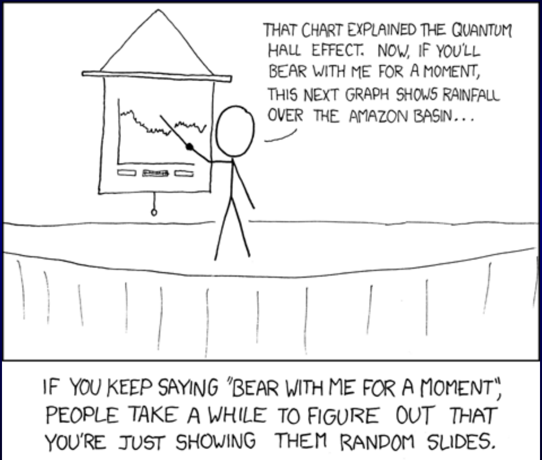
Quantum Mechanics for Engineers

Matt Robinson

Baylor University
Department of Physics


THAT CHART EXPLAINED THE QUANTUM HALL EFFECT. NOW, IF YOU'LL BEAR WITH ME FOR A MOMENT, THIS NEXT GRAPH SHOWS RAINFALL OVER THE AMAZON BASIN...
IF YOU KEEP SAYING "BEAR WITH ME FOR A MOMENT", PEOPLE TAKE A WHILE TO FIGURE OUT THAT YOU'RE JUST SHOWING THEM RANDOM SLIDES.

Outline

1. What is Classical Physics?

Outline

1. What is Classical Physics?
2. What is Quantum Physics?

Outline

Actions and The Principle of Least Action

- Knowing the (scalar) expressions for the kinetic energy $T(\dot{q})$ and the potential energy $V(q)$ for a system, we can define the *Lagrangian* as

$$L(q, \dot{q}) \equiv T - V$$

Actions and The Principle of Least Action

- Knowing the (scalar) expressions for the kinetic energy $T(\dot{q})$ and the potential energy $V(q)$ for a system, we can define the *Lagrangian* as

$$L(q, \dot{q}) \equiv T - V$$

- The *Action* $\mathcal{S}[q]$ is then defined as the integral over time,

$$\mathcal{S}[q] \equiv \int dt \; L(q, \dot{q})$$

Actions and The Principle of Least Action

- The equations of motion are then given by the zeros of the *Euler Lagrange Derivative*,

$$\frac{\delta L}{\delta q} \equiv \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0$$

Actions and The Principle of Least Action

- The equations of motion are then given by the zeros of the *Euler Lagrange Derivative*,

$$\frac{\delta L}{\delta q} \equiv \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0$$

- This is called the principle of *Least Action*.

Projectile Motion

- $T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2), \qquad V = mgy$

Projectile Motion

- $T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2), \qquad V = mgy$
- $L = T - V = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - mgy$

Projectile Motion

- $T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2), \qquad V = mgy$
- $L = T - V = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - mgy$
- $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{d}{dt}m\dot{x} = m\ddot{x}, \quad \frac{\partial L}{\partial x} = 0$

$$\Rightarrow m\ddot{x} = 0$$

Projectile Motion

- $T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2), \qquad V = mgy$
- $L = T - V = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - mgy$
- $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{d}{dt}m\dot{x} = m\ddot{x}, \quad \frac{\partial L}{\partial x} = 0$

$$\Rightarrow m\ddot{x} = 0$$

- $\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} = \frac{d}{dt}m\dot{y} = m\ddot{y}, \quad \frac{\partial L}{\partial y} = -mg$

$$\Rightarrow m\ddot{y} = -mg$$

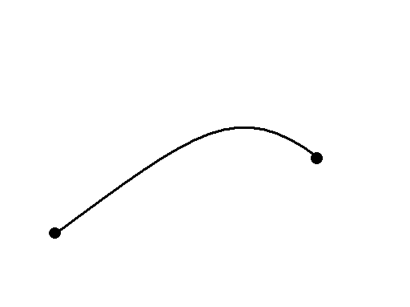
Concluding Thoughts on Classical Physics

Concluding Thoughts on Classical Physics

- For a given initial condition, only *one* path is possible - the path which satisfies the Euler-Lagrange Equation.

Concluding Thoughts on Classical Physics

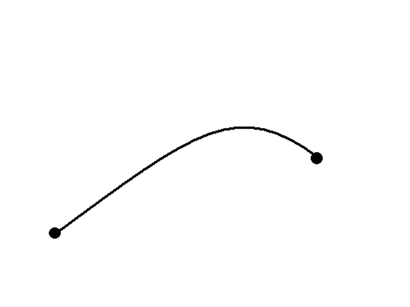
- For a given initial condition, only *one* path is possible - the path which satisfies the Euler-Lagrange Equation.



Concluding Thoughts on Classical Physics

- For a given initial condition, only *one* path is possible - the path which satisfies the Euler-Lagrange Equation.
- The nature of classical physics can be thought of as follows:

INPUT: Ask question.
OUTPUT: Get answer.



So What is Quantum Physics?

Principle of Least Action . . . *Probably*

Principle of Least Action . . . *Probably*

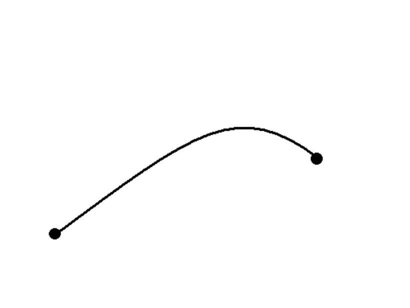
- Classical physics demanded

$$\frac{\delta L}{\delta q} = 0$$

Principle of Least Action . . . *Probably*

- Classical physics demanded

$$\frac{\delta L}{\delta q} = 0$$



Principle of Least Action . . . *Probably*

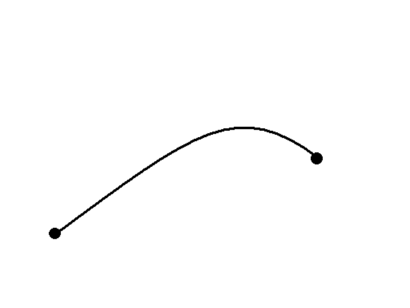
- Classical physics demanded

$$\frac{\delta L}{\delta q} = 0$$

- Quantum physics allows *any* value on the right hand side

$$\frac{\delta L}{\delta q} = \lambda$$

where λ can be anything.



Principle of Least Action . . . *Probably*

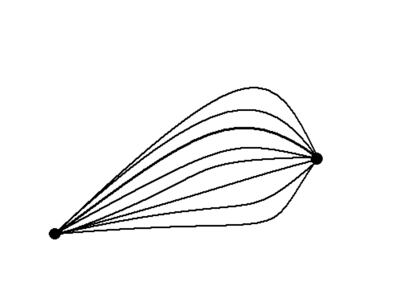
- Classical physics demanded

$$\frac{\delta L}{\delta q} = 0$$

- Quantum physics allows *any* value on the right hand side

$$\frac{\delta L}{\delta q} = \lambda$$

where λ can be anything.



Principle of Least Action . . . *Probably*

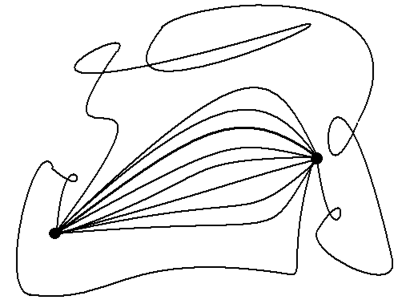
- Classical physics demanded

$$\frac{\delta L}{\delta q} = 0$$

- Quantum physics allows *any* value on the right hand side

$$\frac{\delta L}{\delta q} = \lambda$$

where λ can be anything.



Principle of Least Action . . . *Probably*

- However, not all paths are equally *probable*.

Principle of Least Action . . . *Probably*

- However, not all paths are equally *probable*.
- Each possible path has a statistical weight/probability equal to

$$e^{-\mathcal{S}[q]/\hbar} \qquad \text{or} \qquad e^{i\mathcal{S}[q]/\hbar}$$

(these are the same under the change of variables $dt \longrightarrow idt$)

But it gets worse

Interference

- $q \longrightarrow e^{-\mathcal{S}[q]/\hbar}$

Interference

- $q \quad \longrightarrow \quad e^{-\mathcal{S}[q]/\hbar}$
- $q + \delta q \quad \longrightarrow \quad e^{-\mathcal{S}[q+\delta q]/\hbar}$

Interference

- $q \longrightarrow e^{-\mathcal{S}[q]/\hbar}$
- $q + \delta q \longrightarrow e^{-\mathcal{S}[q+\delta q]/\hbar}$

- $e^{-\mathcal{S}[q+\delta q]/\hbar} = e^{-(\mathcal{S}[q]+\delta q \frac{\delta \mathcal{S}[q]}{\delta q})/\hbar} = e^{-\mathcal{S}[q]/\hbar} e^{-\frac{\delta q}{\hbar}\frac{\delta \mathcal{S}[q]}{\delta q}}$

Interference

- $q \longrightarrow e^{-\mathcal{S}[q]/\hbar}$
- $q + \delta q \longrightarrow e^{-\mathcal{S}[q+\delta q]/\hbar}$
- $e^{-\mathcal{S}[q+\delta q]/\hbar} = e^{-(\mathcal{S}[q]+\delta q\frac{\delta\mathcal{S}[q]}{\delta q})/\hbar} = e^{-\mathcal{S}[q]/\hbar}e^{-\frac{\delta q}{\hbar}\frac{\delta\mathcal{S}[q]}{\delta q}}$
- Most likely path is where

$$\frac{\delta\mathcal{S}[q]}{\delta q} = 0$$

(which is the same as where $\frac{\delta L}{\delta q} = 0$)

Equation of State

- If $\psi \propto e^{i\mathcal{S}/\hbar}$, then

$$\frac{\partial \psi}{\partial t} = \frac{i}{\hbar}\frac{\partial \mathcal{S}}{\partial t}\psi$$

Equation of State

- If $\psi \propto e^{i\mathcal{S}/\hbar}$, then

$$\frac{\partial \psi}{\partial t} = \frac{i}{\hbar}\frac{\partial \mathcal{S}}{\partial t}\psi$$

$$H\psi = i\hbar\frac{\partial \psi}{\partial t}$$

(where $H \equiv T + V$)

Equation of State

- If $\psi \propto e^{i\mathcal{S}/\hbar}$, then

$$\frac{\partial \psi}{\partial t} = \frac{i}{\hbar}\frac{\partial \mathcal{S}}{\partial t}\psi$$

$$H\psi = i\hbar\frac{\partial \psi}{\partial t}$$

(where $H \equiv T + V$)
Solving this differential equation (called Schroedinger's Equation) for ψ will give the (complex) *Wave Function* for the particle

Equation of State

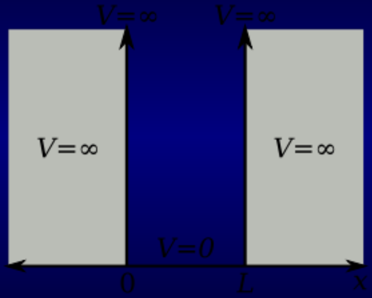
- If $\psi \propto e^{i\mathcal{S}/\hbar}$, then

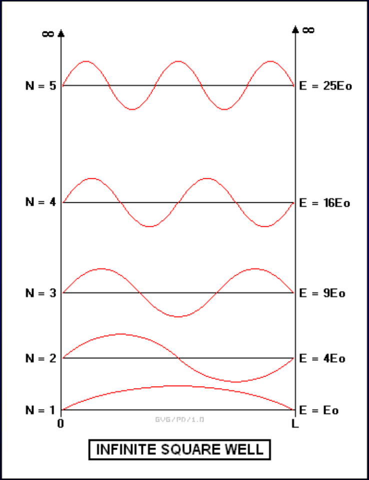
$$\frac{\partial \psi}{\partial t} = \frac{i}{\hbar}\frac{\partial \mathcal{S}}{\partial t}\psi$$

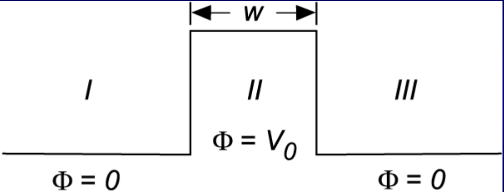
$$H\psi = i\hbar\frac{\partial \psi}{\partial t}$$

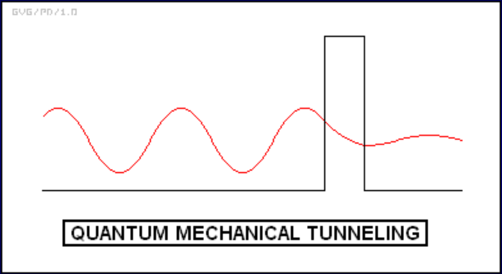
(where $H \equiv T + V$)
Solving this differential equation (called Schroedinger's Equation) for ψ will give the (complex) *Wave Function* for the particle

$$\int_a^b \psi^\star \psi \, dx = \int_a^b |\psi|^2 \, dx$$


V=∞
V=∞
V=∞
V=∞
V=0
0
L
x

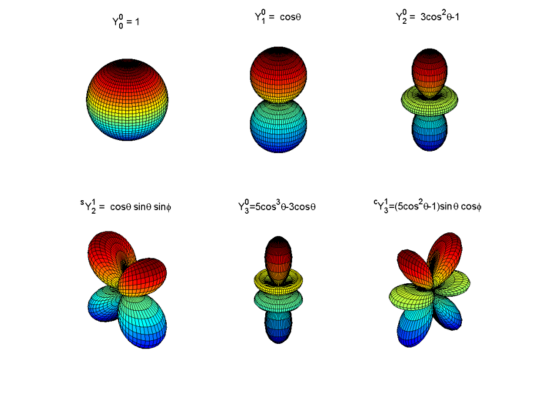

∞
∞
N = 5
E = 25Eo
N = 4
E = 16Eo
N = 3
E = 9Eo
N = 2
E = 4Eo
N = 1
E = Eo
0
L
INFINITE SQUARE WELL


w
I
II
III
Φ = V0
Φ = 0
Φ = 0


GVG/PD/1.0
QUANTUM MECHANICAL TUNNELING

Hydrogen Atom

Hydrogen Atom



Observation

But it gets *even* worse ...

Observation

- A system exists in every possible state it can be in prior to observation (superposition).

Observation

- A system exists in every possible state it can be in prior to observation (superposition).
- These possibilities all interfere with each other.

Observation

- A system exists in every possible state it can be in prior to observation (superposition).
- These possibilities all interfere with each other.
- Once an observation is made, the system "collapses" into one of the possible states, according to some probability distribution which depends on the system.

Concluding Thoughts on Quantum Physics

- For a given initial condition, *any* path/state is possible - no matter how improbable it may be.

Concluding Thoughts on Quantum Physics

- For a given initial condition, *any* path/state is possible - no matter how improbable it may be.
- How often you will *observe* that path/state is determined by the wave function, which is a probability distribution.

Concluding Thoughts on Quantum Physics

- For a given initial condition, *any* path/state is possible - no matter how improbable it may be.
- How often you will *observe* that path/state is determined by the wave function, which is a probability distribution.
- The classical path/state will be the most likely, which is why macroscopic systems appear classical.

Concluding Thoughts on Quantum Physics

- For a given initial condition, *any* path/state is possible - no matter how improbable it may be.
- How often you will *observe* that path/state is determined by the wave function, which is a probability distribution.
- The classical path/state will be the most likely, which is why macroscopic systems appear classical.
- The nature of quantum physics can be thought of as follows:

INPUT: Ask question, suggest an answer.
OUTPUT: How often that answer will be right.

Classical Bits and Quantum Bits

- Classical computer works using bits, which can either be 0 or 1

Classical Bits and Quantum Bits

- Classical computer works using bits, which can either be 0 or 1
- A quantum computer uses *quantum bits*, which can be 0 *and* 1

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Classical Bits and Quantum Bits

- Classical computer works using bits, which can either be 0 or 1
- A quantum computer uses *quantum bits*, which can be 0 *and* 1

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

- Measurement produces:

Classical Bits and Quantum Bits

- Classical computer works using bits, which can either be 0 or 1
- A quantum computer uses *quantum bits*, which can be 0 *and* 1

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

- Measurement produces:

$$0 \text{ with probability } |\alpha|^2$$

Classical Bits and Quantum Bits

- Classical computer works using bits, which can either be 0 or 1
- A quantum computer uses *quantum bits*, which can be 0 *and* 1

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

- Measurement produces:

$$\begin{array}{c} 0 \text{ with probability } |\alpha|^2 \\ 1 \text{ with probability } |\beta|^2 \end{array}$$

Manipulating Q-Bits

Manipulating Q-Bits

- Any (invertible) transformation done to $|\psi\rangle$ will act on every term in the superposition:

Manipulating Q-Bits

- Any (invertible) transformation done to $|\psi\rangle$ will act on every term in the superposition:

$$\text{NOT}|\psi\rangle = \text{NOT}(\alpha|0\rangle + \beta|1\rangle) = \alpha|1\rangle + \beta|0\rangle$$

Manipulating Q-Bits

- Any (invertible) transformation done to $|\psi\rangle$ will act on every term in the superposition:

$$\text{NOT}|\psi\rangle = \text{NOT}(\alpha|0\rangle + \beta|1\rangle) = \alpha|1\rangle + \beta|0\rangle$$

- But, all operations done to a q-bit must be reversible

Illustration of Why Quantum Computing Is So Powerful

Illustration of Why Quantum Computing Is So Powerful

Consider the *Hadamard* transformation H, which takes

Illustration of Why Quantum Computing Is So Powerful

Consider the *Hadamard* transformation H, which takes

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Illustration of Why Quantum Computing Is So Powerful

Consider the *Hadamard* transformation H, which takes

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

and

$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Illustration of Why Quantum Computing Is So Powerful

- Consider n copies of $|0\rangle$.

Illustration of Why Quantum Computing Is So Powerful

- Consider n copies of $|0\rangle$.
- Act on each with H.

Illustration of Why Quantum Computing Is So Powerful

- Consider n copies of $|0\rangle$.
- Act on each with H.
- Result is

Illustration of Why Quantum Computing Is So Powerful

- Consider n copies of $|0\rangle$.
- Act on each with H.
- Result is

$$\frac{1}{\sqrt{2^n}} \sum_{j=1}^{2^n} |j\rangle$$

Illustration of Why Quantum Computing Is So Powerful

- Consider $n = 3$. Acting on three $|0\rangle$'s with H gives

Illustration of Why Quantum Computing Is So Powerful

- Consider $n = 3$. Acting on three $|0\rangle$'s with H gives

$$\frac{1}{\sqrt{8}}(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

Illustration of Why Quantum Computing Is So Powerful

- Consider $n = 3$. Acting on three $|0\rangle$'s with H gives

$$\frac{1}{\sqrt{8}}(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

$$= \frac{1}{\sqrt{8}}\left(|0\rangle|0\rangle|0\rangle + |0\rangle|0\rangle|1\rangle + |0\rangle|1\rangle|0\rangle + \cdots + |1\rangle|1\rangle|1\rangle\right)$$

Illustration of Why Quantum Computing Is So Powerful

- Consider $n = 3$. Acting on three $|0\rangle$'s with H gives

$$\frac{1}{\sqrt{8}}(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

$$= \frac{1}{\sqrt{8}}\left(|0\rangle|0\rangle|0\rangle + |0\rangle|0\rangle|1\rangle + |0\rangle|1\rangle|0\rangle + \cdots + |1\rangle|1\rangle|1\rangle\right)$$

- So, any operation done to this state with n q-bits will produce a superposition of 2^n outcomes.


Threepwood@B3ta
QUANTUM JUNCTION
GET IN BOTH LANES
Max 30^2