American Mathematics Competitions

Practice 8

AMC 8

(American Mathematics Contest 8)

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.

2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.

3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score on this test is the number of correct answers.

5. No aids are permitted other than scratch paper, graph paper, rulers, and erasers. No problems on the test will require the use of a calculator.

6. Figures are not necessarily drawn to scale.

7. Before beginning the test, your proctor will ask you to record certain information on the answer form.

8. When your proctor gives the signal, begin working on the problems. You will have 40 minutes to complete the test.

9. When you finish the exam, *sign your name* in the space provided on the Answer.

1. Cathy's shop class is making a golf trophy. She has to paint 600 dimples on a golf ball. If it takes him 4 seconds to paint one dimple, how many minutes will she need to do her job?
(A) 40 (B) 60 (C) 80 (D) 10 (E) 12

2. I'm thinking of two whole numbers. Their product is 132 and their sum is 23. What is the larger number?
(A) 13 (B) 14 (C) 16 (D) 12 (E) 15

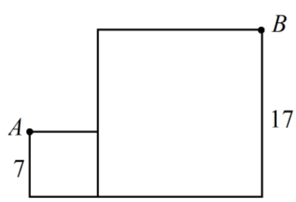
3. Gary has $126. Frank has $4 more than Emily and Emily has two-third as much as Gary. How many dollars does Frank have?
(A) 70 (B) 68 (C) 79 (D) 82 (E) 88

4. The digits 2, 3, 5, 6 and 9 are each used once to form the greatest possible odd five-digit number. The digit in the tens place is
(A) 5 (B) 9 (C) 3 (D) 6 (E) 2

5. Sixteen trees are equally spaced along one side of a straight road. The distance from the first tree to the fifth is 80 feet. What is the distance in feet between the first and last trees?
(A) 90 (B) 300 (C) 305 (D) 320 (E) 240

6. James has 20% more money than Yao, and Bob has 20% less money than James. What percent less money does Bob have than Yao?
(A) 3 (B) 5 (C) 7 (D) 9 (E) 4

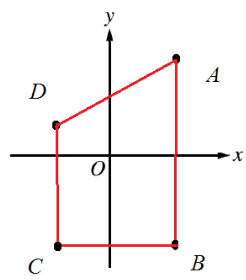
7. Two squares are positioned, as shown. The smaller square has side length 7 and the larger square has side length 17. The length of *AB* is
(A) $13\sqrt{2}$ (B) 25 (C) 26 (D) $13\sqrt{7}$ (E) 24



8. What is the probability that a randomly selected positive factor of 72 is less than 11?
(A) 1/2 (B) 7/11 (C) 2/5 (D) 3/4 (E) 7/12

9. There are 120 different five digit numbers that can be constructed by putting the digits 1, 2, 3, 4 and 5 in all possible different orders. If these numbers are placed in numerical order, from smallest to largest, what is the 73^{rd} number in the list?
(A) 12543 (B) 23145 (C) 32415 (D) 41235 (E) 51325

10. Points *A*, *B*, *C* and *D* have these coordinates: A(3, 5), B(3, −5), C (−3, −5) and D (−3, 2). The area of quadrilateral *ABCD* is
(A) 42 (B) 55 (C) 51 (D) 60 (E) 24



11. Of the 60 students in Robert's class, 14 prefer chocolate pie, 18 prefer apple, and 8 prefer blueberry. Half of the remaining students prefer cherry pie and half prefer lemon. For Robert's pie graph showing this data, how many degrees should she use for cherry pie?
(A) 10 (B) 20 (C) 30 (D) 60 (E) 72

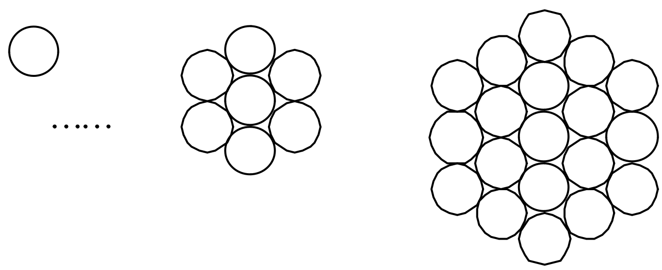
12. Ted has entered a buffet line in which he chooses two kind of meat, three different vegetables and four desserts. If the order of food items is not important, how many different meals might he choose?

Meat: beef, chicken, pork, duck, fish
Vegetables: baked beans, corn, potatoes, tomatoes, broccoli, chives
Dessert: brownies, chocolate cake, chocolate pudding, ice cream, apricot pops
(A) 400 (B) 244 (C) 1000 (D) 800 (E) 144

13. Helen began peeling a pile of 145 potatoes at the rate of 5 potatoes per minute. Five minutes later Charles joined her and peeled at the rate of 7 potatoes per minute. When they finished, how many potatoes had Charles peeled?
(A) 70 (B) 24 (C) 32 (D) 33 (E) 60

14. These circles have the same radius. If the pattern continues, how many circles are therein the 20th figure?

Figure 1 Figure 2 Figure3



(A) 1141 (B) 1142 (C) 2000 (D) 1024 (E) 1000

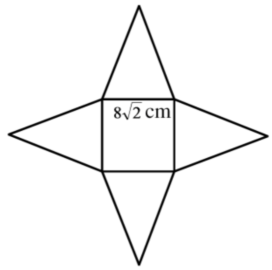
15. Find a positive integer a such that $a = \sqrt{2013^2 + 2013 + 2014}$.
(A) 1002 (B) 2012 (C) 2013 (D) 2014 (E) 1007

16. Three dice are thrown. What is the probability that the product of the three numbers is a multiple of 5?
(A) $\frac{91}{216}$ (B) $\frac{125}{216}$ (C) $\frac{25}{216}$ (D) $\frac{7}{36}$ (E) $\frac{17}{36}$

17. How many ways can the number 10 be written as the sum of exactly three positive and not necessarily different integers if the order in which the sum is written matters? For example, 10 = 1 + 4 + 5 and is not the same as 10 = 4 + 1 + 5.
(A) 10 (B) 16 (C) 27 (D) 36 (E) 30

18. Alex and Bob ride along a circular path whose circumference is 15 km. They start at the same time, from diametrically opposite positions. Alex goes at a constant speed of 35 km/h in the clockwise direction, while Bob goes at a constant speed of 25 km/h in the counter clockwise direction. They both cycle for 3 hours. How many times do they meet?
(A) 12 (B) 13 (C) 14 (D) 15 (E) 10

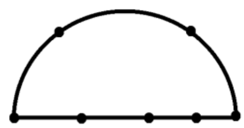
19. Four identical isosceles triangles border a square of side $8\sqrt{2}$ cm, as shown. When the four triangles are folded up they meet at a point to form a pyramid with a square base. If the height of this pyramid is 6 cm, find the area of one triangles.



(A) $8\sqrt{34}$ cm^2 (B) $4\sqrt{34}$ cm^2 (C) 98 cm^2
(D) $18\sqrt{3}$ cm^2 (E) 46 cm^2

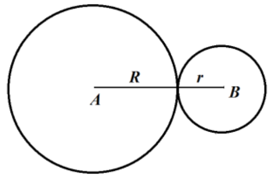
20. There are 52 students in a class. 30 of them can swim. 35 can ride bicycle. 42 can play table tennis. At least how many students can do all three sports?
(A) 3 (B) 4 (C) 12 (D) 5 (E) 7

21. How many triangles can be formed by connecting three points of the figure?
(A) 15 (B) 20 (C) 22 (D) 25 (E) 17



22. You have enough 2¢, 3¢, and 4 ¢ stamps and you want to stick them in a row. How many ways are there to get a total of 10¢?
(A) 11 (B) 15 (C) 16 (D) 17 (E) 19

23. Circle B of radius 2 is rolling around a second circle A of radius 10 without slipping until it returns to its starting point. The number of revolutions the circle B makes is



(A) 3 (B) 4 (C) 8 (D) 6 (E) 7

24. A box contains exactly seven marbles, four red and three white. Marbles are randomly removed one at a time without replacement until all the red marbles are drawn or all the white marbles are drawn. What is the probability that the last marble drawn is white?

(A) 3/10 (B) 2/5 (C) 1/2 (D) 4/7 (E) 7/10

25. A positive integer is randomly selected from all positive integers among 1 and 300 inclusive that are multiples of 3, 4, or 5. What is the probability that the positive integer selected is not divisible by 5?

(A) $\frac{2}{3}$ (B) $\frac{25}{37}$ (C) $\frac{5}{9}$ (D) $\frac{1}{3}$ (E) $\frac{4}{9}$

SOLUTIONS:

1. Solution: (A).
At 4 seconds per dimple, it takes $600 \times 4 = 2400$ seconds to paint them.
Since there are 60 seconds in a minute, he will need $2400 \div 60 = 40$ minutes.

2. Solution: (D).
Since their sum is 23, only positive factors need to be considered.
Number pairs whose product is 132 are (1, 132), (2, 66), (3, 44), (4, 33), (6, 22) , and (12, 11. The sum of the third pair is 23, so the numbers are 12 and 11. The larger one is 12.

3. Solution: (E).
Emily has two-third as much money as Gary, so Emily has \$84.
Frank has \$4 more than Emily, and \$84 + \$4 = \$88.

4. Solution: (E).
To make the number as big as possible, the bigger digits are placed in the higher-value positions.

To make the number odd, we let 3 be the units digit. So we have 96523. The digit in the tens place is 2.

5. Solution: (B).
There are four spaces between the first tree and the fifth tree, so the distance between adjacent trees is 20 feet. There are fifteen spaces between the first and last trees. So the distance is $20 \times 15 = 300$ feet.

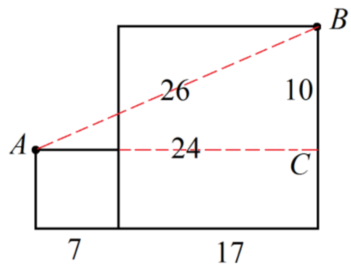
6. Solution: (E).
Let J, Y, and B be the amount of money James, Yao, and Bob have, respectively.

$J = 1.2Y$ (1)

$B = 0.8J$ (2)

Substituting (1) into (2): $B = 0.8\ (1.2Y) = 0.96Y$.
Thus the amount of money Bob has is $1 – 0.96 = 4\%$ less money than Yao's money.

7. Solution: (C).
Connect *AB*. Extend the side of the smaller square from *A* to *C*. Triangle *ABC* is a 10-24-26 (5-12-13) right triangle. So *AB* = 26.



8. Solution: (E).
$72 = 2^3 \times 3^2$ has (3 + 1)(2 + 1) = 12 factors.
The factors less than 11 are 1, 2, 3, 4, 6, 8, and 9. There are 7 of them.
The probability is 7/12.

9. Solution: (D).
If we have 1 as the first digit, we have 4! = 24 numbers with the first number 12345 and the last of them 15432.
If we have 2 as the first digit, we have 4! = 24 numbers.
If we have 3 as the first digit, we have 4! = 24 numbers with the first number 31245 and the last of them 35421.
Now we have 3 × 24 = 71 numbers.
The 73rd number will be 41235.

10. Solution: (C).
The figure is a trapezoid.
The area is $\frac{7+10}{2} \times 6 = 51$ square units.

11. Solution: (D).
Since 14 + 18 + 8 = 40, there are 60 − 40 = 20 children who prefer cherry or lemon pie.
20/ 2 = 10.
$\frac{10}{60} \times 360° = 60°$.

12. Solution: (C).
There are $\binom{5}{2} = 10$ choices for the meat. $\binom{6}{3} = 20$ for vegetables, and $\binom{5}{1} = 5$ for dessert.

The answer is 10 × 20 × 5 = 1000.

13. Solution: (A).
After 5 minutes Helen had peeled 25 potatoes. When Charles joined her, the combined rate of peeling was 12 potatoes per minute, so the remaining 120 potatoes required 10 minutes to peel. In these 10 minutes Charles peeled 70 potatoes.

14. Solution: (A).
Method 1:
We see the pattern for the number of circles in any figure::

Figure 1	Figure 2	Figure 3	Figure *n*
1	2 + ***3*** + 2	3 + 4 + **5** + 4 +3	*n* + …+ (2*n* – 1)+…+ *n*

Thus in figure 20 we have

20 + 21 + …38 + 39 + 38 +...+ 21 + 20 = $2\times\frac{(20+38)\times 19}{2}+39=1141$.

Method 2:

$$\begin{array}{ccccccc}
1 & & 2 & & 3 & & 4 \\
1 & & 7 & & 19 & & 37 \\
 & 6 & & 12 & & 18 & \\
 & & 6 & & 6 & &
\end{array}$$

By Newton's little formula, $a_{20}=1+6\binom{19}{1}+6\binom{19}{2}=1141$.

15. Solution: (D).
$a=\sqrt{2013^2+2013+2014}=\sqrt{2013(2013+1)+2014}=\sqrt{2013(2014)+2014}$
$=\sqrt{2014(2013+1)}=2014$.

16. Solution: (A).

The probability that the product is not a multiple of 5: $\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}=\frac{125}{216}$.

The probability that the product is a multiple of 5: $P=1-\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}=\frac{91}{216}$.

17. Solution: (D).

Method 1:

8 + 1 + 1: $\frac{3!}{2!}=3$ ways.

7 + 2 + 1: 3! = 6 ways.

6 + 3 + 1: 3! = 6 ways.

6 + 2 + 2: $\frac{3!}{2!}=3$ ways.

5 + 4 + 1: 3! = 6 ways.

5+ 3 +2: 3! = 6 ways.

4 + 4 + 2: $\frac{3!}{2!}=3$ ways.

4 + 3 + 3: $\frac{3!}{2!}=3$ ways.

Total 36 ways.

Method 2:

We write 10 as 10 1's. There are nine spaces between these 1's.

1 1 1 1 1 1 1 1 1 1

Any two partitions will generate a division. The partition below shows 10 = 1 + 3 + 6.

1 1 1 1 1 1 1 1 1 1

So the answer will be $\binom{9}{2}=\frac{9\times 8}{2}=36$ ways.

18. Solution: (A)

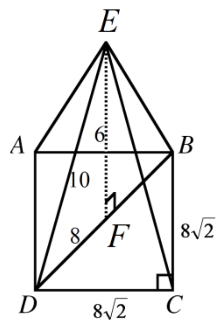
Let Bob does not move at all and Alex moves at a relative speed of (35 + 25) = 60 km/h.

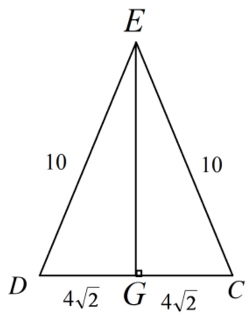
After 3 hours Alex has gone around the track $3 \times 60/15 = 12$ times, so Alex passes Bob 12 times.

19. Solution: (A).
Draw the pyramid and labeled it as shown. Draw a line
EF perpendicular to the square base. In triangle *BCD*, *DB* = 16. So triangle *DEF* is a 6-8-10 right triangle.

In triangle *EDG*, $EG = \sqrt{10^2 - (4\sqrt{2})^2} = \sqrt{68} = 2\sqrt{17}$

The area of the *EDG* is
$\frac{DC \times EG}{2} = \frac{2\sqrt{17} \times 8\sqrt{2}}{2} = 8\sqrt{34}$.





20. Solution: (A).
Method 1:
Number of students who cannot swim: 52 – 30 = 22.
Number of students who cannot ride bicycle: 52 – 35 = 17.
Number of students who cannot play tennis: 52 – 42 = 10.
At most 22 + 17 + 10 = 49 students cannot play at least one of the three activities.
At least 52 – 49 = 3 students can do all three sports.

Method 2: **The tickets method**
Step 1: Give each student a ticket for each activity he or she likes. 30 + 35 + 42 = 107 tickets are given out.

Step 2: Take away the tickets from them. Students who have 2 or more tickets will give back 2 tickets. Students who have less than 2 tickets will give back all the tickets.
Step 3: Calculate the number of tickets taken back: at most 2 × 52 = 104 tickets were taken back.
Step 4: Calculate the number of tickets that are still in the students hands.
107 – 104 = 3.

At this moment, any student who has the ticket will have only one ticket. These students are the ones who like 3 activities. The answer is 3.

21. Solution: (D).
Method 1:
We can either select 2 points from the diameter and 1 point from the circumference or select 1 point from the diameter and 2 points from the circumference

$$\binom{5}{2}\binom{2}{1}+\binom{5}{1}\binom{2}{2}=20+5=25$$

Method 2:
We use indirect way:

$$\binom{7}{3}-\binom{5}{3}=35-10=25.$$

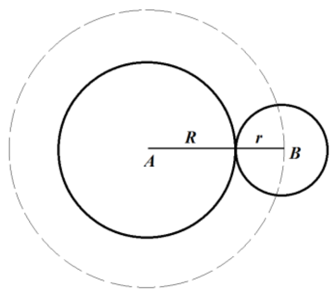
22. Solution: (D).
We need to get N_1, N_2 , N_3, and N_4.

Stairs	# of ways	Note
4	2	(2 + 2 or 4)
3	1	(3)
2	1	(2)
1	0	

With the formula $N_5 = N_3 + N_2 + N_1$, the sequence can be obtained as follows: 0, 1, 1, 2, 2, 4, 5, 8, 11, 17.

23. Solution: (D).
Let N be the number of revolutions the circle B makes.

$$N=\frac{2\pi(R+r)}{2\pi\times r}=\frac{R}{r}+1=\frac{10}{2}+1=6.$$



24. Solution: (D).

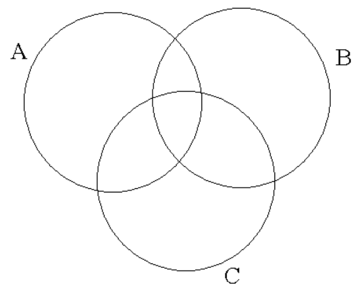
Think of continuing the drawing until all seven marbles are removed form the box. There are $\frac{7!}{4!\times 3!}=\frac{7\times 6\times 5\times 4!}{4!\times 3!}=\frac{7\times 6\times 5}{3!}=35$ possible orderings of the colors

Since we want that last marble drawn is white, so we avoid using all the red marbles in our arrangements (we just use 3 red marbles with 3 white marbles). There are $\frac{6!}{3!\times 3!}=\frac{6\times 5\times 4\times 3!}{3!\times 3!}=\frac{6\times 5\times 4}{3!}=20$ arrangements.

The last marble will be white with probability $P=\frac{20}{35}=\frac{5}{7}$.

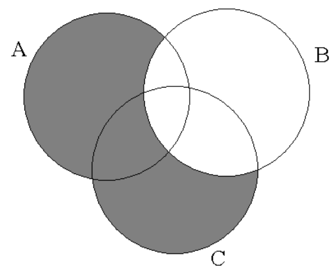
25. Solution: (A).

Let circle *A* represent the set of numbers divisible by 3, circle *B* represent the set of numbers divisible by 5, and circle *C* represent the set of numbers divisible by 4.



We want to find the shaded area in the figure below.

To find the shaded area, we find the union of sets *A*, *B* and *C*, and then subtract that from the set *B* to get the final result.



$$\left\lfloor\frac{300}{3}\right\rfloor+\left\lfloor\frac{300}{4}\right\rfloor+\left\lfloor\frac{300}{5}\right\rfloor-\left\lfloor\frac{300}{3\times 4}\right\rfloor-\left\lfloor\frac{300}{3\times 5}\right\rfloor-\left\lfloor\frac{300}{4\times 5}\right\rfloor+\left\lfloor\frac{300}{3\times 4\times 5}\right\rfloor$$
$$=100+75+60-25-20-15+5=180$$

$$\left\lfloor\frac{300}{5}\right\rfloor=60$$

180 – 60 = 120.

The probability is $P=\frac{120}{180}=\frac{2}{3}$.