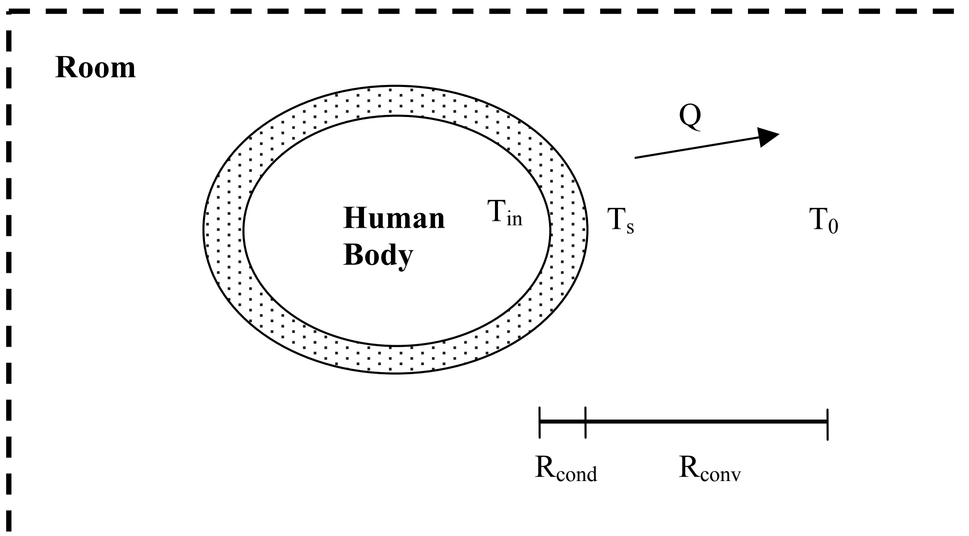
Tutorial # 7

Steady State Conduction

Problem 1 Consider a naked person standing in a room at 20°C with an exposed surface area of 1.7m^2. The deep body temperature of the human body is 37°C, and the thermal conductivity of the human tissue near the skin is about 0.3W/(m·°C). The body is losing heat at a rate of 150W by natural convection to the surroundings. Taking the body temperature 0.5cm beneath the skin to be 37°C, determine the skin temperature of the person and the convective heat transfer coefficient to the surroundings.

Solution:

Step 1: Draw a schematic diagram



Step 2: What to determine?

- The skin temperature of the person, T_s ;
- Convective heat transfer coefficient, h.

Step 3: The information given in the problem statement.

- Deep body temperature, T_{in}=37°C, Room temperature, T_0=20°C;
- Depth beneath the skin L=0.5cm, Exposed surface Area, A=1.7m^2;
- Thermal conductivity of the human tissue, k=0.3W/(m·°C);
- Heat loss rate, $\dot{Q} = 150$ W

Step 4: Assumptions

- Neglect the radiation heat loss to the surroundings;
- The temperature won't change with time(steady problem).

Step 5: Solve

It's a one-dimension steady heat transfer process, and the conductivity resistance is determined as

$$R_{cond} = \frac{L}{kA} = \frac{0.5(cm)}{0.3(W/m{\cdot}^{o}C)\times 1.7(m^2)} = 0.0098(^{o}C/W)$$

The heat loss is

$$\dot{Q}_{tatal} = \dot{Q}_{cond} = \dot{Q}_{conv} = 150\,(\mathrm{W})$$

The conductive heat loss:

$$\dot{Q}_{cond} = \frac{\Delta T}{R_{cond}} = \frac{T_{in} - T_s}{R_{cond}}$$

Thus,

$$T_s = T_{in} - \dot{Q}_{cond}\,R_{cond} = 37(^{o}C) - 150(W)\times 0.0098(^{o}C/W) = 35.5(^{o}C)$$

The convective heat loss to the surroundings is calculated by

$$\dot{Q}_{conv} = \frac{\Delta T}{R_{conv}} = \frac{T_s - T_0}{\frac{1}{hA}}$$

So, the convective heat transfer coefficient,

$$h = \frac{\dot{Q}_{conv}}{A(T_s - T_0)} = \frac{150(W)}{1.7(m^2)\times(35.5-20)(^{o}C)} = 5.69(W/m^{2}\cdot{}^{o}C)$$

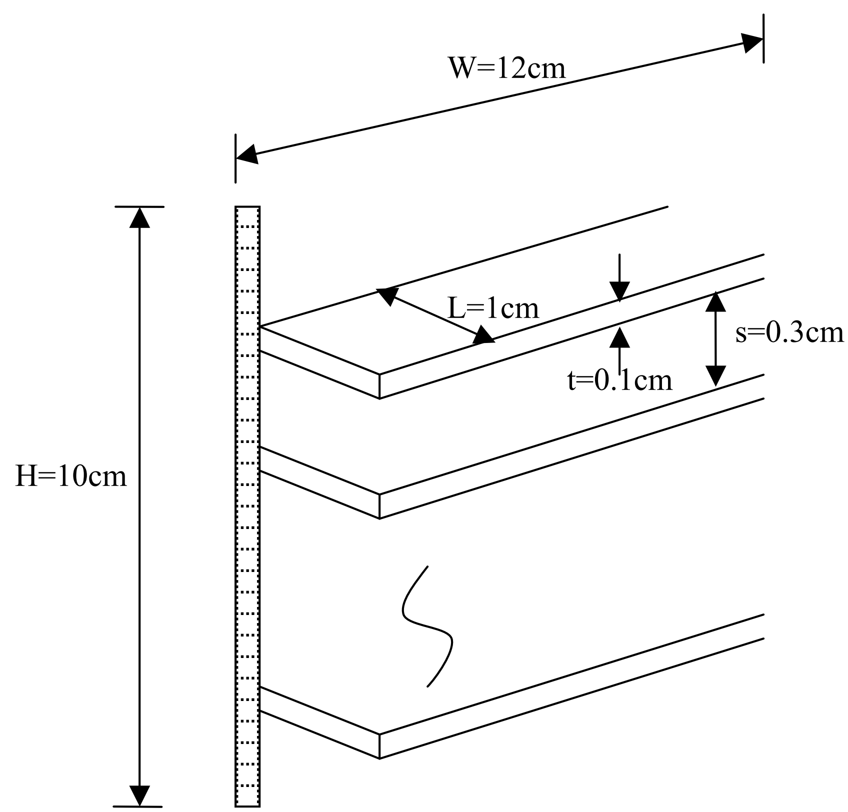
Step 6: Conclusion statement

The skin temperature of the person will be **35.5 °C** and the convective heat transfer coefficient is **5.69 W/(m^2·°C)**

Problem 2 As shown in the figure, steam in a heating system flow through one side of rectangle plane (10cm*12cm). The plane is maintained at a temperature of 150°C. Rectangular aluminium alloy 2024-T6 fins [k=186 W/(m·°C)] with a constant width of 1cm and thickness of 0.1cm are attached to the plane. The space between every two adjacent fins is 0.3cm. The temperature of the surrounding air is 20°C and the combined convective heat transfer is 50W/(m^2·°C), Determine the overall effectiveness of the finned plane.

Solution:

Step 1: Draw a schematic diagram



Step 2: What to determine?

- The overall effectiveness of the finned plane, ε .

Step 3: The information given in the problem statement.

- Plane surface temperature, T_s=150°C, Atmosphere temperature, T_0=20°C;
- Plane geometry: H=10cm, and W=12cm;
- Fin: conductivity: k=186 W/(m·°C), length: L=1cm, thickness: t=0.1cm, space: s=0.3cm, width: W=12cm.
- The combined convective heat transfer: h=50W/(m^2·°C)

Step 4: Assumptions

- Neglect the radiation heat loss to the surroundings;
- The temperature won't change with time(steady problem).

Step 5: Solve

CASE 1. With fins on the plane:

The efficiency of the fins is determined from Fig 10.42 to be

$$\xi = (L + \frac{t}{2})\sqrt{h/kt} = (0.01m + \frac{0.001}{2}m)\sqrt{\frac{50W/m^2 \cdot {}^oC}{(186W/m \cdot {}^oC) \times (0.001m)}} = 0.16$$

which gives us the fin efficiency

$$\eta_{fin} = 0.98$$

the area of a fin is,

$$A_{fin} = 2W(L + \frac{t}{2}) = 2 \times 0.12m \times (0.01m + \frac{0.001}{2}m) = 0.00252m^2$$

So, the actual heat transfer rate through the fins is,

$$\dot{Q}_{fin} = \eta_{fin} h A_{fin}(T_s - T_0) = 0.98 \times (50W/m^2 \cdot {}^oC) \times (0.00252m^2)(150^oC - 20^oC)$$
$$= 16.05\ W$$

For the unfinned region, the area is

$$A_{unfin} = W \cdot s = 0.12m \times 0.003m = 0.00036m^2$$

So, the heat transfer rate through an unfinned area is

$$\dot{Q}_{unfin} = hA_{unfin}(T_s - T_0) = (50W/m^2 \cdot {}^oC) \times (0.00036m^2)(150^oC - 20^oC)$$
$$= 2.34W$$

the number of the fins on the plane:

$$n = \frac{H}{t+s} = \frac{10cm}{0.1cm + 0.3cm} = 25$$

For the total heat transfer rate of the whole plane:

$$\dot{Q}_{total,fin} = n(\dot{Q}_{fin} + \dot{Q}_{unfin}) = 25 \times (16.05\,W + 2.34\,W) = 459.75\,W$$

<u>CASE 2. No fins on the plane</u>

The area of the whole plane is

$$A_{nofin} = H \cdot W = 0.1m \times 0.12m = 0.012m^2$$

and the whole heat transfer rate is

$$\begin{aligned}\dot{Q}_{nofin} &= hA_{nofin}(T_s - T_0)\\ &= (50W/m^2\cdot{}^oC) \times (0.012m^2)(150^oC - 20^oC)\\ &= 78W\end{aligned}$$

<u>The overall effectiveness of the finned plane</u>

$$\varepsilon = \frac{\dot{Q}_{total,fin}}{\dot{Q}_{nofin}} = \frac{459.75\,W}{78\,W} = 5.89$$

Step 6: Conclusion statement

The overall effectiveness of the finned plane is **<u>5.89</u>**, which means the heat transfer rate is enhanced 5.89 times when attaching the fins.