

5-4

Point-Slope Form

You have learned how to write an equation of a line by using its y -intercept. In this lesson, you will learn how to write an equation *without* using the y -intercept.

You can use the slope of a line and any point on the line to write and graph an equation of the line.

KEY CONCEPT: POINT-SLOPE FORM OF A LINEAR EQUATION

The **point-slope form** of an equation of a nonvertical line with slope m and through point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

Where did that come from?

$$(x_2 - x_1)m = \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x_2 - x_1)$$

$$m(x_2 - x_1) = y_2 - y_1$$

$$m(x - x_1) = y - y_1$$

PROBLEM 1: WRITING AN EQUATION IN POINT-SLOPE FORM

Write an equation in point-slope form of the line that passes through the given point and with the given slope. Then write the equation in slope-intercept form.

x_1, y_1
a) $(-3, 6); m = -5$

$$y - y_1 = m(x - x_1)$$

$$y - 6 = -5(x - (-3))$$

$$y - 6 = -5(x + 3)$$

$$y - 6 = -5x - 15$$

$$y = -5x - 9$$

x_1, y_1
b) $(4, 2); m = -\frac{5}{3}$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{5}{3}(x - 4)$$

$$y - 2 = -\frac{5}{3}x + \frac{20}{3}$$

$$y = -\frac{5}{3}x + \frac{26}{3}$$

x_1, y_1
c) $(4, 0); m = -1$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -1(x - 4)$$

$$y = -x + 4$$

d) $(3, -4); m = 6$

$$y - y_1 = m(x - x_1)$$

$$y - (-4) = 6(x - 3)$$

$$y + 4 = 6(x - 3)$$

$$y + 4 = 6x - 18$$

$$y = 6x - 22$$

e) $(8, -4); m = \frac{2}{3}$

$$y - y_1 = m(x - x_1)$$

$$y + 4 = \frac{2}{3}(x - 8)$$

$$y + 4 = \frac{2}{3}x - \frac{16}{3}$$

$$y = \frac{2}{3}x - \frac{28}{3} \leq 9\frac{1}{3}$$

f) $(-2, -7); m = \frac{4}{5}$

$$y + 7 = \frac{4}{5}(x + 2)$$

$$y + 7 = \frac{4}{5}x + \frac{8}{5}$$

$$y = \frac{4}{5}x - \frac{27}{5}$$

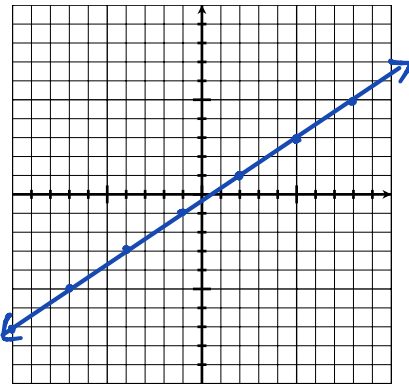
$$-5\frac{2}{5}$$

PROBLEM 2: GRAPHING USING POINT-SLOPE FORM

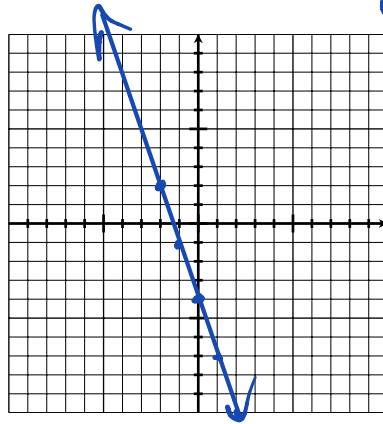
Graph each equation.

$(x_1, y_1); m =$

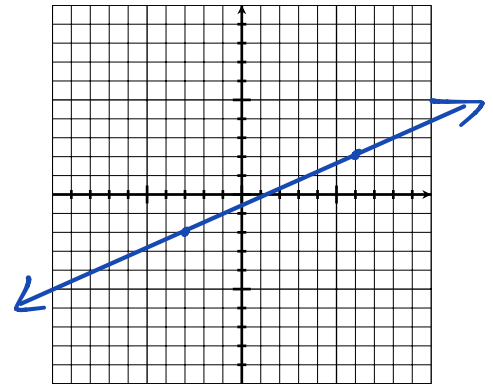
a) $y - 1 = \frac{2}{3}(x - 2)$ $m = \frac{2}{3} = \frac{+2 \uparrow}{+3 \rightarrow} = \frac{-2 \downarrow}{-3 \leftarrow}$ $(2, 1)$



b) $y - 2 = -3(x + 2)$ $m = -3 = \frac{-3 \downarrow}{+1 \rightarrow}$ $(-2, 2)$

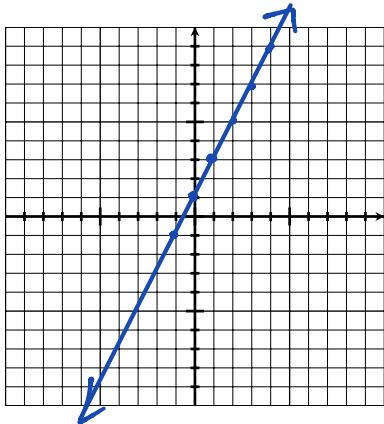


c) $y + 2 = \frac{4}{9}(x + 3)$ $m = \frac{4}{9} = \frac{+4 \uparrow}{+9 \rightarrow}$ $(-3, -2)$



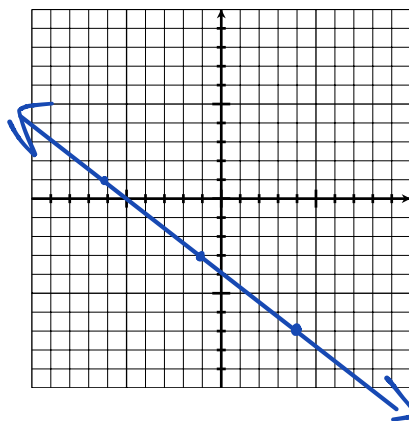
d) $y - 3 = 2(x - 1)$ $m = \frac{2}{1} = \frac{+2 \uparrow}{+1 \rightarrow} = \frac{-2 \downarrow}{-1 \leftarrow}$

$(1, 3)$



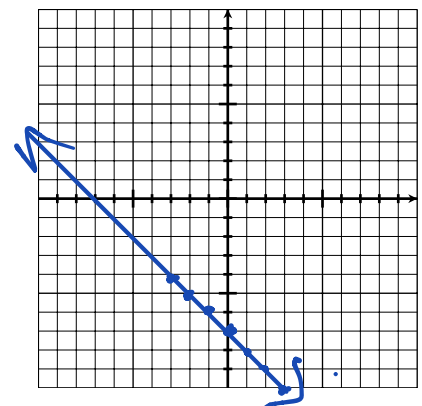
e) $y + 7 = -\frac{4}{5}(x - 4)$ $m = -\frac{4}{5} = \frac{-4 \downarrow}{+5 \rightarrow} = \frac{+4 \uparrow}{-5 \leftarrow}$

$(4, -7)$



f) $y + 5 = -(x + 2)$ $m = -1 = \frac{-1 \downarrow}{+1 \rightarrow}$

$(-2, -5)$



You can write the equation of a line given any two points on the line. First, use the two given points to find the slope. Then use the slope and one of the points to write the equation.

PROBLEM 3: USING TWO POINTS TO WRITE AN EQUATION

Write an equation for the line that passes through the given points.

a) x_1, y_1 and x_2, y_2
 a) $(-2, -3)$ and $(1, 4)$

$$m = \frac{4 - (-3)}{1 - (-2)} = \frac{7}{3}$$

$$y + 3 = \frac{7}{3}(x + 2) \quad \text{or} \quad y - 4 = \frac{7}{3}(x - 1)$$

$$y + 3 = \frac{7}{3}x + \frac{14}{3}$$

$$-3 \quad -3 = \frac{7}{3}$$

$$y = \frac{7}{3}x + \frac{5}{3}$$

d) $(1, 4)$ and $(-1, 1)$

$$m = \frac{1 - 4}{-1 - 1} = \frac{-3}{-2} = \frac{3}{2}$$

$$y - 4 = \frac{3}{2}(x - 1)$$

$$y - 1 = \frac{3}{2}(x + 1)$$

b) x_1, y_1 and x_2, y_2
 b) $(2, 4)$ and $(-3, -6)$

$$m = \frac{-6 - 4}{-3 - 2} = \frac{-10}{-5} = 2$$

$$y - 4 = 2(x - 2)$$

$$y + 6 = 2(x + 3)$$

e) $(5, 0)$ and $(8, 2)$

$$m = \frac{2 - 0}{8 - 5} = \frac{2}{3}$$

$$y - 0 = \frac{2}{3}(x - 5)$$

$$y = \frac{2}{3}(x - 5)$$

$$y - 2 = \frac{2}{3}(x - 8)$$

c) x_1, y_1 and x_2, y_2
 c) $(-6, 6)$ and $(3, 3)$

$$m = \frac{3 - 6}{3 - (-6)} = \frac{-3}{9} = -1$$

$$y - 6 = -(x + 6)$$

$$y - 3 = -(x - 3)$$

f) $(-8, -4)$ and $(0, 6)$

$$m = \frac{6 - (-4)}{0 - (-8)} = \frac{10}{8} = \frac{5}{4}$$

$$y + 4 = \frac{5}{4}(x + 8)$$

$$y - 6 = \frac{5}{4}(x - 0)$$

$$y - 6 = \frac{5}{4}x \Rightarrow y = \frac{5}{4}x + 6$$

PROBLEM 4: USING A TABLE TO WRITE AN EQUATION

a) The table shows the altitude of a hot-air balloon during its linear descent. What equation in slope-intercept form gives the balloon's altitude at any time? What do the slope and y-intercept represent?

$$y - 640 = -\frac{5}{2}(x - 10)$$

$$y - 640 = -\frac{5}{2}x + 25$$

$$+640 \quad +640$$

$$y = -\frac{5}{2}x + 665$$

Hot-Air Balloon Descent

Time, x (s)	Altitude, y (m)	$\frac{\Delta y}{\Delta x}$
10	640	
30	590	$\frac{-50}{20} = -\frac{5}{2}$
70	490	$\frac{-100}{40} = -\frac{5}{2}$
90	440	$\frac{-50}{20} = -\frac{5}{2}$

$$m = -\frac{5}{2}$$

The balloon starts its descent at 665 m and the altitude decreases 2.5 m/s.

b) The table shows the number of gallons of water y in a tank after x hours. The relationship is linear. What is an equation in point-slope form that models the data? What does the slope represent?

$$y - 3320 = 1250(x - 2)$$

The water in the tank is increasing 1250 gal/h.

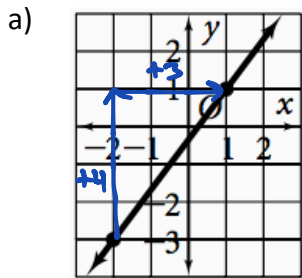
Volume of Water in Tank

Time, x (h)	Water, y (gal)	$\frac{\Delta y}{\Delta x}$
2	3320	
+1	4570	$\frac{+1250 \text{ gal}}{+1 \text{ h}} = 1250$
+2	7070	$\frac{+2500}{+2} = 1250$
+3	10,820	$\frac{+3750}{+3} = 1250$

$$m = 1250 \text{ gal/h}$$

EXTRA STUFF:

a) Write an equation for the lines in point-slope form and slope-intercept form.

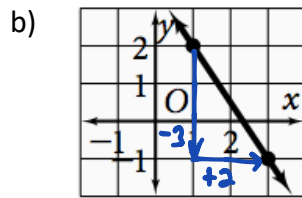


$$m = \frac{+4}{+3} = \frac{4}{3}; (1, 1)$$

$$y - 1 = \frac{4}{3}(x - 1)$$

$$y - 1 = \frac{4}{3}x - \frac{4}{3}$$

$$y = \frac{4}{3}x - \frac{1}{3}$$

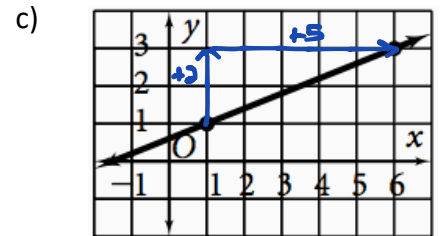


$$m = \frac{-3}{+1} = -3; (1, 2)$$

$$y - 2 = -3(x - 1)$$

$$y - 2 = -3x + 3$$

$$y = -3x + 5$$



$$m = \frac{+2}{+5} = \frac{2}{5}; (6, 3)$$

$$y - 3 = \frac{2}{5}(x - 6)$$

$$y - 3 = \frac{2}{5}x - \frac{12}{5}$$

$$y = \frac{2}{5}x + \frac{3}{5}$$

$$y - 1 = \frac{2}{5}(x - 1)$$

$$y - 1 = \frac{2}{5}x - \frac{2}{5}$$

$$y = \frac{2}{5}x + \frac{3}{5}$$

b) The relationship between altitude and the boiling point of water is linear. At an altitude of 8000 ft, water boils at 197.6°F. At an altitude of 4500 ft, water boils at 203.9°F. Write an equation giving the boiling point b of water, in degrees Fahrenheit, in terms of the altitude a , in feet. What is the boiling point of water at 2500 feet?

$$(8000, 197.6), (4500, 203.9)$$

$$m = \frac{203.9 - 197.6}{4500 - 8000} = \frac{6.3}{-3500} = -.0018$$

$$y - 197.6 = -.0018(x - 8000)$$

$$y - 197.6 = -.0018x + 14.4$$

$$y = -.0018x + 212$$

$$y = -.0018(2500) + 212$$

$$y = 207.5$$

$$207.5^\circ\text{F}$$

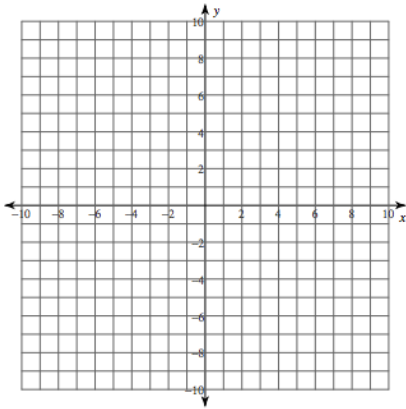
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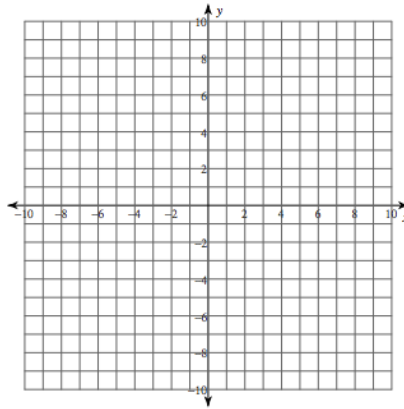
6-5 Practice Worksheet

Graph each equation.

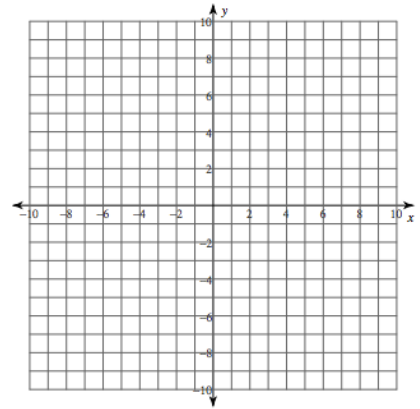
1. $y - 2 = \frac{1}{2}(x - 3)$



2. $y - 2 = 2(x + 3)$



3. $y + 4 = (x - 5)$



Write an equation in point-slope form for the line through the given point that has the given slope.

4. $(3, -4); m = 6$

5. $(1, -8); m = -\frac{1}{5}$

6. $(-5, 2); m = 0$

Write an equation in (a) point-slope form, and (b) slope-intercept form for the line that passes through the given points.

7. $(-6, 6), (3, 3)$

8. $(5, 3), (4, 5)$

9. $(-8, 4), (-4, -2)$

Is the relationship shown by the data linear? If so, model the data with an equation.

10.

x	y
3	1
6	4
9	13
15	49

11.

x	y
-10	-5
-2	19
5	40
11	58

BELL RINGER

- ① Write an equation in point slope form AND slope-intercept form for the line with a slope of -2 that goes through the point $(-3, 4)$.

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -2(x + 3)$$

$$y - 4 = -2x - 6$$

$$y = -2x - 2$$

- ③ Write an equation for the line that passes through the points $(-2, 5)$ and $(3, -1)$
- x_1 y_1 x_2 y_2

$$m = \frac{-1 - 5}{3 - (-2)} = \frac{-6}{5} = -\frac{6}{5}$$

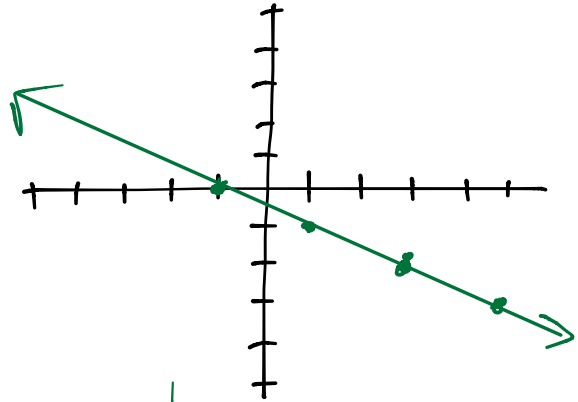
$$y - 5 = -\frac{6}{5}(x + 2)$$

or

$$y + 1 = -\frac{6}{5}(x - 3)$$

$$(5, -3); m = -\frac{1}{2} = \frac{-1}{+2} = \frac{+1}{-2}$$

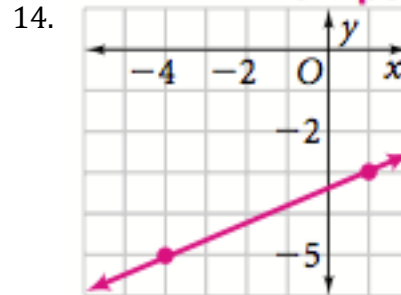
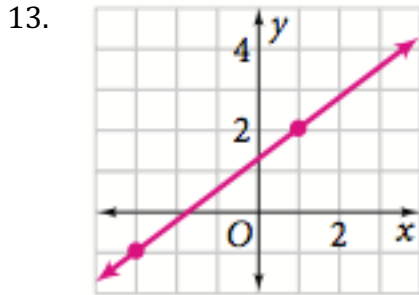
- ② Graph $y + 3 = -\frac{1}{2}(x - 5)$



12. (a) Use the data in the table to write an equation to relate the fine with the speed over the posted speed limit. (b) Rewrite the equation in slope-intercept form. (c) Write a sentence to explain what the slope and y-intercept mean in this situation.

Speed Over Posted Speed Limit (mph)	Fine (\$)
10	75
12	95
15	125
19	165

Write an equation of each line in points slope form AND slope-intercept form.



15. The relationship of degrees Fahrenheit ($^{\circ}\text{F}$) and degrees Celsius ($^{\circ}\text{C}$) is linear. When the temperature is 50°F , it is 10°C . When the temperature is 77°F , it is 25°C .

- (a) Write an equation giving the Celsius temperature C in terms of the Fahrenheit temperature F .
 (b) What is the Celsius temperature when it is 59°F ?

16. Worldwide carbon monoxide emissions are decreasing about 2.6 million metric tons each year. In 1991, carbon monoxide emissions were 79 million metric tons. Use a linear equation to model the relationship between carbon monoxide emissions and time. Let $x = 91$ correspond to 1991.

Write an equation in slope-intercept form of each line described below.

17. The line contains the point $(-3, -5)$ and has the same slope as $y + 2 = 7(x + 3)$

18. The line contains the point $(1, 3)$ and has the same y-intercept as $y - 5 = 2(x - 1)$

19. The line contains the point $(2, -2)$ and has the same x-intercept as $y + 9 = 3(x - 4)$