# Optimization of Train Speed Profiles for a Metro Transit System by Genetic Algorithms

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#### **Abstract**

Traditional Automatic Train Operation (ATO) algorithms are generally designed based on single-train operation with the objective of improving the speed profile of a single train to reduce mechanical energy consumed under operational constraints. For many electrified rail transit systems where energy cost is calculated at the substation level, minimizing energy consumption needs to consider the reuse of regenerative energy from neighboring trains in the same power section. If regenerative energy is considered, when two trains are moving in the same power section, one train adjusts its speed to a different speed profile according to the position, speed and regeneration potential of the other train to reuse the maximum amount of regenerated energy. With the dual objectives of maintaining schedule requirements and optimizing energy efficiency, this paper analyses dynamic and electric performance of two opposing trains operated in the same DC power section. Genetic algorithms have been applied to search for the optimal train speed profiles. Tractive/braking efforts of both trains and energy cost at substation level are defined as strings of chromosome and the fitness function respectively. Simulation through Visual C++ platform demonstrates that the algorithm can provide optimal train speed profiles with better energy performance while satisfying operational constraints. Different synchronization times have different optimization ratios. This research will help facilitate development of on-board train control system logic to analyze energy flow in a multi-train network and reduce overall energy consumption.

### **Keywords**

Metro Transit, Train Optimal Control, Regenerative Braking, Genetic Algorithms

### 1 Introduction

Automatic Train Operation (ATO) systems are used to generate optimal driving commands for train speed regulation. Traditional ATO algorithms are generally designed to reduce the mechanical energy consumption of single train operation by altering a train speed profile while still satisfying operational constraints. But for many electrified rail transit systems where energy cost is calculated at the substation level, minimizing energy consumption is not only a question of reducing the mechanical energy demand of an individual train but also of reducing power peaks in the catenary. Therefore, the current ATO control algorithm based on single trains needs to be improved to consider power flow in the network.

When regenerative braking is available on-board electrified rail transit equipment, optimization of train speed profiles to effectively use the regenerated energy could be a solution to improve overall energy efficiency and lower operating cost. Consider two operating modes for two opposing trains in the same power section: both trains either systematically applies the same speed profiles, or one train adjusts its speed to a different speed profile according to the position, speed and regeneration potential of the other train. With operations synchronized to reuse energy, the latter mode achieves better energy recovery efficiency than the former one.

In addition, through Communications-Based Train Control (CBTC) systems, a train could be made aware of the projected speed profiles of other trains in the same power section before leaving its station (Dingler et al. (2010)). Power flow could be predicted accordingly in real-time, allowing for the possible design of coordinated train control algorithms for ATO systems.

With the dual objectives of maintaining schedule requirements and optimizing energy efficiency, this paper examines the prospects for recovering and reusing energy from regenerative brakes under transit system operating conditions. Assuming that two opposing trains are operated in the same power section, analysis of the dynamic and electric performance of these trains is conducted in accordance with power available on line. To obtain the minimum energy cost at the substation level, genetic algorithms have been applied to search for the optimal train speed profiles. To demonstrate an application of the optimization framework, energy optimization potentials of transit systems with different synchronization time will be compared.

This paper first provides a brief introduction to how the CBTC system may facilitate acquisition of neighbouring train running information for coordinated train control. Previous research on ATO control algorithms is presented in order to identify the proper solution approach. In the next section, the mathematical model of multi-train operation within an electrified network will be presented and enhanced genetic algorithms will be proposed to solve this problem. Finally, a case study will demonstrate the effectiveness of this method through analysis of optimization potentials.

## 2 Architecture of CBTC System

CBTC makes use of bi-directional train-to-wayside data communication to improve safety, efficiency and traffic management. CBTC has been widely applied to light rail, heavy rail and commuter rail systems in many countries. A typical CBTC system architecture is shown in Figure 1 (IEEE Standard for CBTC (1999)).

In the illustrated architecture, the Automatic Train Supervision (ATS) system fills the role of the control center, dedicated to monitoring trains and schedule adherence. Through the data communication network and central ATS system, CBTC subsystems are able to exchange information to perform their functions properly.

Within CBTC, ATO performs part or all of the speed regulation, performance level regulation and other functions otherwise assigned to the train operator. ATO control and status information can be exchanged, through the ATS, between wayside equipment and on-board systems. Via this exchange, information on neighboring train speed profiles can be acquired by each individual train. This information allows the ATO subsystem to estimate and predict energy consumption and voltage fluctuation based on the operational status of trains in the same power section. By choosing the proper acceleration and braking commands to coordinate its speed profile with the other trains, power peaks can be reduced by absorbing regenerative braking energy.

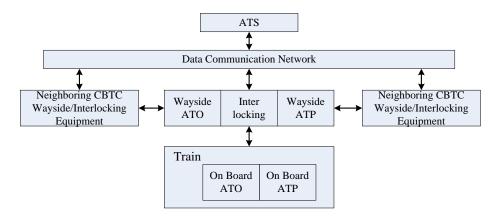


Figure 1 Typical CBTC System

## 3 Previous Research on ATO Control Algorithms

Research on ATO control algorithms began in 1968 when Japanese scholar Ichikwa et al. (1968) solved a simple optimal train control problem by Pontryagin principle. Current research efforts are generally based on single-train operation with the aim of providing an optimal speed profile in order to minimize mechanical energy consumption (mainly tractive energy) under operational constraints such as speed restrictions and scheduled running time. This energy efficient driving problem has been generally modelled based on motion equations applicable to discrete or continuous control. Pontryagin principle is widely used to search for optimal points to make train driving command inputs (Howlett (2000)).

Besides analytical methods, with the rapid development of desktop computing capability to solve large-scale optimization problems, numerical methods are being researched to solve more complicated train operational problems. Investigated methods include dynamic programming (Tang et al. (2013)), fuzzy (Yasunobu et al. (1983)) and evolutionary methods (Chang and Sim (1997)).

Single train based train control optimization discovers that the optimal regimes for single train operation includes full acceleration, cruising, coasting and full braking. For urban transportation system, the cruising stage is unnecessary unless the maximum speed is reached after full acceleration (Thomas, 2008).

Separate from optimization based on single train, running interactions between trains are emphasized in research on coordinated train control and design of multi-train ATO algorithms. By introducing regenerative braking, trains have more opportunities to reduce energy consumption by using braking energy from other trains. In cases where system electrical cost is calculated at the sub-station level, energy exchange between multiple trains may have more influence on electricity cost than mechanical energy consumption of individual trains.

In such problems, headway, synchronization time, dwell time and inter-station running time are principal factors influencing relative train movement and system power

consumption in urban rail transit systems. When the headway becomes shorter, train speed regulation is more likely to be affected by other trains. The gradient method, sequential quadratic programming and dynamic programming are proposed for this headway research (Miyatake et al. (2008, 2010), Tang et al. (2013)). Some studies deal with dwell time to improve regenerative energy absorption by delaying departure of the following train to synchronize acceleration and braking events (Su et al. (2013)). Control methods have applied predictive fuzzy control, search technique and heuristics (Chang et al. (1996), Firpo et al. (1995), Gordon et al. (1998)). Finally, evolutionary methods (such as genetic algorithms) are proposed to look into the influence of running time on energy (Albrecht (2004)).

This paper adds to previous research by studying coordinated train control under the constraint of synchronization time between opposing trains at stations. This problem is a high order, non-linear mathematical model that is difficult to solve by traditional analytical methods. Although evolutionary methods have proven their effectiveness in dealing with similar problems, to improve the convergence performance of traditional GAs, enhanced genetic algorithms are proposed to solve this problem.

## 4 Model Description

### 4.1 Assumptions

The model presented in this paper relies on several key assumptions:

- Inter-station running time and running distance are pre-defined.
- Two opposing trains (one eastbound and one westbound) are running in the same power section.
- Traction energy is provided by power substations at both ends of the power section.
- Resistance in the catenary is evenly distributed.
- Substations are non-reversible. No wayside energy storage devices are included in this problem.

### 4.2 Electrical Network Model

The electrical network model varies according to the current location and status of the two trains within the same power section. As shown in Figure 2, two trains ( $U_{dc1}$  and  $U_{dc2}$ ) are running between two substations ( $U_{d0}$ ).

This model follows the common practice where three-phase AC electricity from the general power supply is converted into DC electricity at a substation that feeds train operation via catenary. In this model, the current conversion at the substation is not considered. Thus, the substation can be described by its external voltage-current characteristics as a Thevenin equivalent voltage source  $U_{d0}$ . In the model Rs is an equivalent resistor and  $I_{u1}$  and  $I_{u2}$  are currents from substations at either ends of the power section.

Two trains are modeled as ideal current sources,  $I_{S1}$  and  $I_{S2}$ . Their actual power during operation varies according to current and voltage level.  $U_{dc1}$  and  $U_{dc2}$  are catenary voltages of the eastbound train and westbound train respectively. Since the catenary resistance is assumed to be uniform, the values of the four catenary equivalent resistors  $R_1$ ,  $R_2$ ,  $R_3$ ,  $R_4$  only depend on the current position of trains within the power section.  $R_0$  is an on-board resistor that is applied during dynamic braking when regenerated electricity cannot be used by other trains.

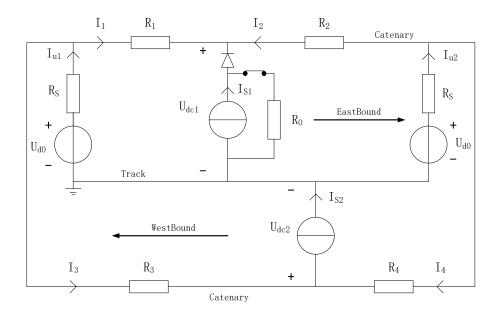


Figure 2: Electrical Model When the Eastbound Train  $(I_{s1})$  is in Braking Status and Westbound Train  $(I_{s2})$  is in Traction Status

During operations, when both trains are in traction status, they take in energy from substations and the currents  $I_{u1}$  and  $I_{u2}$  are positive. In this status, the required energy will rise sharply when either train starts to accelerate. The required energy will decrease when the maximum speed is reached and less power is required during cruise status. When the eastbound train approaches the station ahead, it starts to brake. Regenerative braking is applied and its motor is converted into a generator. Current  $I_{S1}$  is transmitted to the overhead wire and is absorbed by the neighboring train in traction status. When the network is not receptive (regenerative energy cannot be fully absorbed), the excess electricity is dissipated by the resistor  $R_0$  on-board the eastbound train as in dynamic braking. The diode in the electrical network representation of this train is used to restrict the direction of current during dynamic braking. In practice, dynamic braking is not encouraged as it wastes energy and also presents a potential risk of overheating and fire. Thus dynamic braking is not considered as rewarding as regenerative braking in this problem.

## 4.3 Mathematical Model

This paper aims to find the optimal speed profile for both eastbound and westbound trains according to the energy available on the catenary in order to minimize the total energy consumption as measured at the substations.

Model variables are defined as follows:

s<sub>1</sub>, s<sub>2</sub>: The travel distance of eastbound train and westbound train;

v<sub>1</sub>, v<sub>2</sub>: The speed of eastbound train and westbound train;

v<sub>limit</sub>(s): Speed restriction at position s;

P<sub>1</sub>, P<sub>2</sub>: Electrical power of eastbound train and westbound train;

n<sub>1,T</sub>, n<sub>2,T</sub>. Coefficients of tractive effort applied for eastbound and westbound trains;

n<sub>1,B</sub>, n<sub>2,B</sub>: Coefficients of braking effort applied for eastbound and westbound trains;

 $F_{max,\ 1}$  (U,v),  $F_{max,\ 2}$  (U,v): Maximum tractive effort applied on eastbound train and westbound train under the voltage of U, at the speed of v;

 $B_{max, 1}$  (U,v),  $B_{max, 2}$  (U,v): Maximum braking effort applied on eastbound train and westbound train under the voltage of U, at the speed of v;

R<sub>Air</sub>(v): Train resistance;

 $R_G(s)$ : Gradient resistance;

 $R_C(s)$ : Curvature resistance;

 $\Delta$  r: The equivalent resistance of the catenary per unit length;

 $\mu_t$ : Efficiency of driver system at tractive status;

 $\mu_{g}$ : Efficiency of driver system at braking status;

S: Required inter-station distance;

T: Required inter-station running time;

 $\Delta T_{depart}$ : Synchronization time between two train departures from opposite stations;

M: Train weight;

J: Total energy cost at substations.

Note: In the definition of  $F_{max\ 2}$ ,  $B_{max\ 2}$ , maximum effort means tractive or braking effort when the driver's handle is at full level. These values are influenced by how much power can be provided by the power section ( $U_{dc}$ ). The values also vary according to different train speed status (v).

Based on the network description in the previous section, the total energy cost for the power section at the substations can be written as:

$$\min J = \int_0^T \left( U_{d0} \cdot I_{u1} + U_{d0} \cdot I_{u2} \right) dt \ . \tag{1}$$

Four types of constraints are mainly considered in this model: infrastructure, motion equations, electrical constraints and operational constraints. Each will be described in the following sections.

### Infrastructure

Line profile (gradient) and alignment (curvature) are very influential on train performance and energy consumption. These two infrastructure factors are measured at each location by gradient slope and curvature angle respectively. The resulting grade and curvature resistance forces experienced by a train are described in the motion equations in the next section.

Another infrastructure constraint is the maximum allowable speed generally set by curvature, turnouts or other operational conditions. Speed restrictions are related to kilometre posts along the route, and the speed of both trains at any location  $s_i$  must obey corresponding limits:

$$0 \le v_i(s_i) \le v_{\text{limit}}(s_i) \qquad (i = 1, 2).$$
 (2)

### Motion Equations

The motion equation for this problem are established based on a point-mass model of the train. This is a reasonable assumption given the length of most transit train consists relative to the rate of change of alignment and profile geometry. When the train is in traction status, it is experiencing tractive effort, train resistance, grade resistance and curve resistance. When in regenerative braking status, regenerative effort will be applied instead of tractive effort. The expression is shown below:

$$v_i = ds_i / dt \qquad (i = 1, 2) \tag{3}$$

$$M \cdot \dot{v}_{i} = \begin{cases} n_{i,T} F_{\text{max,i}} \left( U_{dci}, v_{i} \right) - R_{Air} \left( v_{i} \right) - R_{G} \left( s_{i} \right) - R_{C} \left( s_{i} \right) \\ n_{i,T} B_{\text{max,i}} \left( U_{dci}, v_{i} \right) - R_{Air} \left( v_{i} \right) - R_{G} \left( s_{i} \right) - R_{C} \left( s_{i} \right) \end{cases}$$
 (i = 1,2). (4)

Where tractive and braking coefficients n<sub>2T</sub> and n<sub>2B</sub> satisfies

$$\begin{cases}
n_{i,T} \in [0,1] & (Traction) \\
n_{i,B} \in [-1,0) & (Braking)
\end{cases} (i = 1,2).$$
(5)

### **Electrical Constraints**

According to Kirchhoff's circuit law, the DC circuit power networks in this optimal control problem are modeled via equality constraints as:

$$\begin{bmatrix} U_{dc2} \\ U_{dc1} \\ I_{u1} \\ I_{u2} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} \bullet \begin{bmatrix} U_{d0} \\ U_{d0} \\ I_{s2} \\ I_{s1} \end{bmatrix}.$$
 (6)

As the internal resistance of catenary is assumed uniform, the equivalent resistances  $R_1$ ,  $R_2$ ,  $R_3$ ,  $R_4$  in Figure 2 are defined by the positions of trains within the power section:

$$R_{1} = s_{1} \cdot \Delta r$$

$$R_{2} = (S - s_{1}) \cdot \Delta r$$

$$R_{3} = (S - s_{2}) \cdot \Delta r$$

$$R_{4} = s_{2} \cdot \Delta r$$
(7)

Train power is calculated in real-time according to the force coefficient, actual speed and the catenary voltage. The train power demand during traction and braking are given in equation (8) respectively.

$$P_{i} = \begin{cases} n_{i,T} \cdot F_{\text{max,i}} \left( U_{dci}, v_{i} \right) \cdot v_{i} \cdot \mu_{t} \\ n_{i,B} \cdot B_{\text{max,i}} \left( U_{dci}, v_{i} \right) \cdot v_{i} / \mu_{g} \end{cases} \quad (i = 1, 2).$$

$$(8)$$

### **Operational Constraints**

Adhering to the operating schedule is essential for urban transit systems. All optimized speed profiles must respect inter-station time and distance requirements. Considering a section between two stations, each train departs from one station and stops at the other. These operational constraints can be described by:

$$v_{1,2}(0) = 0, \quad v_{1,2}(T) = 0$$
  
 $s_{1,2}(0) = 0, \quad s_{1,2}(T) = S$  (9)

As two running directions have been considered in the same section, departure constraint is also required for operational issues.

$$t_2(0) - t_1(0) = \Delta T_{depart}. \tag{10}$$

Equation (10) defines synchronization time, which is the departure time interval (from different stations) between two opposing trains. This constraint ensures within interstation section, the westbound train leaves the station  $\Delta$  T<sub>depart</sub> seconds later than the eastbound train.

## 5 Application of Enhanced Genetic Algorithms

Genetic algorithms (GA) is a global search algorithm technique based on the principle of natural selection. It mimics the evolution of biological organisms to achieve optimal solutions with a given objective function in an artificial system. In GA, a solution to the problem is encoded into strings of digital numbers. Each string (chromosome) represents one possible solution. The collective chromosomes form a set of possible solutions, called the population. GA performs operations like selection, crossover and mutation on chromosomes in the population with a probability based on their corresponding fitness values. Optimal solutions, in the form of high fitness individuals will eventually appear after generations of evolution.

Compared with other optimization techniques, GA has several advantages for large scale optimization problem. First, since it searches from a group of solutions instead of a single point, it avoids being trapped into a local stationary point. Second, it can be applied to various types of problems as the search is carried out based on the fitness function rather than derivatives. Third, probabilistic transition rules are used so that the optimum can be achieved faster with real-time adjustment.

However, traditional genetic algorithms will give rise to premature convergence if a dominant individual occurs in the population. Therefore, by introducing combinational selection method, adaptive probability and dual search loop, an enhanced genetic algorithm is proposed to solve coordinated train optimal control problem to ensure the solution's effectiveness and efficiency.

### 5.1 Problem Coding

A chromosome is defined as the combination of the control variables for the eastbound and westbound trains. Each gene represents the coefficient of the applied force at a control switching point.

In this problem, the control variable is the coefficient of the applied force defined as the percentage of maximum traction/braking force applied by the train. It is discretized in 10-percent increments as  $0, \pm 0.1, \pm 0.2, \ldots, \pm 0.9, \pm 1$ . Positive "+" represents traction, while negative "-" represents braking. Coasting status is represented by "0". During train operation, the control variable can be switched according to different infrastructure parameters, such as speed restrictions, gradient value or curvature value. Thus, the solution can be modeled as a sequence of control variables for trains at specific control switching points (locations).

An example of control switching point is illustrated in Figure 3. Control switching points  $s_1$  to  $s_7$  are based on infrastructure parameters. As the number of control switching points is pre-determined, the length of chromosome is fixed.

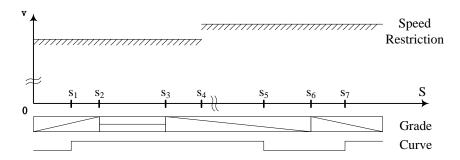


Figure 3 Control Switching Points along the Line

## 5.2 Fitness Function

The objective of this algorithm is to minimize system electrical energy consumption with the restriction of travel time and distance. However, GA is formulated to find a maximum value during the search process, as fitness values are always positive. In addition, as a multi-criterion optimization, a proper combination form needs to be determined to satisfy operational constraints. Several modifications have been made to the traditional GA to deal with the above issues.

Therefore, the fitness function has been formulated in equation (11).

$$J=1/\left(\frac{\sum \left(U_{d0} \cdot \Delta I_{1} + U_{d0} \cdot \Delta I_{3}\right) + w_{S1} \cdot \left|s_{actual,1}\left(T\right) - s_{assigned,1}\left(T\right)\right| + \left|v_{S2} \cdot \left|s_{actual,2}\left(T\right) - s_{assigned,2}\left(T\right)\right| + w_{V1} \cdot \left|v_{actual,1}\left(T\right) - 0\right| + w_{V2} \cdot \left|v_{actual,2}\left(T\right) - 0\right|\right). \tag{11}$$

The first term is the minimization objective for network energy consumption. The following two terms ensure the punctuality and stop accuracy for the arriving train at the next station.  $W_s$  and  $W_v$  are weights for travel distance and speed respectively. These terms have higher weights to ensure operational requirements are satisfied first.

The inversed formulation ensures the minimum energy consumption can be achieved when the maximum fitness value is found by GA.

#### 5.3 Combinational Selection

Selection is the process used to select a group of chromosomes from a population for later breeding based on their fitness values. Individuals with higher fitness values are more likely to be chosen to produce the next generation. Two main selection strategies are applied here: roulette wheel selection and rank selection.

Roulette wheel selection is a fitness-proportionate selection method and is commonly used due to its efficiency in best individual selection. The probability for a chromosome to be selected is proportional to its fitness. However, since this method can quickly eliminate the lower fitted individuals, the solution may inadvertently converge to a local optimum point.

To avoid this potential risk, rank selection is used for population selection in the early stages. Instead of using fitness value, rank selection assigns ranking numbers (from 1 to N) to each chromosome. The worst has 1 and the best has N. The selection probability is then established according to this ranking number. In this way, lower fitted chromosomes have more chances to survive.

The combination of these two methods ensures a variety of species in the early evolution stage and that multiple good solutions will emerge for breeding. As the evolution proceeds, by using roulette wheel selection, better-fitted individuals have a greater chances of selection. Therefore, the later evolution process will be accelerated.

### 5.4 Adaptive Crossover

Crossover is the process to taking more than one parent chromosomes and producing offspring by exchanging part of their gene information. Crossover has two key parameters: crossover probability and crossover operator. The former decides how likely an individual is to be chosen for crossover operation, while the latter decides how parents exchange information.

To ensure the efficiency of evolution, adaptive probability has been applied for crossover probability. According to adaptive probability, higher fitness individuals have lower probability for crossover. This means their good genetic information is preserved for the next generation. On the contrary, lower fitted solutions are have a higher crossover rate and are more likely to be recombined in an effort to improve them. Adaptive probability is defined in (12).

$$P_{c} = \begin{cases} P_{\text{c\_max}} - \left( f' - f_{avg} \right) \cdot \left( P_{\text{c\_max}} - P_{\text{c\_min}} \right) / \left( f_{\text{max}} - f_{avg} \right) & f' > f_{avg} \\ P_{\text{c\_max}} & f' \leq f_{avg} \end{cases} . \tag{12}$$

For crossover operator, traditional two-point crossover is chosen. Everything between the two points is swapped between the parent chromosomes, rendering two child chromosomes.

## 5.5 Adaptive Mutation

Mutation prevents the search from being trapped into a local optimum point by introducing new genes to the selected chromosome. The adaptive method is again used here to decide the mutation probability for each chromosome. Similar to the crossover parameter, the actual mutation probability varies according to the fitness of the chromosome.

## 5.6 Secondary Search

In a departure from traditional genetic algorithms, a secondary search loop has been introduced after the initial search iteration finishes. As the solution found by GA is usually a quasi-optimal point near the global optimal one, a secondary search process through the area neighboring the original result helps find the optimal point.

In this secondary search, the initial population is established through duplication of the best fitted chromosome from the previous search. To ensure the search is restricted to the neighboring area, any new values chosen to replace the current genes at mutation stage are numbers near that gene value. The normal probability distribution has been applied to choose the new mutation value.

## 5.7 Proposed Algorithm Procedure

The proposed algorithm procedure is shown in Figure 4. In this procedure, a reference value will be calculated first. The reference value is the fitness value when both trains use the same single-train-based ATO control algorithm in opposite directions. It is used as the threshold for the two selection methods in the main search loop later on.

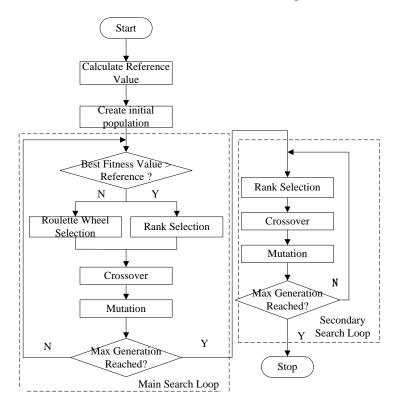


Figure 4 Procedure of Enhanced Genetic Algorithms

After creating an initial population, the search includes two steps: main search loop and secondary search loop. The main search loop tries to find the best-fitted individuals based on randomly initialized population. Two selection methods are used to ensure population diversity during the early stage and efficient convergence during the late

period. Once the main search loop ends, the best solution will be passed to the secondary search loop. A regular genetic algorithm procedure is then followed to search the neighbouring area around the best solution. Final optimal solution will be achieved after the two search loops complete.

## 6 Case Study

In this section, we analyze the obtained simulation results. Simulations have been carried out based on the platform developed by Visual C++ installed on a desktop computer with 8 GB of RAM and a 3.2 GHz i3 processor. The case study solution allows verification of the effectiveness of genetic algorithms for coordinated train control.

The chosen bi-directional simulation section is part of Xi'an metro line of China. The distance between stations is 1,517 meters, and the scheduled travel time is 115 seconds for both directions. Rated voltage for substation is 1,650V. Internal resistance on the catenary is 27 m $\Omega$ /km. The maximum of train acceleration is 1.07m/s². Two trains are included in this simulation. The westbound train leaves the station 70 seconds later than the eastbound train.

A population of 60 randomly generated chromosomes with length of 83 genes is used in the genetic algorithm as the initial population. Maximum crossover rate is 0.8; while minimum crossover rate is 0.4. Maximum mutation rate is 0.1; while minimum mutation rate is 0.001. As shown in Figure 5, the fitness value generally can reach convergence within 150 iterations.

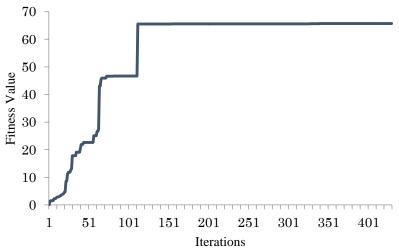


Figure 5 Convergence of Genetic Algorithms

To demonstrate the optimality of coordinated control algorithm, the basic case for the case study uses the speed profiles for the eastbound and westbound trains generated for both under single-train-based ATO algorithm. This means their mechanical energy consumptions are already minimized; they simply don't consider the optimization for regenerative energy receptivity.

The case study is implemented in two steps. In the first step, by using the above parameters, the train speed profiles for eastbound and westbound trains developed through

the coordinated control algorithm are compared to the basic case. The analysis has been carried out at a synchronization time of 60s. In the second step, scenarios with different synchronization times are developed to study the relationship between synchronization time and optimization ratio.

The results of the first step of the case study are shown in Figure 6. The solid blue line is the speed profile of the westbound train in the basic case; the solid orange line is the speed profile of the westbound train in the optimal case; the dash blue line is the speed profile of the eastbound train in the basic case; and the dash orange line is the speed profile of the eastbound train in the optimal case. The four speed profiles are plotted in the same time scale and the westbound train departs 60s later than the eastbound train.

In the basic case, the two trains are under the control of a single-train-based ATO control algorithm. In the optimal case, the eastbound train starts to brake after the departure of the westbound train in order to provide regenerative braking energy for its acceleration. The westbound train deviates from the original profile in basic case, adjusting its actual tractive and braking efforts according to the energy available on the catenary and in order to make use of the regenerative energy from the eastbound train. After the eastbound train stops, the westbound train mainly applies coasting and braking during the rest of the journey, avoiding additional energy consumption.

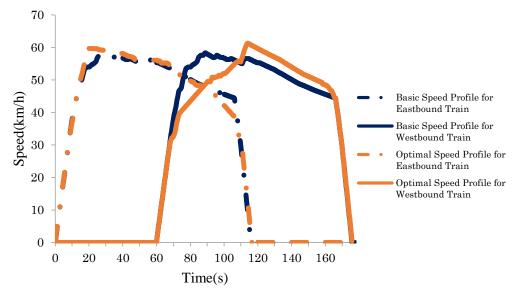


Figure 6 Simulation Results of Speed Profiles

Table 1: Simulation Result for the Westbound Train

	Basic Case	Optimized Case
Sys. Energy (kW•h)	37.37	32.86
Eastbound Train Time (s)	115.8	116.4
Westbound Train Time (s)	115.9	116.6
Eastbound Train Distance (m)	1,516	1,515.37
Westbound Train Distance (m)	1,515.09	1,517.52

As shown by the simulation results in Table 1, by regulating the speed of both eastbound and westbound trains to improve the regenerative receptivity, total system energy consumption at the substation is reduced by 12%. The average time deviation and distance deviation are only1.3% and 0.036% respectively compared with 115s and 1,517 meters requirements.

Another benefit of speed coordination is reduced voltage fluctuation. Frequent voltage fluctuations will do harm to the on-board and substation electrical equipment, impacting system reliability. When a train accelerates and requires energy from a substation, the catenary voltage of the train will drop significantly. However, when part of the traction energy required by the train can be fed from another train in regenerative braking, less energy is required from the power station, and the voltage drop is decreased. The voltage performance of the case study is illustrated in Figure 7.

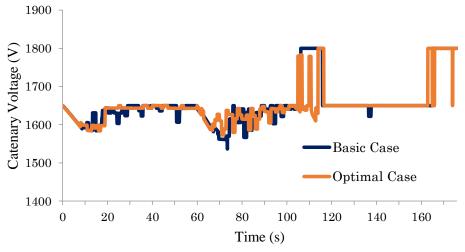


Figure 7 Simulation Results of Catenary Voltage

To further investigate the performance of the coordinated train control algorithm, the energy saving levels at different synchronization times from 20 to 100 seconds have been tested.

Figure 8 shows the energy consumption for both the basic case and the optimized case according to different synchronization times. Figure 9 shows the corresponding ratio of optimized to base energy consumption, termed the "energy optimization ratio". When both trains apply single-train-based ATO algorithms as in the base case, system energy consumption decreases as the synchronization time increases. Without optimization of regenerative receptivity, energy consumption can be reduced by 22% by postponing the westbound train's departure time from 20s to 100s. Through optimization and introducing regenerative energy recovery into the ATO algorithm, further reductions can be achieved for different synchronization times. The benefit of optimization is relatively small when the synchronization time is near 20s and 100s, while the optimization ratio reaches its peak (15.1%) at 80s.

Train speed profiles at different synchronization tine have been shown in Figure 10 in the annex. After the departure of westbound train, the eastbound train gradually increases the braking ratio to provide regenerative energy for westbound train's acceleration; while

the westbound train adjust its traction ratio according to the regenerative energy available on catenary so that the regenerative energy can be fully absorbed. However, this process causes time loss due to eastbound train' early braking and westbound train's partial traction. The longer this adjusting process goes, the more time is lost. To satisfy running time constraints, both trains must accelerate to a higher speed to compensate for this time loss, offsetting some benefits from reuse of regenerative energy. As the synchronization time increases, the position of the westbound train during acceleration is closer to the regenerative braking point of the eastbound train, and it is easier for the westbound train to adjust acceleration without compromising the time constraint. As more regenerative energy is able to be absorbed, the benefits of optimization increase as illustrated by the vertical distance between the two lines plotted in Figure 8. However, at some particular synchronization time (100s in this example), the eastbound train uses regenerative braking at the exact departure time of the westbound train and the regenerative energy can be easily absorbed by simply applying maximum acceleration under the default speed profile. Since further speed profile optimization cannot improve on this scenario facilitated by coincidental timing of maximum acceleration and braking, the basic and optimized energy consumption results converge at this point.

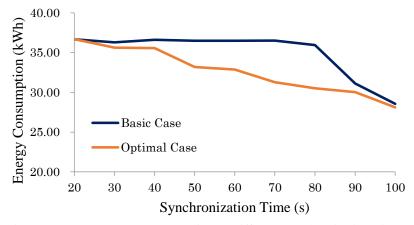


Figure 8 System Energy Consumption at Different Synchronization Time

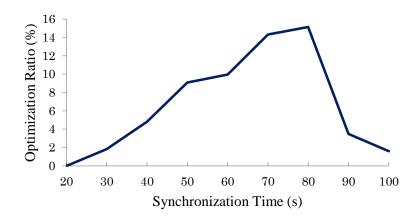


Figure 9 Optimization Ratio at Different Synchronization Time

## 7 Conclusions

In this paper, a mathematical energy consumption model of bi-directional trains running in the same power section has been established based on train operations and electrical theories. Genetic algorithms have been applied to generate an optimal speed profile for the second train to minimize energy consumption at the power substations. Improvements like dual search loops and adaptive probability are introduced to the GA formulation to ensure the efficiency and effectiveness of the algorithm.

Simulation results show that, while respecting operational constraints, the genetic algorithm can successfully reduce total energy consumption and voltage fluctuation. This is accomplished by increasing the use of regenerative braking energy produced by the earlier-departing train. This method can be used for multi-train-based ATO control algorithm design.

Further investigation indicates that synchronization time plays an essential role in system energy consumption. Under the same ATO control algorithm, considerable energy can be saved just by changing the synchronization time. In addition, by applying coordinated train control, different synchronization times have different optimization ratios.

This research will help facilitate development of wayside train control system logic to provide off-line optimal train control profile before train departure from the station and reduce overall energy consumption by reusing the regenerative energy. When implemented, such an algorithm will provide rail transit operators with operational cost saving potentials with given synchronization time of current timetable.

## 8 Acknowledgement

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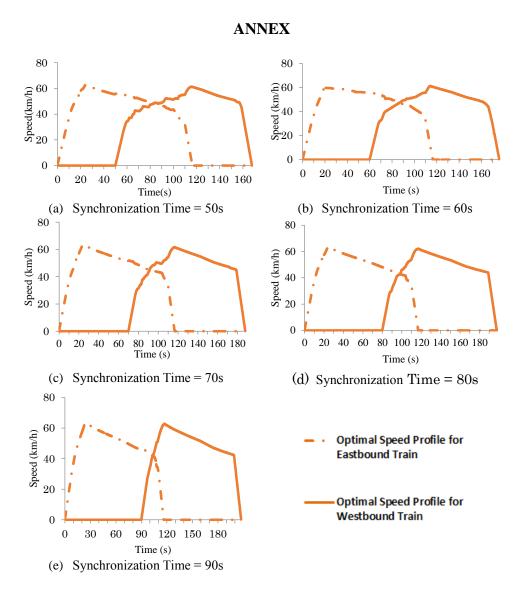


Figure 10 Train Speed Profiles at Different Synchronization Time