LESSON 4.1

Prime Factorization

BEFORE	Now	WHY?
You multiplied whole numbers to find their product.	You'll write a number as a product of prime numbers.	So you can tell whether a Chinese New Year is prime, as in Ex. 51.

Word Watch

prime number, p. 157
composite number, p. 157
prime factorization, p. 158
factor tree, p. 158

Activity **You can make a list of *prime numbers*.**

(1) Write the whole numbers from 2 through 50.

(2) Cross out all multiples of 2 other than 2. (The first row in the list below has been done for you.) Then go to the next remaining number after 2 and cross out all its multiples other than itself. Repeat until you can no longer cross out numbers.

	2	3	~~4~~	5	~~6~~	7	~~8~~	9	~~10~~
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

In the activity, the numbers that are not crossed out are called *prime numbers*. A **prime number** is a whole number greater than 1 whose only whole number factors are 1 and itself. A **composite number** is a whole number greater than 1 that is not prime. For example, 5 is a prime number while $6 = 2 \times 3$ is a composite number. The number 1 is neither prime nor composite.

Poison dart frog from a zoo in New York City

EXAMPLE 1 Writing Factors of a Number

Field Trip A class of 36 students is on a field trip at the zoo. The teacher wants to break the class into groups of the same size. Find all the possible group sizes by writing all the factors of 36.

Solution

$$\begin{aligned} 36 &= 1 \times 36 \\ &= 2 \times 18 \\ &= 3 \times 12 \\ &= 4 \times 9 \qquad \text{36 isn't divisible by 5. Skip to 6.} \\ &= 6 \times 6 \qquad \text{36 isn't divisible by 7 and 8. Skip to 9.} \\ &= 9 \times 4 \qquad \text{Stop when the factors repeat.} \end{aligned}$$

ANSWER The possible group sizes are 1, 2, 3, 4, 6, 9, 12, 18, and 36.

HELP with Review

Need help with divisibility rules? See p. 685.

EXAMPLE 2 Identifying Prime and Composite Numbers

Tell whether the number is *prime* or *composite*.

a. 56 **b.** 11

Solution

a. The factors of 56 are 1, 2, 4, 7, 8, 14, 28, and 56. So, 56 is composite.

b. The only factors of 11 are 1 and 11. So, 11 is prime.

Prime Factorization Expressing a whole number as a product of prime numbers is called **prime factorization**. You can use a diagram called a **factor tree** to write the prime factorization of a number. Use an exponent when a prime factor appears more than once in the prime factorization.

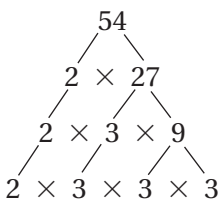
HELP with Reading

To *factor* a number means to write the number as a product of its factors.

EXAMPLE 3 Using a Factor Tree

Use a factor tree to write the prime factorization of 54.

One possible factor tree:



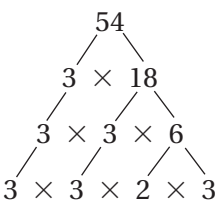
Write original number.

Factor 54 as 2 × 27.

Factor 27 as 3 × 9.

Factor 9 as 3 × 3.

Another possible factor tree:



Write original number.

Factor 54 as 3 × 18.

Factor 18 as 3 × 6.

Factor 6 as 2 × 3.

Both factor trees give the same result: $54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$.

ANSWER The prime factorization of 54 is 2×3^3.

Your turn now **Use a factor tree to write the prime factorization of the number.**

1. 30 **2.** 48 **3.** 44 **4.** 75

4.1 Exercises

More Practice, p. 708

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Getting Ready to Practice

1. Vocabulary In your own words, describe the difference between a prime number and a composite number.

Tell whether the number is *prime* or *composite*. Explain your reasoning.

2. 5 **3.** 10 **4.** 15 **5.** 43 **6.** 22

7. Guided Problem Solving You are a tour guide and want to divide 90 people into the same size tour groups. The ideal tour group size is between 11 and 15 people. How many people should be in each tour group?

(1) Find all the factors of 90.

(2) Use the factors of 90 to find all the possible group sizes.

(3) Is more than one answer possible? Explain your reasoning.

Watch Out!

A number may be divisible by the same prime number multiple times.

Practice and Problem Solving

Write all the factors of the number.

8. 20 **9.** 45 **10.** 24 **11.** 13 **12.** 21

13. 18 **14.** 16 **15.** 54 **16.** 100 **17.** 60

Tell whether the number is *prime* or *composite*. Explain your reasoning.

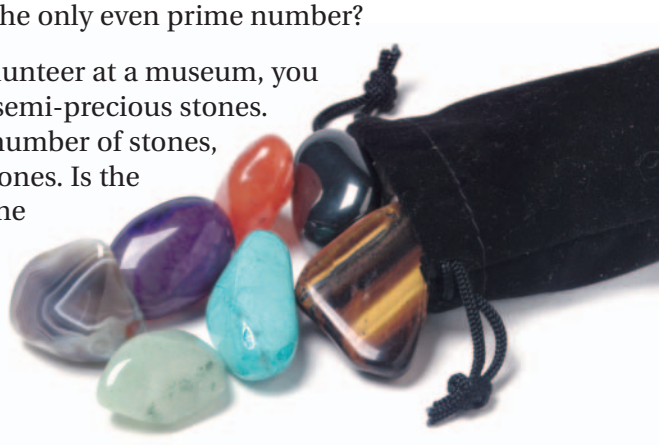
18. 88 **19.** 23 **20.** 61 **21.** 39 **22.** 51

23. 67 **24.** 41 **25.** 99 **26.** 87 **27.** 201

28. List the first 10 prime numbers.

29. Critical Thinking What is the only even prime number?

30. Souvenir Pouches As a volunteer at a museum, you fill souvenir pouches with semi-precious stones. Each pouch has the same number of stones, and there are no leftover stones. Is the total number of stones in the souvenir pouches *prime* or *composite*? Explain your reasoning.



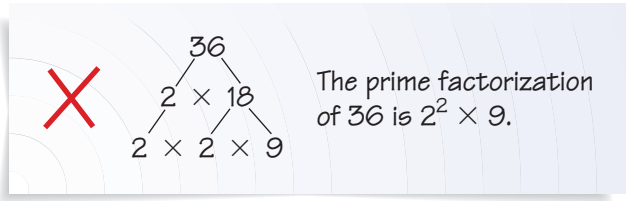
HELP with Homework

Example	Exercises
1	8–17, 52
2	18–27, 51
3	32–44

Online Resources CLASSZONE.COM

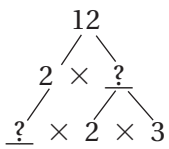
- More Examples
- eTutorial Plus

31. Find the Error Describe and correct the error in writing the prime factorization of 36.



Copy and complete the factor tree. Then write the prime factorization.

32.



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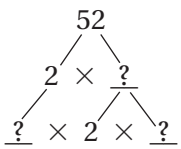
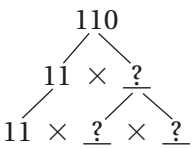


34.



Use a factor tree to write the prime factorization of the number.

35. 26 **36.** 49 **37.** 68 **38.** 50 **39.** 64

40. 144 **41.** 225 **42.** 588 **43.** 612 **44.** 864

45. Writing Explain the difference between finding the factors of a number and finding the prime factorization of a number.

Algebra Tell whether the value of the variable expression is *prime* or *composite*.

46. $6p - 2$ when $p = 5$ **47.** $3x + 1$ when $x = 6$

48. $7n^2 + 3$ when $n = 2$ **49.** $r^3 + 17$ when $r = 4$

50. Goldbach's Conjecture A conjecture is a statement believed to be true but not proved to be true. Christian Goldbach (1690–1764) made this conjecture about prime numbers: Every even number greater than 2 can be written as the sum of two prime numbers. Show that Goldbach's conjecture is true for every even number between 3 and 11.

51. Chinese New Year The year 2019 is the Year of the Pig. Is 2019 prime or composite? Explain your reasoning.

52. Desks in a Classroom A classroom in your school contains 32 desks, and another classroom contains 35 desks. Which classroom allows for more rectangular desk arrangements if you use all the desks? Explain your answer.

Use a factor tree to write the prime factorization of the number.

53. 1764 **54.** 1089 **55.** 1232 **56.** 2310 **57.** 2205

58. Challenge Identify which columns of the table mostly contain composite numbers. Explain why.

	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24

What do you think? History

Chinese New Year

The Chinese New Year, which falls between late January and early February, is associated with an animal. Twelve animals are repeated in 12-year cycles. The year 2019 is the Year of the Pig. What year will be the next Year of the Pig?

Mixed Review

Find the mean, median, mode(s), and range of the data. *(Lesson 3.1)*

59. 4, 6, 4, 7, 8, 3, 9, 4, 3, 2

60. 12, 15, 14, 20, 25, 13, 18, 19, 8

Choose a Strategy **Use a strategy from the list to solve the following problem. Explain your choice of strategy.**

Problem Solving Strategies
- Look for a Pattern
- Estimate
- Make a Table

61. A family of 2 adults and 2 children buys a family season pass to a water park for $249.99. A single adult ticket costs $39.99, and a single child ticket costs $27.99. How many times must the entire family go to the water park for the pass to be worth its cost?

Basic Skills **Test the number for divisibility by 2, 3, 5, 6, 9, and 10.**

62. 144 **63.** 345 **64.** 2040 **65.** 2514

Test-Taking Practice

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66. Short Response Give two different factor trees for 348.

67. Multiple Choice What is the prime factorization of 72?

A. $2^2 \times 3 \times 6$ **B.** $3^3 \times 2^2$ **C.** $2^3 \times 3^2$ **D.** $2^3 \times 9$

BRAIN GAME

What am I?

If a number in the list below is a factor of 2,343,750, write its corresponding letter on a piece of paper. Unscramble the letters to answer the riddle.

2	3	4	5	6	7	8	9	10
C	A	E	H	M	L	N	R	T

Riddle: Take me out and scratch my head,
I am now black, but once was red.
What am I?

Answer: I am a(n) ? .