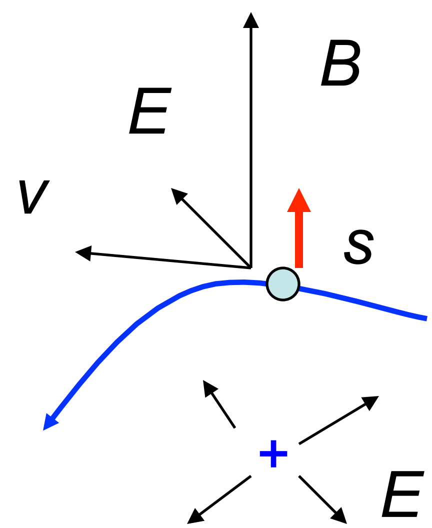
Origin of the spin-orbit interaction



Mott 1927

In a frame associated with the electron: $\mathbf{B} = \frac{1}{c}\mathbf{E} \times \mathbf{v} = \frac{1}{mc}\mathbf{E} \times \mathbf{p}$

Zeeman energy in the SO field: $\hat{H} = \frac{\mu_B}{mc}\vec{\sigma} \bullet (\mathbf{E} \times \hat{\mathbf{p}}) = -\frac{i\hbar^2}{2m^2c^2}\vec{\sigma} \bullet (\vec{\nabla} V \times \nabla)$

1/c expansion of the Dirac equation

$$\hat{H} = \underbrace{\frac{\hat{p}^2}{2m} + eV}_{\text{non-relativistic}} + \underbrace{\frac{\hat{p}^4}{8m^2c^2}}_{\text{K.E. correction}} + \underbrace{\frac{\hbar^2}{8m^2c^2}\nabla^2 V}_{\text{Darwin term}} + \underbrace{\frac{\hbar}{4m^2c^2}\vec{\sigma}\bullet(\vec{\nabla}V \times \hat{\mathbf{p}})}_{\text{SOI}}$$

All three corection terms are of the same order

Smaller by a factor of 2 (“Thomson factor of two”, 1926)
Reason: non-inertial frame

Landau& Lifshits IV: Berestetskii, Lifshits, Pitaevskii
Quantum Electrodynamics, Ch. 33

order of magnitude estimate

$$\mathcal{E}_{SO} \sim \frac{\hbar}{m^2c^2}\, p\, \frac{V}{r} \sim \frac{\hbar^2}{m^2c^2}\frac{V}{r^2}$$

$$V \sim \frac{Ze^2}{r}, \quad r \sim \frac{a_B}{Z} = \frac{\hbar^2}{Zme^2}$$

$$\mathcal{E}_{SO} \sim \frac{\hbar^2}{m^2c^2}\frac{Ze^2}{r^3} = Z^4\left(\frac{e^2}{\hbar c}\right)^2\frac{me^4}{\hbar^2}.$$

This needs to be multiplied by the probability to find an electron at distance r from the nucleus $= |\psi(r)|^2 r^3$.

WKB wavefunction in atomic units $(a_B = 1, \ me^4/\hbar^2 = 1)$

$r \sim 1/Z$; $V \sim Z/r$, $V|_{r\sim 1/Z} \sim Z^2$.

$$|\psi| \sim \frac{1}{r\sqrt{p}} \sim \frac{1}{r|V|^{1/4}} \sim \frac{Z}{(Z^2)^{1/4}} \sim \sqrt{Z}$$

$$|\psi|^2 r^3 \sim Z\frac{1}{Z^3} \sim \frac{1}{Z^2}.$$

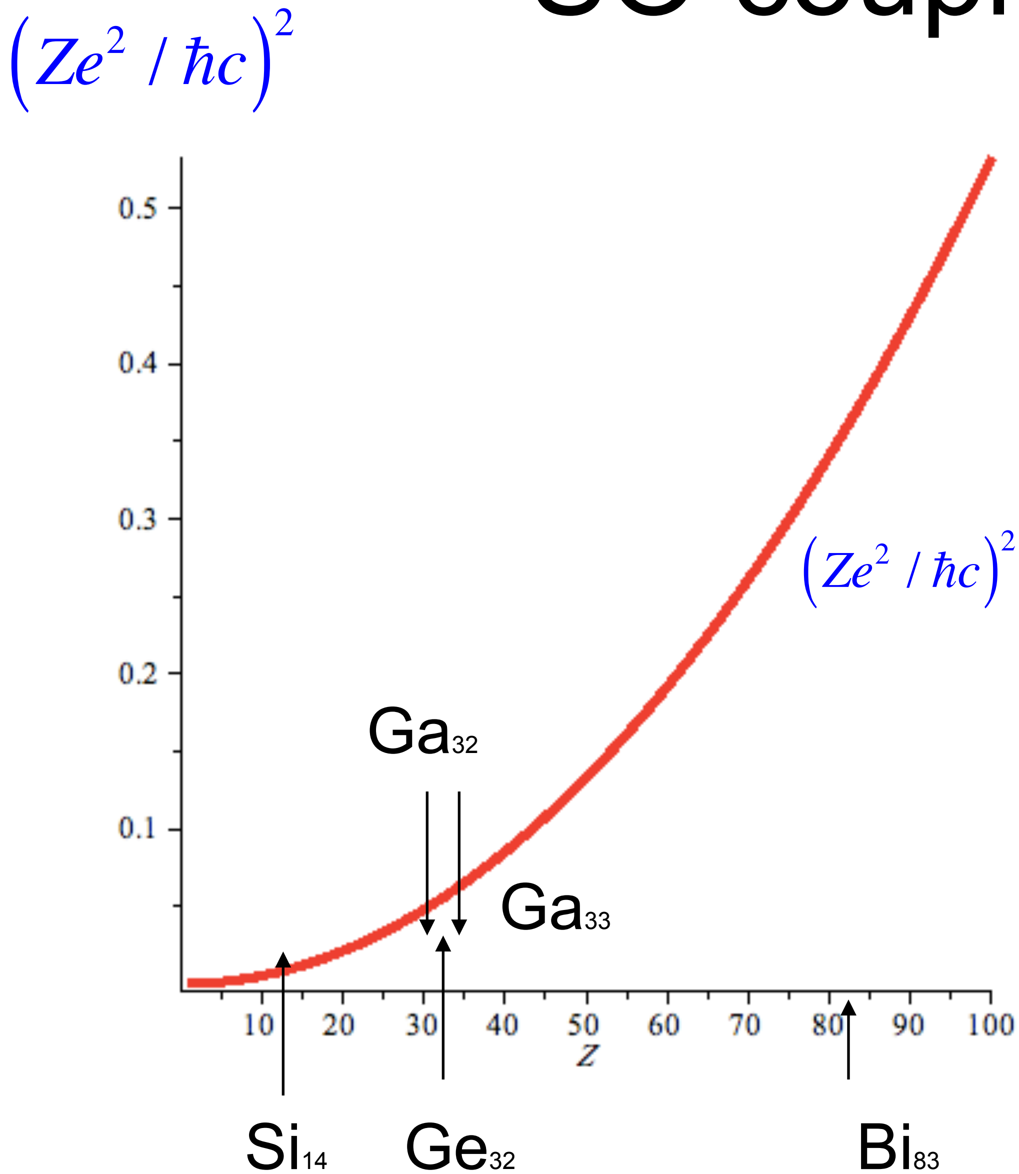
Typical energy of SO interaction

$$\mathcal{E}_{SO}\cdot\frac{1}{Z^2} \sim \left(\frac{Ze^2}{\hbar c}\right)^2 \underbrace{\left(\frac{me^4}{\hbar^2}\right)}_{\sim 10\,eV}$$

$$\frac{Ze^2}{\hbar c} = \frac{Z}{137}$$

^{92}U : $\left(\frac{Z}{137}\right)^2 = 0.45$

SO coupling

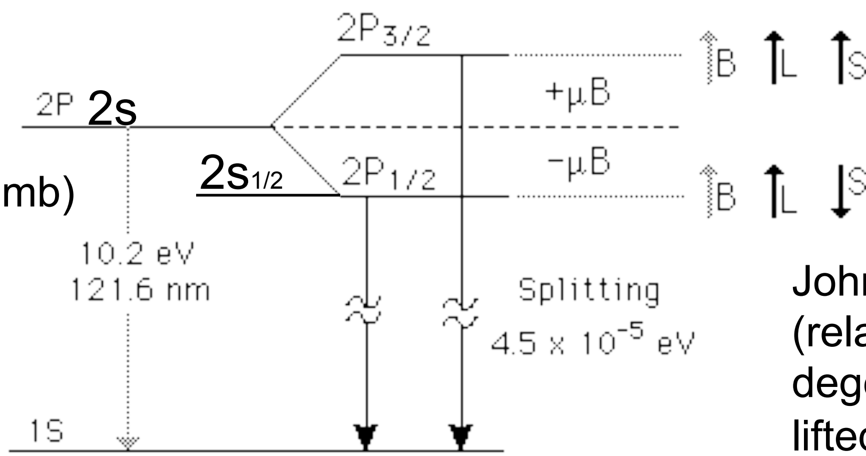


$\left(Ze^2/\hbar c\right)^2$	Si_{14}	Ga_{31}	Ge_{32}	As_{33}	Bi_{83}
	0.01	0.051	0.055	0.058	0.38

Fine structure of atomic levels

Hydrogen

Runge-Lentz
(non-relativistic Coulomb)
+spin degeneracy



Johnson-Lippman
(relativistic Coulomb)
degeneracy
lifted by radiation corrections
(Lamb shift)

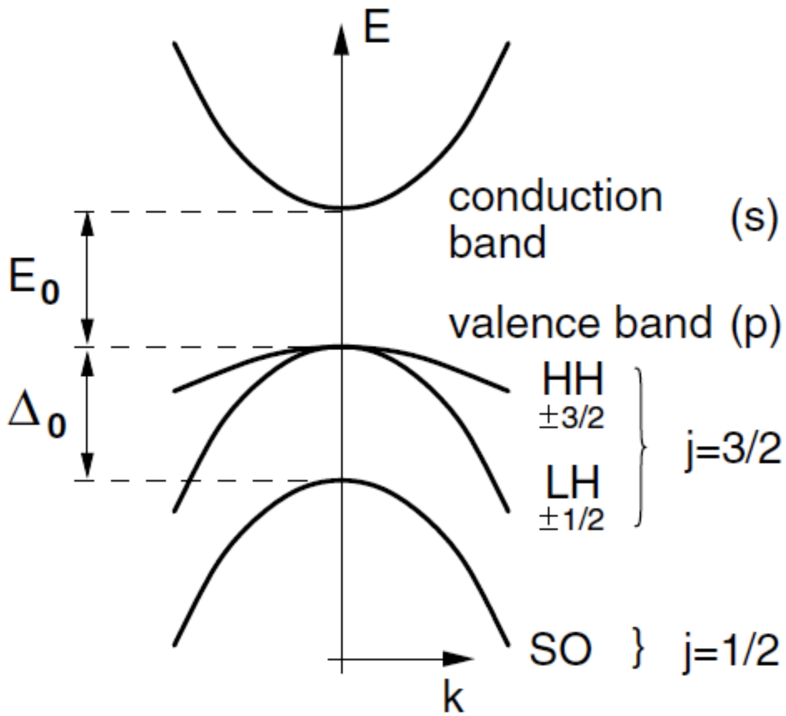
Two types of SOI in solids

1) Symmetry-independent:
   exists in all types of crystals
   stem from SOI in atomic orbitals

2) Symmetry-dependent:
   exists only in crystals without inversion symmetry
   a) Dresselhaus interaction (bulk): Bulk-Induced-Assymetry (BIA)
   b) Bychkov-Rashba (surface): Surface-Induced-Asymmetry (SIA)

Example of symmetry-independent SOI: SO-split-off valence bands

Winkler, Ch. 3



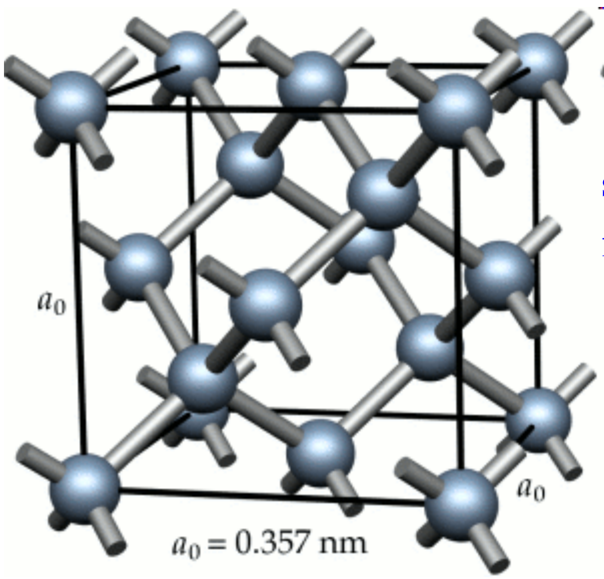
	Compound	Δ_0^{exp} (eV)	Δ_0^{theo} (eV)	f_i
	C	0.006	0.006	0
I	Si	0.044	0.044	0
	Ge	0.29	0.29	0
	α-Sn		0.80	0
	AlN		0.012	0.449
	AlP		0.060	0.307
	AlAs		0.29	0.274
	AlSb	0.75	0.80	0.250
	GaN	0.011	0.095	0.500
	GaP	0.127	0.11	0.327
no I	GaAs	0.34	0.34	0.310
	GaSb	0.80	0.98	0.261
	InN		0.08	0.578
	InP	0.11	0.16	0.421
	InAs	0.38	0.40	0.357
	InSb	0.82	0.80	0.321
	ZnO	−0.005	0.03	0.616
	ZnS	0.07	0.09	0.623
	ZnSe	0.43	0.42	0.630
	ZnTe	0.93	0.86	0.609
	CdS	0.066	0.09	0.685
	CdSe		0.42	0.699
	CdTe	0.92	0.94	0.717
	HgS		0.13	0.79
	HgSe		0.48	0.68
	HgTe		0.99	0.65

$$\left[\frac{(Z_{Ga}+Z_{As})/2}{Z_{Si}}\right]^2 = \left(\frac{Z_{Ge}}{Z_{Si}}\right)^2 = \left(\frac{32}{14}\right)^2 = 5.2$$

$$\Delta_{GaAs}/\Delta_{Si} = .34/0.044 \approx 7.7$$

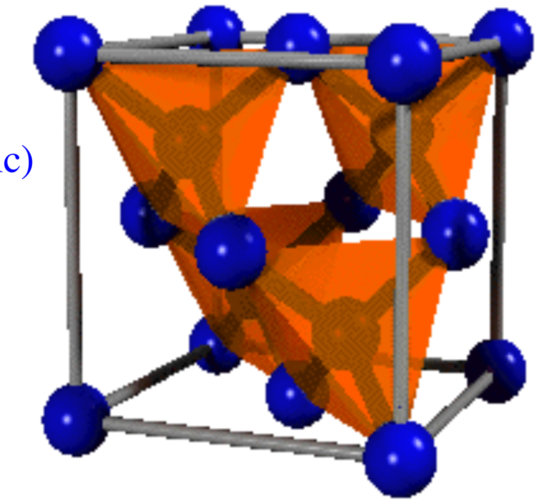
$$\Delta_{GaAs}/\Delta_{Ge} = .34/.029 \approx 1.2$$

Non-centrosymmetric crystals

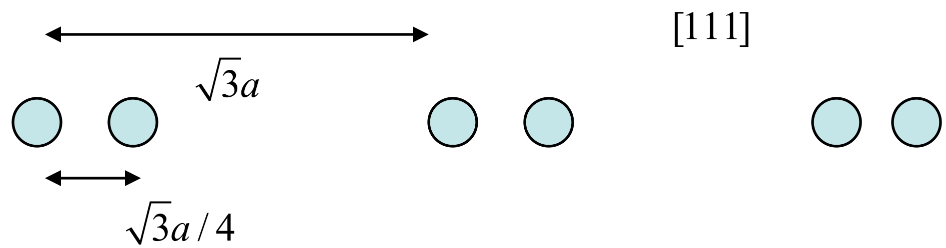


space group: O_h^7 (non-symmorphic)

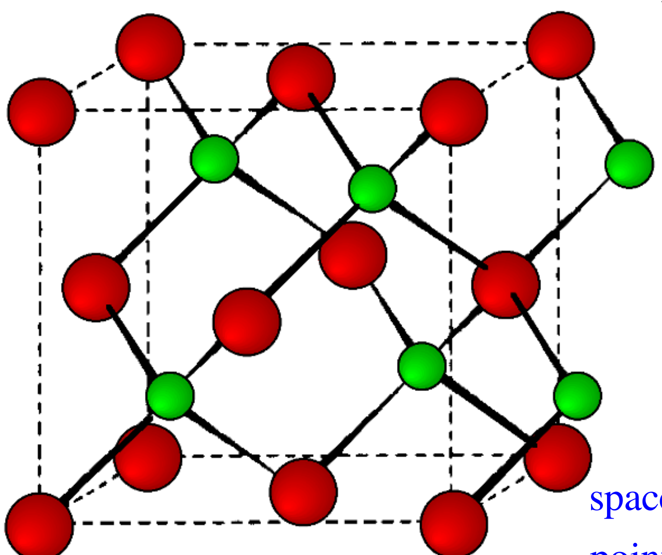
factor group $O_h = T_d \times I$

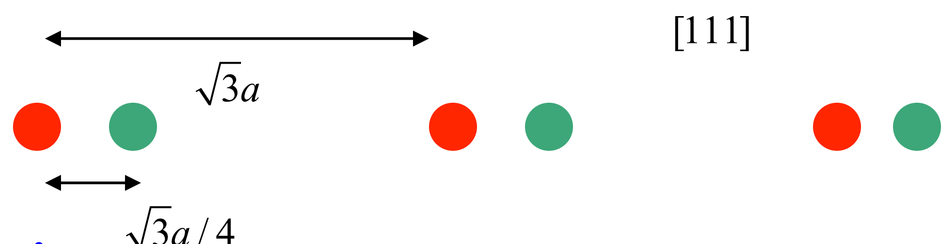


S. Sque (Exter)



Diamond (C): Si, Ge





space group: T_d^2 (symmorphic)

point group: T_d (methane CH_4)

Zincblende (ZnS): GaAs, GaP, InAs, InSb, ZnSe, CdTe …

Additional band splitting in non-centrosymmetric crystals

Kramers theorem: if time-reversal symmetry is not broken, all eigenstates are at least doubly degenerate

if ψ is a solution, ψ^* is also solution

Kramers doublets $\varepsilon_s(\mathbf{k}),\ s=\pm 1$ (not necessary spin projection!)

Time reversal symmetry: **k➔-k,**t➔-t $\varepsilon_s(\mathbf{k})=\varepsilon_{-s}(-\mathbf{k})$

No SOI: $\varepsilon(\mathbf{k})=\varepsilon(-\mathbf{k})$ regardless of the inversion symmetry

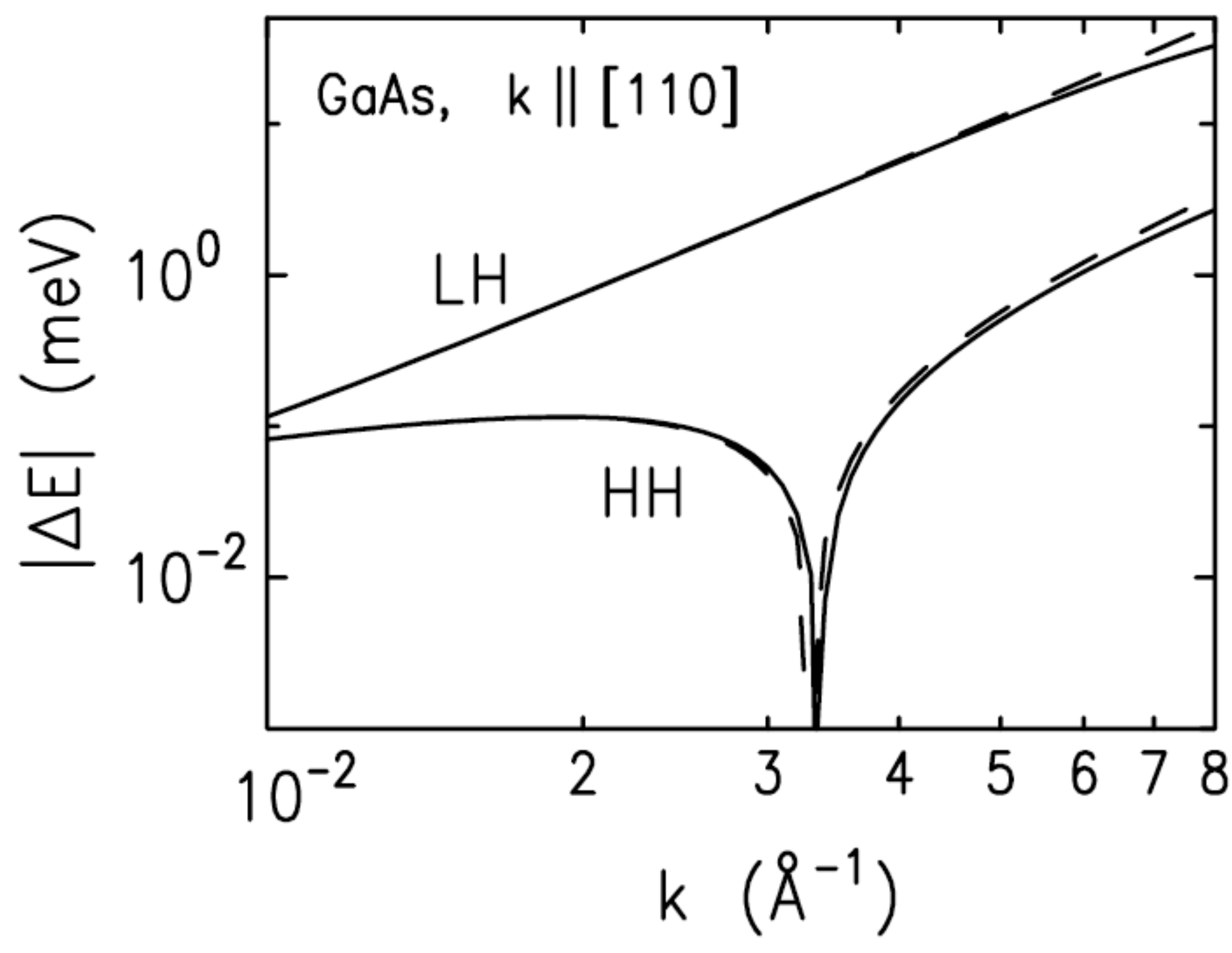
With SOI: i) If a crystal is centrosymmetric

$$\varepsilon_s(\mathbf{k}) \underbrace{=}_{t\to -t} \varepsilon_{-s}(-\mathbf{k}) \underbrace{=}_{\mathbf{k}\to -\mathbf{k}} \varepsilon_{-s}(\mathbf{k}) \Rightarrow \varepsilon_s(\mathbf{k})=\varepsilon_{-s}(\mathbf{k})$$

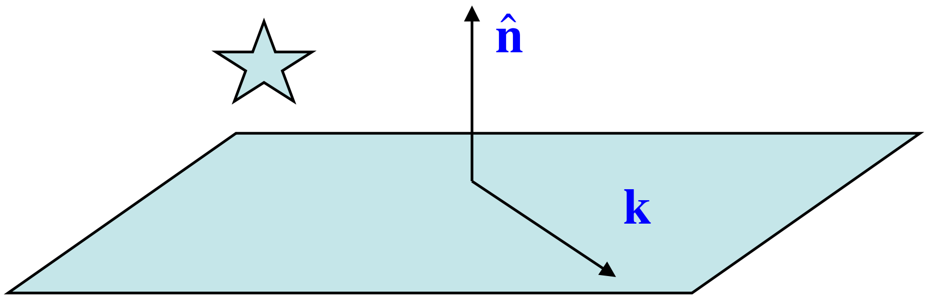
ii) If a crystal is non-centrosymmetric,

$\varepsilon_s(\mathbf{k})\neq\varepsilon_{-s}(\mathbf{k})$ B=0 “spin” splitting

Symmetry-dependent SOI: Dresselhaus band splitting in non-centrosymmetric crystals



Surface-induced asymmetry: Rashba interaction



z>0 and z<0 half-spaces are not equivalent:

$\hat{\mathbf{n}}$ has a specified direction

Three vectors: $\mathbf{k}, \hat{\mathbf{n}}, \vec{\sigma}$

E. I. Rashba and V. I. Sheka: Fiz. Tverd. Tela **3** (1961) 1735; ibid. 1863;
Sov. Phys. Solid State **3** (1961) 1257; ibid. 1357.

How to form a scalar?

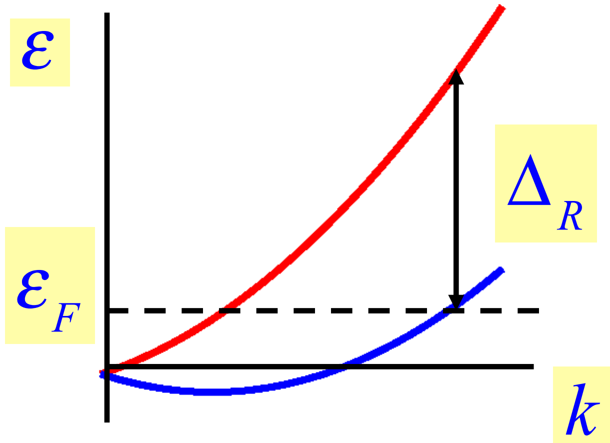
Yu. A. Bychkov and E. I. Rashba, JETP Lett. **39**, 78 (1984);
Yu. A. Bychkov and E. I. Rashba, J. Phys. C: Solid State Phys. **17** (1984) 6039.

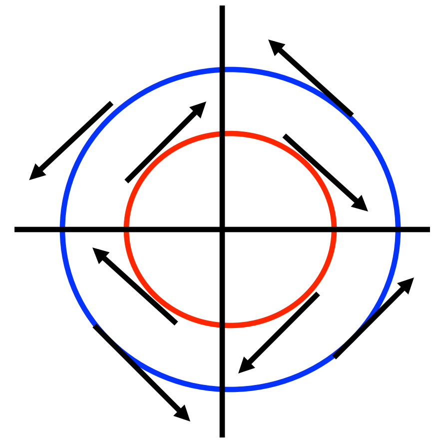
$$\hat{H}_R = \frac{k^2}{2m} + \alpha \hat{\mathbf{n}} \bullet (\vec{\sigma} \times \mathbf{k}) = \frac{k^2}{2m} + \alpha\left(\sigma^x k_y - \sigma^y k_x\right)$$

$$t \to -t : \ \mathbf{k} \to -\mathbf{k}, \sigma \to -\sigma$$

$$H_R = \begin{pmatrix} k^2/2m & \alpha\left(k_y + ik_x\right) \\ \alpha\left(k_y - ik_x\right) & k^2/2m \end{pmatrix} \Rightarrow \varepsilon_\pm = \frac{k^2}{2m} \pm \alpha k$$

Rashba states





$$\varepsilon_{\pm} = \frac{k^2}{2m} \pm \alpha k$$

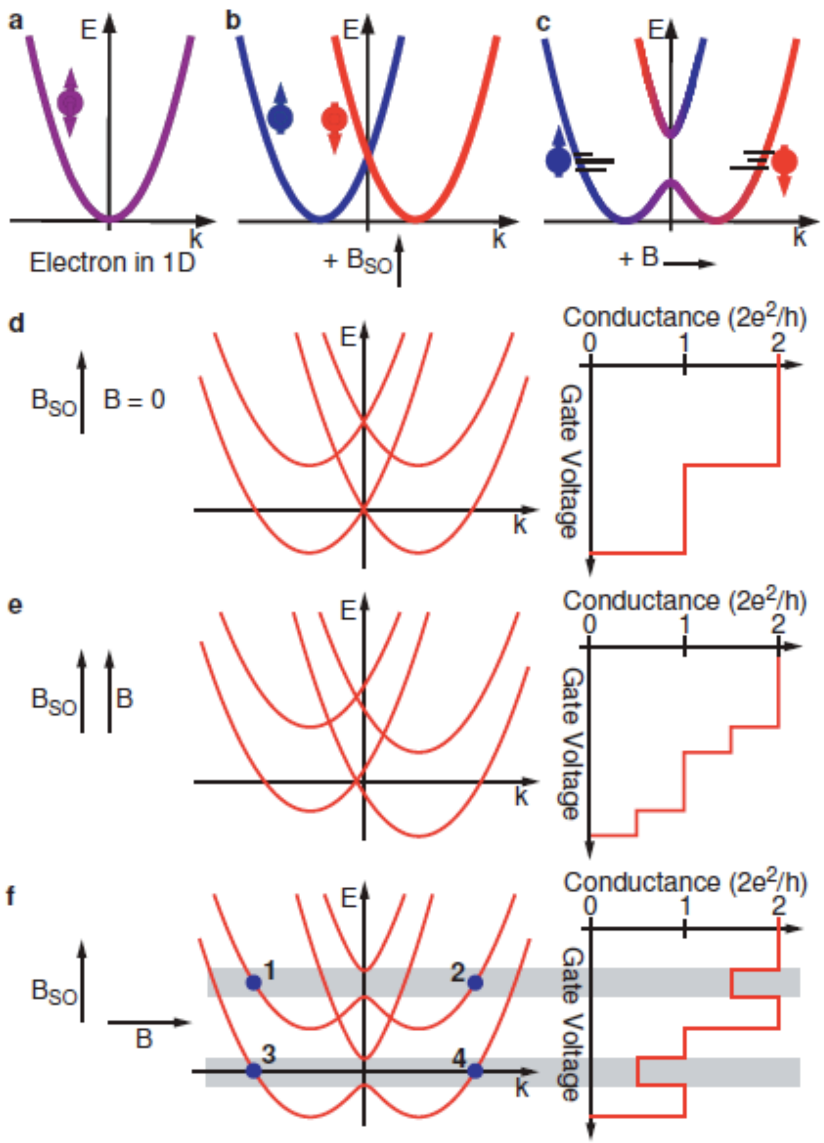
$$\psi_{\pm} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \mp i e^{i\phi_{\mathbf{k}}} \end{pmatrix}$$

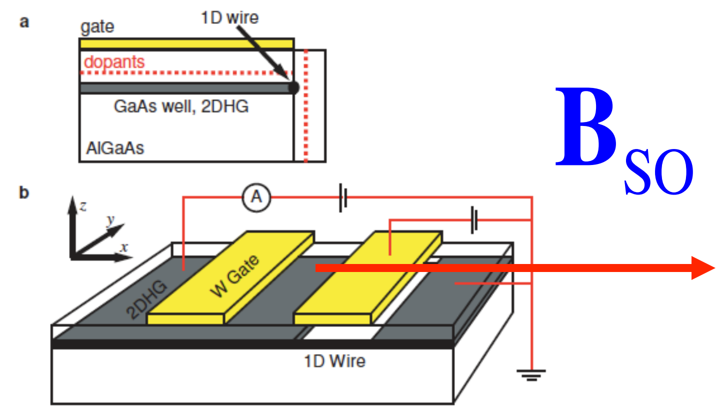
$$\mathrm{S}_{z,\pm} = \frac{1}{2} \bar{\psi}_{\pm} \sigma^z \psi_{\pm} = 0$$

$$S_{x,\pm} = \frac{1}{2} \bar{\psi}_{\pm} \sigma^x \psi_{\pm} = \pm \frac{1}{2} \sin \phi_k ; \; S_{y,\pm} = \frac{1}{2} \bar{\psi}_{\pm} \sigma^y \psi_{\pm} = \mp \frac{1}{2} \cos \phi_k$$

$$\mathbf{k} \bullet \mathrm{S}_{\pm} = 0$$

Conductance of Rashba wires

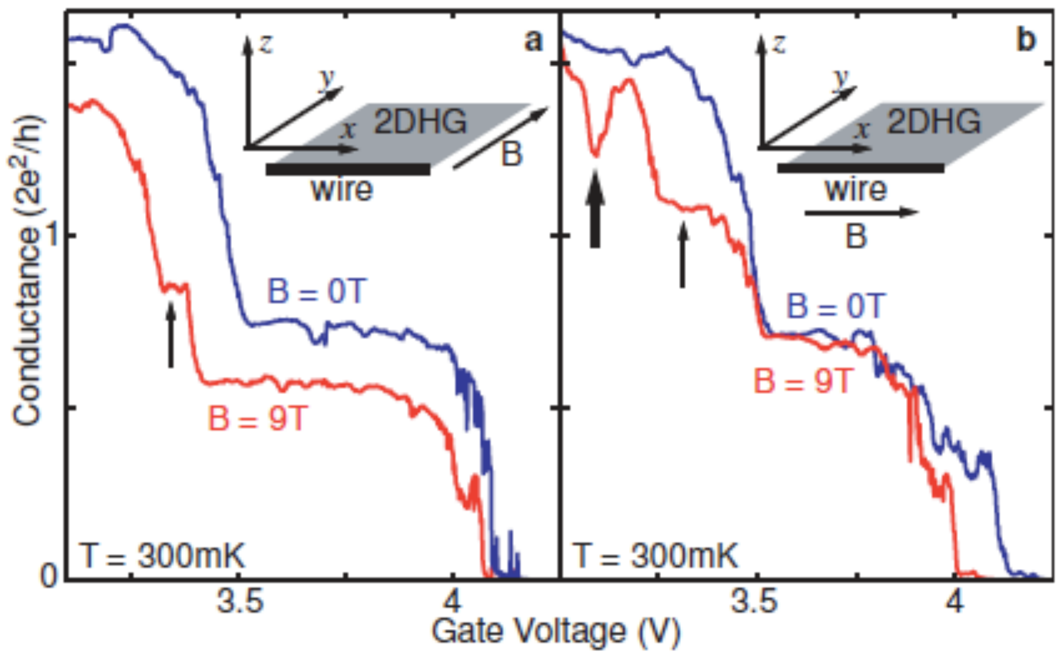


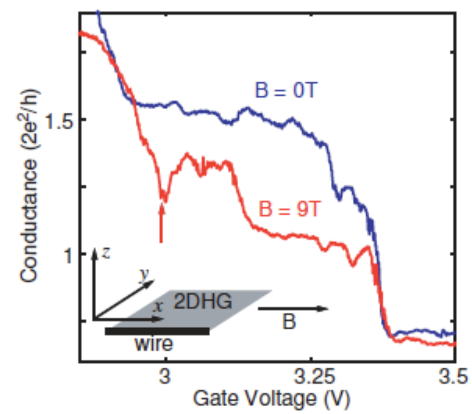


B_SO

$$2D:\ \hat{H}_R = \frac{k^2}{2m} + \alpha\left(\sigma^x k_y - \sigma^y k_x\right)$$

$$1D:\ \hat{H}_R = \frac{k_x^2}{2m} - \alpha\sigma^y k_x$$

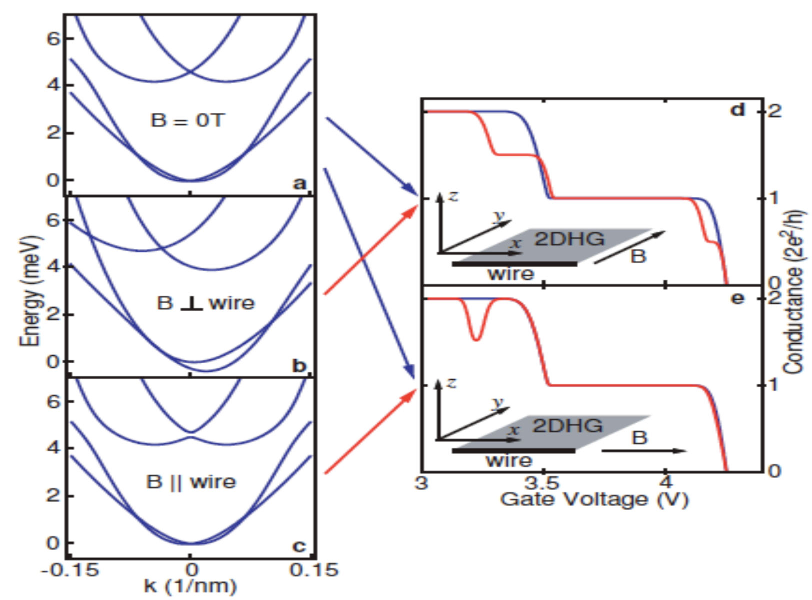




Quay et al. Nature Physics 6, 336 - 339 (2010)

Hole wire

Dresselhaus+Rashba



Dresselhaus Hamiltonian

G. Dresselhaus: Phys. Rev. **100**(2), 580–586 (1955)

zincblende ($A_{III}B_V$) lattice

$$H_D = D\vec{\Omega}(\mathbf{k})\bullet\vec{\sigma}$$

$$\vec{\Omega} = \left(k_x\left(k_y^2 - k_z^2\right), k_y\left(k_z^2 - k_x^2\right), k_z\left(k_x^2 - k_y^2\right)\right)$$

(001) surface (interface) of a $A_{III}B_V$ semiconductor

$$\langle k_z \rangle = 0, \langle k_z^2 \rangle \sim 1/d^2$$

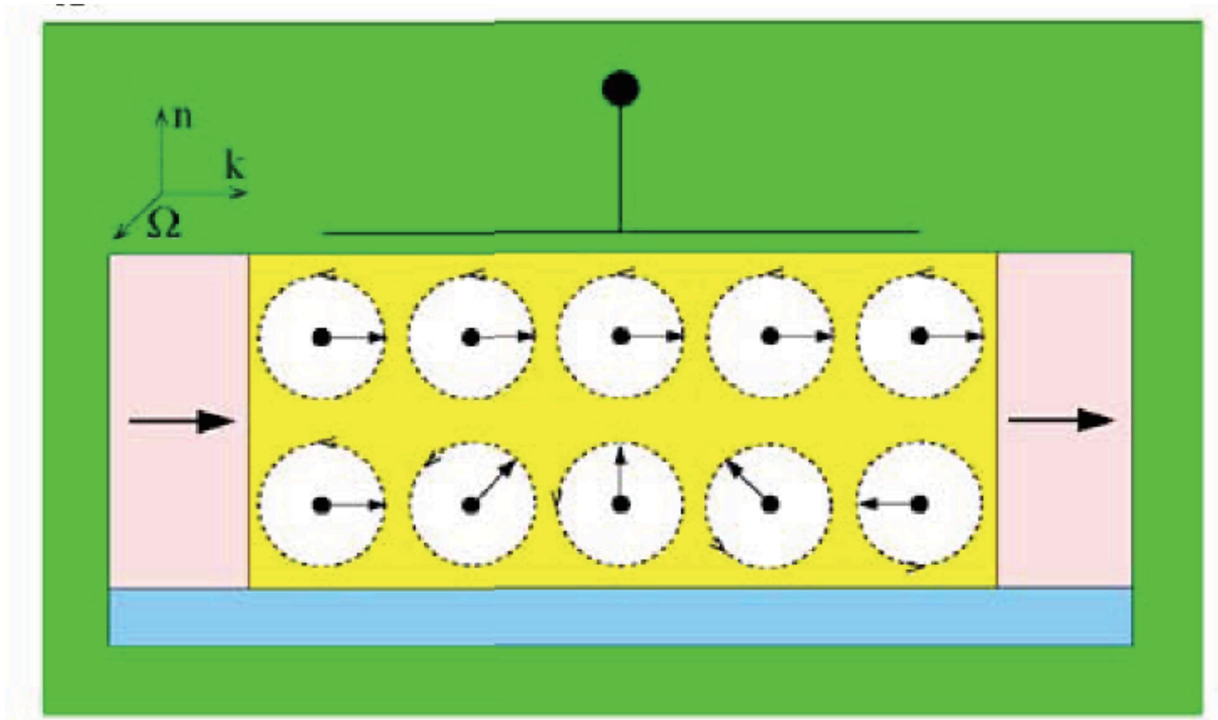
$$\langle\vec{\Omega}\rangle = \left(\underbrace{k_x k_y^2}_{\text{cubic}\to 0} - k_x\langle k_z^2\rangle, k_y\langle k_z^2\rangle - \underbrace{k_y k_x^2}_{\text{cubic}\to 0}, 0\right) = \langle k_z\rangle^2\left(-k_x, k_y, 0\right)$$

$$H_{DS} = \beta\left(\sigma^y k_y - \sigma^x k_x\right)$$

in general: Rashba+Dresselhaus

$$H = \frac{k^2}{2m} + \alpha\left(\sigma^x k_y - \sigma^y k_x\right) + \beta\left(\sigma^y k_y - \sigma^x k_x\right) = \frac{k^2}{2m} + \sigma^x\left(\alpha k_y - \beta k_x\right) + \sigma^y\left(\alpha k_x + \beta k_y\right)$$

Datta-Das Spin Transistor



Datta, S., and B. Das, Electronic analog of the electro-optic modulator, 1990, Appl. Phys. Lett. **56**, 665.

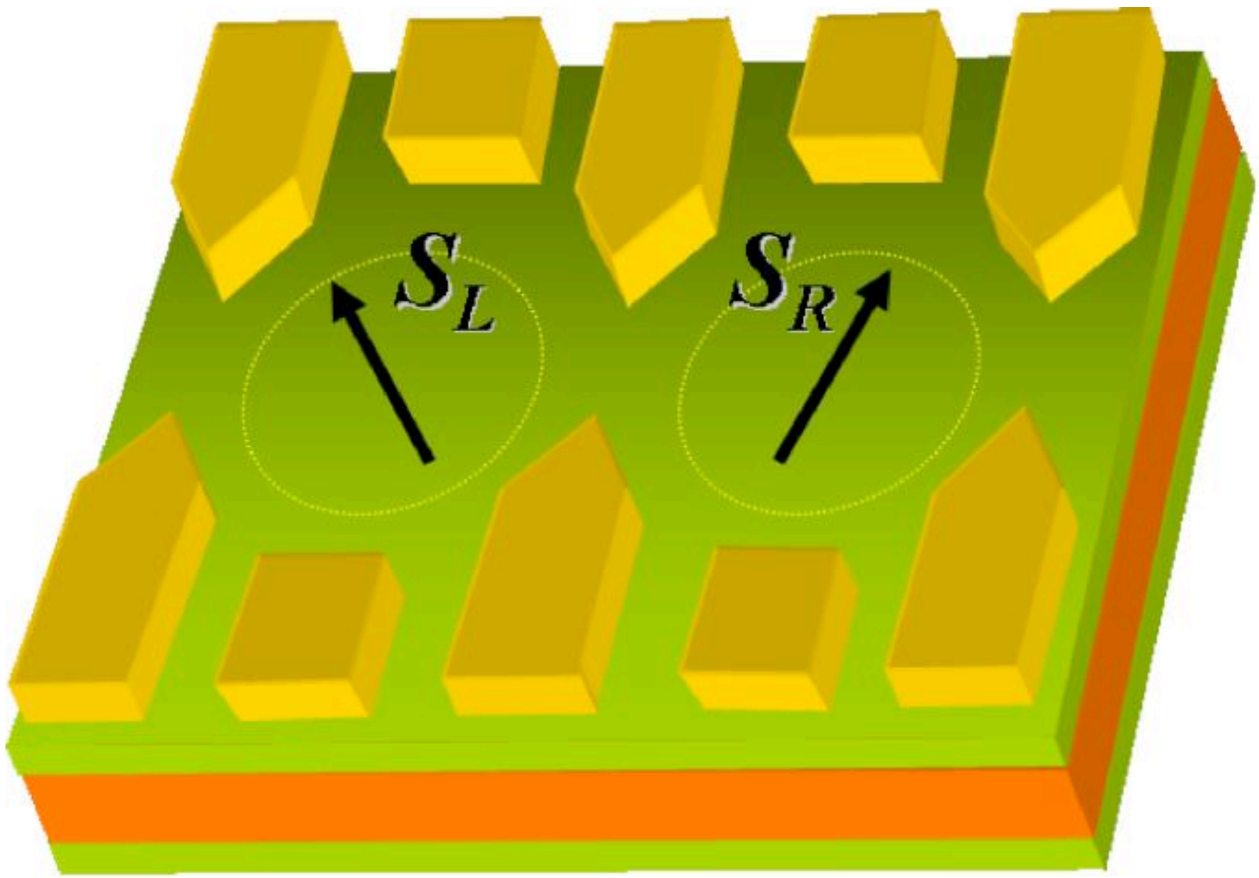
$$\hat{H} = \frac{k^2}{2m} + \frac{g\mu_B}{2}\vec{\sigma}\bullet\mathbf{B}_R(\mathbf{k});\ \mathbf{B}_R\left(\mathbf{k}\right) = \frac{2\alpha}{g\mu_B}\mathbf{k}\times\mathbf{n} \equiv \text{ Rashba field}$$

even number of π rotations: ON

odd number of π rotations: OFF

Loss-DiVincenzo Quantum Computer

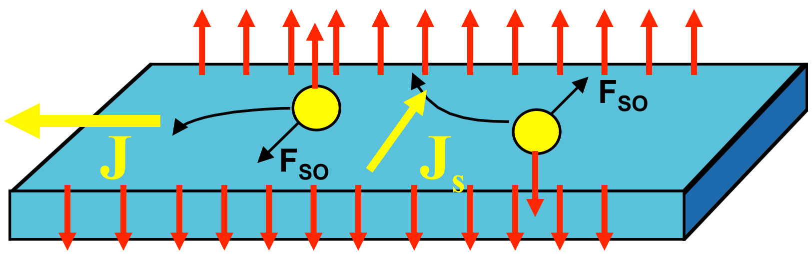
Phys. Rev. A 57, 120 (1998)



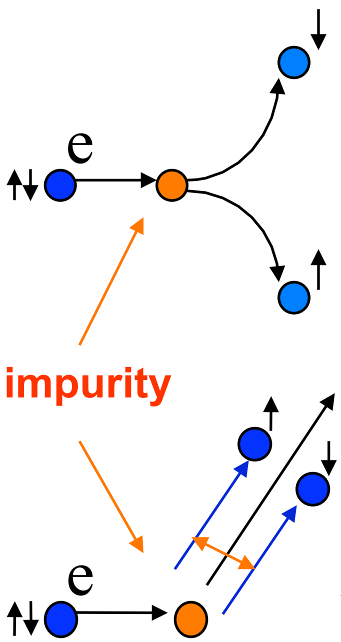
GaAs: strong spin-orbit coupling; strong hyperfine interaction

(Extrinsic) Spin-Hall Effect

D'yakonov, M. I., and V. I. Perel', Feasibility of optical orientation of equilibrium electrons in semiconductors, 1971a, **13**, 206, [JETP Lett. **13**, 144-146 (1971)].



Spin Hall Effect: the regular current (J) drives a spin current (J_s) across the bar resulting in a spin accumulation at the edges.



Skew scattering

More spin up electrons are deflected to the right than to the left (and viceversa for spin down

Side Jump

For a given deflection, spin up and spin down electrons make a side-jump in opposite directions.

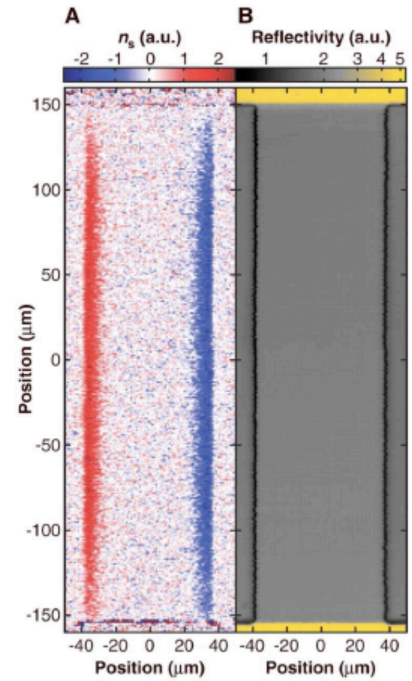


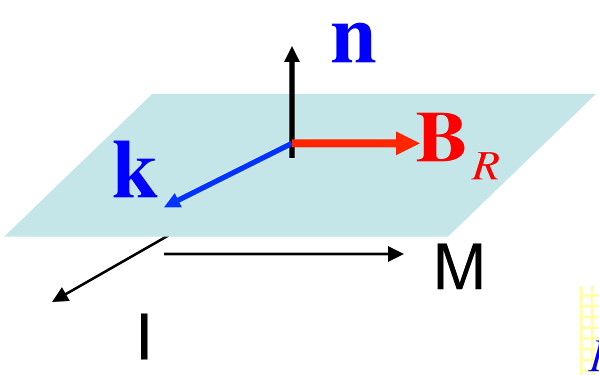
Fig. I.2. (A) Two-dimensional image, obtained by magneto-optic Kerr spectroscopy, of the spin polarization at the edges of a GaAs sample. Red is for positive spin (out or the page), blue for negative (Sih *et al.*, 2005). (B) Spatially resolved reflectance, showing the edges of the sample. The yellow metal contacts are also visible. From Y. K. Kato *et al.*, *Science* **306**, *1910 (2004). Reprinted with permission from AAAS.*

Intrinsic Spin-Hall Effect

No observations as of yet

unbounded 2D: magnetoelectric effect

[V. M. Edelstein, Solid State Comm. 73, 233 (1990).



Can an electric field produce magnetization?

Drift momentum

$$\langle k_x \rangle = eE\tau$$

$$\hat{H} = \frac{k^2}{2m} + \frac{g\mu_B}{2}\vec{\sigma}\bullet\mathbf{B}_R(\mathbf{k});\ \mathbf{B}_R(\mathbf{k}) = \frac{2\alpha}{g\mu_B}\mathbf{k}\times\mathbf{n} \equiv \text{ Rashba field}$$

Current induces steady Rashba field

$$M_y = \mu_B \langle B_R^y \rangle = \frac{2\alpha}{g}\langle k_x \rangle = \frac{2\alpha eE\tau}{g}$$

Impurities are only necessary to maintain steady state

(but forgetting about them leads to incorrect results--"universal spin-Hall conductivity"
Murakami et al. Science **301**, 1348 (2003)
Sinova et al. Phys. Rev. Lett. 92, 126603 (2004)

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http://www.springerlink.com.lp.hscl.ufl.edu/content/g7x071/#section=215878&page=1
available through UF Library

Ch. 1 M.I. Dyakonov, *Basics of Semiconductor and Spin Physics.*
Ch. 8 M. I. Dyakonov and A. V. Khaetskii, *Spin Hall Effect*

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   arXiv:0711.1461
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