

Sequential Logic, Finite State Machines

Instructor: Nick Riasanovsky

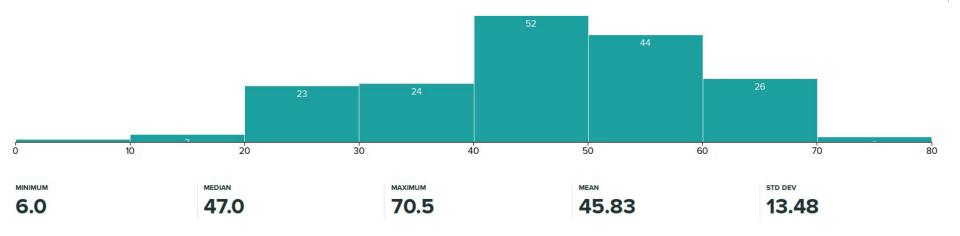
Nick's Lecture Feedback

- 2 Big Takeaways:
 - My lectures are too fast
 - Understood and I will go slower from now on
 - Cool it with the analogies
 - Understood! But with a couple topics you will still see some analogies
 - Only ones with a clear and easy to understand relationship
 - No more Harry Potter explanations

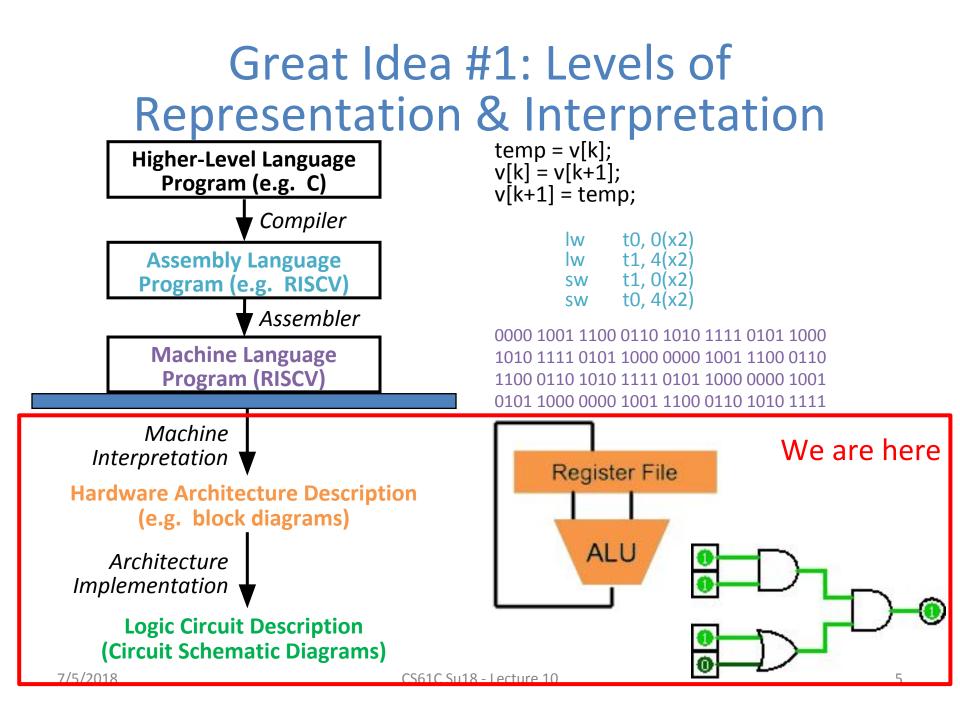
Questions

- Pretty seem overall to be satisfied with the number of questions we answer
- We LOVE to answer your questions but this is some of what make lectures have a time crunch
 - Going too fast is still a me problem I'll fix
- If you have a question that expands beyond the material in the course please consider using the piazza lecture thread

Midterm Grades



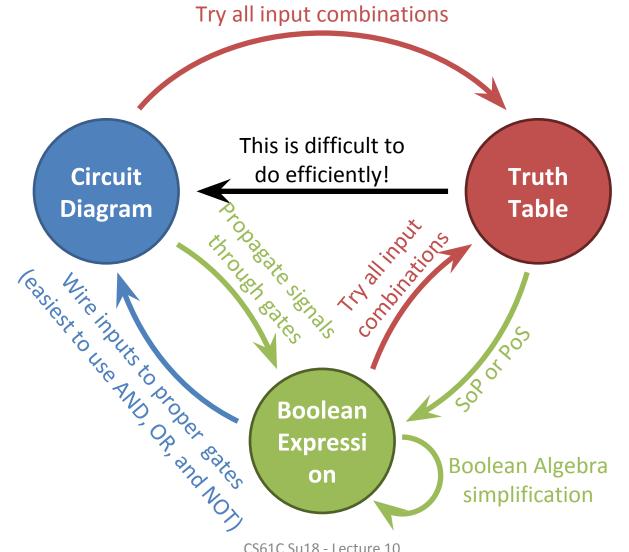
- Midterm Grades are released on gradescope
- Regrades available after lecture, due by Tues
- You did great, this exam was hard



Review

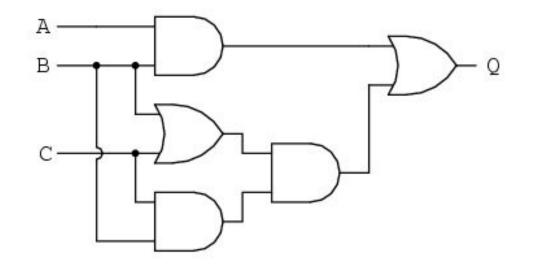
- Hardware is made up of transistors and wires
 - Building blocks of all higher-level blocks
- Synchronous Digital Systems
 - All signals are seen as either 0 or 1
 - Consist of two basic types of circuits:
 - Combinational Logic (CL)
 - AND, OR, XOR
 - Use Truth Tables -> Boolean Algebra
 - Sequential Logic (SL)

Converting Combinational Logic



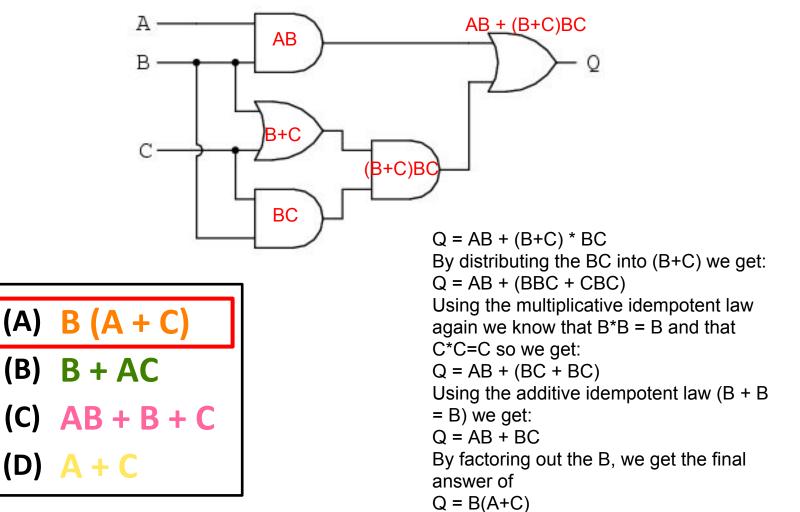


Question: What is the MOST simplified Boolean Algebra expression for the following circuit?





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Review

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Today's Menu

- Sequential Logic (SL)
 - Pulse of a Clock controls flow of information
 - Allows us to keep state (the basis of memory)
 - Understand how to efficiently use our hardware
- Finite State Machines
 - Abstract away combinational and sequential logic into **functions**
 - These functions take in a stream of bits, do something with them, and output something deterministic

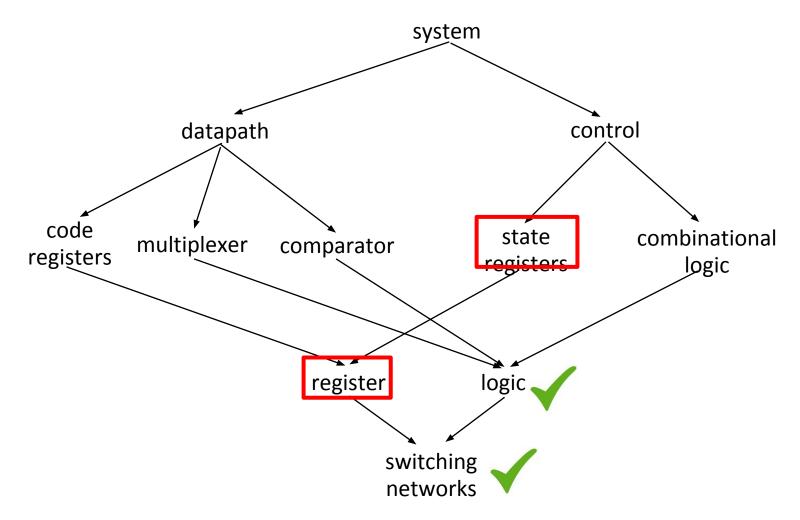
Agenda

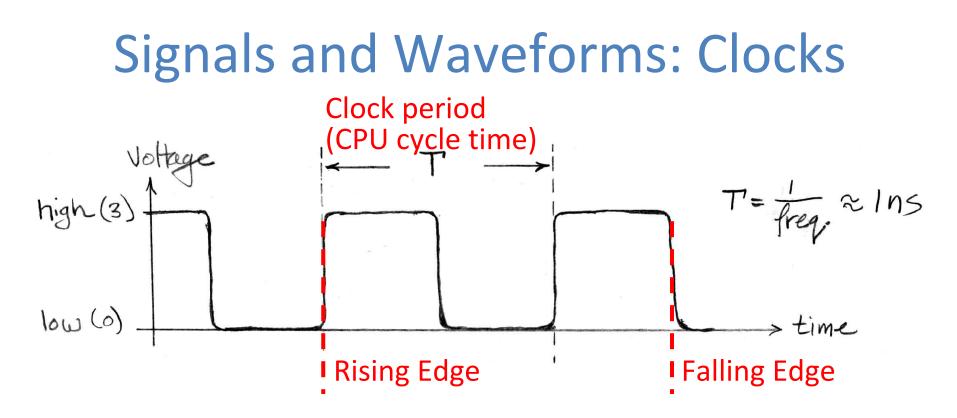
- Transistors, Switching Networks
- Combinational Logic Representations
 - Truth Tables
 - Boolean Algebra
- Meet the Staff
- Sequential Logic
 - Timing Terminology
 - State Elements

Type of Circuits

- Synchronous Digital Systems consist of two basic types of circuits:
 - Combinational Logic (CL)
 - Output is a function of the inputs only, not the history of its execution
 - e.g. circuits to add A, B (ALUs)
 - Sequential Logic (SL)
 - Circuits that "remember" or store information
 - a.k.a. "State Elements"
 - e.g. memory and registers (Registers)

Hardware Design Hierarchy

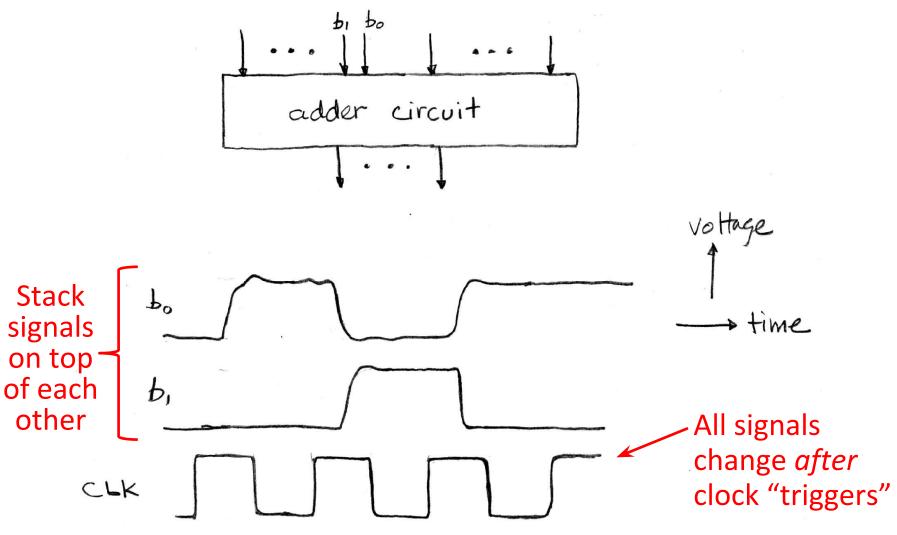


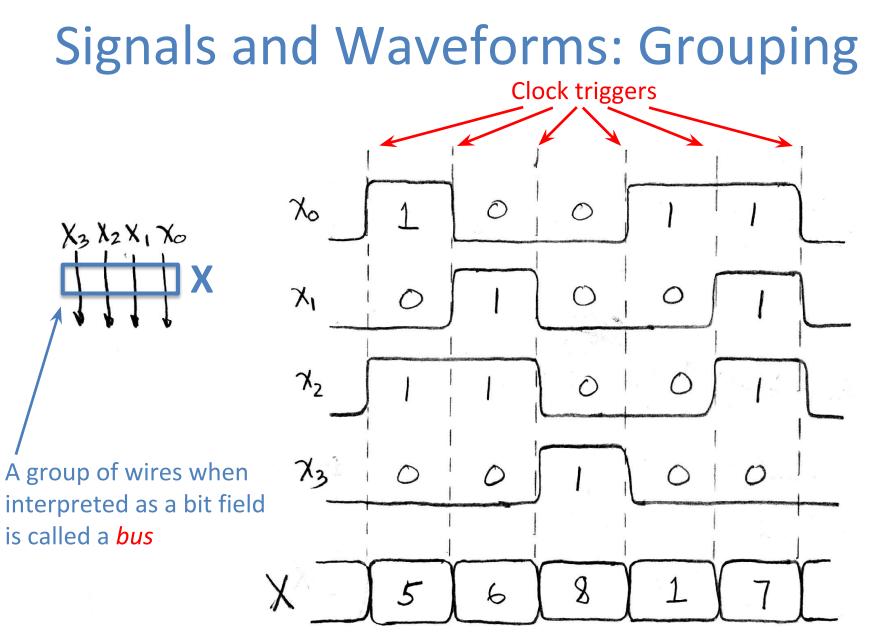


- Signals transmitted over wires continuously
- Transmission is effectively instantaneous
 - Implies that any wire only contains one value at any given time

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Signals and Waveforms





Uses for State Elements

- Place to store values for some amount of time:
 - Register files (like in RISCV)
 - Memory (caches and main memory)
- Help control flow of information between combinational logic blocks
 - State elements are used to hold up the movement of information at the inputs to combinational logic blocks and allow for orderly passage

Accumulator Example

An example of why we would need to control the flow of information.



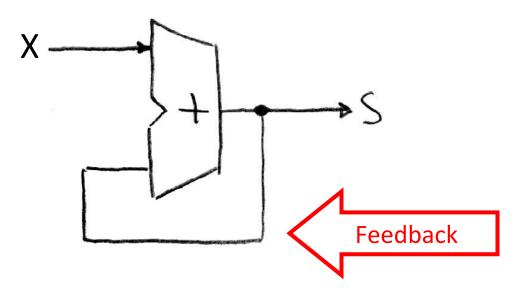
Want: S=0;

for X_1, X_2, X_3 over time... $S = S + X_i$

Assume:

- Each X value is applied in succession, one per cycle
- \bullet The sum since time 1 (cycle) is present on ${\rm S}$

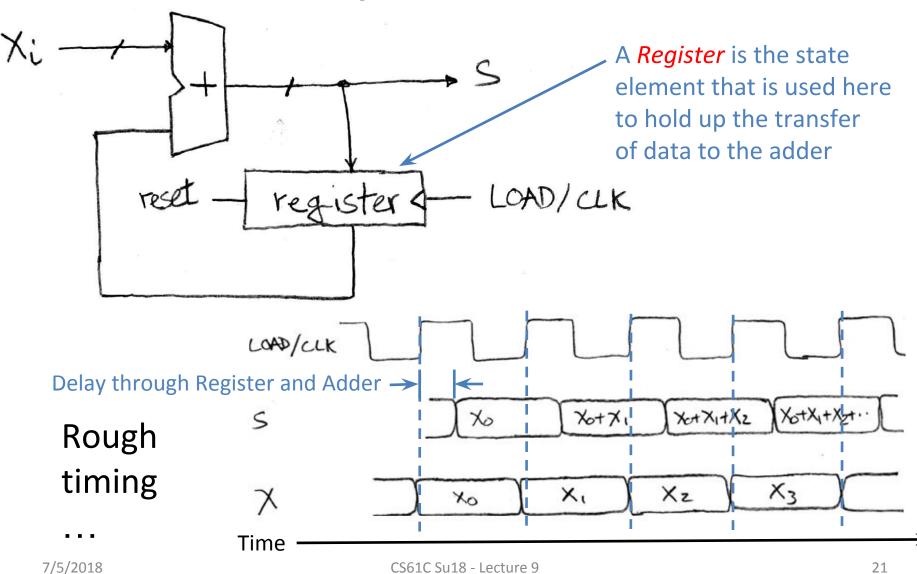
First Try: Does this work?



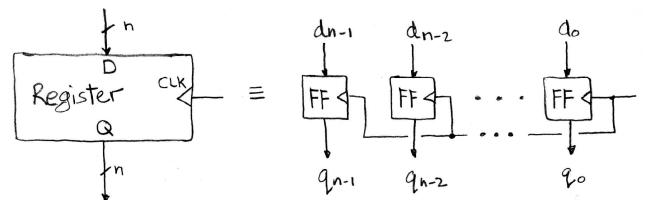
No!

- 1) How to control the next iteration of the 'for' loop?
- 2) How do we say: 'S=0'?

Second Try: How About This?



Register Internals



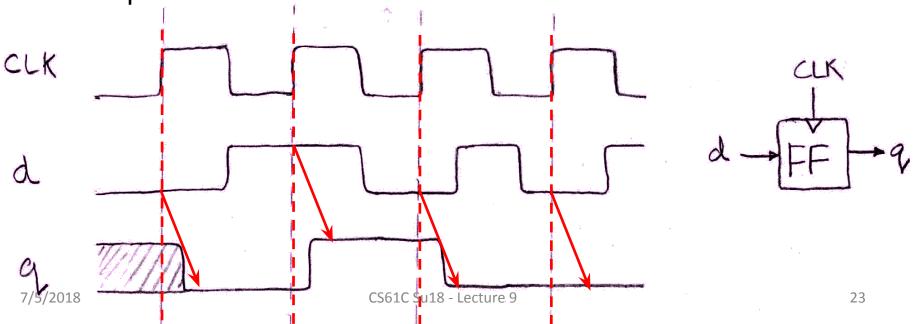
n instances of a "Flip-Flop"

Output flips and flops between 0 and 1

- Specifically this is a "D-type Flip-Flop"
 - D is "data input", Q is "data output"
 - In reality, has 2 outputs (Q and Q), but we only care about 1
- <u>http://en.wikibooks.org/wiki/Practical_Electronics/Flip-flops</u> 7/5/2018 CS61C Su18 - Lecture 9

Flip-Flop Timing Behavior (1/2)

- Edge-triggered D-type flip-flop
 - This one is "positive edge-triggered"
- "On the rising edge of the clock, input d is sampled and transferred to the output. At other times, the input d is ignored and the previously sampled value is retained."
- Example waveforms:



Flip-Flop Timing Terminology (1/3)

- Camera Analogy: non-blurry digital photo
 - Don't move while camera shutter is opening
 - Don't move while camera shutter is closing
 - Check for blurriness once image appears on the display



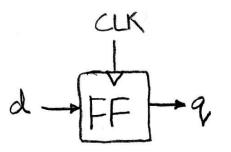
Flip-Flop Timing Terminology (2/3)

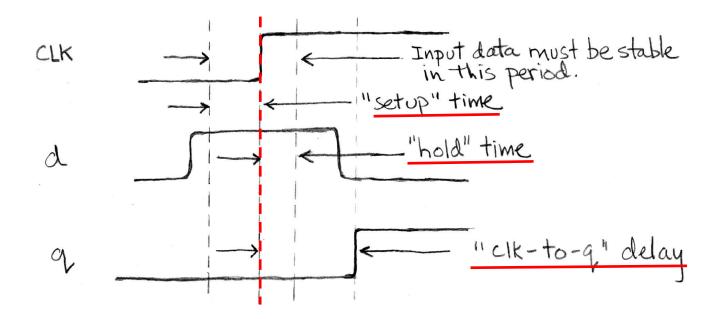
- Camera Analogy: Taking a photo
 - Setup time: don't move since about to take picture (open camera shutter)
 - Hold time: need to hold still after shutter opens until camera shutter closes
 - *Time to data:* time from open shutter until image appears on the output (viewfinder)

Flip-Flop Timing Terminology (3/3)

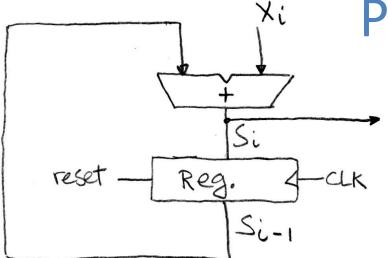
- Now applied to hardware:
 - Setup Time: how long the input must be stable before the CLK trigger for proper input read
 - Hold Time: how long the input must be stable after the CLK trigger for proper input read
 - "CLK-to-Q" Delay: how long it takes the output to change, measured from the CLK trigger

Flip-Flop Timing Behavior

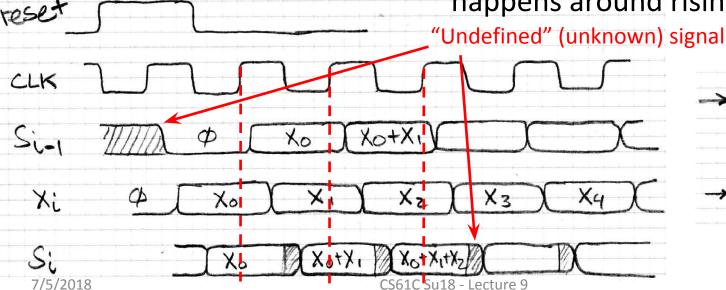


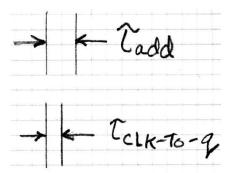


Accumulator Revisited Proper Timing (2/2)



- reset signal shown
- Also, in practice X_i might not arrive to the adder at the same time as S_{i-1}
- S_i temporarily is wrong, but register always captures correct value
- In good circuits, instability never happens around rising edge of CLK





Dealing with Waveform Diagrams

- Easiest to start with CLK on top
 - Solve signal by signal, from inputs to outputs
 - Can only draw the waveform for a signal if *all* of its input waveforms are drawn
- When does a signal update?
 - A *state element* updates based on CLK triggers
 - A combinational element updates ANY time ANY of its inputs changes

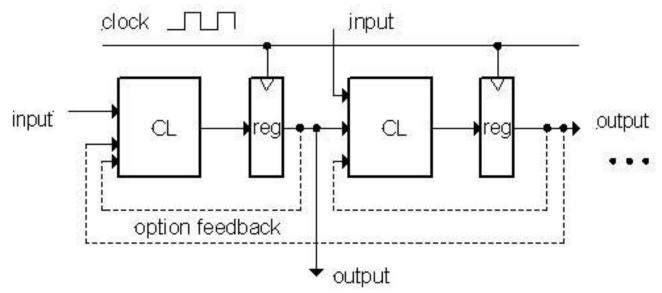
Review of Timing Terms

- **Clock:** steady square wave that synchronizes system
- Flip-flop: one bit of state that samples every rising edge of CLK (positive edge-triggered)
- **Register:** several bits of state that samples on rising edge of CLK (positive edge-triggered); also has RESET
- Setup Time: when input must be stable *before* CLK trigger
- Hold Time: when input must be stable *after* CLK trigger
- CLK-to-Q Delay: how long it takes output to change from CLK trigger

Agenda

- Critical Path and Clock Frequency
- Administrivia
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- Multiplexers
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Model for Synchronous Systems



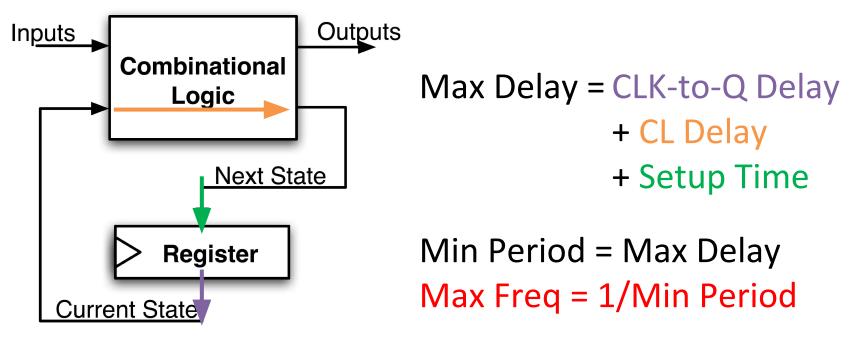
- Combinational logic blocks separated by registers
 - Clock signal connects only to sequential logic elements
 - Feedback is optional depending on application
- How do we ensure proper behavior?
 - How fast can we run our clock?

When Can the Input Change?

- When a register input changes shouldn't violate hold time (t_{hold}) or setup time (t_{setup}) constraints within a clock period (t_{period})
- Let t_{input} be the time it takes for the input of a register to change, measured from the CLK trigger:
 - Then we need $t_{hold} \leq t_{input} \leq t_{period} t_{setup}$
 - Two separate constraints!

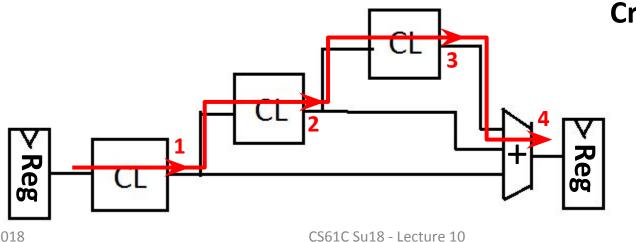
Maximum Clock Frequency

- What is the max frequency of this circuit?
 - Limited by how much time needed to get correct Next State to Register (t_{setup} constraint)



The Critical Path

- The *critical path* is the longest delay between *any* two registers in a circuit
- The clock period must be *longer* than this critical path, or the signal will not propagate properly to that next register



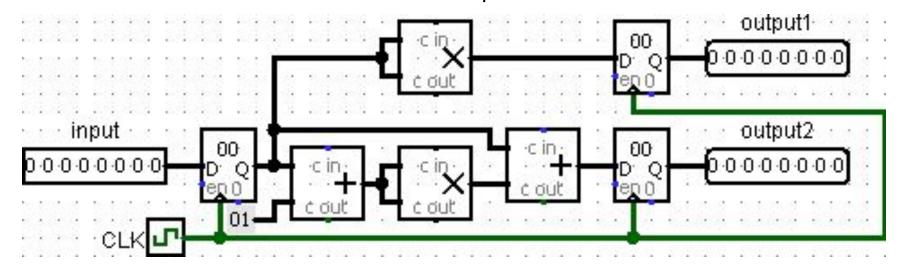
Critical Path = CLK-to-Q Delay + CL Delay 1 + CL Delay 2 + CL Delay 3 + Adder Delay + Setup Time

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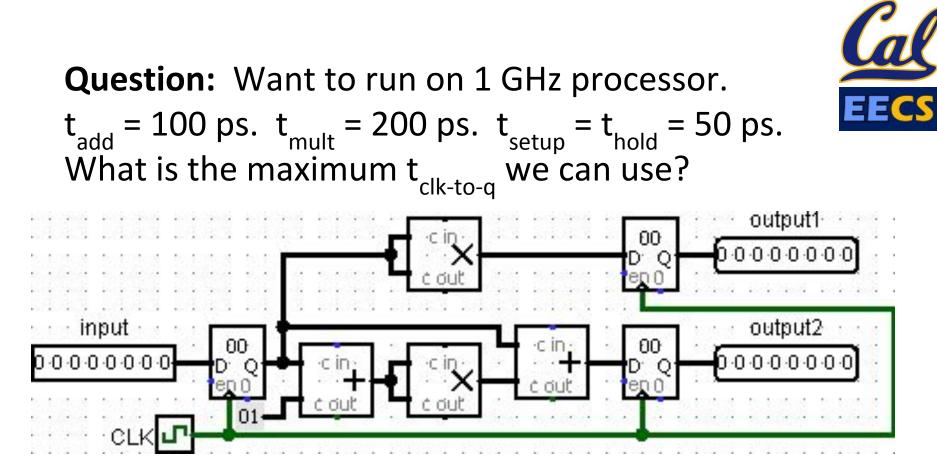


Question: Want to run on 1 GHz processor.

 $t_{add} = 100 \text{ ps. } t_{mult} = 200 \text{ ps. } t_{setup} = t_{hold} = 50 \text{ ps.}$ What is the maximum $t_{clk-to-q}$ there can be?



(A)	550 ps
(B)	750 ps
(C)	500 ps
(D)	700 ps



Bottom path is critical path: $T_clk-t-q + 100 + 200 + 100 + 50 < 1000 \text{ ps} = 1\text{ ns}$ $T_clk-t-q + 450 < 1000 \text{ ps}$ $T_clk-t-q < 550$

Administrivia

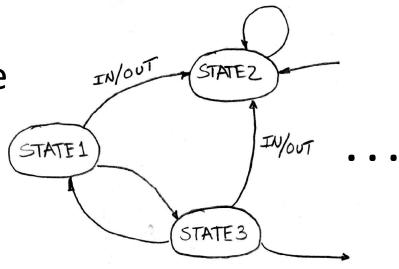
- Homework 2 due tomorrow
- Homework 3 released, due 7/16
- Homework 4 released 7/09, due 7/16
- Proj2-1 due tomorrow
 - Project party tomorrow, 4-6p in the Woz
- Proj2-2 will be released tomorrow
- Please don't leave lecture early!
 - And please DO NOT enter someone else's discussion while the room is still in use

Agenda

- Critical Path and Clock Frequency
- Administrivia
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- Multiplexers
- ALU Design
 - Adder/Subtracter
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Finite State Machines (FSMs)

- A convenient way to conceptualize computation over time
- Function can be represented with a *state transition diagram*
- With combinational logic registers, any FSM can be implemented in hardware!



FSM Overview

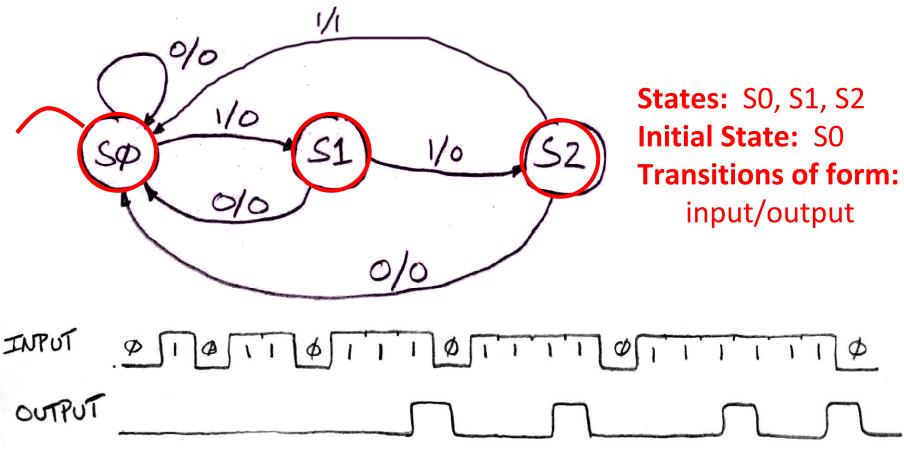
- An FSM (in this class) is defined by:
 - A set of *states* S (circles)
 - An *initial state* s₀ (only arrow not between states)
 - A transition function that maps from the current input and current state to the output and the next state (arrows between states)
- State transitions are controlled by the clock:
 - On each clock cycle the machine checks the inputs and generates a new state (could be same) and

^{7/5/2018} new output

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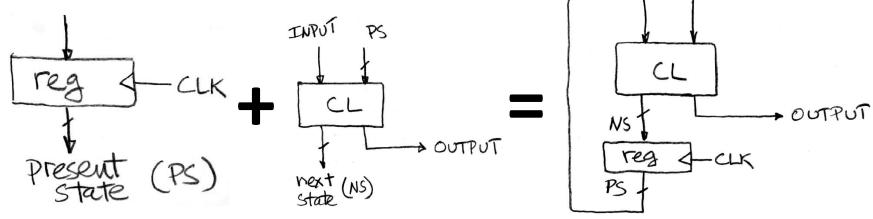
Example: 3 Ones FSM

• FSM to detect 3 consecutive 1's in the Input



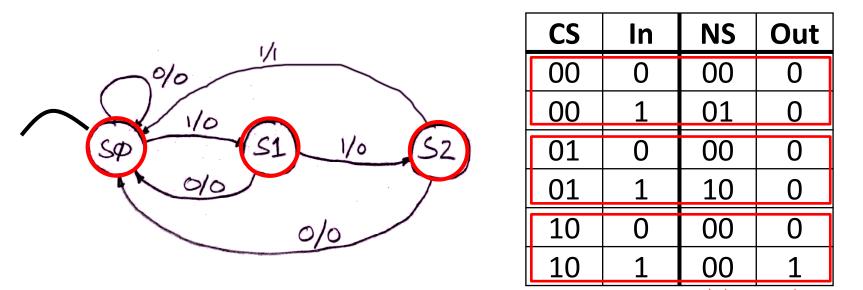
Hardware Implementation of FSM

- Register holds a representation of the FSM's state
 - Must assign a *unique* bit pattern for each state
 - Output is *present/current state* (PS/CS)
 - Input is *next state* (NS)
- Combinational Logic implements transition function (state transitions + output)



FSM: Combinational Logic

- Read off transitions into Truth Table!
 - Inputs: Current State (CS) and Input (In)
 - Outputs: Next State (NS) and Output (Out)



Implement logic for EACH output (2 for NS,¹ for Out)

Unspecified Output Values (1/2)

- Our FSM has only 3 states
 - 2 entries in truth table are undefined/unspecified
- Use symbol 'X' to mean it can be either a 0 or 1
 - Make choice to simplify final expression

CS	In	NS	Out	
00	0	00	0	
00	1	01	0	
01	0	00	0	
01	1	10	0	
10	0	00	0	
10	1	00	1	
11	0	XX	X	
11	1	XX	X	

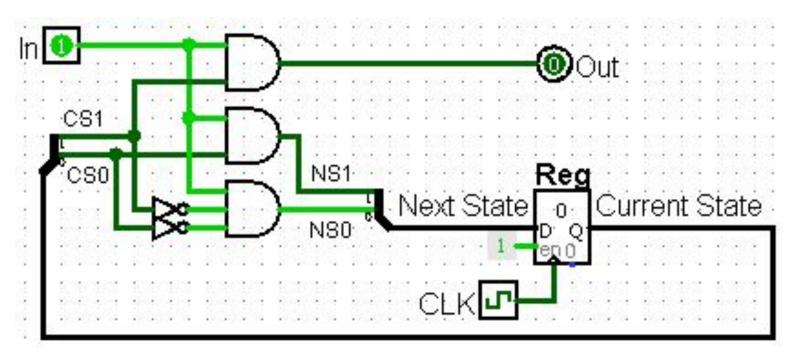
Unspecified Output Values (2/2)

- Let's find expression for NS₁
 - **Recall:** 2-bit output is just a2-bit bus, which is just 2 wires
- Boolean algebra: $-NS_{1} = -CS_{1}CS_{0}In + CS_{0}S_{0}-In + CS_{1}CS_{0}In + CS_{1}CS_{1}In + CS_{1}C$

CS	In	NS	Out	
00	0	00	0	
00	1	<mark>0</mark> 1	0	
01	0	00	0	
01	1	10	0	
10	0	00	0	
10) 1	00	1	
11	0	XX	Х	
11	1	XX	Х	

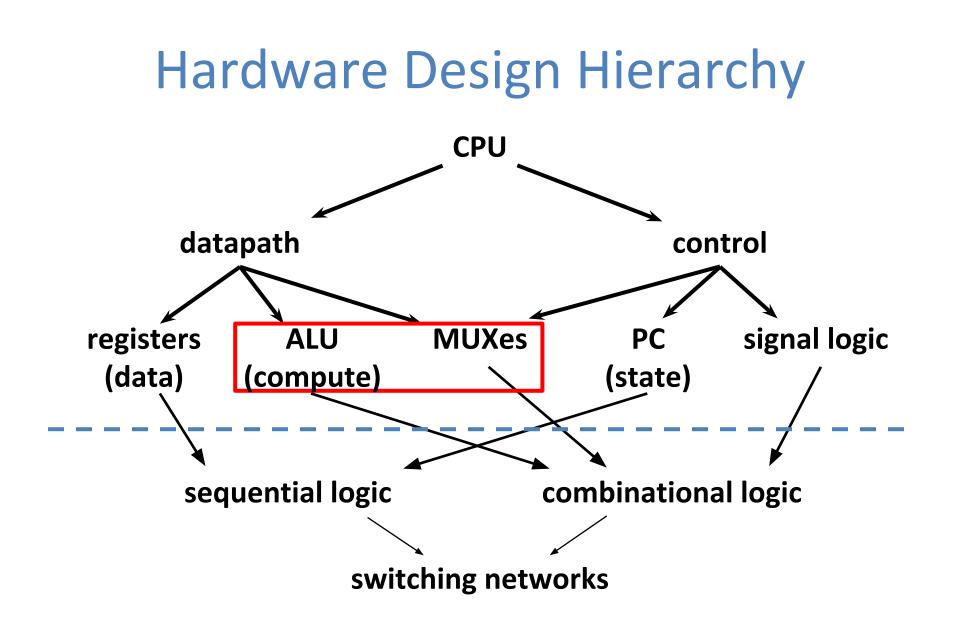
3 Ones FSM in Hardware

- 2-bit **Register** needed for state
- CL: $NS_1 = CS_0In$, $NS_0 = -CS_1 CS_0In$, $Out = CS_1In$



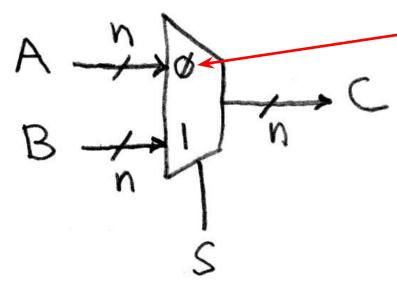
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Data Multiplexor

- Multiplexor ("MUX") is a *selector*
 - Place one of multiple inputs onto output (N-to-1)
- Shown below is an n-bit 2-to-1 MUX
 - Input S selects between two inputs of n bits each



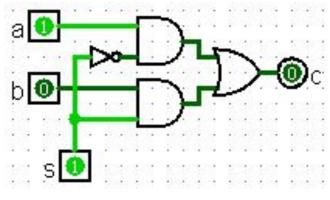
 This input is passed to output if selector bits match shown value

Implementing a 1-bit 2-to-1 MUX

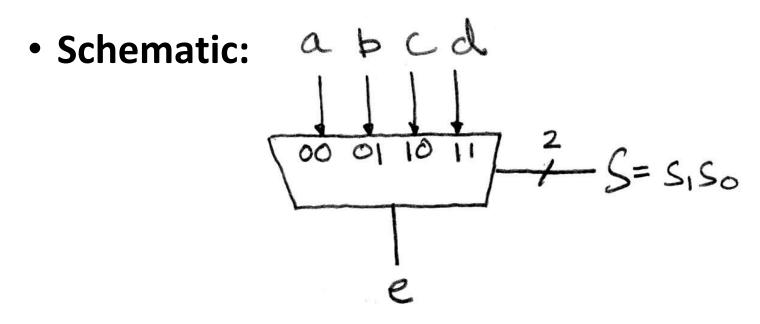
- Truth Table:

			_	
S	а	b	С	
0	0	0	0	
0	0	1	0	•
0	1	0	1	
0	1	1	1	
1	0	0	0	
1	0	1	1	
1	1	0	0	
1	1	1 CS61C S	1 5u18 - Le	cture 10

- Boolean Algebra:
- $c = \overline{s}a\overline{b} + \overline{s}ab + s\overline{a}b + sab$ = $\overline{s}(a\overline{b} + ab) + s(\overline{a}b + ab)$ = $\overline{s}(a(\overline{b} + b)) + s((\overline{a} + a)b)$ = $\overline{s}(a(1) + s((1)b)$ = $\overline{s}a + sb$
- Circuit Diagram:



1-bit 4-to-1 MUX (1/2)



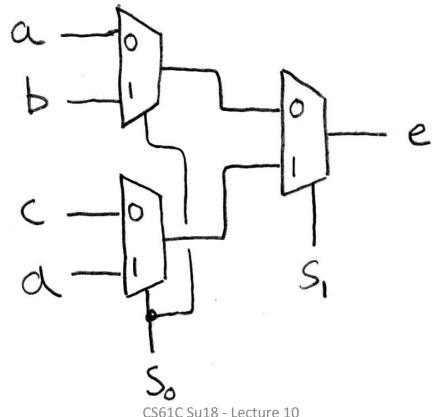
- Truth Table: How many rows? 2⁶
- Boolean Expression:

$$e = -s_1 - s_0 a + -s_1 s_0 b + s_1 - s_0 c + s_1 s_0 d$$

1-bit 4-to-1 MUX (2/2)

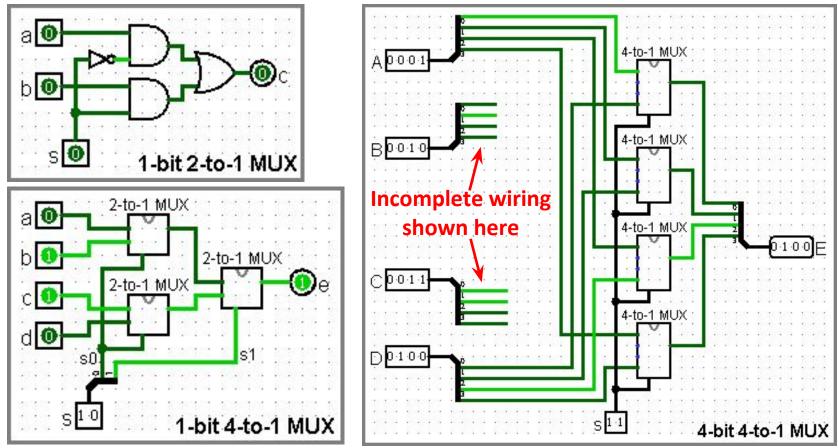
• Can we leverage what we've previously built?

- Alternative hierarchical approach:



Subcircuits Example

- Logisim equivalent of procedure or method
 - Every project is a hierarchy of subcircuits



Meet The Staff

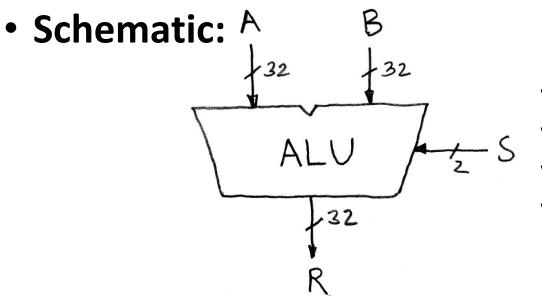
	Damon	Jon
Favorite Villain	Manray	E
Protest	No pineapples on pizza	Bring back L8 night
What are you passionate about?	Music	<u>;;</u>
What would you want to be famous for?	Chess	

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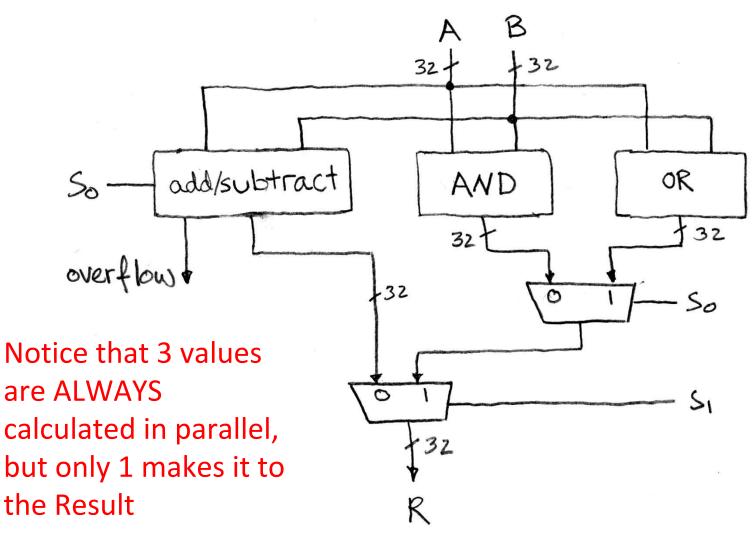
Arithmetic and Logic Unit (ALU)

- Most processors contain a special logic block called the "Arithmetic and Logic Unit" (ALU)
 - We'll show you an easy one that does ADD, SUB, bitwise AND, and bitwise OR



when S=00, R = A + Bwhen S=01, R = A - Bwhen S=10, R = A AND B when S=11, R = A OR B

Simple ALU Schematic



Adder/Subtractor Design

- CL design we've seen before: write out truth table, convert to Boolean, minimize logic, then implement
 - How big might truth table and/or Boolean expression get?
- 2) Break down the problem into smaller pieces that we can cascade or hierarchically layer
 – Let's try this approach instead

Adder/Subtractor: 1-bit LSB Adder

Carry-out bit b_0 S₀ \mathbf{a}_0 C_1 0 0 0 0 1 1 0 0 1 1 0 0 1 1

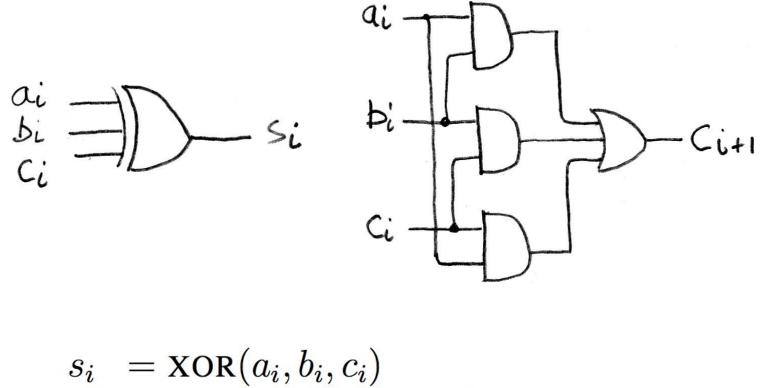
 $s_0 = a_0 \text{ XOR } b_0$ $c_1 = a_0 \text{ AND } b_0$

Adder/Subtractor: 1-bit Adder

				Possible	0.	h	0	G	0
		3		carry-in c_1	a_i	b_i	c_i	S_i	c_{i+1}
	a_3	a_2	a_1	a_0	0	0	0	0	0
+	b_3	b_2	b_1	\mathbf{b}_0	0	0	1	1	0
			- -		0	1	0	1	0
	s_3	s_2	\mathbf{s}_1	\mathbf{s}_0	0	1	1	0	1
1 0 0 1						1	0		
Here defining XOR of many inputs to be 1			1	0	1	0	1		
when an <i>odd</i> number of inputs are 1			1	1	0	0	1		
				1	1	1	1	1	
$s_i = \operatorname{XOR}(a_i, b_i, c_i)$									
c_{i+1} = MAJ $(a_i, b_i, c_i) = a_i b_i + a_i c_i + b_i c_i$									
7/5/20	18			CS61C Su18 - Lo	turo 10				61

Adder/Subtractor: 1-bit Adder

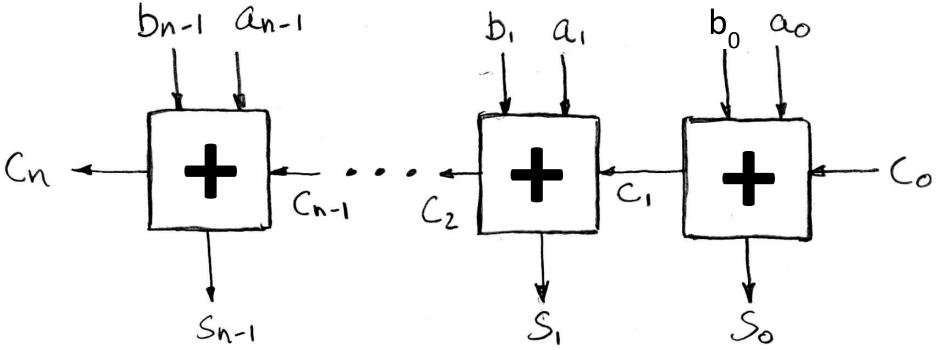
• Circuit Diagrams:



 $c_{i+1} = \operatorname{maj}(a_i, b_i, c_i) = a_i b_i + a_i c_i + b_i c_i$

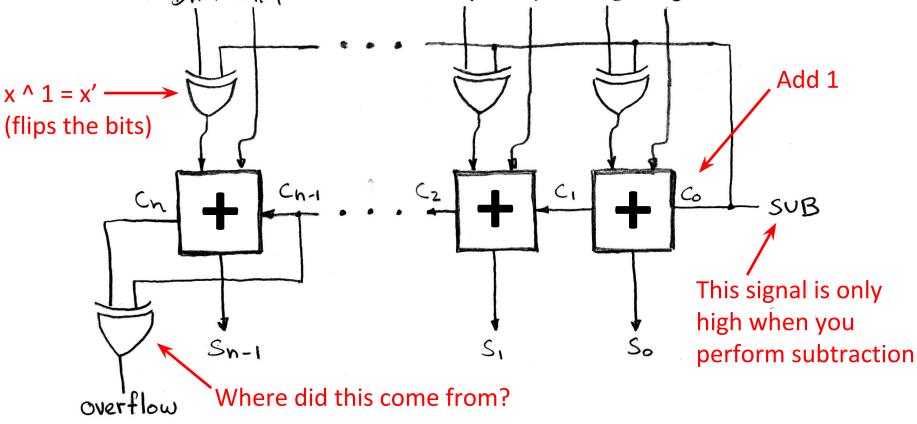
$N \ge 1$ -bit Adders \rightarrow N-bit Adder

Connect CarryOut_{i-1} to CarryIn_i to chain adders:



Two's Complement Adder/Subtractor

• Subtraction accomplished by adding negated number: $b_{n-1} a_{n-1}$ $b_{n-1} a_{n-1}$



Summary

- Hardware systems are constructed from *Stateless* Combinational Logic and *Stateful* "Memory" Logic (registers)
- State registers implemented from Flip-flops

Summary

• Critical (longest) path constrains clock rate

 $-\operatorname{Need} t_{hold} \leq t_{input} \leq t_{period} - t_{setup}$

- Can adjust with extra registers (pipelining)
- Finite State Machines visualize state-based computations

Can implement systems with Register + CL

- Use MUXes to select among input
 - S input bits selects one of 2^s inputs
 - Each input is a bus n-bits wide
- Build n-bit adder out of chained 1-bit adders
 - Can also do subtraction with additional SUB signal

BONUS SLIDES

You are responsible for the material contained on the following slides, though we may not have enough time to get to them in lecture.

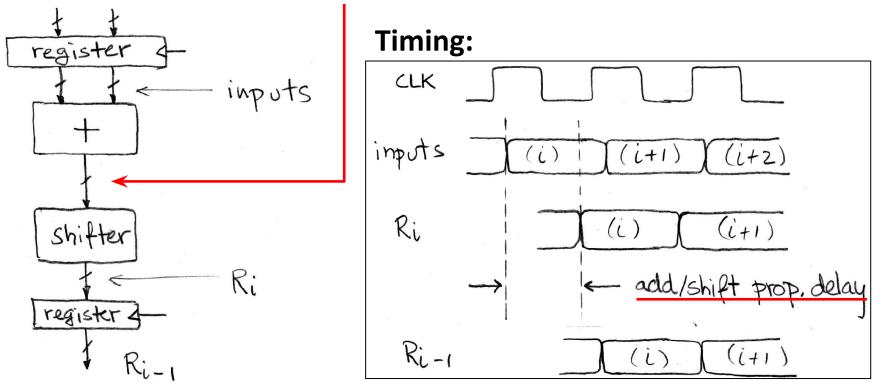
They have been prepared in a way that should be easily readable and the material will be touched upon in the following lecture.

Agenda

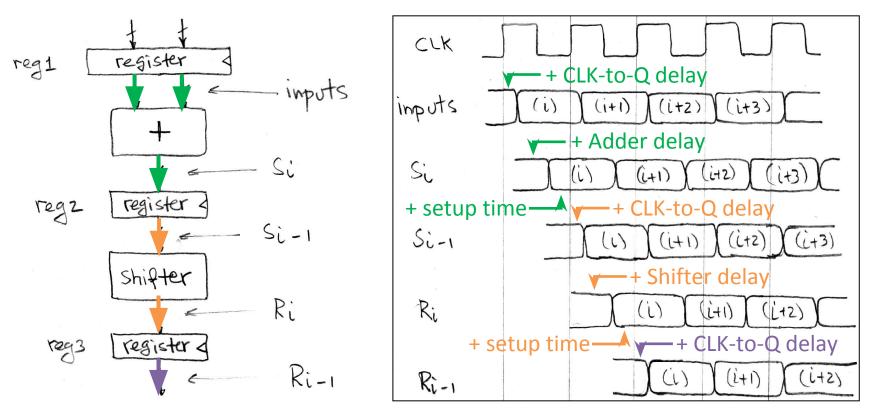
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Pipelining and Clock Frequency (1/2)

- Clock period limited by propagation delay of adder and shifter
 - Add an extra register to reduce the critical path!



Pipelining and Clock Frequency (2/2)



- Reduced critical path \rightarrow allows higher clock freq.
- Extra register \rightarrow extra (shorter) cycle to produce first output

Pipelining Basics

- By adding more registers, break path into shorter "stages"
 - Aim is to reduce critical path
 - Signals take an additional clock cycle to propagate through *each* stage
- New critical path must be calculated
 - Affected by placement of new pipelining registers
 - Faster clock rate \rightarrow higher throughput (outputs)
 - More stages \rightarrow higher startup latency
- Pipelining tends to improve performance
 - More on this (application to CPUs) later

Agenda

- Critical Path and Clock Frequency
- Administrivia
- Finite State Machines
- Multiplexers
- ALU Design
 - Adder/Subtracter
- Bonus:
 - Pipelining intro
 - Handling overflow
 - Logisim Introduction

Detecting Overflow

- Unsigned overflow
 - On addition, if carry-out from MSB is 1
 - On subtraction, if carry-out from MSB is 0
 - This case is a lot harder to see than you might think
- Signed overflow
 - 1) Overflow from adding "large" positive numbers
 - 2) Overflow from adding "large" negative numbers

Signed Overflow Examples (4-bit)

- Overflow from two positive numbers:
 - 0111 + 0111, 0111 + 0001, 0100 + 0100.
 - Carry-out from the 2nd MSB (but not MSB)
 - pos + pos \neq neg
- Overflow from two negative numbers:
 - 1000 + 1000, 1000 + 1111, 1011 + 1011.
 - Carry-out from the MSB (but not 2nd MSB)
 - neg + neg \neq pos
- Expression for signed overflow: C_n XOR C_{n-1}

Agenda

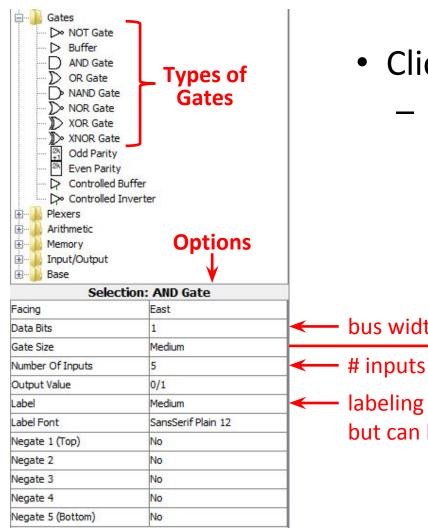
- Critical Path and Clock Frequency
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Logisim

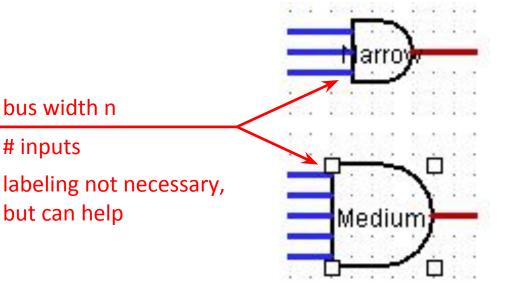
- Open-source (i.e. free!) "graphical tool for designing and simulating logic circuits"
 - Runs on Java on any computer
 - Download to your home computer via class login or the Logisim website (we are using version 2.7.1)
- No programming involved
 - Unlike Verilog, which is a hardware description language (HDL)
 - Click and drag; still has its share of annoying quirks

http://ozark.hendrix.edu/~burch/logisim/

Gates in Logisim

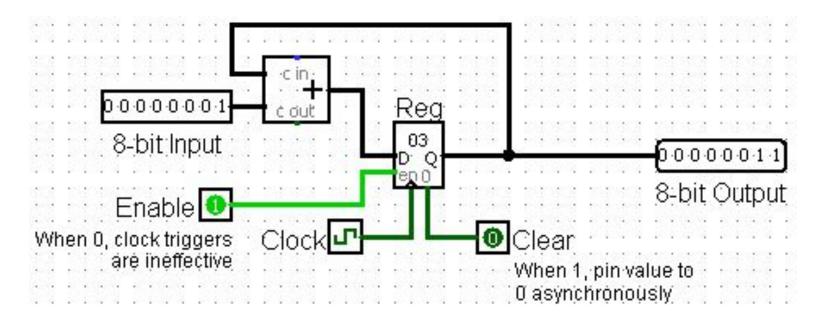


- Click gate type, click to place
 - Can set options *before* placing or select gate *later* to change



Registers in Logisim

- Flip-flops and Registers in "Memory" folder
- 8-bit accumulator:



Wires in Logisim

- Click and drag on existing port or wire
- Color schemes: - Gray: unconnected Input1 - Dark Green: low signal (0) - Light Green: high signal (1) Clock – **Red:** error 3-bit Input 1 0 0 1-bit Outpu - Blue: undetermined signal 1-bit Output -bit Output Orange: incompatible widths • Tunnels: all tunnels with same label are

connected Tunnel Tunnel

Common Mistakes in Logisim

- Connecting wires together
 - Crossing wires vs. connected wires
- Losing track of which input is which
 - Mis-wiring a block (e.g. CLK to Enable)
 - Grabbing wrong wires off of splitter

• Errors:

