ_2 should be selected such that satisfactory closed-loop responses can be achieved. On the basis of extensive simulation studies based on the MATLAB toolbox, the suggested range for λ_2 are $0.1\theta_m 0.8\theta_m$ for small time delay processes, $0.001\theta_m - 0.09\theta_m$ for large time delay and unstable processes.

3.2 Design of the outer loop controller G_{c1}

Based on the nature of the primary process and load disturbance transfer functions, the desired closed-loop complementary sensitivity function is chosen and correspondingly the controller is designed. In fact, for all the cases PID controller in series with lead/lag compensator is obtained. The detailed design procedure is explained below. The loop transfer function for the outer loop is given by

$$L_1(s) = G_{c1}G_{c2}G_{p1} \tag{14}$$

The nominal complementary sensitivity function of the outer loop for disturbance rejection is

$$T_{d_1} = \frac{L_1(s)}{1 + L_1(s)} = \frac{G_{c1}G_{c2}G_{p1}}{1 + G_{c1}G_{c2}G_{p1}}$$
(15)

By following a simple calculation, we get

$$G_{c1} = \frac{T_{d_1}}{1 - T_{d_1}} \times \frac{1}{G_{c2}G_{p1}}$$
(16)

(i)For FOPTD primary process: If the primary process dynamics is $G_{p1} = k_1 e^{-\theta_1 s} / (\tau_1 s + 1)$, in order to get the desired closed-loop performances, the closed-loop complementary sensitivity function is written as

$$T_{d_1} = \frac{1}{(\lambda_1 s + 1)^2} e^{-\theta_m s}$$
(17)

From (8), (13), (16) and (17), we get

$$G_{c1} = \frac{k_2 \left(\lambda_2 s + 1\right) \left(\tau_1 s + 1\right)}{k_1 \left(\tau_2 s + 1\right) \left[\left(\lambda_1 s + 1\right)^2 - e^{-\theta_m s}\right]}$$
(18)

The second order Padé approximation for the time delay (i.e. $e^{-\theta_m s} = (6 - 2s\theta_m)/(6 + 4s\theta_m + s^2\theta_m^2))$ reduces (18) as

$$G_{c1} = \frac{k_2 \left(\lambda_2 s + 1\right) \left(\tau_1 s + 1\right) \left(6 + 4\theta_m s + s^2 \theta_m^2\right)}{k_1 \left(\tau_2 s + 1\right) \left[\left(\lambda_1 s + 1\right)^2 \left(6 + 4\theta_m s + s^2 \theta_m^2\right) \right]}$$
(19)

After following a simple calculation, we get

$$G_{c1} = \frac{k_2 \left(\lambda_2 s + 1\right) \left(\tau_1 s + 1\right) \left(6 + 4\theta_m s + s^2 \theta_m^2\right)}{k_1 s \left[x_4 s^4 + x_3 s^3 + x_2 s^2 + x_1 s + x_0\right]}$$
(20)

where $x_4 = \tau_2 \lambda_1^2 \theta_m^2$, $x_3 = \lambda_1^2 \theta_m^2 + 2\tau_2 \lambda_1 \theta_m^2 + 4\tau_2 \lambda_1^2 \theta_m$, $x_2 = 4\lambda_1^2 \theta_m + 6\tau_2 \lambda_1^2 + 2\lambda_1 \theta_m^2 + \tau_2 \theta_m^2 + 8\tau_2 \lambda_1 \theta_m$, $x_1 = 6\lambda_1^2 + \theta_m^2 + 8\lambda_1 \theta_m + 6\tau_2 \theta_m + 12\tau_2 \lambda_1$ and $x_0 = 6\theta_m + 12\lambda_1$. (20) can be approximated as a PID controller in series with lead/lag compensator in the form of

$$G_{c1} = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \left(\frac{a_2 s^2 + a_1 s + 1}{\left(\frac{b_4 s^4 + b_3 s^3 + b_2 s^2}{+b_1 s + 1} \right)} \right) (21)$$

where

$$\begin{cases} K_c = \frac{6k_2(\tau_1 + \lambda_2)}{k_1 x_0}, \ T_i = \tau_1 + \lambda_2, \ T_d = \frac{\tau_1 \lambda_2}{\tau_1 + \lambda_2} \\ a_2 = \frac{\theta_m^2}{6}, \ a_1 = \frac{2\theta_m}{3} \\ b_4 = \frac{x_4}{x_0}, \ b_3 = \frac{x_3}{x_0}, \ b_2 = \frac{x_2}{x_0}, \ b_1 = \frac{x_1}{x_0} \end{cases}$$
(22)

(ii) For UFOPTD primary process: In order to reject the step load disturbances injected into the primary loop process, an asymptotic constraint

$$\lim_{s \to 1/\tau_1} \left(1 - T_{d_1} \right) = 0 \tag{23}$$

should be satisfied so that the closed-loop internal stability can be achieved. The desired closed-loop complementary sensitivity function is proposed as

$$T_{d_1} = \frac{\beta_1 s + 1}{(\lambda_1 s + 1)^3} e^{-\theta_m s}$$
(24)

where β_1 is a positive number and λ_1 is a tuning parameter for obtaining the desirable closed-loop performances of the outer loop. Substitution of (24) in (23) results in

$$\beta_1 = \tau_1 \left[\left(\frac{\lambda_1}{\tau_1} + 1 \right)^3 e^{\theta_{m/\tau_1}} - 1 \right]$$
(25)

From (9), (13), (16) and (24), we get

$$G_{c1} = \frac{k_2 \left(\lambda_2 s + 1\right) \left(\tau_1 s - 1\right) \left(\beta_1 s + 1\right)}{k_1 \left(\tau_2 s + 1\right) \left[\left(\lambda_1 s + 1\right)^3 - \left(\beta_1 s + 1\right) e^{-\theta_m s}\right]}$$
(26)

Using Padé approximation for the time delay term of (26) gives

$$G_{c1} = \frac{\begin{pmatrix} k_2 (\lambda_2 s + 1) (\tau_1 s - 1) (\beta_1 s + 1) \\ (6 + 4\theta_m s + \theta_m^2 s^2) \end{pmatrix}}{k_1 (\tau_2 s + 1) \begin{bmatrix} (\lambda_1 s + 1)^3 (6 + 4\theta_m s + \theta_m^2 s^2) \\ - (\beta_1 s + 1) (6 - 2\theta_m s) \end{bmatrix}}$$
(27)

After following a simple calculation, we get

c1

$$G_{c1} = -\frac{6k_2}{k_1 m_0} \left(\lambda_2 + \beta_1 + \frac{1}{s} + \lambda_2 \beta_1 s \right) \\ \times \frac{1 + 2\theta_m s/3 + \theta_m^2 s^2/6}{\left(\left(\frac{1 + x_1 s + x_2 s^2 + x_3 s^3 +}{x_4 s^4 + x_5 s^5} \right) / (-\tau_1 s + 1) \right)}$$
(28)

where $x_1 = m_1/m_0$, $x_2 = m_2/m_0$, $x_3 = m_3/m_0$, $x_4 = m_4/m_0$, $x_5 = m_5/m_0$, $m_0 = 18\lambda_1 + 6\theta_m - 6\beta_1$, $m_1 = \theta_m^2 + 12\lambda_1\theta_m + 2\beta_1\theta_m - 6\tau_2\beta_1 + 6\tau_2\theta_m + 18\tau_2\lambda_1 + 18\lambda_1^2$, $m_2 = 3\lambda_1\theta_m^2 + 6\lambda_1^3 + 12\tau_2\lambda_1\theta_m + 12\lambda_1^2\theta_m + 2\tau_2\beta_1\theta_m + \tau_2\theta_m^2 + 18\tau_2\lambda_1^2$, $m_3 = 4\lambda_1^3\theta_m + 3\tau_2\lambda_1\theta_m^2 + 3\lambda_1^2\theta_m^2 + 6\tau_2\lambda_1^3 + 12\tau_2\lambda_1^2\theta_m$, $m_4 = 4\tau_2\lambda_1^3\theta_m + \lambda_1^3\theta_m^2 + 3\tau_2\lambda_1^2\theta_m^2$ and $m_5 = \tau_2\lambda_1^3\theta_m^2$.

(28) can be expressed in the form of a PID controller in series with lead/lag compensator as

$$G_{c1} = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \left(\frac{a_2 s^2 + a_1 s + 1}{b_3 s^3 + b_2 s^2 + b_1 s + 1} \right) (29)$$

where

$$\begin{cases} K_c = -\frac{6k_2 \left(\beta_1 + \lambda_2\right)}{k_1 m_0}, \ T_i = \beta_1 + \lambda_2, \ T_d = \frac{\beta_1 \lambda_2}{\beta_1 + \lambda_2} \\ a_1 = \frac{2\theta_m}{3}, \ a_2 = \frac{\theta_m^2}{6} \\ b_1 = x_1 + \tau_1, \ b_2 = x_2 + b_1 \tau_1, \ b_3 = x_3 + b_2 \tau_1 \end{cases}$$
(30)

The filter parameters b_1 , b_2 and b_3 are obtained by the following method. The parameter b_1 is obtained by taking first derivative of the term

 $(1 + x_1s + x_2s^2 + x_3s^3 + x_4s^4 + x_5s^5)/(-\tau_1s + 1)$ and substituting s = 0. Similarly, the second derivative of the said term and substitution of s = 0 gives the expression for b_2 and so on(Padhan and Majhi (2012)).

The tuning of the control parameter λ_1 aims at the best trade-off between nominal performance of the closed loop and its robust stability. That is, decreasing λ_1 improves the disturbance rejection performance of the closed loop but degrades its robust stability in the presence of process uncertainty. In contrast, increasing λ_1 tends to strengthen the robust stability of the closed loop but degrades its disturbance rejection performance. On the basis of extensive simulation studies based on the MATLAB toolbox, it is observed that the initial value of λ_1 is equal to overall process time delay. The suggested range of the tuning parameter is $\lambda_1 = 0.09\theta_m - \theta_m$ for FOPTD process, $\lambda_1 = 0.5\theta_m - 1.2\theta_m$ for the UFOPTD process.

Remark: From the following block diagram (see Fig. 3), the closed-loop transfer function V(s) can be obtained as

$$V(s) = \frac{1}{\tilde{G}_m} = \frac{1}{G_{c2}\tilde{G}_{p1}}$$
(31)

 G_{c2} is a function of the tuning parameter λ_2 in turn V(s) is also a function of λ_2 . It is to be noted that V(s) primarily helps in improving the overall servo tracking performance of the closed-loop system.



Fig. 3. Block diagram for V(s)



Fig. 4. Nominal responses for example 1: (a) Proposed, (b) Lee et al. (2006)

3.3 Performance

To evaluate the closed-loop performance, we consider two popular performance specifications based on integral error (e(t) = r(t) - y(t)) such as the integral absolute error $(IAE = \int_{0}^{\infty} |e(t)| dt)$ and the integral square error (ISE = $\int_{0}^{\infty} e(t)^2 dt$) criteria.

To evaluate the manipulated input, we compute the total variation (TV) of the input u(t) i.e.TV= $\sum_{i=1}^{\infty} |u_{i+1} - u_i|$, which should be as small as possible. The TV is a good measure of smoothness of a signal (Skogestad (2003)).

4. SIMULATION RESULTS

In this section, to illustrate the usefulness of the proposed cascade control structure and design procedure, three typical simulation examples are presented.

4.1 Example-1

Consider the process and disturbance transfer function models (Lee et al. (2006)) given by $G_{p1} = G_{d1} = e^{-4s}/(20s+1)$ and $G_{p2} = G_{d2} = 1/(10s+1)$. Taking $\lambda_2 = 0.5$, the inner loop controller is obtained as $G_{c2} = (10s+1)/(0.5s+1)$. Choosing the primary controller parameter as $\lambda_1 = 0.5\theta_m$ and using the design formulae (22), the parameters of G_{c1} are obtained as $K_c = 2.5625$, $T_i = 20.5$, $T_d = 0.4878$, $a_2 = 2.6667$, $a_1 = 2.6667$, $b_4 = 13.3333$, $b_3 = 28$, $b_2 = 24.3333$ and $b_1 = 12.1667$. With these controller settings, the performances of the



Fig. 5. Perturbed responses for example 1: (a) Proposed, (b) Lee et al. (2006)

closed loop system is evaluated by introducing a unit step load disturbance at time t = 0. For comparison, the method proposed by Lee et al. (2006) is considered. The closed-loop primary responses for these controllers setting are shown in Fig. 4. In the present work, a + 40% perturbation in the primary process time delay and -40% in the primary and secondary process time constants have been considered and the corresponding responses are shown in Fig. 5. The control efforts(u) are also shown in Fig. 4 and Fig. 5. As the proposed method gives low TV (see Table 1), the control action variation is comparatively smooth. For quantitative comparison, IAE and ISE (for regulatory responses) performance indices are considered here. It is observed from the Table 1 that the proposed method gives low IAE and ISE values. It is evident from the simulation results that the proposed method yields robust and superior control performances.

4.2 Example-2

Consider the following liquefied petroleum gas (GPL) splitter model studied by Rao et al. (2009)

$$G_{p1} = \frac{-0.0067e^{-300s}}{105.8s+1}, \quad G_{d1} = \frac{0.05843e^{-300s}}{115.5s+1}$$
$$G_{p2} = \frac{-5.217}{101.6s+1}, \qquad G_{d2} = \frac{44.15}{109.5s+1}$$

Taking $\lambda_2 = 0.5$ results in

 $G_{c2} = (101.6s + 1)/(-2.608s - 5.217)$. By choosing $\lambda_1 = 0.1\theta_m$, the parameters of G_{c1} are obtained as $K_c = 229.92$, $T_i = 106.3$, $T_d = 0.4976$, $a_2 = 15000$, $a_1 = 200$, $b_4 = 3810000$, $b_3 = 342300$, $b_2 = 10874$ and $b_1 = 179.1$. With these controller settings a step load input of magnitude 100 at t = 0 is introduced and the corresponding closed-loop responses are shown in Fig. 6. To investigate the robustness of the proposed controller, a perturbation of +30% in the primary process and load disturbance time delays is considered and the closed-loop performances are given in Fig. 7. From the simulation results, it is seen that Lee et al. (2006) method produces spikes in a regular interval (see Fig. 6 and Fig. 7). It can be observed (see Table 1) that the proposed method gives smaller performance indices compared to that of Rao et al. (2009)



Fig. 6. Nominal responses for example 2: (a) Proposed, (b) Lee et al. (2006), (c) Rao et al. (2009)



Fig. 7. Perturbed responses for example 2: (a) Proposed, (b) Lee et al. (2006), (c) Rao et al. (2009)

 Table 1. Performance specifications for regulatory responses

Scheme	Nominal System			Perturbed system		
	IAE	ISE	TV	IAE	ISE	TV
Example-1						
(a)	0.28	0.003	1.17	0.35	0.005	1.13
(b)	0.53	0.01	1.28	0.65	0.016	1.24
Example-2						
(a)	63.86	6.16	894.84	94.47	9.05	911.12
(b)	70.39	7.07	$1.49\mathrm{E}4$	105.8	10.04	1.08E4
(c)	108	10.9	1.071E3	116.3	13.12	$1.07\mathrm{E3}$
Example-3						
(a)	1.61	0.11	2.44	1.62^{*}	0.12^{*}	2.59^{*}
				1.63^{+}	0.11^{+}	2.33^{\dagger}
				2.16‡	0.19‡	3.26‡

(a)Proposed, (b) Lee et al. (2006), (c) Rao et al. (2009) (*)-10% change in θ_1 and τ_1 , (†)+10% change in θ_1 and τ_1 and (‡)+10% change in θ_1 and -10% in τ_1

and Lee et al. (2006). Also, it gives better performances for disturbance rejection.



Fig. 8. Regulatory responses for example 3: (a) Nominal, (b) perturbation of -10% in θ_1 and τ_1 , (c) perturbation of +10% in θ_1 and τ_1 and (d)perturbation of +10% in θ_1 and -10% in τ_1

4.3 Example-3

Consider a chemical CSTR (Lee et al. (2002)) of which the primary and secondary process and load disturbance transfer functions $G_{p1} = G_{d1} = e^{-4s}/(20s - 1)$ and $G_{p2} =$ $G_{d2} = 2e^{-2s}/(20s+1)$, respectively. By choosing $\lambda_1 = \theta_m$ and using the design formulae (30), the parameters of G_{c1} are obtained as $K_c = 7.1580, T_i = 22.2317, T_d = 0.02,$ $a_2 = 2.6667, a_1 = 2.6667, b_3 = 48.6843, b_2 = 40.8994$ and $b_1 = 21.9240$. The inner loop controller is obtained as $G_{c2} = (20s + 1)/(0.04s + 2)$. With these controller settings a unit step load disturbance is introduced at time t = 0. The corresponding closed-loop responses are shown in Fig. 8. To illustrate the robustness to parameter variations, perturbations of +10% in the primary process time delay and -10% in the primary process time constant and again $\pm 10\%$ in the primary process time delay and in the primary process time constant have been considered and the corresponding closed-loop responses and control efforts are shown in Fig. 8. The servo responses for perfect and perturbed systems are shown in Fig. 9. It is evident from the simulation results that the proposed cascade scheme gives robust closed-loop performances in terms of the servo tracking and the load disturbance rejection.

5. CONCLUSIONS

The problem of controlling stable and unstable time delayed processes has been tackled by proposing a new parallel cascade control structure. One of the important features of the proposed structure is that it decouples the servo response from the regulatory response in the nominal case. The comparative analysis shows that with less number of controllers, the proposed scheme gives improved closedloop performances. It is shown that both nominal and robust control performances are obtained with the designed controllers.

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- Fig. 9. Servo responses for example 3: (a) Nominal, (b) perturbation of -10% in θ_1 and τ_1 , (c) perturbation of +10% in θ_1 and τ_1 and (d)perturbation of +10% in θ_1 and -10% in τ_1
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