SOA EXAM MLC & CAS EXAM 3L STUDY SUPPLEMENT

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Contents

PREFACE

This study supplement is intended to be an aid to candidates studying for either Exam MLC (Models for Life Contingencies) of the Society of Actuaries or Exam 3L (Life Contingencies and Statistics) of the Casualty Actuarial Society. Each section of this study supplement contains key life contingencies concepts and major life contingencies formulas with **brief** interpretations for specific Exam MLC/3L topics. This study supplement is NOT meant to take the place of an exam's required text book or a proper study guide. For example, there are no detailed explanations of topics that would be found in a text book and no worked examples of the material that would be found in a study guide.

For candidates studying for Exam 3L, this study supplement only covers the life contingencies material on that exam. That is, this study supplement does NOT cover any of the statistics material on Exam 3L.

This study supplement is meant to outline key life contingencies concepts and major life contingencies formulas associated with Exam MLC or Exam 3L, and to provide exercises that are meant to test a candidate's basic knowledge of the material and to improve the candidate's speed in working with certain concepts and formulas. If the candidate can master the exercises in this study supplement, the candidate should be in a good position to tackle actual past Exam MLC and/or Exam 3L questions. Many of the exercises in this study supplement are based on past homework and midterm questions that I used in teaching life contingencies classes at the University of Illinois at Urbana-Champaign. **In some cases, the exercise answer choices are rounded. Choose the letter of the answer choice that is closest to your own answer**.

Once the exercises in this study supplement have been mastered, the candidate can refer to the listing of past Exam MLC and Exam 3L questions at the end of each section for a more advanced test of the candidate's understanding of the material. Web links to past Exam MLC and Exam 3L questions can be found in Appendix A of this study supplement. Also, some Exam MLC exercises in this study supplement will utilize "Multi-State Model Examples" and the "Standard Select Survival Model," each from Dickson et al; these figures are provided in Appendix B of this study supplement.

For candidates enrolled in the Midwestern Actuarial Forum's Exam MLC & 3L Seminar, it is expected that each candidate will have read the required study materials for either Exam MLC or Exam 3L mentioned in the exam's syllabus. The candidate is also expected to have read the relevant sections of this study supplement and also have worked its relevant exercises PRIOR to sitting for the seminar. Furthermore, it is expected that each candidate will have worked as many of the past Exam MLC and Exam 3L questions as possible. Exam 3L candidates should attempt problems from the prior 3L exams first before attempting any problems from prior MLC exams. Also, some sections are marked "Exam MLC Only;" as you may have guessed, candidates preparing for Exam 3L can skip those sections. I will rely heavily on candidate's asking questions regarding these study supplement exercises and past Exam MLC and Exam 3L questions DURING the seminar; often, working exercises and past exam questions identifies issues that once resolved will enhance the candidate's understanding of the material. I would also contend that candidates will only get the most out of this seminar if they have sufficiently studied for their exam beforehand.

Candidates should bring a printed or electronic copy of this study supplement with them to the seminar. Candidates should also print copies of all tables provided during the exam they will be writing (Exam MLC or Exam 3L) - please refer to Appendix A of this study supplement. Candidates should also bring copies of any past Exam MLC and/or Exam 3L questions that they wish to discuss during the seminar.

Despite proofreading, it is possible that this study supplement may contain typos. If there are any questions regarding possible typos, or anything else, please e-mail me at: pjohnson@illinois.edu.

Best of luck in your exam preparation!

Regards:

-Paul H. Johnson, Jr., PhD

AUTHOR BACKGROUND

Paul H. Johnson, Jr. earned a PhD in Actuarial Science, Risk Management and Insurance from the University of Wisconsin-Madison in 2008. He is currently an Assistant Professor of Actuarial Science in the Department of Mathematics at the University of Illinois at Urbana-Champaign. Paul has passed all of the preliminary examinations of the Society of Actuaries (the modern day equivalents of Exams P, FM, M, and C). Since 2003, Paul has taught undergraduate courses meant to prepare students for each the preliminary exams of the Society of Actuaries, and has received various honors and awards for teaching excellence. Most recently, Paul was honored with the N. Tenney Peck Teaching Award in Mathematics for 2011. He has also published in actuarial and risk management journals, including the North American Actuarial Journal and the Risk Management and Insurance Review.

1 SURVIVAL MODELS AND LIFE TABLES

1.1 Key Concepts

Let (x) denote a life aged x.

Future Lifetime:

- T_x = time-until-death for (x), a continuous random variable (in years).

  T_x is also called the future lifetime random variable. T_x may also be written as $T(x)$ or T.

  Special case: T_0 = age-at-death for (0), where (0) denotes a newborn life. Note: T_0 = x + T_x.

- $F_x(t) = {}_tq_x = Pr(T_x \leq t)$

  This is the cumulative distribution function of T_x, "the probability that (x) dies within t years." The q-notation will be used most of the time.

  $F_0(t)$ can also be written more simply as $F(t)$.

- $S_x(t) = {}_tp_x = Pr(T_x > t)$

  This is the survival function function of T_x,"the probability that (x) survives for at least t years." The words "at least" are often omitted. The p-notation will be used most of the time.

  $S_0(t)$ can also be written more simply as $S(t)$ or $s(t)$.

- From above: ${}_tq_x + {}_tp_x = 1$.

  "(x) will either survive or die within t years."

- $S_0(x+t) = S_0(x)\,{}_tp_x$

  "The probability that (0) survives $x+t$ years is equivalent to (0) first surviving x years to age x, and then surviving t additional years to age x + t."

- ${}_{u+t}p_x = ({}_u p_x)({}_t p_{x+u})$

  "The probability that (x) survives $u + t$ years is equivalent to (x) first surviving u years to age x + u, and then surviving t additional years to age x + u + t."

- Be careful: ${}_{u+t}q_x \neq ({}_u q_x)({}_t q_{x+u})$.

  "The right-hand side implies that it is possible for (x) to die within u years, then somehow come back to life at age x + u in order to die again within t years. This, of course, is not possible and cannot be equal to the left-hand side which is the probability (x) dies within $u + t$ years."

- ${}_{u|t}q_x = {}_{u+t}q_x$ - ${}_u q_x = {}_u p_x$ - ${}_{u+t}p_x = ({}_u p_x)({}_t q_{x+u})$

  This is a u-year deferred probability of death, "the probability that (x) dies between ages x + u and x + u + t." Note: ${}_{0|t}q_x = {}_t q_x$.

- Note: ${}_1 q_x$, ${}_1 p_x$, and ${}_{u|1}q_x$ are written as q_x, p_x, and ${}_{u|}q_x$, respectively.

  q_x may be referred to as a <u>mortality rate</u>, and p_x may be referred to as a <u>survival rate</u>.

Force of Mortality:

- $\mu_x = \mu(x) =$ force of mortality at age x, given survival to age x. This is sometimes called the "hazard rate" or "failure rate."

  $\mu_x = -\frac{\frac{d}{dx}[S_0(x)]}{S_0(x)} = -\frac{d}{dx}[\ln S_0(x)]$

  "This is the instantaneous death rate for a life at age x."

- $\mu_{x+t} = \mu_x(t) =$ force of mortality at age x + t, given survival to x + t.

  $\mu_{x+t} = -\frac{\frac{d}{dt}[{}_tp_x]}{{}_tp_x} = -\frac{d}{dt}[\ln {}_tp_x]$

  "This is the instantaneous death rate for a life at age x + t. Here, the variable is time after age x. You could also obtain μ_{x+t} by replacing x in μ_x with x + t."

- ${}_tp_x = \exp[-\int_x^{x+t} \mu_s ds] = \exp[-\int_0^t \mu_{x+s} ds]$

- If $c > 0$, then $\mu^*_{x+s} = c\mu_{x+s} \implies {}_tp^*_x = ({}_tp_x)^c$.

  For constant k, then $\mu^*_{x+s} = \mu_{x+s} + k \implies {}_tp^*_x = (e^{-kt})({}_tp_x)$.

  The constant k should be such that $\mu^*_{x+s} > 0$.

- $f_x(t) = {}_tp_x\mu_{x+t} =$ probability density function of T_x.

  This comes from the above formula for μ_{x+t}, recognizing that $f_x(t) = \frac{d}{dt}[{}_tq_x] = -\frac{d}{dt}[{}_tp_x]$.

- ${}_tq_x = \int_0^t {}_sp_x\mu_{x+s}ds$

- ${}_tp_x = \int_t^\infty {}_sp_x\mu_{x+s}ds$

- ${}_{u|t}q_x = \int_u^{u+t} {}_sp_x\mu_{x+s}ds$

Curtate Future Lifetime:

- K_x = curtate future lifetime for (x), a discrete random variable.

  $K_x = \lfloor T_x \rfloor$ = integer part of T_x. That is, K_x represents the complete number of future years survived by (x), where any fractional time survived in the year of death is ignored. Note: K_x = 0, 1, 2...

  K_x may also be written as $K(x)$ or K.

- ${}_{k|}q_x = Pr(K_x = k) = Pr(k \leq T_x < k+1)$ for k = 0, 1, 2...

  This is the probability mass function of K_x, "the probability that (x) dies in the $(k + 1)$st year, between ages x + k and x + k + 1."

- ${}_{k+1}q_x = {}_{0|}q_x + {}_{1|}q_x + ... + {}_{k|}q_x$

  "The probability that (x) dies within $k + 1$ years is the sum of the probabilities that (x) dies in the first year, the second year, ..., the $(k + 1)$st year."

Other Features of T_x and K_x Distributions:

- $\mathring{e}_x = E(T_x) = \int_0^\infty t({}_tp_x)(\mu_{x+t})dt = \int_0^\infty {}_tp_x dt$

  This is the complete expectation of life for (x), the average time-until-death for (x). That is, (x) is expected to die at age x + $\mathring{e}_x$.

- $Var(T_x) = E(T_x^2)$ - $[E(T_x)]^2 = 2\int_0^\infty t\,{}_tp_x dt$ - $[\mathring{e}_x]^2$

- $\mathring{e}_{x:\overline{n}|} = E[min(T_x, n)] = \int_0^n {}_tp_x dt$

  This is the n-year temporary complete life expectancy for (x), the average number of years out of the next n years that (x) survives.

  This expectation helps define the recursion: $\mathring{e}_x = \mathring{e}_{x:\overline{n}|} + {}_np_x\mathring{e}_{x+n}$.

  "The average number of future years that (x) survives is the average number of years out of the first n years that (x) survives plus the average number of years (x) survives after the first n years (accounting for the probability that (x) survives the first n years)."

  Similarly: $\mathring{e}_{x:\overline{m+n}|} = \mathring{e}_{x:\overline{m}|} + {}_mp_x\mathring{e}_{x+m:\overline{n}|}$.

- The 100α-th percentile of the distribution of T_x, π_α, is such that:

  ${}_{\pi_\alpha}q_x = \alpha$ for $0 \leq \alpha \leq 1$.

  Special case: $\alpha = 0.50$; $\pi_{.50}$ is called the median future lifetime for (x).

- $e_x = E(K_x) = \sum_{k=0}^{\infty} k({}_{k|}q_x) = \sum_{k=1}^{\infty} {}_kp_x$

  This is the curtate expectation of life for (x), the average curtate future lifetime for (x).

- $\mathring{e}_x \approx e_x + \frac{1}{2}$

- $Var(K_x) = E(K_x^2)$ - $[E(K_x)]^2 = \sum_{k=1}^{\infty}(2k-1)_kp_x$ - $[e_x]^2$

- $e_{x:\overline{n}|} = E[min(K_x, n)] = \sum_{k=1}^{n} {}_kp_x$

  This is the n-year temporary curtate life expectancy for (x).

  This expectation helps define the recursions: $e_x = e_{x:\overline{n}|} + {}_np_xe_{x+n}$ and

  $e_{x:\overline{m+n}|} = e_{x:\overline{m}|} + {}_mp_xe_{x+m:\overline{n}|}$.

Special Mortality Laws:

de Moivre's Law: T_x has a continuous uniform distribution.

The limiting age is ω such that $0 \leq x \leq x + t \leq \omega$.

- $\mu_x = \frac{1}{\omega - x}$ (Note: $x \neq \omega$)
- $S_0(x) = \frac{\omega - x}{\omega}$
- $F_0(x) = \frac{x}{\omega}$
- ${}_tp_x = \frac{\omega - x - t}{\omega - x}$
- ${}_tq_x = \frac{t}{\omega - x}$
- ${}_{u|t}q_x = \frac{t}{\omega - x}$
- $f_x(t) = {}_tp_x \mu_{x+t} = \frac{1}{\omega - x}$ (Note: $x \neq \omega$)
- $\mathring{e}_x = \frac{\omega - x}{2}$
- $\mathring{e}_{x:\overline{n}|} = n\,{}_np_x + \frac{n}{2}\,{}_nq_x$

  "(x) can either survive n years with probability ${}_np_x$, or die within n years with probability ${}_nq_x$. Surviving n years contributes n to the expectation. Dying within n years contributes $\frac{n}{2}$ to the expectation as future lifetime has a uniform distribution - (x), on average, would die halfway through the n-year period."
- $Var(T_x) = \frac{(\omega - x)^2}{12}$
- $e_x = \frac{\omega - x - 1}{2}$
- $Var(K_x) = \frac{(\omega - x)^2}{12}$ - $\frac{1}{12}$

Modified/Generalized de Moivre's Law: T_x has a beta distribution.

The limiting age is ω such that $0 \leq x \leq x + t \leq \omega$. Also, there is a parameter $\alpha > 0$.

- $\mu_x = \frac{\alpha}{\omega - x}$ (Note: $x \neq \omega$)
- $S_0(x) = \left(\frac{\omega - x}{\omega}\right)^\alpha$
- $F_0(x) = 1 - \left(\frac{\omega - x}{\omega}\right)^\alpha$
- ${}_tp_x = \left(\frac{\omega - x - t}{\omega - x}\right)^\alpha$
- ${}_tq_x = 1 - \left(\frac{\omega - x - t}{\omega - x}\right)^\alpha$

  Note: ${}_tq_x \neq \left(\frac{t}{\omega - x}\right)^\alpha$.
- $\mathring{e}_x = \frac{\omega - x}{\alpha + 1}$
- $Var(T_x) = \frac{\alpha(\omega - x)^2}{(\alpha+1)^2(\alpha+2)}$

Note: $\alpha = 1$ results in uniform distribution/de Moivre's Law.

Constant Force of Mortality: T_x has an exponential distribution, $x \geq 0$. There is another parameter that denotes the force of mortality: $\mu > 0$.

- $\mu_x = \mu$
- $S_0(x) = e^{-\mu x}$
- $F_0(x) = 1 - e^{-\mu x}$
- ${}_tp_x = e^{-\mu t} = (p_x)^t$
- ${}_tq_x = 1 - e^{-\mu t}$
- $\mathring{e}_x = \frac{1}{\mu}$
- $\mathring{e}_{x:\overline{n}|} = \frac{1-e^{-\mu n}}{\mu}$
- $Var(T_x) = \frac{1}{\mu^2}$
- $e_x = \frac{p_x}{q_x}$
- $Var(K_x) = \frac{p_x}{(q_x)^2}$

Note1: A constant force of mortality implies that "age does not matter." This can easily be seen from ${}_tp_x = e^{-\mu t}$; x does not appear on the right-hand side.

Note2: T_x has an exponential distribution implies that K_x has a geometric distribution.

Gompertz's Law:

- $\mu_x = Bc^x$ for $x \geq 0$, $B > 0$, $c > 1$
- $S_0(x) = \exp[-\frac{B}{\ln c}(c^x - 1)]$
- $F_0(x) = 1$ - $\exp[-\frac{B}{\ln c}(c^x - 1)]$
- ${}_tp_x = \exp[-\frac{B}{\ln c}c^x(c^t - 1)]$
- ${}_tq_x = 1$ - $\exp[-\frac{B}{\ln c}c^x(c^t - 1)]$

Makeham's Law:

- $\mu_x = A + Bc^x$ for $x \geq 0$, $A \geq$ -B, $B > 0$, $c > 1$
- $S_0(x) = \exp[-Ax - \frac{B}{\ln c}(c^x - 1)]$
- $F_0(x) = 1$ - $\exp[-Ax - \frac{B}{\ln c}(c^x - 1)]$
- ${}_tp_x = \exp[-At - \frac{B}{\ln c}c^x(c^t - 1)]$
- ${}_tq_x = 1$ - $\exp[-At - \frac{B}{\ln c}c^x(c^t - 1)]$

  Note: $A = 0$ results in Gompertz's Law.

Weibull's Law: T_x has a Weibull distribution.

- $\mu_x = kx^n$ for $x \geq 0$, $k > 0$, $n > 0$
- $S_0(x) = \exp[-\frac{k}{n+1}x^{n+1}]$
- $F_0(x) = 1$ - $\exp[-\frac{k}{n+1}x^{n+1}]$
- ${}_tp_x = \exp[-\frac{k}{n+1}((x+t)^{n+1} - x^{n+1})]$
- ${}_tq_x = 1$ - $\exp[-\frac{k}{n+1}((x+t)^{n+1} - x^{n+1})]$

Life Tables:

Given a survival model with survival probabilities ${}_tp_x$, one can construct a life table, also called a mortality table, from some initial age x_0 (usually age 0) to a maximum age ω (a limiting age).

- Let l_{x_0}, the radix of the life table, represent the number of lives age x_0.

  l_{x_0} is an arbitrary positive number.

- $l_\omega = 0$.

- $l_{x+t} = (l_x)({}_tp_x)$ for $x_0 \leq x \leq x + t \leq \omega$.

  l_{x+t} represents the expected number of survivors to age x + t out of l_x individuals aged x.

- ${}_td_x = l_x$ - $l_{x+t} = (l_x)({}_tq_x)$ for $x_0 \leq x \leq x + t \leq \omega$.

  ${}_td_x$ represents the expected number of deaths between ages x and x + t out of l_x lives aged x.

  Note 1: ${}_1d_x$ is written as d_x.

  Note 2: If n = 1, 2..., then ${}_nd_x = d_x + d_{x+1} + ... + d_{x+n-1}$.

- ${}_td_{x+u} = l_{x+u}$ - $l_{x+u+t} = (l_x)({}_{u|t}q_x)$.

The Illustrative Life Table is the life table that is provided to the candidate taking Exam MLC or Exam 3L. Some questions from either exam will involve Illustrative Life Table calculations. A web link to this table (and ALL exam tables) is provided for each exam in Appendix A of this study supplement.

Fractional Age Assumptions:

Life Tables are usually defined for integer ages x and integer times t. For a quantity that involves fractional ages and/or fractional times, one has to make an assumption about the survival distribution between integer ages; that is, one has to interpolate the value of the quantity within each year of age. Two common interpolation assumptions follow.

Uniform Distribution of Deaths (UDD):

One linearly interpolates within each year of age. For integer age x and 0 $\leq s \leq s+t \leq 1$:

- $l_{x+s} = l_x$ - $sd_x = (1-s)l_x + (s)l_{x+1}$. This is a linear function for s.
- ${}_sq_x = sq_x$
- ${}_sp_x = 1$ - sq_x
- $\mu_{x+s} = \frac{q_x}{1-sq_x}$ (does not hold at $s = 1$)
- $f_x(s) = {}_sp_x\mu_{x+s} = q_x$ (does not hold at $s = 1$)
- ${}_sq_{x+t} = \frac{sq_x}{1-tq_x}$
- $\mathring{e}_x = e_x + \frac{1}{2}$
- $Var(T_x) = Var(K_x) + \frac{1}{12}$
- $\mathring{e}_{x:\overline{n}|} = e_{x:\overline{n}|} + \frac{1}{2}{}_nq_x$
- Note: uniform distribution/de Moivre's Law has the property of UDD across all ages up to the limiting age ω.

  Furthermore, uniform distribution/de Moivre's Law may be expressed as $l_x = k(\omega - x)$ for $0 \leq x \leq \omega$ where $k > 0$.

Constant Force of Mortality (Exam MLC Only):

One exponentially interpolates within each year of age. For integer age x and $0 \leq s \leq s+t \leq 1$:

- $l_{x+s} = l_x p_x^s \implies \ln[l_{x+s}] = (1-s)\ln[l_x] + s\ln[l_{x+1}]$
- ${}_s p_x = p_x^s$
- ${}_s q_x = 1$ - p_x^s
- $\mu_{x+s} =$ -$\ln p_x = \mu_x$ (does not hold at $s = 1$)
- $f_x(s) = {}_s p_x \mu_{x+s} =$ -$\ln p_x (p_x^s)$ (does not hold at $s = 1$)
- ${}_s q_{x+t} = 1$ - p_x^s

1.2 Exercises

1.1 Suppose: $F_0(t) = 1$ - $(1 + 0.00026t^2)^{-1}$ for $t \geq 0$.

Calculate the probability that (30) dies between ages 35 and 40.

(A) 0.056 (B) 0.058 (C) 0.060 (D) 0.062 (E) 0.064

1.2. You are given: $s(x) = \frac{10{,}000 - x^2}{10{,}000}$ for $0 \leq x \leq 100$. Calculate: q_{49}.

(A) 0.009 (B) 0.011 (C) 0.013 (D) 0.015 (E) 0.017

1.3. Suppose: $S_0(t) = \exp[-\frac{t^2}{2500}]$ for $t \geq 0$.

Calculate the force of mortality at age 45.

(A) 0.036 (B) 0.039 (C) 0.042 (D) 0.045 (E) 0.048

1.4. The probability density function of the future lifetime of a brand new machine is: $f(x) = \frac{4x^3}{27c}$ for $0 \leq x \leq c$.

Calculate: $\mu(1.1)$.

(A) 0.06 (B) 0.07 (C) 0.08 (D) 0.09 (E) 0.10

1.5. You are given:

(i) The probability that (30) will die within 30 years is 0.10.

(ii) The probability that (40) will survive to at least age 45 and that another (45) will die by age 60 is 0.077638.

(iii) The probability that two lives age 30 will both die within 10 years is 0.000096.

(iv) All lives are independent and have the same expected mortality.

Calculate the probability that (45) will survive 15 years.

(A) 0.90 (B) 0.91 (C) 0.92 (D) 0.93 (E) 0.94

1.6. You are given:

(i) $e_{50} = 20$ and $e_{52} = 19.33$

(ii) $q_{51} = 0.035$

Calculate: q_{50}.

(A) 0.028 (B) 0.030 (C) 0.032 (D) 0.034 (E) 0.036

1.7. For a population of smokers and non-smokers:

(i) Non-smokers have a force of mortality that is equal to one-half the force of mortality for smokers at each age.

(ii) For non-smokers, mortality follows a uniform distribution with $\omega = 90$.

Calculate the difference between the probability that a 55 year old smoker dies within 10 years and the probability that a 55 year old non-smoker dies within 10 years.

(A) 0.20 (B) 0.22 (C) 0.24 (D) 0.26 (E) 0.28

1.8. You are given:

(i) The standard probability that (40) will die prior to age 41 is 0.01.

(ii) (40) is now subject to an extra risk during the year between ages 40 and 41.

(iii) To account for the extra risk, a revised force of mortality is defined for the year between ages 40 and 41.

(iv) The revised force of mortality is equal to the standard force of mortality plus a term that decreases linearly from 0.05 at age 40 to 0 at age 41.

Calculate the revised probability that (40) will die prior to age 41.

(A) 0.030 (B) 0.032 (C) 0.034 (D) 0.036 (E) 0.038

1.9. You are given:

(i) T_x denotes the time-until-death random variable for (x).

(ii) Mortality follows de Moivre's Law with limiting age ω.

(iii) The variance of T_{25} is equal to 352.0833.

Calculate: $\mathring{e}_{40:\overline{10|}}$.

(A) 7.5 (B) 8.0 (C) 8.5 (D) 9.0 (E) 9.5

1.10. You are given: $\mu_x = \frac{1}{\sqrt{80-x}}$ for $0 \leq x < 80$.

Calculate the median future lifetime for (40).

(A) 4.0 (B) 4.3 (C) 4.6 (D) 4.9 (E) 5.2

1.11. You are given:

$$\mu_x = \begin{cases} 0.04 & \text{for } 0 \leq \text{x} < 40 \\ 0.05 & \text{for } 40 \leq \text{x} \end{cases}$$

Calculate: $\mathring{e}_{25}$.

(A) 22 (B) 23 (C) 24 (D) 25 (E) 26

1.12. You are given: ${}_{k|}q_0 = 0.10$ for $k = 0, 1, ..., 9$.

Calculate: ${}_2p_5$.

(A) 0.50 (B) 0.55 (C) 0.60 (D) 0.65 (E) 0.70

1.13. For the current model of Zingbot:

(i) $s(x) = \frac{\omega - x}{\omega}$ for $0 \leq x \leq \omega$

(ii) $var[T(5)] = 102.083333$.

For the proposed model of Zingbot, with the same ω as the current model:

(1) $s^*(x) = (\frac{\omega - x}{\omega})^\alpha$ for $0 \leq x \leq \omega$, $\alpha > 0$

(2) $\mu^*_{10} = 0.0166667$.

Calculate the difference between the complete expectation of life for a brand new proposed model of Zingbot and the complete expectation of life for a brand new current model of Zingbot.

(A) 5.9 (B) 6.1 (C) 6.3 (D) 6.5 (E) 6.7

1.14. Mortality for Frodo, age 33, is usually such that:

${}_tp_x = \left(\frac{110 - x - t}{110 - x}\right)^2$ for $0 \leq t \leq 110 - x$.

However, Frodo has decided to embark on a dangerous quest that will last for the next three years (starting today). During these three years only, Frodo's mortality will be revised so that he will have a constant force of mortality of 0.2 for each year. After the quest, Frodo's mortality will once again follow the above expression for ${}_tp_x$.

Calculate Frodo's revised complete expectation of life.

(A) 15.2 (B) 15.4 (C) 15.6 (D) 15.8 (E) 16.0

1.15. You are given:

(i) $\mu(x) = B(1.05)^x$ for $x \geq 0$, $B > 0$.

(ii) $p_{51} = 0.9877$

Calculate: B.

(A) 0.001 (B) 0.002 (C) 0.003 (D) 0.004 (E) 0.005

1.16. You are given:

(i) The force of mortality for Vivian is $\mu_x^V = \mu$ for $x \geq 0$, $\mu > 0$.

(ii) The force of mortality for Augustine is $\mu_x^A = \frac{1}{90-x}$ for $0 \leq x < 90$.

Calculate μ so that ${}_{10}p_{30}$ is the same for Vivian and Augustine.

(A) 0.016 (B) 0.018 (C) 0.020 (D) 0.022 (E) 0.024

1.17. Consider the following life table, where missing entries are denoted by "—":

$\mathbf{x}$	$\mathbf{q_x}$	$\mathbf{l_x}$	$\mathbf{d_x}$
48	—	90,522	—
49	0.007453	89,900.9286	—

Calculate the expected number of deaths between ages 48 and 50.

(A) 1280 (B) 1290 (C) 1300 (D) 1310 (E) 1320

1.18. You are given the following life table, where missing entries are denoted by "—":

x	l_x	q_x	e_x
65	79,354	0.0172	—
66	—	0.0186	—
67	—	—	—
68	74,993	—	14.89
69	—	—	14.22

Calculate the expected number of deaths between ages 67 and 69.

(A) 2800 (B) 2900 (C) 3000 (D) 3100 (E) 3200

1.19. You are given:

(i) $l_x = 1000(\omega - x)$ for $0 \le x \le \omega$

(ii) $\mu_{30} = 0.0125$

Calculate: $\mathring{e}_{40:\overline{20}|}$.

(A) 17.1 (B) 17.6 (C) 18.1 (D) 18.6 (E) 19.1

1.20. You are given the following life table, where missing values are indicated by "—":

x	l_x	d_x	p_x
0	1000.0	—	0.875
1	—	125.0	—
2	—	—	—
3	—	—	0.680
4	—	182.5	—
5	200.0	—	—

Calculate ${}_{2|}q_0$.

(A) 0.16 (B) 0.17 (C) 0.18 (D) 0.19 (E) 0.20

1.21. Woolhouse is currently age 40. Woolhouse's mortality follows 110% of the Illustrative Life Table; that is, the probability that Woolhouse dies between ages x and x + 1 is 110% of the probability of death between ages x and x + 1 in the Illustrative Life Table for x = 40, 41, ..., 110.

Calculate Woolhouse's 4-year temporary curtate life expectancy.

(A) 3.94 (B) 3.95 (C) 3.96 (D) 3.97 (E) 3.98

1.22. Suppose mortality follows the Illustrative Life Table, and deaths are uniformly distributed within each year of age.

Calculate: ${}_{4.5}q_{40.3}$.

(A) 0.0141 (B) 0.0142 (C) 0.0143 (D) 0.0144 (E) 0.0145

1.23. Suppose mortality follows the Illustrative Life Table, where deaths are assumed to be uniformly distributed between integer ages.

Calculate the median future lifetime for (32).

(A) 41 (B) 43 (C) 45 (D) 47 (E) 49

1.24. Suppose mortality follows the Illustrative Life Table with the assumption that deaths are uniformly distributed between integer ages.

Calculate: ${}_{0.9}q_{60.6}$.

(A) 0.0130 (B) 0.0131 (C) 0.0132 (D) 0.0133 (E) 0.0134

1.25. You are given the mortality rates:
$q_{30} = 0.020$, $q_{31} = 0.019$, $q_{32} = 0.018$.

Assume deaths are uniformly distributed over each year of age.

Calculate the 1.4-year temporary complete life expectancy for (30).

(A) 1.36 (B) 1.37 (C) 1.38 (D) 1.39 (E) 1.40

1.26. **(Exam MLC Only:)** Suppose:

(i) $q_{70} = 0.04$ and $q_{71} = 0.05$.

(ii) Let UDD denote a uniform distribution of deaths assumption within each year of age, and let CF denote a constant force of mortality within each year of age.

Calculate the probability that (70.6) will die within the next 0.5 years under UDD minus the probability that (70.6) will die within the next 0.5 years under CF.

(A) 0.00008 (B) 0.00010 (C) 0.00012 (D) 0.00014 (E) 0.00016

1.27. **(Exam MLC Only:)** You are given:

(i) The force of mortality is constant between integer ages.

(ii) ${}_{0.3}q_{x+0.7} = 0.10$

Calculate: q_x.

(A) 0.24 (B) 0.26 (C) 0.28 (D) 0.30 (E) 0.32

Answers to Exercises

1.1. E

1.2. C

1.3. A

1.4. B

1.5. C

1.6. B

1.7. A

1.8. C

1.9. D

1.10. B

1.11. A

1.12. C

1.13. E

1.14. D

1.15. A

1.16. B

1.17. B

1.18. E

1.19. A

1.20. D

1.21. D

1.22. E

1.23. C

1.24. A

1.25. C

1.26. A

1.27. D

1.3 Past Exam Questions

- Exam MLC, Spring 2012: #2 (MLC Only)
- Exam 3L, Spring 2012: #1, 2, 3
- Exam MLC, Sample Questions: #13, 21, 22, 28, 32, 59, 65, 98, 106, 116, 120, 131, 145, 155, 161, 171, 188, 189, 200, 201, 207, 219, 223, 267 (MLC Only), 276
- Exam 3L, Fall 2011: #1, 2
- Exam 3L, Spring 2011: #1, 2, 3
- Exam 3L, Fall 2010: #1, 2, 3
- Exam 3L, Spring 2010: #1, 2, 3, 4
- Exam 3L, Fall 2009: #1, 2, 3
- Exam 3L, Spring 2009: #1, 3
- Exam 3L, Fall 2008: #12, 13, 14
- Exam 3L, Spring 2008: #13, 14, 15, 16
- Exam MLC, Spring 2007: #1, 21