Singapore Math: A Visual Approach to Word Problems

Model Drawing in *Math in Focus*™

Since the early 1980s, a distinguishing characteristic of the math taught in Singapore—a top performing nation as seen on the Trends in International Math and Science Study (TIMSS) reports of 1995, 1999, 2003, and 2007—is the use of "model drawing". Model drawing, often called "bar modeling" in the U.S., is a systematic method of representing word problems and number relationships that is explicitly taught beginning in second grade and extending all the way to secondary algebra. Students are taught to use rectangular "bars" to represent the relationship between known and unknown numerical quantities and to solve problems related to these quantities.

In Singapore, 86% of primary schools use the math series *My Pals Are Here! Maths.* In ***Math in Focus,*** the U.S. edition of *My Pals Are Here! Maths,* students learn to use the bars to model problems that involve the four operations both with whole numbers, fractions, and ratios. The use of the rectangular bars and the identification of the unknown quantity with a question mark help students visualize the problem and know what operations to perform—in short, viewing all problems from an algebraic perspective beginning in early elementary grade levels.

The problems might be as simple as:

Jane has 10 cookies and Joe has 12, how many do they have altogether? (second grade)

Or as complex as:

Jessica and Lillian had the same amount of money. Jessica gave $1,140 to a charity and Lillian gave $580 to a different charity. In the end Lillian had 9 times as much money as Jessica. How much money did each girl have at first?

What makes model drawing so effective is less about the specific model—the rectangles—than the systematic and consistent way it is taught. Each grade level addresses distinct operations and number relationships—addition and subtraction in second grade, multiplication and division in third, fractions and ratios in fourth and fifth—so students can visualize and solve increasingly complex problems.

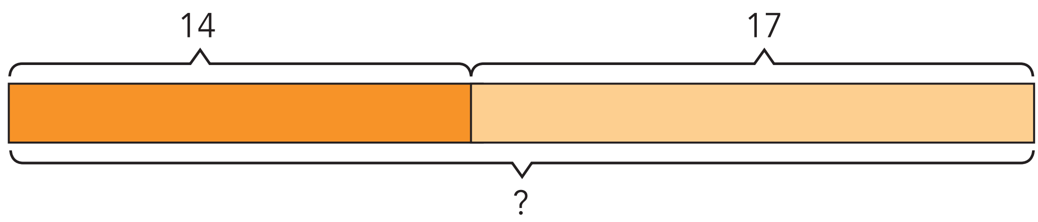
Typically, students in the U.S. are taught a variety of strategies for problem solving, including "draw a picture." But usually this entails drawing objects, animals, or counters. It is not very efficient when you move to larger numbers. In Singapore, students learn to represent these objects with rectangles that enable them to see the number relationships, rather than focusing on the objects of the problems. Rectangles are used because they are easy to draw, divide, represent larger numbers, and display proportional relationships.

Bar Modeling: Pictorial Understanding

Students are first introduced to model drawing in second grade to represent part/part whole situations that can be solved with addition or subtraction. The first problem, introduced in second grade, might be as simple as:

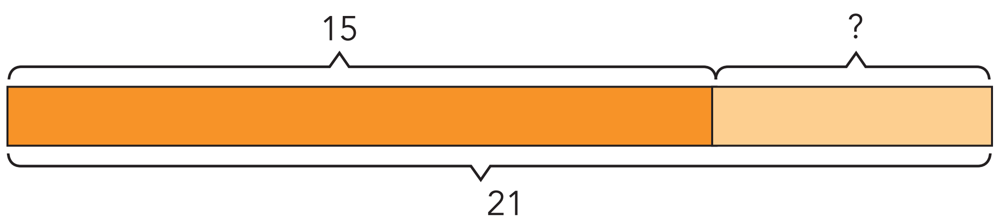
Helen has 14 breadsticks. Her friend has 17. How may do they have altogether?

Students would draw one bar, divided into two parts, one slightly longer than the other. In this problem the two parts are "known," and the student must add to find the whole or the "unknown."



But the next problem is:

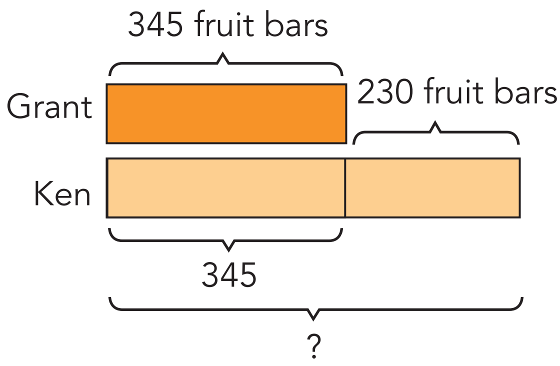
There are 21 fish in a bowl. Fifteen are from students. The rest are from the school. How many are from the school?



Notice in this problem, the student knows the whole and one part, and can solve for the missing part either by adding up or subtracting, so students understand the relationship between addition and subtraction. Students solve for an unknown variable at a pictorial stage, which aids the transition into the abstract.

While part/part whole models can be used to represent many subtraction problems, they cannot be used to represent comparison problems—how many more or fewer is one quantity compared to another. Such a problem might be:

Grant buys 345 fruit bars. Ken buys 230 more fruit bars than Grant. How many fruit bars does Ken buy?



Notice how visually clear these comparison problems become when the two rectangles are drawn. Even when the problems become more complex—for instance asking students how much Ken and Grant have altogether—the visual representation helps students realize they must first figure out how many Ken has and then how much they have altogether. With just these two models, students can solve most multi-step, complex addition and subtraction problems.

Multiplication, Division, and Fractions

At the end of second grade and more thoroughly in third grade, students are taught to model problems that are solved by multiplication and division. Again, students distinguish between part/whole problems and comparisons. The first might be a problem like this: *A box has 12 pencils. How many pencils are in 5 boxes?* Students would draw a bar with five parts, each labeled as 12.

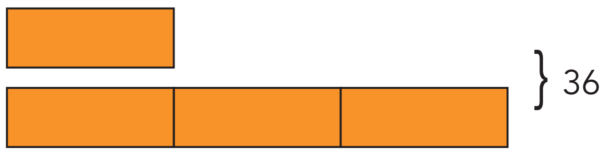
The comparison problems might be as simple as:

Jim has $15. Tom has twice as much. How much does Tom have?

Or as challenging as:

The sum of two numbers is 36. The larger number is three times the smaller number. Find the two numbers.

Imagine drawing the smaller number as a rectangle. Then the larger number would be three of them and the sum of the two is 36.

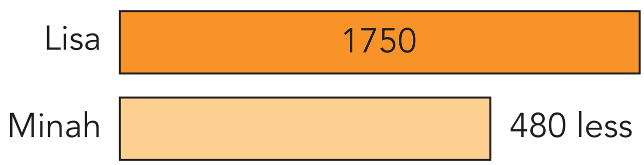


The student quickly visualizes that the sum of the four bars is 36, and that 36 ÷ 4 = 9 for the smaller number and 27 for the larger one.

Students are gently led from simple problems with easily manipulated numbers to more complex ones that require more arithmetic and multiple steps. A comparable multiplication problem in fourth grade is:

Lisa had 1750 stamps. Minah had 480 fewer stamps than Lisa. Lisa gave some stamps to Minah. Now Minah has 3 times as many stamps as Lisa.

How many stamps did Minah have at first? How many stamps does Lisa have now?



Minah = 1750 - 480 = 1270 at first

Minah + Lisa = 1270 + 1750 = 3020 total

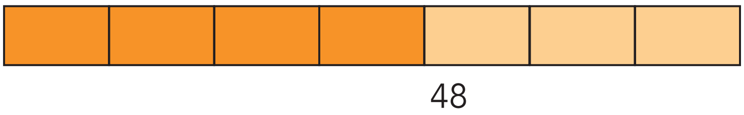
Lisa = 3020 ÷ 4 = 755   Minah = 3 x 755 = 2265

Lisa now has 755 stamps.

Fractions and Ratios

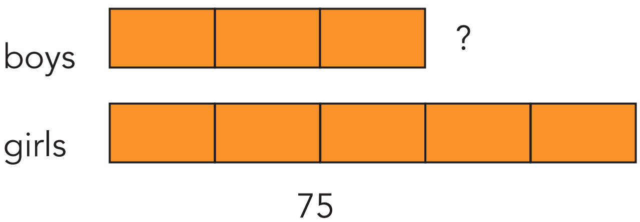
Finally, in fourth and fifth grade students use models to understand and solve problems that involve fractions and proportional thinking. Once again, some problems involve part/whole problems like this:

Vincent spent 4/7 of his money on a pair of shoes. The shoes cost $48. How much money did he have at first?



The comparison problem might read:

There are 3/5 as many boys as girls. If there are 75 girls, how many boys are there?



Conclusion: The Road to Algebra

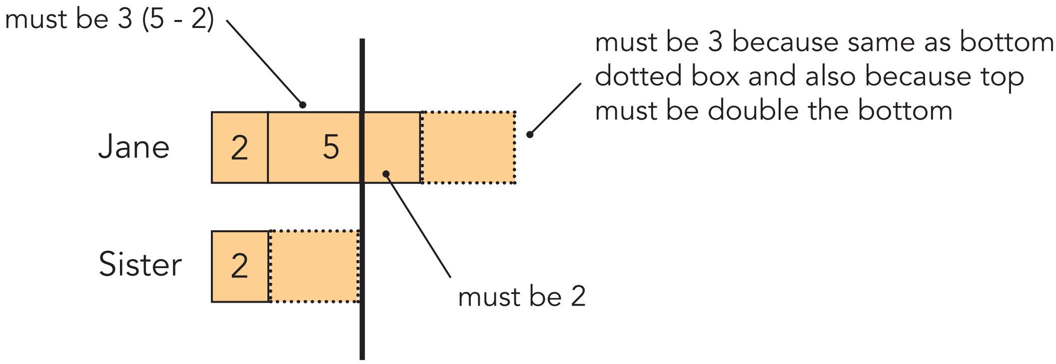
By now, you probably recognize that these rectangular bars, including the fractional units shown on page 5, are gradually leading students to the beginning of a concept of a variable and an unknown. These rectangles, which will become variable expressions in algebra, enable students to construct more abstract representations of problems as they continue in mathematics. Representing number relationships, comparisons, proportions, and changes becomes second nature as students do this from grade level to grade level.

By fifth grade, students may be trying problems as complex as:

A group of people pay $720 for admission tickets to an amusement park. The price of an adult ticket is $15, and a child ticket is $8. There are 25 more adults than children. How many children are in the group?

Or problems that involve changing quantities or situations that change:

Jane had $7 and her sister had $2. Their parents gave them each an equal amount of money. Then Jane had twice as much money as her sister. How much money did their parents give each of them?



What is most exciting is that rather than using the usual student favorite—guess and check, or at least guess—students tackle word problems with efficient and strategic visual models that lead to generalizations. In summary, this puts them on the road to algebra and future success in higher-level mathematics.

The development of successful problem solving skills is a key part of mathematics learning. At the core of ***Math in Focus*** is the systematic development of these skills tied directly to the arithmetic. With ***Math in Focus*** and its model drawing approach, students gain the skills they need to tackle more and more complicated word problems from grade level to grade level. The program fosters both good number sense and the ability to solve complex problems. We can all agree these should be the goals of any good math program.

About the Author

Andy Clark is a former elementary and junior high school teacher. He recently retired as the K–12 Math Coordinator for Portland Public Schools, an urban district that outperformed the state of Oregon and closed the achievement gap. Clark is coauthor of a number of math programs, including ***Every Day Counts®: Calendar Math*** (Pre-K–6), ***Algebra Readiness*** (6 & Up), ***Partner Games*** (K–6), and ***Math in Focus*** (K–5).

Math in Focus™

Singapore Math

by Marshall Cavendish

Math in Focus: Singapore Math by ***Marshall Cavendish*** is the U.S. edition of *My Pals are Here! Maths,* the world-class program most widely used in Singapore classrooms today. **Marshall Cavendish** math programs have contributed to Singapore's consistent top performance on the Trends in International Math and Science Study (TIMSS) since 1995.

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Marshall Cavendish Education

800.289.4490
hmheducation.com/mathinfocus