mathematics vision project

Transforming Mathematics Education

ALGEBRA II

An Integrated Approach

MODULE 4

Polynomial Functions

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

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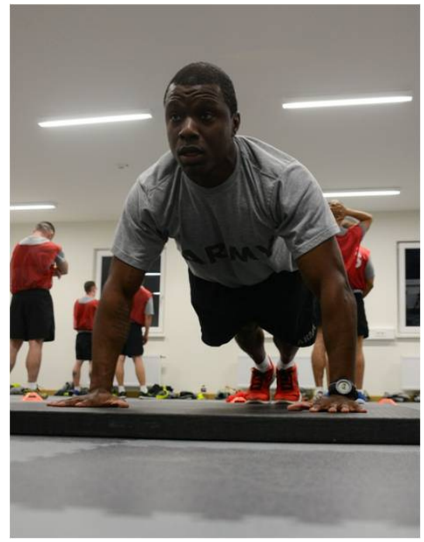
POLYNOMIAL FUNCTIONS

4.1 Scott's March Madness

A Develop Understanding Task



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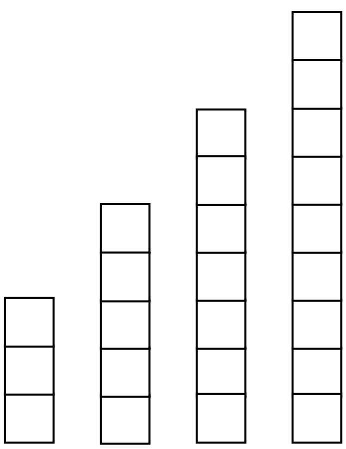
Each year, Scott participates in the "Macho March" promotion. The goal of "Macho March" is to raise money for charity by finding sponsors to donate based on the number of push-ups completed within the month. Last year, Scott was proud of the money he raised, but was also determined to increase the number of push-ups he would complete this year.

Part I: Revisiting the Past

Below is the bar graph and table Scott used last year to keep track of the number of push-ups he completed each day, showing he completed three push-ups on day one and five push-ups (for a combined **total** of eight push-ups) on day two. Scott continued this pattern throughout the month.



n Days	$f(n)$ Push-ups each **day**	$g(n)$ **Total** number of pushups in the month
1	3	3
2	5	8
3	7	15
4	9	24
5	11	35
…	…	
n		

1. Write the recursive and explicit equations for the number of push-ups Scott completed **on any given day** last year. Explain how your equations connect to the bar graph and the table above.

2. Write the recursive and explicit equation for the **accumulated total number of push-ups Scott completed by any given day** during the "Macho March" promotion last year.

Part II: March Madness

This year, Scott's plan is to look at the total number of push-ups he *completed for the month* last year ($g(n)$) and do that many push-ups each day ($m(n)$).

n Days	$f(n)$ Push-ups each **day last year**	$g(n)$ **Total** number of pushups in the month	$m(n)$ Push-ups each **day this year**	$T(n)$ **Total** push-ups completed for the month
1	3	3	3	
2	5	8	8	
3	7	15	15	
4	9	24		
5				
…	…			
n				

3. How many push-ups will Scott complete on day four? How did you come up with this number? Write the recursive equation to represent the total number of push-ups Scott will complete for the month on any given day.

4. How many **total** push-ups will Scott complete for the month on day four?

5. Without finding the explicit equation, make a conjecture as to the type of function that would represent the explicit equation for the total number of push-ups Scott would complete on any given day for this year's promotion.

6. How does the rate of change for this explicit equation compare to the rates of change for the explicit equations in questions 1 and 2?

7. Test your conjecture from question 5 and justify that it will always be true (see if you can move to a generalization for all polynomial functions).

READY, SET, GO! Name Period Date

READY

Topic: Completing inequality statements

For each problem, place the appropriate inequality symbol between the two expressions to make the statement true.

$If\ a > b, then:$	$If\ x > 10, then:$	$If\ 0 < x < 1$
1. $3a$ ___ $3b$	4. x^2 ___ 2^x	7. x ___ x^2
2. $b - a$ ___ $a - b$	5. $\sqrt{x}$ ___ x^2	8. $\sqrt{x}$ ___ x
3. $a + x$ ___ $b + x$	6. x^2 ___ x^3	9. x ___ $3x$

SET

Topic: Classifying functions

Identify the type of function for each problem. Explain how you know.

10

x	$f(x)$
1	3
2	6
3	12
4	24
5	48

11.

x	$f(x)$
1	3
2	6
3	9
4	12
5	15

12.

x	$f(x)$
1	3
2	9
3	18
4	30
5	45

13.

x	$f(x)$
1	7
2	9
3	13
4	21
5	37

14.

x	$f(x)$
1	-26
2	-19
3	0
4	37
5	98

15.

x	$f(x)$
1	-4
2	3
3	18
4	41
5	72

16. Which of the above functions are NOT polynomials?

GO

Topic: Recalling long division and the meaning of a factor

Find the quotient without using a calculator. If you have a remainder, write the remainder as a whole number. Example: $21\overline{)149}^{\,7}$ **remainder 2**

17. $30\overline{)510}$

18. $13\overline{)8359}$

19. Is 30 a factor of 510? How do you know?

20. Is 13 a factor of 8359? How do you know?

21. $22\overline{)14857}$

22. $952\overline{)40936}$

23. Is 22 a factor of 14587? How do you know?

24. Is 952 a factor of 40936? How do you know?

25. $92\overline{)3405}$

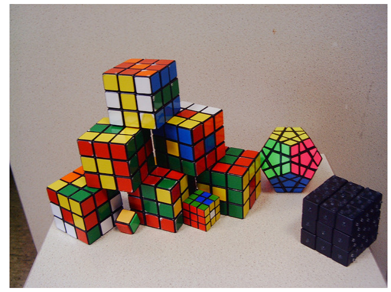
26. $27\overline{)3564}$

27. Is 92 a factor of 3405?

28. Is 27 a factor of 3564?

4.2 You-mix Cubes

A Solidify Understanding Task

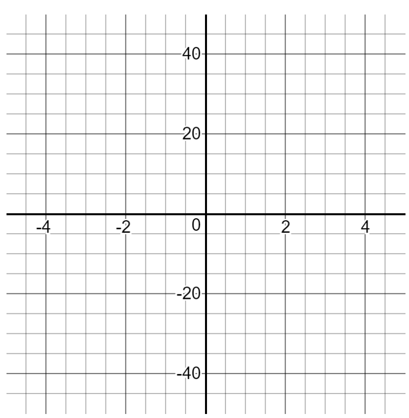


In Scott's March Madness, the function that was generated by the sum of terms in a quadratic function was called a **cubic function**. Linear functions, quadratic functions, and cubic functions are all in the family of functions called **polynomials**, which include functions of higher powers too. In this task, we will explore more about cubic functions to help us to see some of the similarities and differences between cubic functions and quadratic functions.

To begin, let's take a look at the most basic cubic function, $f(x) = x^3$. It is technically a **degree 3 polynomial** because the highest exponent is 3, but it's called a cubic function because these functions are often used to model volume. This is like quadratic functions which are **degree 2** polynomials but are called quadratic after the Latin word for square. Scott's March Madness showed that linear functions have a constant rate of change, quadratic functions have a linear rate of change, and cubic functions have a quadratic rate of change.

1. Use a table to verify that $f(x) = x^3$ has a quadratic rate of change.

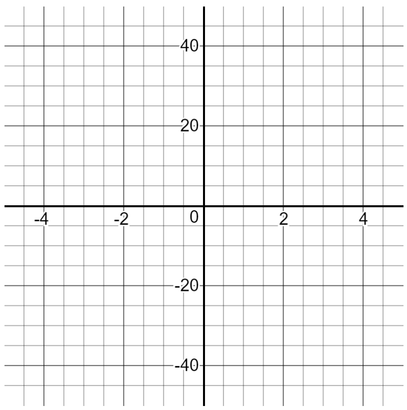
2. Graph $f(x) = x^3$.



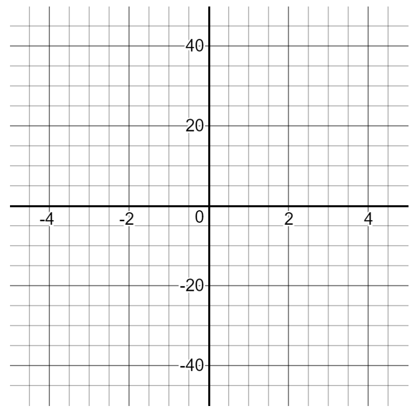
3. Describe the features of $f(x) = x^3$ including intercepts, intervals of increase or decrease, domain, range, etc.

4. Using your knowledge of transformations, graph each of the following without using technology.

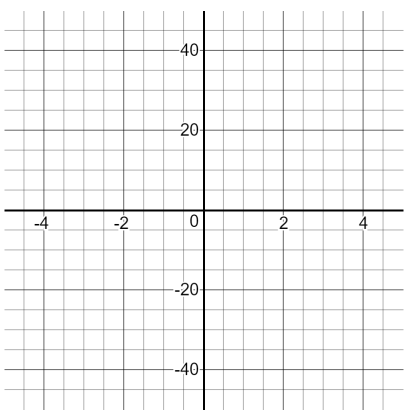
a) $f(x) = x^3 - 3$



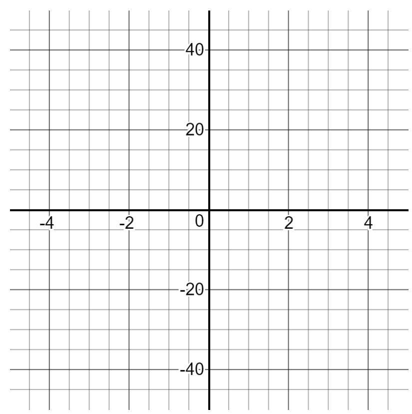
b) $f(x) = (x + 3)^3$



c) $f(x) = 2x^3$



d) $f(x) = -(x - 1)^3 + 2$



5. Use technology to check your graphs above. What transformations did you get right? What areas do you need to improve on so that your cubic graphs are perfect?

6. Since quadratic functions and cubic functions are both in the polynomial family of functions, we would expect them to share some common characteristics. List all the similarities between $f(x) = x^3$ and $g(x) = x^2$.

7. As you can see from the graph of $f(x) = x^3$, there are also some real differences in cubic functions and quadratic functions. Each of the following statements describe one of those differences. Explain <u>why</u> each statement is true by completing the sentence.

a) The range of $f(x) = x^3$ is $(-\infty, \infty)$, but the range of $g(x) = x^2$ is $[0, \infty)$ because:

____

b) For $x > 1, f(x) > g(x)$ because: ______________________________________

____

c) For $0 < x < 1, g(x) > f(x)$ because: ___________________________________

____

READY, SET, GO! Name Period Date

READY

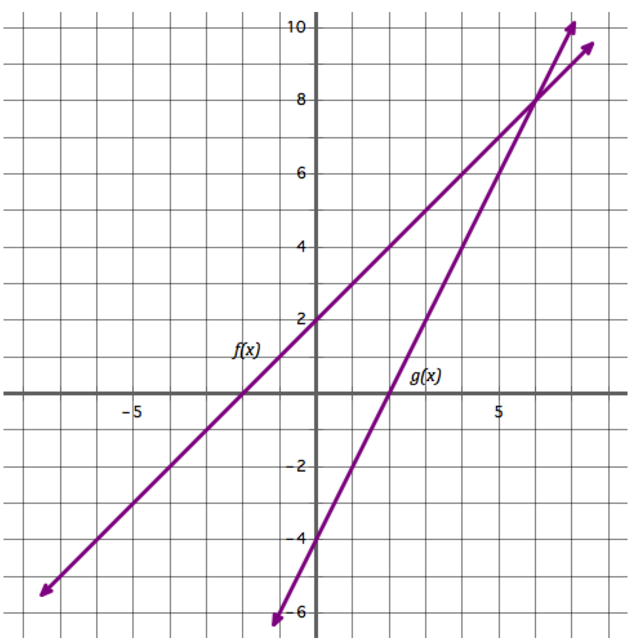
Topic: Adding and subtracting binomials

Add or subtract as indicated.

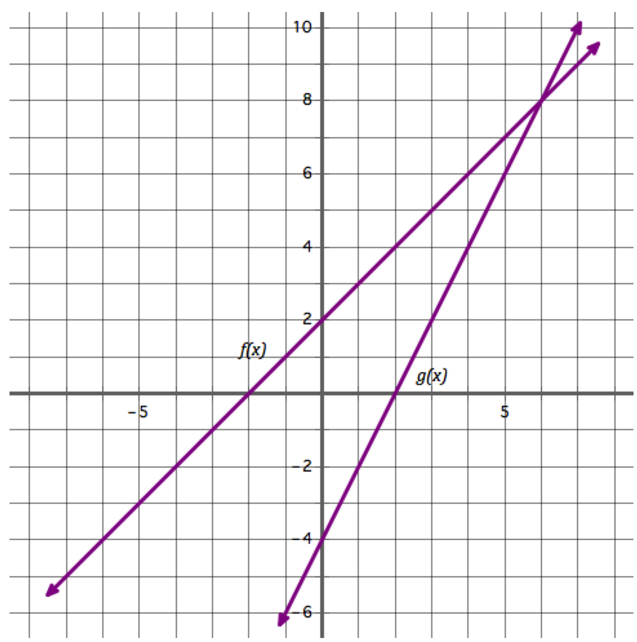
1. $(6x + 3) + (4x + 5)$
2. $(x + 17) + (9x - 13)$
3. $(7x - 8) + (-2x + 9)$
4. $(4x + 9) - (x + 2)$
5. $(-3x - 1) - (2x + 5)$
6. $(8x + 3) - (-10x - 9)$
7. $(3x - 7) + (-3x - 7)$
8. $(-5x + 8) - (-5x + 7)$
9. $(8x + 9) - (7x + 9)$

10. Use the graphs of $f(x)$ and $g(x)$ to sketch the graphs of $f(x) + g(x)$ and $f(x) - g(x)$.

$f(x) + g(x)$



$f(x) - g(x)$.



SET

Topic: Comparing simple polynomials

11. Complete the tables below for $y = x$ *and* $y = x^3$ *and* $y = x^5$

x	$y = x$
---1	
0	
1	

x	$y = x^3$
–1	
0	
1	

x	$y = x^5$
–1	
0	
1	

12. What assumption might you be tempted to make about the graphs of $y = x$, $y = x^3$ *and* $y = x^5$ based on the values you found in the 3 tables above?

13. What do you really know about the graphs of $y = x$ *and* $y = x^3$ *and* $y = x^5$ despite the values you found in the 3 tables above?

14. Complete the tables with the additional values.

x	$y = x$
-1	
$-1/2$	
0	
$1/2$	
1	

x	$y = x^3$
-1	
$-1/2$	
0	
$1/2$	
1	

x	$y = x^5$
-1	
$-1/2$	
0	
$1/2$	
1	

15. Graph $y = x\ and\ y = x^3\ and\ y = x^5$ on the interval $[-1, 1]$, using the same set of axes.

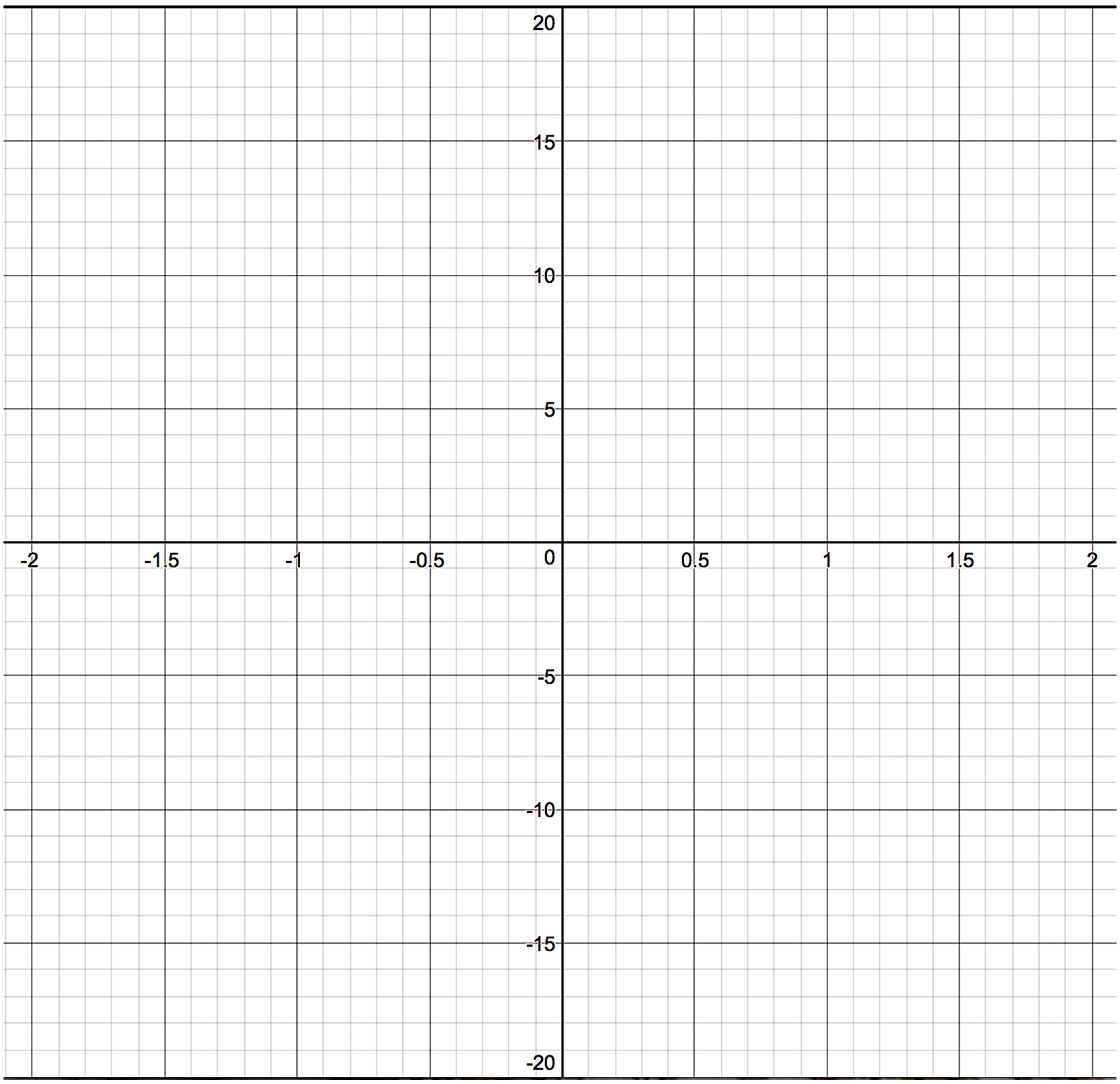
16. Complete the tables with the additional values.

x	$y = x$
-2	
-1	
$-1/2$	
0	
$1/2$	
1	
2	

x	$y = x^3$
-2	
-1	
$-1/2$	
0	
$1/2$	
1	
2	

x	$y = x^5$
-2	
-1	
$-1/2$	
0	
$1/2$	
1	
2	

17. Graph $y = x$ *and* $y = x^3$ *and* $y = x^5$ on the interval [-2, 2], using the same set of axes.



GO

Topic: Using the exponent rules to simplify expressions

Simplify.

18. $x^{1/3} \cdot x^{1/6} \cdot x^{1/4}$

19. $a^{2/5} \cdot a^{3/10} \cdot a^{2/15}$

20. $m^{4/7} \cdot m^{3/14} \cdot m^{5/28}$

4.3 Building Strong Roots

A Solidify Understanding Task

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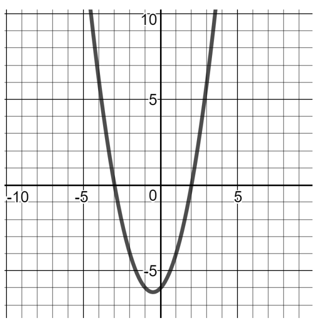
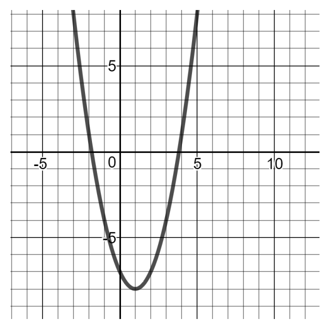
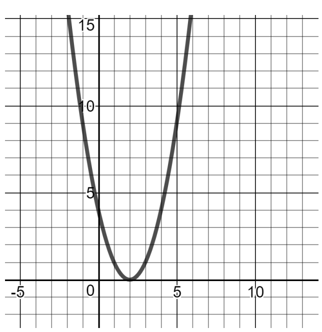
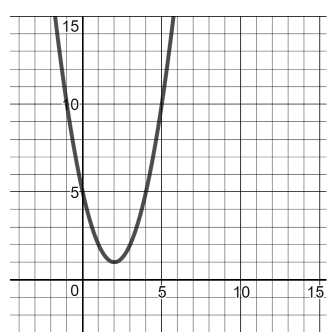
When working with quadratic functions, we learned the Fundamental Theorem of Algebra:
An n^{th} degree polynomial function has n roots.
In this task, we will be exploring this idea further with other polynomial functions.

First, let's brush up on what we learned about quadratics. The equations and graphs of four different quadratic equations are given below. Find the roots for each and identify whether the roots are real or imaginary.

1.

a) $f(x) = x^2 + x - 6$ 	b) $g(x) = x^2 - 2x - 7$ 
Roots:	Roots:
Type of roots:	Type of roots:
c) $h(x) = x^2 - 4x + 4$ 	d) $k(x) = x^2 - 4x + 5$ 
Roots:	Roots:
Type of roots:	Type of roots:

2. Did all of the quadratic functions have 2 roots, as predicted by the Fundamental Theorem of Algebra? Explain.

3. It's always important to keep what you've previously learned in your mathematical bag of tricks so that you can pull it out when you need it. What strategies did you use to find the roots of the quadratic equations?

4. Using your work from problem 1, write each of the quadratic equations in factored form. When you finish, check your answers by graphing, when possible, and make any corrections necessary.

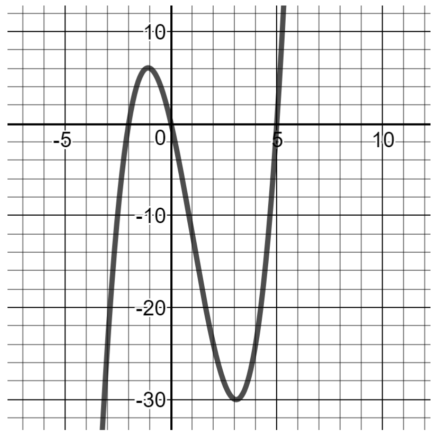
a) $f(x) = x^2 + x - 6$	b) $g(x) = x^2 - 2x - 7$
Factored form:	Factored form:
c) $h(x) = x^2 - 4x + 4$	d) $k(x) = x^2 - 4x + 5$
Factored form:	Factored form:

5. Based on your work in problem 1, would you say that roots are the same as x-intercepts? Explain.

6. Based on your work in problem 4, what is the relationship between roots and factors?

Now let's take a closer look at cubic functions. We've worked with transformations of $f(x) = x^3$, but what we've seen so far is just the tip of the iceberg. For instance, consider:

$g(x) = x^3 - 3x^2 - 10x$



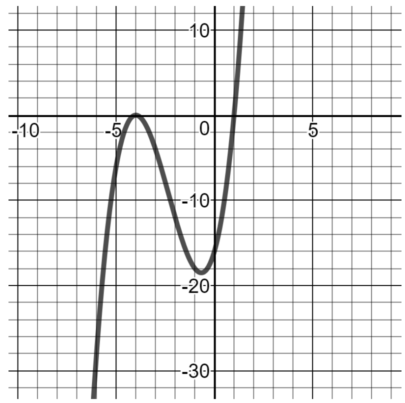
7. Use the graph to find the roots of the cubic function. Use the equation to verify that you are correct. Show how you have verified each root.

8. Write $g(x)$ in factored form. Verify that the factored form is equivalent to the standard form.

9. Are the results you found in #7 consistent with the Fundamental Theorem of Algebra? Explain.

Here's another example of a cubic function.

$f(x) = x^3 + 7x^2 + 8x - 16$

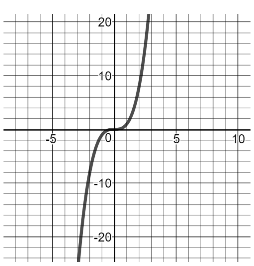


10. Use the graph to find the roots of the cubic function.

11. Write $f(x)$ in factored form. Verify that the factored form is equivalent to the standard form. Make any corrections needed.

12. Are the results you found in #10 consistent with the Fundamental Theorem of Algebra? Explain.

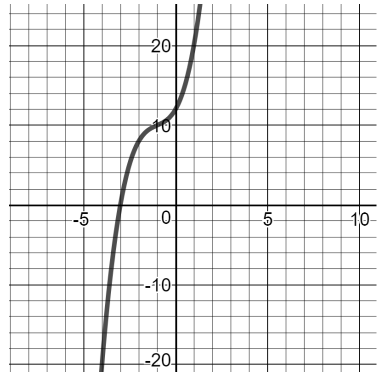
13. We've seen the most basic cubic polynomial function, $h(x) = x^3$ and we know its graph looks like this:



Explain how $h(x) = x^3$ is consistent with the Fundamental Theorem of Algebra.

14. Here is one more cubic polynomial function for your consideration. You will notice that it is given to you in factored form. Use the equation and the graph to find the roots of $p(x)$.

$$p(x) = (x+3)(x^2+4)$$



15. Use the equation to verify each root. Show your work below.

16. Are the results you found in #14 consistent with the Fundamental Theorem of Algebra? Explain.

17. Explain how to find the factored form of a polynomial, given the roots.

18. Explain how to find the roots of a polynomial, given the factored form.

READY, SET, GO! Name Period Date

READY

Topic: Practicing long division on polynomials

Divide using long division. (These problems have no remainders. If you get one, try again.)

1. $(x+3)\overline{)5x^3+2x^2-45x-18}$

2. $(x-6)\overline{)x^3-x^2-44x+84}$

3. $(x-5)\overline{)3x^3-15x^2+12x-60}$

4. $(x+2)\overline{)x^4+6x^3+7x^2-6x-8}$

SET

Topic: Applying the Fundamental Theorem of Algebra

Predict the number of roots for each of the given polynomial equations. (Remember that the Fundamental Theorem of Algebra states: An n^{th} degree polynomial function has n roots.)

5. $a(x) = x^2 + 3x - 10$

6. $b(x) = x^3 + x^2 - 9x - 9$

7. $c(x) = -2x - 4$

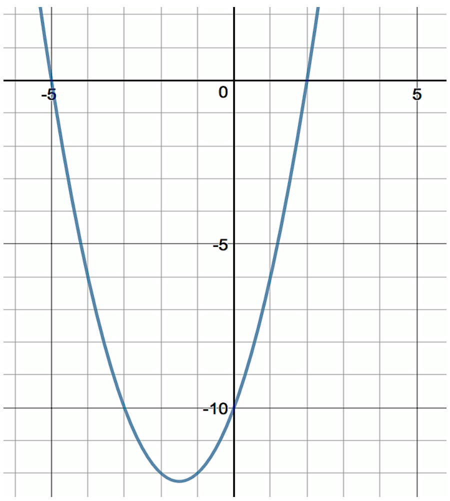
8. $d(x) = x^4 - x^3 - 4x^2 + 4x$

9. $f(x) = -x^2 + 6x - 9$

10. $g(x) = x^6 - 5x^4 + 4x^2$

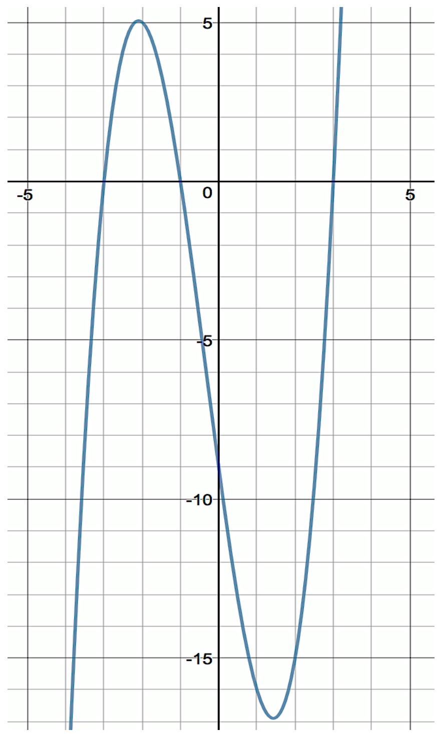
Below are the graphs of the polynomials from the previous page. Check your predictions. Then use the graph to help you write the polynomial in factored form.

11. $a(x) = x^2 + 3x - 10$



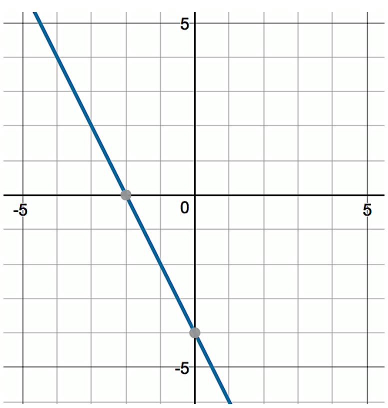
Factored form:

12. $b(x) = x^3 + x^2 - 9x - 9$



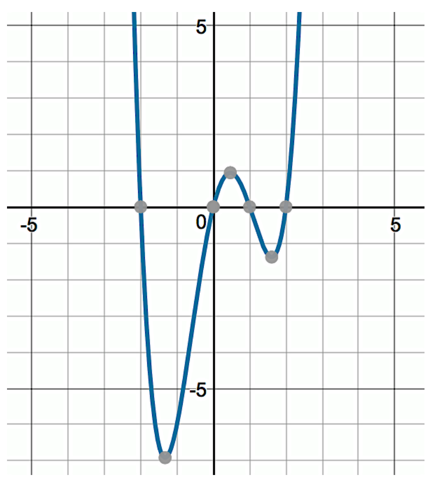
Factored form:

13. $c(x) = -2x - 4$



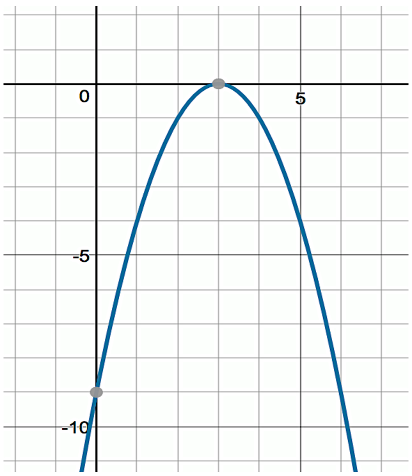
Factored form:

14. $d(x) = x^4 - x^3 - 4x^2 + 4x$



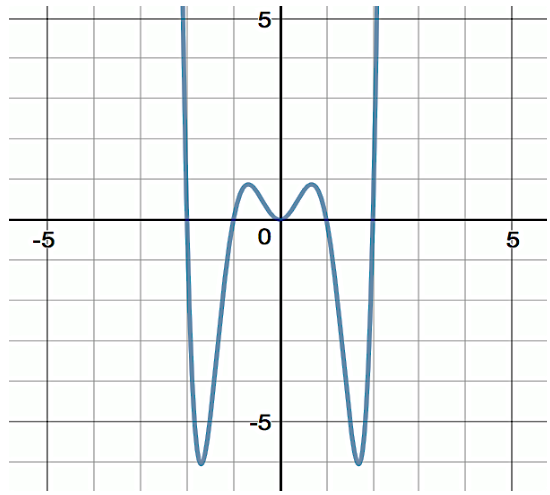
Factored form:

15. $f(x) = -x^2 + 6x - 9$



Factored form:

16. $g(x) = x^6 - 5x^4 + 4x^2$



Factored form:

17. The graphs of #15 and #16 don't seem to follow the Fundamental Theorem of Algebra, but there is something similar about each of the graphs. Explain what is happening at the point (3, 0) in #15 and at the point (0,0) in #16.

GO

Topic: Solving quadratic equations

Find the zeros for each equation using the quadratic formula.

18. $f(x) = x^2 + 20x + 51$

19. $f(x) = x^2 + 10x + 25$

20. $f(x) = 3x^2 + 12x$

21. $f(x) = x^2 - 11$

22. $f(x) = x^2 + x - 1$

23. $f(x) = x^2 + 2x + 3$

4.4 Getting to the Root of the Problem

A Solidify Understanding Task

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In *4.3 Building Strong Roots,* we learned to predict the number of roots of a polynomial using the Fundamental Theorem of Algebra and the relationship between roots and factors. In this task, we will be working on how to find all the roots of a polynomial given in standard form.

Let's start by thinking again about numbers and factors.

1. If you know that 7 is a factor of 147, what would you do to find the prime factorization of 147? Explain your answer and show your process here:

2. How is your answer like a polynomial written in the form: $P(x) = (x-7)^2(x-3)$?

The process for finding factors of polynomials is exactly like the process for finding factors of numbers. We start by dividing by a factor we know and keep dividing until we have all the factors. When we get the polynomial broken down to a quadratic, sometimes we can factor it by inspection, and sometimes we can use our other quadratic tools like the quadratic formula.

Let's try it! For each of the following functions, you have been given one factor. Use that factor to find the remaining factors, the roots of the function, and write the function in factored form.

3. Function: $f(x) = x^3 + 3x^2 - 4x - 12$ Factor: $(x+3)$ Roots of function:

Factored form:

4. Function: $f(x) = x^3 + 6x^2 + 11x + 6$ Factor: $(x + 1)$ Roots of function:

Factored form:

5. Function: $f(x) = x^3 - 5x^2 - 3x + 15$ Factor: $(x - 5)$ Roots of function:

Factored form:

6. Function: $f(x) = x^3 + 3x^2 - 12x - 18$ Factor: $(x - 3)$ Roots of function:

Factored form:

7. Function: $f(x) = x^4 - 16$ Factor: $(x - 2)$ Roots of function:

Factored form:

8. Function: $f(x) = x^3 - x^2 + 4x - 4$ Factor: $(x - 2i)$ Roots of function:

Factored form:

9. Is it possible for a polynomial with real coefficients to have only one imaginary root? Explain.

10. Based on the Fundamental Theorem of Algebra and the polynomials that you have seen, make a table that shows all the number of roots and the possible combinations of real and imaginary roots for linear, quadratic, cubic, and quartic polynomials.

READY, SET, GO! Name Period Date

READY

Topic: Ordering numbers from least to greatest

Order the numbers from least to greatest.

1. 100^3 $\sqrt{100}$ $log_2 100$ 100 2^{10}

2. 2^{-1} $-\sqrt{100}$ $log_2\left(\frac{1}{8}\right)$ 0 $(-2)^1$

3. 2^0 $\sqrt{25}$ $log_2 8$ $2(x^0), x \neq 0$ $(2)^{-\frac{1}{2}}$

4. $log_3 3^3$ $log_5 5^{-2}$ $log_6 6^0$ $log_4 4^{-1}$ $log_2 2^3$

Refer to the given graph to answer the questions.

Insert $>, <, or =$ in each statement to make it true.

5. $f(0)$ ________ $g(0)$

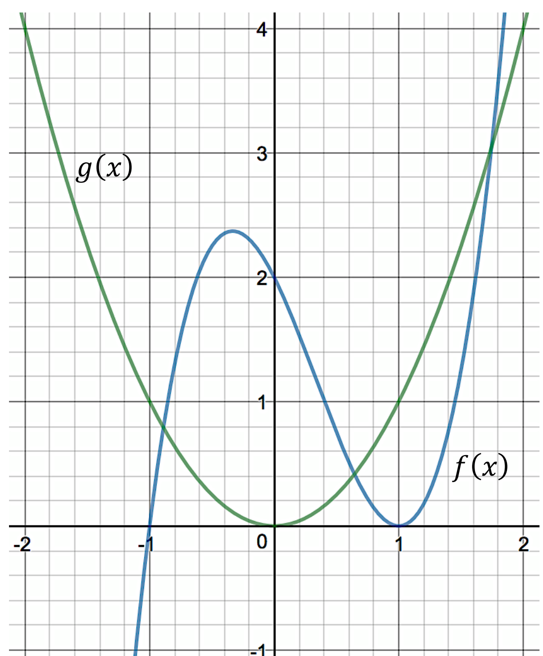
6. $f(2)$ ________ $g(2)$

7. $f(-1)$ ________ $g(-1)$

8. $f(1)$ ________ $g(-1)$

9. $f(5)$ ________ $g(5)$

10. $f(-2)$ ________ $g(-2)$



SET

Topic: Finding the roots and factors of a polynomial

Use the given root to find the remaining roots. Then write the function in factored form.

Function	Roots	Factored form
11. $f(x) = x^3 - 13x^2 + 52x - 60$	$x = 5$	
12. $g(x) = x^3 + 6x^2 - 11x - 66$	$x = -6$	
13. $p(x) = x^3 + 17x^2 + 92x + 150$	$x = -3$	
14. $q(x) = x^4 - 6x^3 + 3x^2 + 12x - 10$	$x = \sqrt{2}$	

GO

Topic: Using the distributive property to multiply complex expressions

Multiply using the distributive property. Simplify. Write answers in standard form.

15. $(x-\sqrt{13})(x+\sqrt{13})$

16. $(x-3\sqrt{2})(x+3\sqrt{2})$

17. $(x-4+2i)(x-4-2i)$

18. $(x+5+3i)(x+5-3i)$

19. $(x-1+i)(x-1-i)$

20. $(x+10-\sqrt{2}\,i)(x+10+\sqrt{2}\,i)$

4.5 Is This The End?

A Solidify Understanding Task

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In previous mathematics courses, you have compared and analyzed growth rates of polynomial (mostly linear and quadratic) and exponential functions. In this task, we are going to analyze rates of change and end behavior by comparing various expressions.

Part I: Seeing patterns in **end behavior**

1. In as many ways as possible, compare and contrast linear, quadratic, cubic, and exponential functions.

2. Using the graph provided, write the following functions vertically, from **greatest to least for** $\boldsymbol{x = 0}$. Put the function with the greatest value on top and the function with the smallest value on the bottom. Put functions with the same values at the same level. An example, $l(x) = x^7$, has been placed on the graph to get you started.

$f(x) = 2^x$	$p(x) = x^3 + x^2 - 4$	$g(x) = x^2 - 20$
$h(x) = x^5 - 4x^2 + 1$	$k(x) = x + 30$	$m(x) = x^4 - 1$
$r(x) = x^5$	$n(x) = \left(\frac{1}{2}\right)^x$	$q(x) = x^6$

3. What determines the value of a polynomial function at $x = 0$? Is this true for other types of functions?

4. Write the same expressions on the graph in order from **greatest to least** when x represents a very large number (this number is so large, so we say that it is approaching positive infinity). If the value of the function is positive, put the function in quadrant 1. If the value of the function is negative, put the function in quadrant IV. An example has been placed for you.

5. What determines the end behavior of a polynomial function for very large values of x?

6. Write the same functions in order from **greatest to least** when x represents a number that is approaching negative infinity. If the value of the function is positive, place it in Quadrant II, if the value of the function is negative, place it in Quadrant III. An example is shown on the graph.

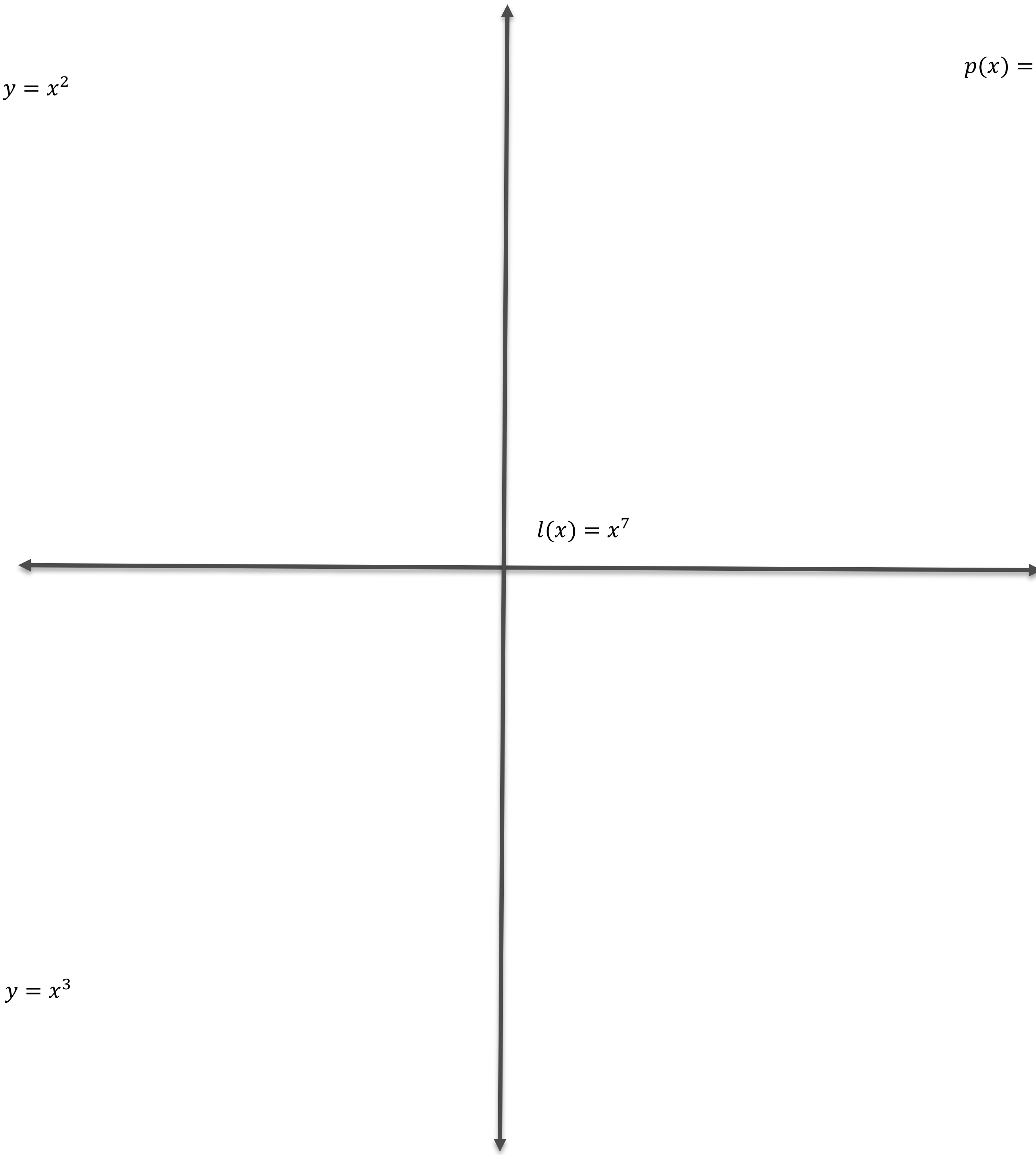
7. What patterns do you see in the polynomial functions for x values approaching negative infinity? What patterns do you see for exponential functions? Use graphing technology to test these patterns with a few more examples of your choice.

8. How would the end behavior of the polynomial functions change if the lead terms were changed from positive to negative?

$\boldsymbol{x \to -\infty}$ $\boldsymbol{x = 0}$ $\boldsymbol{x \to \infty}$

$y = x^2$

$p(x) = x^7$



$y = x^3$

READY, SET, GO! Name Period Date

READY

Topic: Recognizing special products

Multiply.

1. $(x+5)(x+5)$
2. $(x-3)(x-3)$
3. $(a+b)(a+b)$

4. In problems 1 – 3 the answers are called **perfect square trinomials**. What about these answers makes them be a **perfect square trinomial**?

5. $(x+8)(x-8)$
6. $(x+\sqrt{3})(x-\sqrt{3})$
7. $(x+b)(x-b)$

8. The products in problems 5 – 7 end up being binomials, and they are called the **difference of two squares**. What about these answers makes them be the **difference of two squares**?

   Why don't they have a middle term like the problems in 1 – 3?

9. $(x-3)(x^2+3x+9)$
10. $(x+10)(x^2-10x+100)$
11. $(a+b)(a^2-ab+b^2)$

12. The work in problems 9 – 11 makes them feel like the answers are going to have a lot of terms. What happens in the work of the problem that makes the answers be binomials?

    These answers are called the **difference of two cubes** (#9) and the **sum of two cubes** (#10 and #11.) What about these answers makes them be the **sum or difference of two cubes**?

SET

Topic: Determining values of polynomials at zero and at $\pm\infty$. (End behavior)

State the y-intercept, the degree, and the end behavior for each of the given polynomials.

13. $f(x) = x^5 + 7x^4 - 9x^3 + x^2 - 13x + 8$

y- intercept:

Degree:

End behavior:

As $x \to -\infty,\ f(x) \to$ ________

As $x \to +\infty,\ f(x) \to$ ________

14. $g(x) = 3x^4 + x^3 + 5x^2 - x - 15$

y- intercept:

Degree:

End behavior:

As $x \to -\infty,\ g(x) \to$ ________

As $x \to +\infty,\ g(x) \to$ ________

15. $h(x) = -7x^9 + x^2$

y- intercept:

Degree:

End behavior:

As $x \to -\infty,\ h(x) \to$ ________

As $x \to +\infty,\ h(x) \to$ ________

16. $p(x) = 5x^2 - 18x + 4$

y- intercept:

Degree:

End behavior:

As $x \to -\infty,\ p(x) \to$ ________

As $x \to +\infty,\ p(x) \to$ ________

17. $q(x) = x^3 - 94x^2 - x - 20$

y- intercept:

Degree:

End behavior:

As $x \to -\infty,\ q(x) \to$ ________

As $x \to +\infty,\ q(x) \to$ ________

18. $y = -4x + 12$

y- intercept:

Degree:

End behavior:

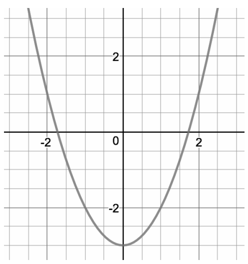
As $x \to -\infty,\ y \to$ ________

As $x \to +\infty,\ y \to$ ________

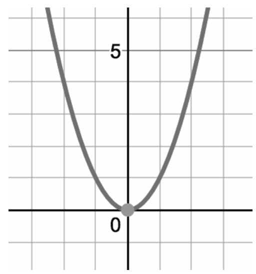
Topic: Identifying even and odd functions

19. Identify each function as even, odd, or neither.

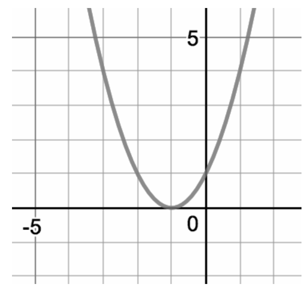
a) $f(x) = x^2 - 3$



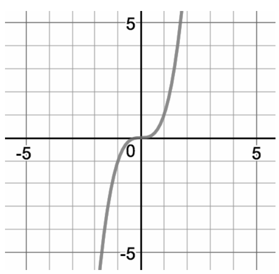
b) $f(x) = x^2$



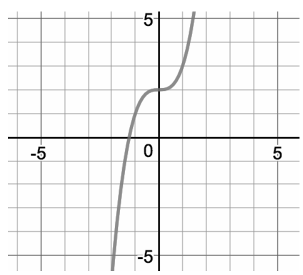
c) $f(x) = (x+1)^2$



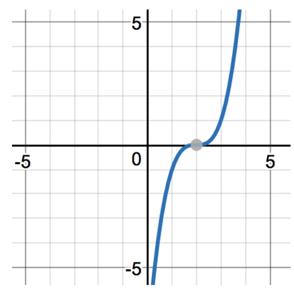
d) $f(x) = x^3$



e) $f(x) = x^3 + 2$



f) $f(x) = (x-2)^3$



GO

Topic: Factoring special products

Fill in the blanks on the sentences below.

20. The expression $\boldsymbol{a^2 + 2ab + b^2}$ is called a **perfect square trinomial**. I can recognize it because the first and last terms will always be perfect ____________________________.

The middle term will be 2 times the ____________________ and ____________________.

There will always be a ____________ sign before the last term.

It factors as (____________)(____________).

21. The expression $\boldsymbol{a^2 - b^2}$ is called the **difference of 2 squares**. I can recognize it because it's a binomial and the first and last terms are perfect ______________________________.
The sign between the first term and the last term is always a __________________ .
It factors as (____________)(____________).

22. The expression $\boldsymbol{a^3 + b^3}$ is called the **sum of 2 cubes**. I can recognize it because it's a binomial and the first and last terms are ______________________. The expression $a^3 + b^3$ factors into a binomial and a trinomial. I can remember it as a *short* (____) and a *long* (__________).
The sign between the terms in the binomial is the _____________ as the sign in the expression. The first sign in the trinomial is the ________________ of the sign in the binomial. That's why all of the middle terms cancel when multiplying.
The last sign in the trinomial is always ________.
It factors as (____________)(______________________).

Factor using what you know about special products.

23. $25x^2 + 30 + 9$

24. $x^2 - 16$

25. $x^3 + 27$

26. $49x^2 - 36$

27. $x^3 - 1$

28. $64x^2 - 240 + 225$

Part II: Using end behavior patterns

For each situation:

- Determine the function type. If it is a polynomial, state the degree of the polynomial and whether it is an even degree polynomial or an odd degree polynomial.
- Describe the end behavior based on your knowledge of the function. Use the format:
  As $x \rightarrow -\infty, f(x) \rightarrow$ _____ $and \quad as\ x \rightarrow \infty\ f(x) \rightarrow$ _____

1. $f(x) = 3 + 2x$

   Function type:

   End behavior: As $x \rightarrow -\infty, f(x) \rightarrow$ _____

   End behavior: As $x \rightarrow \infty, f(x) \rightarrow$ _____

2. $f(x) = x^4 - 16$

   Function type:

   End behavior: As $x \rightarrow -\infty, f(x) \rightarrow$ _____

   End behavior: As $x \rightarrow \infty, f(x) \rightarrow$ _____

3. $f(x) = 3^x$

   Function type:

   End behavior: As $x \rightarrow -\infty, f(x) \rightarrow$ _____

   End behavior: As $x \rightarrow \infty, f(x) \rightarrow$ _____

4. $f(x) = x^3 + 2x^2 - x + 5$

   Function type:

   End behavior: As $x \rightarrow -\infty, f(x) \rightarrow$ _____

   End behavior: As $x \rightarrow \infty, f(x) \rightarrow$ _____

5. $f(x) = -2x^3 + 2x^2 - x + 5$

   Function type:

   End behavior: As $x \rightarrow -\infty, f(x) \rightarrow$ _____

   End behavior: As $x \rightarrow \infty, f(x) \rightarrow$ _____

6. $f(x) = log_2 x$

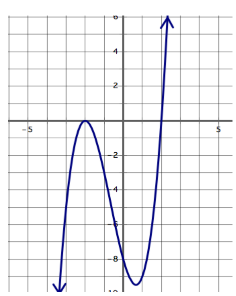
   Function type:

   End behavior: As $x \rightarrow -\infty, f(x) \rightarrow$ _____

   End behavior: As $x \rightarrow \infty, f(x) \rightarrow$ _____

Use the graphs below to describe the end behavior of each function by completing the statements.

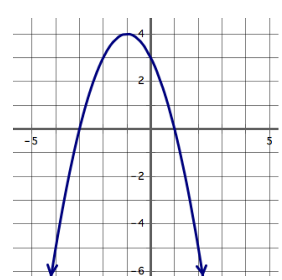
7.



End behavior: As $x \rightarrow -\infty, f(x) \rightarrow$ _____

End behavior: As $x \rightarrow \infty, f(x) \rightarrow$ _____

8.



End behavior: As $x \rightarrow -\infty, f(x) \rightarrow$ _____

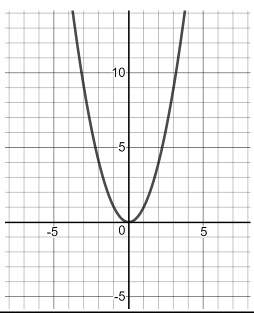
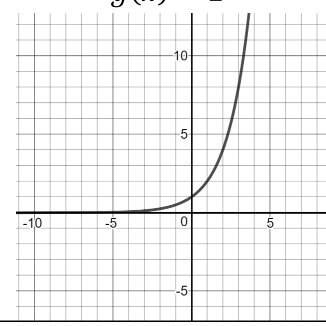
End behavior: As $x \rightarrow \infty, f(x) \rightarrow$ _____

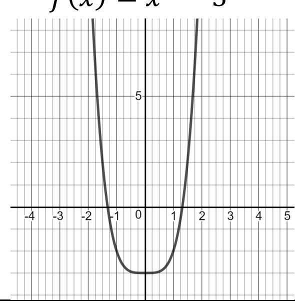
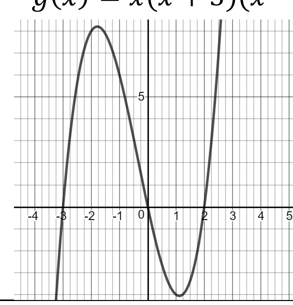
9. How does the end behavior for quadratic functions connect with the number and type of roots for these functions? How does the end behavior for cubic functions connect with the number and type of roots for cubic functions?

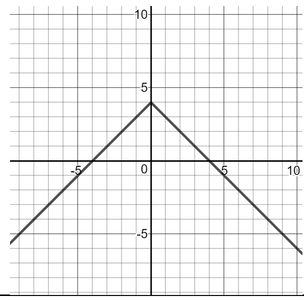
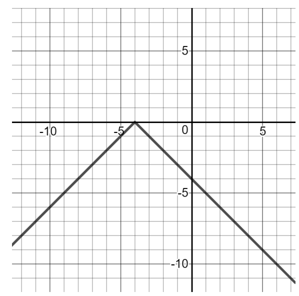
Part III: Even and Odd Functions

Some functions that are not polynomials may be categorized as even functions or odd functions. When mathematicians say that a function is an even function, they mean something very specific.

1. Let's see if you can figure out what the definition of an even function is with these examples:

<table>
<tr><td>Even function:
$f(x) = x^2$
</td><td>Not an even function:
$g(x) = 2^x$
</td></tr>
<tr><td colspan="2">Differences:</td></tr>
</table>

<table>
<tr><td>Even function:
$f(x) = x^4 - 3$
</td><td>Not an even function:
$g(x) = x(x+3)(x-2)$
</td></tr>
<tr><td colspan="2">Differences:</td></tr>
</table>

<table>
<tr><td>Even function:
$f(x) = -|x| + 4$

</td><td>Not an even function:
$g(x) = -|x + 4|$

</td></tr>
<tr><td colspan="2">Differences:</td></tr>
<tr><td>Even function:
$f(2) = 5 \; and \; f(-2) = 5$</td><td>Not an even function:
$g(2) = 3 \; and \; g(-2) = 5$</td></tr>
<tr><td colspan="2">Differences:</td></tr>
</table>

2. What do you observe about the characteristics of an even function?

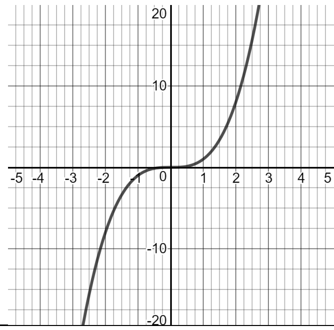
3. The algebraic definition of an even function is:
   $f(x)$ is an even function if and only if $f(x) = f(-x)$ for all values of x in the domain of f.
   What are the implications of the definition for the graph of an even function?

4. Are all even-degree polynomials even functions? Use examples to explain your answer.

5. Let's try the same approach to figure out a definition for odd functions.

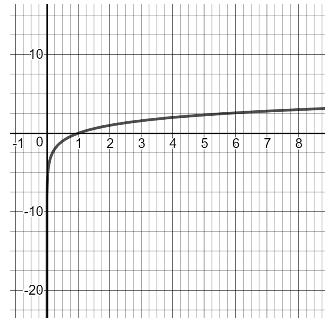
Odd function:

$$f(x) = x^3$$



Not an odd function:

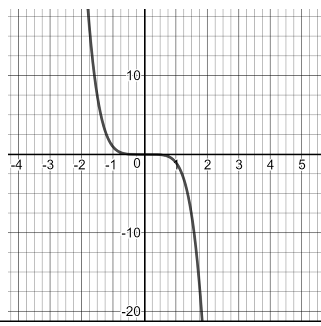
$$g(x) = \log_2 x$$



Differences:

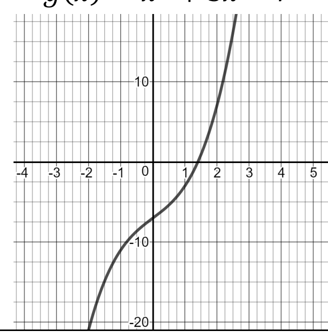
Odd function:

$$f(x) = -x^5$$



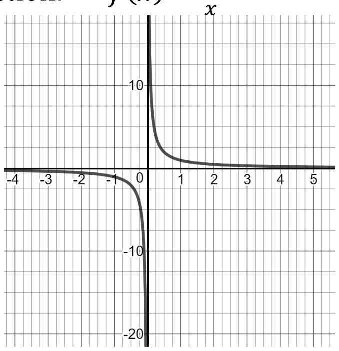
Not an odd function:

$$g(x) = x^3 + 3x - 7$$



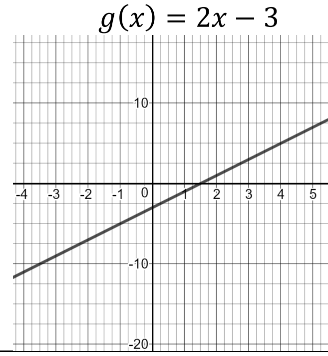
Differences:

Odd function: $f(x) = \frac{1}{x}$



Not an odd function:

$$g(x) = 2x - 3$$



Differences:

Odd function:

$$f(2) = 3 \; and \; f(-2) = -3$$

Not an odd function:

$$g(2) = 3 \; and \; g(-2) = 5$$

Differences:

6. What do you observe about the characteristics of an odd function?

7. The algebraic definition of an odd function is:

   $f(x)$ is an odd function if and only if $f(-x) = -f(x)$ for all values of x in the domain of f.

   Explain how each of the examples of odd functions above meet this definition.

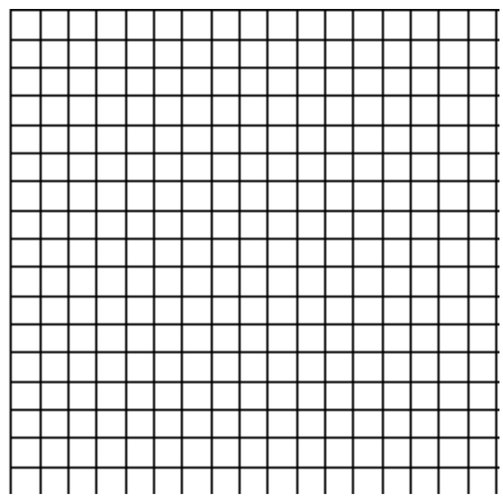
8. How can you tell if an odd-degree polynomial is an odd function?

9. Are all functions either odd or even?

4.6 Puzzling Over Polynomials

A Practice Understanding Task

For each of the polynomial puzzles below, a few pieces of information have been given. Your job is to use those pieces of information to complete the puzzle. Occasionally, you may find a missing piece that you can fill in yourself. For instance, although some of the roots are given, you may decide that there are others that you can fill in.

1.	**Function (in factored form)** **Function (in standard form)** **End behavior:** $as \quad x \to -\infty, \qquad f(x) \to$ ____ $as \quad x \to \infty, \qquad f(x) \to$ ____ **Roots (with multiplicity):** -2, 1, and 1 **Value of leading co-efficient:** -2 **Degree:** 3	Graph: 

2.

Function (in factored form)

Function (in standard form)

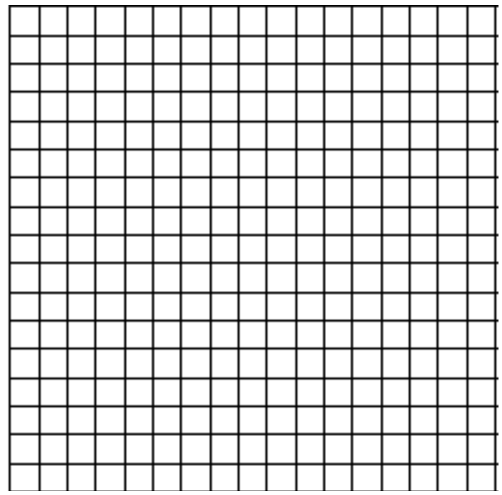
End behavior:
$as \quad x \to -\infty, \qquad f(x) \to$ ____
$as \quad x \to \infty, \qquad f(x) \to$ ____

Roots (with multiplicity):
$2+i, 4, 0$

Value of leading co-efficient:
1

Degree: 4

Graph:



3.

Function:
$f(x) = 2(x-1)(x+3)^2$

End behavior:
$as \quad x \to -\infty, \qquad f(x) \to$ ____
$as \quad x \to \infty, \qquad f(x) \to$ ____

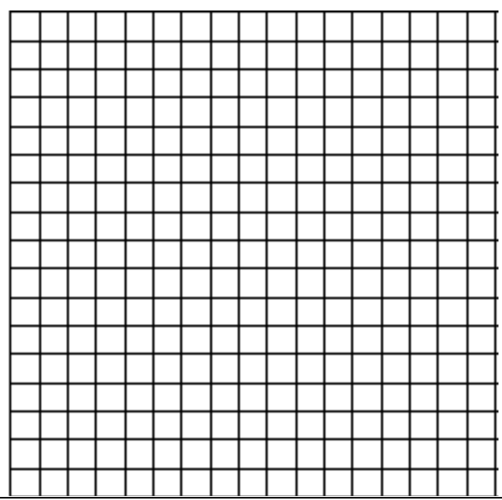
Roots (with multiplicity):

Value of leading co-efficient:

Domain:

Range: All Real numbers

Graph:



4.

Function:

End behavior:
$as\quad x \to -\infty, \qquad f(x) \to \infty$
$as\quad x \to \infty, \qquad f(x) \to$ ____

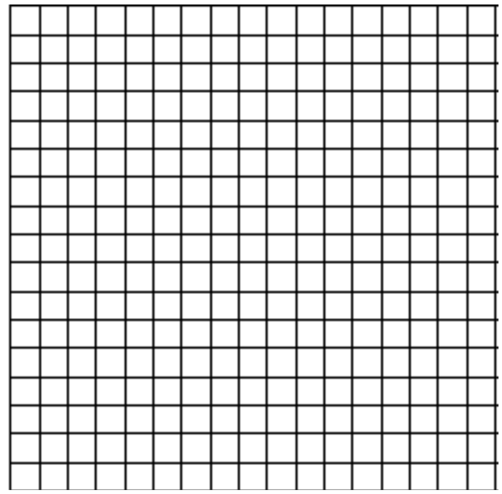
Roots (with multiplicity):
(3,0) m: 1;
(-1,0) m: 2
(0,0) m: 2

Value of leading co-efficient: -1

Domain:

Range:

Graph:



5.

Function:

End behavior:
$as\quad x \to -\infty, \qquad f(x) \to$ ____
$as\quad x \to \infty, \qquad f(x) \to$ ____

Roots (with multiplicity):

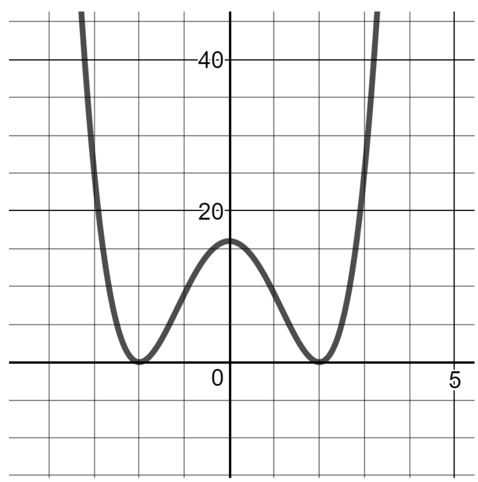
Value of leading co-efficient: 1

Domain:

Range:

Other: $f(0) = 16$

Graph:



6.

Function (in standard form):
$f(x) = x^3 - 2x^2 - 7x + 2$

Function (in factored form):

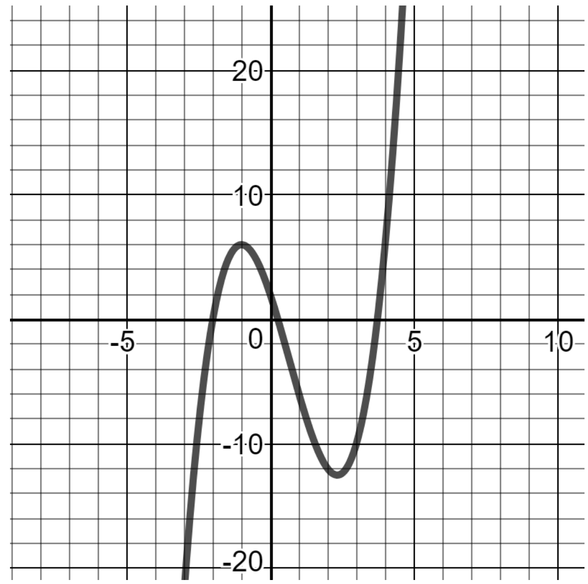
End behavior:
$as\ \ x \to -\infty, \quad f(x)\ \to$ ____
$as\ \ x \to\ \ \infty, \quad f(x)\ \to$ ____

Roots (with multiplicity):
-2

Domain:

Range:

Graph:



7.

Function (in standard form):
$f(x) = x^3 - 2x$

Function (in factored form):

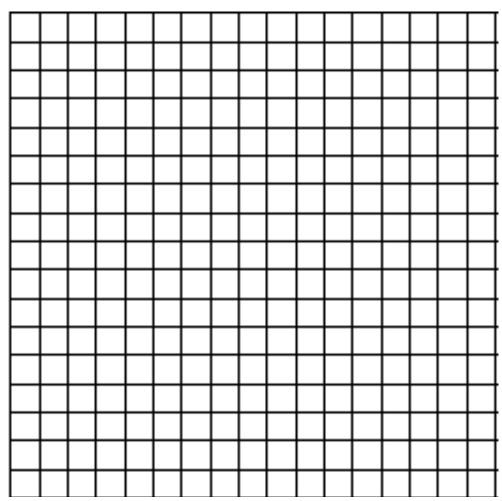
End behavior:
$as\ \ x \to -\infty, \quad f(x)\ \to$ ____
$as\ \ x \to\ \ \infty, \quad f(x)\ \to$ ____

Roots (with multiplicity):

Domain:

Range:

Graph:



READY, SET, GO! Name Period Date

READY

Topic: Reducing rational numbers and expressions

Reduce the expressions to lowest terms. (Assume no denominator equals 0.)

1. $\frac{3x}{6x^2}$

2. $\frac{2\cdot 5\cdot x\cdot x\cdot x\cdot y}{3\cdot 5\cdot x\cdot y\cdot y}$

3. $\frac{7ab^2}{7ab^2}$

4. $\frac{(x+2)(x-9)}{(x+2)(x-9)}$

5. $\frac{(3x-5)(x+4)}{(x-1)(3x-5)}$

6. $\frac{(2x-11)(3x+17)}{(2x-11)(3x-5)}$

7. $\frac{(8x-7)(x+3)}{8x(x+3)(2x-3)}$

8. $\frac{3x(2x+7)(x-1)(6x-5)}{x(2x+7)(x-1)(6x-5)}$

9. Why is it important that the instructions say to assume that no denominator equals 0?

SET

Topic: Reviewing features of polynomials

Some information has been given for each polynomial. Fill in the missing information.

10.

Graph:

Function: $f(x) = x^3$

Function in factored form:

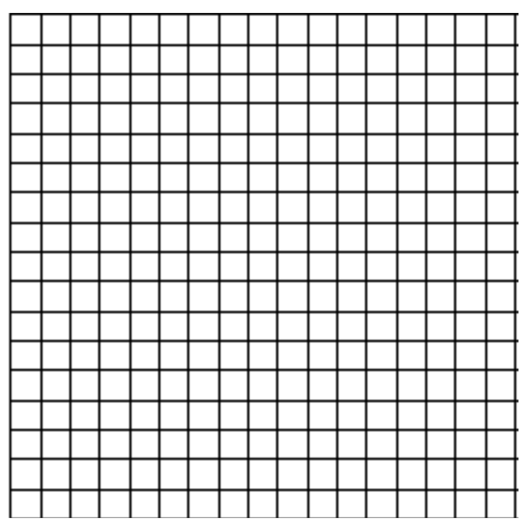
End behavior:

As $x \to -\infty, f(x) \to$ ______ As $x \to \infty, f(x) \to$ ______

Roots (with multiplicity):

Degree:

Value of leading co-efficient:



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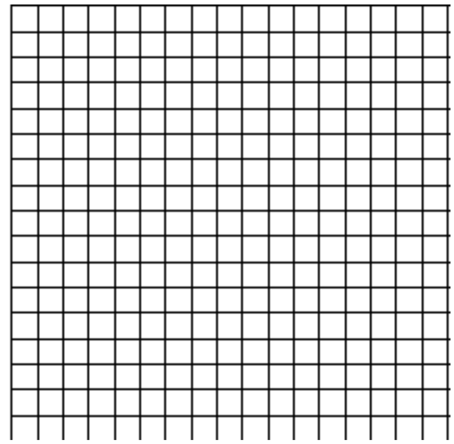
11.
Graph:



Function in standard form:

Function in factored form: $g(x) = -x(x-2)(x-4)$

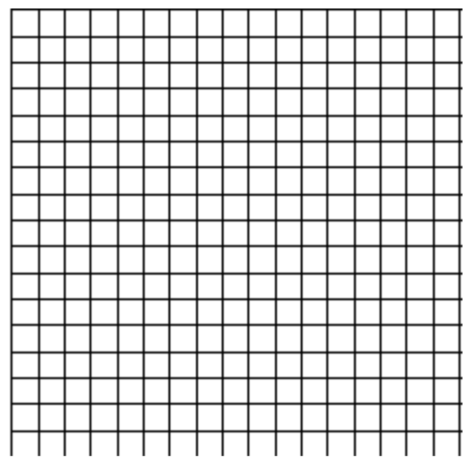
End behavior:
As $x \to -\infty, g(x) \to$ ______ As $x \to \infty, g(x) \to$ _____

Roots (with multiplicity):

Degree:

Value of leading co-efficient:

12.
Graph:



Function in standard form: $h(x) = x^3 - 2x^2 - 3x$

Function in factored form:

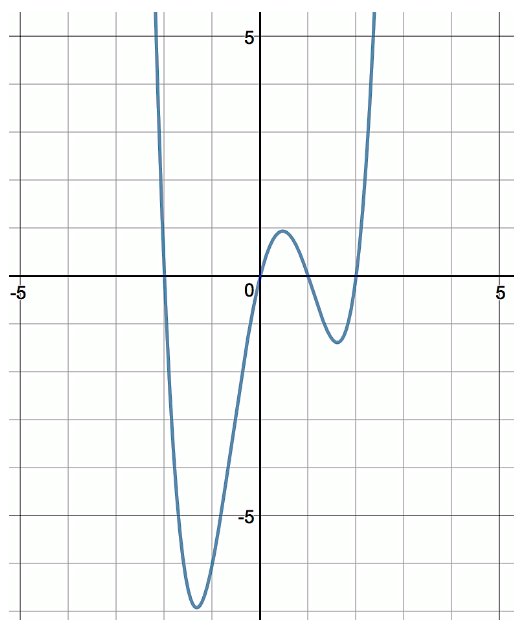
End behavior:
As $x \to -\infty, h(x) \to$ ______ As $x \to \infty, h(x) \to$ _____

Roots (with multiplicity):

Degree:

Value of $h(2)$:

13.
Graph:



Function in standard form:

Function in factored form:

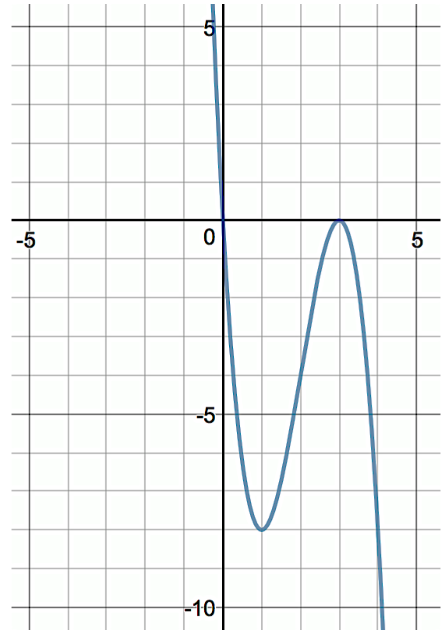
End behavior:
As $x \to -\infty, f(x) \to$ ______ As $x \to \infty, f(x) \to$ _____

Roots (with multiplicity):

Degree:

y-intercept:

14. **Graph:**



Function in standard form:

Function in factored form:

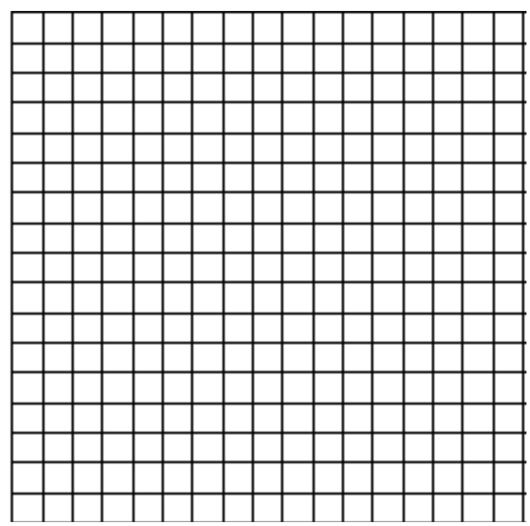
End behavior:
As $x \to -\infty, p(x) \to$ ______ As $x \to \infty, p(x) \to$ _____

Roots (with multiplicity):

Degree:

Value of leading coefficient:

15. **Graph:**



Function in standard form: $q(x) = x^3 + 2x^2 + x + 2$

Function in factored form:

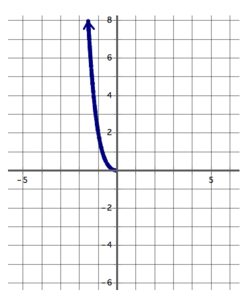
End behavior:
As $x \to -\infty, q(x) \to$ ______ As $x \to \infty, q(x) \to$ _____

Roots (with multiplicity):
$x = i$

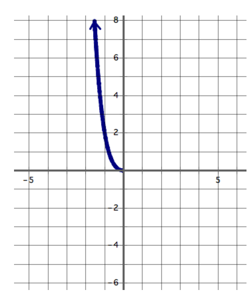
Degree:

y-intercept:

16. Finish the graph if it is an even function.



17. Finish the graph if it is an odd function.



GO

Topic: Writing polynomials given the zeros and the leading coefficient

Write the polynomial function in standard form given the leading coefficient and the zeros of the function.

18. Leading coefficient: 2; $roots$: $2, \sqrt{2}, -\sqrt{2}$

19. Leading coefficient: -1; $roots$: $1, 1+\sqrt{3},\ 1-\sqrt{3}$

20. Leading coefficient: 2; $roots$: $4i, -4i$

Fill in the blanks to make a true statement.

21. If $f(b) = 0$, then a factor of $f(b)$ must be ______________________________.

22. The rate of change in a linear function is always a ________________________________.

23. The rate of change of a quadratic function is _______________________________________.

24. The rate of change of a cubic function is ____.

25. The rate of change of a polynomial function of degree n can be described by a function of degree

____________________.