Q1.

A golfer practising on a range with an elevated tee 4.9 m above the fairway is able to strike a ball so that it leaves the club with a horizontal velocity of 20 m s^{-1}. (Assume the acceleration due to gravity is 9.80 m s^{-2}, and the effects of air resistance may be ignored unless otherwise stated.)

Tee → 20 m s^{-1}

4.9 m

Fairway

a How long after the ball leaves the club will it land on the fairway?

b What horizontal distance will the ball travel before striking the fairway?

c What is the acceleration of the ball 0.5 s after being hit?

d Calculate the speed of the ball 0.80 s after it leaves the club.

e With what speed will the ball strike the ground?

A1.

a $x = ut + 0.5at^2$
then 4.9 m = 0 + 0.5(9.8 m s^{-2})t^2
and t = 1.0 s

b x = (average speed)(time) = (20 m s^{-1})(1.0 s) = 20 m

c The acceleration of the ball is constant at any time during its flight, and is equal to the acceleration due to gravity
= 9.8 m s^{-2} down

d After 0.80 s, the ball has two components of velocity:
v_x = 20 m s^{-1}
and v_y = 0 + (9.8 m s^{-2})(0.80 s) = 7.84 m s^{-1}
The speed of the ball at 0.80 s is given by:
[(20 m s^{-1})^2 + (7.84 m s^{-1})^2]^{½} = 21.5 m s^{-1}

e The ball will strike the ground 1.0 s after it is struck.
Then v_x = 20 m s^{-1}
and v_y = 0 + (9.8 m s^{-2})(1.0 s) = 9.8 m s^{-1}
The speed of the ball at 1.0 s is given by:
[(20 m s^{-1})^2 + (9.8 m s^{-1})^2]^{½} = 22.3 m s^{-1}

Q2.

A bowling ball of mass 7.5 kg travelling at 10 m s^{-1} rolls off a horizontal table 1.0 m high. (Assume the acceleration due to gravity is 9.80 m s^{-2}, and the effects of air resistance may be ignored unless otherwise stated.)

a Calculate the ball's horizontal velocity just as it strikes the floor.

b What is the vertical velocity of the ball as it strikes the floor?

c Calculate the velocity of the ball as it reaches the floor.

d What time interval has elapsed between the ball leaving the table and striking the floor?

e Calculate the horizontal distance travelled by the ball as it falls.
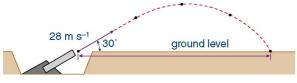
A2.

a The horizontal velocity of the ball remains constant and v_x = 10 m s^{-1}.

b $v^2 = u^2 + 2ax$
and $v_y{}^2 = 0^2 + 2(9.8$ m s^{-2})(1.0 m)
and v_y = 4.4 m s^{-1} down

c v = [(10 m s^{-1})^2 + (4.43 m s^{-1})^2]^{½} = 10.9 m s^{-1} at 24° to the horizontal,
where the angle is determined from
tan θ = 4.43 m s^{-1}/10 m s^{-1} = 0.443 and θ = 24°

d $x = ut + 0.5at^2$
and 1.0 m = 0 + 0.5(9.8 m s^{-2})t^2
so t = 0.45 s

e Horizontal distance = (horizontal speed)(time)
= (10 m s^{-1})(0.45 s) = 4.5 m

QUESTIONS 3 - 8

A senior physics class conducting a research project on projectile motion constructs a device that can launch a cricket ball. The launching device is designed so that the ball can be launched at ground level with an initial velocity of 28 m s^{-1} at an angle of 30° to the horizontal.

Q3.

Calculate the horizontal component of the velocity of the ball:

a   initially
b   after 1.0 s
c   after 2.0 s.

A3.

a $v_x = (28$ m s$^{-1})$ cos 30° = 24.2 m s^{-1} north and remains constant throughout the flight.
b   24.2 m s^{-1} north
c   24.2 m s^{-1} north

Q4.

Calculate the vertical component of the velocity of the ball:

a   initially
b   after 1.0 s
c   after 2.0 s.

A4.

a $v_y = (28$ m s$^{-1})$ sin 30° = 14 m s^{-1} up
b $v_y = 14$ m s$^{-1} - (9.8$ m s$^{-2})(1.0$ s$) = 4.2$ m s^{-1} up
c   The time for the ball to reach its maximum height is determined from $v = u + at$.
Then at maximum height, the vertical velocity of the ball = 0
and $0 = 14$ m s$^{-1} - (9.8$ m s$^{-2})t$
and $t = 1.43$ s
Therefore at $t = 2.0$ s the ball is 0.57 s into its downward flight.
$v_y = 0 + (9.8$ m s$^{-2})(0.57$ s$) = 5.6$ m s^{-1} down

Q5.

a   At what time will the ball reach its maximum height?
b   What is the maximum height that is achieved by the ball?
c   What is the acceleration of the ball at its maximum height?

A5.

a   The time for the ball to reach its maximum height is determined from $v = u + at$.
Then at maximum height, the vertical velocity of the ball = 0
and $0 = 14$ m s$^{-1} - (9.8$ m s$^{-2})t$
and $t = 1.43$ s
b $v^2 = u^2 + 2ax$
then $0 = (14$ m s$^{-1})^2 - (9.8$ m s$^{-2})x$
and $x = 10$ m
c   The acceleration of the ball is constant at any time during its flight, and is equal to the acceleration due to gravity = 9.8 m s^{-2} down.

Q6.

a   At which point in its flight will the ball experience its minimum speed?
b   What is the minimum speed of the ball during its flight?
c   At what time does this minimum speed occur?

A6.

a   The minimum speed will occur when the vertical components of the ball's velocity = 0, i.e. at the maximum height.
b   The minimum velocity of the ball during its flight occurs at the maximum height, and is equal to the horizontal component of the ball's velocity = 24.2 m s^{-1} horizontally.
c   The minimum speed of the ball during its flight occurs at the maximum height at $t = 1.43$ s.

Q7.

a   At what time after being launched will the ball return to the ground?
b   What is the velocity of the ball as it strikes the ground?
c   Calculate the horizontal range of the ball.

A7.

a   The flight of the ball is symmetrical. Therefore the time for it to reach the ground after launching = 2(1.43 s) = 2.86 s.
b   The flight of the ball is symmetrical. Therefore the ball will strike the ground at the same velocity as that when it was launched: 28 m s^{-1} at an angle of 30° to the horizontal.
c   Horizontal range = (horizontal speed)(time)
= (24.2 m s^{-1})(2.86 s) = 69.2 m

Q8*.

If the effects of air resistance were taken into account, which one of the following statements would be correct?

A The ball would have travelled a greater horizontal distance before striking the ground.

B The ball would have reached a greater maximum height.

C The ball's horizontal velocity would have been continually decreasing.

A8.

C is the correct answer. Air resistance is a force that would be acting in the opposite direction to the horizontal velocity of the ball, thereby producing a horizontal deceleration of the ball during its flight.

Q9.

A softball of mass 250 g is thrown with an initial velocity of 16 m s^{-1} at an angle θ to the horizontal. When the ball reaches its maximum height, its kinetic energy is 16 J.

a What is the maximum height achieved by the ball from its point of release?

b Calculate the initial vertical velocity of the ball.

c What is the value of θ?

d What is the speed of the ball after 1.0 s?

e What is the displacement of the ball after 1.0 s?

f How long after the ball is thrown will it return to the ground?

g Calculate the horizontal distance that the ball will travel during its flight.

A9.

a The initial E_K of the ball = 0.5(0.250 kg)(16 m s^{-1})^2 = 32 J
At its maximum height E_K = 16 J,
then loss in E_K = 32 J − 16 J = 16 J = (0.250 kg)(9.8 m s^{-2})h
and maximum height h = 6.53 m

b $v^2 = u^2 + 2ax$
and 0 = u^2 − 2(9.8 m s^{-2})(6.5306 m)
then the initial vertical velocity u = 11.3 m s^{-1} up

c At its maximum height the velocity of the ball is equal to its horizontal component v_x and E_K = 16 J = 0.5(0.250 kg)v_x^2

and v_x = 11.31 m s^{-1} = (16 m s^{-1}) cos θ
then cos θ = 0.70625 and θ = 45°

d v_x = (16 m s^{-1}) cos 45° = 11.31 m s^{-1}
v_y = 11.31 − (9.8 m s^{-2})(1.0 s) = 1.51 m s^{-1}
then the speed of the ball at t = 1.0 s
= [(11.31 m s^{-1})^2 + (1.51 m s^{-1})^2]$^{\frac{1}{2}}$ = 11.4 m s^{-1}

e The horizontal displacement of the ball at t = 1.0 s
= (11.31 m s^{-1})(1.0 s) = 11.31 m,
while the vertical displacement at t = 1.0 s
= (11.31 m s^{-1})(1.0 s) − 0.5(9.8 m s^{-2})(1.0 s)^2 = 6.41 m
The resultant displacement after 1.0 s =
[(11.31 m)^2 + (6.41 m)^2]$^{\frac{1}{2}}$ = 13.0 m
The angle θ of the displacement from the horizontal is given by:
tan θ = 6.41 m/11.31 m = 0.5668
and θ = 30°

f $v = u + at$
then at maximum vertical height v_y = 0
therefore 0 = 11.31 m s^{-1} − (9.8 m s^{-2})t
and t = 1.154 s
Since flight is symmetrical, time of flight T = 2(1.154 s) = 2.31 s.

g Horizontal distance = (horizontal speed)(time)
= (11.31 m s^{-1})(2.31 s) = 2.61 m

Q10*.

During training, an aerial skier takes off from a ramp that is inclined at 40.0° to the horizontal and lands in a pool that is 10.0 m below the end of the ramp. If she takes 1.50 s to reach the highest point of her trajectory, calculate:
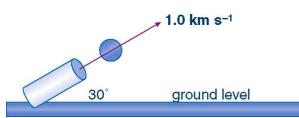
a the speed at which she leaves the ramp

b the maximum height above the end of the ramp that she reaches

c the time for which she is in mid-air.

A10.

a Taking down as positive and the top of the ramp as the zero position. Using the vertical component and finding initial velocity by analysing motion to maximum height:
a = 9.80 m s^{-2}, t = 1.5 s, v = 0, u = ?
$v = u + at$
0 = u + 9.80 × 1.50
u_v = −14.7 m s^{-1}; i.e. 1.47 m s^{-1} up
Trigonometry can be used to determine initial speed: $v = u_v/\sin 40° = 14.7/0.643$
= 22.9 m s^{-1}

b Using vertical component: $a = 9.80 \text{ m s}^{-2}$, $t = 1.50 \text{ s}$, $u = -14.7 \text{ m s}^{-1}$, $v = 0$, $x = ?$
$x = \frac{1}{2}(u + v)t = 0.5 \times -14.7 \times 1.50 = 11.0 \text{ m}$

c Using vertical component: $u = -14.7 \text{ m s}^{-1}$, $a = 9.8 \text{ m s}^{-2}$, $x = 10 \text{ m}$, $t = ?$
First find final vertical velocity (to avoid quadratic equation): $v = ?$
$v^2 = u^2 + 2ax = (-14.7)^2 + 2 \times 9.80 \times 10 = 412$
$v = 20.3 \text{ m s}^{-1}$
Now find total flight time: $t = ?$
$v = u + at$
$20.3 = -14.7 + 9.80 \times t$
$t = 3.57 \text{ s}$

QUESTIONS 11 - 13

1.0 km s⁻¹

30°   ground level

The diagram shows a projectile being launched at a velocity of 1.0 km s⁻¹ at an angle of 30° to the horizontal. Assume that $g = 9.8 \text{ m s}^{-2}$ and that air resistance is negligible.

Q11.
a What is the time of flight of the projectile?
b What is the speed of the projectile at its maximum height?
c Determine the acceleration of the projectile 1.0 s after it is launched.

A11.
a $v = u + at \Rightarrow 0 = [(1000 \text{ m s}^{-1})(\sin 30°)] - (9.8 \text{ ms}^{-2})t \Rightarrow t = 51.02 \text{ s}$
The time of flight T (from launch until it strikes the ground) is given by:
$T = 2t = 2(51.02 \text{ s}) = 102 \text{ s}$
b At maximum height the projectile only has the horizontal component of velocity.
Then speed = the initial horizontal component
$= (1000 \text{ m s}^{-1})(\cos 30°) = 870 \text{ m s}^{-1}$

c After the projectile is launched the only acceleration it will experience is that due to gravity: $a = 9.8 \text{ m s}^{-2}$ ↓

Q12.
a Determine the maximum vertical height that the projectile will reach.
b Calculate the horizontal range of the projectile.
c What is the minimum speed of the projectile during its flight?

A12.
a $v^2 = u^2 + 2ax \Rightarrow 0 = [(1000 \text{ m s}^{-1})(\sin 30°] - 2(9.8 \text{ m s}^{-2}) x \Rightarrow x = 13 \text{ km}$
b $x = [(1000 \text{ m s}^{-1})(\cos 30°)](102.04 \text{ s}) = 88 \text{ km}$
c The minimum speed of the projectile occurs at its maximum vertical height because the projectile only has a horizontal component at this height $\Rightarrow v = 870 \text{ m s}^{-1}$.

Q13.
a Calculate the vertical component of the projectile's velocity 1.0 s after it is launched.
b What is the horizontal component of the projectile's velocity 1.0 s after it is launched?
c Determine the speed of the projectile 1.0 s after it is launched.

A13.
a $v = u + at = [(1000 \text{ m s}^{-1})(\sin 30°)] - (9.8 \text{ ms}^{-2})(1.0 \text{ s}) = 490 \text{ m s}^{-1}$
b $(1000 \text{ m s}^{-1})(\cos 30°) = 870 \text{ m s}^{-1}$
c Speed $= [(490 \text{ m s}^{-1})^2 + (870 \text{ m s}^{-1})^2]^{\frac{1}{2}} = 9.9 \times 10^2 \text{ m s}^{-1}$

Q14.
A projectile is launched with an initial velocity of 1.5 km s⁻¹ at an angle θ. The target is located at a distance of 3000 m and on the same level. The projectile strikes the target after 4.0 s.
a Determine the horizontal velocity of the projectile.
b Calculate the value of θ.

A14.
a $v_x = x/t = (3000 \text{ m})/4.0 \text{ s} = 750 \text{ m s}^{-1}$
b $(1500 \text{ m s}^{-1})(\cos \theta) = 750 \text{ m s}^{-1} \Rightarrow \theta = 60°$

Q15.
Consider a projectile launched with initial velocity v at an angle θ to the horizontal. Discuss the difference between the horizontal and vertical components of the projectile's velocity during the flight. Ignore air resistance.

A15.
Note that these components act *independently* of each other. Since we assume that the only force acting on the projectile is gravity, then the horizontal component of the projectile's velocity will remain constant during its flight. The vertical component of the projectile's velocity will decrease at a constant rate until it reaches its maximum vertical height. It will then begin to descend. During its descent, the vertical component of the projectile's velocity will increase at a constant rate.

QUESTIONS 16 – 19
Consider a projectile launched with an initial velocity v at an angle θ to the horizontal.
Assume that g = 9.8 m s^{-2} and that air resistance is negligible.

Q16. Which one or more of the following statements are correct?
A   The acceleration of the projectile remains constant during its upward flight.
B   The acceleration of the projectile remains constant during its downward flight.
C   The acceleration of the projectile increases during its downward flight.
D   The acceleration of the projectile decreases during its upward flight.

A16.
A and B are correct since the acceleration of the projectile at any time during its flight is equal to g = 9.8 m s^{-2}.

Q17.
Which one or more of the following statements are correct?

A   The vertical component of the projectile's velocity remains constant.
B   The vertical component of the projectile's velocity is never zero.
C   The horizontal component of the projectile's velocity remains constant.
D   The horizontal component of the projectile's velocity is never zero.

A17.
C and D are correct. Since there are no horizontal forces acting on the projectile, the horizontal component of the projectile's velocity will not change. Since the projectile does not actually stop during its flight, the horizontal component of the projectile's velocity is never zero.

Q18.
Which one or more of the following statements are correct?
A   The minimum speed of the projectile during its flight is zero.
B   The minimum speed of the projectile during its flight is equal to its initial vertical component of velocity.
C   The minimum speed of the projectile during its flight is equal to its initial horizontal component of velocity.
D   The minimum speed of the projectile during its flight is never zero.

A18.
C and D are correct. The minimum speed of the projectile during its flight occurs when it has reached its maximum vertical height when the vertical component of its velocity is equal to zero and it is travelling with its initial horizontal component of velocity.

Q19.
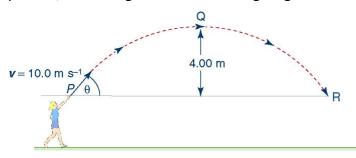Which one or more of the following statements are correct concerning the upward motion of the projectile?
A   The vertical component of the projectile's velocity decreases at a constant rate.
B   The vertical component of the projectile's velocity decreases at a non-constant rate.
C   The horizontal acceleration of the projectile is zero.
D   The vertical acceleration of the projectile is zero.

A19.
A and C are correct. The only vertical force acting is gravity, which will produce a constant deceleration of 9.8 m s^{-2}. There are no horizontal forces acting on the projectile.

QUESTIONS 20 - 23
The diagram shows the trajectory of a Vortex after it has been thrown with an initial speed of 10.0 m s^{-1}. The Vortex reaches its maximum height at point Q; 4.00 m higher than its starting height.

Q20. What is the value of the angle θ that the initial velocity vector makes with the horizontal?

A20.
At point Q the vertical velocity = 0
Then if u = initial vertical velocity of Vortex,
$0 = u^2 – 2(9.8 \text{ m s}^{-2})(4.0 \text{ m})$
and u = 8.854 m s^{-1}
Since u = (10 m s^{-1}) sin θ,
then θ = 62.3°

Q21.
What is the speed of the Vortex at point Q?

A21.
At point Q the speed of the Vortex is equal to the horizontal component of velocity
= (10 m s^{-1}) cos 62.3° = 4.6 m s^{-1}

Q22.
What is the acceleration of the Vortex at point Q?

A22.
The acceleration of the Vortex at point Q = acceleration due to gravity = 9.8 m s^{-2} down

Q23.
How far away is the Vortex when it reaches point R?

A23.
Horizontal distance X = (4.6 m s^{-1})T,
where T = time of flight
From $v = u + at$,
$0 = 8.854 \text{ m s}^{-1} – (9.8 \text{ m s}^{-2})t$
and t = 0.903 s
and T = 2(0.903 s) = 1.806 s
Then X = (4.65 m s^{-1})(1.806 s) = 8.4 m

Q24.
Two identical tennis balls X and Y are hit horizontally from a point 2.0 m above the ground with different initial speeds: ball X has an initial speed of 5.0 m s^{-1} while ball Y has an initial speed of 7.5 m s^{-1}.
a   Calculate the time it takes for each ball to strike the ground.
b   Calculate the speed of ball X just before it strikes the ground.
c   What is the speed of ball Y just before it strikes the ground?
d   How much further than ball X does ball Y travel in the horizontal direction before bouncing?

A24.
a   Using the formula $x = ut + 0.5at^2$,
then 2.0 m = 0 + 0.5(9.8 m s^{-2})t^2
and t = 0.64 s
b   For ball X: horizontal velocity = 7.5 m s^{-1},
vertical velocity on striking ground
= 0 + (9.8 m s^{-2})(0.639 s) = 6.26 m s^{-1}
then the speed of ball X just before it strikes the ground
= [(5.0 m s^{-1})^2 + (6.26 m s^{-1})^2]$^{\frac{1}{2}}$ = 8.0 m s^{-1}
c   For ball Y: horizontal velocity = 7.5 m s^{-1},
vertical velocity on striking ground
= 0 + (9.8 m s^{-2})(0.639 s) = 6.26 m s^{-1}
then the speed of ball Y just before it strikes the ground
= [(7.5 m s^{-1})^2 + (6.26 m s^{-1})^2]$^{\frac{1}{2}}$ = 9.8 m s^{-1}
d   Distance = (7.5 m s^{-1} – 5.0 m s^{-1})(0.639 s) = 1.6 m