Physics 136-1: General Physics Lab
Laboratory Manual - Mechanics

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Version 1.1b

June 21, 2019

Contents

Foreward

Welcome to the general physics laboratory! This laboratory experience is designed to guide your learning of fundamental concepts of experimentation and data collection, delivered through the medium of hands-on experiments on why and how objects with mass move and interact with other objects. As a student, you should be aware that you and your colleagues will have a broad set of backgrounds in math, science, and writing and a similarly broad set of career trajectories. Even with the diversity of participants in a course such as this, everyone can share an appreciation of the scientific process. It is our job as instructors (TAs, faculty, and other assistants) to help facilitate this learning independent of your preparation level. Some will find this easier than others, but we will have done our job if you, regardless of background, walk away appreciating a little more deeply what it means for a scientist to claim that "I know something" based on experiments.

Some passages of text have been emphasized and color coded to make finding them later more convenient.

Checkpoint

Checkpoints are intended to cause the student to be sure he is understanding and remembering the material before continuing to waste his time.

Helpful Tip

Helpful tips offer the student an opportunity to learn a shortcut or otherwise to make better use of his effort.

Historical Aside

A Historical Aside informs the student of some of the history associated with the discussion topic. History itself is not helpful in performing the experiment or in understanding the physics; however, it sometimes helps the student understand why we do things as we do.

WARNING

Warnings are exactly what they seem. Defying warnings can result in some personal injury (likely not serious), in some disruption of the apparatus (time-consuming to repair), etc.

General Information

General Information is usually very helpful with respect to understanding the discussion topic in the broader context of the physical world.

We hope these decorations improve the student's experience and help him/her to learn to experiment more effectively.

Chapter 1

Introduction to the Laboratory

Physics is an experimental science. As part of a basic education in Physics, students learn both physical principles and problem solving (130/135 lecture) and concepts of experimental practice and analysis (136). Physics 136-1 is designed to provide an introduction to experimental techniques in the laboratory, focused on experiments on *forces, masses, and motion*. We will build on algebra, geometry, trigonometry, and world experience to introduce laws of motion and conservation that seem to permeate the physical world. These laws are expressed using algebra and trigonometry and we will utilize these equations as examples of how scientists gather and process data to test an idea's validity. The process of using a set of tools to yield data and then of analyzing the data to reach conclusions is the same for life-sized mechanics as for more abstract subject areas that will follow in later courses.

The primary purpose of the Physics Laboratory is NOT to duplicate the concepts of lecture, although reinforcement is certainly beneficial and intended. This lab is an independent course from lecture covering independent concepts. The topics of the lecture serve as examples that we will explore in the lab to learn how to trust and to believe in physical principles. The schedule of topics in each lecture may not correspond directly with the material in the lab, which will be focused on *observing and measuring* physical phenomena. These two components complement each other, but they seldom track each other. Taken together, the 130/135 and 136 physics courses should provide the knowledge, problem solving skills, intuition, and practical experience with apparatus and data collection expected of a first year in college-level physics.

1.1 Objectives of Introductory Physics Laboratories

In this course, students should expect to advance several learning goals that are broadly relevant in science, technology, and general understanding of human knowledge. These objectives are outlined by the American Association of Physics Teachers at
www.physics.usu.edu/dennison/3870-3880/References/AAPT%20Lab%20Goals.pdf :

- Develop **experimental and analytical skills** for both theoretical problems and data.
- Appreciate the **"Art of Experimentation"** and what is involved in designing and analyzing a data-driven investigation, including inductive and deductive reasoning.

- Reinforce the **concepts of physics** through conceptual and experiential learning.
- Understand the **role of direct observation** as the basis for knowledge in physics.
- Appreciate **scientific inquiry** into exploring creatively how the world works.
- Facilitate **communication skills** through informative, succinct written reports.
- Develop **collaborative learning skills** through cooperative work.

1.2 Calculus vs Non-Calculus Based Physics

The same set of experiments are given to students in both calculus-based and algebra-based physics courses. The work in this laboratory is designed to be independent of calculus, but it is natural that students with more math background can better appreciate the subtleties of the physics probed in these experiments. Calculus is never required in this course, and your grading will not be affected by your knowledge of calculus (or lack thereof).

For completeness the physical laws and principles will be presented in their most general form and that typically does require calculus; however, the student will receive the same grade if he simply ignores these derivations and goes directly to the solutions while keeping necessary assumptions and approximations in mind. These solutions frequently contain algebra and trigonometry but they can always be understood intuitively without resorting to calculus.

1.3 What to Bring to the Laboratory

You should bring the following items to each lab session, including the first session of the course. There is no additional textbook.

1) A bound quadrille ruled lab notebook. You must have your own, and you cannot share with your lab partner. A suitable version is sold by the Society of Physics Students in Dearborn B6. This lab notebook can be reused for future physics labs or salvaged by SPS. A scientist's notebook is his most reliable long-term memory, his evidence that he performed the work and when, and his instructions for how to reproduce the work.
2) This Physics Laboratory 1st Quarter lab manual. A printed copy of each relevant experiment must be brought to class each week; but the student may choose to print it himself or to purchase the printed copy from the Norris bookstore. The cost of ink and paper is commensurate with the manual's purchase price.
3) A scientific calculator.
4) An ink pen.
5) You will need to transfer electronic data files and figures from lab to your lab reports. The lab's computers are designed to use 'box' for this purpose; however this

can be done by email, another cloud storage account, or a USB drive.

6) Periodically, hard documents need digitized. Each lab has a document scanner that also can utilize the students' box accounts, but students with phone cameras might prefer to use those. TAs require signed data to accompany each report.

1.4 Lab Reports

You will write lab reports and submit them electronically. The purpose of this exercise is both to demonstrate your work in lab and to guide you to think a bit more deeply about what you are doing. The act of technical writing also helps improve your communication skills, which are broadly relevant far beyond the physics lab.

Appendix E of this lab manual provides some guidance on how best to prepare these reports. You should keep in mind that these are not publishable manuscripts, but concise and clear descriptions of your experiments. They will follow a clear format to communicate your work best. They are not meant to be long. In the past, similar reports were written in class in about 30 minutes. . . these at-home reports are a bit more involved than that, but not by much, and the electronic format allows the students to begin utilizing word processors for technical writing. Students should walk out of the lab with Data and Analysis sections mostly complete and several ideas and details to incorporate into their Purpose, Procedure, and Conclusions. One additional hour should flesh out these skeletons into report submissions.

In addition to background material, details of apparatus function, and instructions for gathering data, each chapter of this lab manual suggests ideas you should consider while assembling your reports. Be certain to read each chapter carefully up to the Procedures before class. You will be tasked with taking an online quiz about this material before class and prior knowledge will help you perform efficiently and correctly while in the laboratory. It is also a good idea to scan the Procedures and to examine the Analysis and Conclusions for the kinds of physics good data will demonstrate.

Chapter 2

Understanding Errors and Uncertainties in the Physics Laboratory

2.1 Introduction

We begin with a review of general properties of measurements and how measurements affect what we, as scientists, choose to believe and to teach our students. Later we narrow our scope and dwell on particular strategies for estimating what we know, how well we know it, and what else we might learn from it. We learn to use statistics to distinguish which ideas are consistent with our observations and our data.

2.1.1 Measurements, Observations, and Progress in Physics

Physics, like all natural sciences, is a discipline driven by observation. The concepts and methodologies that you learn about in your lectures are not taught because they were first envisioned by famous people, but because they have been observed always to describe the world. For these claims to withstand the test of time (and repeated testing in future scientific work), we must have some idea of how well theory agrees with experiment, or how well measurements agree with each other. Models and theories can be invalidated by conflicting data; making the decision of whether or not to do so requires understanding how strongly data and theory agree or disagree. Measurement, observation, and data analysis are key components of physics, equal with theory and conceptualization.

Despite this intimate relationship, the skills and tools for quantifying the quality of observations are distinct from those used in studying the theoretical concepts. This brief introduction to errors and uncertainty represents a summary of key introductory ideas for understanding the quality of measurement. Of course, a deeper study of statistics would enable a more quantitative background, but the outline here represents what everyone who has studied physics at the introductory level should know.

Based on this overview of uncertainty, you will perhaps better appreciate how we have come to trust scientific measurement and analysis above other forms of knowledge acquisition, **precisely because we can quantify what we know and how well we know it**.

2.2 Some References

The study of errors and uncertainties is part of the academic field of statistics. The discussion here is only an introduction to the full subject. Some classic references on the subject of error analysis in physics are:

- Philip R. Bevington and D. Keith Robinson, *Data Reduction and Error Analysis for the Physical Sciences*, McGraw-Hill, 1992.
- John R. Taylor, *An Introduction to Error Analysis; The Study of Uncertainties in Physical Measurements*, University Science Books, 1982.
- Glen Cowan, *Statistical Data Analysis*, Oxford Science Publications, 1998

2.3 The Nature of Error and Uncertainty

Error is the difference between an observation and the true value.

$$\text{Error} = \text{observed value} - \text{true value}$$

The "observation" can be a direct measurement or it can be the result of a calculation that uses measurements; the "true" value might also be a calculated result. Even if we do not know the true value, its existence defines our "error"; but in this case we will also be unable to determine our error's numeric value. The goal of many experiments, in fact, is to estimate the true value of a physical constant using experimental methods. When we do this our error cannot be known, so we study our apparatus and estimate our error(s) using knowledge of our measurement uncertainties.

Example: Someone asks you, what is the temperature? You look at the thermometer and see that it is 71° F. But, perhaps, the thermometer is mis-calibrated and the actual temperature is 72° F. There is an *error* of -1° F, but you do not know this. What you *can* figure out is the reliability of measuring using your thermometer, giving you the *uncertainty* of your observation. Perhaps this is not too important for casual conversation about the temperature, but knowing this uncertainty would make all the difference in deciding if you need to install a more accurate thermometer for tracking the weather at an airport or for repeating a chemical reaction exactly during large-scale manufacturing.

Example: Suppose you are measuring the distance between two points using a meter stick but you notice that the 'zero' end of the meter stick is very worn. In this case you can greatly reduce your likely error by sliding the meter stick down so that the '10 cm' mark is aligned with the first point. This is a perfectly valid strategy; however, you must now subtract 10 cm from the remaining point(s) location(s). A similar strategy applies (in reverse) if your ruler's zero is not located at the end and you must measure into a corner; in this case you must *add* the extra length to your measurement(s).

Another common goal of experiments is to try to verify an equation. To do this we alter the apparatus so that the parameters in the equation are different for each "trial". As an example we might change the mass hanging from a string. If the equation is valid, then the apparatus responds to these variations in the same way that the equation predicts. We then use graphical and/or numerical analysis to check whether the responses from the apparatus (measurements) are consistent with the equation's predictions. To answer this question we must address the uncertainties in how well we can physically set each variable in our apparatus, how well our apparatus represents the equation, how well our apparatus is isolated from external (i.e. not in our equation) environmental influences, and how well we can measure our apparatus' responses. Once again we would prefer to utilize the errors in these parameters, influences, and measurements but the true values of these errors cannot be known; they can only be estimated by measuring them and we must accept the uncertainties in these measurements as preliminary estimates for the errors.

2.3.1 Sources of Error

No real physical measurement is exactly the same every time it is performed. The uncertainty tells us how closely a second measurement is expected to agree with the first. Errors can arise in several ways, and the uncertainty should help us quantify these errors. In a way the uncertainty provides a convenient 'yardstick' we may use to estimate the error.

- **Systematic error:** Reproducible deviation of an observation that biases the results, arising from procedures, instruments, or ignorance. Each systematic error biases every measurement in the same direction, but these directions and amounts vary with different systematic errors.
- **Random error:** Uncontrollable differences from one trial to another due to environment, equipment, or other issues that reduce the repeatability of an observation. They may not actually be random, but deterministic (if you had perfect information): dust, electrical surge, temperature fluctuations, etc. In an ideal experiment, random errors are minimized for precise results. Random errors are sometimes positive and sometimes negative; they are sometimes large but are more often small. In a sufficiently large sample of the measurement population, random errors will average out.

Random errors can be estimated from statistical repetition and systematic errors can be estimated from understanding the techniques and instrumentation used in an observation; many systematic errors are identified while investigating disagreement between different experiments.

Other contributors to uncertainty are *not* classified as 'experimental error' in the same scientific sense, but still represent difference between measured and 'true' values. The challenges of estimating these uncertainties are somewhat different.

- **Mistake, or 'illegitimate errors':** This is an error introduced when an experimenter does something wrong (measures at the wrong time, notes the wrong value).

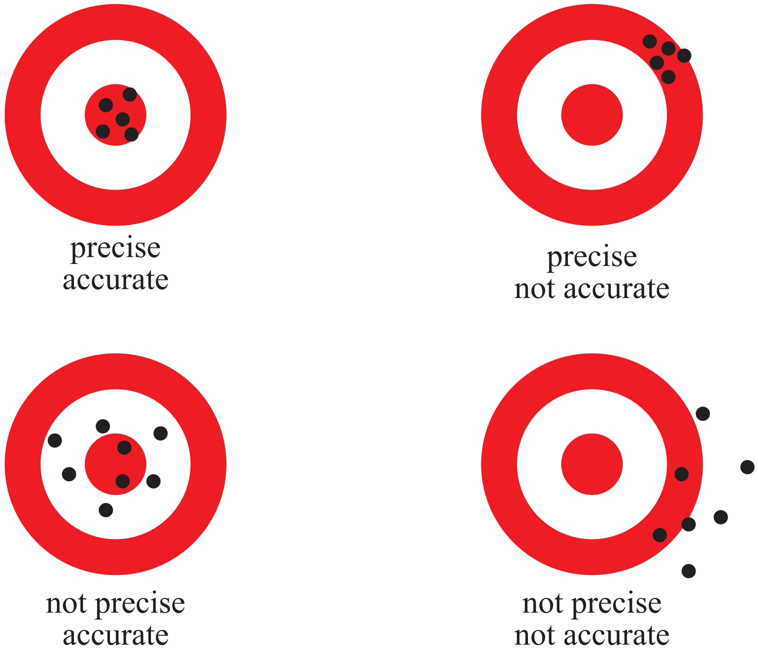


Figure 2.1: *Several independent trials of shooting at a bullseye target illustrate the difference between being accurate and being precise.*

These should be prevented, identified, and corrected, if possible, and ideally they should be completely eliminated. Lab notebooks can help track down mistakes or find procedures causing mistakes.

- **Fluctuations:** Sometimes, the variability in a measurement from its average is not a random error in the same sense as above, but a physical process. Fluctuations can contain information about underlying processes such as thermal dynamics. In quantum mechanics these fluctuations can be real and fundamental. They can be treated using similar statistical methods as random error, but there is not always the desire or the capacity to minimize them. When a quantity fluctuates due to underlying physical processes, perhaps it is best to redefine the quantity that you want to measure. (For example, suppose you tried to measure the energy of a single molecule in air. Due to collisions this number fluctuates all over the place, even if you could identify a means to measure it. So, we invent a new concept, the *temperature*, which is related to the average energy of molecules in a gas. Temperature is something that we can measure, and assign meaningful uncertainties to. Because of physical fluctuations caused by molecular collisions, temperature is a more effective measurement than one molecule's energy. Temperature reflects the aggregate average of all of the molecules and, as such, fluctuates far less.

2.3.2 Accuracy vs. Precision

Errors and uncertainties have two independent aspects:

- **Accuracy:** Accuracy is how closely a measurement comes to the 'true' value. It describes how well we eliminate systematic error and mistakes.
- **Precision:** Precision is how exactly a result is determined without referring to the 'true' value. It describes how well we suppress random errors and thus how well a sequence of measurements of the same physical quantity agree with each other.

It is possible to acquire two precise, but inaccurate, measurements using different instruments that do not agree with each other at all. Or, you can have two accurate, but imprecise, measurements that are very different numerically from each other, but statistically cannot be distinguished.

2.4 Notation of Uncertainties

There are several ways to write various numbers and uncertainties, but we will describe our data using **Absolute Uncertainty:** The magnitude of the uncertainty of a number in the same units as the result. We use the symbol δx for the uncertainty in x, and express the result as $x \pm \delta x$.
Example: For an uncertainty $\delta x = 6\,\text{cm}$ in a length measurement L of $x = 2\,\text{meters}$, we would write $L = (2.00 \pm 0.06)\,\text{m}$. Note that x and δx have the same number of digits after the decimal point. In fact, δx tells us how many digits in x are truly measurable and allows us to discard the noise; because of this x and δx always have the same number of decimal places.

2.5 Estimating Uncertainties

The process of estimating uncertainties requires practice and feedback. Uncertainties are always due to the measuring tool and to our proficiency with using it.

2.5.1 Level of Uncertainty

How do you actually estimate an uncertainty? First, you must settle on what the quantity δx actually means. If a value is given as $x \pm \delta x$, what does the range $\pm\delta x$ mean? This is called the *level of confidence* of the results.

Assuming no systematic biases, $x - \delta x <$ true value $< x + \delta x$ 68% of the time. There are valid reasons to specify tolerances much greater than the statistical uncertainty. For example,

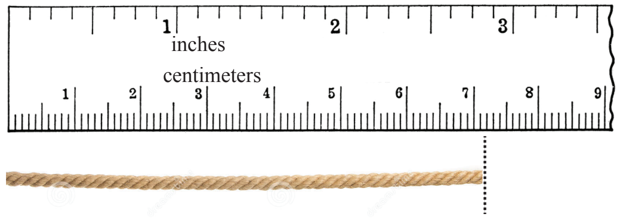


Figure 2.2: *Measuring a string with a ruler. A reasonable measurements from this might be reported as* 7.15 ± 0.05 *cm.*

manufacturers cannot afford to have 32% of their products returned. But scientists generally use 68% confidence levels.

Helpful Tip

Frequently, students list "human error" among the reasons why predictions disagree with measurements. Actually, it is the tools we use that have limitations. The "human error" in reading e.g. a meter stick *should* be included in the measurement tolerance and compounded with other measurement uncertainties. In this case, the sigma for the comparison contains these "human errors" and *cannot* be the reason the difference is greater than the sigma. Humans can design micrometers and interferometers to measure better than meter sticks. Our tools are limited, but humans are more versatile.

2.5.2 Reading Instrumentation

Measurement accuracy is limited by the tools used to measure. In a car, for example, the speed divisions on a speedometer may be only every 5 mph, or the digital readout of the odometer may only read down to tenths of a mile. To estimate instrumentation accuracy, assume that the uncertainty is one half of the smallest division that can be unambiguously read from the device. **Instrumentation accuracy must be recorded during laboratory measurements.** In many cases, instrument manufacturers publish specification sheets that detail their instrument's errors more thoroughly. In the absence of malfunction, these specifications are reliable; however, 'one half of the smallest division' might not be very reliable if the instrument has not been calibrated recently.

2.5.3 Experimental precision

Even on perfect instruments, if you measure the same quantity several times, you will obtain several different results. For example, if you measure the length of your bed with a ruler several times, you typically find a slightly different number each time. The bed and/or the ruler could have expanded or contracted due to a change in temperature or a slightly different amount of tension. Your eye might not be properly aligned with the ruler and the bed so that parallax varies the measurements. These unavoidable uncertainties are always present to some degree in an observation. In fact, if you get the same answer every time, you probably need to estimate another decimal place (or even two). Even if you understand their origin, the randomness cannot always be controlled. We can use statistical methods to quantify and to understand these random uncertainties. Our goal in measuring is not to get the same number every time, but rather to acquire the most accurate and precise measurements that we can.

2.6 Quantifying Uncertainties

Here we note some mathematical considerations of dealing with random data. These results follow from the central limit theorem in statistics and analysis of the normal distribution. This analysis is beyond the scope of this course; however, the distillation of these studies are the point of this discussion.

2.6.1 Mean, Standard Deviation, and Standard Error

Statistics are most applicable to very large numbers of samples; however, even 5-10 samples benefit from statistical treatment. Statistics greatly reduce the effect of truly random fluctuations and this is why we utilize this science to assist us in understanding our observations. But it is still imperative that we closely monitor our apparatus, the environment, and our precision for signs of systematic biases that shift the mean.

With only a few samples it is not uncommon $\left(\frac{1}{2^N}\right)$ for all samples to be above (or below) the distribution's mean. In these cases the averages that we compute are biased and not the best representation of the distribution. These samples are also closely grouped so that the standard error is misleadingly small. This is why it is important to develop the art of estimating measurement uncertainties in raw data as a sanity check for such eventualities.

The mean

Suppose we collect a set of measurements of the *same* quantity x, and we label them by an integer index i: $\{x_i\} = (x_1, x_2, \ldots x_N)$. What value do we report from this set of identical measurements? We want the **mean**, μ, of the population from which such a data set was randomly drawn. We can approximate μ with the *sample mean* or average of this particular set of N data points:

$$\mu \approx \boxed{\bar{x} = \frac{1}{N}\sum_i x_i} \tag{2.1}$$

Of course, this is not the true mean of the population, because we only measured a small subset of the total population. But it is our best guess and, statistically, it is an *unbiased predictor* of the true mean μ.

The standard deviation

How precisely do we know the value of x? To answer this question of statistical uncertainty based on the data set $\{x_i\}$, we consider the squared deviations from the sample mean $\bar{x}$. The **sample variance** s_x^2 is the sum of the squared deviations divided by the 'degrees of freedom' (DOF). For N measurements the DOF for variance is $N-1$. (The origin of the $N-1$ is a subtle point in statistics. Ask if you are interested.) The **sample standard deviation**, s_x, is the square root of the sample variance of the measurements of x.

$$s_x = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{\mathrm{DOF}}} \tag{2.2}$$

The sample standard deviation is our best 'unbiased estimate' of the true statistical standard deviation σ_x of the population from which the measurements were randomly drawn; thus it is what we use for a 68% confidence interval for *one* measurement (i.e. each of the x_i).

The standard error

If we do not care about the standard deviation of one measurement but, rather, how well we can rely on a calculated average value, $\bar{x}$, then we should use the **standard error** or standard deviation of the mean $s_{\bar{x}}$. This is found by dividing the sample standard deviation by $\sqrt{N}$:

$$s_{\bar{x}} = \frac{s_x}{\sqrt{N}}. \tag{2.3}$$

If we draw *two* sets of random samples from the same distribution and compute the two means, the two standard deviations, and the two standard errors, then the two means will agree with each other within their standard errors 68% of the time.

2.6.2 Reporting Data

Under normal circumstances, the best estimate of a measured value x predicted from a set of measurements $\{x_i\}$ is given by $x = \bar{x} \pm s_{\bar{x}}$. Statistics depend intimately upon large numbers of samples and none of our experiments will obtain so many samples. Our uncertainties will have 1-2 significant figures of relevance and the uncertainties tell us how well we know our measurements. Therefore, **we will round our uncertainties, δm, to 1-2 significant figures** and then **we will round our measurements, m, to the same number of decimal places**; (3.21 ± 0.12) cm, (434.2 ± 1.6) nm, etc.

2.6.3 Error Propagation

One of the more important rules to remember is that the measurements we make have a range of uncertainty so that any calculations using those measurements also must have a commensurate range of uncertainty. After all the result of the calculation will be different for each number we should choose even if it is within range of our measurement.

We need to learn how to propagate uncertainty through a calculation that depends on several uncertain quantities. Final results of a calculation clearly depend on these uncertainties, and it is here where we begin to understand how. Suppose that you have two quantities x and y, each with an uncertainty δx and δy, respectively. What is the uncertainty of the quantity $x + y$ or xy? Practically, this is very common in analyzing experiments and statistical analysis provides the answers disclosed below.

For this course we will operate with a set of rules for uncertainty propagation. It is best not to round off uncertainties until the final result to prevent accumulation of rounding errors. Let x and y be measurements with uncertainty δx and δy and let c be a number with negligible uncertainty. We assume that the errors in x and y are *uncorrelated*; when one value has an error, it is no more likely that the other value's error has any particular value or trend. We use our measurements as described below to calculate z and the propagated uncertainty in this result (δz).

- **Multiplication by an exact number:** If $z = c\,x$, then

$$\delta z = c\,\delta x \tag{2.4}$$

- **Addition or subtraction by an exact number:** If $z = c + x$, then

$$\delta z = \delta x \tag{2.5}$$

- **Addition or subtraction:** If $z = x \pm y$, then

$$\delta z = \sqrt{(\delta x)^2 + (\delta y)^2} \tag{2.6}$$

- **Multiplication or division:** If $z = xy$ or $z = \frac{x}{y}$, then

$$\frac{\delta z}{z} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2} \tag{2.7}$$

- **Power:** If $z = x^c$, then

$$\frac{\delta z}{z} = c\,\frac{\delta x}{x} \tag{2.8}$$

The important pattern in these rules is that when you combine multiple uncertainties, you do not add them directly, but rather you square them, add, and then take the square root. The reason for this is intuitive: if one error is randomly positive, the other one is sometimes negative, which *reduces* the total error. Therefore, it is incorrect to estimate the combination of two uncertainties as their sum since this overestimates the average size of the *combined* error.

2.6.4 (Essential) Significant Figures

WARNING

Failure to adhere to the following protocol *will* result in point deductions.

The *significant figures* of a number are the digits in its representation that contribute to the precision of the number. In practice, we assume that all digits used to write a number are significant (except leading zeroes). Therefore, completely uncertain digits should not be used in writing a number and results should be rounded to the appropriate significant figure. The noise in our measurements should be discarded. For example, you should not express your height as 70.056 inches if your uncertainty is ± 0.1 inch. It would more appropriately be written as 70.1 inches. Uncertainties specified using only significant digits are always ± 5 times a power of 10; the least significant displayed digit was the result of rounding up or down by as much as 0.5 of that digit. Usually we know our uncertainty to be something close to this but yet different. Further, results of simple calculations should not increase the number of significant digits. Calculations transform our knowledge; they do not increase our knowledge. The rounding should be performed at the final step of a calculation to prevent rounding errors at intermediate steps from propagating through your work but one or two extra digits suffice to prevent this.

Zeros are also considered significant figures. If you write a number as 1,200, we assume there are four significant digits. If you only mean to have two or three, then it is best to use scientific notation: 1.2×10^3 or 1.20×10^3. Leading zeros are not considered significant: 0.55 and 0.023 have just two significant figures. After some time the decimal point frequently gets obscured, but the '0' and the space allows us to realize that this is 0.55 *not* '55.'

There are some guidelines for tracking significant figures throughout mathematical manipulation. This is useful as a general method to keep track of the precision of a number so as not to carry around extra digits of information, but you should generally be using more formal error estimates from Sections 2.5 and 2.6 for reporting numbers and calculations in the physics lab.

- **Addition and Subtraction:** The result is known to the decimal place of the *least* precise input number.

  Example: $45.37 + 10 = 55$, not 55.37 or 55.4
  Why? $\delta = \sqrt{0.005^2 + 0.5^2} = 0.5$

  Where we used the sum formula Equation (2.6).

- **Multiplication and Division:** The result is known to as many significant figures as are in the least precise input number.

  Example: $45.4 \times 0.25 = 11$, not 11.4
  Why? $\delta = 11\sqrt{\left(\frac{0.05}{45}\right)^2 + \left(\frac{0.005}{0.25}\right)^2} = 0.2 > 0.05$

Where we used the product formula Equation (2.7).

Example: If you measure a value on a two-digit digital meter to be 1.0 and another value to be 3.0, it is *incorrect* to say that the ratio of these measurements is 0.3333333, even if that is what your calculator screen shows you. The two values are measurements; they are not exact numbers with infinite precision. Since they each have two significant digits, the correct number to write down is 0.33. If this is an intermediate result, then 0.333 or 0.3333 are preferred, but the final result must have two significant digits.

For this lab, you should use proper significant figures for all reported numbers including those in your notebook. We will generally follow a rule for significant figures in reported numbers: **calculate your uncertainty to two significant figures, if possible, using the approach in Sections 2.5 and 2.6, and then use the same level of precision in the reported error and measurement.** This is a rough guideline, and there are times when it is more appropriate to report more or fewer digits in the uncertainty. However, it is always true that the result must be rounded to the same decimal place as the uncertainty. The uncertainty tells us how well we know our measurement.

2.7 How to Plot Data in the Lab

Plotting data correctly in physics lab is somewhat more involved than just drawing points on graph paper. First, you must choose appropriate axes and scales. The axes must be scaled so that the data points are spread out from one side of the page to the other. **Axes must always be labeled** with physical quantity plotted and the data's units. Then, plot your data points on the graph. **Ordinarily**, you must add error bars to your data points, but we forgo this requirement in the introductory labs. Often, we only draw error bars in the vertical direction, but there are cases where it is appropriate to have both horizontal and vertical error bars. In this course, we *would* use one standard deviation (standard error if appropriate for the data point) for the error bar. This means that 68% of the time the 'true' value should fall within the error bar range.

Do *not* connect your data points by line segments. Rather, fit your data points to a model (often a straight line), and then add the best-fit model curve to the figure. The line, representing your theoretical model, is the best fit to the data collected in the experiment. Because the error bars represent just one standard deviation, it is fairly common for a data point to fall more than an error bar away from the fit line. *This is OK!* Your error bars are probably too large if the line goes through all of them! Since 32% of your data points are more than $1\,\sigma$ away from the model curve, you can use this fact to practice choosing appropriate uncertainties in your raw data.

Some of the fitting parameters are usually important to our experiment as measured values. These measured parameters and other observations help us determine whether the fitting model agrees or disagrees with our data. If they agree, then some of the fitting parameters might yield measurements of physical constants.

2.8 Fitting Data (Optional)

Fully understanding this section is not required for Physics 136. You will use least-squares fitting in the laboratory, but we will not discuss the mathematical justifications of curve fitting data. Potential physics and science majors are encouraged to internalize this material; it will become an increasingly important topic in upper division laboratory courses and research and it will be revisited in greater detail.

In experiments one must often test whether a theory describes a set of observations. This is a statistical question, and the uncertainties in data must be taken into account to compare theory and data correctly. In addition, the process of 'curve fitting' might provide estimates of parameters in the model and the uncertainty in these parameter estimations. These parameters tailor the model to your particular set of data and to the apparatus that produced the data.

Curve fitting is intimately tied to error analysis through statistics, although the mathematical basis for the procedure is beyond the scope of this introductory course. This final section outlines the concepts of curve fitting and determining the 'goodness of fit'. Understanding these concepts will provide deeper insight into experimental science and the testing of theoretical models. **We will use curve fitting in the lab, but a full derivation and statistical justification for the process will not be provided in this course**. The references in Section 2.2, Wikipedia, advanced lab courses, or statistics textbooks will all provide a more detailed explanation of data fitting.

2.8.1 Least-Squares and Chi-Squared Curve Fitting

Usually data follows a mathematical model and the model has adjustable parameters (slope, y-intercept, etc.) that can be optimized to make the model fit the data better. To do this we compute the vertical distance between each data point and the model curve, we add together all of these distances, and then we adjust all of the parameters to minimize this sum. This strategy is the "least squares" algorithm for fitting curves.

Scientific curve fits benefit from giving more precise data points a higher weight than points that are less well-known. This strategy is "chi squared" curve fitting. These algorithms may be researched readily on the internet.

2.9 Strategy for Testing a Model

2.9.1 A Comparison of Measurements

If we have two independent measurements, $X_1 = x_1 \pm \delta x_1$ and $X_2 = x_2 \pm \delta x_2$, of the *same* physical quantity and having the same physical units, then we will conclude that they agree if the smaller plus its δ overlaps with the larger minus its δ.

A disagreement *could* mean that the data contradicts the theory being tested, but it could also just mean that one or more assumptions are not valid for the experiment; perhaps we should revisit these. Disagreement could mean that we have underestimated our errors (or even have overlooked some altogether); closer study of this possibility will be needed. Disagreement could just mean that this one time the improbable happened. These possibilities should specifically be mentioned in your Analysis according to which is most likely, but further investigation will await another publication.

Helpful Tip

One illegitimate source of disagreement that plagues students far too often is simple math mistakes. When your data doesn't agree and it isn't pretty obvious why, hide your previous work and repeat your calculations very carefully to make sure you get the same answer twice. If not, investigate where the two calculations began to differ.

Chapter 3

Experiment 1: A Modern Galileo

The study of motion was one of the first successful applications of the scientific method advocated by early scientists such as Galileo (1564 - 1642). In fact, the experiment we will perform here is very similar to one designed by Galileo to quantify the nature of the motion of objects. We have, of course, updated the apparatus considerably; we will supplement the low-tech inclined plane of Galileo with an electronic timer instead of a water clock and nearly frictionless air pucks in place of a rolling ball. Our results will be as convincing as Galileo's, demonstrating that there is a general principle underlying the motion of objects due to gravity. The underlying modern description of this physics took another century for Newton to formulate (it will take you another week or so).

The famous inclined plane experiments of Galileo showed that *for an object moving under the influence of gravity, there is a precise mathematical relationship between the distance traveled and the time that has passed*. Galileo wanted to observe and to test this relationship, but falling objects accelerate too quickly for him to perform accurate experiments with his crude instrumentation. The inclined plane solved Galileo's problem: it slowed the motion down so that his measurement tools could be effective in testing his hypotheses. This short description of Galileo's experiment already illuminates several of the primary challenges of experiments in physics: measurement, limits of instrumentation, accurate collection of data, creativity, problem solving, and rigorous hypothesis testing.

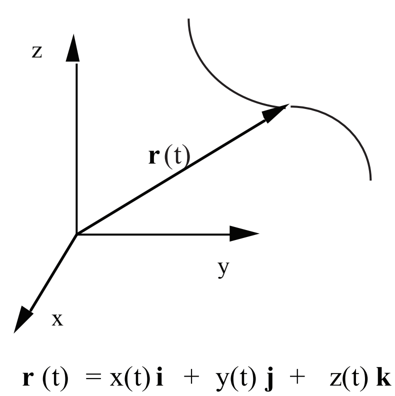


Figure 3.1: *Plot of the position vector, $r(t)$; this curve is called the trajectory.*

Galileo explored the relationship between two physical variables — position and time. In mechanics, the mathematical description of motion is called **kinematics**. The origin of this motion is studied in **dynamics**, which we will explore later. Our first experiment will familiarize you with the kinematical quantities used to describe the motion of point-like objects — *position*, *velocity*, and *acceleration*. Each of these can change with *time*. If we wish to record motion quantitatively, we need to measure both the positions of a body and the times when the body was at each position. You are already intuitively familiar with the basic procedure of these measurements. Position is measured with respect to some reference coordinate system. For example, a ruler can measure the distance between two points: 1) $y = 0$ and 2) the object's location. Time is measured using some form of a "clock"; all clocks begin with pulses at very regular time intervals. The specific approach to implementing these observations are the details provided by an experimenter such as yourself or Galileo. Figure 3.1 shows one possible representation of the motion of an object, defined by the coordinates x, y, and z.

3.1 Background: The Mathematics of Kinematics

Time (s)	Position (cm)
0	1.00
1	5.00 ± 0.05
2	7.00
3	7.00
4	5.00

Table 3.1: *Sample position vs. time data presented in a labeled table.*

As background to the experiment, we first develop the basic mathematics of motion described using vectors. Motion in three dimension can be described in the most general way using three coordinates. The position of an object's center of mass is represented by the vector $\mathbf{r}(t)$, which changes in time along a continuous line called its *trajectory*. This trajectory can be represented at specific times by noting the coordinates of the position at each time. Table 3.1 is an example of the data for one-dimensional motion expressed in tabular form. Practically, when recording data in the laboratory, you will write down a table of values that are measured. It is often convenient to plot the position $s(t)$ of the object as a function of time t, shown for example in Figure 3.2.

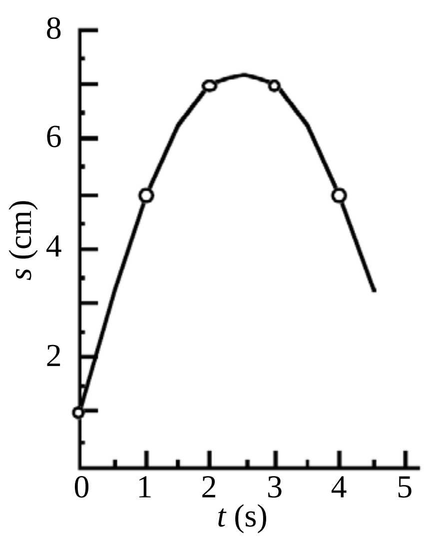


Figure 3.2: *Plot of position s vs. time, with data from Table 3.1. The lines are only guides to the eye.*

We can now describe the other kinematic quantities from these data points. We define the **average velocity**, $\bar{v}$, of the body during a particular time interval, $\Delta t_{1,2} = t_2 - t_1$, as the ratio of the change of position $\Delta s_{1,2} = s_2 - s_1$ to $\Delta t_{1,2}$:

$$\bar{v} = \frac{\Delta s_{1,2}}{\Delta t_{1,2}} = \frac{s_2 - s_1}{t_2 - t_1} \tag{3.1}$$

For the special path in which an object leaves a point and then returns to the same position the average velocity is zero. During the motion, however, the average velocity calculated between any two time points will vary. An accurate description of the motion requires a more powerful descriptive tool than the average velocity. The instantaneous velocity is defined using calculus by taking the limit of Equation (3.1).

$$v(t) = \frac{\mathrm{d}s}{\mathrm{d}t} = \lim_{\Delta t \to 0} \frac{s(t + \Delta t) - s(t)}{(t + \Delta t) - t} \tag{3.2}$$

The symbol $\frac{\mathrm{d}s}{\mathrm{d}t}$ is the "derivative, with respect to time, of the function $s(t)$." This concept is from calculus, and it is a measure of the rate at which the function $s(t)$ changes. These expressions generalize to vectors. For students in Physics 130-a, the calculus origins of these expressions are not crucial, but the meaning of velocity as being the rate that a position vector changes is the point of the discussion and certainly important.

If the velocity is uniform, the derivative of the position is a constant over all time and the instantaneous velocity is the same as the average velocity. However, if the velocity is not uniform, then the velocity is itself a function of time. We can similarly define the average acceleration, $\bar{a}$, as the change in the instantaneous velocity, $\Delta v_{1,2} = v_2 - v_1$, divided by the time interval $\Delta t_{1,2} = t_2 - t_1$ over which the change occurs:

$$\bar{a} = \frac{\Delta v_{1,2}}{\Delta t_{1,2}} = \frac{v_2 - v_1}{t_2 - t_1} \tag{3.3}$$

The instantaneous acceleration at time t is also obtained following the calculus limit process as above:

$$a(t) = \frac{\mathrm{d}v}{\mathrm{d}t} = \lim_{\Delta t \to 0} \frac{v(t + \Delta t) - s(t)}{(t + \Delta t) - t} \tag{3.4}$$

Here, $\frac{\mathrm{d}v}{\mathrm{d}t}$ is the derivative, with respect to time, of the function $v(t)$. Since the velocity $v(t)$ is already the derivative of the position $s(t)$, the acceleration can be obtained from the position function by applying the derivative process twice. The symbol $\frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$ is called the "second derivative," with respect to time, of the function $s(t)$. The graphical or geometrical interpretation of the acceleration in relation to velocity is similar to that of the velocity in relation to position: the instantaneous acceleration at some time t is equal to the slope (of the tangent) of the velocity versus time curve at t.

Kinematics defines the quantities $s(t)$, $v(t)$, and $a(t)$ which completely characterize the motion of the center of mass of any object. Given any one of these functions and appropriate initial values, differential calculus (or plane geometry plus ingenuity) allows us to calculate the other two functions. As noted previously, kinematics does not tell us where these functions come from, only how they are related. If the velocity changes uniformly, the acceleration is a constant over all time. In this case, the instantaneous acceleration is the same as the average acceleration. This kinematic condition, the constant acceleration that Galileo measured, is one characteristic of gravity near earth's surface.

Checkpoint

In studying the motion of an object, what does the trajectory represent?

3.2 The Experiment

Here, you will reproduce Galileo's seminal inclined plane experiment. You will find experimental evidence for very specific rules that govern motion. From lecture, you are likely already familiar with these rules. By performing the experiment yourself, you will learn to record data, to understand how "good" your measurements are, to notice your experiment's environment, and to gain experience in "fitting" a model to these data. Your own data will make a strong case, likely as strong as Galileo's own experiment from the 17^{th} century, of this fundamental observation: *the distance traveled under the influence of gravity is proportional to the square of the time that elapses*. It may seem simple, but proving this hypothesis with real data was one of the earliest applications of the modern scientific method and the basis for the later development of Newton's First Law of Motion. And simple or not, prior to Galileo this was a controversial point.

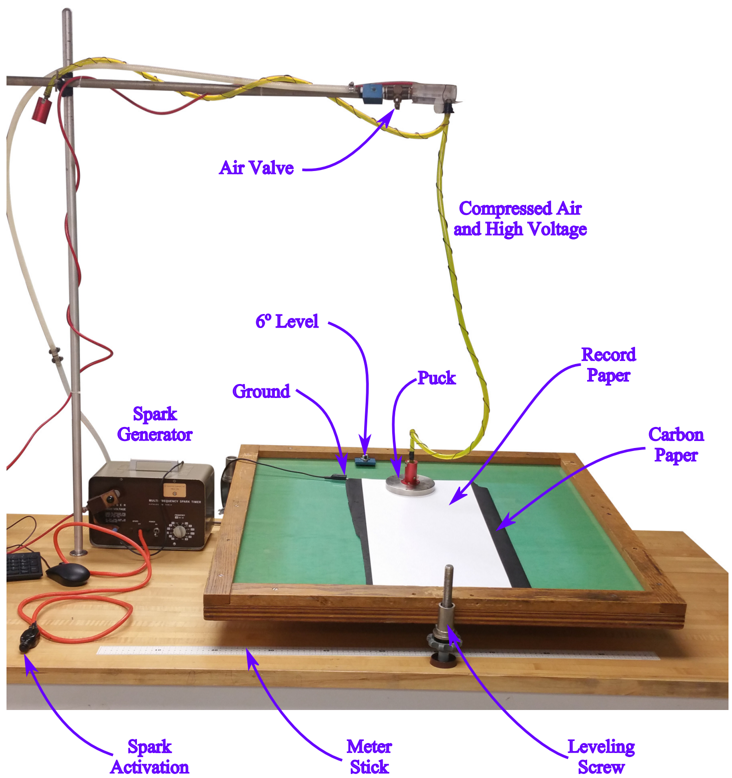


Figure 3.3: *A photograph of the apparatus showing the relevant controls.*

3.2.1 The Apparatus

In this experiment we will use an "air hockey puck" to study the motion of an object accelerating under the influence of gravity. Figure 3.3 shows a photograph of the experiment. Pressurized air keeps a disk, or "puck", of stainless steel afloat above a glass surface so that it can move practically without friction. A pointed electrode protruding from a hollow chamber

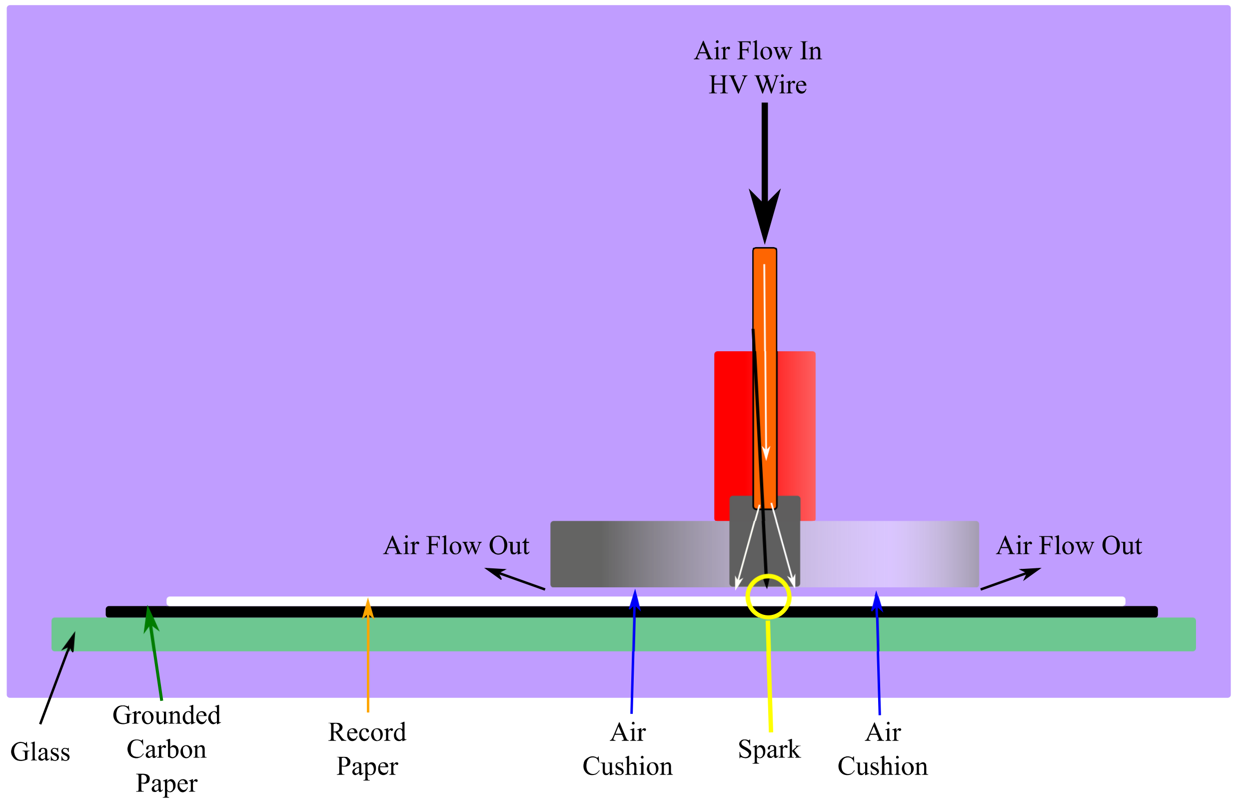


Figure 3.4: *Pressurized air and a high-voltage wire inside the hose float the puck on a cushion of air and excite a spark through the record paper.*

open to the bottom of the puck, is aligned along the axis of the puck (see Figure 3.4).

Checkpoint

Why do you float the puck used in this experiment on an air cushion? If the mass of the puck were doubled, what effects would this have on the experiment?

This electrode sits just above a special sheet of carbon paper which covers the surface of the table. The embedded carbon turns this paper into a fair conductor of electricity. A sheet of white paper – the record paper – is placed between the carbon sheet and the floating puck. The pulse generator – when activated by pressing the hand-held pushbutton – applies a series of high-voltage pulses to the puck electrode. With each high voltage pulse an electric discharge takes place between the carbon sheet and the pointed electrode in the center of the puck. This electrical discharge leaves a mark – a black dot – caused by carbon evaporation. The marks are made on the *underside* of the record paper and each one reflects the position of the puck at the time of the spark. In our laboratory equipment, the electrode is pulsed by a high-voltage pulse generator operating at a constant frequency of 60 cycles per second, or 60 Hertz (Hz). This means that every 1/60 sec a black dot will mark the position of the projected center of mass of the moving object (the center of the puck's bottom surface). This trajectory contains all the necessary information (position as a function of time) necessary to determine the velocity and acceleration of the puck.

Helpful Tip

Do not turn the air supply on all the way. It will damage the air hose. Only turn it on a little way. The puck will float well.

WARNING

Do not drop the puck. The table top is glass! The puck should be on the white record paper and not on the carbon paper whenever the push-button activating the pulse generator is pressed. The white paper should be on the carbon paper. If you touch the high voltage terminal and ground at the same time, you may get a shock – harmless, but unpleasant! Always make sure that the ground clip is properly connected to the carbon sheet before activating the pulser.

Checkpoint

A pulse generator is used in an experiment which operates at 60 kHz (60×10^3 oscillations/sec). What time interval will pass between successive dots?

Historical Aside

The carbon paper we are using has a trade name: *Teledeltos*. It was developed and patented around 1934 by Western Union. It was originally used to transmit newspaper images over the telegraph lines (as an early fax machine). The receiving end of the "Wirephoto" system operated on the same principle as our laboratory equipment. The cylinder of a drum was covered first with a sheet of Teledeltos paper, and then with a sheet of white record paper. A pointed electrode triggered by signals transmitted over the telegraph lines would then reconstruct the image by varying the density of black dots on the record paper. The image was then scaled photographically.

3.3 Procedure

Helpful Tip

There is a video on Canvas regarding this first experiment. Viewing it before class will make your experiment proceed faster and with fewer missteps.

3.3.1 Collecting position data

Using the inclined level, set the table bed to an inclination angle of 6° by adjusting the screws until the air bubble is centered in the level while the level is squared in the wood frame. Clear away eraser crumbs and other debris. Place a sheet of carbon paper on the air table and connect it with the alligator clip to the pulse generator. Place a sheet of record paper on top of the carbon sheet, set the puck on the white paper, and turn on the pressurized air. Adjust the air flow rate just a little more than enough to free up the puck's motion. Too much air pressure will cause the puck to rebound like water pressure pushes a garden hose; but, too little air pressure will not remove enough friction. Rotate the valve about 5° more than just enough for the puck to move on its own. Hold the puck with its top edge near the top end of the record paper. Turn on the power of your pulse generator. If your activation fob has a rocker switch, turn that switch on also. Activate the pulse and release the puck simultaneously. Hold the button down until the puck reaches the bottom of the paper. The puck will slide down the incline accelerating under the influence of gravity and the sparks will record its motion.

Helpful Tip

Your TA or lab staff should already have leveled the inclined table at the beginning of the week. You should check it is correct, but it is not likely that you will need to readjust the incline unless an earlier student modified it or one of the glide cups has been removed from under the legs.

Release the push-button as soon as the puck hits the frame of the air table. A dotted line will be generated on the *underside* of the record paper as shown in Figure 3.5. By looking at the separation between the dots you can see that the puck is accelerating; the distances between the dots along the motion are increasing while the time interval is a constant 1/60 s.

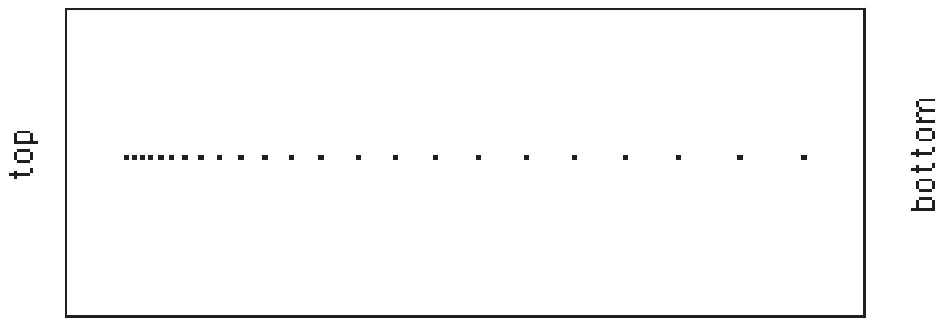


Figure 3.5: *Typical evaporated carbon dots on the record paper.*

3.3.2 Compute the velocity as a function of time

Now you will construct a basic graph *by hand* of these data in your lab notebook. Graphing paper can make this more accurate if it is available. Introduce a coordinate system on your paper that you can use to measure the puck's position; a single axis (x or y) along the straight trajectory will suffice. Choose the origin of the coordinate system to be near the beginning of the puck's motion. Draw a line perpendicular to the dots at this point. Begin with a dot near this reference but far enough along the trajectory that the separation between the dots is evident. Measure the distance between the reference line and the first discernible dot (we can refer to this dot as #1). Count down through seven dots (6 intervals) and measure the distance from the reference line to this dot (dot #7). Continue to measure distances between the reference line and each successive dot following six intervals. That is, note the distances to dot #1, #7, #13, #19, #25 … $\#6n + 1$. These are the positions of your puck in your chosen coordinate system. Make a table of these values similar to Table 3.1. You will need columns (and column headings) for time (s), position (cm), displacement (cm), and velocity (cm/s). Note and record your experimental uncertainties in the body of your table.

Checkpoint

- How accurately can you position the zero on the meter stick at the reference mark?
- How well can you read the dots' positions from the meter stick? Together these are the uncertainties in your position measurement.
- What determines the time? Can you decide what the uncertainties in the times should be?

Helpful Tip

Your Procedure in your report should describe how you measured position and time.

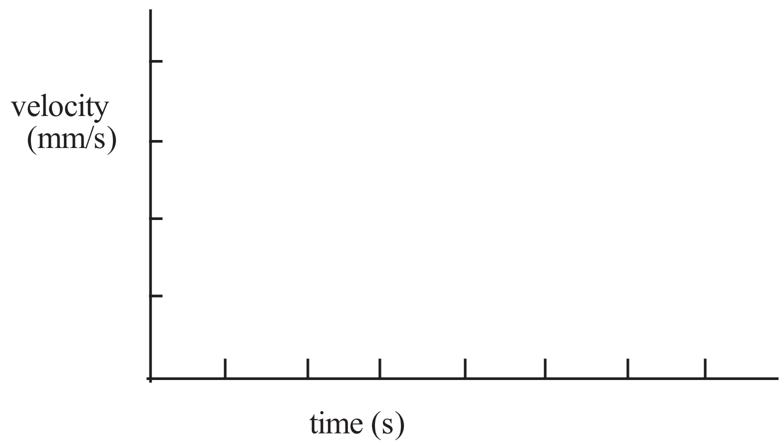


Figure 3.6: *Example of setting up graph axes, with appropriate labels and units.*

The average velocity is given by the distance between two consecutive dots divided by the time needed for the puck to travel this distance. This time is $1/60\,\mathrm{s}$ between adjacent dots; we have measured the distance between the first and seventh dots or over six intervals, so the time interval is $6 \times (1/60) = 0.10\,\mathrm{s}$. Calculate the velocity of the puck by subtracting each position coordinate value from the value above it in the position column, x. Label the new column displacement or Δx. (Don't forget to put the units in the heading of the column.) Calculate the displacements for all of your data. Calculate the velocity for each interval by dividing the displacement, $\Delta x = x_{i+1} - x_i$, by the time interval, $0.1\,\mathrm{s}$, for each entry. Since the time interval is $0.10\,\mathrm{s}$, you can merely transfer entries from the previous column by moving the decimal point one place to the right ...that is by multiplying by $10\,\mathrm{s}^{-1}$. Do the calculation ($v = \Delta x/\Delta t$) for one set of data if you are not already convinced of this. These calculations can always be done using a spreadsheet program, but is unnecessary here.

Now you should plot your results. The first time, it can be most instructive to do this by hand using an entire page of your lab notebook or an entire page of graph paper. Plot the velocity data points as a function of time t. Label the velocity vs. time plot, with the proper units as shown in Figure 3.6. Note that the time interval between our chosen points is $1/10\,\mathrm{s}$. You might wonder where in the interval should the velocity point be plotted, at $t = 0\,\mathrm{s}$, $0.10\,\mathrm{s}$, or $0.05\,\mathrm{s}$...? Technically, the data point should be plotted midway in the interval, so that the velocity for the interval $0 - 0.10\,\mathrm{s}$ should be plotted at $0.05\,\mathrm{s}$, and the next at $0.15\,\mathrm{s}$, etc.. However, such a horizontal translation doesn't change the slope and our choice of $t = 0$ was arbitrary anyway, so there is really no wrong choice as long as the same point is chosen for every interval.

Helpful Tip

To improve your plotting accuracy, scale both axes (velocity and time) so that your data occupy *at least* half of your page. You can always conveniently double the space between $0\,\mathrm{s}$ and $0.1\,\mathrm{s}$...

3.3.3 Calculate the puck acceleration

Do your points lie in a straight line? Remember that measurements have experimental uncertainty, so there is likely to be fluctuations above and below the best line. If one or more points vary greatly, check to be sure that the points were measured and calculated correctly. Maybe the spark timer missed a spot? If so you must count this spot despite its absence; the time did pass, after all, even if the voltage was too low to mark it.

Once you are happy with your data, you should estimate the acceleration from your measurements. For this introductory lab, we typically do this by hand first. The acceleration is given by the slope of the straight line that best fits your data. With the straight edge of a ruler choose a line that seems best to fit all the data. It helps to lower your eye near the line and to view the page edge on. Draw this line, extend it all the way across the page, and choose two points that lie on the line (NOT the data points), one on either end, near the extreme ends of the line. If possible choose points for which x and y values can be easily read. Use these values to calculate the slope, a, of the line. Don't forget to include your units. This procedure mimics a data fitting algorithm, but relies on your eye and brain to estimate the best line.

3.3.4 Estimate the uncertainty in the slope

The value of every measured quantity in an experiment is only a small part of the required information that should be recorded about a measurement. Every measurement has units that must be recorded, since every physical value is compared to some scale. Also, an experimenter *must* determine the expected range of values which might reasonably be expected if a similar experiment were repeated. This last piece of information communicates *how precise* the measurement is. Obviously, using modern equipment, your measurements likely can be more precise than Galileo's (but perhaps he spent more time getting it perfect than your two hours…). If you want to claim that your modern tools are better, you must give some estimate of how precise you expect your measurement to be. This is an important part of experimentation, and it is why there is an entire chapter of this manual (Chapter 2) devoted to it.

Similar to data collection, your estimate of the acceleration as the slope is also subject to uncertainty. When looking at your data, representative lines that you might draw should pass among the set of data points; however, we can easily see the influence of random perturbations on the graph. Choose a new line using your straight edge that passes among your data points and deviates as much as it reasonably might from your earlier ("best fit") line. Try to increase or decrease the slope of the line as much as you can and still be close to most of the data points. This is not an exact measurement, only an estimate. Calculate the slope of this new line and compare it with the previous measurement by taking the difference between the two numbers. How different are the slopes of these two new lines? Report this difference as the uncertainty, δa, in your measurement of acceleration, a: acceleration $= a \pm \delta a$. (Don't forget the units of both a and δa.)

Checkpoint

To build some intuition for your measurement precision, you could calculate the percentage uncertainty relative to the best fit value. Would you describe your fitting procedure as a precise determination of the acceleration? How precise is it?

3.3.5 (Optional) Fitting the data

Is the slope of your velocity graph constant? if so, this means that the acceleration is constant as time passes. Using calculus we can reverse the process of differentiation in Section 3.1 and obtain the position from the acceleration. If the acceleration is not constant, this can be a harder math problem. But, if the acceleration is constant a, then the position function $x(t)$ can be obtained using basic kinematic methods:

$$x(t) = x_0 + v_{x0}\, t + \frac{1}{2} a t^2 \tag{3.5}$$

where x_0 and v_{x0} are initial position and initial velocity at time $t = 0$, respectively. Note that this kinematics procedure is entirely mathematical; there is no rule that says that motion *must* have constant acceleration, only that if it does, its position looks like Equation (3.5).

Plot your position data in the computer program of your choice.

Checkpoint

Do your data look like a parabola? Said another way. . . do your data *look* consistent with the relationship between x and t given by Equation (3.5)?

Now fit your position data to Equation (3.5) using the program of your choice (see the Appendices). It is important that your fitting procedure give you the uncertainty of your fit parameters. The meaning of this uncertainty from Least Squares fitting is discussed in Chapter 2. The procedure here is outlined using Vernier's Graphical Analysis software.

Run Vernier Software's Graphical Analysis program. Rename the x column to 'time' by double-clicking the gray column header. Type the new column name into the name edit control and type the units (s?) into the units edit control. OK and repeat for the y column to prepare it to hold the position data. Type your measured positions and times into the table and note that the points are graphed as you type them.

Equation (3.5) is the equation of a parabola. Do your data points look like a section of a parabola? In that case draw a box around your data points with your mouse so that every row of the data table turns black. From the menu choose Analyze/Curve Fit. . . , select the parabola model, click "Try Fit", and verify that the curve passes through your data points. The computer should draw a black parabola section through your data points. If it does not do so, ask your instructor to help you. The computer will choose values for 'A', 'B', and 'C' to make the math model fit your data points as well as possible. Once the model curve passes through your data points, OK and drag the parameters box away from your data points. Copy and paste your data table and graph into Word®, save it to Box Sync\PhysLabs, and print a copy for your notebook. Compare the model function to Equation (3.5) and note that

$$\tfrac{1}{2}a = A \qquad \text{so} \qquad a = 2A. \tag{3.6}$$

Is this acceleration measurement similar to the slope of your other graph?

Helpful Tip

Be certain to save your fit in an electronic format for submission with your report!

3.4 Analysis

The procedures in Sections 3.3.1–3.3.4 exemplify the best analysis that Galileo would have been able to apply to extract an acceleration. Do your results support the same conclusions that Galileo achieved? The 'by hand' methods outlined above are obviously rather crude. With computers, we can apply more rigorous algorithms to get the best measurement of the acceleration and its uncertainty provided by your experiment. In your analysis of this experiment, you will go beyond the tools available to Galileo and use modern computers to obtain quantitative results. In this way, you can understand the statistical significance of your conclusions. We will build on these computer analysis approaches throughout the course.

Helpful Tip

You may be tempted to collect your data and to leave lab since you have access to computers outside of lab. This could cost you significant time in preparing your reports without the guidance of your TA. You are likely better off taking advantage of the full assigned lab time to finalize your figures and analysis before writing the actual summary report outside of class. It will likely go much faster.

In the lab, you have access to Microsoft Excel®and Vernier Software's Graphical Analysis program. They both have their benefits. You can use what is most comfortable to you. In the end, you will need to submit a clear, well-labeled graph of your results with your report. Regardless of what software tool you use, the program will be performing a statistical procedure known as Least Squares fitting to obtain the best line possible from your data. (See Section 2.8 for more information.)

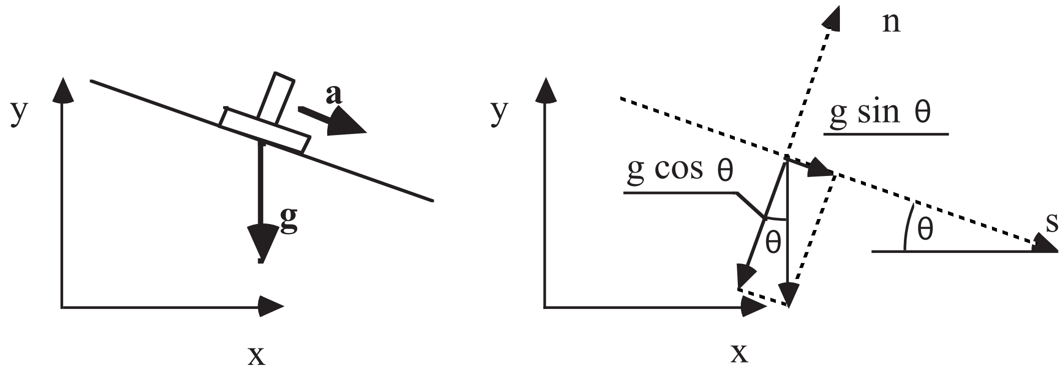


Figure 3.7: *Schematic diagrams illustrating the relation between slope of incline and acceleration of gravity.*

3.4.1 Prediction of acceleration from vector analysis

Solve for the acceleration due to gravity in the inclined plane. You should perform this calculation in your lab notebook. Introduce a coordinate system with one axis along the plane of the incline (call the axes of the new system surface, s, and normal, n, instead of x and y). The acceleration vector $\mathbf{g}$ must then be resolved into components along the s and n axes. This is done with the help of the trigonometric relations given in Figure 3.7. The acceleration of the puck, a, measured in this experiment is the component of $\mathbf{g}$ along the s axis of the incline. This quantity is related to $g = |\mathbf{g}|$ by

$$|\mathbf{a}| = g \sin\theta. \tag{3.7}$$

(Note that the steeper the incline, the larger the acceleration of the puck for the same g. Can you see this from the last equation? Also note that $\theta = 90°$ means the incline is vertical and $a = g$. Is this what you would expect?) Use this analysis to "predict" the acceleration of gravity at the Earth's surface, g, from your measurement of the acceleration on the inclined plane. Use the same equation to calculate the uncertainty in your value of g from your uncertainty in a. Write it in the notebook in the form $g \pm \delta g$. (Don't forget to include your units in each of these values; since units multiply the numbers, it is common practice to use the same units and to factor them outside: $g = (981.4 \pm 1.2)\,\mathrm{cm/s^2}$.)

Checkpoint

Does your experimental value for gravity agree with the well-known value? What does it mean to "agree"?

3.5 Guidelines

Your TA will be looking at the work that you perform in class as well as your communication of these results in a *short* written report. Remember that there is a template which will make preparation of this short writeup simpler.

These guidelines detail the elements that your TA expects to find in your in-class notebook. Your grade will be based in part on completing these tasks, but also on the *quality* of the tasks. Therefore, merely having these elements does not guarantee a good score. Your TA also has discretionary points in the grade not tied to specific sections in which he or she will be able to rate how well your understanding of the material is communicated both in lab and in writing.

3.5.1 Lab Notebook

Your Lab Notebook should contain the following:

- A table of data collected in the lab (printed or hand written).
- A hand-drawn figure of your plotted position and velocity data.
- Estimated 'best fit' lines for the acceleration of your puck.
- Relevant sample calculations for vector analysis of the inclined plane.

3.5.2 Lab Report Guidelines

Your Lab Report should contain the following information. Your goal with the reports is to communicate the experiment to the reader in a short write-up. These general guidelines will not necessarily be repeated for every experiment, since they do not change. Refer to Appendix E for writing instructions and the 'Example.pdf' on CANVAS.

1. **Introductory Information**
    - An informative title for your report
    - Your name and your partner's name(s)
    - Your lab section number and the date of the experiment
2. **Purpose** Explain in your own words what the purpose of this lab work was. Why should we perform the experiment? Why should others contribute (financially and/or otherwise) to see the experiment completed? Be as brief and clear as possible. What did you try and measure? What physics will each measurement illustrate and/or test? What else might we learn whose usefulness we might not know yet (esp. new equipment, strategies, etc.)? We are always interested in *new* information and capabilities.

3. **Procedure** Refer to Appendix E for writing instructions and the 'Example.pdf' on CANVAS for generally applicable instruction. Your procedure for *this* experiment should answer:

   - What object were you watching?
   - Why did the object move?
   - How did you obtain the positions and correlate them to times?
   - How did you compute velocities and measure acceleration?
   - How might your experiment measure g?

   Two concise paragraphs can answer all of these, but a bullet list is also acceptable. Illustrations might help. Manufacturer names and model numbers will convey much information with few words.

4. **Data and Results** Refer to Appendix E for writing instructions and the 'Example.pdf' on CANVAS. All raw data, M, must contain 1) the quantity measured (m), 2) a reasonable estimate of experimental uncertainty (δm), and 3) the correct units multiplying the measurement and uncertainty: $M = (m \pm \delta m)$ units.

   For this specific report:

   - A table of t, s, and v. This might be located in your appendix and yet should be addressed by name here.
   - A graph of velocity vs. time fit to a line. This might be located in your appendix and yet should be addressed by name here.
   - (optional) A graph of position vs. time fit to a parabola.
   - Examples of velocity and acceleration calculations.
   - Your measured $g \pm \delta g$ with units. This might be in the next section instead.

   Unlike many lab reports, you are not required to write a Theory section in this class. You can assume that your reader knows and understands the theory as if such a section existed. You may refer to a calculation that appears in your lab notebook without reproducing it in your report **if** you append images of your notes after your Conclusions. *The lab report is not the correct venue for derivations! Occasional sample calculations may be appropriate, but not full derivations.* Your notebook will contain derivations periodically.

5. **Analysis of Results** Refer to Appendix E for writing instructions and the 'Example.pdf' on CANVAS. For this specific report:

   - A comparison between your measured g and the accepted value discerned from previous measurements. (See Section 2.9.)
   - A graph showing your best fit of the velocity curve. This probably will be *located* in the previous section or your appendix, but its implications should be discussed here.
   - "Other sources of error".

Ultimately, this section should convince your reader that your conclusions are valid; this section should *not* **be** your conclusions.

6. **Conclusion** Refer to Appendix E for writing instructions and the ‘Example.pdf’ on CANVAS. The conclusions for this experiment should:
   - Clearly and completely specify your measured g.
   - Characterize the puck’s motion in kinematic terms.
   - State which equations your data support, contradict, or neither.
   - Suggest applications for part or all of your apparatus and/or procedure.
   - Suggest likely improvements to the experiment.

   Generally, Purpose and Conclusions address the same topics. Reviewing Data and Analysis sometimes reveals physical constants to report.

Chapter 4

Experiment 2: Kinematics and Projectile Motion

Galileo is famous for several early mechanics experiments. In our first experiment, we reproduced the simple inclined plane to show that objects follow uniform acceleration in gravity. This conclusion about gravity is of course related to Galileo's famous "leaning tower of Pisa" experiment.

Galileo also used an inclined plane to launch objects into the air to observe their projectile motion, or motion in two dimensions under the influence of gravity. In Galileo's time, the recent invention of cannons prompted deeper understanding of projectile motion for improved warfare. In modern times, our interest in mechanics is a bit more general and fundamental. Here, we will use the same inclined plane apparatus from Experiment 1 to explore motion in two-dimensions; since this requires two position coordinates, we must use $\Big(x(t), y(t)\Big)$ instead of $s(t)$. Our approach will be different from Galileo's, involving more sophisticated computer data acquisition, but the conclusions about mechanics will be the same: *objects in motion in two dimensions with constant acceleration will follow a parabolic path.* In this experiment, we will investigate the vector acceleration in more detail than before. We will utilize computer analysis, and we will also note how an object will recoil from an elastic spring in preparation for our upcoming study of collisions. Just as the incline slows down gravity's acceleration, the elastic band slows down the collision with the wooden frame so that we may study it in more detail.

4.1 Background: Kinematics

The mathematics of kinematics that was covered in the previous experiment is still relevant to this experiment, and you should refer to the material in Experiment 1 for a refresher. The definitions of position, velocity, and acceleration used here are all the same, except in this experiment we will need to distinguish between motion in more than one dimension. The concept of vectors, although relevant to Experiment 1, is now required. Since we have already shown that the acceleration due to gravity on an inclined plane is a constant along the plane, Equation (3.5) will be particularly important as well to describe motion under gravity.

The primary insight that distinguishes dynamics in more than one dimension from the simpler one-dimensional case is that each direction is distinct and the governing equations can be written independently. This surprising structure is inherent in the vector form of Newton's laws. Of course, motion in different directions can be coupled in many ways, but the laws of classical mechanics can always be written on separate orthogonal axes using vector components.

4.2 Hooke's Law

In 1660 Robert Hooke realized that when a spring is compressed or stretched the effort needed grew larger as the amount the spring was deformed increased. Additionally, the effort (force) needed to compress the spring was opposite to the effort needed to stretch the spring. Hooke succinctly expressed this relation using the equation

$$F_s = -kx \tag{4.1}$$

where x is the signed amount that the spring is stretched and k is the constant of proportionality that characterizes each particular spring. The negative sign summarizes the fact that the spring tries to return to its undeformed shape and exerts a force, F_s, itself to achieve this. Since this force will affect the puck's motion while the puck is in contact with the elastic cord, we will find that the acceleration is not constant during these times. The student might recall that we omitted from consideration (and possibly did not record it at all) all of the puck's motion after it contacted the wooden frame in the previous experiment. The puck's motion was also greatly affected by that interaction and in the next few weeks we will begin to understand this more clearly. The elastic cord greatly slows down this reversal in motion so that we can investigate it. This strategy is not unlike using the inclined plane to slow down the effect of gravity itself.

4.3 Apparatus

WARNING

Whenever the push-button which activates the pulse generator is pressed, the puck should be on the white record paper and not on the carbon paper. The white paper should be on the carbon paper. If you touch the high voltage terminal and ground at the same time, you may get a shock – harmless but unpleasant! Always make sure that the ground clip is properly connected to the carbon sheet before activating the pulser.

The apparatus we use in this experiment is nearly identical to that used in Experiment 1. The primary difference is the addition of an elastic cord across the table (see Figure 4.1). Operation of the remaining components such as the 'air hockey puck', the Teledeltos paper, and the 60 Hz pulse generator are familiar.

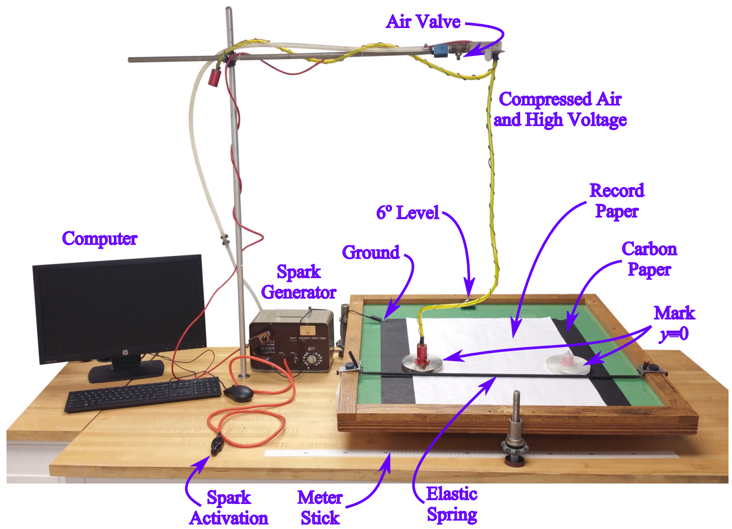


Figure 4.1: *A photograph showing the apparatus and the relevant controls. The 'ghostly' puck image at the right is the second location we need to mark $y = 0$; the first location is illustrated by the puck itself.*

4.4 Procedure

4.4.1 The puck's positions

Consider the motion as taking place in one dimension. Consequently, the position is simply the displacement of the puck with respect to the origin of the coordinate system, or in other words, a measure of how far away (and in which direction) the puck is from the reference line (See Figure 4.2). The location of the origin and the orientation of the axes of a coordinate system is ours to choose as we find convenient. With the help of the inclined level, set the table at an angle of 6° by adjusting the leveling screws until the air bubble is centered in the level and the level is squared in the frame. To avoid introducing an error due to parallax, position your eyeball directly above the level.

Helpful Tip

The lab technician has probably already oriented the incline, but you should check his work since your data depends upon it.

Place a sheet of carbon paper on the air table and connect it with the alligator clip to the pulse generator. Place a sheet of record paper on top of the carbon sheet, set the puck on the air table near the left side of the paper just barely touching the elastic cord, turn on the power to the pulse generator, and briefly press the sparker button. Move the puck near the right side of the paper just barely touching the cord and briefly press the sparker button. A straight line connecting these two dots will be our x-axis and will separate the motion into sections having a spring force and having no spring force.

Helpful Tip

Since these reference points are already on your paper, be sure not to disturb the record paper after you begin making these measurements until you have finished recording all of your data.

Turn on the pressurized air until the puck just starts to move freely. Open the valve another 5° or so; if there are leaks in your air hose you might need to open the valve a little more still. Now your puck should rebound almost to the same height from which it was released. Make sure the puck does not hit the wood frame; release from a lower height or ask your teaching assistant to adjust your elastic cord if necessary. Allow the puck to slide down the incline from some position near the top of the paper, bounce off of the elastic cord, and come to rest again near the top of the paper. It will be helpful to give the puck a small horizontal velocity before releasing it so that the upward trajectory does not cover up the downward trajectory. After practicing a few times, activate the spark timer with the push button trigger as you release the puck; keep the button pressed until the puck comes to rest again near the top of the paper. Dots will be recorded every $1/60^{\text{th}}$ of a second but will appear on the underside of the paper.

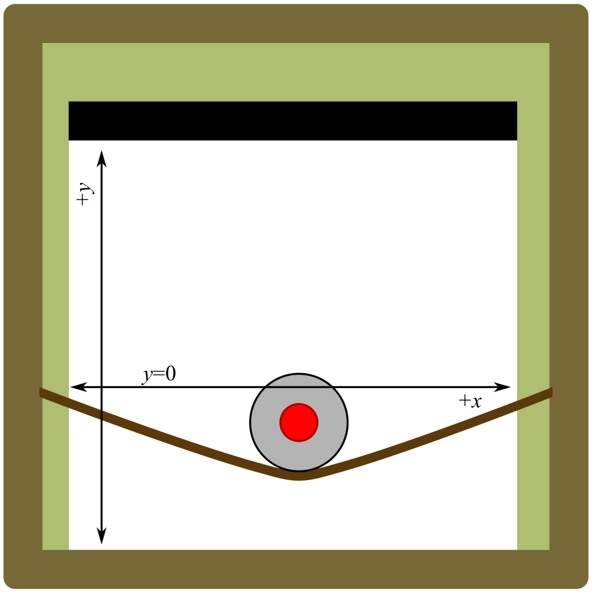


Figure 4.2: *An illustration of the preferred coordinate system for the analysis. It is convenient to separate the data into a class of points where the puck was accelerated by the elastic cord and a class where the puck was not. These coordinates separate those points by $y < 0$ and $y > 0$, respectively.*

WARNING

Do not turn the air supply on all the way. It will damage the air hose. Only turn it on a little way. The puck will float well. Do not drop the puck. The table top is glass!

Remove the record paper and turn it over. Remember that flipping the paper interchanges the left and right sides. Use a straight edge to connect the points on the left and right of the paper where the puck first contacts the cord; call this line $y = 0$. Beginning with the first discernible individual dot at the beginning of the puck's motion, circle every other dot (1, 3, 5, etc.). Measure the vertical distance from the $y = 0$ reference line to each of the circled dots in turn giving dots above the axis $+$ and those below the axis $-$. The distance we need is from the center of the line to the center of the dot when the ruler is perpendicular to the line. Estimate the uncertainties in 1) positioning the ruler's 0 at the center of the line and 2) reading the location of the dots' centers. How confident are you that the ruler is perpendicular to the line? Will this affect your uncertainty estimate? If the cord was not carefully placed horizontally, will this affect your uncertainty estimate? Your ability to draw your reference line through the centers of the dots is also relevant. How might we estimate the uncertainties in our times?

We will enter these data into the Graphical Analysis ('Ga3') computer program from Vernier. The computer will do the tedious job of calculating the velocity and acceleration. The Ga3 program has a data window with columns for data entry. Double-click the first column header and label it as 't' with units of seconds ('s'). Calculate the entries by enabling the "Generate Values" option in the dialog box. Set the first time entry to 0, the interval to $\frac{1}{30} = 0.03333$, and the end time to about 2 seconds. Enter the position measurements into the second column labeling it as 'y' with the units of your measurements: m, cm, etc. As the data is entered the computer will plot the position as a function of time. The plot should resemble the trajectory of dots on the paper.

4.4.2 Calculate and plot the instantaneous velocity

This is the same task as in the first kinematics lab. You can focus on the vertical velocity so that it is a one-dimensional problem. We can calculate the instantaneous velocity by subtracting displacements of consecutive intervals $\left(\frac{1}{30}\,\mathrm{s}\right)$. Actually, the result is the average velocity for the interval; however, such approximations are the only information available to experiments. There is always at least one instant within this interval (usually near the center) for which this average is the instantaneous velocity. Note that on the downward path consecutive positions decrease in value giving negative differences, and thus negative velocities. On the upward path consecutive positions get larger, giving positive differences and thus, positive velocities.

Helpful Tip

A common mistake is to forget that the velocity depends on the direction of the object's motion. The puck slides down and yields negative velocity values. And then the puck bounces back up and generates positive values of velocity!

In the previous lab you took differences of position and divided by the time interval for each successive pair of position points. You can instruct the computer to perform this task by choosing "Data/New Calculated Column..." from the menu. Selecting this option will bring up a window in which you can specify the parameters of a new column. Title the column 'v' in units of velocity appropriate for your measurements (cm/s. . . etc.). In the "Equation:" edit control, select "Functions/delta" and "Variables (Columns)" position (y?). Next, divide by ('/') "Functions/delta" and "Variables (Columns)" time (t?). Click "Done" and a new velocity column will be generated — actually this is average velocity for the $\frac{1}{30}$ s intervals, but this is the best we can do without calculus... Click the position graph and use the sizing handles to move the bottom up to about $\frac{1}{3}$ of the screen; we want to display the velocity and the acceleration both under the position. "Insert/Graph" from the menu, adjust its height to about $\frac{1}{3}$ of the screen, adjust its width to match the position graph, and use the dark border to position it under the position graph. If the y-axis is not already "Velocity", click the y-axis label and choose velocity (v?). Similarly make sure the x-axis is time.

Now repeat this process to generate an acceleration column by dividing "delta("v")" by "delta("t")". Don't forget your column title and units and place acceleration on the third and bottom graph. Adjust the time axis scales and graph widths and positions so that the times are vertically aligned for all three graphs.

Checkpoint

Does the velocity decrease uniformly before the puck hits the spring? Does the velocity resume a uniform rate of decrease again after the puck leaves the spring?

4.4.3 Observe your graphs

Do you see any points on any of the graphs that seem out-of-place? If so, re-measure the positions near this time to be sure you have entered all of them correctly. With the time axis of each plot aligned (all minimum and maximum times are the same for the three axes) make some observations. Note the relative changes in each plot. For instance, when the puck slides down, it will do so at some constant acceleration (as in your first kinematics experiment). When it contacts the spring it is decelerated to a stop; i.e. the acceleration is opposite to the velocity or positive in this case. Note the relative sizes of the acceleration of gravity alone versus the spring force plus gravity. Is the spring force (acceleration) constant? Is it consistent with Hooke's law ($ma = F = -ky$)? After the puck stops, the cord continues to accelerate the puck in the positive direction making the new velocities positive. The puck bounces back and loses contact with the spring. Finally, its acceleration depends only on gravity again and will again be a (negative) constant. Draw a box around these constant acceleration velocities before the spring and "Analyze/Linear Fit" to get a measured value of acceleration. If necessary right-click on the fit parameters box, "Linear Fit Options...", and enable "Show Uncertainty" so that your measured acceleration is complete with its uncertainty. (It is also possible to draw a box around the constant acceleration points and "Analyze/Statistics" to find their mean and standard deviation.) To avoid mixing up your printout with those of your classmates, enter your name in the Text Window; you might also consider entering other information about your data such as the error in your measurements of y and t. Obtain a hard copy of your table and plot for each of your lab notebooks (one for each member of your group) by using "File/Page Setup..." and "Landscape" orientation. Next "File/Print...", click "OK", and enter 2 (or 3) in the "number of copies to print" edit control.

Helpful Tip

You can copy and paste the graph and/or the table into a Word®document that you can build into your report. You can also print to the pdf printer and have an electronic copy of your figure. This may be the most useful way of preparing to submit your report for grading. In this case print only one copy.

4.4.4 Indicate where the puck was in contact with the spring

Indicate on the $y(t)$ graph, by a vertical line through all three graphs, the times at which the puck made contact with the spring ($y = 0$). Once the times when the puck first made contact with the spring and second left the spring are determined, note any worthwhile observations in your lab notebook.

Checkpoint

If you press the spring with your finger, does its reaction have the correct sign? Does the force get bigger as y gets bigger? This is the minimum necessary for the cord to be consistent with Hooke's law.

4.4.5 (Optional) Two-dimensional motion

Now that you have repeated your 1D analysis of the vertical motion with the spring, you can record the horizontal positions as well. Draw a y-axis perpendicular to the x-axis; you can utilize the 3-4-5 right triangle to do this more precisely. Now measure the x locations of the same dots and enter them into the "x" column. Once again, the dots' x coordinates are signed. You can now plot a *trajectory*, which is a plot of y vs. x. Except for scale, the trajectory should be identical to the dots on the paper.

Historical Aside

So far we have analyzed kinematics as position as a function of time. This is natural for our mathematical expressions in dynamics are functions of time. But, historically and practically, the trajectory shape (position vs. position), is often what one actually wants to know. In the case of Galileo and his contemporaries, cannon shot trajectories were of particular importance. Battleships of WWII had complex (and large) mechanical computers that performed calculations so that they could fire 2700 pound projectiles accurately toward a target 25 miles away.

4.4.6 (Optional) Hooke's Law

We wish to observe evidence of Hooke's law in our data.[1] Soon we will learn about Newton's second law of motion, the net force equals mass times acceleration ($F = ma$). When the puck is stretching the elastic band, the force obeys Hooke's law (see Equation (4.1)) so that

$$ma = F = -ky \tag{4.2}$$

and the acceleration is proportional to the position. Insert a new graph of acceleration versus position and observe the result for $y < 0$. Is the result consistent with Hooke's law and Newton's second law of motion?

4.5 Analysis

As in Experiment 1's optional exercise, you should use computer software to fit your data and extract model parameters. These parameters will come with mathematical uncertainties that you can use to compare your measured accelerations of gravity to each other and/or to other measurements. (See Section 2.9.1.)

The most important part of this experiment is to examine the three curves for $y(t)$, $v(t)$, and $a(t)$ all aligned on the same sheet of graph paper or computer plot, and to recognize the following relations predicted by our study of kinematics:

4.6 Lab Notebook Guidelines

Your grade will be based on two components: your in-class performance (including your lab notes) and your *short* written report communicating your work and its implications. Refer to Appendix E.

Your Lab Notebook should contain the following:

- A table of data collected in the lab (printed or hand written),
- Plots of the position, the velocity, and acceleration,
- One or two least-squares fit(s) for velocity,
- (Option) Graph of y vs. x depicting the trajectory,
- (Option) Graph of a vs. y illustrating Hooke's law,
- Other measurements, ideas, and observations.

[1] Thanks to Jonathan Trossman for this idea.

4.6.1 Calculus predictions for kinematics

Our study of kinematics predicts that your three graphs should have the following properties:

- The value of the puck coordinate $y(t)$
  - decreases until the bumper spring reverses the vertical velocity,
  - reaches a minimum value when the velocity is zero,
  - increases again until the end of the trajectory,
  - is minimum at the same time as acceleration is maximum.
- The velocity $v(t)$
  - increases in the negative direction at a uniform rate until after the puck comes into contact with the spring,
  - reaches maximum negative value (algebraic minimum) when acceleration is zero,
  - begins increasing (less and less negative and finally positive) after the minimum,
  - increases further becoming more and more positive, until just before the puck loses contact with the spring,
  - from there on the velocity decreases again (at a uniform rate only after the puck leaves the spring).
- The acceleration $a(t)$
  - is constant from the moment the puck is released until it comes in contact with the spring,
  - increases as the puck stretches the spring,
  - goes to zero as the velocity becomes minimum in value,
  - becomes positive and much larger than the acceleration due to gravity alone,
  - is maximum at the same time $y(t)$ is minimum,
  - goes to zero again as the velocity becomes maximum in value,
  - becomes constant again after the puck leaves the spring,
- Generally,
  - When the puck hits the bumper spring, – its trajectory $y(t)$ starts to flatten out, – its instantaneous velocity curve $v(t)$ does not immediately stop decreasing, and – its acceleration curve $a(t)$ does not immediately change sign.
  - When the puck is at the maximum compression point of the bumper spring, – its trajectory has reached a minimum, – its velocity goes through zero, and – its acceleration is at a maximum.

Draw conclusions about our kinematic relations from these observations and note these observations in your discussion.

4.6.2 Thoughts on the written report

Refer to Appendix E. Some topics for discussion include:

- What physical relations have your data tested?
- Is Hooke's law ($F = -ky$) feasible?
- How about Newton's second law ($F = ma$)? Is acceleration greatest when applied force is greatest?
- What do your data say about calculus' prediction that a function is extreme (maximum or minimum) when its derivative is zero?
- Recall that position's derivative is velocity and that velocity's derivative is acceleration. Both pairs of curves are predicted to have this behavior.
- Did you make any measurements worthy of reporting in your Conclusions?
- Is there an advantage to having the trajectory as a graph of y vs. x in addition to the dots on the record paper?
- Do you have a prediction for what a graph of x vs. t might look like?

Each of these topics should be addressed in 2-3 sections, but the particular sections vary with the topics. Refer to Appendix E.

Chapter 5

Experiment 3: Newton's Second Law

The relationship between force and motion was first addressed by Aristotle (384 - 332 B.C.). He argued that the natural state of an object was to be at rest, and a force was not only required to put an object into motion, but a continued force was required to keep the body in motion. This may at first seem to correspond well with our everyday experiences, but it is certainly not what is taught in physics.

Galileo Galilei (1564 -1642), in addition to his postulates on uniform gravitational acceleration, proposed that a body at rest is a special case of a more general state of constant motion (i.e. constant velocity). He understood that without friction acting on a body to slow it down, it might indeed continue to move in a straight line forever. Galileo proposed that bodies remain at rest or in a state of constant motion if no force acts to change this motion. Friction is just another example of a force.

Isaac Newton (1642 -1727) formalized the relationship between force and motion in his *Principia* (published in 1687). Newton proposed that the acceleration of an object is directly proportional to the net force acting on an object and inversely proportional to the mass of the object. The Law is summarized in the vector formula $\mathbf{F} = m\mathbf{a}$. In this laboratory, we will verify this relationship quantitatively. This law describes our understanding of the *dynamics* of classical mechanics.

5.1 Background: Forces, Energy, and Work

The concepts of work and energy can be derived from Newton's Second Law in mechanics. The details of these quantities will be covered in detail in lecture and in later lab exercises. For the purposes of the laboratory, basic relevant definitions are given here.

In the case of a constant force, the physical work done by this force is defined simply. If a position changes by a displacement Δx under a constant force F_x along that direction, then the *work done by the force* is

$$W = F_x \Delta x = F \Delta x \cos\theta \tag{5.1}$$

where θ is the angle between the direction of the force and the direction of displacement. This can be succinctly written as a vector dot product:

$$W = \mathbf{F} \cdot \Delta\mathbf{x} \tag{5.2}$$

Kinematics equations for uniform acceleration can be manipulated to obtain a useful relationship between physical work and the change in kinetic energy known as the **Work-Energy Theorem**:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \Delta KE = W = \mathbf{F} \cdot \Delta\mathbf{x} \tag{5.3}$$

The Work-Energy Theorem is only valid when W is the total work done by all forces acting on an object. Since all forces can change kinetic energy, it is important to be able to know all of the forces acting on the object under study. In particular, we often would like to eliminate friction as a relevant force since its magnitude can rarely be directly measured (often resulting in energy lost from the system, according to the Work-Energy Theorem). This may not be entirely possible, and so you should be aware that this may influence your results despite the considerable expense and effort that has been made to minimize friction in constructing the laboratory equipment.

Historical Aside

One might take pause to appreciate the fact that Galileo and Newton were able to discover the principles of mechanics (forces, energy, etc.) without the benefit of technically advanced equipment. Galileo rolled spheres and cylinders down inclines and dropped objects from the Tower of Pisa. Newton extended Galileo's observations to the motion of planets and moons of our solar system. Yet, they were able mentally to extract the kernel of truth from such an environment and to recognize the universality of the laws of motion.

5.2 Apparatus

We will be using an air track for this experiment. It consists of a hollow extruded aluminum beam with small holes drilled into the upper surface. Compressed air is pumped into the beam and released through the holes. This forms a cushion of air that supports a glider on a nearly frictionless surface.

Helpful Tip

Do not move the air track. It is leveled and difficult to readjust.

Attached to the air track is a sonic motion sensor. The computer signals the range finder to emit a sound pulse. The pulse reflects off the plastic card attached to the glider and returns an echo to the motion sensor. The computer receives the signal and calculates the position of the glider from the time delay between sending the pulse and receiving the echo and the known speed of sound waves in air. Computer software plots the data and can use the data to calculate velocity and acceleration. The setup is shown in Figure 5.1.

Helpful Tip

Do not disturb the rangefinder. Your data depends upon its proper alignment.

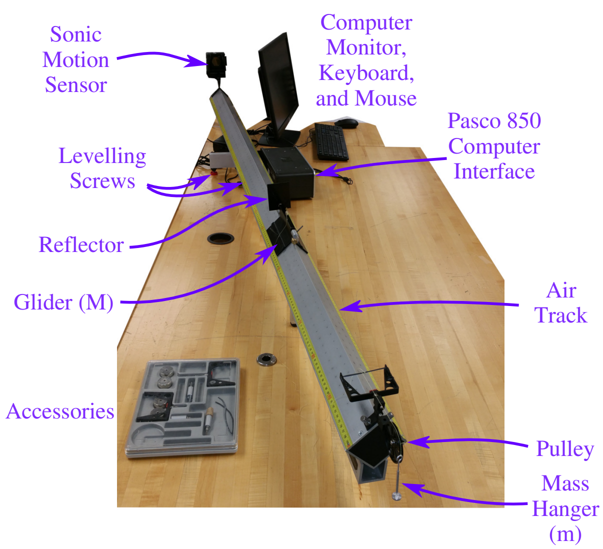


Figure 5.1: *Photograph of the air track, glider, weights, and motion sensor used to examine Newton's second law of motion.*

The computer is actually doing the same measurement you did in the first laboratory when you determined the positions of the air puck by tediously measuring the distance from a reference line to each of the dots laid down by the spark timer. The computer's fast speed enables it to process more position data while you concentrate on the physics involved rather than the calculations. The computer is also doing the same calculations as you did when you found average velocity from displacements and time intervals.

The glider has a string attached to it which runs over a pulley at the end of the track opposite the range finder. At the other end of the string is a weight holder. Weights can be added to vary the accelerating force on the glider. The vertical force of gravity acting on the weights is transferred via the pulley to a horizontal tension applied to the glider. If friction and a few other small forces can be neglected, only this gravitational force accelerates the glider and the hanging masses at the same rate. The string's length maintains a constant distance between the two so the time derivatives must also be the same. You can draw the force diagrams and solve Newton's equations for the expected acceleration before coming to lab for a better understanding of this.

Helpful Tip

Be sure that the string touches only the glider, the round pulley, and the weight hanger; otherwise, the string will experience a large frictional force that is not included in your data analysis.

We are using Pasco's Capstone program with their 850 Universal (computer) Interface. We have already prepared Capstone to gather your data and we have saved the setup for you to load. Open the "Newtons Law.cap" file from the lab's website:

'http://groups.physics.northwestern.edu/lab/newtons-law.html'

Before proceeding we must verify that the track is level. Each day the tracks are pre-adjusted by the Laboratory Assistant. This is a delicate adjustment and should only rarely need to be done if the track is not moved around on the table and the table is not moved around on the floor. To test the level, momentarily detach the string and weight holder from the glider. With the glider near the center of the range of motion on the track, turn on the air supply, and verify that the glider remains (mostly) at rest when free to move along the track. Be careful that slight gusts of air from other sources are not affecting the motion of the glider. Be careful not to bump the air track or the table. If the air track appears to be out of level (noticeable and sustained acceleration), let the Teaching Assistant know before attempting any adjustments. With the permission of the Teaching Assistant you may adjust the level screws on the legs of the air track to bring the track to level.

Helpful Tip

Random or back-and-forth motion is not an indication of being unlevel; only continuous acceleration indicates that the track needs leveling. Even a mild breath will move the glider!

WARNING

Small adjustments make a *big* difference; since friction is so low, even a *tiny* component of $\mathbf{g}$ along the track will cause acceleration.

5.3 Procedure

We will hang masses under their gravitational weights to provide known external forces. This force will accelerate the air track cart. Table 5.1 shows the measured masses of these weights.

5.3.1 Acceleration vs. Unbalanced External Force

In this experiment we will measure the acceleration of the glider under conditions of varying accelerating force provided by varying the weights hanging from the pulley. Choose an initial weight for the weight holder at the end of the string. The masses of the various weights is shown in Table 5.1. You will want to use total mass combinations in the range from 2 g to 22 g. Start with an initial mass of 2.0 g or 4.0 g. Make repeated runs with at least 5 different mass combinations up to 22 g.

Table 5.1: *Measured masses of hanging weights and weight holder.*

Description	Mass (g)
Holder	1.984 ± 0.035
Small Black	0.961 ± 0.019
Large Black	1.9430 ± 0.0087
Small Silver	4.926 ± 0.039
Large Silver	9.941 ± 0.045

For each run start by moving the glider away from the pulley until the weight hanger is near the pulley; hold it there. Placing a finger in contact with the air track and glider simultaneously provides enough friction to hold the glider. Be sure the software is at the point where the velocity graph is visible. To begin taking data, click the "Record" button at the bottom left and release the glider. Be sure not to obstruct the path the range finder's sound wave needs to travel or reflections from you, the video monitor, etc. will confuse your data. If this happens merely delete the data run and retake the data. If your data is noisy, check that the reflector above the glider is perpendicular to the track and that the visual image reflected from the Motion Sensor verifies that it is pointed correctly. Ask your instructor for assistance.

When the glider hits the end of the track or the hanger hits the floor, click the "Stop" button. (The "Record" button turns into the "Stop" button when pressed and vice versa.) You should now see a plot of the glider's position and velocity displayed as a function of time. You can choose to retake the data by deleting the data (button at the bottom right) and repeating or just leave the bad results and take new data over the old. If everything seems okay, proceed to analyze the data run. Let the mouse cursor hover on the velocity plot so that the toolbar appears above the velocity vs. time graph. Click the 'Data Selection Tool' () on the toolbar and size it to highlight the appropriate section of data points. Select the linear fit option from the curve fitting tool () and enable the tool. Optionally, from the position graph choose the quadratic fit after selecting the appropriate data points. A square will appear with the pertinent fit results. If necessary, right-click the parameters box and enable the "Show Uncertainties" option.

Checkpoint

What uncertainties should you record for your hanger's and weights' masses?

Note the mass of the hanging weight $(m \pm \delta m)$ g and the corresponding fit parameter $(a \pm \delta a)\,\mathrm{m/s^2}$ in a nice table for later use. Now change the hanging mass by adding and/or removing weights on the hanger and repeat the experiment. Repeat the experiment at least five times with at least five different hanging weights.

Since we want to check whether $F = Ma$, run Vernier Software's Graphical Analysis 3.4 (Ga3) program. We have already prepared "Newtons Law.ga3" and stored it on the website as well

'http://groups.physics.northwestern.edu/lab/newtons-law.html'

You will need to right-click the link from the FireFox browser and to save the template (Downloads, Documents, or Box Sync/Physlabs would be good places). Next, click the blue arrow at the top right and select the "Newtons Law.ga3" file. Do your data points seem linear? If so, then it is reasonable to fit them to a line.

Checkpoint

What does Newton's law $F = Ma$ predict for F versus a for this experiment? What do we expect for the line's slope and y-intercept?

As you enter "Hanging Mass" values in the designated units, the computer enters the calculated force in the calculated column. You should check one or two of these values to make sure you entered the correct number and that you understand how the force was calculated. You can double-click the "Force" column heading to see how the title, units, and formula are entered; you can also see the formula that the computer uses.

Finally, we need to plot F vs. a instead of m vs. a. If necessary, click the "Hanging Mass" on the vertical axis and select "Force" instead. Now your Ga3 file is the same as ours for Part 1; feel free to save yours frequently to avoid losing your work.

Now let us try to fit our data points to the $F = Ma$ model suggested by Newton. If your mass column is not sorted numerically, choose "Data/Sort Data Set/Force vs. Acceleration" from the menu, sort using any column. Select the data points you want to analyze by drawing a box around them with your mouse, choose "Analyze/Curve Fit/Proportional" from the menu. Click "Try Fit" and verify that the black model line passes through your data points; click "OK." The computer will choose values for the proportionality constant (A) to minimize the vertical distances between your data points and the straight line given by $y = Ax$. The process the computer uses is called the "Least Squares Fit."

Historical Aside

A Linear Regression is a particular least squares fit that can be solved exactly; this yields a set of equations which determine slope and intercept ($y = Mx + B$) of a straight line that best represents the set of (x, y) values. These equations were derived by using calculus to find the minimum of the sum of the squares of the deviations of y value of the line at each x-value of data from the corresponding y-value of data. These equations have been written into Ga3 and are used for the 'Data/Linear Fit' option. The solution is available on the web.

Record the slope of the best fit line, its uncertainty, and its units in your notebook. We can represent experimental uncertainties in either of two equivalent ways. For example, Newton's second law predicts our glider's mass to be $M = (0.210 \pm 0.006)\,\mathrm{kg}$. We can also express the uncertainty as a percent (%) of the measurement $M = 0.210\,\mathrm{kg} \pm 3\%$. If Ga3 does not show the uncertainties in slope and intercept, right-click on the parameters box, "Fit Options...", and select the "Show Uncertainties" option. You should write the values you measured in your lab notebook. You should weigh your glider and record its mass, uncertainty, and units in your lab notebook. The value we get using Newton's second law should be close to this value.

Helpful Tip

The scale can measure the gliders' masses to four significant figures; however, it can measure the 1 g, 2 g, 5 g, and 10 g masses to little more than one significant figure. You can improve this by measuring them in groups of ten and dividing the result by 10, but a far better strategy is to use the values in Table 5.1 that were obtained with a more sensitive (and more fragile) scale.

5.3.2 Acceleration Proportional to Inverse of the Mass

In this experiment we will measure the acceleration of the glider with a fixed hanging weight on the weight holder at the end of the string while varying the mass of the glider. The initial mass of the glider is about 0.20 kg. You can check your glider's mass with the electronic scale in the lab. Don't forget to record your units. The mass of the glider can be changed by adding weights to the thin rods extending from each side and the top of the glider. The cylindrical weights which fit over these rods each have a mass of $(50.00 \pm 0.01)\,\mathrm{g}$. A maximum of four weights can be added giving you five possible mass values between 0.2000 kg and 0.4000 kg. Keep the glider balanced across the beam by adding equal weights to each side; odd weights can be added to the top.

Checkpoint

What value of uncertainty should you record for the scale's reading?

Obtain values for the acceleration of the glider for each of five values of glider masses using the same procedure followed in Section 5.3.1. Use a mass of 10 g on the 2.0 g weight holder for a total of $(11.925 \pm 0.057)\,\mathrm{g}$. This will produce an accelerating force of

$$mg = (0.011925\,\mathrm{kg})(9.807\,\mathrm{m/s^2}) = (0.11695 \pm 0.00056)\,\mathrm{N}.$$

You can use that data point from the first experiment, and save yourself some time. Make a table of the results. Plot the acceleration as a function of the mass of the glider.

Checkpoint

Do you get a linear relationship? If not, what does the plot suggest?

Helpful Tip

Determine a way to obtain a linear plot by rearranging Newton's second law.

You should use the Least Squares Fitting Program (Ga3) again to find the slope of the line, the uncertainty in the slope, and the units of both. "Newtons Law.ga3" is already setup to process this data also; just select "Page 2" instead of "Page 1" from the toolbar. Alternatively, you can adapt your previous setup by renaming the columns, changing the "Equation" formula, and selecting the correct columns for your graph.

Checkpoint

What measurement corresponds to the slope of *this* graph?

5.3.3 (optional) Test of the Work-Energy Theorem

In this experiment we will measure the position and velocity of the glider at two separate points and compare the change in kinetic energy with the work done by the force of the string on the glider.

You may rerun the experiment for a specific set of accelerating weights and glider mass or you can use the data from the last run. Whatever the source of data, be sure that you measure the position (x) and the velocity (v) at the same times. If you move the mouse cursor to one of the graphs, you will find that a toolbar will appear above the graph. On the toolbar will be a button to "Show Coordinates and access Delta Tool" ( ). When you push the button, a pair of dotted (x, y) axes will appear at the top left of the graph. Grab the square at the 'origin' with the mouse and drag it to your data points. The coordinates of the data point, (t, x) or (t, v), will be displayed in a box. After you drop the origin, you can grab this text box and move it around with your mouse if you choose; it might be necessary to uncover the origin so that you can move the 'origin' to the next point you wish to study.

Select two points, one near the start of the motion of the glider and one where the glider is near the end of the track. Be sure to choose points on the constant acceleration section of the graph (and be sure the time for velocity is the same as the time for position.) Record the readings of x-position and corresponding velocity (v) of the glider for these two points.

Using the velocities calculate the change of kinetic energy experienced by the glider between these two points. Find the change in position by taking the difference between the two position measurements. Use this with the value of the accelerating force (the mass of the weight holder and weights multiplied by g) to calculate the work done on the glider by the tension of the string pulling it.

Compare the two numbers and decide if the Work-Energy Theorem has been verified. If there is a significant discrepancy, can it be explained? Good predictions (i.e. small Difference) indicates that Newton's law works well for this purpose whereas large differences might indicate that the law has a problem, that our data has a problem, or that our assumptions are not realized by our experiment. Check your calculations if your agreement is very bad but note that we have not considered our measurement uncertainties and, thus, have no quantitative expectation for agreement.

5.4 Analysis

Using the values you recorded in Section 5.3.1, compute the difference (ΔM) between your measurement of glider mass (using the mass scale) and that predicted by Newton's second law from the slope of your graph. When using the mass scale, be sure to weigh all of the mass that was being accelerated by the various forces. Refer to Section 2.9.1.

Using the values you recorded in Section 5.3.2, compute the difference (ΔF) between your measurement of applied force (from the constant hanging mass) and that predicted by Newton's second law from the slope of your graph.

(optional) Using the values you recorded in Section 5.3.3, compute the difference (ΔE) between the change in kinetic energy and the work done by gravity.

Are we confident that the air track was exactly level? Did we eliminate all friction? If not, where are places we might have missed? Is the glider's mass the *only* mass accelerated by the hanging weight? If not, can you think of a way to estimate a compensation? Did we include the uncertainties in our mass measurements in our σ? Would these uncertainties increase our σ and make our Δ a smaller multiple of σ? What total mass was moving and thus contributing to kinetic energy? What other sources of experimental error have we excluded (by choice or by accident) from our analysis?

5.5 Guidelines

Your grade will be based on two components: your in-class performance (including your lab notes) and your brief written report communicating your work and its implications.

5.5.1 Notebook

Your Lab Notebook should contain the following:

- Two tables of data collected in the lab (printed or hand written).

- A figure (graph) showing the acceleration of the cart with a fixed weight of the cart versus the various mass hanging from the string.
- A figure (graph) showing the acceleration of the cart versus the reciprocal cart mass when the hanging weight was not changed.
- (optional) A calculation and comparison of the work and change in kinetic energy.

5.5.2 Report

Your written report should address the following physics:

- Does your data support Newton's Second Law of Motion?
- (optional) Does your data support the Work-Energy Theorem?
- Since the Work-Energy Theorem follows directly from Newton's Second Law, what does your answer to the second part imply about the first part?
- "Yes" and "No" are *terrible* answers to these questions. Use these questions to guide your discussions and report structure.
- Don't forget to label your figures and tables (including units); don't forget to discuss each figure and each table in your text.
- Your report's 'Analysis', 'Discussion of Results', etc., should closely follow the Analysis in your notebook as described Section 5.4. See Appendix E.
- Your report should always summarize the physics that your data supports or contradicts and all physical constants that you have measured in your 'Conclusions'. Any suggestions for improvements to the experiment and/or applications of what you observed or used are also welcome.

Chapter 6

Experiment 4: Conservation of Energy

For this experiment, we will further our understanding of energy and work in Newton's laws of mechanics. We pick up from Equation (5.3). The derivations of the Work-Energy theorem there are still valid in this discussion.

The Work-Energy Theorem presents a way of dealing with kinematic quantities in mechanics without regard for vector direction. These directionless quantities, such as kinetic energy, are called *scalars*.

Historical Aside

It turns out that scalar quantities played an important role historically in the development of classical and analytic mechanics by early luminaries such as Joseph-Louis Lagrange (1736-1813) and William Rowan Hamilton (1805-1865). Scalar quantities are often much easier to work with than vectors, and the concepts of mechanics using scalars implied by the Work-Energy theorem translate naturally to quantum mechanics even when the vector approach of Newton's Laws does not. If this short aside tantalizes you, consider becoming a physics major and learning more in the advanced physics courses!

All forces are vectors and all forces do work when applied to a moving object; however, the work done by some forces is independent of the path the object takes in its motion. The force of gravity is one such force. The work done by the force of gravity moving an object from point A to point B separated by a displacement $\Delta\mathbf{r}$ is

$$W_g = m\mathbf{g} \cdot \Delta\mathbf{r} = mg\,|\Delta\mathbf{r}|\cos\theta = -mg\Delta y \tag{6.1}$$

where θ is the angle between the displacement vector and the gravitational force (straight down). But $|\Delta\mathbf{r}|\cos\theta$ is just the difference in height, $-\Delta y$. This gives a positive value for downward motion, $W_g = -mg\Delta y = mgy_\mathrm{i} - mgy_\mathrm{f}$. (Having the y-axis point upward opposite $m\mathbf{g}$ means that $y_\mathrm{f} < y_\mathrm{i}$ for downward motion.) Even if we travel around the world, this *conservative* force does negative work every time we go up, positive work every time we go down, and zero work when we go sidewise such that the total is just $W_g = -mg\Delta y$.

If we make the approximation that the force of gravity is everywhere uniform for a given mass, m, and choose the y-direction to be up, then the work done against the force of gravity in elevating the mass above the ground is recoverable by letting the mass return to the ground. We think of the elevated mass as having potential to do work or "potential energy." To cement this idea further, we take the work done by the force of gravity and move it to the energy side of the work-energy theorem

$$mg(y_\mathrm{f} - y_\mathrm{i}) + \frac{1}{2}mv_\mathrm{f}^2 - \frac{1}{2}mv_\mathrm{i}^2 = W = \mathbf{F} \cdot \Delta\mathbf{x} \tag{6.2}$$

The last term on the right side of this equation still represents all external forces *except gravity* acting on the mass m. One can regroup the terms on the left to recognize a total energy consisting of potential and kinetic energy.

$$mgy_\mathrm{f} + \frac{1}{2}mv_\mathrm{f}^2 - \left(mgy_\mathrm{i} + \frac{1}{2}mv_\mathrm{i}^2\right) = E_\mathrm{f} - E_\mathrm{i} = W = \mathbf{F} \cdot \Delta\mathbf{x} \tag{6.3}$$

General Information

In this form we see that the change in *total* energy and not just kinetic energy equals the work that we have not already considered using potential energy functions.

The **Principle of Conservation of Energy** is expressed as

$$mgy_\mathrm{f} + \frac{1}{2}mv_\mathrm{f}^2 = \left(mgy_\mathrm{i} + \frac{1}{2}mv_\mathrm{i}^2\right) + \mathbf{F} \cdot \Delta\mathbf{x} \tag{6.4}$$

or

$$E_\mathrm{f} = E_\mathrm{i} + W; \tag{6.5}$$

the final energy is the initial energy plus the energy we add as work. (We can also "add" negative work to remove energy.) Forces that have a potential energy function are known as conservative forces. Every conservative force has a potential energy function whose final value is added on the left and whose initial value is added on the right.

If *only* conservative forces act on a set of objects, then the total amount of kinetic and potential energy is a constant of the motion. A loss of kinetic energy must be accompanied by an equivalent gain of potential energy and vise-versa. Since we are free to choose the initial and final points on our trajectory, the Principle of Conservation of Energy applies to every point on the trajectory or, equivalently, to all instants of time.

A force which can be treated in terms of potential energy is one in which the work done by the force depends only on the starting and ending points of the path along which the mass is moved and not on the specific trajectory or path along which the mass travels. Another feature of forces having potential energy functions is that the total work done by such a force around any closed path, where the starting point is the same as the ending point, is zero; everything is as if the mass had not moved at all.

Friction is *not* a conservative force. When friction does work, that energy loss cannot be recovered by returning to the original position. On the other hand, the force of a spring is a conservative force. In general, the work done by a conservative force is the same as the energy lost by the potential energy function, $W = -\Delta PE$. The potential energy function of a spring having spring constant, k, and compressed by x is

$$PE_S = \frac{1}{2}kx^2 \tag{6.6}$$

In summary, there are three forms of mechanical energy to consider in the motion of the glider in this experiment:

$$KE = \frac{1}{2}mv^2 \tag{6.7}$$

$$PE_G = mgy \tag{6.8}$$

$$PE_S = \frac{1}{2}kx^2 \tag{6.9}$$

If there are no other external forces acting on the system and doing work, the sum of these three forms of mechanical energy is conserved.

General Information

Friction is a non-conservative force, so the energy it removes from our system of objects cannot be returned to the kinetic energy of the objects' motions. However, we believe that even this energy is still present somewhere in the universe. In the particular case of friction, no macroscopic object can be perfectly smooth; as the two microscopically rough surfaces bounce on each other, their atoms and molecules start to shake internally like a baby playing with the mobile above his crib. We measure this internal shaking as the objects' temperatures. The energy lost to friction becomes thermal energy inside the two objects, $\Delta E = mC\Delta T$.

6.1 Background – Hooke's Law

We introduced Hooke's law earlier in Equation (4.1). A direct result of this conservative spring force is the spring's potential energy function in Equation (6.9). To calculate the potential energy of the spring one also needs to know the spring constant k, which is a measure of the stiffness of the spring. Hooke's force law governs the force versus compression

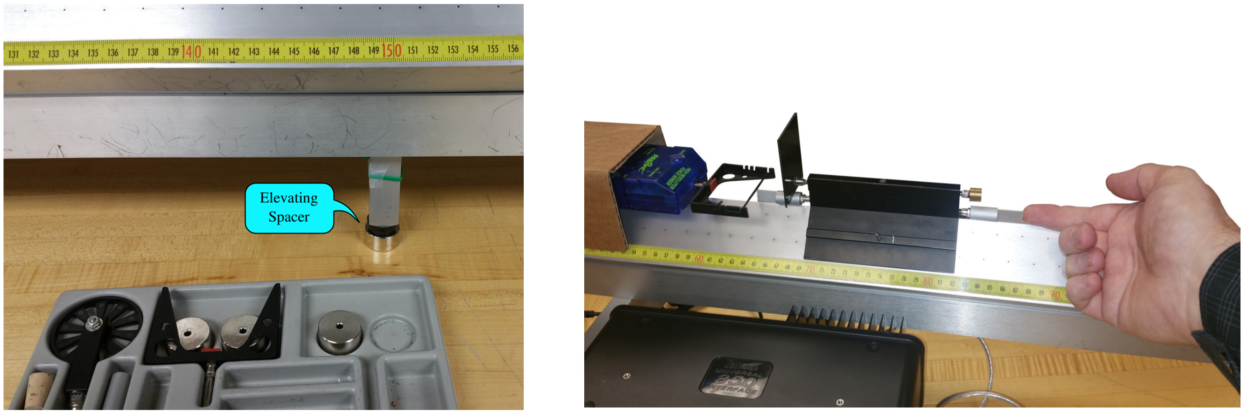


Figure 6.1: *Before measuring the spring constant, remove the elevating spacer so the track is level. Execute the Hooke's Law Capstone setup and slowly press the cart against the elastic rubber bumper. Slowly release the spring and stop Capstone. The motion sensor measures the cart position (x) and the force sensor measures the spring force F_s.*

of springs

$$F_s = -kx.$$

Ideally, this equation applies to *our experiment*. We will use Pasco's force sensor to measure F_s and simultaneously we will use their sonic motion sensor to measure x.

6.2 Apparatus

The apparatus shown in Figure 6.2 is very similar to what we used in the last lab. It consists of a hollow extruded aluminum beam with small holes drilled into the upper surface. Compressed air is pumped into the beam and is released under pressure through the holes. This forms a cushion of air between the beam and a glider and allows the glider to move along the beam with almost no friction. The glider literally floats on the air cushion.

WARNING

Do not move the air track. It is leveled and difficult to readjust.

Attached to the air track is a sonic motion sensor which is controlled by the computer. The computer signals the motion sensor to emit a sound pulse. The pulse travels through the air at the speed of sound, reflects off the plastic card attached to the glider, and returns an echo to the motion sensor. The computer receives the signal and calculates the position of the glider from the time delay between sending the pulse and receiving the echo and the speed of sound in air.

WARNING

Do not touch the rangefinder. Good data requires that it point directly at the cart. Avoid obstructing the path to the glider; sound will reflect from your hand, your notebook, your video monitor, etc.

We are using Pasco's 850 Universal (computer) Interface and their Capstone control software to plot the data and to calculate velocity. Double-click on the Capstone icon on the desktop. We have already prepared Capstone for use with this experiment and published it on the lab's website as 'Conservation Energy.cap'

'groups.physics.northwestern.edu/lab/';

open this file now. Now we are ready to take data.

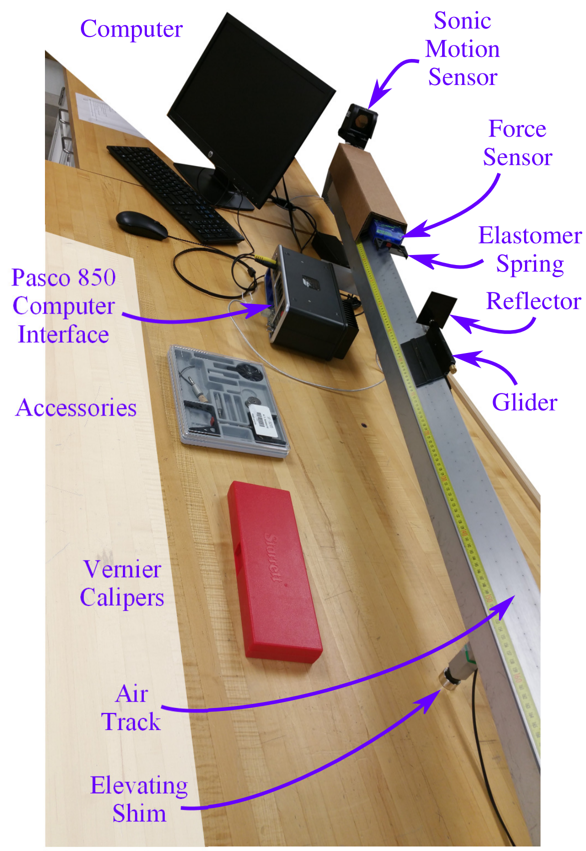


Figure 6.2: *Photograph of the apparatus ready for energy conservation. Note the shim used to elevate one end of the air track through an accurately measurable angle from a leveled air track.*

6.3 Procedure

6.3.1 Measuring the angle

Before proceeding we must set and determine the angle of the air track's incline. This is made simple by the fact that the feet of the air track are spaced 1.0000 m apart. Remove the shim that has been placed under the single foot at one end and measure the shim's thickness, t. This is the amount the shim will elevate the single air track foot. Before replacing the shim, verify that the air track is level by turning on the air pressure and seeing that the glider does not accelerate. If necessary, adjust the leveling screws by equal amounts to keep the beam balanced and level the track; **small adjustments make a *big* difference.** As you replace the shim under the foot, note that the entire air track rotates about the axis defined by the two places where the leveling adjustments contact the table. Without the shim the air track is horizontal and to insert the shim we must first rotate the air track about the pivot by the angle θ. (See Figure 6.3.) We form a right triangle with opposite side having the shim's thickness and hypotenuse defined by the bottoms of the air track's feet. We can find the

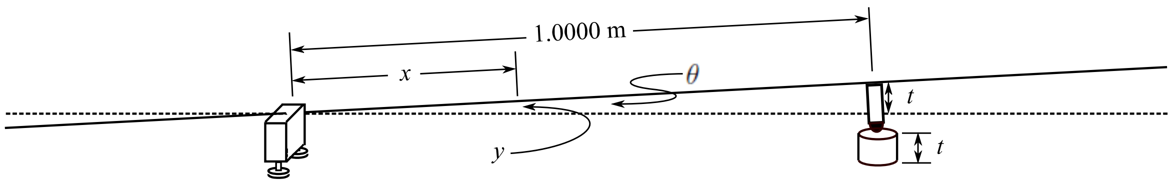


Figure 6.3: *A sketch of how to use the geometry to convert glider position into height for the tilted air track.*

angle of inclination, θ, using

$$\sin\theta = \frac{t}{1.0000 \text{ m}} \tag{6.10}$$

We will find it convenient to define the x-axis to coincide with the air track and the origin to be when the glider just barely touches the rubber spring. We will also define the glider's height at this point to be the gravitational potential energy zero. When the glider is at position x on our coordinate system, then, x is the hypotenuse of a similar right triangle having opposite side y above our gravitational potential energy zero. In this coordinate system, $\hat{\mathbf{y}}$ is *not* perpendicular to $\hat{\mathbf{x}}$. The height of the glider above its height at the origin is given by trigonometry to be

$$y = x\sin\theta = \frac{xt}{1.0000 \text{ m}} \tag{6.11}$$

Capstone reports x in m, so the units of y are the same as the units of t. What should you report for the uncertainties in x? Using these coordinates, it is easy to represent the gravitational potential energy function as

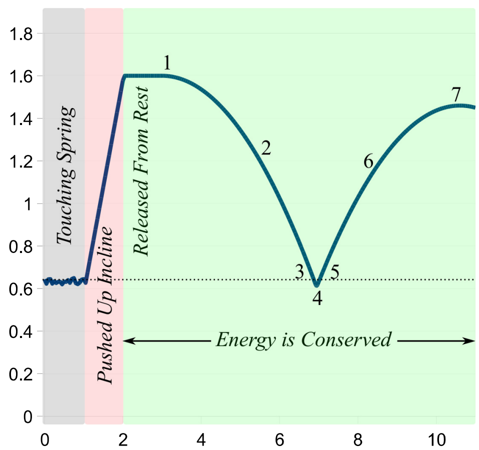
$$PE_g(x) = mgy(x) = \frac{mgxt}{1 \text{ m}} \tag{6.12}$$

If t is expressed in m, m is expressed in kg, and $g = 9.807\,\text{m/s}^2$, then PE_g will have units of Joules (J=N·m=kg·m^2/s^2).

6.3.2 Determining the spring constant

Before we can compute the energy stored in the stretched spring, we need to determine the spring constant k. First, turn on the air pump and remove the elevating shim from beneath the air track foot. (See Figure 6.1.) Now download the 'Hooke's Law' Capstone setup file from the lab's website and allow Capstone to execute it. With nothing touching the spring, press the 'Zero' button on the force sensor to calibrate its zero. Hold the glider cart as shown in Figure 6.1, press 'start' at the bottom left of the display, gradually push the cart into the spring harder and harder, gradually release the force, and press 'stop'.

Does the spring force seem proportional to the position? The specific character of this



N	PE_g (J)	PE_s (J)	KE (J)	E_T (J)
1	_	0	0	_
2	_	0	_	_
3	0	0	_	_
4	_	_	0	_
5	0	0	_	_
6	_	0	_	_
7	_	0	0	_

Figure 6.4: *Simulated data showing points and areas of interest. A suggested data table to analyze the energy at several instants of time.*

graph is suitable for Analysis discussion. To find the spring constant, fit the data to a line and note that the slope is the spring constant. You can copy the data into Ga3 to perform this analysis or you can analyze it in Capstone. What are the units of your spring constant and uncertainty?

6.3.3 Conservation of Energy

In this experiment we will measure the various energies of the glider as it moves along the air track changing height along the way and sum all of the energies to arrive at a total energy. In the end, we will see whether the total energy remains the same. For each run start by touching the glider to the bumper spring at the lower end of the track and holding it there. DO *NOT* COMPRESS THE SPRING; simply touch it. You can hold the glider steady by touching a finger simultaneously to the air track and the glider's bottom. To begin taking data, click the "Record" button at the bottom left and wait a short moment with the glider touching the bumper; this will identify x_0 to be this glider location. When $x > x_0$ the spring is not compressed and when $x < x_0$ the spring is compressed by $x_0 - x$. After a brief moment when you see that the program has marked $x = x_0$ clearly, move the glider quickly to a spot about half-way up the track and release the glider *from rest*. The glider will accelerate down the slope and bounce off the bumper.

Be sure to move out of the way of the motion sensor. Also be sure the computer's video monitor and other items in the lab are at least a foot away from the sound's path. Otherwise, the motion sensor might mistake echoes off these items as signals from the glider and yield garbage for some of your data. If this happens merely retake the data being more careful. Continue taking data as the glider bounces off the lower bumper and rebounds back up the track. When the glider comes to rest momentarily and starts back down you can click the "Stop" button using the mouse. The "Record" button turns into the "Stop" button (and vice versa) when it is clicked.

You should see a plot of the position of the glider as shown in Figure 6.4 and the velocity displayed as a function of time. You can choose to retake the data by deleting the data using the button at the bottom right of the screen and by repeating or just by leaving the bad results and by taking new data over the old. If everything seems okay, proceed to the analysis.

First, we need to determine the motion sensor reading at x_0. Move the mouse cursor to the *position* graph and hover there. A toolbar will appear above the graph containing an icon for selecting data points for analysis ( ). Click this selector button to bring up a pale area surrounded by eight sizing squares. You can drag the selected area with the mouse and you can change its shape by dragging one of the sizing squares. Shape and position the selected area so that only the data having the x_0 position at the first of the run are selected. Compute the statistics for this data using the "Σ" button on the toolbar ( ). If the mean ($\bar{x}_0$), the std. dev. (s_{x_0}), and the number of data points (N) are not all displayed, click the little down triangle to the right of the "Σ" button and check all that are absent. You may optionally un-check those statistics that we do not need. If necessary, review Section 2.6.1 to learn how to report your measurement.

6.3.4 (optional) Measure the Impulse

As an option, we can measure the impulse of the glider colliding with the spring and then we can compare this to the change in the glider's momentum. For this purpose, execute the "Impulse.cap" template and release the cart from the top of the incline. Don't forget to press the 'Zero' button on the force sensor first. Copy the force versus time data from the table and paste it into Ga3. Draw a box around the pulse and 'Analyze/Integral' (or press the 'Integral' toolbar button) to find the area under the force pulse

$$\mathbf{I} = \int \mathbf{F}\ \mathrm{d}t. \tag{6.13}$$

Next week we will learn that this impulse integral equals the cart's change in momentum

$$\mathbf{I} = \mathbf{p}_\mathrm{f} - \mathbf{p}_\mathrm{i} = M\Big(\mathbf{v}_\mathrm{f} - \mathbf{v}_\mathrm{i}\Big). \tag{6.14}$$

Don't forget to include the signs of the velocities when computing this difference.

6.4 Analysis

OPTION 1: If you move the mouse cursor to one of the graphs, you will find that a toolbar will appear above the graph. On the toolbar will be a button to "Show Coordinates and access Delta Tool" ( ). When you push the button, a pair of dotted (x, y) axes will appear at the top left of the graph. Grab the square at the 'origin' with the mouse and drag it to your data points. The coordinates of the data point, (t, x) or (t, v), will be displayed in a box. After you drop the origin, you can grab this text box and move it around with your mouse if you choose; it might be necessary to uncover the origin so that you can move the origin to the next point you wish to study.

Figure 6.4 also shows specific times along the position plot numbered 1, 2,. . . ,7. Note: at the points designated with number 1 and 7, the glider is at the top of its trajectory where the velocity is zero. The glider will have no kinetic energy and no spring potential energy at these points; all of the energy is gravitational potential energy. For data points number 2 - 3 and 5 - 6, the glider will have both kinetic and potential energy but still no spring potential energy. At the point 5, the glider will have no kinetic energy and almost no gravitational potential energy, only spring potential energy.

Find convenient points near the suggested number positions in Figure 6.4 to obtain position and velocity data from your plots. Read the numbers using the measurement tool; the ordered pairs are $(t,\, x_{\text{ms}})$ in the position graph and $(t,\, v)$ in the velocity graph. Record both the position and velocity for the *same* time coordinate at these strategic points. Be sure to obtain the position at 4 even though gravitational potential energy is minimal at that point; in this case, $x = x_{\text{ms}} - x_0$ will be used to determine the spring's potential energy.

Make a table like Figure 6.4 showing the kinetic (6.7), gravitational potential (6.8), spring potential (6.9), and total energies for each of the positions 1 through 7. Using the appropriate formulas for the appropriate energy calculate the entries to the table. (This table is transposed – i.e. incorrectly formatted – but the student will find it more convenient for adding the total energies.)

Be sure to be consistent with units. The total energy is the sum of the other three entries; compare the total energies through the motion. Is energy conserved? Graph the nine total energies to allow visual comparison; the horizontal axis can be numbers 1-7. Look for trends especially for data that was calculated in a similar manner. Be aware of measurement uncertainties.

Checkpoint

Note that the spring potential energy was obtained from a radically different method. Does it fit in with the trends set by the other energy totals? How well do you trust that number?

OPTION 2: We can also let the computer analyze all of our data. Find the row in Capstone's data table that corresponds to the instant we released the glider from rest. Click this time entry in the data table, use the scrollbars to move to the bottom of the table, hold the "Shift" key while clicking the last position entry, and ctrl+c to copy all entries after we released the glider to the Windows clipboard.

Execute Vernier Software's Graphical Analysis 3.4 (Ga3) program. We have prepared a Ga3 template to process all of your data. Open 'Conservation Energy.ga3' at 'http://groups.physics.northwestern.edu/lab/'.

Click the first row under "time" and ctrl+v or "Edit/Paste" to paste your (t, x) points into Ga3. Ga3 should immediately fill in most of the remaining columns with default parameters. You need to repair the "Equations" for *your* experiment by replacing the constants (m, k, t (thickness), and x_0) by the values you measured for your apparatus. Double-click "KE" and repair the glider mass. Double-click "PEg" and repair the glider mass, the axis zero ($\bar{x}_0$), and the shim thickness (t). Double-click "E1" and repair the spring constant (k) and the axis zero (x_0).

Since the spring is not compressed for most of your data, the spring's potential energy must be handled a little differently. Scroll through your data and identify which rows have $x < \bar{x}_0$. These rows and *only* these rows have the correct values for spring potential energy in column "E1". Select the column "E1" data for these rows. The easiest way to select the range is to click the first entry you want to copy, to use the scrollbar to find the last entry you want to copy, and to hold down the 'Shift' key while clicking this last entry; now only the data to be copied is selected. Verify that your selection is correct by using the scrollbar again to find the first entry again. Now copy them with ctrl+c or "Edit/Copy", click the "Es" column at the top of this range of rows, and paste the spring potential energy into this range of rows using ctrl+v or "Edit/Paste". This will replace the zeros that are in the template by default.

Now Ga3 should show the total energy (E) vs. the time in the graph window. Click the lowest number on the energy axis and enter "0" for the minimum value.

Checkpoint

Does it look like total energy is a constant plus some random fluctuations?

Print three graphs that compare the individual energy of each 'store' (KE, PE_G, and PE_S) to the total energy. For each of the graphs, select one individual energy column and the total energy simultaneously. If the y-axis does not specify which columns are plotted, then you must do so (i.e. using the 'Text' box). Study these graphs until you understand how the total energy changes its form in the course of motion and yet remains constant. Print each of these to the .pdf driver and/or copy each to your Word® document also for illustrating your written report.

6.5 Discussion

Discuss what your data says about the Principle of Energy Conservation. Are there particular places when/where your total energy changed noticeably? Was the change larger than the uncertainty in your total energy? What was the glider doing at these times? If necessary, repeat the experiment while watching the glider and the video monitor to correlate the motion with the data points. This is a clue to the cause of the change(s). Can you identify any causes for the change(s)? What other forms of energy might the losses have become? What mechanism (force × distance) might have allowed the losses?

Helpful Tip

Use Ga3 to find the average total energy and its standard deviation before a noticeable change and again after the change. This will allow you to decide objectively whether the change is statistically significant. Do you remember how to do this? If not, review Chapter 2 on errors.

Discuss how your measurement uncertainties might contribute to the variations in total energy. What other sources of error have we not considered? If these other energy forms were added into our total and all of our errors were considered, is it possible that total energy is conserved?

6.6 Guidelines

Is energy conserved in your data? If some of the other energy forms discussed in Analysis were measured and added to your total energy, might energy be conserved? How well does your data support this last conclusion? We might want to use our spring again... what is its spring constant? (Units? Uncertainty?) Can you think of a way to estimate the friction between the air track and glider using your data?

6.6.1 Notebook

Your Lab Notebook should contain the following:

- Complete specifications for the glider mass, the shim thickness, and the axis zero.
- Example Position vs. time and velocity vs. time graphs.
- Force vs. Position graph fit to Hooke's law.
- A labeled data table of energies. (at least nine rows, but NOT more than one screenful.)
- Details about the force sensor and position sensor.
- Three example calculations to verify that KE, PE_G, and PE_S are each computed correctly.
- An objective analysis of each noticeable energy change.
- Three graphs comparing separate energies to total energy.
- One graph of total energy vs. time.
- Other thoughts and/or observations.

6.6.2 Report

Referring to Appendix E, our written report should contain the following:

- Labeled Position vs. time and Velocity vs. time graphs.
- Specifications of m, t, k, and x_0.
- OPTIONAL: Three graphs comparing separate energies to total energy.
- One graph of total energy vs. time.
- Objective discussion of specific time(s) total energy changed (if it did) and reasonable explanations for why. Enumerate any observations that support these possibilities. Detailed calculations are not desired in your report; however, you should be prepared to defend your statements with your notebook.
- A clear statement concluding whether your data supports or contradicts the Law of Conservation of Energy. (Maybe your data does neither?) Did you measure any constants worth reporting? Can you think of any applications for anything you have used or observed? How might the experiment be improved?

Chapter 7

Experiment 5: Conservation of Momentum

Conservation laws such as the one we studied in the previous experiment lead to interesting insights and general principles. Isaac Newton (1642 - 1727) formalized the relationship between force and motion in his *Principia* (published in 1687) in which he proposed his most quoted third law "For every action there is an equal and opposite reaction". This law is the source of a second conservation law which actually has broader application than the energy conservation law. It also represents a departure from our previous treatments in that we will consider two masses m_1 and m_2 that can *interact* with each other. This is a dramatic departure – it is the first step to treating objects as having spatial volume composed of many particles.

Historical Aside

Conservation laws are so general that they are instrumental in determining the existence of many subatomic particles. For example, large particle colliders will accelerate particles and smash them into each other. Detectors then monitor what comes out of these collisions. We place so much trust in the conservation laws that when momentum or energy are missing, we expect that there may be an undiscovered particle present. Intense searches then led to discoveries of these particles.

7.1 Background – Collisions

If m_1 acts with a force F_{21} on m_2 to accelerate it such that $F_{21} = m_2\, a_2$, then by Newton's 3^{rd} Law m_2 should respond with an equal but opposite force F_{12} acting on m_1 such that $F_{12} = m_1\, a_1$. The acceleration is defined as a change in velocity over a change in time $a = \Delta v/\Delta t$. There is every reason to believe that F_{12} only acts while F_{21} is acting and that both forces act over the same interval of time Δt. Starting from Newton's 3^{rd} Law which can now be written formally as

$$F_{12} = -F_{21} \tag{7.1}$$
$$m_1\, a_1 = -m_2\, a_2 \tag{7.2}$$
$$m_1 \frac{\Delta v_1}{\Delta t} = -m_2 \frac{\Delta v_2}{\Delta t} \tag{7.3}$$
$$m_1 \Delta v_1 = -m_2 \Delta v_2 \tag{7.4}$$

which we can unravel to $m_1(v_{1\mathrm{f}} - v_{1\mathrm{i}}) = -m_2(v_{2\mathrm{f}} - v_{2\mathrm{i}})$. Reorganizing by causality yields

$$m_1\, v_{1\mathrm{i}} + m_2\, v_{2\mathrm{i}} = m_1\, v_{1\mathrm{f}} + m_2\, v_{2\mathrm{f}} \tag{7.5}$$

We observe a repeating form, mv, which we define to be momentum, p, which is properly a vector:

$$\mathbf{p} = m\,\mathbf{v} \tag{7.6}$$

The above Equation (7.5) simply stated, makes note that the total momentum of a system of masses is a conserved quantity,

$$\sum_{i=1}^{N} \mathbf{p}_i = \text{ constant} \tag{7.7}$$

Newton had actually formulated his *second* law in terms of momentum. The equation took the form

$$\mathbf{F} = \frac{d\,\mathbf{p}}{d\,t}. \tag{7.8}$$

Here we recognize that momentum is a vector quantity. This expression is a more inclusive rule which reduces to the more familiar form of Newton's 2^{nd} Law when we include the assumption that mass, m, is a constant of motion,

$$\mathbf{F} = \frac{d\,(m\mathbf{v})}{d\,t} = m\frac{d\,\mathbf{v}}{d\,t} = m\,\mathbf{a} \tag{7.9}$$

Checkpoint

In the previous experiment we saw that in the case of energy conservation friction was not so easy to eliminate. Although energy conservation was a nice idea in an ideal world, it was difficult to achieve in practicality. Will we encounter a similar constraint with the application of conservation of momentum?

Whereas work W is defined as a force F multiplied by a distance Δx over which the force acts, we now define *impulse* I to be a force F multiplied by an interval of time Δt during which the force acts,

$$W = \mathbf{F}\cdot\Delta\mathbf{x} \;\text{ and }\; \mathbf{I} = \mathbf{F}\Delta t \tag{7.10}$$

In a similar way in which the case of zero external work brought about the conditions for the Principle of Conservation of Mechanical Energy, we can say that having zero external impulse for a system gives rise to the Principle of Conservation of Momentum (Equation 7.7).

Newton's 3rd Law says that for each force there is an equal and opposite reaction force. These forces encompass any and all internal forces between any set of masses. Any external forces must originate from different sources than the masses we are studying. Interestingly enough, friction may be included as an internal force acting between the masses as they interact. Such a force would preclude conservation of energy, and yet it does *not* destroy the action–reaction relationship and momentum conservation will still hold. Such a class of problems are generally referred to as collision problems. We will study the energy and momentum of two masses as they collide with and without friction.

7.1.1 Elastic Collisions

A collision that occurs without loss of kinetic energy is called a perfectly elastic collision. An elastic collision between two masses, m_1 and m_2, can be completely specified by two equations, one for conservation of momentum and a second for conservation of kinetic energy. We use the prime symbol to indicate velocities after the collision,

$$m_1\,v_1 + m_2\,v_2 = m_1\,v_1' + m_2\,v_2' \tag{7.11}$$

$$\frac{1}{2}m_1\,v_1^2 + \frac{1}{2}m_2\,v_2^2 = \frac{1}{2}m_1\,v'^2_1 + \frac{1}{2}m_2\,v'^2_2 \tag{7.12}$$

These two equations allow for prediction of the unambiguous result of the collision

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2}v_1 + \frac{2m_2}{m_1 + m_2}v_2 \tag{7.13}$$

$$v_2' = \frac{2m_1}{m_1 + m_2}v_1 + \frac{m_2 - m_1}{m_1 + m_2}v_2 \tag{7.14}$$

General Information

On the way to deriving these equations one comes upon an interesting side issue in the equation $v_1 - v_2 = v_2' - v_1'$. Basically this equation states that the velocity at which the masses approach each other before the collision will be the same as they leave each other after the collision. If you were riding on one of the carts watching the other cart approach, you would see the cart depart at the same speed after the collision. Essentially it would bounce off at the same speed as it came in.

7.1.2 Inelastic Collisions

A collision in which some energy is lost to friction is an inelastic collision. The outcome of such a collision is difficult to predict with simple physics principles. Equation (7.12) is not applicable, and the single momentum conservation equation alone is not sufficient to determine the resulting two velocities of two masses uniquely. Another equation for the two velocities is required. Sometimes we may state that a certain fraction or percent of the initial kinetic energy survives the collision. This thinking leads to a ‘coefficient of restitution’. Other times we may find that the masses fuse together so that both final velocities are the same, $v_1' = v_2' = v'$, and the outcome is again predictable,

$$m_1\, v_1 + m_2\, v_2 = (m_1 + m_2)\, v' \tag{7.15}$$

$$v_1' = v_2' = v' = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \tag{7.16}$$

We will test these Equations, (7.11), (7.12), and (7.16), in the following series of experiments.

General Information

Explosions are closely related to collisions, but in explosions we have *more* kinetic energy after the explosion than we did before. This can only happen when some form of stored energy gets released as kinetic energy. In a grenade, chemical energy in gunpowder gets released as one notable example. We will not study explosions, but we admit the similarities between explosions and what we will do.

7.2 Apparatus

We will again be using the air track to minimize external impulses. The set up shown in Figure 7.1 is very similar to what we used in the last lab. It consists of a hollow extruded aluminum beam with small holes drilled into the upper surface. Compressed air is pumped into the beam and released through the holes. This forms a cushion of air supporting two gliders on a nearly frictionless surface. The glider can move with almost frictionless horizontal motion. The gliders can be attached with rubber band bumpers, knife edges, or clay pots to explore different collision conditions.

WARNING

Do not move the air track. It is leveled and difficult to readjust.

Attached to the air track are two sonic motion sensors which are controlled by the computer. The computer signals a motion sensor to emit a sound pulse. The pulse reflects off the plastic card attached to the glider and returns an echo to the motion sensor. The computer receives the signal and calculates the position of the glider from the time delay

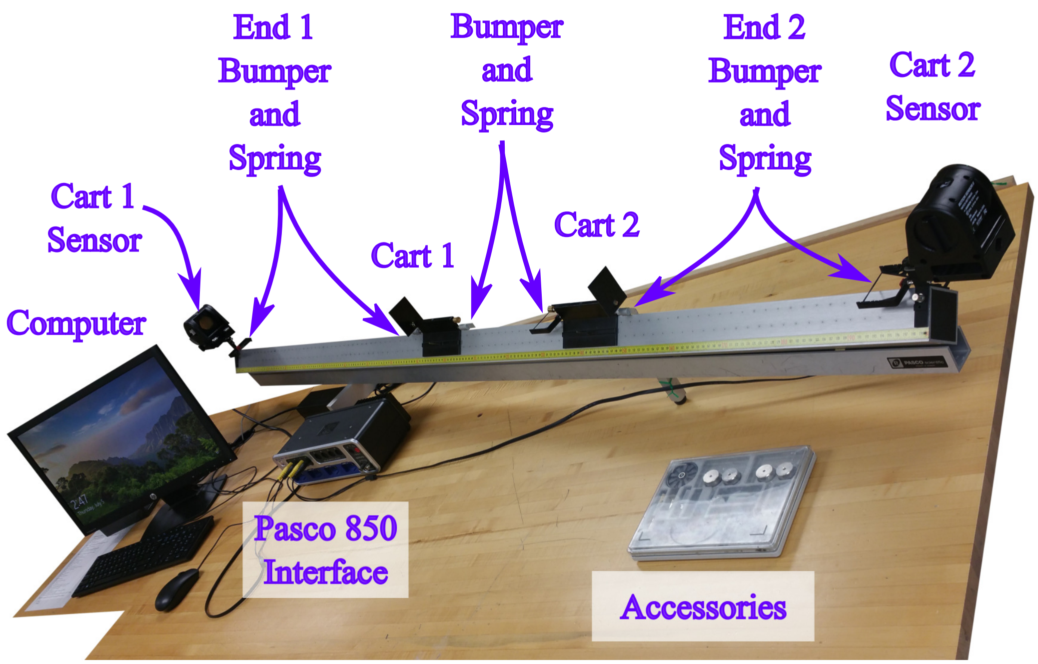


Figure 7.1: *A photograph of an apparatus used to study momentum conservation. The elastomer spring and bumper between the carts also conserves kinetic energy; a needle and clay pot instead sticks the carts together. The accessories have different masses, so changing accessories should always precede weighing the carts.*

between sending the pulse and receiving the echo using the average speed of sound in air. Computer software plots the data and can use the data to calculate velocity and acceleration.

WARNING

Do not touch the rangefinder. The quality of your data depends upon its pointing directly at the glider. Do not obstruct the path between the sensors and gliders with your hand, the video monitor, etc.

7.3 Procedure

We are using Pasco's Capstone with their 850 Universal (computer) Interface. We have published a configuration file for you to use 'Conservation Momentum.cap' at

'http://groups.physics.northwestern.edu/lab/';

open this file now. Once the air track is under pressure, the gliders have the masses you want, and the gliders are closing on each other, click "Record" at the bottom left. Once the button has been clicked, it turns into the “Stop” button you will need to end the data collection phase. After the collision, allow the gliders to move for a few more seconds and click “Stop”. If you don’t have several seconds worth of data before *and* after the collision, “Delete Last” run at bottom right, adjust the experiment to get better data, and repeat the experiment.

Helpful Tip

Take care to note which velocity graph corresponds to which glider. If you get the glider masses and/or their velocities confused, your results will be wrong and you will probably need to repeat the experiment to get this right.

Checkpoint

Both velocities before the collision should be constant and both velocities after the collision should be constant; however, there will usually be a few data points *during* the collision when neither velocity is constant.

All of these equivalent velocities can be treated statistically and reported as described in Section 2.6.1. For each of the three experiments, report your measured velocities to be $v_k = (\bar{v} \pm \delta v)$ U. “U” is the units of velocity; m/s in this case. k is some subscript like “1b” for the velocity of mass 1 before the collision or “1i” for initial velocity of mass 1. Choose your subscripts so that *you* will know what they mean; decide on a subscript strategy early and be consistent or you will get confused. It might help to write down explicitly in your notes what you have chosen your symbols to mean.

7.3.1 Elastic collisions

In this experiment we will measure the speeds of two air carts before and after they collide with each other. First, attach the rubber band bumper and knife edge to the gliders. Carefully weigh both gliders with attachments and record their masses in your data. Don’t forget to estimate your measurement uncertainty and to record your units. Be sure to note which measurement corresponds to which cart.

Note that the motion sensors face *in opposite directions*; rightward motion will have positive velocity when measured by the left sensor but negative velocity when measured by the right sensor. Set one cart at rest near the center of the air track. You may simultaneously touch the glider and the track with one finger to keep it at rest. Give the second cart a velocity directing it away from the first cart. The cart will bounce off the end of the track and rebound reaching a stable speed before colliding with the first cart. Be sure to release the resting cart prior to the collision by sliding your finger perpendicularly to the allowed motion. The extra time while the cart is moving away from the resting cart will give you a chance to hit "Record" before the collision occurs. Try to click "Record" at the same time the cart hits the end stop. Alternatively, it is permissible to press "Record" before setting the cart into motion; just don't get confused by the extra data points that begin such a run. Be sure the plastic cards on both gliders are perpendicular to the track so the sound will be reflected directly to the motion sensor. Orient the gliders so that the plastic reflector card is closer to the sensor than the center of mass; otherwise, the echo from the glider's front will sometimes confuse the sensor. Be sure to keep your hands, your body, and other items in the lab at least a foot away from the motion sensors' sound paths; they might mistake reflections off of these items as signals from the glider. If this happens merely retake the data. Let the computer continue to take data as the gliders bounce off one another and until one of them hits the end of the track. Hit the "Stop" button using the mouse.

Scan the data and look for the collision event which should show a sudden exchange of velocity between the carts. Hover the mouse cursor on a velocity graph so that a toolbar appears above that graph. The toolbar will contain a data selection tool icon (). Click this selector button to bring up a pale area surrounded by eight sizing squares. You can drag the selected area with the mouse and you can change its shape by dragging one of the sizing squares. Shape and position the selected area so that only the data having the v_i velocity at the first of the run are selected. Compute the statistics for this data using the "Σ" button () on the toolbar. If the mean ($\bar{v}_\mathrm{i}$), the std. dev. (s_{v_i}), and the number of data points (N) are not all displayed, click the little down triangle to the right of the "Σ" button and check all that are absent. You may optionally un-check those statistics that we do not need.

Use these statistics to record the mean and the deviation of the mean as described in Section 2.6.1; note that some of the velocities you need are on the other graph making it necessary to obtain another selection tool for those points. If you need more significant digits for some particular mean, you can right-click the parameters box, 'Properties...', and change the significant digits that are displayed.

Find the change in the two masses' velocities

$$\Delta v_\mathrm{i} = v'_\mathrm{i} - v_\mathrm{i} \tag{7.17}$$

and combine the uncertainties in the velocities' changes using

$$\delta(\Delta v_\mathrm{i}) = \sqrt{(\delta v'_\mathrm{i})^2 + (\delta v_\mathrm{i})^2}. \tag{7.18}$$

Calculate the change in momentum for each mass in the system using Equation (7.4),

$$\Delta p_1 = m_1 \Delta v_1 \quad \text{and} \quad \Delta p_2 = m_2 \Delta v_2 \tag{7.19}$$

The negative sign in Equation (7.4) was already provided by the opposite facing motion sensors. Estimate the uncertainties in these changes in momentum using

$$\delta\Delta p_1 = \sqrt{(m_1\,\delta\Delta v_1)^2 + (\Delta v_1\,\delta m_2)^2} \quad \text{or} \quad \delta\Delta p_2 = \sqrt{(m_2\,\delta\Delta v_2)^2 + (\Delta v_2\,\delta m_2)^2} \tag{7.20}$$

Calculate the changes in kinetic energy for the two masses using

$$\Delta KE_1 = \frac{1}{2}m_1(v'^2_1 - v_1^2) \text{ and } \Delta KE_2 = \frac{1}{2}m_2(v'^2_2 - v_2^2) \tag{7.21}$$

Calculate the uncertainty in the *moving* mass' kinetic energy before the collision using

$$\delta KE_{\text{mov}} = \frac{1}{2}\sqrt{(v^2\delta m)^2 + (2mv\delta v)^2} \tag{7.22}$$

Helpful Tip

The uncertainties in these calculated measurements are determined as specified in Section 2.6.3. All three experiments have four velocities and two masses with about the same uncertainties as in the first experiment. All of these measurements and uncertainties must be recorded; however, we will assume that $\delta\Delta p_1$, $\delta\Delta p_2$, δKE_1, and δKE_2 are the same for all three experiments. **This *is* a source of error!** for the second and third experiments, but it also saves substantial time.

We will estimate the uncertainties in Equations (7.21) by assuming that the uncertainty in each of the four kinetic energies (two masses before and two masses after) are approximately the same as we found using Equation (7.22). *This is wrong!* But this is all we will take time to do.

$$\delta(\Delta KE_1) \sim \delta(\Delta KE_2) \sim \sqrt{(\delta KE_{\text{mov}})^2 + (\delta KE_{\text{mov}})^2} = \delta KE_{\text{mov}}\sqrt{2} \tag{7.23}$$

We should repeat these uncertainty calculations (Equation (7.22) and Equation (7.20)) for every mass and for every experiment to follow; however, we would see that they are similar within a factor of 2 or so. Instead of repeating these uncertainty calculations we will use our time to perform more experiments and we will simply assume that every experiment will have the same uncertainties for Δp_i and for ΔKE_i $(i = 1, 2)$.

7.3.2 Moving Target, Elastic Collision

Keep the elastic bumpers and repeat the above procedure for both gliders in motion. Since neither mass changed, it is not necessary to weigh the gliders; just remember which glider has which mass. Add masses to one of the gliders while keeping the glider balanced; note the new masses. Record all four velocities and uncertainties as described by Equation (**??**). (We will continue to *use* the uncertainties in momentum calculated in Part 1; however, we still want to *record* the correct velocity specifications.) Compute the change in momentum for the two masses using

$$\Delta p_1 = m_1(v'_1 - v_1) \text{ and } \Delta p_2 = m_2(v'_2 - v_2) \tag{7.24}$$

Checkpoint

Do these have opposite signs? Don't forget that the sensors face in opposite directions...

Another way to state the conservation of momentum is to say that the momentum lost by 1 must be gained by 2 and vice versa. The same can be said for the gains and losses in kinetic energy. Compute the changes in kinetic energies for the two masses using

$$\Delta KE_1 = \frac{1}{2}m_1(v'^2_1 - v_1^2) \text{ and } \Delta KE_2 = \frac{1}{2}m_2(v'^2_2 - v_2^2) \tag{7.25}$$

Now, just record the same uncertainties for these values as you have already recorded above (using Equation (7.20) and Equation (7.23)). Copying the uncertainties from our previous measurements *is wrong*, but it is much quicker than computing them again.

7.3.3 Completely Inelastic Collision

Repeat Experiment 2 for a completely inelastic collision. To do this, replace the bumper on one glider with the needle-point and the knife edge on the other glider with the clay pot receptacle. This will change the masses of both gliders, so it will be necessary to weigh them again. You might as well decide what additional mass you want your gliders to have and weigh the lumped totals. Since time is short and the effect is small, just use the same uncertainties for the masses that you already have at hand. The needle and clay pot should cause the gliders to stick together when they collide. If this is not the case, ask your TA to re-pack the clay in the pot. Allow one or both masses to move prior to colliding and have the computer to record their velocities before and after the collision. Record all four velocities and uncertainties as described by Equation (**??**). Determine the changes in momentum and in kinetic energy for each of the masses during the collision. Estimate the uncertainties in these changes to be the same as the first experiment.

7.4 Analysis

Calculate the difference (Δ) between the absolute values of the two changes in momentum and the uncertainty (σ) of these differences. Review Section 2.9.1. Since statistics is normalized to σ, we will find it convenient to convert the units of our Δ into σ. Now it is easy for us to assess the probability that our prediction is statistically different from our measurement. Compute the difference and uncertainties in the kinetic energies as well.

7.5 Guidelines

What external forces have we omitted from our considerations that might affect the momentum of one or both of our masses? What forces (internal or external) have we omitted from our considerations that might perform work on one or both of our masses? What other forms of energy might have been generated during the collisions? Can you think of sources of experimental error that we have not included in our total uncertainties? Are our differences so much bigger than our uncertainties that these other considerations are unlikely to explain them? If so, make sure you have not made mistakes while processing your data.

7.5.1 Notebook

Your Lab Notebook should contain the following:

- Observed details about commercial equipment.
- Clear sketches (velocities and masses) of each experiment.
- Table(s) containing glider masses, two initial, two final velocities, correct units, and correct uncertainties for each experiment.
- Table(s) of all momenta and energies (and their uncertainty estimates) before and after the collisions.
- Calculations of relevant uncertainties for comparing before and after the collisions (esp. Part 1).
- Calculation of before vs. after comparison (Δ) and total composite uncertainty (σ).
- Table of Results showing Δ and σ in momentum (energy) units as well as Δ with σ units.

7.5.2 Report

Your report should address:

- What physical relationships do your data support?
- Clear illustrations (velocities and masses) of each experiment.
- How did we reduce external forces? Measure mass? Velocity?
- Summarize (and produce) the tables in the notebook.
- Summarize other important sources of error.
- When is momentum conserved?
- When is kinetic energy conserved?
- Have we measured anything worth reporting in our Conclusions?
- How might we improve our experiment?

Chapter 8

Experiment 6: Collisions in Two Dimensions

Last week we introduced the Principle of Conservation of Momentum and we demonstrated it experimentally in linear collisions. This week we will extend this demonstration to include two-dimensional collisions. It turns out that multi-dimensional collisions are one of our main sources of information about sub-atomic and other fundamental particles, so understanding momentum and energy conservation in these situations has broad significance to physics.

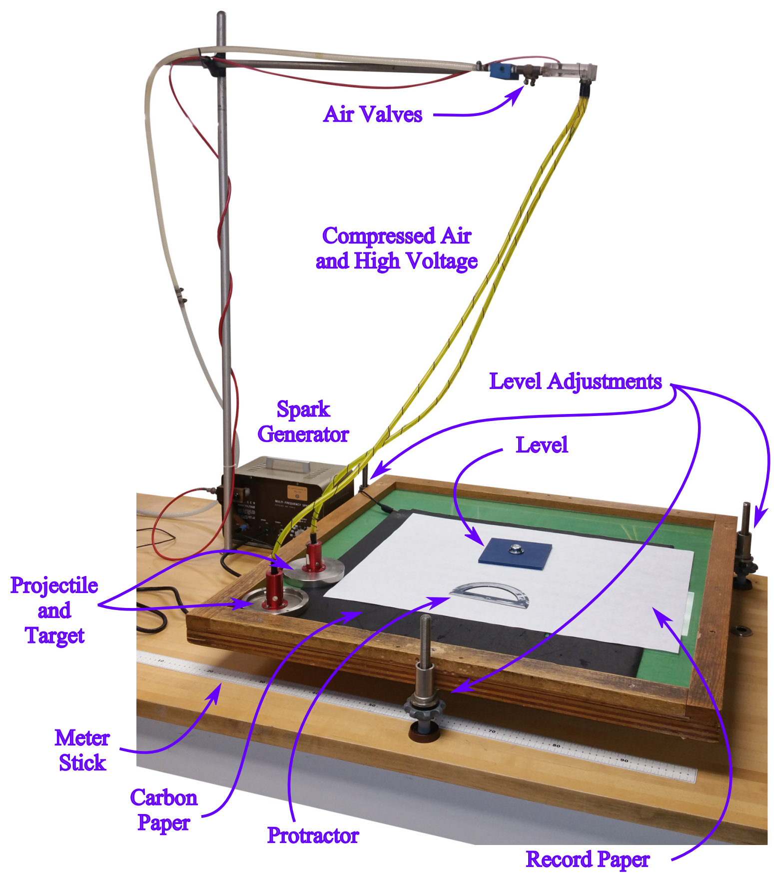


Figure 8.1: *A photograph of the apparatus detailing relevant controls.*

Historical Aside

Conservation of energy and momentum in collisions (and associated particle decays) are exceptionally important in the discovery and identification of new sub-

Historical Aside

atomic particles. In 1930, the *neutrino* was proposed by Wolfgang Pauli to account for the lack conservation of energy and momentum in beta particle decay. The neutrino was directly detected in 1956 and observed to obey the characteristics predicted by the conservation laws (and eventually its detection received a Nobel Prize). More recently, in 2012, the Higgs boson was discovered at the Large Hadron Collider at CERN (also prompting a Nobel Prize). This apparatus collides two high-energy protons together to produce a stream of sub-atomic particles, including the Higgs, which was detected by looking at the energy of other emitted particles from the collision. Clearly the physics of sub-atomic particles are far more complex than Newton's Laws, but the principles of conservation of energy and momentum in collisions are key ingredients for understanding the output of high-energy collisions.

In this experiment the elastic collision between two air hockey pucks is recorded. The information obtained is used to confirm the conservation of momentum in 2D and of kinetic energy in elastic collisions. As illustrated in Figure 8.1, we once again utilize the pucks and table from our first two experiments. This time the table is level, however, and we use two pucks.

8.1 Background

We wish to confirm the principle of the conservation of momentum; in other words we wish to confirm that

$$\mathbf{p}_{\text{before}} = \mathbf{p}_{\text{after}} \tag{8.1}$$

where the subscripts are with respect to the time of the collision. Note that in our current approach, we are considering *vector* momenta, which means that *each component of the vector* and not just the magnitude must be conserved. Figure 8.2 represents these two situations. Two vectors are equal only when their components are identical. To test Equation (8.1) we must determine and then compare the momentum components of the total system (projectile and target) before and after the collision. If the subscripts 'p' and 't' indicate the projectile and the target pucks respectively, and if the primed quantities refer to the velocities after the collision, then Equation (8.1) can be written as follows,

$$m_{\mathrm{p}}\mathbf{v}_{\mathrm{p}} + m_{\mathrm{t}}\mathbf{v}_{\mathrm{t}} = \mathbf{p}_{\mathrm{p}} + \mathbf{p}_{\mathrm{t}} = \mathbf{p}'_{\mathrm{p}} + \mathbf{p}'_{\mathrm{t}} = m_{\mathrm{p}}\mathbf{v}'_{\mathrm{p}} + m_{\mathrm{t}}\mathbf{v}'_{\mathrm{t}} \tag{8.2}$$

We choose the velocity of the t-puck to be zero and if we define the $\mathbf{v}_{\mathrm{p}}$ to be along the x-axis; we can rewrite the last equation in components along the x and y axes,

$$m_{\mathrm{p}}v_{\mathrm{p}} = p_{\mathrm{p}x} + p_{\mathrm{t}x} = p'_{\mathrm{p}x} + p'_{\mathrm{t}x} = m_{\mathrm{p}}v'_{\mathrm{p}}\cos\theta'_{\mathrm{p}} + m_{\mathrm{t}}v'_{\mathrm{t}}\cos\theta'_{\mathrm{t}} \tag{8.3}$$

$$0 = p_{\mathrm{p}y} + p_{\mathrm{t}y} = p'_{\mathrm{p}y} + p'_{\mathrm{t}y} = m_{\mathrm{p}}v'_{\mathrm{p}}\sin\theta'_{\mathrm{p}} + m_{\mathrm{t}}v'_{\mathrm{t}}\sin\theta'_{\mathrm{t}}. \tag{8.4}$$

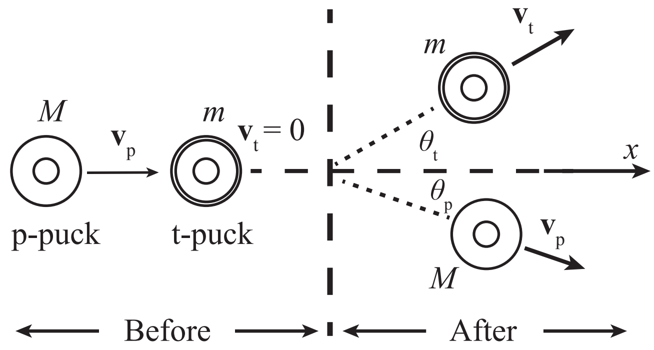


Figure 8.2: *Sketch of two puck trajectories before and after elastic collision in 2D. The recommended x-axis is shown and the dashed vertical line is the recommended y-axis.*

We will also wish to determine if the kinetic energy, KE, is conserved and that consequently it is an elastic collision. We will wish to confirm that

$$\frac{1}{2}m_{\mathrm{p}}v_{\mathrm{p}}^2 = KE_{\mathrm{before}} = KE_{\mathrm{after}} = \frac{1}{2}m_{\mathrm{p}}v'^2_{\mathrm{p}} + \frac{1}{2}m_{\mathrm{t}}v'^2_{\mathrm{t}} \tag{8.5}$$

Please notice that energy is a scalar (not a vector, such as momentum $\mathbf{p}$ or velocity $\mathbf{v}$) and consequently you do not need to use components.

8.2 Apparatus

The apparatus sketched in Figure 8.1 has been used previously in labs 1 and 2, so we will forgo most of the apparatus' description. The two air pucks are the solid puck, which is the one used in the first and second labs, and a less massive rimmed puck. The first one is used as a projectile (p) and the second one as a target (t). The table and gravity combine to constrain both pucks to move in a plane. To qualify for the Conservation of Momentum using measured velocities, we must eliminate all external force components in the plane of motion; this means leveling the table so that gravity is perpendicular to the plane of motion and reducing friction as much as possible.

Checkpoint

What conditions must be met in order for momentum to be conserved in a three dimensional collision?

Each of the pucks is also fitted with a pressurized air hose to push a cushion of air under the pucks to minimize friction and a pointed electrode periodically pulsed with high voltage to record the positions of the pucks at the times of these pulses. Knowledge of these positions and the times of the pulses will allow us to determine the pucks' velocities. The masses of p and t pucks are

$$m_{\mathrm{p}} = 1.20\,\mathrm{kg} \quad \text{and} \quad m_{\mathrm{t}} = 0.680\,\mathrm{kg} \tag{8.6}$$

but this does *not* include the attached hoses and wires.

WARNING

Do not turn the air supply on all the way. It will damage the air hose. Only turn it on a little way. The puck will float well. Do not drop the puck. The table top is glass!

WARNING

The puck should be on the white record paper and not on the carbon paper whenever the push-button activating the pulse generator is pressed. The white paper should be on the carbon paper. If you touch the high voltage terminal and ground at the same time, you may get a shock – harmless, but unpleasant! Always make sure that the ground clip is properly connected to the carbon sheet before activating the pulser.

8.3 Procedure

Carefully level the air table, first with the level and then by trying to keep the floating t-puck stationary at the center of the air table. It will also help to lift the hose vertically with only a little slack. With the t-puck at rest near the center of the table, practice making a two-dimensional collision in which the projectile puck is given a velocity of about 0.5 m/s and the angle between the two pucks, after the collision, is about 45°. Now make a record of the collision by holding the spark timer button throughout the relevant motion. Be sure to note whether the projectile is the heavy or the light puck in your Data. Also note both masses, units, and uncertainties.

Draw arrows on the record paper that show the directions of the pucks' velocities before and after the collision. Since the target puck was not moving prior to the collision (hopefully), only three vectors will be shown. If this is done correctly, the projectile puck will be moving inward prior to the collision and both pucks will be moving outward after the collision. Sketch your x-axis parallel to the incoming velocity and your y-axis perpendicular to the x-axis. Perfection is neither needed nor desired at this point. Rotate the paper until the x-axis points to the right as it should and draw the y-axis upward as it should be.

Remove the record paper from the apparatus and transfer your velocities and axes to the back side where the puck locations are recorded. Note that the puck trajectories are usually not perfectly straight. We desire the velocity vectors immediately prior to and immediately following the collision; however, to get accurate speeds we must measure reasonably long distances ($\Delta x \geq 3\,\mathrm{cm}$) so that our measurement uncertainties are a smaller fraction of our measurements themselves. Using a straight edge, draw three lines along the three velocities as close to the instant of collision as you can guess. These lines will need to extend about six inches on each side of the collision and they will need to extend back until they cross the x-axis. Draw arrows on the correct ends of these lines to indicate the correct orientation of the velocity vectors. If necessary, check your original notes on the front side of the paper to make sure you get these right.

Using the straight edge, carefully extend the incoming velocity across the entire page and label it to be the x-axis. Referring to the front side of the page, carefully construct the y-axis perpendicular to this x-axis. Since we are working on the back of the page, the y-axis direction and our angles will be opposite to what we normally experience; positive angles will now be clockwise from $+x$ and negative angles will now be counterclockwise from $+x$. Use the protractor to measure the angles of the outgoing velocity vectors and record them in your notebook's Data section. The angles you want to measure have the tips of the vector arrows out by the protractor scale and the tail of the vector arrows crossing the x-axis.

Checkpoint

How accurately can you construct these vectors? How accurately can you read the protractor? Let these questions guide you when estimating the experimental uncertainties in your angle measurements.

Don't forget to *record* the units and the uncertainties of these angles in a table. While you're at it, record their sines and cosines to four significant digits beside the angles.

Begin with the trajectory with the closest dot spacing. Use a ruler to measure the distance between two dots at least 3 cm apart and count the number of *spaces*, N, between these two endpoints. Now measure the length of the same number, N, of spaces for the other trajectories. Frequently, the dot closest to the collision along each of the pucks' trajectories might have occurred before or after the collision with equal probability. If you cannot be certain whether the closest dot occurred before or after the collision, omit it from further consideration. Also, once in a while the sparker skips a dot; you must count this skipped dot to obtain correct measurements; circle the place you think the dot should be. As long as the same number of spaces is used on all vectors, the interval ($\Delta t = \frac{N}{60}$ s) will cancel from our calculations. Record the distances the three pucks moved in this time interval in a nice table in your Data section. Don't forget to estimate your measurement uncertainties and to record your units. If your target trajectory before the collision was not a point, you will need to ask your instructor to help you estimate its initial momentum so that you can include it below as well.

First, calculate the pucks' speeds:

$$v_{\rm p} = \frac{\Delta s_{\rm p}}{\Delta t} \qquad v'_{\rm p} = \frac{\Delta s'_{\rm p}}{\Delta t} \tag{8.7}$$

$$v_{\rm t} = \frac{\Delta s_{\rm t}}{\Delta t} \stackrel{?}{=} 0 \qquad v'_{\rm t} = \frac{\Delta s'_{\rm t}}{\Delta t} \tag{8.8}$$

and use these to compute the x and y components of each of the three (four?) momenta:

$$\begin{aligned} p_{{\rm p}x} &= m_{\rm p} v_{\rm p}, & p_{{\rm t}x} &= m_{\rm t} v_{\rm t} \cos\theta_{\rm t} \stackrel{?}{=} 0 \\ p'_{{\rm p}x} &= m_{\rm p} v'_{\rm p} \cos\theta'_{\rm p}, & p'_{{\rm t}x} &= m_{\rm t} v'_{\rm t} \cos\theta'_{\rm t} \\ p_{{\rm p}y} &= 0, & p_{{\rm t}y} &= m_{\rm t} v_{\rm t} \sin\theta_{\rm t} \stackrel{?}{=} 0 \\ p'_{{\rm p}y} &= m_{\rm p} v'_{\rm p} \sin\theta'_{\rm p}, & p'_{{\rm t}y} &= m_{\rm t} v'_{\rm t} \sin\theta'_{\rm t} \end{aligned}$$

and estimate their uncertainties using the appropriate uncertainty equations. We have provided an Excel® worksheet to propagate the uncertainties '2D Momentum.xlsx' at

'http://groups.physics.northwestern.edu/lab/'.

This is necessary since none of the formulas (Equations 2.4-2.8) accommodate trigonometric functions. The student may also enter formulas and allow the spreadsheet to perform his calculations of momentum components or he can simply enter the correct numbers. The results may be printed or copied to a Word® document for inclusion in the report.

Checkpoint

If these calculations were done correctly, all of the x components (except possibly the initial target momentum) should be positive and the y components should have opposite signs. This is insufficient to assure correctness but might indicate simple mistakes.

Find the total momentum before and again after the collision using

$$\begin{aligned} p_x &= p_{{\rm t}x} + p_{{\rm p}x} & p'_x &= p'_{{\rm t}x} + p'_{{\rm p}x} \\ p_y &= p_{{\rm p}y} + p_{{\rm t}y} & p'_y &= p'_{{\rm t}y} + p'_{{\rm p}y}. \end{aligned}$$

Find the initial and final kinetic energies using

$$\begin{aligned} KE &= \frac{1}{2} m_{\rm p} v_{\rm p}^2 + \frac{1}{2} m_{\rm t} v_{\rm t}^2 \\ KE' &= \frac{1}{2} m_{\rm p} v_{\rm p}'^2 + \frac{1}{2} m_{\rm t} v_{\rm t}'^2. \end{aligned}$$

8.4 Analysis

Can you say something about the frictional forces in the air puck system? Why do the three velocity vectors not intersect at a point? Calculate the differences between 1) the initial and final x momentum, 2) the initial and final y momentum, and 3) the initial and final kinetic energy. Instead of computing the uncertainties in these total momentum components in detail, we might accept that the y-momentum is conserved. In this case, the difference between before and after the collision would entirely be due to our measurement uncertainties (and other possible error sources...) Since the *same* measurements are used to compute the x-momentum, we should expect about the same disagreement. If

$$\Delta p_x \approx \sigma_y = \Delta p_y \tag{8.9}$$

within a factor of 2-3, then we can reasonably conclude that this line of thought supports momentum conservation.

What does our study of statistics indicate about our experiment and the conservation of momentum and kinetic energy?What external forces can you think of that might have affected the momentum of one or both pucks? What forces (internal or external) can you think of that might have performed work on one or both pucks? Can you think of any evidence to support this? What other forms of energy might the initial kinetic energy have become? What other sources of experimental error can you think of that we have not incorporated into our total errors, σ? Is it likely in your estimation that these considerations are large enough to explain the differences between the theories of momentum and energy conservation and your data? How might sliding friction between the pucks (an internal force) affect: 1) the intersections of the velocity lines, 2) the conservation of momentum, 3) the conservation of kinetic energy, and 4) the rotation (and the rotational kinetic energy) of the disks?

8.5 Report Guidelines

Is $\mathbf{p}_{\text{before}} = \mathbf{p}_{\text{after}}$? How well can you make this comparison? (what is the uncertainty?) Did you verify all 3D components? How might we apply this to alpha rays interacting with a gold nucleus? Was kinetic energy conserved?

Your Lab Notebook should contain the following:

- A sketch of the observed collision.
- A table of puck masses, dot distances, and trajectory angles.
- Calculation of momentum (both components).
- Table of 6-8 before and after momentum components; total momentum components before and after.
- A comparison of (x, y) momentum components complete with error analysis.

- Kinetic energy (total) before and after.
- A comparison of the energies and their error analysis.
- Observations regarding error sources, external forces (impulses), other forms of energy excited by the collision.

Your Report should contain the following:

- A complete description of the apparatus and method (or a suitable reference to previous work and a description of similarities and differences).
- A schematic sketch of your collision.
- Data tables showing your raw data, your momentum components, and comparisons between initial and final momentum components.
- Data table summarizing initial and final kinetic energies and their comparisons; can be appended to previous table.
- Discussion of error sources, external forces, work, etc.
- Discussion introducing raw data and leading to total momentum components and kinetic energy.
- A statement of conclusion telling what physics your data supports, contradicts, or neither.
- Possible improvements.

Chapter 9

Experiment 7: Newton's Second Law for Rotational Motion

Isaac Newton (1642 -1727) formalized the relationship between force and motion in his *Principia* (published in 1687) in which he proposed that the acceleration of an object is directly proportional to the net force acting on an object and inversely proportional to the mass of the object. The Law is summarized in the succinct vector formula $\mathbf{F} = m\mathbf{a}$. This law can be straightforwardly translated from a single particle to a collection of particles (an object). This composite object can have its center of mass move, but it can also rotate (with all constituent particles moving together) about an axis. In Classical Mechanics, Newton's Laws must still govern the motion of all of these particles that make the object. As such, Newton's Second Law can be generalized to rotating objects. For the rotation of a rigid body about a fixed axis, Newton's second law takes on the slightly different, yet similar, form $\tau = I\alpha$ where these terms will be explained below.

9.1 Background

The measure of rotation is the angular position θ analogous to linear position $\mathbf{x}$. The counterpart of linear velocity $\mathbf{v} = \Delta\mathbf{x}/\Delta t$ is angular velocity $\omega = \Delta\theta/\Delta t$. Angular acceleration $\alpha = \Delta\omega/\Delta t$ is similar to linear acceleration $\mathbf{a} = \Delta\mathbf{v}/\Delta t$.

When the measure of rotation is given in units of radians then the relationship between linear terms and their rotational counterparts take the following forms: $s = r\theta$, $v_T = r\omega$, $a_T = r\alpha$, where s is the arc length swept out by the radius vector rotating through the angle θ and v_T and a_T are tangential velocity and tangential acceleration. $\mathbf{r}$ might point to any particle in the object.

The rotational counterpart of force is *torque*, τ, defined in terms of the Force, $\mathbf{F}$, and a radius vector, $\mathbf{r}$, which locates the point of application of the force with respect to the axis of rotation,

$$\tau = \mathbf{r} \times \mathbf{F} \tag{9.1}$$

Newton's Second Law takes the form

$$\tau = I\alpha \tag{9.2}$$

where I is called the moment of inertia and plays the role of inertial mass in rotating rigid bodies. I depends not merely on the amount of mass rotating but also on how the mass is distributed with respect to the axis of rotation. The total moment of inertia, I, of a body consisting of a finite number of mass elements, m_i, located by their distance from the axis of rotation, r_i, is given by

$$I = \sum_{i=1}^{N} m_i r_i^2 \quad \text{or} \quad I = \int_M r^2 \, \mathrm{d}m \tag{9.3}$$

The moment of inertia of a disk of mass M and radius R is calculated as

$$I_{\text{disk}} = \frac{1}{2} M R^2 \tag{9.4}$$

Checkpoint

Newton's Second Law is fundamentally a *vector* relationship. If you think carefully about the axis of rotation, can you modify the rotation law $\tau = I\alpha$ to be a vector law? Analogous to linear motion, this vector description would perhaps make signs and directions of torques a more intuitive concept for problem solving.

The motion of a rigid body can be described completely in terms of the linear equations of motion of the center of mass of the object and the equations of rotational motion of the body about its center of mass. We have previously studied the motion of the center of mass of rigid bodies and in this laboratory we will quantitatively verify Newton's Second Law in it's rotation form.

General Information

We emphasize that this is *not* a new law of physics. What we do now is also a direct result of Newton's Second Law but applied to all of the pieces that make up a rigid body.

9.2 Apparatus

We will be using an air table for this experiment. Our version consists of a hollow aluminum plate with small holes drilled into the upper surface. Compressed air is pumped into the plate and released through the holes. This forms a cushion of air supporting a disk on a

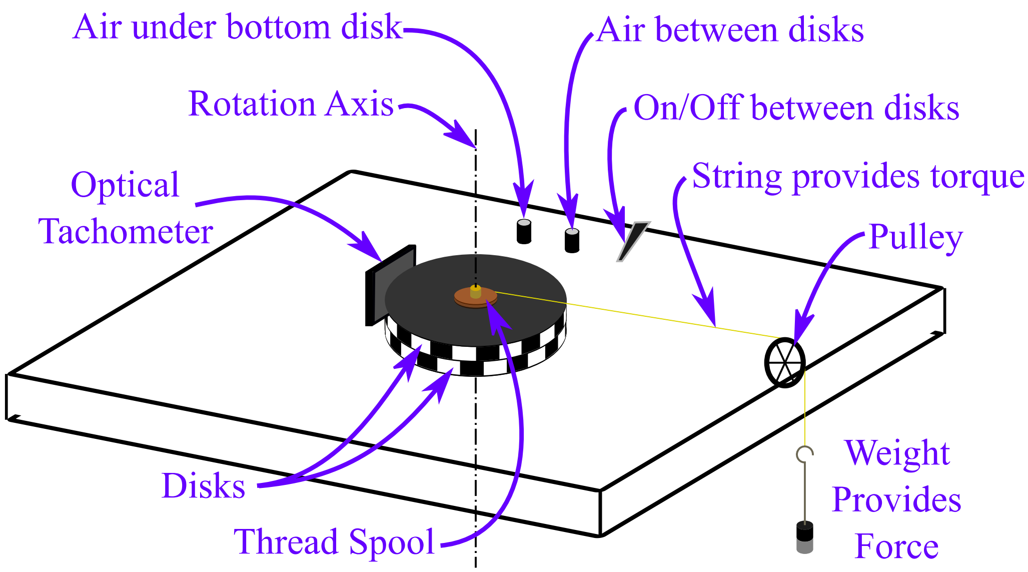


Figure 9.1: *Sketch of lab apparatus showing the rotating disks, the thread spool that provides torque, the weights that provide force, the compressed air controls, and the optical tachometer that measures disks' orientations.*

nearly frictionless surface. The disk can rotate about a vertical axis with almost frictionless rotational motion. A manifold of tubes and valves allows us to direct this compressed air under the bottom disk only to allow both disks to rotate synchronously, between the two disks only to allow only the top disk to rotate, or under both disks to allow the two disks to rotate independently from each other.

Mounted on the disk is a spindle which has a string wound around it. The other end of the string passes over a pulley at the edge of the lab table and has a weight holder attached to the end of the string. Since the radius of the spindle separates the string's tension force from the disks' center of rotation, the tension exerts a torque on the top disk and the spindle. This torque causes the disk's rotation to change at constant acceleration as described by Equation (9.2).

Two optical encoders will count black and white stripes painted on the edges of the disks. The stripes are illuminated by red LEDs selected to emit light of the color that generates the best sensitivity for the encoders. Since there are 24 stripes, each is $360°/24 = 15°$ wide. By counting the stripes that pass, we measure the angle the disk rotates. By measuring the time needed for a stripe to pass, we can compute a measured angular velocity. Differences in angular velocity in the same time interval allow us also to compute a measured angular acceleration.

All of these measurements and calculations are performed by an 'Arduino' microcontroller circuit that has been mounted to the bottom of the apparatus main board. Ten times per second, the Arduino also sends the latest time, position, velocity, and acceleration that it measured for each of the two disks through the USB to the computer.

A computer terminal simulation program called "TeraTerm" will receive this data and allow us to copy it and to paste it into Vernier Software's Ga3 graphical analysis software.

We will use Ga3 to plot the angular positions, angular velocities, and (possibly) angular accelerations versus time and to fit them to kinematic models.

WARNING

Do not force the valves closed. Turn them off gently! The valves will fully close before they stop turning.

WARNING

Do not drop the disks. They are heavy! Dropped from the tabletop, one might easily break your foot. Furthermore, the dents, scratches, and dings reduce their performance.

9.3 Procedure

First, remove the spindle and measure its diameter from the *center of the string* on either side using the Vernier calipers. With the Vernier scale's help, you can estimate the spindle's diameter to a small fraction of a millimeter. Record half this distance and half its estimated uncertainty in your Data as the radius, r, needed to relate the string's tension to the torque that it exerts.

The tension, T, in the string is obtained from the mass on the weight holder as well as the mass of the holder itself. Multiply this mass by the acceleration of gravity. Since the string leaves the spindle at right angles to the radius (see Figure 9.2), the torque is the simple product of the tension and the radius of the spindle. Equation (9.1) reduces to

$$|\boldsymbol{\tau}| = |\mathbf{r}|\,|\mathbf{F}| = rgm \tag{9.5}$$

Since neither r nor g will change today, you might find it convenient to store their product in a calculator memory.

9.3.1 Angular Acceleration Proportional to Unbalanced External Torque

In this experiment we will measure the angular acceleration of one disk under conditions of varying torques. This will allow us to decide whether angular acceleration is indeed directly proportional to torque and also whether the constant of proportionality is the disk's moment of inertia. Choose an initial weight for the hanging mass between 2 g and 30 g.

For each run start by winding the string on the spindle such that the weight holder is

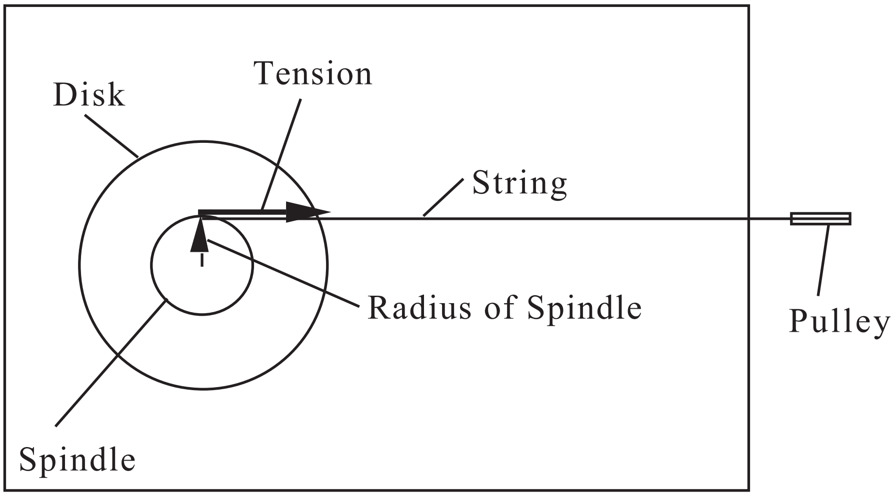


Figure 9.2: *Top view of apparatus showing how string tension is converted to torque by the spindle. Note the right angle between the radius vector and the tension force.*

hanging over the pulley near the top of its travel and holding the disk keeps it there. If the string is not straight on the pulley, try winding the string in the opposite direction; viewed from above, the disk should rotate clockwise after being released from rest. Be sure that the string does not touch anything except the spindle, the pulley, and the mass hanger since any resulting friction will skew your results. Be sure the TeraTerm program is ready to take data. Press any keyboard key and release the disk. The Arduino will send data for 10 s and then stop automatically. This data will auto-scroll up TeraTerm's window. Once the mass reaches its bottom, stop the disk and rewind the string for the next run.

Use the mouse to "Edit/Select All" in TeraTerm. Use the mouse to "Edit/Copy Table" in TeraTerm. Save 'Angular Momentum.ga3' from the website at

'http://groups.physics.northwestern.edu/lab/'

and execute it using the blue arrow at Firefox' upper right. Click Row 1 under the Time column and paste your data with ctrl+v or "Edit/Paste". Save your data in Ga3 format, analyze the data with relevant curve fit(s) and save the data again. Record the angular acceleration, uncertainty, and units in an appropriate data table. Delete the old data from TeraTerm using "Edit/Clear Buffer". Repeat this procedure until you have at least five combinations of hanging mass (to get torque) vs. the resulting angular acceleration. The measured masses of the weights and weight holder is shown in Table 5.1.

Once data from all five torques has been gathered, click the down triangle beside "1: Kinematics" on the toolbar and select "2: v Torque" instead. This changes Ga3's display page so that now your data table and graph should be ready for you to enter each total hanging mass and the corresponding angular acceleration. Each pair of entries automatically plots the point on the graph.

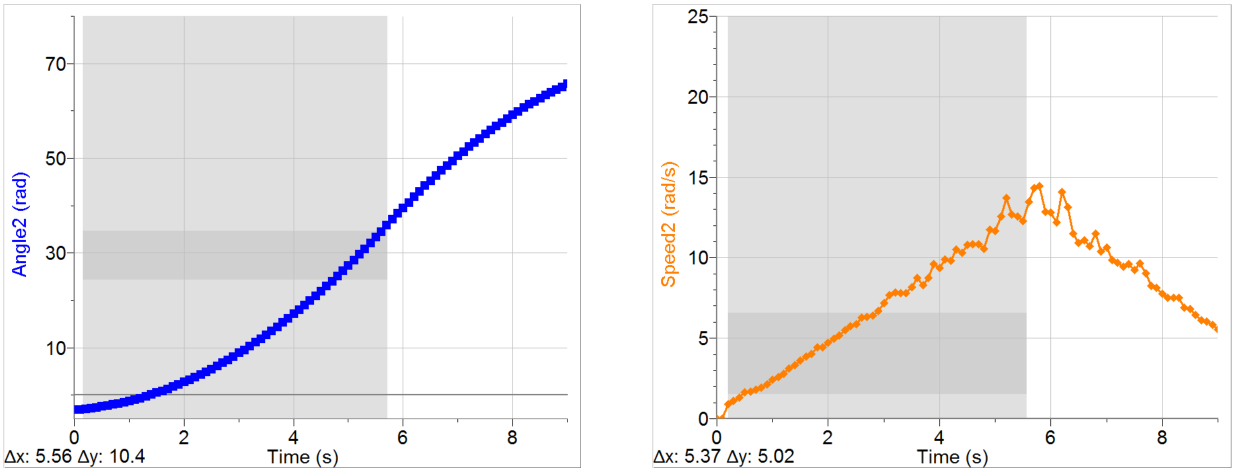


Figure 9.3: *Example data showing disk angle vs. time and angular speed vs. time. We are interested only in the constant acceleration portion prior to six seconds; this cut-off time will vary. After this time the weights are lifted causing constant deceleration* $\alpha < 0$*). If the string is long enough, the weights will be at rest on the floor for a constant velocity period between these.*

Helpful Tip

If you want, you can switch back and forth between these two pages. Analyze a parabola or a line to get acceleration on page "1: Kinematics" and then switch to page "2: v Torque" to add the acceleration and hanging mass to the plot. This strategy will immediately alert you to a bad data point so that you can retake the data immediately.

Once all five data points have been plotted, drag the mouse cursor across the graph to select all five points, "Analyze/Curve Fit.../Proportional", "Try Fit", and if it looks good "OK". The constant of proportionality (A) should be the moment of inertia of your disk about the rotation axis. If the uncertainty is not shown, right-click the parameters box, "Fit Properties...", and enable "Show Uncertainties".

Record the slope of the best fit line, the units of the slope, and the uncertainty of the slope. Determine the moment of inertia of the disk by measuring the radius of the disk and calculating the moment using the formula for moment of inertia of a disk given in Equation (9.4). (The aluminum disk has mass $M_{\text{aluminum}} = (0.517 \pm 0.001)\,\text{kg}$ and the spindle has mass $M_{\text{spindle}} = (13.6 \pm 0.1)\,\text{g}$.

WARNING

When weighing the disks, place the disk *gently* on the scale or you will break the scale's force sensor.

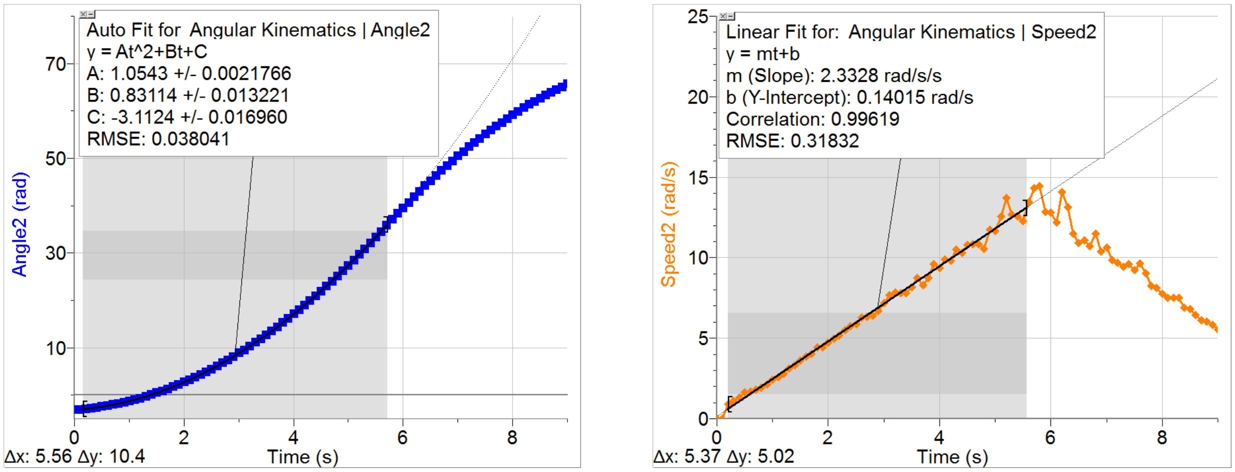


Figure 9.4: *Example data with curve fits. The angle vs. time data is parabolic and the speed vs. time data is linear. The fit to angle is more accurate and precise; however, the '$A \pm \delta A$' parameter must be doubled in this case.*

Use their respective masses and radii in Equation (9.4) and then add the results. You might find that the spindle and the pulley are negligible relative to the disk and its uncertainty.) Speaking of uncertainty in calculated moment of inertia, we can find it from the uncertainties in R and M. Review Chapter 2 to learn to do this.

9.3.2 Angular Acceleration Inversely Proportional to the Moment of Inertia

In this experiment we will measure the angular acceleration of the disk with a fixed hanging weight at the end of the string. We will do this several times while varying the moment of inertia of the disk. This will show whether angular acceleration is inversely proportional to moment of inertia and whether the constant of proportionality is the torque. Obtain values for the angular acceleration of the combination of disks for each of four values of moments of inertia using Equation (9.4).

Three disks of exactly the same dimensions are provided; two are made from steel and one is made from aluminum. The mass of the steel disks is $M_{\text{steel}} = (1.480 \pm 0.001)\,\text{kg}$ and the mass of the aluminum disk is $M_{\text{aluminum}} = (0.517 \pm 0.001)\,\text{kg}$. The two different masses give two moments of inertia, but the apparatus is designed to allow only the aluminum, only the steel, the steel and aluminum, or two steel disks to spin for a total of four different moments of inertia. Place the heaviest weight used in Section 9.3.1 above on the weight hanger. Since the combination of this mass and only the aluminum disk was measured above, it is not necessary to repeat that experiment; merely use the data gathered above. Use this same hanging weight to measure the angular acceleration of the remaining three combinations of disks (aluminum and steel, steel alone, and two steel disks).

Always reattached the spindle to the top disk. Make a table of the results. Enter the data into Ga3 page "3: v Inertia". Do your data lie in a straight line? If not, can you rearrange Equation (9.2) to make your plot linear? You might find that a suitable calculated column is provided... Once again fit to a proportionality model; review the end of Section 9.3.1 if you don't recall how to do that. Record the slope, its uncertainty, and its units in your notebook; this is the torque that Newton's Second Law predicts for the weight on your string.

Use Equation (9.1), the spindle radius, and the mass hanging from the string to calculate a measured torque (slope of the line). The computer has already computed this torque on page "2: v Torque"; but it might be a good idea to check its work. Use the uncertainties in hanging mass and spindle radius to estimate the uncertainty in this prediction. Don't forget to include your units as well.

An even more thorough procedure would be to recognize that the power, n, of the moment of inertia relation I^n can be obtained by applying the natural log to the expression $\alpha = \tau I^n$ to give $\ln(\alpha) = \ln(\tau) + n\ln(I)$. This is the equation of a straight line ($y = a + bx$; $a = y$-intercept, $b =$ slope) where n is slope of the line and $\ln(\tau)$ is the y-intercept. You can take the natural log (ln[ ]) of your angular acceleration – moment of inertia pairs and feed them into Ga3. Ga3 can then fit them to a straight line, "a + n * x", from which you can deduce the power, n, and the torque, e^a. If our hypothesis is correct, then the ideal exponent should be $n = -1$. Selecting "Analysis/Regression" from the menu will provide uncertainties for the fit parameters. Alternatively, Ga3 can fit your original data to the power law directly if you enter "T * x^ n" into the fit model edit box. The program will do the calculation for you. This is so common that a "Power law" fit $y = Ax^B$ is already provided.

9.4 Analysis

What does statistics say about how well our data agrees with Newton's second law in rotational form? Record reasons for any discrepancies or deviations from your best fit lines in your discussion of results. Can you think of any forces or torques that might exist in reality and that we have omitted from your analysis?

Can you think of sources of error that we have not incorporated into your total errors? Is the string tension truly $T = mg$ or is there a correction that we should have applied? Are the data points we gave Ga3 linear? What does this imply about Newton's second law?

9.5 Report Guidelines

Your Lab Notebook should contain the following:

- Measurements of the spindle and disk radii and masses of disks and weights (with uncertainties and units).
- Calculation of torque (and uncertainty?).
- Example of ω vs. t (or θ vs. t) and measurement of α.

- Calculation of the relevant moments of inertia (and uncertainty?).
- Plots of the angular acceleration vs. torque and the angular acceleration vs. moments of inertia.
- Differences and expected errors.
- Other thoughts and observations.

We expect that students should already have begun learning to compose reports for themselves. We do advise that they keep Appendix E, their past reports, their graders' comments, and the example report foremost in their attention.

Chapter 10

Experiment 8: Oscillations and the Pendulum

Periodic motion is one of the most important concepts in physics, and it is fitting that we end the first quarter of a physics lab on mechanics by setting the stage for describing a phenomenon that prevails in essentially *all* areas of physics. Here, we will treat the mechanical motion of a physical pendulum. If you continue in physics, it will become clear that this motion is directly analogous to a great many physical processes, and the description of oscillations is of fundamental importance.

Historical Aside

Oscillations truly are ubiquitous in physics. A thermodynamic system near equilibrium can oscillate. Vibrations of a solid are oscillations. Light can be considered as oscillations of the electromagnetic field. Quantum mechanics is described by oscillating wavefunctions. Atomic clocks are periodic oscillations of atomic states. The list goes on and on...

In this laboratory you will observe the motion of a pendulum to decide whether Newton's laws predict its motion correctly. You will time the period of oscillation using two technologies, you will evaluate the quality of these measurements, and you will consider how measurement strategies might be improved. This lab relies heavily on what you have learned in the third lab about torque and moment of inertia. If you are not thoroughly familiar with these concepts, read the previous lab manual chapter again before attending this lab. We will further begin to understand how oscillations and periodic motion can result from conservative force(s) acting on a mass.

10.1 Background

A pendulum is a rigid body mounted on a fixed horizontal axis, about which it is free to rotate under the influence of gravity. In the schematic representation of the pendulum shown

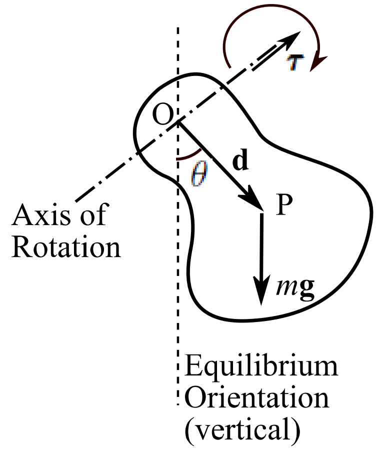


Figure 10.1: *Sketch of a physical pendulum showing relevant physical parameters.*

in Figure 10.1, O represents a point through which the axis of rotation passes, and P is the center of mass. The line OP makes an instantaneous angle, θ, with the vertical line, which gives the position of equilibrium of the pendulum. The weight of the pendulum of mass, m, is the force applied to its CM. A torque of magnitude

$$\tau = |\boldsymbol{\tau}| = |\mathbf{d} \times m\mathbf{g}| = mgd \sin\theta \tag{10.1}$$

will force the pendulum to rotate around the axis through O. The torque direction is determined by the "right hand" rule. The CM will describe an arc of a circle so the motion is a circular one. The angle θ will fully describe the position of the pendulum with respect to its position of equilibrium as time goes on. Because one parameter, θ, is sufficient to describe the motion, it is said that the system has one 'degree of freedom'. (The number of degrees of freedom of a system is the minimum number of parameters necessary to describe its motion. Linear motion along the x-axis is characterized by 1 parameter: the position x; this motion has one degree of freedom. Translational motion in a plane can be characterized by two parameters, (x, y); this motion has two degrees of freedom, etc.). To derive the equation of motion, $[\theta(t)]$, we must solve Newton's law for rotational motion using the expression for torque acting on a pendulum:

$$I\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = I\alpha = \tau = -mgd \sin\theta \tag{10.2}$$

We make the assumption that the angle of oscillation, θ, is small so that $\sin\theta$ can be replaced by θ in radians. Then Newton's law for the equation of motion of the pendulum is

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{mgd}{I}\theta = 0 \tag{10.3}$$

This *differential equation* can be solved and has solution given by

$$\theta(t) = \theta_{\max} \cos(\omega t + \varphi) \;\; \text{where} \;\; \omega^2 = \frac{mgd}{I} \tag{10.4}$$

Since the argument of cos has radian units and parameter t has units of seconds (s), parameter ω must have units rad/s. From the units of m, g, d, and I, we see that ω has units 1/s; however, radians are arc length/radius whose length units cancel from the division resolving this seeming contradiction. We must be careful to distinguish between the parameter ω and the angular velocity of the pendulum's rigid body. We call parameter ω angular (or radian) frequency, which enhances the tendency to confuse; however, ω is the angular frequency of cosine's argument only in this case even though in other situations the rigid body might spin continuously with a constant angular velocity of the other sort instead of oscillating back and forth as in this case. It is, perhaps, unfortunate that ω has these two different definitions, but it is also because both kinds of motion utilize the sine and cosine functions for their descriptions that we name this parameter ω for both kinds of motion. Constants θ_{max} and φ are constants of integration and can be uniquely determined from the angular position and angular velocity of the pendulum at $t = 0$. These initial conditions are the angular position, $\theta(0)$, and the angular velocity, $\frac{\mathrm{d}\theta}{\mathrm{d}t}(0)$, when the clock was first started. These integration constants can sometimes be determined uniquely from angular position specified at two different times, $\theta(t_0)$ and $\theta(t_1)$.

This equation describes harmonic motion, and the pendulum is called a harmonic oscillator. This is true only when the oscillations are small. We might note that replacing cos by sin also solves the problem and one frequently sees this alternate representation. Since sin and cos are each periodic with period 2π rad, we can introduce a period of time, T, that is the period of our equation of motion,

$$2\pi = \omega T_0 \ \text{ so } \ T_0 = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}} \tag{10.5}$$

We use T_0 for the motion of a pendulum that is harmonic. . . only for $\theta_{\mathrm{max}} << 1$ rad; however, the motion is periodic for $\theta_{\mathrm{max}} < \pi$ rad. We use T for this more general periodic motion. When we discuss periodic motion, we describe it using its angular frequency and its period; however, we also sometimes describe it using its frequency, f, defined to be how many back and forth oscillations occur in each second of time. It is perhaps not surprising that any one of these three parameters completely determines this aspect of periodic motion and that there are relations that allow us to determine all three once any one becomes known,

$$\omega = 2\pi f \ \text{ and } \ f = \frac{1}{T} \tag{10.6}$$

We will not prove so here, but the period of a pendulum's motion for all amplitudes is given by

$$T = \frac{2T_0}{\pi} K\left(\sin\frac{\theta_{\mathrm{max}}}{2}\right) \approx T_0 + a\theta_{\mathrm{max}}^2 \tag{10.7}$$

The special function $K(x)$ is the complete elliptic function of the first kind; for amplitudes larger than $\frac{\pi}{4}$ or so, we will need to evaluate this function instead of using the approximation in cases where the period needs to be predicted accurately.

The pendula shown in Figure 10.1 and Figure 10.3 are called physical pendula. Often the

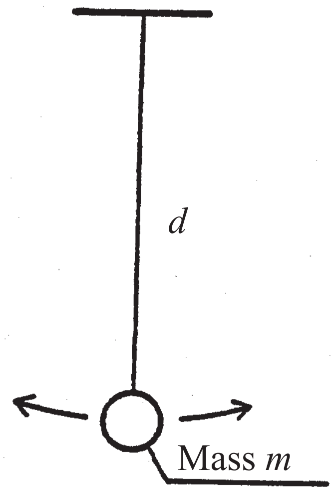


Figure 10.2: *A sketch of a simple pendulum made of a single point mass.*

actual pendulum consists of a long lightweight bar or cord, which serves as a support for a small, massive bob. The idealization of this into a point mass at the end of a weightless rod of length d is called a simple pendulum. In a simple pendulum, the length of the support, d (Figure 10.2), becomes identical to the distance between the axis of rotation and the center of mass of Figure 10.1. The moment of inertia, I, equals md^2 for a point mass. Equation (10.5) for the period then becomes

$$T_{\text{simple}} = 2\pi\sqrt{\frac{d}{g}} \tag{10.8}$$

The period of motion of a pendulum for small amplitudes is therefore independent of its amplitude, and depends only on the geometry of the pendulum and on the local value of g, the acceleration due to gravity. Pendula have therefore been used as the key element in clocks, or inversely to measure g or to measure the moment of inertia, I, of an object.

The analysis of the pendulum motion based on energy considerations is very interesting. When you displace the bob from its vertical position of equilibrium to some angle θ, you do a certain amount of work. This work is stored in the pendulum in the form of gravitational potential energy. When you now release the pendulum this energy oscillates from potential energy (parameterized by position) into kinetic energy (parameterized by speed) as it swings back and forth. The friction of the pendulum, in the air and at the point of suspension, will slowly dissipate the energy. You can view the oscillation as a cyclic transformation of energy from one form (potential) into another (kinetic) and back again. It is indeed always true in physics that oscillations (in mechanical systems, electrical circuits, or in optical devices) take place only when you have two energy reservoirs, and a mechanism whereby energy is transferred from one system to the other, repeatedly back and forth, until the motion is completely damped out.

10.2 Apparatus

The experimental apparatus for this experiment consists of two copies of a mass on a bar that each act as a physical pendulum (Figure 10.3). These bars are attached to a metal frame. Angle demarcations are present to read the pendulum amplitudes.

WARNING

Do not force the pendulum in any direction other than the plane of oscillation. It will break the suspension!

WARNING

Do not move the pendulum frame. The pendulum frame is level. Unstable oscillation will break the suspension!

We will measure the pendulum's period using a stopwatch application whose link may be found on the lab's website.

Next, we will measure the period automatically using Pasco's Capstone program and 850 Universal Interface, which is connected to a photoelectric system that includes a light emitting diode (LED) and a phototransistor. These two devices are located inside the two gold cylinders mounted on the pendulum base. The LED generates a light beam which hits the phototransistor. When the pendulum swings between the LED and phototransistor, it blocks the beam, separates the LED - phototransistor couple, and starts Capstone's clock. After a complete oscillation the pendulum will interrupt the beam and signal Capstone to measure the past cycle and to begin a new cycle. The photocouple was developed and built in the electronics shop of the High Energy Physics group at Northwestern University especially for this lab.

10.3 Procedure

10.3.1 Calculate the Moment of Inertia

Calculate the mass, m, and the moment of inertia, I, of one of the two identical pendula of your setup. Each pendulum, shown in Figure 10.3, consists of a shaft and a bob, both made of aluminum, with cylindrical shape. The moment of inertia of the entire pendulum will be the sum of the contributions from the shaft and from the bob,

$$I_{\text{pendulum}} = I_{\text{shaft}} + I_{\text{bob}} \tag{10.9}$$

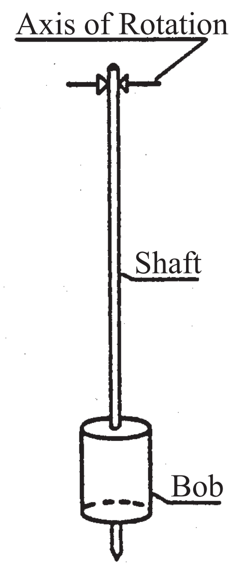


Figure 10.3: *Illustration of the pendulum in our experiment.*

Appendix 10.6 has a drawing showing the physical dimensions of the pendula and shows how to calculate I_{bob} as an example. To determine I_{pendulum} you must now calculate I_{shaft}. In order to have enough time during the lab to complete the experiments, **these calculations must be completed before you come to the lab.** (Note that all of the information is supplied in the appendix.)

10.3.2 Predict the Period

Calculate the period, T_0, of one of the two identical pendula of your setup using Equation (10.5) for a physical pendulum. To calculate T_0, you need the mass, m, and the moment of inertia, I, of the pendulum. You also need to know the distance, d, between the axis of rotation and the CM of the shaft-bob system. Appendix 10.6 also shows you how to determine the location of the CM.

Helpful Tip

Perform these calculations at home prior to attending the lab.

10.3.3 Simple Pendulum Period

Helpful Tip

Perform these calculations at home prior to attending the lab.

Calculate the period, T_{simple}, of one of the two identical pendula in your setup, using Equation (10.8) for the simple pendulum. Again, T_{simple} and T_0 should be calculated before the lab, with all the information needed available in Appendix 10.6.

10.3.4 Time the Pendulum Period

WARNING

To prevent damaging the suspension, do not displace the pendulum outside of the vertical plane. Do not put marks on the scale indicating the angle of displacement, θ_{max}.

Measure the period, T, of one of the two identical pendula of your setup. First, access the stopwatch application from the lab website. How might you use the stopwatch to measure one period of the pendulum's motion? Practice this strategy a few times and then measure the period nine times. Take turns with your partner(s) to minimize any systematic bias to start/stop early or late. Enter these measurements into Ga3, plot them, and evaluate their mean and standard deviation. What do you imagine to be the primary reason why your measurements vary from try to try?

Can you invent a strategy to improve your measurement of period? Such a strategy must reduce the *effect* of your primary error. For example, you might envision dividing your primary error among several periods instead of concentrating it all in a single period. If the period itself varies substantially, this strategy will fail or (possibly) mislead you. Once you invent a strategy, use this method to measure the period nine more times, enter these measurements into Ga3, plot them, and evaluate their mean and standard deviation.

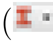
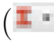
The standard deviation is the typical uncertainty in *each* single measurement. Knowing this compare the two standard deviations above and decide whether your strategy succeeded. If you failed to reduce your uncertainty in period by 5× to 10×, improve your strategy further or try to invent a more effective strategy. Once you have a strategy that works well, express your best estimate of measured period as

$$T = (\overline{m} \pm s_{\overline{m}})\,\mathrm{U}. \tag{10.10}$$

If necessary, review Section 2.6.1 to learn how to do this.

10.3.5 Automated Timing: The Photo-gate

Let the computer automatically measure the period, T_θ. We first need to execute the 'Pendulum.cap' from the lab's website. This should setup Capstone to gather pendulum period. Hold the pendulum at rest at the desired oscillation amplitude and click the "Record" button at the bottom left; release the pendulum and allow it to oscillate for at least 10 complete periods. If Capstone is configured correctly, it will plot the period for each oscillation on the y-axis and the time the measurement was completed on the x-axis. Press "Stop" after at least 10 data points are gathered to complete this sequence of measurements. The "Record" button changes into the "Stop" button and vice versa when it is clicked.

Hover the mouse cursor over the graph so that the toolbar will appear at the top. The toolbar will contain an icon () for selecting data points. Click this selector button to bring up a pale area surrounded by eight sizing squares. You can drag the selected area with the mouse and you can change its shape by dragging one of the sizing squares. Shape and position the selected area so that all of the data points are selected. Compute the statistics for this data using the statistics button () on the toolbar. If the mean ($\bar{T}_\theta$), the standard deviation (s_{T_θ}), and the number of data points (N) are not all displayed, click the little down triangle to the right of the () button and check all that are absent. You may optionally un-check those statistics that we do not need.

Use these statistics to record the mean and the deviation of the mean

$$T = \left(\bar{T} \pm \frac{s_T}{\sqrt{N}}\right) \text{ U} \tag{10.11}$$

If you need more significant digits, you can right-click the parameters box, 'Properties. . . ', and change the significant digits that are displayed. Round the uncertainty to one-two significant digits and round the mean to match.

We might imagine that our data could be described by a simple pendulum. The data in the Appendix gives $d = (0.613 \pm 0.001)\,\mathrm{m}$. Use this and Equation (10.8) to predict a simple pendulum period and estimate its uncertainty. Don't forget to include the units for your predicted period.

10.3.6 Extrapolate to Zero Amplitude

Enter your data table into Vernier Software's Ga3 graphical analysis program. Double-click the column headers to correct the column labels and to add correct units. Plot the periods on the y-axis and the amplitudes on the x-axis. Draw a box around the data points with the mouse. Once all data points are selected, "Analyze/Curve Fit. . . " and fit the data to Equation (10.7). Scroll to the last few fit models and look for 'Pendulum'. If you find it, verify the function to be

$$\text{T0 + a * x ^ 2} \tag{10.12}$$

If the equation is anything else, click its radio button, delete it, create a new fit function, type Expression 10.12 into the edit box, type "Pendulum" into the name, and press "OK". Now, select the 'Pendulum' function by clicking its radio control and "Try Fit". If the model curve passes through your data points, "OK". If the fitting parameter T0's uncertainty is not displayed, right-click on the parameters box, "Fit Properties. . . ", and select "Show Uncertainties". Record your T0, its uncertainty, and its units in your notebook. This is an extrapolated estimate for the limit of your pendulum's period as its amplitude approaches zero. In this limit, $\sin\theta = \theta$ exactly; and Equations (10.2) and (10.3) are exactly the same.

10.3.7 Coupled Pendula

Helpful Tip

This section describes a very advanced experiment. We include it for the Integrated Sciences Program and to explain the purpose of the unused pendulum.

In nature there are many fascinating examples of systems which have two degrees of freedom (this means that two parameters are sufficient to describe the time dependence of the motion).

General Information

Some of the most beautiful examples of coupled oscillators involve molecules (for instance, ammonia NH_3) and elementary particles (the system of neutral K°, $\bar{K}^\circ$ mesons). To study them thoroughly requires a knowledge of quantum mechanics. Many examples of coupled systems with two degrees of freedom can also be found in electrical circuits.

There is no set procedure for a coupled pendulum experiment provided here. If you have time in the lab after completing your data analysis, you may explore this more complicated system with two degrees of freedom. Here are some thoughts to consider with two pendula:

- You have a system of two pendula. How many different, distinct "modes" of oscillation can you identify? Would 'symmetric' and 'antisymmetric' have any meaning in describing the modes you identify?
- How can you make the system oscillate in one or the other mode?
- Which is larger: the period of the symmetric or antisymmetric mode?

10.4 Analysis

A thorough consideration of the dimensions in the appendix and of their uncertainties leads to a predicted $T_0 = (1.5549 \pm 0.0012)\,s$.

Does Equation (10.4) describe the pendulum's motion that you observed? What does your data say about the periods of pendula? Is this true for both the simple and the physical pendulum? Use complete sentences and define all symbols in your equation(s). Find the difference between the value of T_0 predicted by Equation (10.5) and the value measured. Compare your measured T_0 to the simple pendulum prediction of Equation (10.8). Which periods (if either) agree with your best measurement?

What do your observations say about the equation of motion we found using our mathematical studies of differential equations? Do you think it likely that a function of time chosen at random would perform as well? If so, go ahead and produce one.

Can you think of any forces (i.e. torques) that acted on the pendulum (other than the gravity that we included in our equation of motion)? What other forms of energy might the original gravitational potential energy have become? What might be the effect of these forces (torques) on our pendulum's motion?

What sources of experimental error have we omitted from the total error calculated above? For each one elaborate on the probability that it is negligibly small compared to those we have included and on the ease with which its error might be evaluated numerically. Consider the complexity of the pendulum's motion and prediction using Newton's second law of rotational dynamics while discussing the accuracy and efficacy of Newton's second law.

10.5 Report Guidelines

Your Lab Notebook should contain the following:

- Details about the stopclock, pendulum, photogate, and timer.
- A calculation of the moment of inertia of the physical pendulum.
- A calculation of the periods for the physical pendulum and the simple pendulum.
- Stopclock measurements of the pendulum period.
- Stastics of stopclock measurements.
- A table of period vs. amplitude.
- A plot of period vs. amplitude (with fit to model).
- Uncertainty estimates and discussion.
- Comparison of predicted period of simple *and* physical pendulum period to measured T0; respective error analysis.
- Discussion of losses and other error sources.

We expect that students should already have begun learning to compose reports for themselves. We do advise that they keep Appendix E, their past reports, their graders' comments, and the example report foremost in their attention.

10.6 APPENDIX: Moment of Inertia

10.6.1 Moment of Inertia

Figure 10.4 is a drawing showing the definitions and shape of the pendulum used in this laboratory experiment. Figure 10.5 shows all of the relevant dimensions of the pendulum components. The concept of moment of inertia was introduced in the seventh lab. The moment of inertia of a cylinder whose base is a circle of radius, R, and whose height is h, is given in Equation (10.13), in the case that the cylinder is rotating around an axis through its center of mass and perpendicular to the symmetry axis of the cylinder, as shown in Figure 10.4(a).

$$I_{\mathrm{CM}} = \frac{M}{12}(3R^2 + h^2) \tag{10.13}$$

If the axis of rotation is parallel to the one in Figure 10.4a, but at a distance X as shown in Figure 10.4b, then the parallel axis theorem gives

$$I = I_{\mathrm{CM}} + MX^2 \tag{10.14}$$

(M is the mass of the cylinder). If the bob were a point mass (the radius of a point is zero), then $I_{\mathrm{CM}} = 0$ and this becomes the moment of inertia of a *simple pendulum*. The moment of inertia of the physical pendulum is

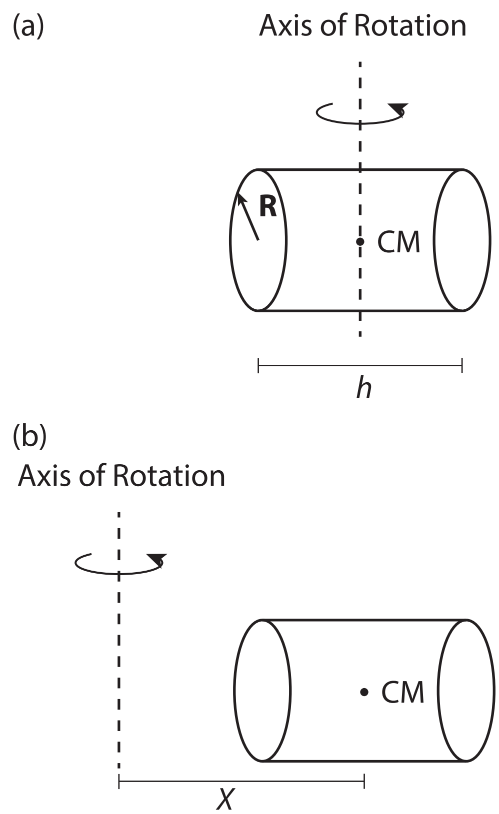


Figure 10.4: *Drawing detailing coordinates of this pendulum.*

$$I_{\mathrm{pendulum}} = I_{\mathrm{shaft}} + I_{\mathrm{bob}} \tag{10.15}$$

General Information

Any strategy that allows the rotation of the object to be neglected will result in a simple pendulum. One example includes isolating the massive bob from the light shaft using quality ('frictionless') bearings. Another is to utilize *bifilar* suspension to prevent rotation.

10.6.2 The Bob

We calculate as an example the bob's moment of inertia, I_{bob}. The bob has a cylindrical shape with radius R and it has a hole of radius r (for the shaft) along its symmetry axis. The I of the bob, relative to its CM, is calculated with Equation (10.13) as illustrated in Figure 10.6. The bob is a solid cylinder with a hole drilled down its axis so its moment of inertia is the moment of inertia of a solid cylinder minus the moment of inertia of the material that was drilled out of the hole.

To calculate these quantities (I_{solid} and I_{hole}) we need the mass of the two aluminum cylinders, their radii, and their height. The dimensions of the two cylinders are included in Figure 10.5 and the mass density of aluminum can be found in tables to be $\rho_{\text{Al}} = 2.7 \times 10^3$ kg/m^3. Since density is defined to be mass/volume, we can find the masses of the cylinders from their volumes and aluminum's density:

$$\begin{aligned} M_{\text{solid}} &= \rho V_{\text{solid}} = \rho \pi R^2 h \\ &= (2.7 \times 10^3 \text{ kg/m}^3)\pi(2.54 \times 10^{-2} \text{ m})^2(0.0953 \text{ m}) \\ &= 0.52152 \text{ kg} \end{aligned}$$

is the mass of the solid cylinder before the hole was drilled and

$$\begin{aligned} M_{\text{hole}} &= \rho V_{\text{hole}} = \rho \pi R_{\text{hole}}^2 h \\ &= (2.7 \times 10^3 \text{ kg/m}^3)\pi(3.97 \times 10^{-3} \text{ m})^2(0.0953 \text{ m}) \\ &= 0.01274 \text{ kg} \end{aligned}$$

is the mass removed to drill the hole. We can now use these masses, the dimensions in Figure 10.5, and Equation (10.13) to find the moments of inertia of these two cylinders about their centers-of-mass;

$$\begin{aligned} I_{\text{solid}} &= \frac{M_{\text{solid}}}{12}(3R_{\text{solid}}^2 + h^2) \\ &= \frac{0.52152 \text{ kg}}{12}[3(2.54 \times 10^{-2} \text{ m})^2 + (0.0953 \text{ m})^2] \\ &= 4.7882 \times 10^{-4} \text{ kg m}^2 \end{aligned}$$

for the solid cylinder and

$$\begin{aligned} I_{\text{hole}} &= \frac{M_{\text{hole}}}{12}(3R_{\text{hole}}^2 + h^2) \\ &= \frac{0.01274 \text{ kg}}{12}[3(3.97 \times 10^{-3} \text{ m})^2 + (0.0953 \text{ m})^2] \\ &= 9.692 \times 10^{-6} \text{ kg m}^2 \end{aligned}$$

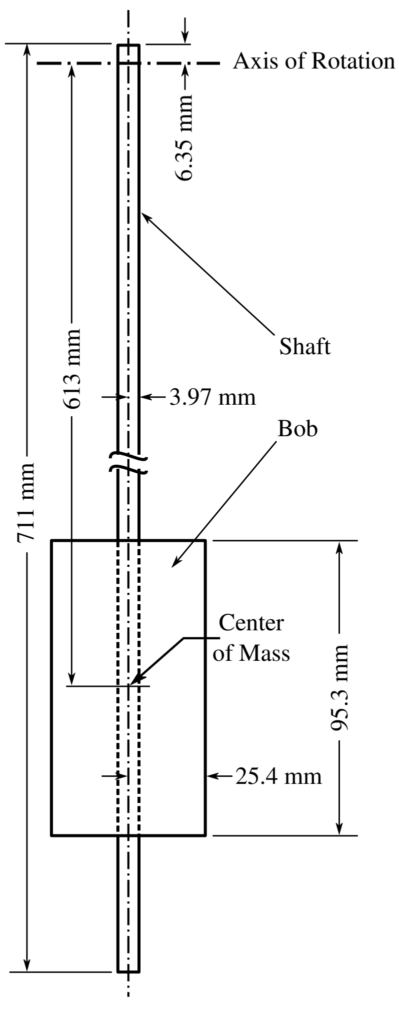


Figure 10.5: *Illustration of the pendulum in our experiment, showing relevant dimensions.*

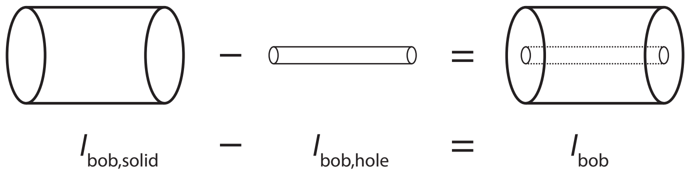


Figure 10.6: *Illustration of the strategy to calculate the moment of inertia of a cylinder with coaxial hole.*

if the mass that was removed from the hole were to be re-assembled into its original cylinder. If we review Figure 10.6 we can find the moment of inertia of the bob containing the hole as it stands by subtracting the moment of inertia of the material that filled the hole prior to drilling from the moment of inertia of the solid cylinder;

$$I_{\text{bob}}^{\text{CM}} = I_{\text{solid}} - I_{\text{hole}} = (4.7882 - 0.0969) \times 10^{-4}\ \text{kg m}^2 = 4.691 \times 10^{-4}\ \text{kg m}^2 \qquad (10.16)$$

is the moment of inertia of the bob about its own center-of-mass. Unfortunately, the axis of rotation is some distance, X_{bob}, away from the bob's center-of-mass and perpendicular to the axis of the bob's axis of symmetry. Recall that Equation (10.13) also applies to an axis perpendicular to the cylinders' axis of symmetry (see Figure 10.4) so that the axis of rotation is parallel to the axis for which $I_{\text{bob}}^{\text{CM}}$ is the moment of inertia for the bob. This is the required condition for us to use the parallel axis theorem in Equation (10.15). The mass that we need to use is the mass of the bob not the mass of the solid cylinder and not the mass of the hole but, rather, the difference between them,

$$M_{\text{bob}} = M_{\text{solid}} - M_{\text{hole}} = 0.52152\ \text{kg} - 0.01274\ \text{kg} = 0.50878\ \text{kg}$$

and

$$I_{\text{bob}} = I_{\text{bob}}^{\text{CM}} + M_{\text{bob}} X_{\text{bob}}^2 = 4.691 \times 10^{-4}\ \text{kg m}^2 + (0.50878\ \text{kg})(0.613\ \text{m})^2 = 0.1917\ \text{kg m}^2$$

is the moment of inertia of the bob about the axis of rotation.

Checkpoint

We might note that after all of our hard work, $I_{\text{bob}}^{\text{CM}}$ only changed the least significant digit from 2 to 7; the vast bulk of the moment of inertia of the bob is due to the center-of-mass being located far from the axis of rotation. For most rotational systems we will not need so much accuracy in which case we could have treated the bob as a point mass. It turns out, however, that oscillating systems like our pendulum are remarkably accurate so that even this miniscule mistake would have resulted in a noticeable oversight. The accuracy of a pendulum is the reason they are chosen to be time standards for clocks.

10.6.3 The Shaft

It is now **left as an exercise for the student** to perform a subset of this exercise to calculate the moment of inertia of the cylindrical shaft about the rotation axis and to add that value to the moment of inertia of the bob shown above. Since the shaft is a simple solid cylinder, it will not be necessary to subtract out a hole; the remaining steps will be required, however.

10.6.4 Calculating the Center of Mass

Consider the torques on a continuous or distributed mass system as shown in Figure 10.1. Each piece of mass experiences a gravitational force and tends to rotate the rigid body about the axis of rotation. Larger masses will experience larger weights and correspondingly larger torques. Continuous bodies can be treated using calculus; however, we are also aware now that all matter is made from atoms and molecules so that we can also treat each atom as a point particle exactly like we will do here. Since each particle experiences a torque due to its gravity, we can find the total torque by adding all of these torques together,

$$\tau = \sum_{i=1}^{N} x_i m_i g \tag{10.17}$$

We might also consider the possibility that we can pretend all of the masses were at the same point and ask the question "Where should that point be if its weight is to provide the same torque as the extended body?" In this case the torque is simply

$$x_{\mathrm{CM}} M g = \tau = \sum_{i=1}^{N} x_i m_i g \tag{10.18}$$

where $M = \sum_{i=1}^{N} m_i$ is the total mass. For these torques to be equal we must have

$$x_{\mathrm{CM}} = \frac{1}{M} \sum_{i=1}^{N} m_i x_i \tag{10.19}$$

We call x_{CM} the "center of mass" or the "center of gravity" of the object. Referring to Figure 10.1 again, we admit the possibility that the axis of rotation might be in the left-right direction instead of into the page. In this case the torques about the axis of rotation would concern the y coordinate instead of x; however, exactly the same line of reasoning would have led us to

$$y_{\mathrm{CM}} = \frac{1}{M} \sum_{i=1}^{N} m_i y_i \tag{10.20}$$

in that case. We also note that where we place the mass along the vertical line passing through (x, y) does not affect the torque about either axis, so we might as well define

$$z_{\mathrm{CM}} = \frac{1}{M}\sum_{i=1}^{N} m_i z_i \tag{10.21}$$

as well. In fact, we can combine all of these into a single vector equation

$$\mathbf{r}_{\mathrm{CM}} = \frac{1}{M}\sum_{i=1}^{N} m_i \mathbf{r}_i. \tag{10.22}$$

We need to use Equation (10.5) to predict the period of our pendulum, but we do not know yet the value of parameter d. Studying Figure 10.1, however, in view of the discussion of center of mass above we see that $d = |\mathbf{r}_{\mathrm{CM}}|$. Figure 10.5 details the locations of the centers-of-mass of the two masses making up our pendulum so we can find that

$$\begin{aligned} d &= \frac{M_{\mathrm{bob}}\, x_{\mathrm{bob}} + M_{\mathrm{shaft}}\, x_{\mathrm{shaft}}}{M_{\mathrm{bob}} + M_{\mathrm{shaft}}} \\ &= \frac{(0.50878\,\mathrm{kg})(0.613\,\mathrm{m}) + (0.09505\,\mathrm{kg})(0.3492\,\mathrm{m})}{0.50878\,\mathrm{kg} + 0.09505\,\mathrm{kg}} \\ &= 0.57147\,\mathrm{m} \end{aligned} \tag{10.23}$$

The CM of the pendulum is just 4 cm above the CM of the bob. If you had substituted the CM of the bob for that of the pendulum (bob + shaft), the error would have been small. The accuracy of our measurement would have detected this discrepancy, however.

Appendix A

Physical Units

In science, we describe processes in Nature using mathematics. Math is very concise, structured, and adaptable so that once we have mathematical models that mimic Nature, we can make very specific and accurate predictions about what will happen in other situations. Engineers use this fact to very good effect to obtain useful results in society.

Physical laws are usually expressed in terms of physical quantities that have dimensions. The definition of units that are used to describe these dimensions is the raison d'être of the National Institute of Standards and Technology (NIST). An example that we all should have seen by now is Newton's second law,

$$\mathbf{F}(\mathbf{a}) = m\mathbf{a} \tag{A.1}$$

Given a mass whose "value" is m, to get motion whose acceleration is $\mathbf{a}$ we must apply a push or a pull whose "value" is $\mathbf{F}$. Since a push or a pull is an *entirely different entity* than a mass, we include in $\mathbf{F}$ a different multiplicative variable (N, dyne, pounds, etc.) than we include in m's value (kg, g, slugs, etc.). The specific point that we need to address now is "What about the parameter $\mathbf{a}$?" What intrinsic multiplicative variable should we include in $\mathbf{a}$'s "value" so that the model is mathematically self-consistent? We can use algebra to figure this out.

Let the unknown units be x and consider the specific case of $F = 1\,\mathrm{N}$ and $m = 1\,\mathrm{kg}$. To solve our problem we substitute these values into the model and solve for our unknown, x,

$$1\,\mathrm{N} = F = ma = (1\,\mathrm{kg})(1\,x) \quad \text{so} \quad x = \frac{\mathrm{N}}{\mathrm{kg}} \tag{A.2}$$

If we multiply all of $\mathbf{a}$'s pure numbers by the units N/kg , then the unknowns (and unknowables) that we call units will always cancel in such a way that forces will always have units N, masses will always have units kg, and accelerations will always have units N/kg. Because all of these are always true, our model is mathematically self-consistent. Of course, kinematics has taught us that $\mathrm{N/kg} = \mathrm{m/s^2}$ and that we can also determine acceleration by measuring the distances that objects move, the rates that objects move, and the times needed for these motions and rates to occur. One of the most beautiful and compelling aspects of physics is the frequency that a single physical entity (acceleration in this case) can be determined in more than one way (sometimes many, many more than one).

Let us now move on to a more general case. Let us pretend that we have developed a model represented by

$$f(x) = ax + bx^3 \tag{A.3}$$

Without more information the units of f, x, a, and b are ambiguous; so, let us stipulate that f is force (N) and that x is position (m). Is this enough information for us to determine the units of a and b? Our first instinct might be to answer that "No, one equation cannot yield two unknowns"; however, the rules of algebra must be obeyed as well. When we add the two quantities ax and bx^3, they must have the same units. Furthermore, these common units will be assigned to f, so the results of these multiplications had better yield units of N! Now our problem has reduced to exactly the same problem as we solved above; we just have to solve it twice in the present case. Let A be the units of a, let B be the units of b, and substitute into the model:

$$1\,\mathrm{N} = f = ax = (1\,\mathrm{A})(1\,\mathrm{m}) \quad \text{so} \quad \mathrm{A} = \frac{\mathrm{N}}{\mathrm{m}} = \frac{\mathrm{kg}}{\mathrm{s}^2} \tag{A.4}$$

$$1\,\mathrm{N} = f = bx^3 = (1\,\mathrm{B})(1\,\mathrm{m})^3 \quad \text{so} \quad \mathrm{B} = \frac{\mathrm{N}}{\mathrm{m}^3} = \frac{\mathrm{kg}}{\mathrm{s}^2\,\mathrm{m}^2} \tag{A.5}$$

Models having any number of terms can be handled in exactly this same way; we just have to solve the problem once for each term as above. Most students will also soon find short-cuts that will make this much more expedient than it may seem just now.

Converting units

Before looking at more complex situations, we will summarize how to convert between different physical units. Over the years many, many systems of units have been conceived and used for specifying distance. Historically, it has been important for economic, political, legal, personal, and scientific reasons that we have the ability to specify distance concisely and accurately. From a need to know our height or shoe size to a need to know the area of a land tract or to know how far away is the center of government, we need the ability to measure and to communicate distances. The problem is that different purposes benefit from somewhat different units of measure. It does not make sense to measure the distance to Los Angeles in human hair-widths, for example, even though you often hear small things being referred to "the thickness of a human hair" (~ 100 micrometers, by the way). To compare 'apples' to 'apples', we invariably have to convert between different units of measure.

The simplest strategy to convert between systems of units is to place the unit for distance in one system beside the unit for distance in the other system and simply to see how they compare. We use one of the "yardsticks" to measure the length of the other "yardstick". As an example, we might use a meter stick to measure the length of a foot long ruler; when we do so we will find that $1\,\mathrm{ft} = 0.3048\,\mathrm{m}$. We could also use the foot ruler to measure the meter stick and to find that $1\,\mathrm{m} = 3.2808\,\mathrm{ft}$. Of course, these numbers are inverses of each other, which is required for the measurements to be consistent. This implies that

$$0.3048\,\frac{\mathrm{m}}{\mathrm{ft}} = 1 = 3.2808\,\frac{\mathrm{ft}}{\mathrm{m}} \tag{A.6}$$

Let us suppose that we are given some distance $d = 33.24\,\mathrm{ft}$ and that we need to communicate this distance to Paris, France, where no one has heard of "ft"; over there everyone measures distance in "m". We can multiply anything by 1 without changing its value. This applies to d as well. We see that $1 = 0.3048\,\mathrm{m/ft}$ so that multiplying by $0.3048\,\mathrm{m/ft}$ is just multiplying by 1.

$$d = 33.24\,\mathrm{ft} \;= 33.24\,\mathrm{ft} \times 1 = 33.24\,\mathrm{ft} \times 0.3048\,\frac{\mathrm{m}}{\mathrm{ft}} = 10.13\,\mathrm{m} \tag{A.7}$$

Dimensionless Functions

Frequently we find expressions like

$$f(x) = f_0 e^{ax} \text{ or } g(x) = g_0 \ln\left(\frac{x}{x_0}\right) \tag{A.8}$$

These functions belong to the class of exponential functions (recall that $y = e^x$ if and only if $x = \ln y$). What must the units of f_0, a, g_0, and x_0 be in order that f and g be force (N) and that x be distance (m)? First, realize that $e \approx 2.7183$ is just a number, no different than 3, 8, or 1 as far as units are concerned. Multiplying pure numbers together does not (and cannot) cause units to appear; therefore, raising e to a power (multiplying e by itself over and over) also results in a pure number without units. Then f_0 and g_0 must have the same units as f and g, respectively.

An exponent necessarily has no units. The only way to convert between units is to multiply by ratios of units and only raising to higher exponents $(x^a)^b = x^{ab}$ yields products of exponents. Units are algebraic unknowns so that having units in an exponent is equivalent to having the exponent to be unknown… not to know how many times to multiply the base. We might invent a rule or a convention simply to ignore the units – simply pretend that they are not there – just raise the base to whatever number is there. But that would mean that

$$1.6332 = 5^{0.3048\,\mathrm{m}} = 5^{1.000\,\mathrm{ft}} = 5 \tag{A.9}$$

which is clearly not true. On the other hand, requiring that exponents have no units works out a little better. To show this we re-arrange our english-metric conversions a little

$$1 = 3.2808\,\frac{\mathrm{ft}}{\mathrm{m}} \quad \text{and} \quad \frac{\mathrm{ft}}{\mathrm{m}} = 0.3048 \tag{A.10}$$

and manipulate them similar to the way we did above. Let b be some arbitrary number and note that

$$b = b^1 = b^{3.2808\,\frac{\mathrm{ft}}{\mathrm{m}}} = b^{(3.2808)(0.3048)} = b^1 \tag{A.11}$$

which is self-consistent.

Exponents must be pure numbers. For similar reasons the arguments of logarithms must also be unitless, pure numbers. To see this recall that

$$\ln\left(\frac{x}{x_0}\right) = \ln(x) - \ln(x_0) \tag{A.12}$$

If x and x_0 have the same units, the units cancel on the left and leave a pure number to be the logarithm's argument. If they have different units, then any unit conversion applied to x_0 on the right would change the value subtracted from $\ln(x)$ thus implying that the quantity on the left can simultaneously be equal to both values. So, how do we deal with the right side of Equation (A.12) if x and x_0 have units? We understand that we *actually* mean something different:

$$\ln(x) - \ln(x_0) = \ln(x) - \ln(x_0) + \ln(a) - \ln(a) = \ln(x/a) - \ln(x_0/a) = \ln\frac{x}{x_0} \tag{A.13}$$

The constant a must have the same dimensions as x and x_0, and although it cancels entirely from the expression, it must be there to make the logarithm's dimensionless argument make sense. Incidentally, this is also why logarithmic scales like the Richter scale for earthquakes and the dB scale for sound always have a reference intensity (a definite value with units), equivalent to x_0 or a in the discussion above.

Sine, cosine, and tangent are trigonometric functions whose arguments are usually derived from angles. Trigonometric functions arguments are dimensionless, but they are not unitless; these arguments must have the units of their angular measure. What this means is that if you scale physical dimensions, the angles do not change... if you change your angular measure, then the angle's numerical value (degrees or radians) does change. Frequently, we deal with expressions like

$$x(t) = x_0\cos(\omega t + \phi) \tag{A.14}$$

If x has units of m and t has units of s, what are the units of x_0, ω, and ϕ? The cosine function itself has no units so x_0 has the units of x. The argument of the cosine is a polynomial, so each term must have the angular units that the cosine function needs. The natural units of angle are radians so that a circle has 2π radians. The units of ω must be rad/s. You might notice a similarity between logarithms, exponentials, and trigonometric functions: their arguments are all dimensionless. This is not an accident. Mathematics typically concerns functions applied to *numbers*. Thus, mathematical expressions tend to be defined in a way such that they have no dimensions. It is only in physics, where we have to compare real measurements of different quantities, where the reference units appear. Therefore, in these mathematical functions, we must convert the argument to pure numbers for the expressions to make sense.

Appendix B

Using Vernier Graphical Analysis

Additional guidance is generally provided in the lab experiment chapters.

Helpful Tip

There are two version of this software on the lab computers. The older one, 'Gax' is not well-liked and has caused many TA and student headaches throughout the years. The newer version, 'Graphical Analysis 3.4' (Ga3) is more functional and reliable but requires a few more steps to get it to do what you need it to do. Please use Graphical Analysis 3.4 in this course. Excel and Gax are always available if you prefer, however.

When we perform a least squares fit to a mathematical model, we are effectively compiling all of the knowledge gained by our data into a much smaller set of fitting parameters. To the extent that our data applies to this particular model, the experimental errors that are inevitable in measurements are averaged out so that we might utilize the fitting parameters to improve upon a prediction of our model beyond the data points themselves. This does not mean that the fitting parameters have no uncertainties associated with them; indeed they do have and it is essential that we specify each uncertainty when we discuss the parameter.

Many times Graphical Analysis 3.4 does not show the uncertainties in fitting parameters by default. We can always ask it to do so by right-clicking the parameters box, "Properties...", and select "Show Uncertainties". We can also specify other properties in the fitting parameters such as the number of significant digits.

Not only is the older Gax somewhat unstable, but it also offers meaningful estimates of fitting parameter uncertainties *only* for the separate 'linear regression'; custom models have no convenient way to get uncertainty estimates. Since fitting parameters are effectively measurements that the computer has deduced from the data, it is essential that we be able to specify the uncertainties and the units that must accompany them. These are the reasons why we prefer Ga3 that has a less obvious user interface.

Appendix C

Using Microsoft Excel

C.1 Creating plots and curve fits

Helpful Tip

The primary advantage of Excel is that students have access to it at home. However, it is more challenging to perform statistical-based fits in Excel unless you write your own spreadsheet or macros. It is recommended that you perform your data analysis with Graphical Analysis 3.4 in the lab. However, you always have access to Excel in many locations in case you want to plot outside the lab.

This short list of resources will help guide you in creating figures in Excel, if you decide to use it.

- A good introduction to basic graphing in Excel is provided by the LabWrite project:

  https://www.ncsu.edu/labwrite/res/gt/graphtut-home.html

- Curve fitting in Excel can be performed, but sometimes it is much more painful than using a dedicated data analysis program. For least-squares curve fitting, this link details how to use Excel:

  http://www.jkp-ads.com/articles/leastsquares.asp

  Neither do Excel's least-squares fitting parameters come specified with uncertainty estimates even after you go through these acrobatics.

- We can use Excel's "LINEST" function to provide complete statistics on straight lines. See 'linest' in Excel's Help files to learn more about its usage.

C.2 Performing Calculations

All calculations in spreadsheets begin with "=". First, select the cell that will hold the result of the calculation, type "=", and enter the formula to be evaluated. Typing "=5+9" and 'Enter' will yield "14" in the cell; the cell beneath will then be selected. If you need a cell to contain '=' instead the results of a calculation, precede it with a single quote '''... anything you type after that will be displayed verbatim. Even numeric characters won't be numbers after '''.

If this was the extent of their capabilities, spreadsheets would not be very popular. Each spreadsheet cell is at the intersection of a column having alphabetic label at the top and a row having numeric label at the left. 'A1' is the cell at the top left corner of the sheet, 'C8' is at the eighth row and third column, and 'Z26' is at the 26^{th} row and the 26^{th} column. After column 'Z' comes column 'AA' to 'AZ', 'BA' to 'BZ',..., 'ZA' to 'ZZ'. If even more columns are needed, three, four, and five characters can be used. Column 'numbers' count up just like decimal digits, but column designations have base 26 instead of base 10.

As an exercise we could put the numbers 1-10 in cells A1:A10. Then in, say, B7 we could type '=A1+A2+A3+A4+A5+A6+A7+A8+A9+A10' to add the numbers (55). After we type '=', we could click on cell A1 to get the computer to place the 'A1' in the formula. This comes in handy in a large project when you don't remember the cell designation and it is not visible on the screen.

This is pretty nice and the total (cell B7) will automatically update anytime one or more of the numbers in A1:A10 changes. However, this would mean a lot of typing if the sum of, say, A1:A1000 were needed instead. It turns out that spreadsheets have a 'sum' function for adding the contents of cells. "=sum(A1:A10)" would also add the contents of these cells. ANY block of cells can be specified using "range" designators. "=sum(a1:z6)" will add the first six rows of the first 26 columns. The function names and column designators are not sensitive to case. The cell into which the result is to be placed MUST NOT be part of the range or an error will result. The 'A1:A10' or 'A1:Z6' can be generated by the computer by dragging the mouse across the desired range of cells.

In addition to 'sum', we could 'average', 'stdev', 'count', etc. to perform statistics on a range. Many other functions are part of the standard spreadsheet application. A partitioned list and usage instructions can be generated using the 'function' toolbar button; usually the icon has 'f', 'f_x', '$f(x)$', or such.

Frequently, we want to generate graphs. To do this we first need to generate the independent (x-axis) values. Enter '10' into say C3 and '=0.1+C3' into C4. Now, go back to C4 and ctrl+c to copy this formula. Next, drag the mouse from C5 to C103 to select this range (the cells turn black) and ctrl+v to paste the formula into all of the selected cells. Now, C3=10, C4=10.1,..., C103=20. Click on one of these cells and look at the formula in the formula bar. The computer has automatically replaced the '3' in 'C3' with the correct number to add 0.1 to the cell above this one! This always happens by default when you copy and paste the formulas from one cell to another unless you "Edit/Paste Special..." and specify the details you want pasted.

For relatively smooth functions, these 101 values will yield a very nice graph. Smaller increments can also be generated at need for more quickly changing functions. Uniformly spaced numbers are needed so often that a shortcut exists. Type 10 into C3 and 10.1 into C4. Next, drag from C3 to C4 to select both and release the mouse button. The bottom right corner of the selected cells is a drag handle that can be used to extend the pattern 10.0, 10.1, 10.2,... by dragging this handle downward with the left mouse button. Rows of incrementing numbers can be generated similarly.

Now that we have our abscissa, we need to generate our ordinate. Click on D3 and enter '=3*sin(C3)'. Go back to D3 and drag the drag handle at the bottom right downward to D103. Release the mouse button and note that the result is a sinusoid with amplitude 3. This is OK! but we can generate **families** of curves with varying amplitudes, frequencies, or phases quite easily. Enter '2' into D2, '2.2' into E2,..., '3' into I2. Now enter "=D$2*cos($C3)" into D3. Go back to D3, drag the drag handle across to I3, and release the mouse button. Now, drag the drag handle down to I103 and note that we have generated six sinusoids with amplitudes given by row 2. The '$' prevented the '2' in 'D2' and the 'C' in 'C3' from changing. Using 'A1' or 'B3' would insert the number in A1 or B3 into all of the formulas; this would allow you to tweak these two numbers by hand to optimize all of the curves at once.

Practice generating various abscissas and ordinates so that you can remember these basics. Also practice plotting the results and pasting the plots into a Word document. You will want 'scatter plots' so that you can choose the (x, y) points.

Appendix D

Using Microsoft Word

Helpful Tip

A nice template containing a report outline and hints for performing many of the functions science reports, journal publications, and books often employ can be downloaded from each lab's website. All word processing programs have similar capabilities and many can import the template... although perhaps not seamlessly. Hopefully, this will get you started using word processing software.

For your electronic write-ups, you can embed figures directly in the Word files, or you can upload them separately. If you want to embed in Microsoft Word while preparing your document, here are some useful sources of assistance.

- To embed an Excel figures directly into Word, you can follow the instructions here:

  https://support.office.com/en-us/article/Insert-a-chart-from-an-Excel-spreadsheet-into-Word-0b4d40a5-3544-4dcd-b28f-ba82a9b9f1e1

- If you want to embed a pdf figure into a Word document, you can follow the instructions here:

  https://support.office.com/en-za/article/Add-a-PDF-to-a-document-9a293b43-45de-4ad2-a0b7-55a734cf6458

- Another set of instructions for embedding pdfs in Word, with a focus on preserving quality, can be found here:

  https://pagehalffull.wordpress.com/2012/11/20/quick-and-easy-way-to-insert-a-pdf-graph-or-diagram-into-a-microsoft-word-document-without-losing-too-much-quality/

- For embedding more general images in Word, and using word wrapping, see here:

  https://support.microsoft.com/en-us/kb/312799

With Microscoft Word, there are usually many ways to accomplish the same task. Some produce better results than others. This is why you always have the option of uploading figures and images separately from the text of your write-up.

Helpful Tip

Your goal for writing in this lab is clarity in communication, not professional quality documents. Beginning students have much difficulty including enough background information about their experiment and apparatus and they tend to include too much mundane detail that applies only to their specific apparatus instead. Eventually our work will be read by scientists all over the world, so it is essential that we practice including enough information about our apparatus and procedure that everyone in the world can understand our data while simultaneously avoiding these mundane details that will tend to confuse readers without our apparatus in front of them.

Appendix E

Composing a Report

E.1 Title and Partner Credit

Each report must have a relevant title. Each report must list all lab partners with the report author listed first. Technical reports begin at a general viewpoint discussing widely known science and angle toward more specific aspects of *this* experiment and the particular science that is within its scope.

E.2 Purpose or Introduction

We typically suggest 3–5 brief sentences that note the Purposes of the experiment. What physics will the experiment test? Usually physics is expressed as equations; however, frequently these equations are 'named'. The widely-known name is a preferred mode of address, but a forward citation to your Theory is second-best. (In this case Theory is absent so use the lab manual equation number.)

What new OEM instruments and devices will we learn to use? What physical quantities will we measure?

E.3 Theory

The Theory section would be quite similar to the mathematical development in this manual. Because of this, we do *not* require a Theory section in your reports. You may cite the equation numbers from this manual as if you had typed the relevant development into your report. In professional papers you *will* need to demonstrate that your apparatus and procedure are accurate representations of the science you wish to test; in these cases you may refer to similar developments in this manual for ideas in how to proceed.

E.4 Apparatus and Procedure

What specific instruments and devices were used? Manufacturer names and model numbers allow the reader to seek technical specifications directly from the manufacturer. How were the devices connected and/or utilized? What data did each device report? What system state did each device control? If the Data do not make it redundant, what Procedure was followed to generate the data? This section will answer these questions, but they can usually be answered in pairs and triplets by well structured sentences. Content is important, but length is not important.

Frequently, a sketch, a schematic diagram, or a photograph can convey much information and quickly. Students may copy and paste such illustrations from the lab manual, the website, etc.; however, he *must* cite the source in each case. **Citations must lead the reader directly and completely to the cited object.** A book name, publisher, and page number (or figure label) is a valid citation. A url and figure label is another. Furthermore, we can give a student permission to copy only the illustrations that we own. Copying from other sources is not allowed at all without prior permission from those authors.

E.5 Data and Results

Briefly state general observations and publish data tables and graphs to portray your measurements. Graphs usually will also include a fitted model of the physics equation(s). Frequently, there will also be 'singleton' measurements that do not logically fit into any of the tables. Sometimes a 'miscellaneous' measurements table can house these and other times these measurements are worked into paragraphs. The visibility and tidiness of tables and graphs make these favorite places to portray data.

All measurements have three pieces:

1. The best estimate m for the measured value,
2. An objective estimate δm of measurement uncertainty, and
3. Units (U) multiplying the best estimate and uncertainty.

No measurement is complete until all three pieces are included: $M = (m \pm \delta m)\,\mathrm{U}$. Measurements in paragraphs are disclosed using this format. Measurements in tables have the name (M) and units (U) in the table column headers and the best estimate and uncertainty ($m \pm \delta m$) in the respective table column. Graphs have the best estimate represented as a location on the graph, the uncertainty represented by error bars, and the measurement name and units in the axis titles. We will not specify error bars, but we will repeat some of the graph's measurements in suitable tables that do specify uncertainty. Professional reports will utilize only one of these formats for each data set; but students are not yet professionals.

As described in Section 2.5, the uncertainty tells us how well we know the measured value so we need only 1-2 significant digits in the uncertainty. Since the uncertainty does tell us how well we know the measured value, **it is necessary that these two numbers have *exactly* the same number of decimal places:** (10.2 ± 0.1), (13.935 ± 0.025), and (1250 ± 10) are all correctly specified (except for absent units); however, (10.2 ± 0.05), (10.225 ± 0.2), and (2.2258 ± 0.1254) **are all incorrect** at this level.

Usually the raw measurements will be used in physics model equations to predict other measurements. Show one example of how you calculated each of these predictions from the raw data; we prefer that you show the formula with symbols, the numbers substituting for the symbols, and the result with correct units. In 2–3 experiments during the quarter, your TA will also specify that you use the methods in Section 2.6.3 to specify the uncertainties in these derived results; please show one example for each of these as well when required. Additionally, specify which table columns contain raw data and which contain derived results.

E.6 Analysis of Results

In the Analysis section, we will decide whether our data prove anything and, if so, what they prove. First, do the model curves fit the data points? If so, this is strong evidence that the data supports the model *even if some of the model parameters are different than expected.*

To determine whether two numbers are distinguishable by the experiment, we rely heavily upon statistics to decide, but we also simply use the relevant statistics results without proof. The strategy (see Section 2.9.1) is to form NULL hypotheses by subtracting results computed using a model under test from a direct measurement taken to check the model

$$\Delta M = |m_{\mathrm{P}} - m_{\mathrm{M}}| \,. \tag{E.1}$$

We also need the acceptable error (σ or 'sigma') in this difference

$$\sigma_M = \sqrt{(\delta m_{\mathrm{P}})^2 + (\delta m_{\mathrm{M}})^2}. \tag{E.2}$$

General Information

The following observations are also discussed in Section 2.9.1.

Statistics tells us that 68.3% of the time $\Delta M < \sigma_M$ if M_{P} and M_{M} are taken from the same distribution. Similarly, 95.4% of the time $\Delta M < 2\sigma_M$ and 99.73% of the time $\Delta M < 3\sigma_M$. These probabilities get progressively more certain as agreement requirements get poorer. We may also turn this argument around. If we stated that $\Delta M > \sigma_M$ means that the prediction and measurement are different, then this statement would be wrong 32% of the time (about 1/3 of the time). Insisting that $\Delta M > 2\sigma_M$ means the numbers are different would make us wrong 4.6% of the time (about 1/22 of the time). Having $\Delta M > 3\sigma_M$ happens due to statistics alone only 0.27% of the time (about 1/370 of the time).

Being wrong 1/3 of the time is considered unsavory among scientists. Being wrong 1/22 of the time, even, is undesired. Thus we generally expect and state that when $\Delta M < 2\sigma_M$ the two numbers probably do come from the same distribution and any disagreement is due to "other sources of error." Contrarily, having $\Delta M > 3\sigma_M$ happens only 1/370 of the time so we usually expect that this difference probably has a **non-statistical** cause; we reject the hypothesis that these two numbers are taken from the same distribution. This does *not* necessarily mean that the physics model is incorrect, but this does remain one possibility. Other possibilities include 1) one or more of the assumptions needed for the model to be valid are not met, 2) we have underestimated (or completely overlooked) some of our uncertainties, 3) the environment has perturbed the experiment, 4) the experiment's performance was substandard, etc.

We also need to enumerate several of the most likely "other sources of error" that might cause any observed discrepancies however small. Our considerations of how well the model curves fit the data points, of how well the parameters match what we should expect, of how well other observations match the model's predictions are all relevant to the conclusions we draw.

E.7 Conclusions

This section should be very brief, but it also should be self-reliant. Which of our original hypotheses do our data support? Which do they contradict? Which are neither supported nor contradicted? We do *not* say why; (everything above says why.) Equations may be referenced via citation or accepted name (Newton's law, conservation of energy, Equation (4), etc.). What physical measurements have you made? These are constants independent of the experiment that we (or others) might need one day. Can you think of any applications for what we have observed? Finally, how might we improve the experiment if we should perform it again?

Appendix F

Submitting a Report in Canvas

Preparation

Your lab report should be written in either Microsoft Word or be converted to a text-readable pdf. Every student at Northwestern has access to Microsoft Word and Excel for lab report preparation. JPEG format is not allowed for the main text.

Both Word and pdf formats allow your TA to provide helpful feedback using Canvas' SpeedGrader system. Experience has shown that this online feedback is popular to students in the 136 labs. This formatting requirement is therefore beneficial to everyone involved.

If possible, it is easiest if all of your files are merged into a single document. In Word, images and figures can be embedded directly into the text file. If you are submitting a pdf prepared in another method, it is possible to attach images and figures saved in the lab as pdf documents into a single pdf file. Adobe Acrobat Professional can accomplish this. Alternatively, you can merge pdf files online. One possible provider is PDFMerge (http://www.pdfmerge.com/).

If you cannot put all of your content into a single file, it is accessible to load figures as separate files (such as jpeg files). If you submit as separate files, you should refer to each uploaded figure in some other part of your Lab Report. Your TA is not required to view or grade any document that is not referred to in other graded sections of your report. It is recommended that you name your files "Figure X" where X is a number. Then you can refer to each figure by name. Alternatively, you can upload figures as a single document such as a pdf, and have the appropriate labels on the figures in each document. Remember: a lab report is practice in communication of ideas and activities. Failure to make references to uploaded figures clear to your TA can result in lost points.

Submission

To submit your lab report, open the assignment in the Module on Canvas. Click the "Submit Assignment" link.

Under the File Upload tab, select the “Choose File” button and select the appropriate file. You can press “Add Another File” to create another upload link. When all files are ready and you agree to the TurnItIn pledge, you can press “Submit Assignment”. You are able to re-submit your assignment to Canvas, although if it is late it will be flagged. You should also be able to view your TurnItIn similarity score a few minutes after submission.

Your TA will be able to see all submitted files in the Canvas grading system. It is probably easiest on your TA if the first file that you upload is your main lab report text, followed by figure files in order. Do not make life harder for your TAs as they grade your work!