Prediction of Heavy-Oil Viscosities With a Simple Correlation Approach

A. Bahadori, Southern Cross University, and **M. Mahmoudi** and **A. Nouri,** University of Alberta

Summary

Heavy-oil development is becoming increasingly important because of the continuous decline in conventional-oil production. For heavy-oil reservoirs, the oil viscosity usually varies dramatically during production processes such as in thermal processes. When producing heavy oil, the high viscosity is a major impediment to recovery. Oil viscosity is often correlated directly to the reserves estimate in heavy-oil formations and can determine the success or failure of a given enhanced-oil-recovery scheme. As a result, viscosity is an important parameter for performing numerical simulation and determining the economics of a project.

In this work, a simple-to-use correlation has been developed to correlate the viscosity of heavy oil to temperature and to a simple correlating parameter that can be used for heavy-oil characterization. The reported results are the product of the analysis of heavy-oil data collected from the open literature for various heavy-oil fields around the world. The tool developed in this study can be of immense practical value for petroleum engineers, providing a method for quick assessment of the viscosity of heavy oils. In particular, petroleum and production engineers would find the proposed correlation to be user-friendly, with transparent calculations involving no complex expressions.

The new proposed correlation shows consistently accurate results. This consistency could not be matched by any of the widely accepted existing correlations within the investigated range. For all conditions, the new correlation provided better results than existing correlations in the literature.

Introduction

The continuing decline in conventional crude-oil reserves combined with the continuing high worldwide demand for oil has led to the increased role of unconventional resources, especially heavy crude oil, in the world (Pedersen and Fredenslund 1984; Butler 1991; Al-Maamari et al. 2006). There are many challenges to the successful exploitation of these resources (Willman et al. 1961). One of the major issues is the high viscosity of the heavy crude oils, which makes production and processing difficult (Monnery et al. 1995; Mehrotra et al. 1996). Heavy crude oil and tar-sand oil are petroleum or petroleum-like liquids or semisolids occurring naturally in porous media. These oils are characterized by viscosity and density (Puttagunta et al. 1993).

The viscosity of heavy oils is a critical property in predicting oil recovery (Mehrotra 1990) and selecting a production approach. Several viscosity correlations are available in the literature (Mehrotra 1991a, b; Miadonye et al. 1992; Puttagunta et al. 1992; Miadonye et al. 1993; Puttagunta et al. 1993; Singh et al. 1993a, b; De Ghetto et al. 1995; Petrosky and Farshad 1995; Wakabayashi 1997; Bennison 1998; Miadonye and Puttagunta 1998), and are mainly a function of density (or °API) and temperature. These correlations are used when viscosity measurements are not available. Considerable errors may be introduced when these correlations are used for assessing heavy-oil viscosity. To increase accuracy, compositional terms, such as the percent of saturates, aromatics, resins, and asphaltenes, should be used in the correlation (Al-Maamari et al. 2006). Therefore, we use a new correlating parameter called corrected API (CAPI), as proposed by Al-Maamari et al. (2006), which can be used for heavy-oil characterization.

The results in this paper are the product of analysis of heavy-oil data collected from the open literature for various heavy-oil fields around the world. Distinctive parameters that have been considered are crude-oil gravity (API) and compound class distributions [i.e., saturated hydrocarbons (*Sa*), aromatic hydrocarbons (*Ar*), resins (*Re*), and asphaltenes (*As*)] (Al-Maamari et al. 2006):

$$CAPI = API\left(\frac{Sa}{Ar+Re+As}\right). \quad \text{.....(1)}$$

The units in Eq. 1 for *Sa, Ar, Re,* and *As* are mass fraction. In view of the aforementioned issues and the importance of viscosity in heavy-oil production and processing, it is necessary to develop an accurate and simple correlation for predicting the viscosity of heavy crude oil as a function of temperature and for correlating parameters that can be easily assessed. This paper discusses the formulation of such a predictive tool in a systematic manner. The proposed method is an exponential function that leads to well-behaved (i.e., smooth and nonoscillatory) equations, enabling more-accurate and nonoscillatory predictions.

In the majority of cases, existing correlations indicated a good prediction of crude-oil viscosity for the oils from which they were derived. However, when used with other crude oils from other regions, these correlations are, in most cases, not accurate and certain modifications are needed to obtain acceptable viscosity predictions.

This correlation is only for dead-oil viscosity; therefore, the viscosity/API correlation is the focus of this paper. A new correlating parameter, CAPI, is used for heavy-oil characterization. It is not a function of pressure. The reported results are the product of analysis of data from many heavy oils collected from the open literature for various heavy-oil fields around the world.

Methodology for the Development of a Novel Correlation

The primary purpose of the present study is to accurately correlate the viscosity of heavy crude oil as a function of temperature, and as a function of a simple correlating parameter that can be used for heavy-oil characterization.

The Vandermonde matrix is a matrix with the terms of a geometric progression in each row (i.e., an $m \times n$ matrix) (Bair et al. 2006):

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Coefficient	Value	Coefficient	Value
A_1	$8.1200497192428 \times 10^{1}$	A_3	$2.6082312548726 \times 10^{3}$
B_1	$-6.6627637648617 \times 10^{-1}$	B_3	$-2.2671850189606 \times 10^{1}$
C_1	$1.8084634786642 \times 10^{-3}$	C_3	$6.2726567869170 \times 10^{-2}$
D_1	$-1.6288179364299 \times 10^{-6}$	D_3	$-5.6442619718476 \times 10^{-5}$
A_2	$-2.4138852681554 \times 10^{2}$	A_4	$-3.1680306723558 \times 10^{3}$
B_2	3.2215900957370	B_4	$2.6172339326323 \times 10^{1}$
C_2	$-1.0769759542352 \times 10^{-2}$	C_4	$-7.0094351138107 \times 10^{-2}$
D_2	$1.0860461931835 \times 10^{-5}$	D_4	$6.1573901588549 \times 10^{-5}$

Table 1—Tuned coefficients used in Eqs. 11 through 14.

$$V = \begin{bmatrix} 1 & \alpha_1 & \alpha_1^2 & \dots & \alpha_1^{n-1} \\ 1 & \alpha_2 & \alpha_2^2 & \dots & \alpha_2^{n-1} \\ 1 & \alpha_3 & \alpha_3^2 & \dots & \alpha_3^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha_m & \alpha_m^2 & \dots & \alpha_m^{n-1} \end{bmatrix} \quad \text{.....(2)}$$

or

$$V_{i,j} = \alpha_i^{j-1} \quad \text{.....(3)}$$

for all indices i and j. The determinant of a square Vandermonde matrix (where $m = n$) can be expressed as (Bair et al. 2006)

$$\det(V) = \prod_{1 \le i < j \le n} (\alpha_j - \alpha_i). \quad \text{.....(4)}$$

The Vandermonde matrix evaluates a polynomial at a set of points. Formally, it transforms coefficients of a polynomial $a_0 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1}$ to the values that the polynomial takes at the points α_i. The nonvanishing of the Vandermonde determinant for distinct points α_i shows that for distinct points, the map from coefficients to values at those points is a one-to-one correspondence, and thus that the polynomial interpolation problem is solvable with a unique solution. This result is called the unisolvence theorem (Fulton and Harris 1991). They are thus useful in polynomial interpolation because solving the system of linear equations $Vu = y$ for u, with V being an $m \times n$ Vandermonde matrix, is equivalent to finding the coefficients u_j of the polynomial(s) (Bair et al. 2006; Fulton and Harris 1991; Horn and Johnson 1991).

$$P(x) = \sum_{j=0}^{n-1} u_j x^j. \quad \text{.....(5)}$$

For degree $\le n-1$, which has the property

$$P(\alpha_i) = y_i, \text{ for } i = 1, \dots, m, \quad \text{.....(6)}$$

the Vandermonde matrix can easily be inverted in terms of Lagrange basis polynomials—each column is the coefficient of the Lagrange basis polynomial, with terms in increasing order going down. The resulting solution to the interpolation problem is called the Lagrange polynomial.

Suppose that the interpolation polynomial is in the form (Bair et al. 2006; Fulton and Harris 1991; Horn and Johnson 1991):

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_2x^2 + a_1x + a_0. \quad \text{.....(7)}$$

The statement that P interpolates the data points means that

$$P(x_i) = y_i, \text{ for all } i \in \{0,1,\dots,n\}. \quad \text{.....(8)}$$

If we substitute Eq. 3 into Eq. 8, we obtain a system of linear equations in the coefficients a_k. The system in matrix/vector form reads (Fulton and Harris 1991; Horn and Johnson 1991; Bair et al. 2006)

$$\begin{bmatrix} x_0^n & x_0^{n-1} & x_0^{n-2} & \dots & x_0 & 1 \\ x_1^n & x_1^{n-1} & x_1^{n-2} & \dots & x_1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ x_n^n & x_n^{n-1} & x_n^{n-2} & \dots & x_n & 1 \end{bmatrix} \begin{bmatrix} a_n \\ a_{n-1} \\ \vdots \\ a_0 \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_n \end{bmatrix}. \quad \text{.....(9)}$$

We have to solve this system for a_k to construct the interpolant $P(x)$. The matrix on the left is commonly referred to as a Vandermonde matrix (Fulton and Harris 1991; Horn and Johnson 1991; Bair et al. 2006).

Development of the Correlation. The data required to develop this correlation include the viscosity of heavy oil as a function of temperature and the corrected API (CAPI) (Henshaw et al. 1998; Al-Maamari et al. 2006). The following methodology (Bahadori and Vuthaluru 2009; Bahadori 2010, 2011) has been applied to develop this correlation by use of Matlab® technical computing language (Matlab 2008):

1. Correlate the viscosities of heavy oil as a function of the CAPI for a given temperature.
2. Repeat Step 1 for other temperatures.
3. Correlate the corresponding polynomial coefficients, which were obtained for different temperatures, vs. temperature: $a = f(T)$, $b = f(T)$, $c = f(T)$, $d = f(T)$ (see Eqs. 11 through 14].

Eq. 10 represents the proposed governing equation in which four coefficients are used to correlate the viscosity of heavy oil as a function of temperature and as a function of the CAPI:

$$\ln(\eta) = a + \frac{b}{CAPI} + \frac{c}{(CAPI)^2} + \frac{d}{(CAPI)^3}, \quad \text{.....(10)}$$

where

$$a = A_1 + B_1T + C_1T^2 + D_1T^3, \quad \text{.....(11)}$$

$$b = A_2 + B_2T + C_2T^2 + D_2T^3, \quad \text{.....(12)}$$

$$c = A_3 + B_3T + C_3T^2 + D_3T^3, \quad \text{.....(13)}$$

and

$$d = A_4 + B_4T + C_4T^2 + D_4T^3, \quad \text{.....(14)}$$

with the relevant coefficients reported in **Table 1.**

Field	°API	CAPI	Kinematic Viscosity (mm^2/s)						
			40°C	50°C	60°C	70°C	100°C	135°C	177°C
Cymric	8.7	1.69	23 280	6420	2260	1000	145.5	32.6	10.37
Cymric	9.4	1.95	22 380	5990	2050	890	126.9	31.2	10.18
Midway Sunset	10.4	2.01	12 190	3670	1380	610	103.9	26.3	9.43
Midway Sunset	10.5	2.03	9530	3200	1220	540	91.5	24.2	8.72
Cymric	9.2	2.20	9170	2950	1170	540	97.7	24.4	8.71
Cymric	10	2.22	13 020	3720	1350	580	94.2	23.3	8.01
Cymric	9.7	2.31	9340	3100	1150	510	84.8	21.1	7.59
Cymric	10.2	2.44	9420	2920	1120	510	86	23.4	8.62
Cymric	10.9	2.45	4400	1520	640	310	61.6	17.4	6.62
Coalinga	10.3	2.79	9270	2830	1080	480	79.7	22.2	7.99
Coalinga	9.7	2.80	9890	2950	1100	490	78.3	22	7.83
Cymric	11.7	2.80	2070	800	370	190	42	13.6	5.48
Kern River	11.9	3.41	3360	1250	550	280	58	17.1	6.86
Kern River	13.1	3.58	1840	740	340	180	41.7	13.7	5.52
Kern River	12.5	3.60	3050	1140	510	260	54.3	16.3	6.87
Kern River	12.3	3.75	2170	890	420	220	52.1	15.8	6.49
Kern River	14.5	4.03	710	330	170	100	26.6	10	4.4
Kern River	14.1	4.17	600	280	150	90	24.9	7.4	4.15
Kern River	13.4	4.84	1150	490	240	130	33.6	12.9	4.92
Coalinga	14.4	5.34	380	190	100	60	19.1	7.3	3.4

Table 2—Viscosity data used in developing the new correlation (Al-Maamari et al. 2006; Henshaw et al. 1998).

$$\mu = \eta \times SG, \quad \text{.....(15)}$$

where μ is the dynamic viscosity in cp, η is the kinematic viscosity measured in cSt, and SG is specific gravity. These optimum tuned coefficients help to cover temperatures up to 180°C and CAPI as low as 6. The optimum tuned coefficients given in Table 1 can be further refined according to the proposed approach if more data become available in the future.

In this work, our efforts were directed toward formulating a correlation that can be expected to assist engineers for rapid calculation of the heavy-oil viscosity as a function of temperature and the CAPI. The proposed tool is simple and novel. The selected exponential function to develop the tool leads to well-behaved (i.e., smooth and nonoscillatory) equations, enabling reliable and accurate predictions.

Results

Table 2 summarizes the data used to develop this correlation. **Fig. 1** shows the proposed correlation curves in comparison with the literature data (Henshaw et al. 1998; Al-Maamari et al. 2006) that were used to calibrate the correlation. Note that the data that were used in developing and calibrating the correlation had gravities less than 15 °API [corrected API (CAPI) less than 6] and temperatures in the range of 40 to 177°C. **Figs. 2 and 3** show the results from the proposed method and its smooth performance in the prediction of the viscosity of heavy oil as a function of temperature and CAPI.

The performance of the proposed correlation, as well as other correlations in the literature, was examined against the literature data. **Table 3** compares the performance of various exsiting correlations with that of the proposed correlation in predicting the viscosity of crude oil. It should be noted that the same set of data that was used for the correlation calibration was used for the performance assessment, which does not allow an unbiased performance comparison. It shows that the existing correlations can result in significant errors when it comes to predicting the viscosity of heavy crude oils because the correlations do not involve the composition of heavy oil. The inclusion of the heavy-oil composition in the proposed correlation ensures a more-accurate viscosity assessment because it recognizes the difference in the viscosity of oils that have the same gravity in °API but have different compositions. Further viscosity and compositional data are required to make a more-objective performance comparison. **Table 4** summarizes the error assessments for various correlations at different temperatures. It can be seen from the table that most correlations result in higher errors at lower temperatures.

It is expected that our efforts in formulating the tool will pave the way for arriving at an accurate prediction of the viscosity of heavy crude oil. The tool developed in this study can be of great practical value for experts and engineers, providing a method for obtaining a quick assessment of the viscosity of heavy crude oil. In particular, petroleum engineers would find the approach to be user-friendly, with transparent calculations involving no complex expressions.

Conclusions

In this work, simple-to-use equations are presented for the estimation of heavy-oil viscosity as a function of temperature, and of a simple correlating parameter that can be used for heavy-oil characterization (corrected API). The performance of the proposed correlation was examined against the performance of several correlations in the literature. It was found that the proposed correlation resulted in the most-accurate predictions for the data used in that exercise. Note that the correlation was developed with viscosity data in the range of 8.7 to 14.5 °API and in the temperature range of 40 to 177°C. Therefore, care must be exercised when the correlation is used in the assessment of viscosity (in °API) and temperature ranges outside of those of the data used in calibrating the correlation.

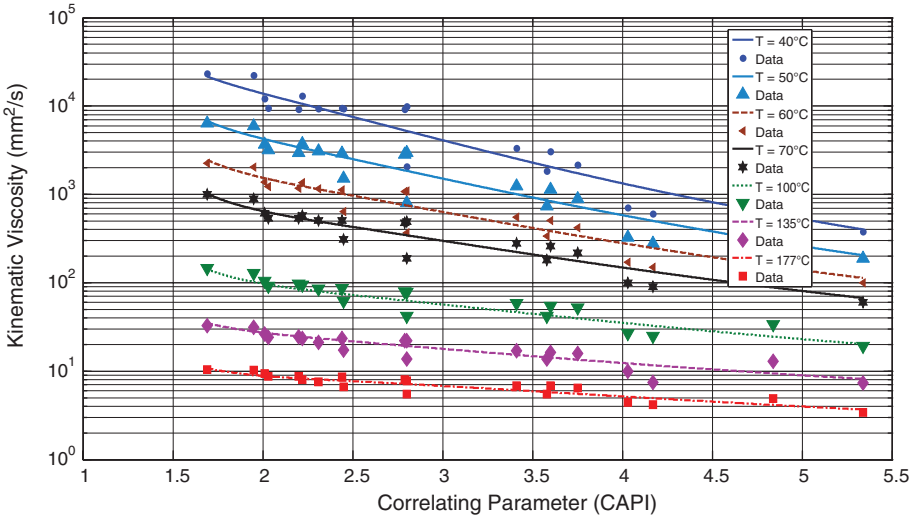

Fig. 1—The calibration of the predictive tool for the estimation of viscosity of heavy oil (Al-Maamari et al. 2006; Henshaw et al. 1998).

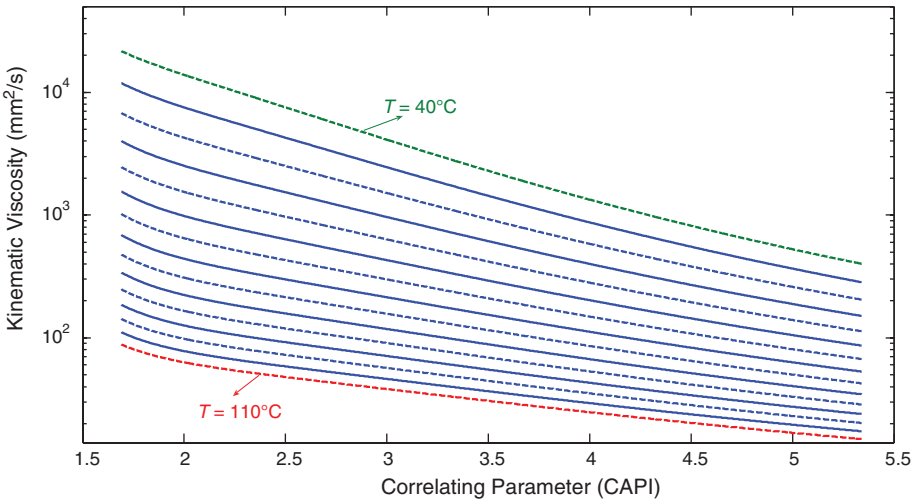

Fig. 2—The smooth results of the predictive tool in estimating the viscosity of heavy oils for temperatures less than 110°C.

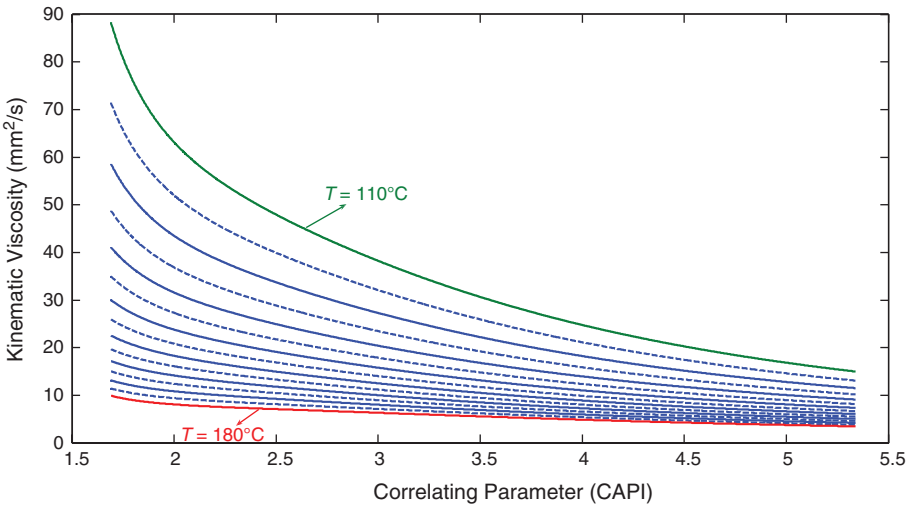

Fig. 3—The smooth results of the predictive tool in estimating the viscosity of heavy oils for temperatures greater than 110°C.

Developer	Equation	Maximum Error (%)	Minimum Error (%)	Average Error (%)
Beal (1946)	$\mu_{od} = \left[0.32 + \frac{1.8(10^7)}{API^{4.53}}\right]\left(\frac{360}{T+200}\right)^a$ $a = \text{antilog}(0.43 + 8.33/API)$	723	0.7	108.3
Beggs and Robinson (1975)	$\mu_{od} = 10^x - 1$ $x = y(T-460)^{-1.163}$ $y = 10^z$ $z = 3.0324 - 0.02023(API)$	1,536	12.4	396.3
Glaso (1980)	$\mu_{od} = \left[3.141(10^{10})\right](T-460)^{-3.444}\left[\log(API)\right]^a$ $a = 10.313\left[\log(T-460)\right] - 36.447$	347	0.8	69.6
Labedi (1992)	$\mu_{od} = (10)^{9.224}/(API)^{4.7013} \cdot (T_R)^{0.6739}$	1,048	1.5	133.3
Elsharkawy and Alikhan (1999)	$\mu_{od} = \text{antilog}_{10}(x) - 1.0$ $x = \text{antilog}_{10}(y)$ $y = 2.16924 - 0.02525(API) - 0.68875\left[\log_{10}(T)\right]$	626.6	1.24	100.1
Hossain (2005)	$\mu_{od} = 10^{(-0.71523*API+22.13766)} * T^{(0.269024*API-8.26)}$	212	1.6	31.3
Kartoatmodjo (1990)	$\mu_{od} = 16(10^8)(T_f)^{-2.8177}(\log API)^x$ $x = 5.7526\left[\log(T_f)\right] - 26.9718$	99.5	52.4	90.8
Petrosky and Farshad (1995)	$\mu_{od} = 2.3511*(10^7)*(T)^{-2.10255}(\log API)^x$ $x = 4.59388*\left[\log(T)\right] - 22.82792$	99.5	3.6	82
Proposed Correlation	$\ln(\eta) = a + \frac{b}{CAPI} + \frac{c}{(CAPI)^2} + \frac{d}{(CAPI)^3}$	88	0	22

Table 3—Comparison of the prediction error for different correlations.

Unlike complex mathematical approaches for estimating the viscosity of heavy oils, the proposed correlation is straightforward and would be greatly helpful for engineers, especially those dealing with petroleum engineering and heavy-oil production. Additionally, the level of mathematical formulations associated with the estimation viscosity of heavy oil can be easily handled by an engineer or practitioner without any in-depth mathematical abilities. The proposed method has a clear numerical background, wherein the relevant coefficients can be retuned quickly if more data become available in the future.

Nomenclature

a = correlating parameter in the viscosity correlation
A = first tuned coefficient
API = oil gravity, (API = $145/\gamma_o$, 135)
Ar = aromatic hydrocarbons
As = asphaltenes
b = correlating parameter in the viscosity correlation
B = second tuned coefficient
c = correlating parameter in the viscosity correlation
C = third tuned coefficient
$CAPI$ = correlating parameter
d = correlating parameter in the viscosity correlation
D = fourth tuned coefficient
i = index
j = index
m = matrix row index for $m \times n$ matrix
n = matrix column index for $m \times n$ matrix
P = polynomial
Re = resins
Sa = saturated hydrocarbons
SG = specific gravity
T = temperature, K
u = coefficient of polynomial
V = Vandermonde matrix
x = correlating parameter in viscosity correlation
X = data point
y = correlating parameter in viscosity correlation
Y = data point
z = correlating parameter in viscosity correlation
α = matrix element
$\gamma_{o,R}$ = oil specific gravity at reservoir conditions
η = kinematic viscosity, mm^2/s
μ = oil dynamic viscosity, cp

Developer	Error (%)	40°C	50°C	60°C	70°C	100°C	135°C	177°C	All Data
Beal (1946)	Max	723.07	273.03	106.51	42.60	67.39	78.60	80.81	723.07
	Min	85.24	17.42	2.17	0.71	39.07	41.14	12.59	0.71
	Average	376.55	139.48	45.27	17.13	52.57	64.77	62.65	108.35
Beggs and Robinson (1975)	Max	1,536.07	1391.23	1143.65	962.01	481.45	235.97	116.73	1,536.07
	Min	24.88	65.50	77.48	83.44	70.98	34.43	12.45	12.45
	Average	599.45	623.27	565.39	491.96	278.24	143.68	72.71	396.38
Glaso (1980)	Max	347.05	130.43	50.24	61.37	79.81	87.10	89.85	347.05
	Min	6.72	4.51	0.77	1.46	43.77	56.60	62.54	0.77
	Average	164.91	56.57	21.75	28.76	62.46	74.46	78.20	69.59
Labedi (1992)	Max	1048.43	242.29	53.43	69.69	92.68	97.82	99.16	1,048.43
	Min	44.83	7.64	1.48	32.46	82.80	92.42	96.07	1.48
	Average	468.47	112.09	20.33	50.61	87.98	95.72	97.98	133.31
Kartoatmodjo (1990)	Max	89.03	91.97	95.25	96.88	98.67	99.32	99.56	99.56
	Min	52.36	77.66	86.26	90.42	95.44	96.77	97.42	52.36
	Average	71.93	85.13	90.98	93.86	97.23	98.33	98.73	90.88
Petrosky and Farshad (1995)	Max	79.33	86.21	90.69	93.97	97.93	99.10	99.51	99.51
	Min	3.56	52.44	76.23	84.94	94.47	96.74	97.78	3.56
	Average	38.02	69.72	83.48	89.73	96.27	98.14	98.80	82.02
Elsharkawy and Alikhan (1999)	Max	626.68	349.08	194.84	116.21	50.47	66.12	74.28	626.68
	Min	6.19	12.15	3.76	1.64	1.24	44.51	60.19	1.24
	Average	257.35	157.11	90.70	49.32	23.94	54.23	68.28	100.13
Hossain (2005)	Max	212.85	109.04	62.44	61.70	55.73	48.45	50.66	212.85
	Min	3.96	4.10	2.86	1.61	4.33	3.38	3.46	1.61
	Average	65.94	37.98	26.73	23.89	23.95	17.42	23.44	31.34
Proposed correlation (2012)	Max	88.33	65.65	53.67	45.75	36.07	36.79	23.68	88.33
	Min	3.49	0.04	0.89	1.17	1.45	1.56	0.81	0.04
	Average	35.89	28.32	24.77	22.09	17.77	14.10	10.83	21.97

Table 4—Maximum prediction error for different correlations at several temperatures.

μ_{od} = dead-oil viscosity, cp
ρ = density, lbm/ft^3

References

Al-Maamari, R.S., Houache, O., and Abdul-Wahab, S.A. 2006. New Correlating Parameter for the Viscosity of Heavy Crude Oils. *Energy & Fuels* **20** (6): 2586–2592. http://dx.doi.org/10.1021/ef0603030.

Bahadori, A. 2010. Determination of Well Placement and Breakthrough Time in Horizontal Wells for Homogeneous and Anisotropic Reservoirs. *Journal of Petroleum Science and Engineering* **75** (1–2): 196–202. http://dx.doi.org/10.1016/j.petrol.2010.11.007.

Bahadori, A. 2011. Estimation of Combustion Flue Gas Acid Dew Point During Heat Recovery and Efficiency Gain. *Applied Thermal Engineering* **31** (8–9): 1457–1462. http://dx.doi.org/10.1016/j.applthermaleng.2011.01.020.

Bahadori, A. and Vuthaluru, H.B. 2009. A Novel Correlation for Estimation of Hydrate Forming Condition of Natural Gases. *Journal of Natural Gas Chemistry* **18** (4): 453–457. http://dx.doi.org/10.1016/S1003-9953(08)60143-7.

Bair, E., Hastie, T., Paul, D. et al. 2006. Prediction by Supervised Principal Components. *Journal of the American Statistical Association* **101** (473): 119–137. http://dx.doi.org/10.1198/016214505000000628.

Beal, C. 1946. The Viscosity of Air, Water, Natural Gas, Crude Oils and Its Associated Gases at Oil Field Temperatures and Pressures. *Transactions of the AIME* **165** (01): 94–115. SPE-946094-G. http://dx.doi.org/10.2118/946094-G.

Beggs, H.D. and Robinson, J.R. 1975. Estimating the Viscosity of Crude Oil Systems. *J Pet Technol* **27** (09): 1140–1141. SPE-5434-PA. http://dx.doi.org/10.2118/5434-PA.

Bennison, T. 1998. Prediction of Heavy Oil Viscosity. Presented at the IBC Heavy Oil Field Development Conference, London, England, 2–4 December.http://www.pe.tamu.edu/barrufet/public_html/PETE310/Papers/Visco%20models.pdf.

Butler, R.M. 1991. *Thermal Recovery of Oil and Bitumen*. Englewood Cliffs, New Jersey: Prentice-Hall.

De Ghetto, G., Paone, F., and Villa, M. 1995. Pressure-Volume-Temperature Correlations for Heavy and Extra Heavy Oils. Presented at the SPE International Heavy Oil Symposium, Calgary, Alberta, 19–21 June. SPE-30316-MS. http://dx.doi.org/10.2118/30316-MS.

Elsharkawy, A.M. and Alikhan, A.A. 1999. Models for Predicting the Viscosity of Middle East Crude Oils. *Fuel* **78:** 891–903. http://dx.doi.org/10.1016/S0016-2361(99)00019-8.

Fulton, W. and Harris, J. 1991. *Representation Theory: A First Course*. New York City, New York: Graduate Texts in Mathematics: Readings in Mathematics Series, No. 129, Springer-Verlag.

Glaso, O. 1980. Generalized Pressure-Volume-Temperature Correlations. *J Pet Technol* **32** (05): 785–795. SPE-8016-PA. http://dx.doi.org/10.2118/8016-PA.

Henshaw, P.C., Carlson, R.M.K., Pena, M.M. et al. 1998. Evaluation of Geochemical Approaches to Heavy Oil Viscosity Mapping in San Joaquin Valley, California. Presented at the SPE Western Regional Meeting, Bakersfield, California, 10–13 May. SPE-46205-MS. http://dx.doi.org/10.2118/46205-MS.

Horn, R.A. and Johnson, C.R. 1991. *Topics in Matrix Analysis*, Chap. 6, Section 6.1, pp. 383–406. Cambridge, England: Cambridge University Press. http://dx.doi.org/10.1017/CBO9780511840371.

Hossain, M.S. 2005. *Assessment and Development of Heavy Oil Viscosity Correlations.* MS thesis, the University of Tulsa, Tulsa, Oklahoma.

Kartoatmodjo, R.S.T. 1990. *New Correlations for Estimating Reservoir Fluids Physical Properties*. MS thesis, University of Tulsa, Tulsa, Oklahoma.

Labedi, R. 1992. Improved Correlations for Predicting the Viscosity of Light Crudes. *Journal of Petroleum Science and Engineering* **8** (3): 221–234. http://dx.doi.org/10.1016/0920-4105(92)90035-Y.

Matlab®, version 7.6.0.324. 2008. Natick, Massachusetts: The MathWorks, Inc.

Mehrotra, A.K. 1990. Modeling the Effects of Temperature, Pressure, and Composition on the Viscosity of Crude Oil Mixtures. *Ind. Eng. Chem. Res.* **29** (7): 1574–1578. http://dx.doi.org/10.1021/ie00103a073.

Mehrotra, A.K. 1991a. A Generalized Viscosity Equation for Pure Heavy Hydrocarbons. *Ind. Eng. Chem. Res*. **30** (2): 420–427. http://dx.doi.org/10.1021/ie00050a021.

Mehrotra, A.K. 1991b. Modeling Temperature and Composition Dependence for the Viscosity of Diluted Bitumens. *Journal of Petroleum Science and Engineering* **5** (3): 261–272. http://dx.doi.org/10.1016/0920-4105(91)90042-L.

Mehrotra, A.K., Monnery, W.D., and Svrcek, W.Y. 1996. A Review of Practical Calculation Methods for Viscosity of Liquid Hydrocarbons and Their Mixtures. *Fluid Phase Equilibria* **117** (1–2): 344–355. http://dx.doi.org/10.1016/0378-3812(95)02971-0.

Miadonye, A. and Puttagunta, V.R. 1998. Modeling the Viscosity-Temperature Relationship of Nigerian Niger-Delta Crude Petroleum. *Petroleum Science and Technology* **16** (5 & 6): 627–638. http://dx.doi.org/10.1080/10916469808949803.

Miadonye, A., Puttagunta, V.R., and Singh, B. 1993. Prediction of the Viscosity of Crude Oil Fractions from a Single Measurement. *Chem. Eng. Commun.* **122:** 195–199. http://dx.doi.org/10.1080/00986449308936156.

Miadonye, A., Singh, B., and Puttagunta, R.V. 1992. One-Parameter Correlation in the Estimation of Crude Oil Viscosity. SPE-26206-MS. https://www.onepetro.org/download/general/SPE-26206-MS?id=general%2FSPE-26206-MS.

Monnery, W.D., Svrcek, W.Y., and Mehrotra, A.K. 1995. Viscosity: A Critical Review of Practical Predictive and Correlative Methods. *Can. J. Chem. Eng.* **73** (1): 3–40. http://dx.doi.org/10.1002/cjce.5450730103.

Pedersen, K.S. and Fredenslund, A. 1984. Viscosity of Crude Oils. *Chemical Engineering Science* **39** (6): 1011–1016. http://dx.doi.org/10.1016/0009-2509(84)87009-8.

Petrosky, G.E. and Farshad, F.F. 1995. Viscosity Correlations for Gulf of Mexico Crude Oils. Presented at the SPE Production Operations, Symposium, Oklahoma City, Oklahoma, 2–4 April. SPE-29468-MS. http://dx.doi.org/10.2118/29468-MS.

Puttagunta, V.R., Miadonye, A., and Singh, B. 1992. Viscosity Temperature Correlation for Prediction of Kinematic Viscosity of Conventional Crude. *Trans. Inst. Chem. Eng*. **70:** 627–631.

Puttagunta, V.R., Miadonyea, A., and Singh, B. 1993. Simple Concept Predicts Viscosity of Heavy Oil and Bitumen. *Oil Gas J.* **91** (18): 71–73.

Puttagunta, V.R., Singh, B., and Miadonye, A. 1993. Correlation of Bitumen Viscosity with Temperature and Pressure. *Can. J. Chem. Eng.* **71** (3): 447–450. http://dx.doi.org/10.1002/cjce.5450710315.

Singh, B., Miadonye, A., and Puttagunta, V.R. 1993a. Modeling the Viscosity of Middle-East Crude Oil Mixtures. *Ind. Eng. Chem. Res*. **32** (9): 2183–2186. http://dx.doi.org/10.1021/ie00021a044.

Singh, B., Miadonye, A., and Puttangunta, V.R. 1993b. Heavy Oil Viscosity Range from One Test. *Hydrocarbon Processing* **12** (5): 157–162.

Wakabayashi, T. 1997. Viscosity Correlation with Specific Gravity and Molecular Weight of Crude Oil Fractions. *Fuel* **76** (11): 1049–1056. http://dx.doi.org/10.1016/S0016-2361(97)00090-2.

Willman, B.T., Valleroy, V.V., Runberg, G.W. et al. 1961. Laboratory Studies of Oil Recovery by Steam Injection. *J Pet Techol* **13** (07): 681–690. SPE-1537-G-PA. http://dx.doi.org/10.2118/1537-G-PA.

Alireza Bahadori is a lecturer in the School of Environment, Science, and Engineering at Southern Cross University, Lismore, New South Wales, Australia. During the past 20 years, he has held various process- and petroleum-engineering positions and was involved in many large-scale projects at the National Iranian Oil Company, Petroleum Development Oman, and Clough AMEC. Bahadori is the author of more than 200 articles, including a number of SPE papers. His 10 books have been published by multiple major publishers, including Elsevier's *Natural Gas Processing*, Wiley's *Corrosion and Materials Selection for Petroleum Industries,* and Springer's *Pollution Control in Oil and Gas Industries.* Bahadori holds a PhD degree from Curtin University, Perth, Western Australia.

Mahdi Mahmoudi is a PhD degree candidate at the University of Alberta. Previously, he worked as a completion engineer for National Iranian South Oil Company. Mahmoudi's research interests include petroleum geomechanics, wellbore hydraulics, and heavy-oil-production technologies. He holds an MSc degree in petroleum engineering from the University of Louisiana at Lafayette.

Alireza Nouri is an associate professor of petroleum engineering at the University of Alberta and a consultant to the petroleum industry in areas related to petroleum geomechanics, including physical and numerical modeling of sand production, numerical simulation of hydraulic fracturing, and caprock-integrity assessment. He has 5 years of industrial experience and has been the author of nearly 70 technical papers, 30 of which were published in SPE-affiliated journals or conference proceedings. Nouri holds a PhD degree in sand-production control from Dalhousie University. He is a member of SPE and is a registered professional engineer in Alberta.