New PID designs for sampling control and batch process optimization

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Outline

1. Introduction
2. PID design for sampled control systems
3. Learning PID design for batch optimization
4. Conclusions & Outlook

Introduction

Some facts of PID controllers

Advantages:

- **Simple structure and low cost;**
- **Easily understood and commanded by users;**
- **Most widely used and commercialized in industrial applications.**

Disadvantages [1, 2]:

- **Generally not optimal in control performance;**
  (often used as an inferior to demonstrate other advanced control designs)
- **Difficult to analyze the robust stability against system uncertainties.**
  (lowly valued for theoretical contribution in top control-relevant journals)

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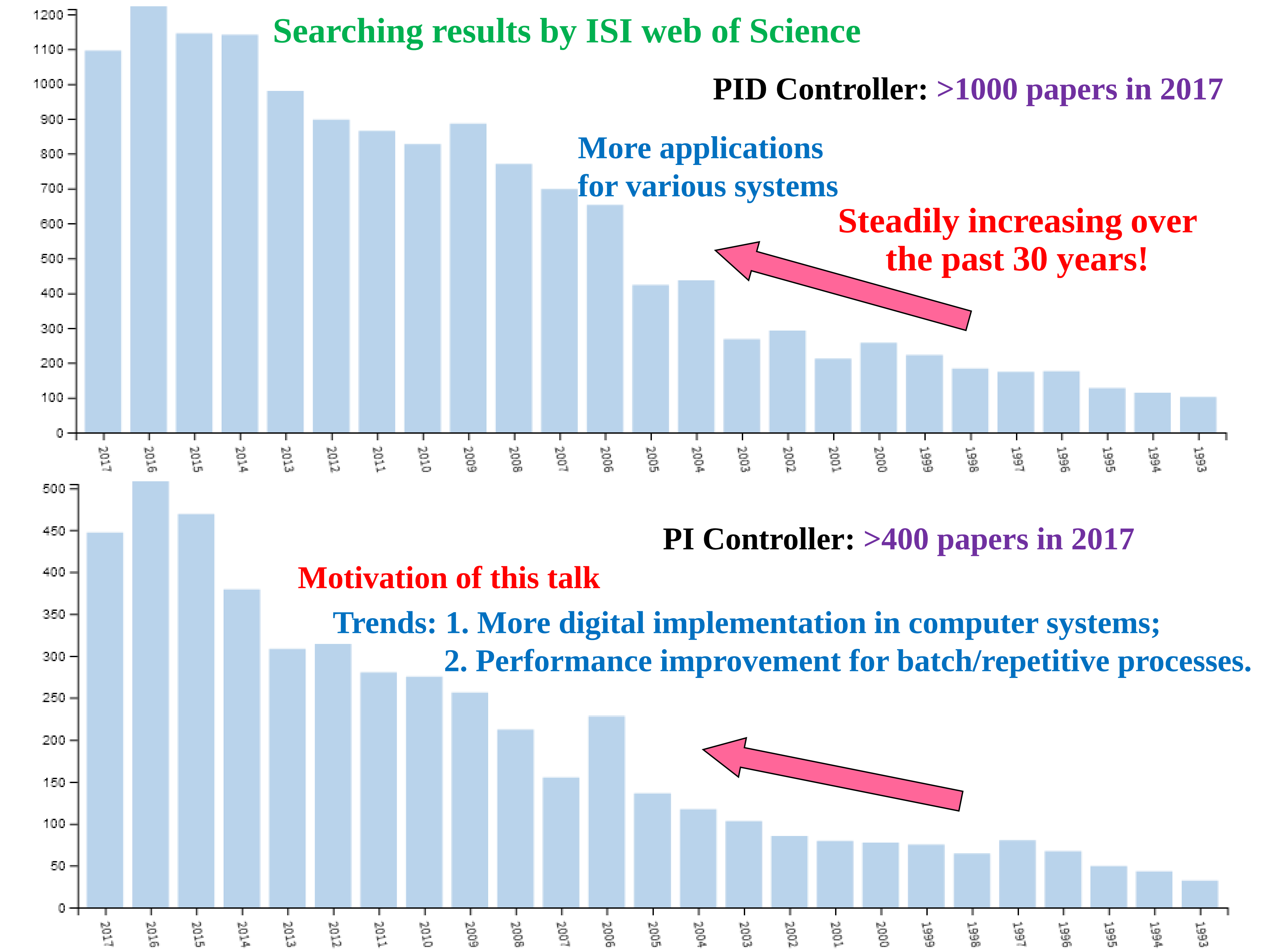
[1] Karl J. Åström, P.R. Kumar. Control: A perspective. ***Automatica***, 2014, 50(1), 3-43.
[2] Tao Liu, Furong Gao. Industrial Process Identification and Control Design: Step-test and Relay-Experiment-Based Methods. London UK: ***Springer***, 2012.


Searching results by ISI web of Science
PID Controller: >1000 papers in 2017
More applications
for various systems
Steadily increasing over
the past 30 years!
PI Controller: >400 papers in 2017
Motivation of this talk
Trends: 1. More digital implementation in computer systems;
2. Performance improvement for batch/repetitive processes.
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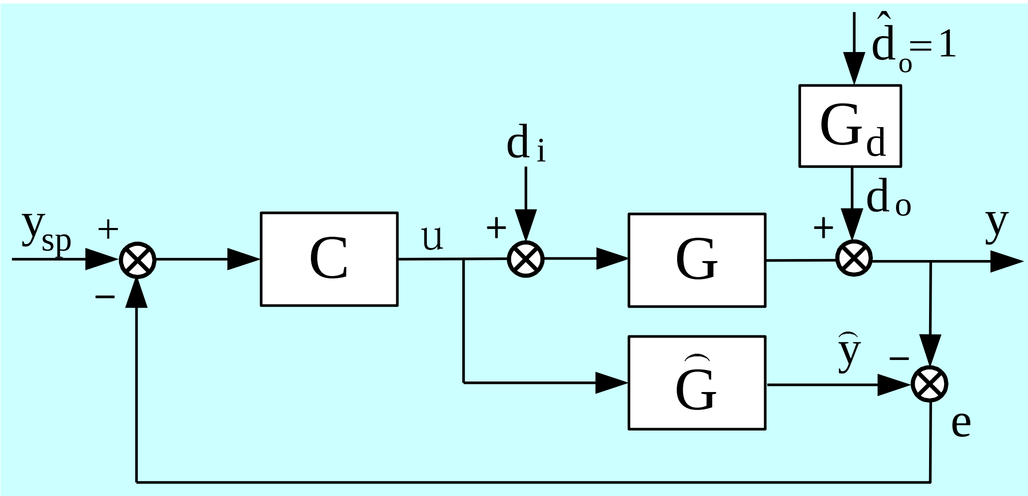
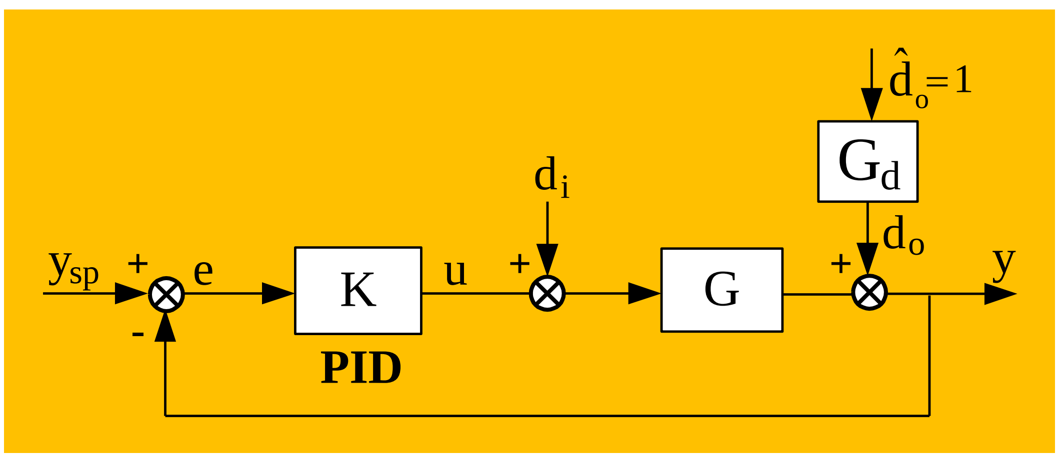
PID design for sampled control systems

Relationship between the PID control and the internal model control (IMC)



The internal model control (IMC) structure



The unity feedback control structure

Process model: $\hat{G} = \dfrac{B(z)}{A(z)} z^{-d}$

Frequency domain: $\hat{G} = \dfrac{B(s)}{A(s)} e^{-\theta s}$

IMC: applicable for stable processes

Equivalent relationship

$$K = \frac{C}{1 - \hat{G}C}$$

PID: applicable for stable, integrating, and unstable processes

Review of the IMC design in frequency domain

Step 1. Decompose the model into the minimum-phase (MP), non-MP, and all-pass parts.

For example: $\widehat{G} = \dfrac{k_p(-\tau_0 s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)} e^{-\theta s}$ ⇒ $\widehat{G} = \widehat{G}_{mp}\widehat{G}_{nmp}\widehat{G}_{ap}$ $\widehat{G}_{ap} = e^{-\theta s}$

where $\tau_0 > 0$, $\tau_1 > 0$, $\tau_2 > 0$.

$\widehat{G}_{mp} = \dfrac{k_p}{(\tau_1 s + 1)(\tau_2 s + 1)}$ $\widehat{G}_{nmp} = -\tau_0 s + 1$

Step 2. Specify the desired closed-loop transfer function for set-point tracking.

$$T = F_1 F_2 \widehat{G}_{nmp}\widehat{G}_{ap}$$

where F_1 **and** F_2 **are two low-pass filters.**

For a stable process described as above,

$F_1 = \dfrac{1}{(\lambda s + 1)^3}$ ⇒ **ensure** $F_1\widehat{G}_{mp}$ **strictly proper**

$F_2 = \dfrac{1}{\tau_0 s + 1}$ ⇒ **ensure** $F_2\widehat{G}_{nmp}$ **all-pass, i.e.** $\dfrac{-\tau_0 s + 1}{\tau_0 s + 1} = e^{-2\tau_0 s}$

Desired closed-loop transfer function $T = \dfrac{1}{(\lambda s + 1)^3} \dfrac{-\tau_0 s + 1}{\tau_0 s + 1} e^{-\theta s}$

single tuning parameter λ

No overshoot
Minimal ISE

Discrete-time domain IMC design

Step 1. Decompose the model into the minimum-phase (MP), non-MP, and all-pass parts.

For example:

$$\widehat{G} = \frac{k_{\mathrm{p}}(z-b_1)(z-b_2)}{(z-a_1)(z-a_2)} z^{-d}$$

where $|a_1|<1,\ |a_2|<1,\ |b_1|<1,\ |b_2|>1.$

$$\widehat{G} = \widehat{G}_{\mathrm{mp}}\widehat{G}_{\mathrm{nmp}}\widehat{G}_{\mathrm{ap}} \qquad \widehat{G}_{\mathrm{ap}} = z^{-d}$$

$$\widehat{G}_{\mathrm{mp}} = \frac{k_{\mathrm{p}}(z-b_1)}{(z-a_1)(z-a_2)} \qquad \widehat{G}_{\mathrm{nmp}} = z-b_2$$

Step 2. Specify the desired closed-loop transfer function for set-point tracking.

$$T = F_1F_2\widehat{G}_{\mathrm{nmp}}\widehat{G}_{\mathrm{ap}}$$

where F_1 **and** F_2 **are two low-pass filters.**

For a stable process described as above,

$$F_1 = \frac{(1-\lambda_{\mathrm{c}})^2}{(z-\lambda_{\mathrm{c}})^2}$$

→ **ensure** $F_1\widehat{G}_{\mathrm{mp}}$ **strictly proper**

$$F_2 = \frac{1-b_2^{-1}}{(1-b_2)(z-b_2^{-1})}$$

→ **ensure** $F_2\widehat{G}_{\mathrm{nmp}}$ **all-pass, i.e.** $\dfrac{(1-b_2^{-1})(z-b_2)}{(1-b_2)(z-b_2^{-1})}$

Desired closed-loop transfer function

$$T = \frac{(1-\lambda_{\mathrm{c}})^2}{(z-\lambda_{\mathrm{c}})^2}\frac{(1-b_2^{-1})(z-b_2)}{(1-b_2)(z-b_2^{-1})}z^{-d}$$

Discrete-time domain IMC design

Step 3. Determine the IMC controller.

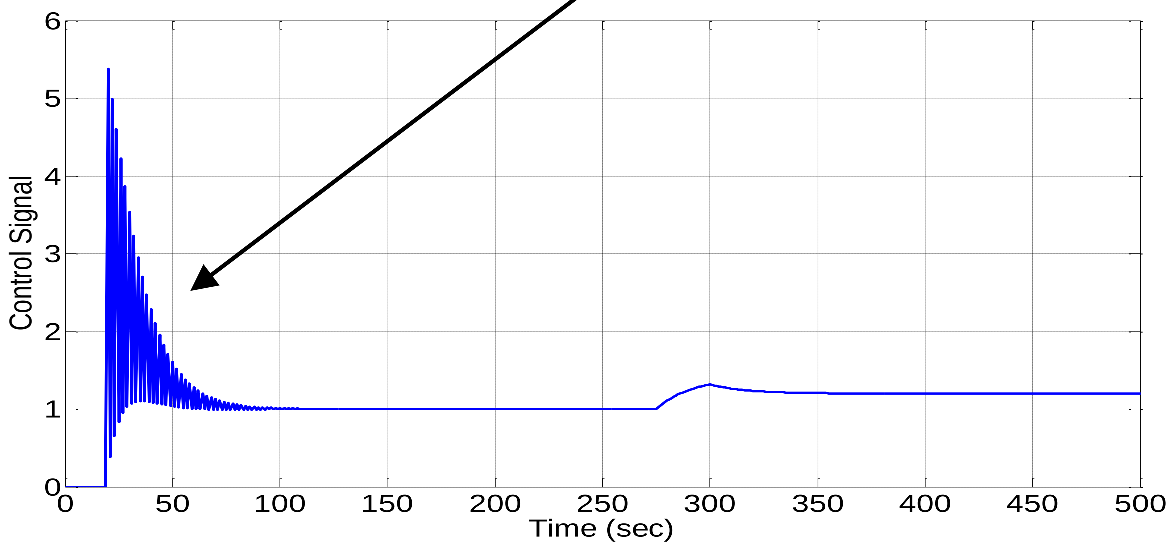
$$C_{\text{IMC}}(z) = \frac{T(z)}{\widehat{G}(z)}$$ ← $T = GC$

For a stable process described as above,

single tuning parameter λ_c

$$C_{\text{IMC}}(z) = \frac{(z-a_1)(z-a_2)(1-\lambda_c)^2}{k_p(z-b_1)(z-\lambda_c)^2} \frac{(1-b_2^{-1})}{(1-b_2)(z-b_2^{-1})}$$

In case $-1 < b_1 < 0$, it could provoke inter-sample ringing in the control signal !



Discrete-time domain IMC design

Solution: Introduce another filter to remove such a zero for implementation

$$F_3(z) = z^{-1}\frac{z-b_1}{1-b_1} \Rightarrow C_{\mathrm{RIMC}}(z) = F_3(z)C_{\mathrm{IMC}}(z)$$

Control performance assessment

For a stable process described by

$$G_1(z) = \frac{K_{\mathrm{p1}}}{z-z_{\mathrm{p}}}z^{-d}$$

$$R(z) = \frac{z}{z-1}$$

Step response in time domain

$$T_{\mathrm{d}}(z) = \frac{1-\lambda_{\mathrm{c}}}{z-\lambda_{\mathrm{c}}} \Rightarrow Y(z) = \frac{1-\lambda_{\mathrm{c}}}{z-\lambda_{\mathrm{c}}}z^{-d}R(z) \Rightarrow y(kT_{\mathrm{s}}) = \begin{cases} 0, & k \le d, \\ 1-\lambda_{\mathrm{c}}^{\,k\text{-}d}, & k > d. \end{cases}$$

Set-point tracking error

$$IAE_{\mathrm{r}} = \sum_{k=d+1}^{\infty}[1-y(kT_{\mathrm{s}})] = \sum_{k=d+1}^{\infty}\lambda_{\mathrm{c}}^{k-d} = \lim_{n\to\infty}\frac{\lambda_{\mathrm{c}}(1-\lambda_{\mathrm{c}}^{n})}{1-\lambda_{\mathrm{c}}} = \frac{\lambda_{\mathrm{c}}}{1-\lambda_{\mathrm{c}}}$$

Disturbance rejection error to a step change

$$IAE_{\mathrm{d}} = \sum_{k=d+1}^{\infty}y(kT_{\mathrm{s}}) = \frac{K_{\mathrm{p1}}}{(1-\lambda_{\mathrm{c}})(1-z_{\mathrm{p}})}$$

Quantitative tuning

IMC-based PID design in frequency domain

Step 4. Determine the equivalent controller in a unity feedback control structure.

$$K = \frac{C}{1-\widehat{G}C}$$

For $$\widehat{G} = \frac{k_\mathrm{p}(-\tau_0 s+1)}{(\tau_1 s+1)(\tau_2 s+1)}e^{-\theta s}$$ ➡ $$K = \frac{(\tau_1 s+1)(\tau_2 s+1)}{k_\mathrm{p}[(\lambda s+1)^3(\tau_0 s+1)-(-\tau_0 s+1)e^{-\theta s}]}$$

Step 5. Determine the PID controller by the Taylor approximation [1].

Since $\lim_{s\to 0} K(s) = \infty$, **let** $K(s) = \dfrac{M}{s}$

$$K(s) = \frac{1}{s}\left[M(0)+M'(0)s+\frac{M''(0)}{2!}s^2+\cdots\right]$$

Important advantage: single tuning parameter

$$K(s) = k_\mathrm{C} + \frac{1}{\tau_\mathrm{I} s} + \frac{\tau_\mathrm{D} s}{\tau_\mathrm{F} s+1}$$

$$\begin{cases} k_\mathrm{C} = M'(0) \\ \tau_\mathrm{I} = 1/M(0) \\ \tau_\mathrm{D} = M''(0)/2 \end{cases}$$

$$\tau_\mathrm{F} = (0.01-0.1)\tau_\mathrm{D}$$

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[1] Lee, Y., Park, S., Lee, M., & Brosilow, C. PID controller tuning for desired closed-loop responses for SI/SO systems. ***AIChE Journal***, 44, 106-115, 1998.

IMC-based PID design in frequency domain

Alternatively, the delay term in the process model or the IMC controller may be rationally approximated to derive a PID, e.g.,

The first-order Taylor approximation [2]: $e^{-\theta s} \approx 1-\theta s$

Pade approximation [3]: $e^{-\theta s} \approx (1-\frac{\theta}{2}s)/(1+\frac{\theta}{2}s)$

Model reduction [4, 5]: $\frac{1}{(\tau_1 s+1)(\tau_2 s+1)} \approx \frac{1}{(\tau_1+\frac{\tau_2}{2})s+1}e^{-\frac{\tau_2}{2}s}$ **'Half rule'**

Frequency response fitting [6]: $\min \sum_{i=1}^{m} \left| K_{\text{PID}}(\mathrm{j}\omega_i) - K(\mathrm{j}\omega_i) \right|$ $\omega_i \in (0.1,\ 1)\omega_{\text{cb}}$

[2] Rivera, D. E., Morari, M., Skogestad, S. Internal model control. 4. PID controller design. ***Ind. Eng. Chem. Res.***, 25, 252-265, 1986.

[3] Fruehauf P. S., Chien I.-L., Lauritsen M. D. Simplified IMC-PID tuning rules. ***ISA Transactions***, 1994, 33, 43-59.

[4] Skogestad, S. Simple analytical rules for model reduction and PID controller tuning. ***Journal of Process Control***, 2003, 13, 291-309.

[5] Lee, J. Cho, W. Edgar, T. F. Simple analytic PID controller tuning rules revisited, ***Industrial & Engineering Chemistry Research***, 53(13), 5038-5047, 2014.

[6] Wang, Q. G., Yang, X. P. Single-loop controller design via IMC principles. ***Automatica***, 37, 2041-2048, 2001.

IMC-based PID design in discrete-time domain

For sampling control implementation, the first-order differentiation is generally used to discretize the above frequency domain design.

For direct PID design in discrete-time domain, let the equivalent controller in a unity feedback control structure be

$$K(z)=\frac{M(z)}{z-1} \quad \Leftarrow \quad K=\frac{C}{1-\widehat{G}C}$$

owing to $\lim_{z\to 1} K(z)=\infty$ **.**

A PID controller is determined by the Taylor approximation [1, 2].

$$K(z)=\frac{1}{z-1}\left[M(1)+M'(1)(z-1)+\frac{M''(1)}{2!}(z-1)^2+\ldots\right]$$

$$K_{\mathrm{PID}}(z)=k_c\left[1+\frac{1}{\tau_{\mathrm{I}}(z-1)}+(1-\alpha\tau_{\mathrm{D}})\frac{\tau_{\mathrm{D}}(z-1)}{z-\alpha\tau_{\mathrm{D}}}\right]$$

$$\begin{cases} k_{\mathrm{C}}=M'(1) \\ \tau_{\mathrm{I}}=M'(1)/M(1) \\ \tau_{\mathrm{D}}=M''(1)/2M'(1) \end{cases}$$

single tuning parameter λ

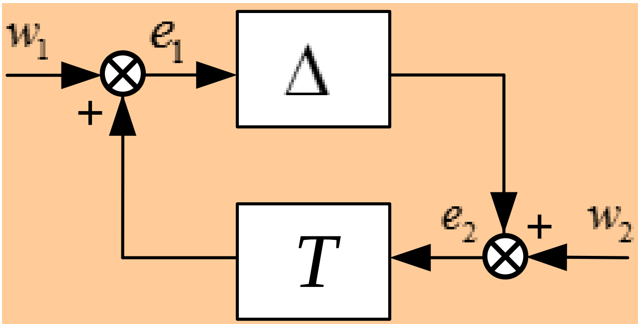
$$\alpha\in(0.01,\ 0.1)$$

[1] Wang, D., **Liu T.***, Sun X., Zhong Chongquan. Discrete-time domain two-degree-of-freedom control for integrating and unstable processes with time delay. ***ISA Transactions***, 63, 121-132, 2016.

[2] Cui J., Chen Y., **Liu T.***. Discrete-time domain IMC-based PID control design for industrial processes with time delay. ***The 35th Chinese Control Conference (CCC)***, Chengdu, China, 5946-5951, 2016.

IMC-based PID design in discrete-time domain

Closed-loop system robust stability analysis



The $T-\Delta$ structure

Small gain theorem

$$\left\|T(z)\Delta_{\mathrm{m}}(z)\right\|_{\infty}<1$$

$$T(z)=\frac{G_{\mathrm{p}}(z)C_{\mathrm{PID}}(z)}{1+G_{\mathrm{p}}(z)C_{\mathrm{PID}}(z)}$$

$$\Delta_{\mathrm{m}}(z)=[G(z)-G_{\mathrm{m}}(z)]/G_{\mathrm{m}}(z)$$

Magnitude/Phase margin, Maximal sensitivity index M_s

No analytical solution!

For a stable process described by $G_1(z)=\frac{k_{\mathrm{p}}}{z-\tau}z^{-d}$

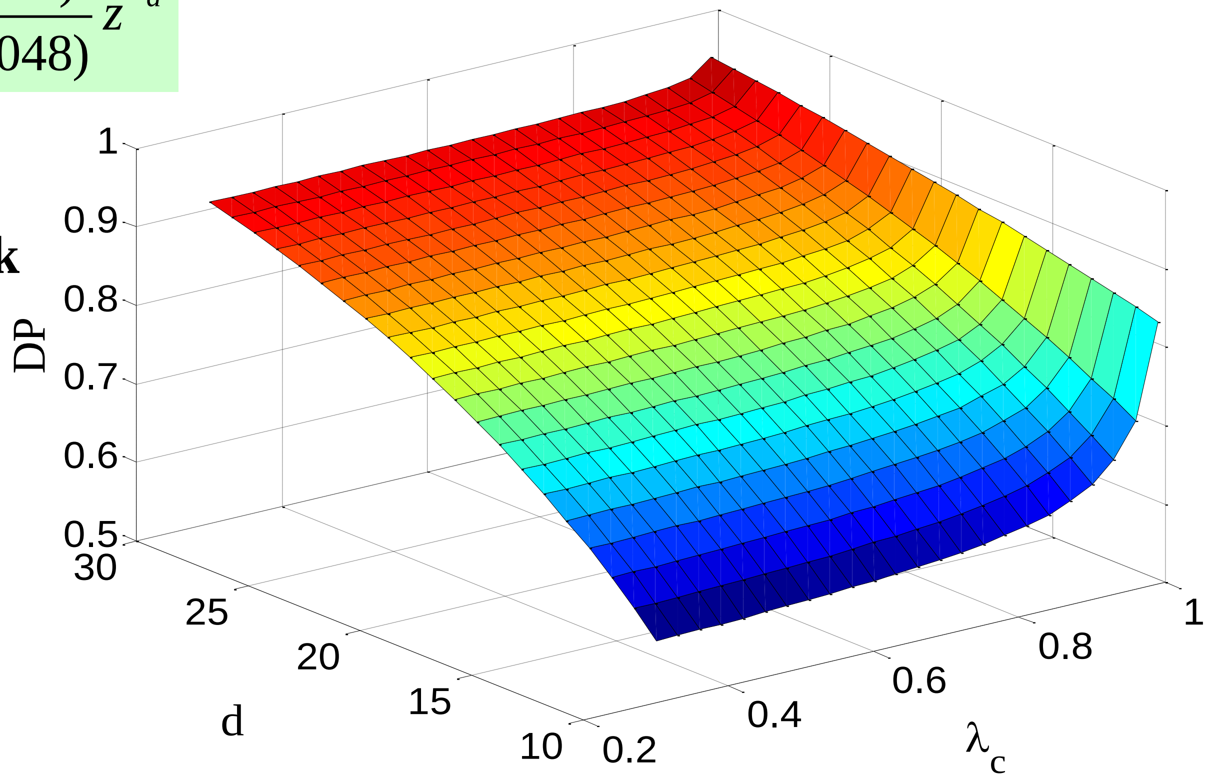
$$\left\|\frac{\frac{k_{\mathrm{p}}}{z-\tau}z^{-d}K\left[1+\frac{1}{T_{\mathrm{i}}(z-1)}+(1-\alpha T_{\mathrm{d}})\frac{T_{\mathrm{d}}(z-1)}{z-\alpha T_{\mathrm{d}}}\right]}{1+\frac{k_{\mathrm{p}}}{z-\tau}z^{-d}K\left[1+\frac{1}{T_{\mathrm{i}}(z-1)}+(1-\alpha T_{\mathrm{d}})\frac{T_{\mathrm{d}}(z-1)}{z-\alpha T_{\mathrm{d}}}\right]}\right\|_{\infty}<\frac{1}{\left\|\Delta_{\mathrm{m}}(z)\right\|_{\infty}}$$

IMC-based PID design in discrete-time domain

Disturbance rejection performance

$$G(z) = \frac{0.0065904(z+0.9222)}{(z-0.8669)(z-0.9048)} z^{-d}$$

Disturbance response peak



Tuning guideline: **The single adjustable parameter λ_c can be monotonically increased or decreased in a range of $\lambda_c \in (0.8, 1)$ to meet a good trade-off between the closed-loop control performance and its robust stability.**

IMC-based PID design in discrete-time domain

An illustrative example:

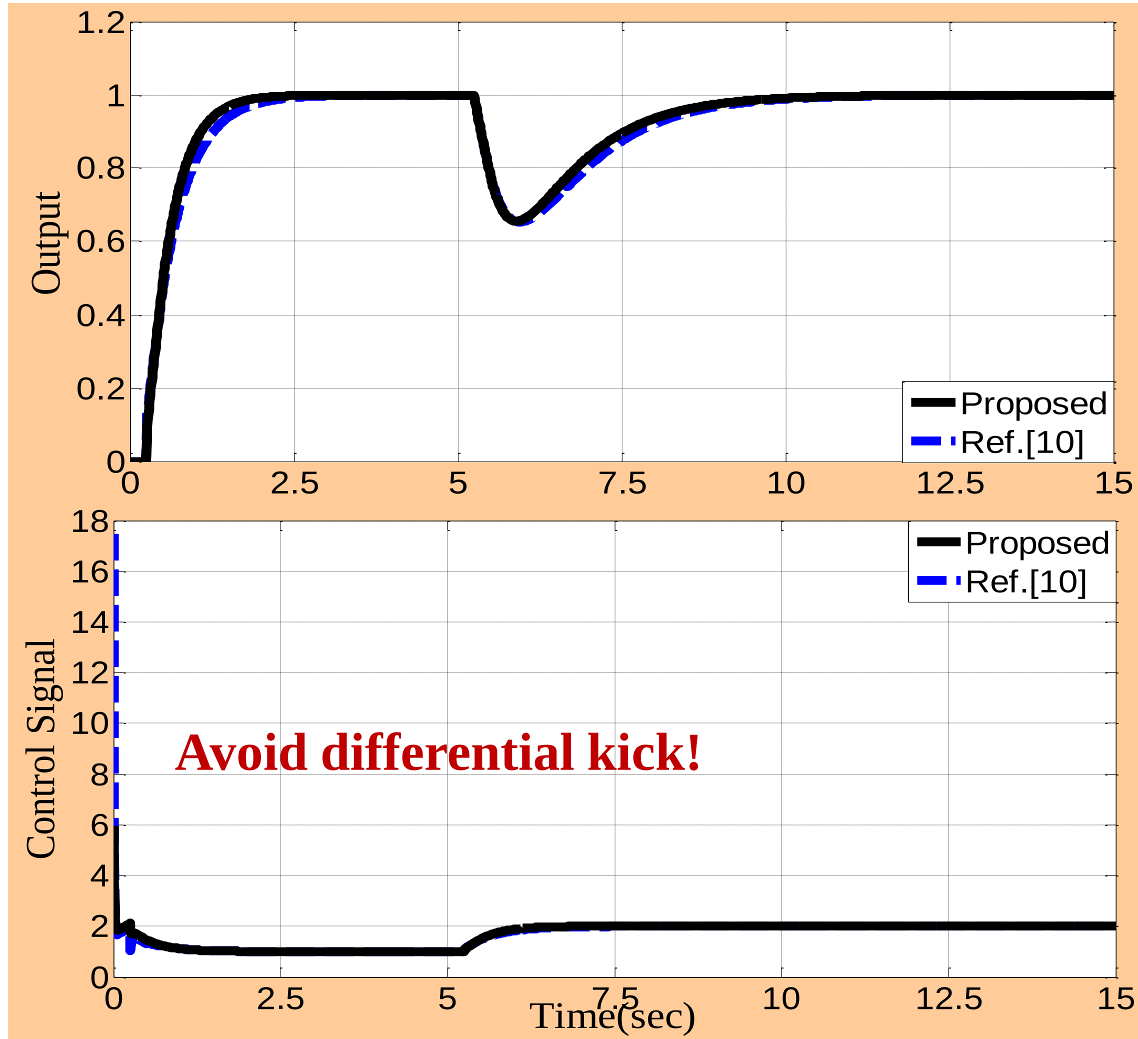
$$G(s)=\frac{1}{s+1}e^{-0.25s}$$

Sampling period: $T_s = 0.02(s)$

$$G(z)=\frac{0.0003947(z+0.9868)}{(z-1)(z-0.9608)}z^{-25}$$

$$C_{\mathrm{IMC}}(z)=\frac{(z-0.99)(1-\lambda_c)}{0.00995(z-\lambda_c)}$$

$$C_{\mathrm{PID}}(z)=1.7049\left[1+\frac{1}{105.2388(z-1)}+\frac{2.4956(z-1)}{z-0.4791}\right]$$



[10] Panda R.C. Synthesis of PID tuning rule using the desired closed-loop response, ***Industrial & Engineering Chemistry Research***, 47(22), 8684-8692, 2008.

Improved IMC-based PID design for disturbance rejection

For a slow process described by $G = \frac{k_{\mathrm{p}} e^{-s}}{\tau_{\mathrm{p}} s + 1}$ **with a large time constant** τ_{p} **,**

Discrete-time domain model: $\mathrm{G}(z) = \frac{k_{\mathrm{p}}}{z - z_{\mathrm{p}}} z^{-d}$ **with a pole** $|z_{\mathrm{p}}| < 1$ **close to the unit circle**

Load disturbance transfer function

(optimal $1 - GC = 1 - T$ **by IMC)**

$$\frac{y}{d_{\mathrm{i}}} = G(1 - GC) \qquad \frac{y}{\hat{d}_{\mathrm{o}}} = G_{\mathrm{d}}(1 - GC)$$

The slow pole of G **or** G_{d} **affects the disturbance rejection performance !**

Solution: Introduce an asymptotic constraint to remove the effect of slow dynamics.

Modify the desired transfer function:

$$T_{\mathrm{RIMC}} = \frac{(\alpha s + 1) e^{-\theta s}}{(\lambda_{\mathrm{f}} s + 1)^2} \qquad T_{\mathrm{d}}(z) = \frac{(1 - \lambda_{\mathrm{c}})^{n_{\mathrm{d}}} (\beta_0 + \beta_1 z)}{(z - \lambda_{\mathrm{c}})^{n_{\mathrm{d}}}}$$

$$\alpha = \tau_{\mathrm{p}} [1 - (\frac{\lambda_{\mathrm{f}}}{\tau_{\mathrm{p}}} - 1)^2 e^{-\frac{\theta}{\tau_{\mathrm{p}}}}] \Leftarrow \lim_{s \to -1/\tau_{\mathrm{p}}} (1 - T_{\mathrm{RIMC}}) = 0$$

Discrete-time domain: $\lim_{z \to 1} (1 - T_{\mathrm{d}}) = 0$ $\lim_{z \to -1/z_{\mathrm{p}}} (1 - T_{\mathrm{d}}) = 0$ $\Rightarrow$

$$\begin{cases} \beta_1 = \frac{(z_{\mathrm{p}} - \lambda_{\mathrm{c}})^{n_{\mathrm{d}}} - (1 - \lambda_{\mathrm{c}})^{n_{\mathrm{d}}}}{(z_{\mathrm{p}} - 1)(1 - \lambda_{\mathrm{c}})^{n_{\mathrm{d}}}} \\ \beta_0 = 1 - \beta_1 \end{cases}$$

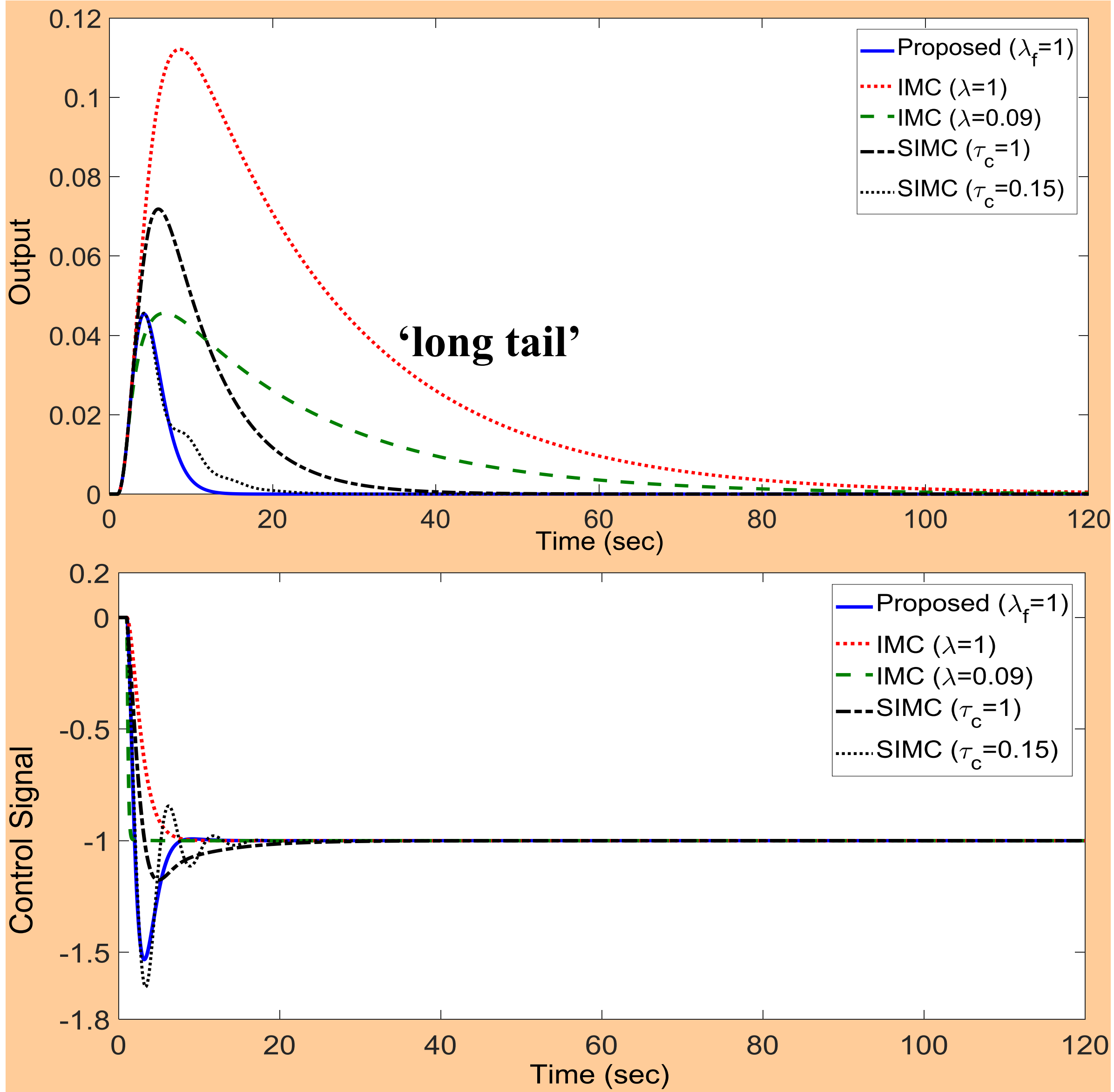
Improved IMC-based PID design for disturbance rejection

An illustrative example:

$$G = \frac{e^{-s}}{(20s+1)(2s+1)}$$

Slow pole: $s = -\frac{1}{20}$

Comparison with the standard IMC and SIMC by Skogestad (JPC, 2003)

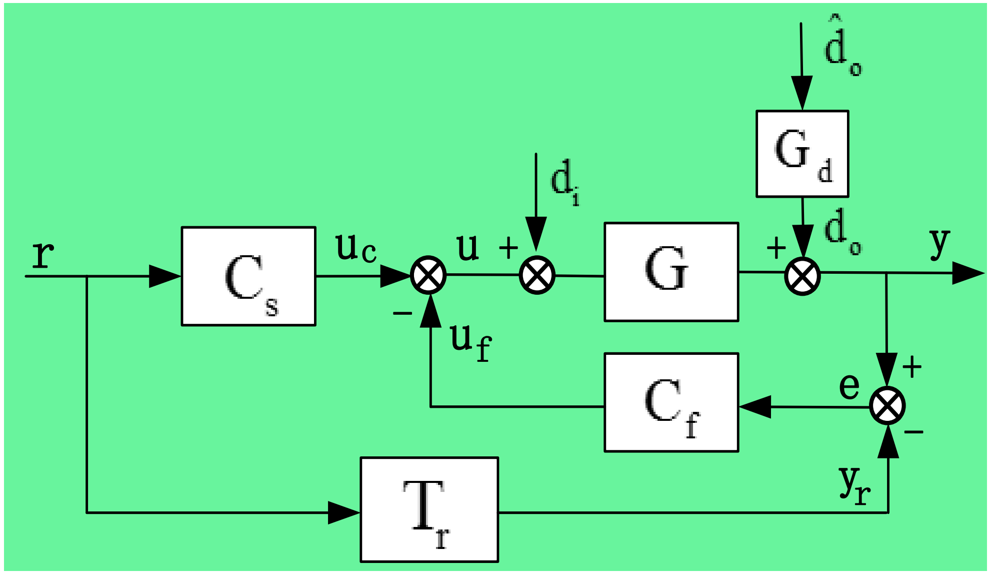


[1] Tao Liu, Furong Gao. New insight into internal model control filter design for load disturbance rejection. ***IET Control Theory & Application***, 4 (3), 448-460, 2010.

Discrete-time PID design for integrating and unstable processes

Problem: The classical PID control structure could not suppress large overshoot for set-point tracking ! There exists severe water-bed effect!

Solution: A two-degree-of-freedom (2DOF) control structure



Advantage: the set-point tracking is decoupled from load disturbance rejection.

Set-point tracking: open-loop control IMC design: $T_r = GC_s$

Disturbance rejection: closed-loop control using a PID controller, C_f

The desired transfer function for set-point tracking is the same as the standard IMC design

The desired transfer function for disturbance rejection :

$$T_d(z) = (\beta_0 + \beta_1 z + \cdots \beta_m z^m)\frac{(1-\lambda_f)^{m+1}}{(z-\lambda_f)^{m+1}} z^{-d}$$

Discrete-time PID design for integrating and unstable processes

Integrating process:

$$\widehat{G}_{\mathrm{I}}(z)=\frac{k_{\mathrm{p}}(z-z_0)}{(z-1)(z-z_{\mathrm{p}})}z^{-d}$$

Unstable process:

$$\widehat{G}_{\mathrm{U}}(z)=\frac{k_{\mathrm{p}}(z-z_0)}{(z-z_{\mathrm{u}})(z-z_{\mathrm{p}})}z^{-d}$$

The following asymptotic tracking constraints must be satisfied,

$$\lim_{z\to 1}(1-T_{\mathrm{d}})=0$$

$$\lim_{z\to \eta}(1-T_{\mathrm{d}})=0$$

$\eta = z_{\mathrm{u}}$ **or** $\eta = z_{\mathrm{p}}$ **(close to the unit circle)**

$$\lim_{z\to 1}\frac{d}{dz}(1-T_{\mathrm{d}})=0$$

$$T_{\mathrm{d}}=\frac{\widehat{G}C_{\mathrm{f}}}{1+\widehat{G}C_{\mathrm{f}}}$$

Derive the closed-loop controller:

$$C_{\mathrm{f}}=\frac{T_{\mathrm{d}}}{1-T_{\mathrm{d}}}\cdot\frac{1}{G}$$

Taylor approximation → **PID**

Discrete-time PID design for integrating and unstable processes

Integrating process:

$$G(s)=\frac{0.1e^{-5s}}{s(5s+1)}$$

Frequency domain design:

$$C_{s}(s)=\frac{s(5s+1)}{0.1(\lambda_{c}s+1)^{2}}$$

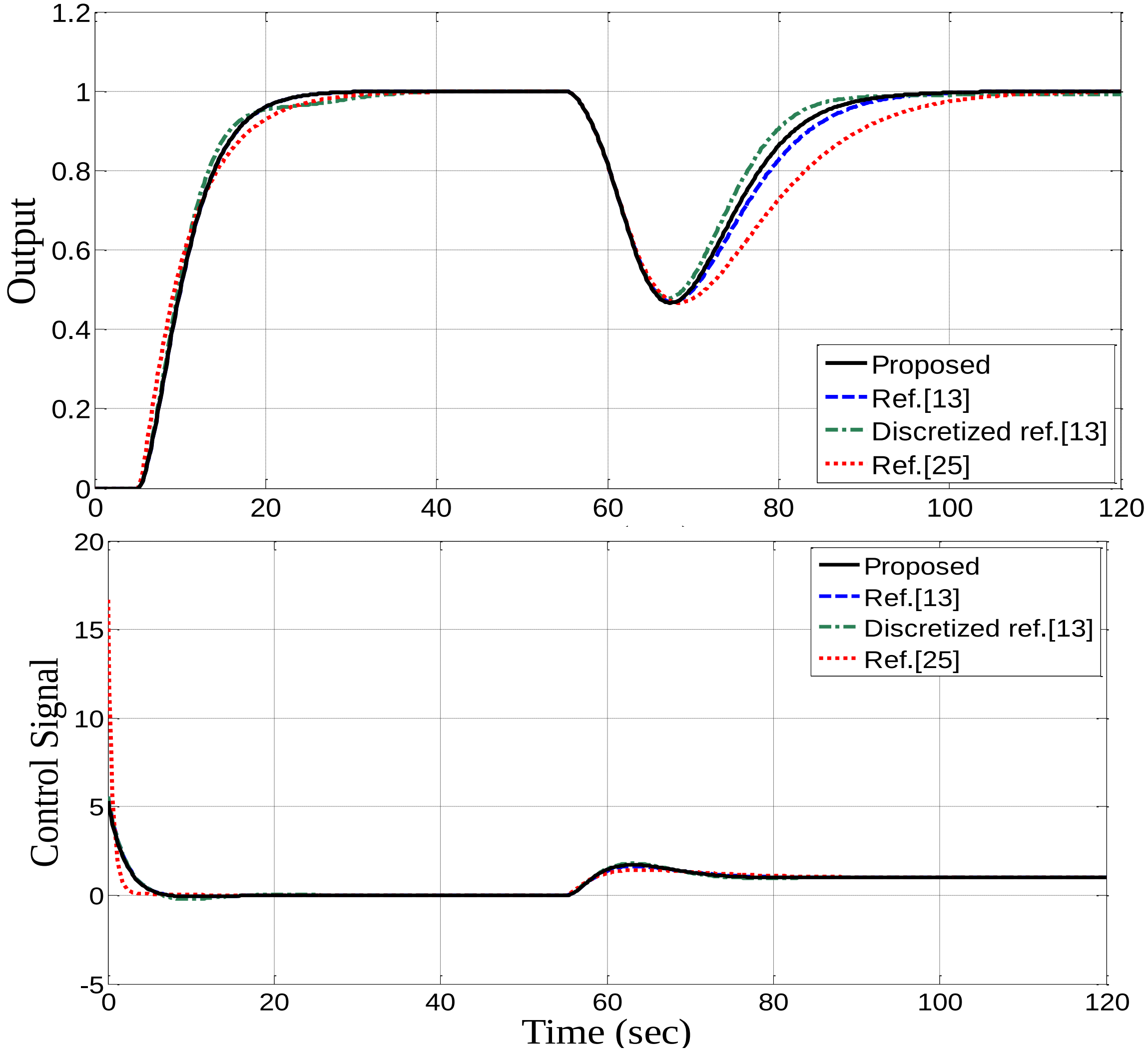
$$T_{r}(s)=\frac{e^{-5s}}{(\lambda_{c}s+1)^{2}}$$

$$C_{f}(s)=\frac{1}{G(s)}\cdot\frac{T_{d}(s)}{1-T_{d}(s)}$$

$$T_{d}(s)=\frac{\eta_{2}s^{2}+\eta_{1}s+1}{(\lambda_{f}s+1)^{4}}e^{-5s}$$

$$\eta_{1}=4\lambda_{f}+5$$

$$\eta_{2}=5\eta_{1}+25(0.2\lambda_{f}-1)^{4}e^{-1}-25$$



[1] Wang, D., **Liu T.***, Sun X., Zhong Chongquan. Discrete-time domain two-degree-of-freedom control for integrating and unstable processes with time delay. ***ISA Transactions***, 63, 121-132, 2016.

[13] **Liu T.***, Gao, F. Enhanced IMC design of load disturbance rejection for integrating and unstable processes with slow dynamics. ***ISA Transactions***, 50 (2), 239-248, 2011.

Discrete-time PID design for integrating and unstable processes

Sampling period: $T_s = 0.2(s)$

$$G(z) = \frac{0.0003947(z+0.9868)}{(z-1)(z-0.9608)} z^{-25}$$

$$C_s(z) = \frac{(z-1)(z-0.9608)(1-\lambda_c)^2}{0.0007842(z-\lambda_c)^2}$$

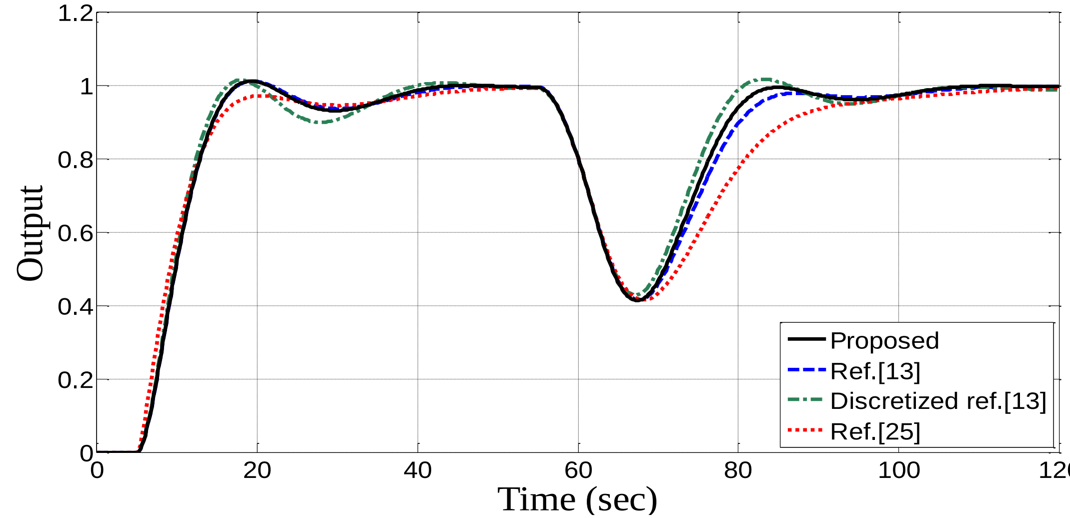
$$T_d(z) = \frac{(\beta_0+\beta_1 z+\beta_2 z^2)(1-\lambda_f)^4}{(z-\lambda_f)^4} z^{-25}$$

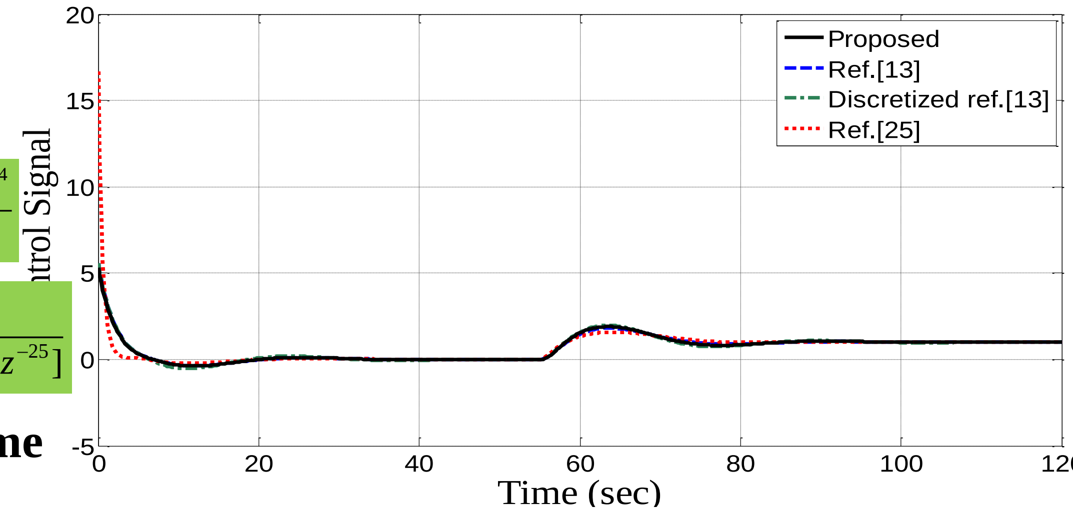
$$\beta_1 = d + \frac{4}{1-\lambda_f} - 2\beta_2$$

$$\beta_2 = \frac{(\tau_p-\lambda_f)^4 \tau_p{}^d + 4(1-\lambda_f)^3(1-\tau_p) + (d - d\tau_p - 1)(1-\lambda_f)^4}{(1-\lambda_f)^4(\tau_p-1)^2}$$

$$C_f(z) = \frac{(1-\lambda_f)^4(\beta_0+\beta_1 z+\beta_2 z^2)(z-1)(z-\tau_p)}{k_p z(1-z_0)[(z-\lambda_f)^4 - (\beta_0+\beta_1 z+\beta_2 z^2)(1-\lambda_f)^4 z^{-25}]}$$

Perturbation: the process gain and time constant are actually 20% larger

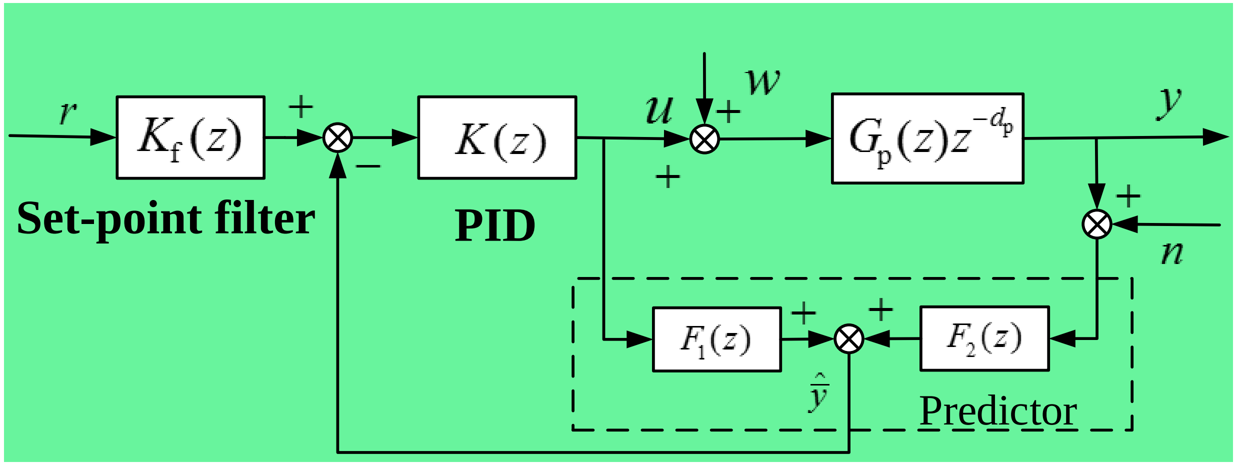




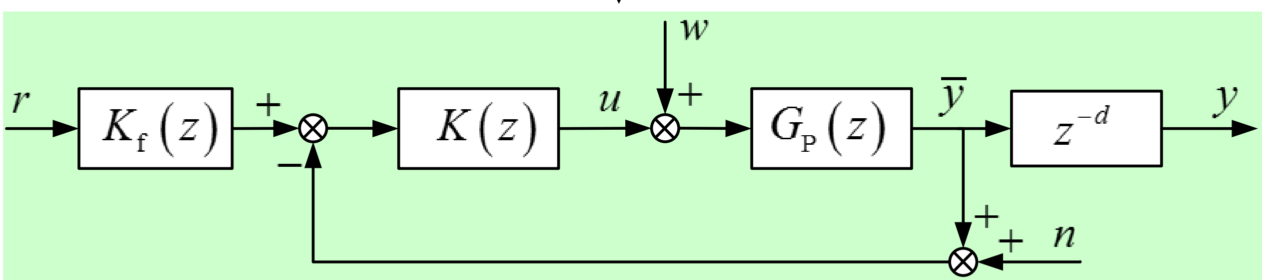
[25] Torrico B.C., Cavalcante M.U., Braga A.P.S., Normey-Rico J.E., Albuquerque A.A.M. Simple tuning rules for dead-time compensation of stable, integrative, and unstable first-order dead-time processes. ***Ind. Eng. Chem. Res.***, 52, 11646-11654, 2013.

Discrete-time PID design for long time delay processes

Predictor based control structure



Delay-free output prediction



[1] **Tao Liu***, Pedro García, Yueling Chen, Xuhui Ren, Pedro Albertos, Ricardo Sanz. New Predictor and 2DOF Control Scheme for Industrial Processes with Long Time Delay. ***IEEE Transactions on Industrial Electronics***, 65 (5), 4247-4256, 2018.
[2] Yueling Chen, **Tao Liu***, Pedro García, Pedro Albertos. Analytical design of a generalized predictor-based control scheme for low-order integrating and unstable systems with long time delay. ***IET Control Theory & Application***, 10(8), 884-893, 2016.

Discrete-time PID design for long time delay processes

The nominal system transfer function

$$y(z) = K_{\mathrm{f}}(z)\frac{K(z)G_{\mathrm{p}}(z)}{1+K(z)\hat{G}(z)}z^{-d_{\mathrm{p}}}r(z) + \frac{G_{\mathrm{p}}(z)}{1+K(z)\hat{G}(z)}\left[1+K(z)F_{1}(z)\right]z^{-d_{\mathrm{p}}}w(z)$$

Closed-loop transfer function for disturbance rejection

$$\frac{u(z)}{w(z)} = T_{\mathrm{d}}(z) = \frac{K(z)G(z)}{1+K(z)G(z)}$$

The desired closed-loop transfer function (free of time delay)

$$T_{\mathrm{d}}(z) = \frac{(1-\lambda_{\mathrm{c}})^{n_{\mathrm{d}}}}{(z-\lambda_{\mathrm{c}})^{n_{\mathrm{d}}}}\sum_{i=0}^{l}\beta_{i}z^{i}$$

$$\sum_{i=0}^{l}\beta_{i} = 1$$

$$K(z) = \frac{T_{\mathrm{d}}(z)}{1-T_{\mathrm{d}}(z)}\cdot\frac{1}{G_{\mathrm{p}}(z)}$$

Taylor approximation → **PID**

Discrete-time PID design for long time delay processes

Example: $$G(s) = \frac{e^{-27.5s}}{52.5s+1}$$

Sampling period: $T_s = 0.5(s)$

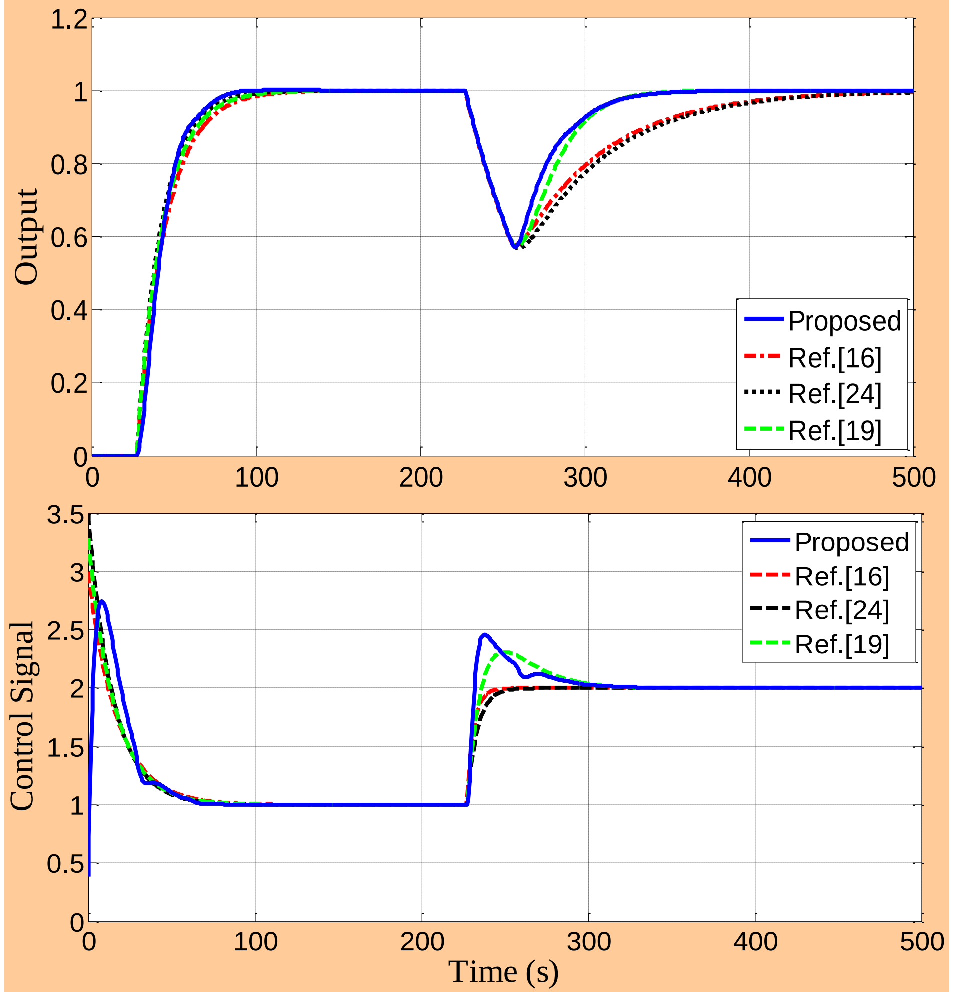
$$G(z) = \frac{0.009479}{z-0.9905} z^{-55}$$

PI controller: $$K(z) = \frac{(1-\lambda_c)^2(\beta_1 z + \beta_0)}{0.009479(z-1)}$$

$$\beta_0 = 1 - \beta_1$$

$$\beta_1 = (z_p - \lambda_c)^2 - (1-\lambda_c)^2 / (z_p - 1)(1-\lambda_c)^2$$

$$K_f(z) = \frac{(z-\lambda_c)^2(1-\lambda_f)^2 z}{(1-\lambda_c)^2(\beta_0 + \beta_1 z)(z-\lambda_f)^2}$$



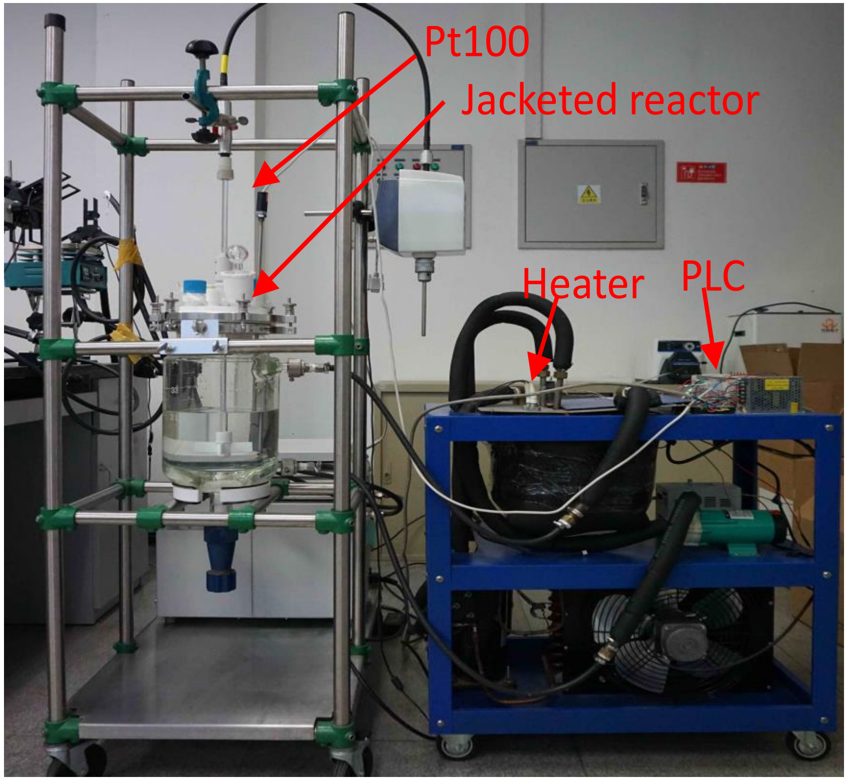
[16] Kirtania, K.M. Choudhury, A.A.S. A novel dead time compensator for stable processes with long dead times. ***J. Process Control***, 22(3), 612-625, 2012.

[24] Zhang, W., Rieber, J.M., Gu, D. Optimal dead-time compensator design for stable and integrating processes with time delay. ***J. Process Control***, 18(5), 449-457, 2008.

[19] Normey-Rico J.E., Camacho E. F. Unified approach for robust dead-time compensator design. ***J Process Control***, 19: 38-47, 2009.

Discrete-time PID design for long time delay processes

A temperature control system of a 4-liter jacketed reactor for pharmaceutical crystallization



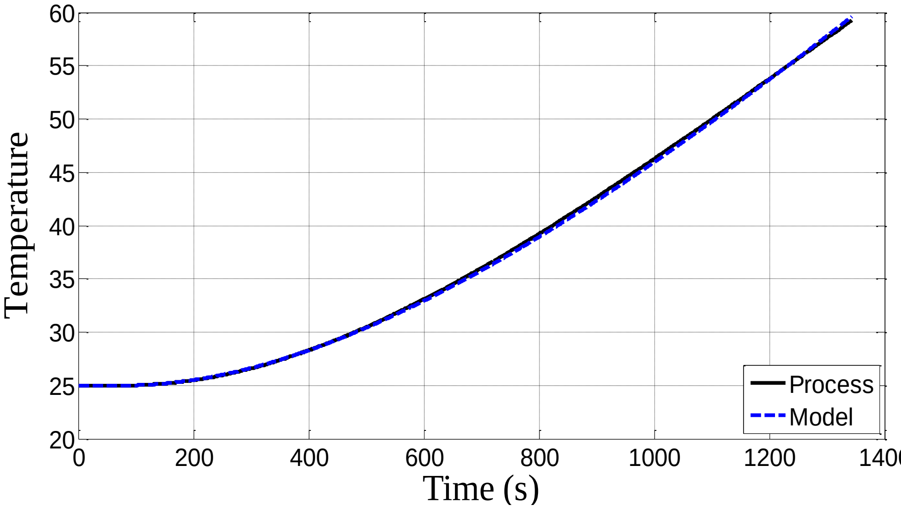
Sampling period: $T_s = 3(s)$

Step response identification:

Integrating type process model:

$$G(s) = \frac{0.0004529}{s(760.40s+1)} e^{-100.25s}$$

$$G(z) = \frac{2.6765 \times 10^{-6}(z+0.9989)}{(z-1)(z-0.9961)} z^{-34}$$



Closed-loop controller:

$$K(z) = \frac{(1-\lambda_f)^4(\beta_0 + \beta_1 z + \beta_2 z^2)}{4.9557 \times 10^{-6} z(z-1)(z-0.9899)}$$

$$\beta_0 = 1 - \beta_1 - \beta_2 \quad \beta_1 = 4/(1-\lambda_f) - \beta_2$$

$$\beta_2 = \frac{(0.9961-\lambda_f)^4}{(0.9961-1)^2(1-\lambda_f)^4} - \frac{4}{(1-0.9961)(1-\lambda_f)} - \frac{1}{(0.9961-1)^2}.$$

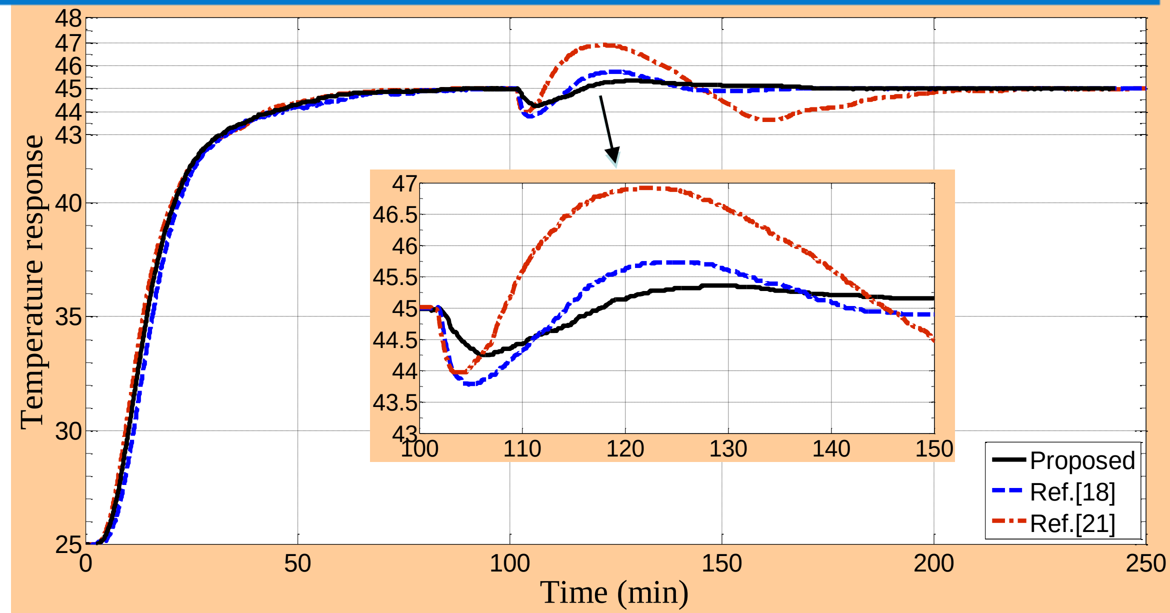
Set-point filter:

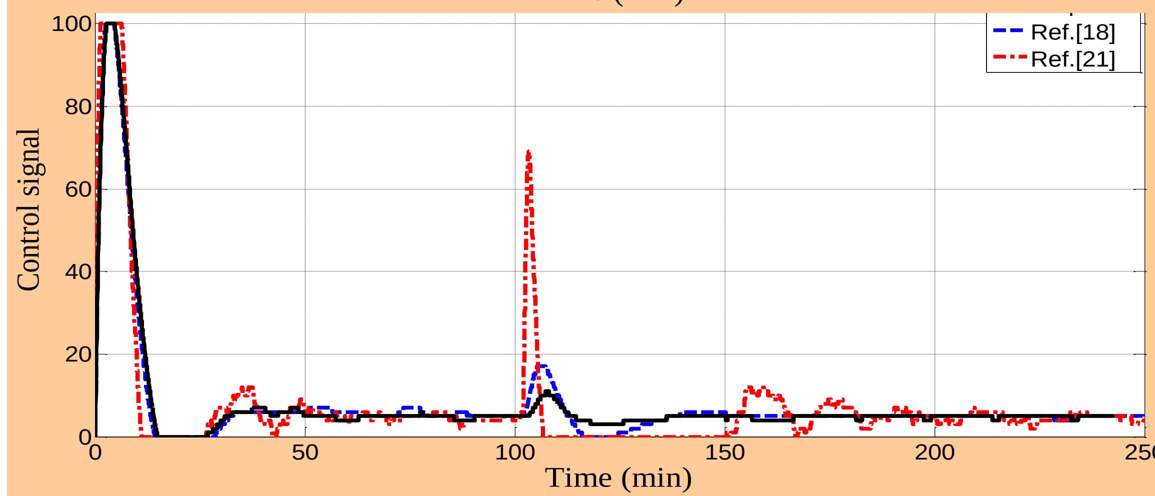
$$K_f(z) = \frac{z(z-\lambda_f)^4(1-\lambda_s)^3}{(1-\lambda_f)^4(\beta_0+\beta_1 z+\beta_2 z^2)(z-\lambda_s)^3}$$

Discrete-time PID design for long time delay processes

Heat up the aqueous solution from the room temperature (25°C) to 45°C.
Load disturbance arises from feeding 200(ml) solvent of distilled water.

Input constraint: $0 \le u \le 100$
regulate the heating power





Compared with the filtered PID method [21] based on delayed output feedback, more than 30 minutes are saved for recovering the solution temperature to the operation zone of (45±0.1)°C against the load disturbance.

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[18] Normey-Rico J.E, Camacho E.F. Unified approach for robust dead-time compensator design. ***J. Process Control***, 19, 38-47, 2009.

[21] Jin, Q.B., Liu, Q. Analytical IMC-PID design in terms of performance/robustness tradeoff for integrating processes: From 2-Dof to 1-Dof. ***J. Process Control***, 24(3), 22-32, 2014.

Our recent publications on discrete-time domain PID design

- ✓ Tao Liu*, Pedro García, Yueling Chen, Xuhui Ren, Pedro Albertos, Ricardo Sanz. New Predictor and 2DOF Control Scheme for Industrial Processes with Long Time Delay. *IEEE Transactions on Industrial Electronics*, 2018, 65 (5), 4247-4256.
- ✓ Yueling Chen, Tao Liu*, Pedro García, Pedro Albertos. Analytical design of a generalized predictor-based control scheme for low-order integrating and unstable systems with long time delay. *IET Control Theory & Application*, 2016, 10(8), 884-893.
- ✓ Dong Wang, Tao Liu*, Ximing Sun, Chongquan Zhong. Discrete-time domain two-degree-of-freedom control for integrating and unstable processes with time delay. *ISA Transactions*, 2016, 63, 121-132.
- ✓ Tao Liu*, Furong Gao. Enhanced IMC design of load disturbance rejection for integrating and unstable processes with slow dynamics. *ISA Transactions*, 2011, 50 (2), 239-248.
- ✓ Tao Liu, Furong Gao*. New insight into internal model control filter design for load disturbance rejection. *IET Control Theory & Application*, 2010, 4 (3), 448-460.
- ✓ Jiyao Cui, Yueling Chen, Tao Liu*. Discrete-time domain IMC-based PID control design for industrial processes with time delay. *The 35th Chinese Control Conference (CCC)*, *Chengdu*, *China*, July 27-29, 2016, 5946-5951.
- ✓ Hongyu Tian, Xudong Sun, Dong Wang, Tao Liu*. Predictor based two-degree-of-freedom control design for industrial stable processes with long input delay. *The 35th Chinese Control Conference (CCC)*, *Chengdu*, *China*, July 27-29, 2016, 4348-4353.

PID design for batch process optimization

Industrial batch processes, e.g., injection molding machines, chemical reactors

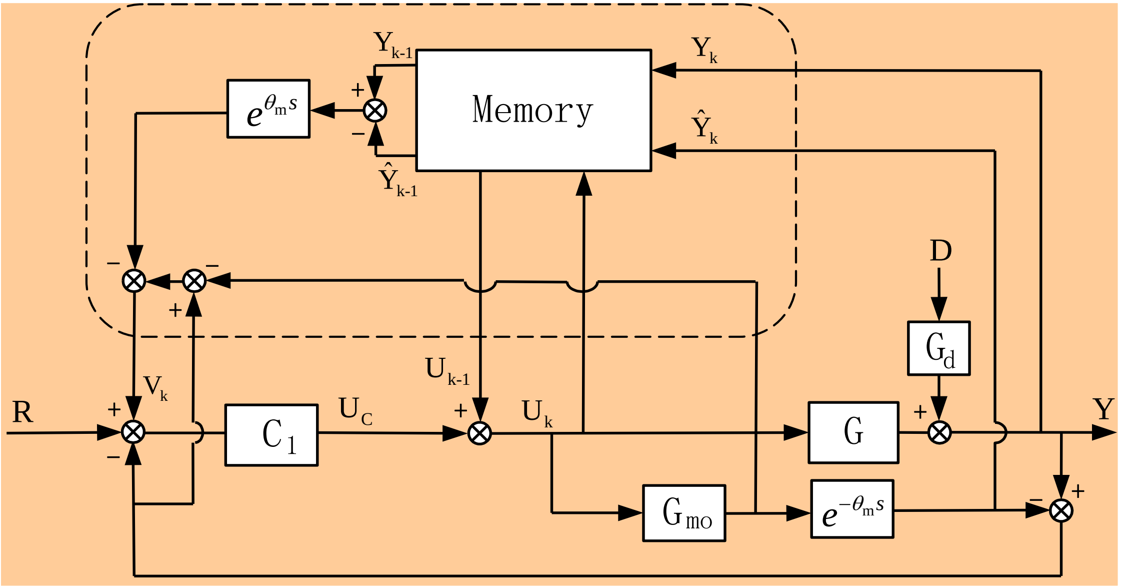
Features: 1. Repetitive operation for production;

2. Historical cycle information for progressively improving system performance;

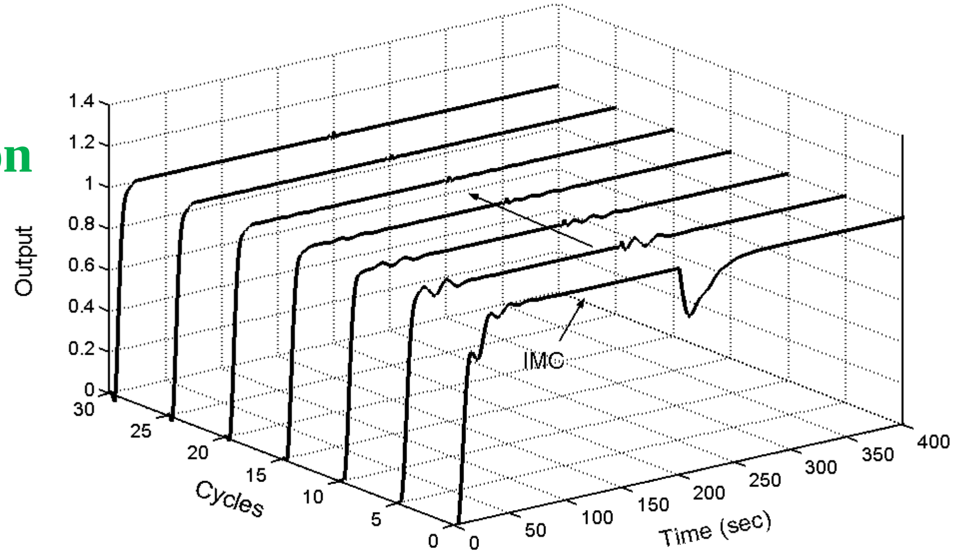
3. Time or batch varying uncertainties.

PID design for batch process optimization

IMC-based iterative learning control (ILC) for perfect tracking using historical cycle information



Advantage: batch control optimization

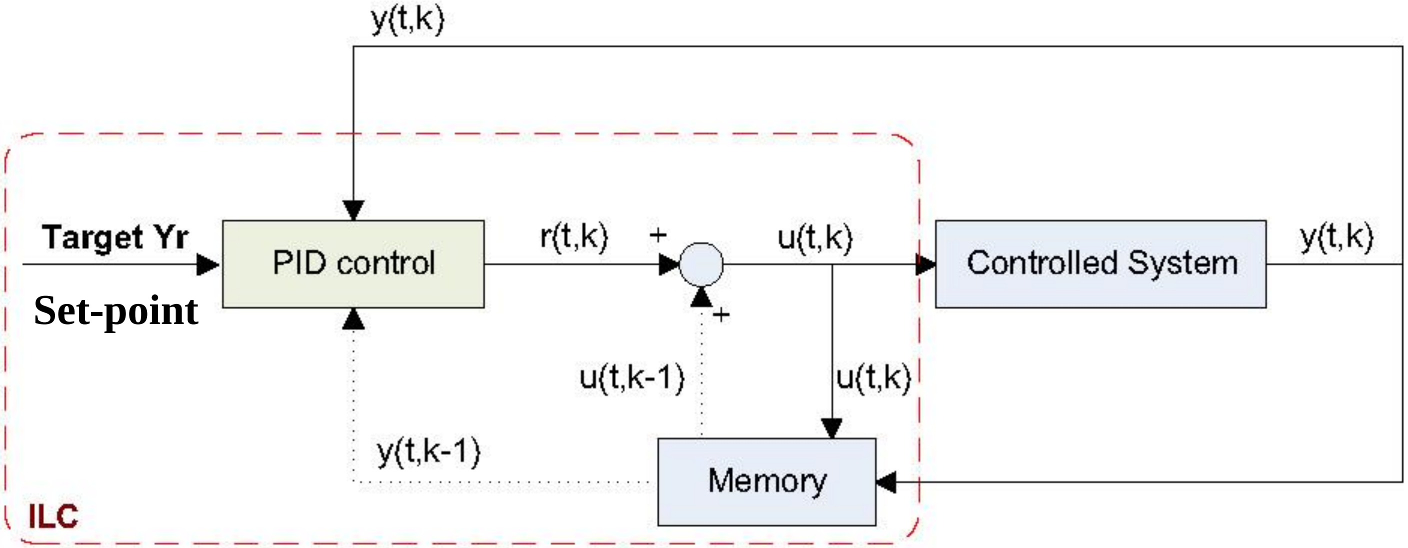


Tao Liu, Furong Gao, Youqing Wang. IMC-based iterative learning control for batch processes with uncertain time delay. ***Journal of Process Control***, 20 (2), 173-180, 2010.

PID design for batch process optimization

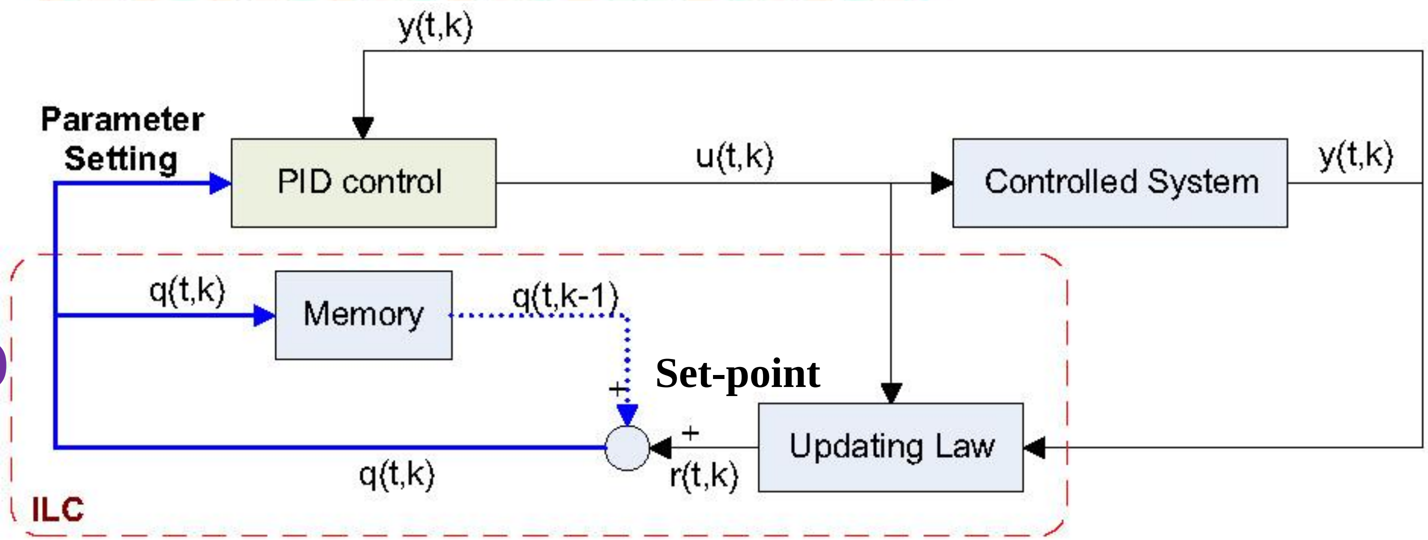
Type I: PID-based ILC system

(modify the input error for batch operation)



Type II: ILC-based PID control system

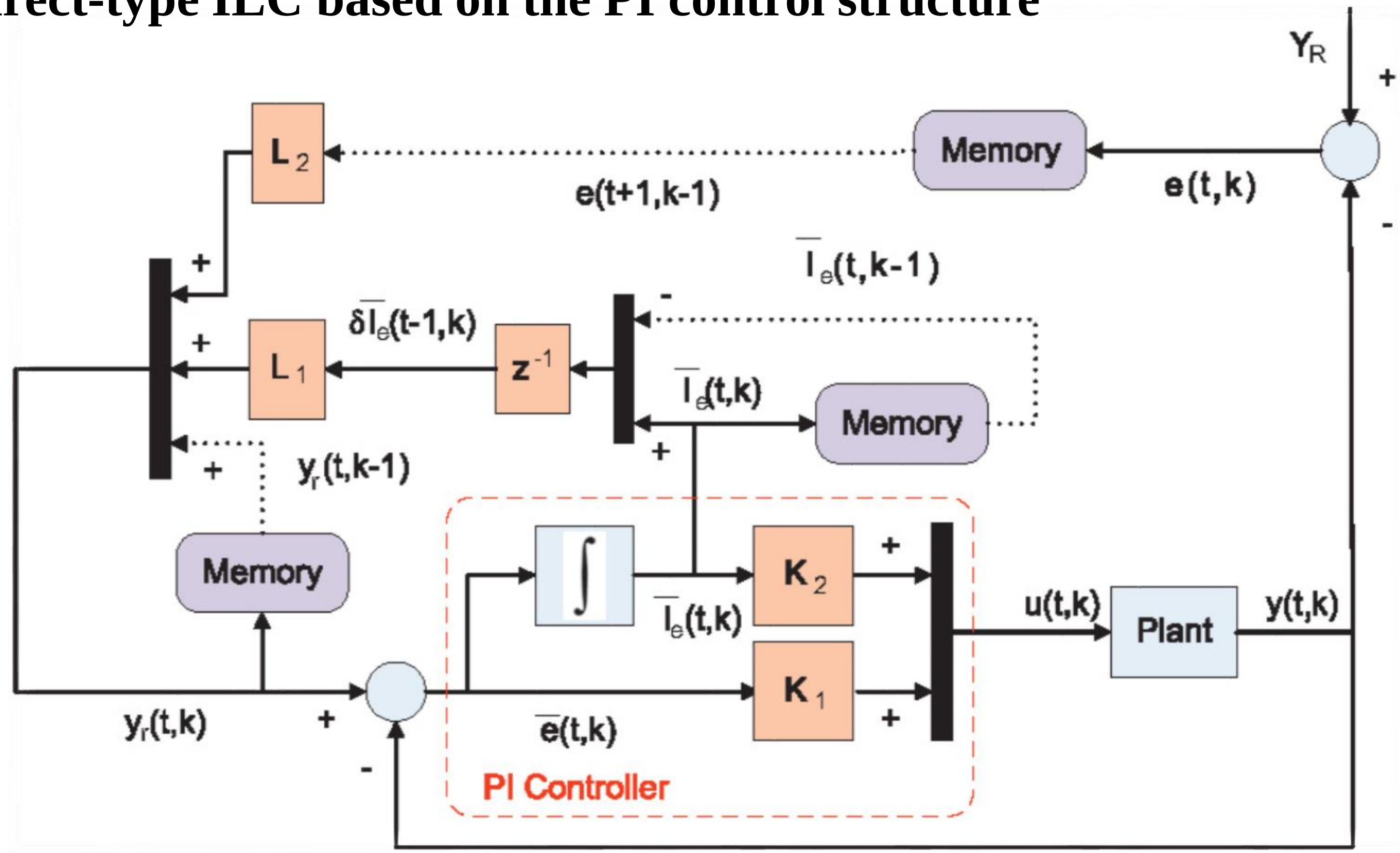
(controller retuning for batch operation)



Wang, Y., Gao, F., Doyle F.III., Survey on iterative learning control, repetitive control, and run-to-run control. ***J. Process Control***, 19(10), 1589-1600, 2009.

PID design for batch process optimization

Indirect-type ILC based on the PI control structure

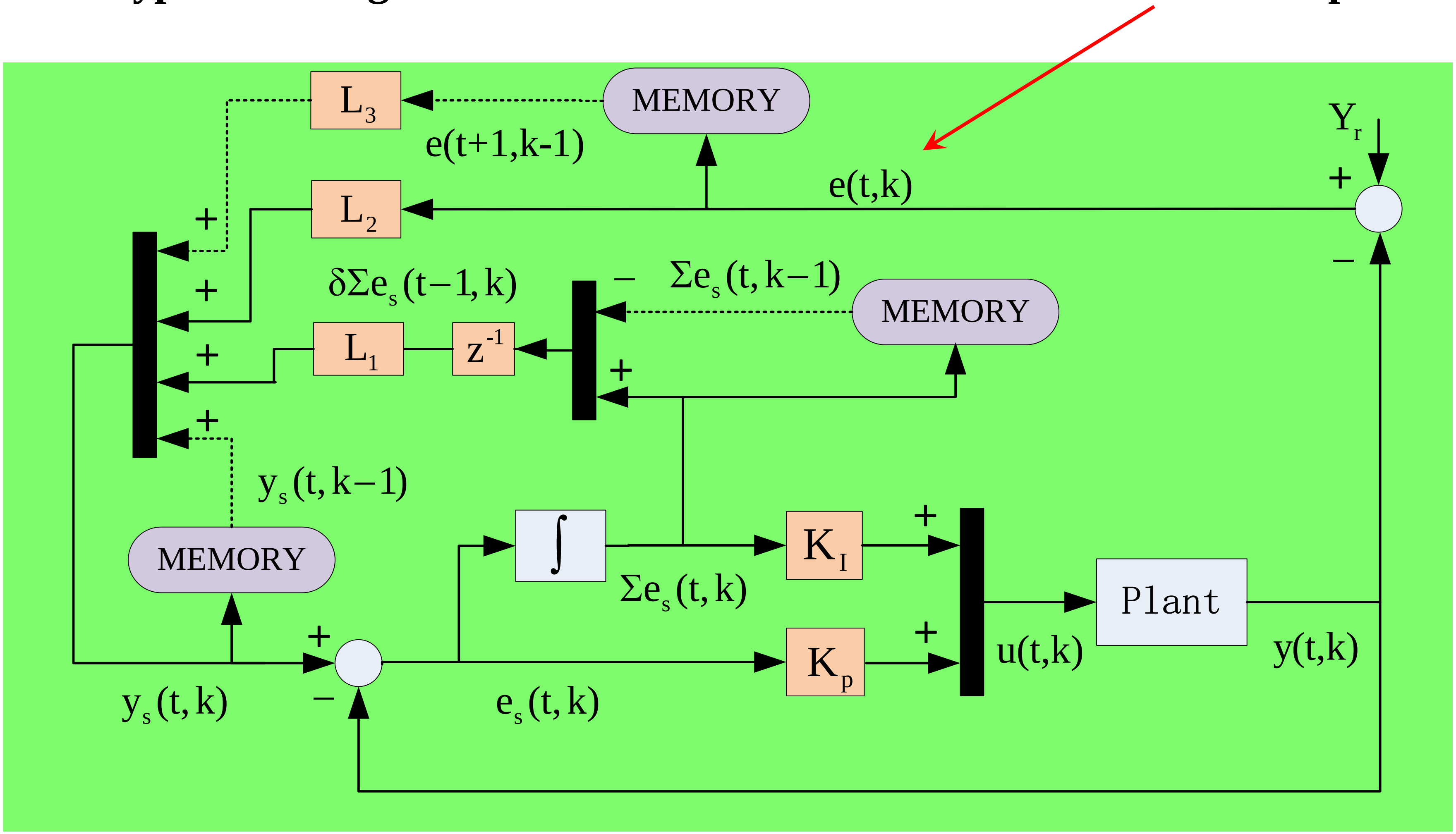


Advantage: No need to modify the closed-loop PI controller for ILC design, i.e., The PI controller and ILC updating law can be separately designed.

Youqing Wang, **Tao Liu**, Zhong Zhao. Advanced PI control with simple learning set-point design: Application on batch processes and robust stability analysis. ***Chemical Engineering Science***, 71(1), 153-165, 2012.

PID design for batch process optimization

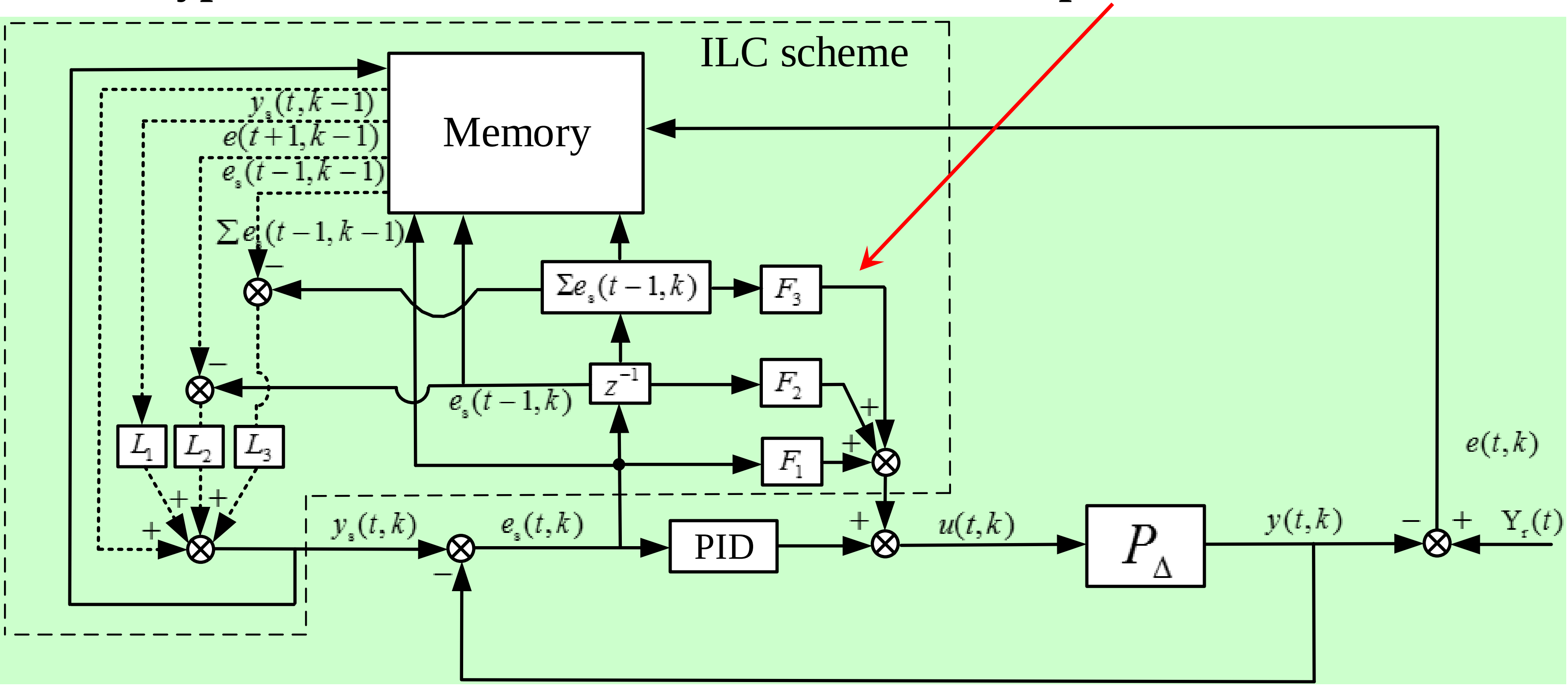
Indirect-type ILC design based on the PI control structure and current output error



Shoulin Hao, **Tao Liu***, Qing-Guo Wang. PI based indirect-type iterative learning control for batch processes with time-varying uncertainties A 2D FM model based approach. ***J. Process Control***, submitted, 2018.

PID design for batch process optimization

Indirect-type ILC based on the PID control structure plus feedforward control



P_Δ :

$$
\begin{cases}
x(t+1,k) = [A_m + \Delta\tilde{A}(t,k)]x(t,k) + [B_m + \Delta\tilde{B}(t,k)]u(t,k) + \omega(t,k) \\
y(t,k) = Cx(t,k), \quad 0 \le t \le T_p; \\
x(0,k) = x(0), \quad k=1,2,\cdots.
\end{cases}
$$

t : **time index**

k : **batch index**

T_p : **batch period**

Tao Liu, Xue Z. Wang, Junghui Chen. Robust PID based indirect-type iterative learning control for batch processes with time-varying uncertainties. ***Journal of Process Control***, 24 (12), 95-106, 2014.

Robust PID tuning for indirect-type ILC

A PID control law in discrete-time domain

$$u_{\mathrm{PID}}(t) = k_{\mathrm{P}} e(t) + k_{\mathrm{I}} \sum e(t) + k_{\mathrm{D}}[e(t+1) - e(t)]$$

$$e(t,k) = \Upsilon_{\mathrm{r}}(t) - y(t,k)$$

approximate

$$e(t) - e(t-1)$$

$$[e(t) + e(t-2) - 2e(t-1)]/2$$

State-space closed-loop PID system description

$$\begin{cases} \begin{bmatrix} x(t+1) \\ \sum e(t) \end{bmatrix} = \begin{bmatrix} \tilde{A} + \tilde{B}[\hat{k}_{\mathrm{D}}(C\mathrm{I} - A_{\mathrm{m}}) - \hat{k}_{\mathrm{P}} C] & \tilde{B}\hat{k}_{\mathrm{I}} \\ -C & \mathrm{I} \end{bmatrix} \begin{bmatrix} x(t) \\ \sum e(t-1) \end{bmatrix} + \begin{bmatrix} \mathrm{I} \\ \mathbf{0} \end{bmatrix} \omega(t) \\ y(t) = \begin{bmatrix} C & \mathbf{0} \end{bmatrix} \begin{bmatrix} x(t) \\ \sum e(t-1) \end{bmatrix} \end{cases}$$

where

$$\tilde{A} = A_{\mathrm{m}} + \Delta\tilde{A}(t)$$

$$\tilde{B} = B_{\mathrm{m}} + \Delta\tilde{B}(t)$$

$$\hat{k}_{\mathrm{P}} = (\mathrm{I} + k_{\mathrm{D}} C B_{\mathrm{m}})^{-1}(k_{\mathrm{P}} + k_{\mathrm{I}})$$

$$\hat{k}_{\mathrm{I}} = (\mathrm{I} + k_{\mathrm{D}} C B_{\mathrm{m}})^{-1} k_{\mathrm{I}}$$

$$\hat{k}_{\mathrm{D}} = (\mathrm{I} + k_{\mathrm{D}} C B_{\mathrm{m}})^{-1} k_{\mathrm{D}}$$

Robust PID tuning for indirect-type ILC

The H infinity control objective for closed-loop system robust stability

$$\|e(t)\|_2 < \gamma_{\mathrm{PID}} \|\omega(t)\|_2$$

where γ_{PID} **denotes the robust performance level.**

Theorem 1: The PID control system is guaranteed robustly stable if there exist $P_{11} > 0$ $P_{22} > 0$ **matrices** P_{12} **,** R_1 **,** R_2 **, and positive scalars** ε_1**,** ε_2**, such that**

$$\begin{bmatrix} -P + \varepsilon_1 \Phi_{\mathrm{A1}} \Phi_{\mathrm{A1}}^T + \varepsilon_2 \Phi_{\mathrm{B1}} \Phi_{\mathrm{B1}}^T & \Gamma & D_{\mathrm{g}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & -P & \mathbf{0} & PH^T C^T & P\Phi_{\mathrm{A2}}^T & P\Phi_{\mathrm{B2}}^T \\ * & * & -\gamma_{\mathrm{PID}} \mathrm{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\gamma_{\mathrm{PID}} \mathrm{I} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -\varepsilon_1 \mathrm{I} & \mathbf{0} \\ * & * & * & * & * & -\varepsilon_2 \mathrm{I} \end{bmatrix} < 0$$

where $D_{\mathrm{g}} = [\mathrm{I} \quad \mathbf{0}]^T$ $\quad H = [\mathrm{I} \quad \mathbf{0}]$ $\quad \Phi_{\mathrm{A1}} = [\Delta \bar{A}_1^T, \ \mathbf{0}]^T$ $\quad \Phi_{\mathrm{A2}} = [\Delta \bar{A}_2 P_{11}, \ \Delta \bar{A}_2 P_{12}]$

$$\Phi_{\mathrm{B1}} = [\Delta \bar{B}_1^T, \ \mathbf{0}]^T$$

$$\Phi_{\mathrm{B2}} = [\Delta \bar{B}_2 R_1, \ \Delta \bar{B}_2 R_2]$$

$$P = \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix} \qquad \Gamma = \begin{bmatrix} A_{\mathrm{m}} P_{11} + B_{\mathrm{m}} R_1 & A_{\mathrm{m}} P_{12} + B_{\mathrm{m}} R_2 \\ -CP_{11} + P_{12}^T & -CP_{12} + P_{22} \end{bmatrix}$$

Robust PID tuning for indirect-type ILC

Correspondingly, the PID controller is determined by

$$[\hat{k}_{\mathrm{D}}(C\mathrm{I} - A_{\mathrm{m}}) - \hat{k}_{\mathrm{P}}C \quad \hat{k}_{\mathrm{I}}] = [R_1 \quad R_2]P^{-1}$$

$$k_{\mathrm{I}} = \hat{k}_{\mathrm{I}}(\mathrm{I} + k_{\mathrm{D}}CB_{\mathrm{m}})$$

$$k_{\mathrm{P}} = \hat{k}_{\mathrm{P}}(\mathrm{I} + k_{\mathrm{D}}CB_{\mathrm{m}}) - k_{\mathrm{I}}$$

where k_{D} **is user specified for implementation. If** $k_{\mathrm{D}} = 0$**, it is a PI controller.**

An optimal program for tuning PID to accommodate for the uncertainty bounds,

$$\underset{\Delta\tilde{A}(t),\ \Delta\tilde{B}(t)}{\mathrm{Min}} \gamma_{\mathrm{PID}}$$

Guideline: A smaller value of γ_{PID} **leads to faster output response with a more aggressive control action, and vice versa.**

Tao Liu, Xue Z. Wang, Junghui Chen. Robust PID based indirect-type iterative learning control for batch processes with time-varying uncertainties. ***Journal of Process Control***, 24 (12), 95-106, 2014.

Robust PI tuning for indirect-type ILC

Another robust tuning of PI controller by assigning the closed-loop system poles to a prescribed circular region, $D(\alpha, r)$ centered at $(\alpha, 0)$ with radius r and $|\alpha| + r < 1$, i.e.,

$$\lambda(\tilde{A}) \subset D(\alpha, r)$$

while the closed-loop transfer function $H(z) = \hat{C}(zI - \tilde{A})^{-1}\hat{D}$ satisfies

$$\| H(z) \|_\infty < \gamma_{\text{PI}}$$

Theorem 2: The PI control system is guaranteed robustly D-stable if there exist matrices $P_1 > 0$, $P_3 > 0$, $P_2 = P_2^{\mathrm{T}}$, R_1, R_2, and positive scalar ε, such that

$$\begin{bmatrix} \Lambda_1 & 0 & \Lambda_2 & P\hat{C}^{\mathrm{T}} & 0 & P\hat{F}^{\mathrm{T}} \\ * & -\beta_1^{-1}\gamma_{\text{PI}}^2 I & \hat{D}^{\mathrm{T}} & 0 & \hat{D}^{\mathrm{T}} & 0 \\ * & * & \Lambda_3 & 0 & 0 & 0 \\ * & * & * & -\beta_1^{-1} I & 0 & 0 \\ * & * & * & * & -\beta_2 P & 0 \\ * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0$$

where $\beta_1 = 1 - |\alpha|$ $\quad \beta_2 = (\beta_1^{-1} - 1)^{-1}$ $\quad \Lambda_1 = -\alpha P\hat{A}^{\mathrm{T}} - \alpha\hat{A}P + (\alpha^2 - r^2)P + \varepsilon\alpha^2\hat{E}\hat{E}^{\mathrm{T}}$

$$P = \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} \quad P\hat{F}^{\mathrm{T}} = \begin{bmatrix} P_1 F_A^{\mathrm{T}} - R_1^{\mathrm{T}} F_B^{\mathrm{T}} \\ P_2^{\mathrm{T}} F_A^{\mathrm{T}} - R_2^{\mathrm{T}} F_B^{\mathrm{T}} \end{bmatrix} \quad \hat{A}P = \begin{bmatrix} AP_1 - BR_1 & AP_2 - BR_2 \\ -CP_1 + P_2^{\mathrm{T}} & -CP_2 + P_3 \end{bmatrix}$$

$$\Lambda_2 = P\hat{A}^{\mathrm{T}} - \varepsilon\alpha\hat{E}\hat{E}^{\mathrm{T}}$$

$$\Lambda_3 = -P + \varepsilon\hat{E}\hat{E}^{\mathrm{T}}$$

Robust PI tuning for indirect-type ILC

Correspondingly, the PI controller is determined by

$$\left[(K_{\mathrm{P}}+K_{\mathrm{I}})C \quad -K_{\mathrm{I}}\right]=\left[R_1 \quad R_2\right]P^{-1}$$

$$(R_1 \quad R_2)P^{-1}=(\hat{K}_{\mathrm{P}} \quad \hat{K}_{\mathrm{I}})$$

$$K_{\mathrm{I}}=-\hat{K}_{\mathrm{I}}$$
$$K_{\mathrm{P}}=\hat{K}_{\mathrm{P}}C^{\mathrm{T}}(CC^{\mathrm{T}})^{-1}+\hat{K}_{\mathrm{I}}$$

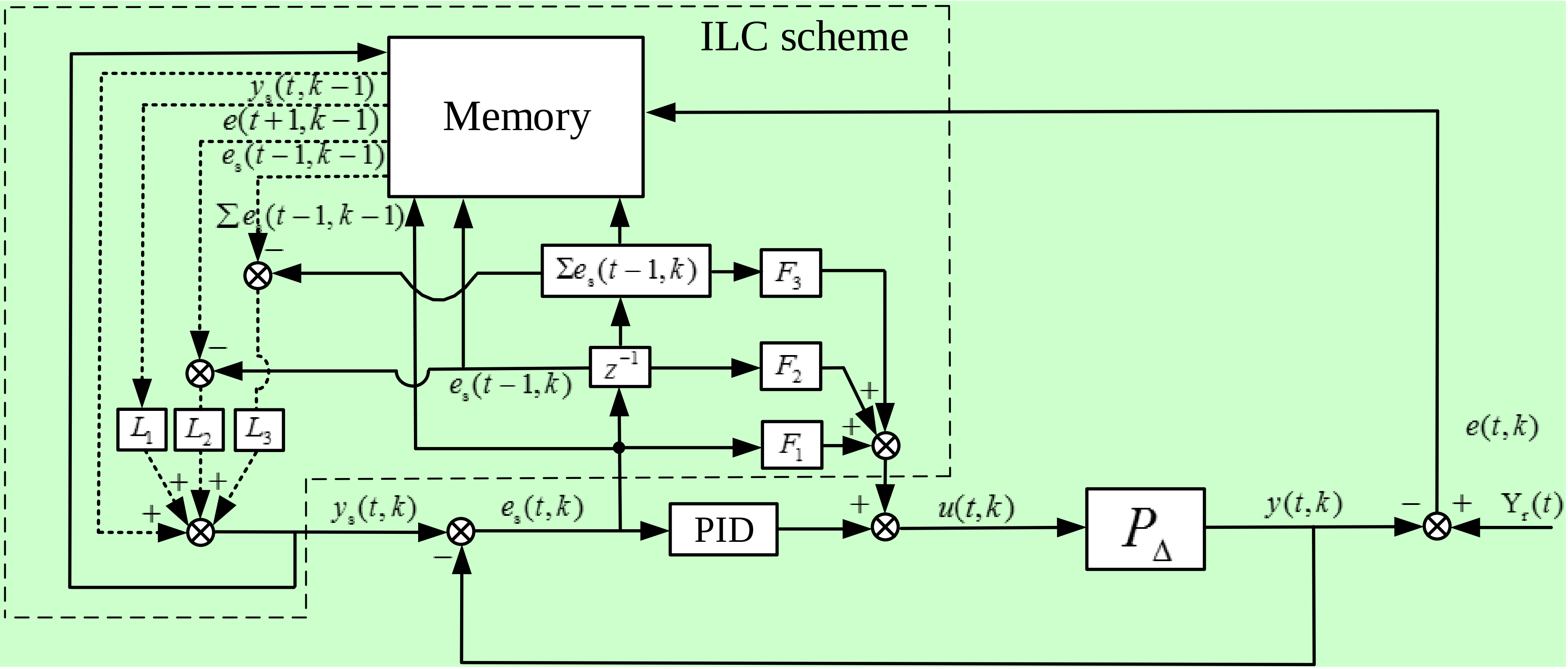
To optimize the robust H infinity control performance, the PI controller can be determined by solving the following optimization program,

$$\underset{\Delta\tilde{A}(t),\ \Delta\tilde{B}(t)}{\mathrm{Min}}\ \gamma_{\mathrm{PI}}$$

Guideline: A smaller value of γ_{PI} leads to faster output response with a more aggressive control action, and vice versa.

Shoulin Hao, **Tao Liu***, Qing-Guo Wang. PI based indirect-type iterative learning control for batch processes with time-varying uncertainties A 2D FM model based approach. ***J. Process Control***, submitted, 2018.

PI based set-point learning design for indirect-type ILC



Based on the PI or PID closed-loop, a learning set-point command is designed as

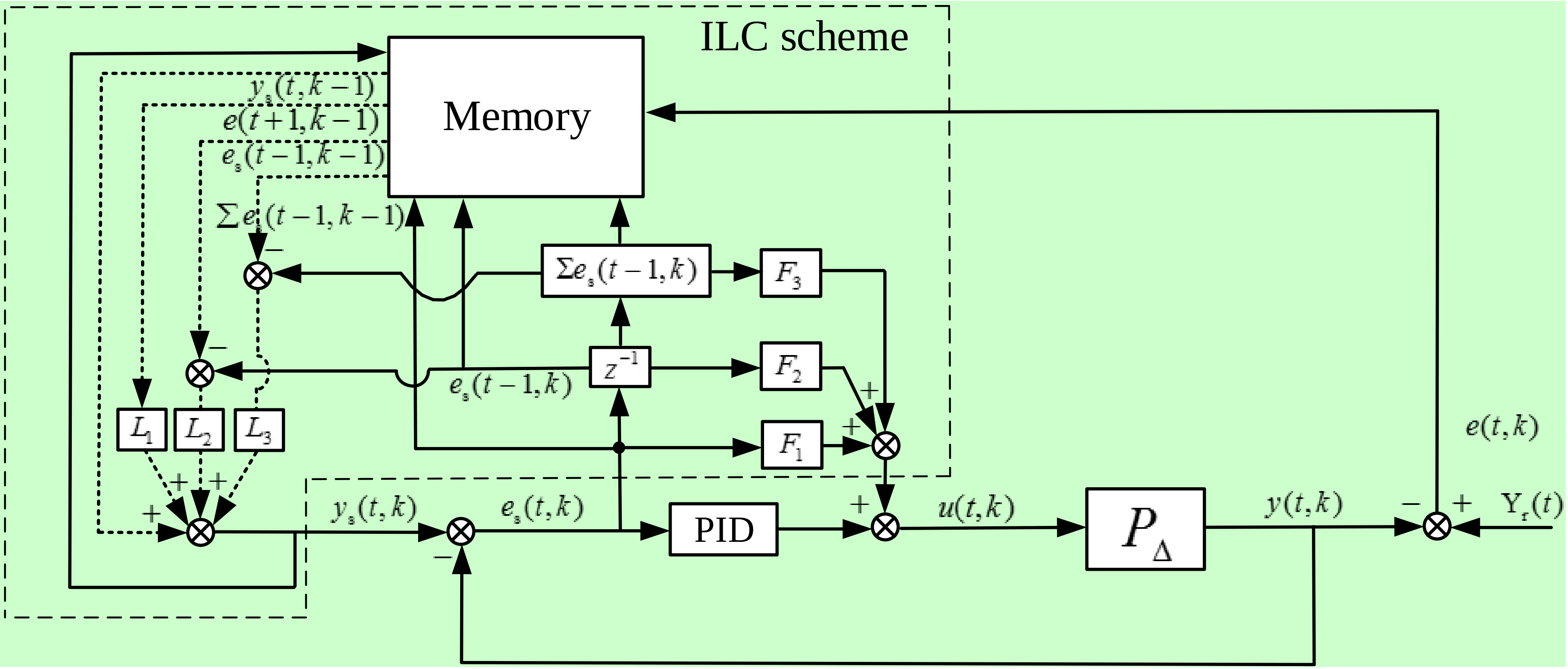
$$y_s(t,k) = y_s(t,k-1) + L_1 e(t+1,k-1) + L_2 \delta e_s(t-1,k) + L_3 \delta \sum e_s(t-1,k)$$

where $y_s(t,k)$ **is the set-point command in the previous cycle,**

Like PI

$$\delta e_s(t-1,k) = e_s(t-1,k) - e_s(t-1,k-1) \qquad \delta \sum e_s(t,k) = \delta \sum e_s(t-1,k) + \delta e_s(t,k)$$

PI based set-point learning design for indirect-type ILC



Feedforward controllers are used to adjust the process input

$$u(t,k) = u_{\mathrm{PID}}(t,k) + F_1 e_s(t,k) + F_2 e_s(t-1,k) + F_3 \sum e_s(t-1,k)$$

Note: The setpoint tracking errors at the current moment, one-step ahead moment, and the error integral in the current cycle are used to construct the feedforward control.

PI based set-point learning design for indirect-type ILC

Two-dimensional (2D) system description of the indirect ILC scheme

$$\begin{cases} \begin{bmatrix} \delta x(t+1,k) \\ \delta e_s(t,k) \\ \delta \sum e_s(t,k) \\ e(t+1,k) \end{bmatrix} = \tilde{\Psi} \begin{bmatrix} \delta x(t,k) \\ \delta e_s(t-1,k) \\ \delta \sum e_s(t-1,k) \\ e(t+1,k-1) \end{bmatrix} + D_w \varpi(t) \\ \varsigma(t,k) = G \begin{bmatrix} \delta x(t,k) \\ \delta e_s(t-1,k) \\ \delta \sum e_s(t-1,k) \\ e(t+1,k-1) \end{bmatrix} \end{cases}$$

where

$$\varsigma(t,k) = e(t+1,k-1)$$

$$G = [\mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \boldsymbol{I}]$$

$$D_w = [\boldsymbol{I} \quad \mathbf{0} \quad \mathbf{0} \quad -C^T]^T$$

$$\tilde{\Psi} = \begin{bmatrix} \tilde{A}-\tilde{B}(k_p+k_i+k_d+F_1)C & \tilde{B}[(k_p+k_i+k_d+F_1)L_2+F_2-k_d] & \tilde{B}[(k_p+k_i+k_d+F_1)L_3+F_3+k_i] & \tilde{B}(k_p+k_i+k_d+F_1)L_1 \\ -C & L_2 & L_3 & L_1 \\ -C & L_2 & \boldsymbol{I}+L_3 & L_1 \\ -C\tilde{A}+C\tilde{B}(k_p+k_i+k_d+F_1)C & -C\tilde{B}[(k_p+k_i+k_d+F_1)L_2+F_2-k_d] & -C\tilde{B}[(k_p+k_i+k_d+F_1)L_3+F_3+k_i] & \boldsymbol{I}-C\tilde{B}(k_p+k_i+k_d+F_1)L_1 \end{bmatrix}$$

PI based set-point learning design for indirect-type ILC

The control objectives for robust tracking from batch to batch

$$J_{\mathrm{BP}} = \sum_{t=0}^{N_1 = T_{\mathrm{p}}} \sum_{k=0}^{N_2 \to \infty} (\gamma_{\mathrm{ILC}}^{-1} \| \varsigma(t,k+1) \|_2^2 - \gamma_{\mathrm{ILC}} \| \varpi(t,k+1) \|_2^2) < 0$$

2D Roesser's system stability [1]:

$$\begin{cases} \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = \begin{bmatrix} A_{11}+\Delta A_{11} & A_{12}+\Delta A_{12} \\ A_{21}+\Delta A_{21} & A_{22}+\Delta A_{22} \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + \omega(i,j) \\ y(i,j) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \\ i,j = 0,1,2,\cdots. \end{cases}$$

Robust stability condition [1]:

$$\tilde{A}^T P \tilde{A} - P < 0$$

$$\tilde{A} = \begin{bmatrix} A_{11}+\Delta A_{11} & A_{12}+\Delta A_{12} \\ A_{21}+\Delta A_{21} & A_{22}+\Delta A_{22} \end{bmatrix}$$

$$P = diag\{P_1,\ P_2\}$$

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[1] Rogers, E., Galkowski, K. & Owens, D. H. Control Systems Theory and Applications for Linear Repetitive Processes, Berlin: Springer, 2007.

PI based set-point learning design for indirect-type ILC

Define

$$x^h(t,k)=\begin{bmatrix}\delta x(t,k)\\ \delta e_s(t-1,k)\\ \delta\sum e_s(t-1,k)\end{bmatrix}$$

$$x^v(t,k)=e(t+1,k)$$

Lyapunov-Krasovskii function used for analyzing 2D asymptotic stability

$$\Delta V=V_Q\begin{bmatrix}x^h(t+1,k)\\ x^v(t,k)\end{bmatrix}-V_Q\begin{bmatrix}x^h(t,k)\\ x^v(t,k-1)\end{bmatrix}$$

The objective function of robust batch operation for minimization

$$J_{\mathrm{BP}}=\sum_{t=0}^{N_1=T_p}\sum_{k=0}^{N_2\to\infty}\left(\gamma_{\mathrm{ILC}}^{-1}\left\|\varsigma(t,k+1)\right\|_2^2-\gamma_{\mathrm{ILC}}\left\|\varpi(t,k+1)\right\|_2^2+\Delta V\right)-\sum_{t=0}^{N_1=T_p}\sum_{k=0}^{N_2\to\infty}\Delta V<0$$

$$\begin{cases}\delta x(0,0)=\delta x(0,1)=\delta x(1,0)=0;\\ \delta e_s(0,0)=\delta e_s(0,1)=\delta e_s(1,0)=0;\\ \delta\sum e_s(0,0)=\delta\sum e_s(0,1)=\delta\sum e_s(1,0)=0;\\ e(0,0)=e(0,1)=e(1,0)=0.\end{cases}$$

$\Rightarrow$

$$\sum_{t=0}^{N_1=T_p}\sum_{k=0}^{N_2\to\infty}\Delta V>0$$

PI based set-point learning design for indirect-type ILC

Theorem 3: The 2D control system is guaranteed robustly stable with a H infinity control performance level, γ_{ILC}, if there exist $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Q_4 > 0$, matrices $\hat{F}_2$, $\hat{F}_3$, $\hat{L}_1$, $\hat{L}_2$, $\hat{L}_3$, and positive scalars ε_1, ε_2, such that

$$\begin{bmatrix} -Q+\varepsilon_1\Omega_{\text{A1}}\Omega_{\text{A1}}^T+\varepsilon_2\Omega_{\text{B1}}\Omega_{\text{B1}}^T & \Pi & D_{\text{w}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & -Q & \mathbf{0} & QG^T & P\Omega_{\text{A2}}^T & P\Omega_{\text{B2}}^T \\ * & * & -\gamma_{\text{ILC}}\text{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\gamma_{\text{ILC}}\text{I} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -\varepsilon_1\text{I} & \mathbf{0} \\ * & * & * & * & * & -\varepsilon_2\text{I} \end{bmatrix} < 0$$

where $Q = diag\{Q_1,\ Q_2,\ Q_3,\ Q_4\}$ $\quad D_{\text{g}} = [\text{I} \quad \mathbf{0}]^T$ $\quad H = [\text{I} \quad \mathbf{0}]$

$\Omega_{\text{A1}} = [\Delta\bar{A}_1^T,\ \mathbf{0},\ \mathbf{0},\ -\Delta\bar{A}_1^T C^T]^T$ $\quad \Omega_{\text{A2}} = [\Delta\bar{A}_2,\ \mathbf{0},\ \mathbf{0},\ \mathbf{0}]$ $\quad \Omega_{\text{B1}} = [\Delta\bar{B}_1^T,\ \mathbf{0},\ \mathbf{0},\ -\Delta\bar{B}_1^T C^T]^T$

$$\Omega_{\text{B2}} = \Big[-\Delta\bar{B}_2(k_{\text{p}}+k_{\text{i}}+k_{\text{d}}+F_1)C,\quad \Delta\bar{B}_2[(k_{\text{p}}+k_{\text{i}}+k_{\text{d}}+F_1)\hat{L}_2+\hat{F}_2-k_{\text{d}}],\quad \Delta\bar{B}_2[(k_{\text{p}}+k_{\text{i}}+k_{\text{d}}+F_1)\hat{L}_3+\hat{F}_3+k_{\text{i}}]\quad \Delta\bar{B}_2(k_{\text{p}}+k_{\text{i}}+k_{\text{d}}+F_1)\hat{L}_1\Big]$$

$$\Pi = \begin{bmatrix} A_{\text{m}}Q_1 - B_{\text{m}}(k_{\text{p}}+k_{\text{i}}+k_{\text{d}}+F_1)CQ_1 & B_{\text{m}}(k_{\text{p}}+k_{\text{i}}+k_{\text{d}}+F_1)\hat{L}_2+B_{\text{m}}\hat{F}_2-B_{\text{m}}k_{\text{d}}Q_2 & B_{\text{m}}(k_{\text{p}}+k_{\text{i}}+k_{\text{d}}+F_1)\hat{L}_3+B_{\text{m}}\hat{F}_3+B_{\text{m}}k_{\text{i}}Q_3 & B_{\text{m}}(k_{\text{p}}+k_{\text{i}}+k_{\text{d}}+F_1)\hat{L}_1 \\ -CQ_1 & \hat{L}_2 & \hat{L}_3 & \hat{L}_1 \\ -CQ_1 & \hat{L}_2 & Q_3+\hat{L}_3 & \hat{L}_1 \\ -CA_{\text{m}}Q_1+CB_{\text{m}}(k_{\text{p}}+k_{\text{i}}+k_{\text{d}}+F_1)CQ_1 & -CB_{\text{m}}(k_{\text{p}}+k_{\text{i}}+k_{\text{d}}+F_1)\hat{L}_2-CB_{\text{m}}\hat{F}_2+CB_{\text{m}}k_{\text{d}}Q_2 & -CB_{\text{m}}(k_{\text{p}}+k_{\text{i}}+k_{\text{d}}+F_1)\hat{L}_3-CB_{\text{m}}\hat{F}_3-CB_{\text{m}}k_{\text{i}}Q_3 & Q_4-CB_{\text{m}}(k_{\text{p}}+k_{\text{i}}+k_{\text{d}}+F_1)\hat{L}_1 \end{bmatrix}$$

PI based set-point learning design for indirect-type ILC

Correspondingly, the PI type ILC controller is determined by

$$\begin{cases} L_1 = \hat{L}_1 Q_4^{-1} \\ L_2 = \hat{L}_2 Q_2^{-1} \\ L_3 = \hat{L}_3 Q_3^{-1} \end{cases}$$

The feedforeward controller is determined by

$$\begin{cases} F_2 = \hat{F}_2 Q_2^{-1} \\ F_3 = \hat{F}_3 Q_3^{-1} \end{cases}$$

To optimize the set-point tracking performance, the PI type ILC controller can be determined by solving the following optimization program,

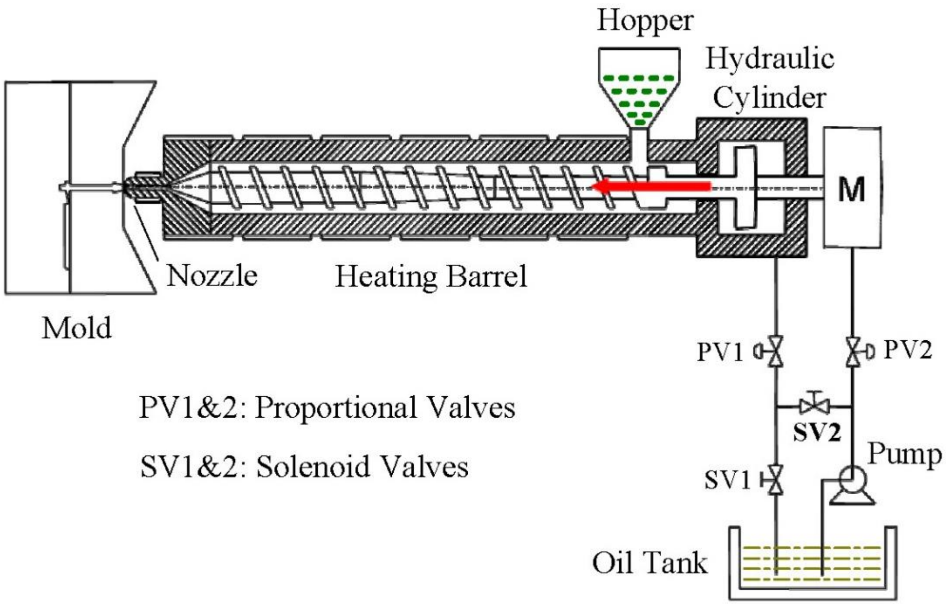
$$\underset{\Delta\tilde{A}(t),\ \Delta\tilde{B}(t)}{\text{Min}} \gamma_{\text{ILC}}$$

Guideline: A smaller value of γ_{ILC} leads to faster output response with a more aggressive control action, and vice versa.

Shoulin Hao, **Tao Liu***, Qing-Guo Wang. PI based indirect-type iterative learning control for batch processes with time-varying uncertainties A 2D FM model based approach. ***J. Process Control***, submitted, 2018.

PI based indirect-type ILC for batch injection molding

injection molding machine



The nozzle pressure response to the hydraulic valve input was modeled [1] by

$$y(t,k+1)=\frac{1.239(\pm5\%)z^{-1}-0.9282(\pm5\%)z^{-2}}{1-1.607(\pm5\%)z^{-1}+0.6086(\pm5\%)z^{-2}}u(t,k+1)+\omega(t,k+1)$$

Shi, J., Gao, F., Wu, T.-J. Integrated design and structure analysis of robust iterative learning control system based on a two-dimensional model. ***Ind. Eng. Chem. Res.,*** 44, 8095-8105, 2005.

PI based indirect-type ILC for batch process opimtization

Equivalently, the process model is rewritten in a state-space form

$$\begin{cases} x(t+1,k+1) = \left(\begin{bmatrix} 1.607 & 1 \\ -0.6086 & 0 \end{bmatrix} + \Delta\tilde{A}\right)x(t,k+1) + \left(\begin{bmatrix} 1.239 \\ -0.9282 \end{bmatrix} + \Delta\tilde{B}\right)u(t,k+1) + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\omega(t,k+1) \\ y(t,k+1) = \begin{bmatrix} 1, & 0 \end{bmatrix} x(t,k+1) \end{cases}$$

time-varying uncertainties

$$\Delta\tilde{A}(t) = \begin{bmatrix} 0.0804\delta(t) & 0 \\ -0.0304\delta(t) & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \delta(t) & 0 \\ 0 & \delta(t) \end{bmatrix}\begin{bmatrix} 0.0804 & 0 \\ -0.0304 & 0 \end{bmatrix}$$

$$\Delta\tilde{B}(t) = \begin{bmatrix} 0.062\delta(t) \\ -0.0464\delta(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \delta(t) & 0 \\ 0 & \delta(t) \end{bmatrix}\begin{bmatrix} 0.062 \\ -0.0464 \end{bmatrix}$$

$$|\delta(t)| \le 1$$

Robustly tuned PI controllers: $k_{\mathrm{p}} = 1.2889 \quad k_{\mathrm{i}} = 0.0336$

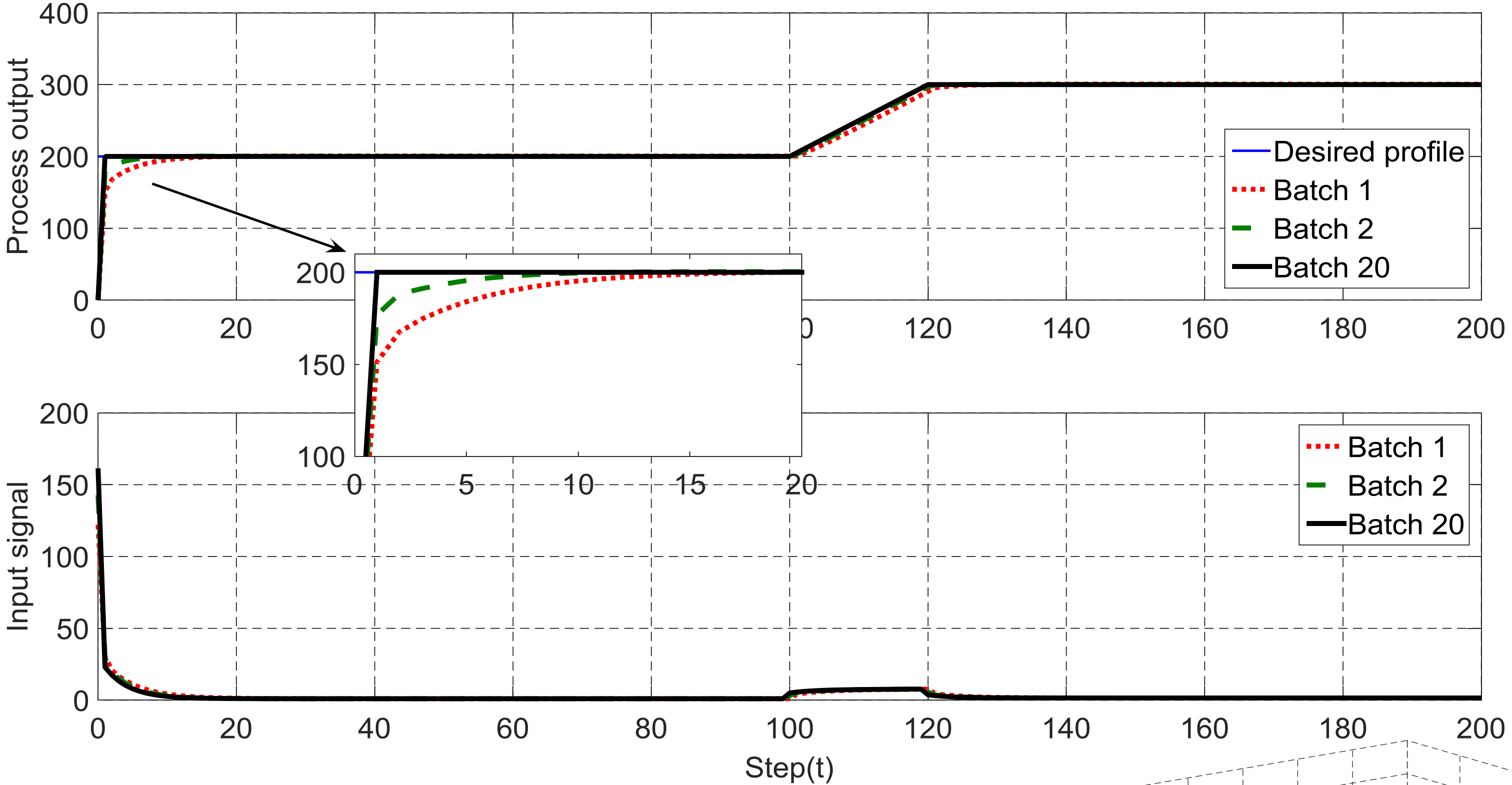
ILC controllers:

$F_1 \in [-1.3,\ 0.1] \Rightarrow \lambda[A_{\mathrm{m}} - B_{\mathrm{m}}(k_{\mathrm{p}} + k_{\mathrm{i}} + k_{\mathrm{d}} + F_1)C] < 1$

$F_2 = 0 \quad F_3 = -0.0097$

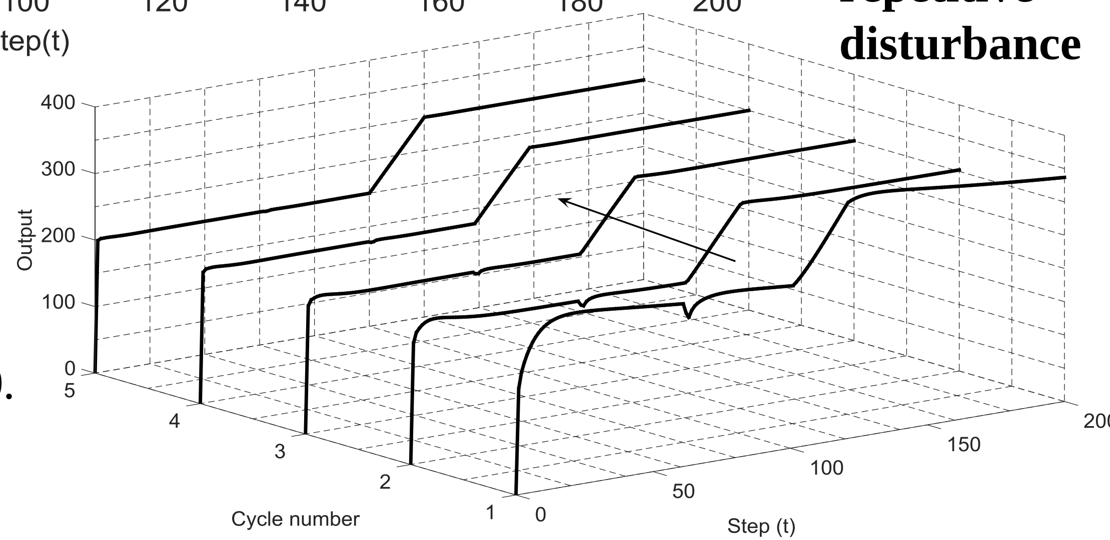
$L_1 = 0.1776 \quad L_2 = 0 \quad L_3 = -0.029$

PI based indirect-type ILC for batch process opimtization



Case 1: time-invariant uncertainties

Case 2: repetitive disturbance



Desired output profile

$$
Y_r = \begin{cases} 200, & 0 \le t \le 100; \\ 200+5(t-100), & 100 < t \le 120; \\ 300, & 120 < t \le T_p = 200. \end{cases}
$$

PI based indirect-type ILC for batch process opimtization

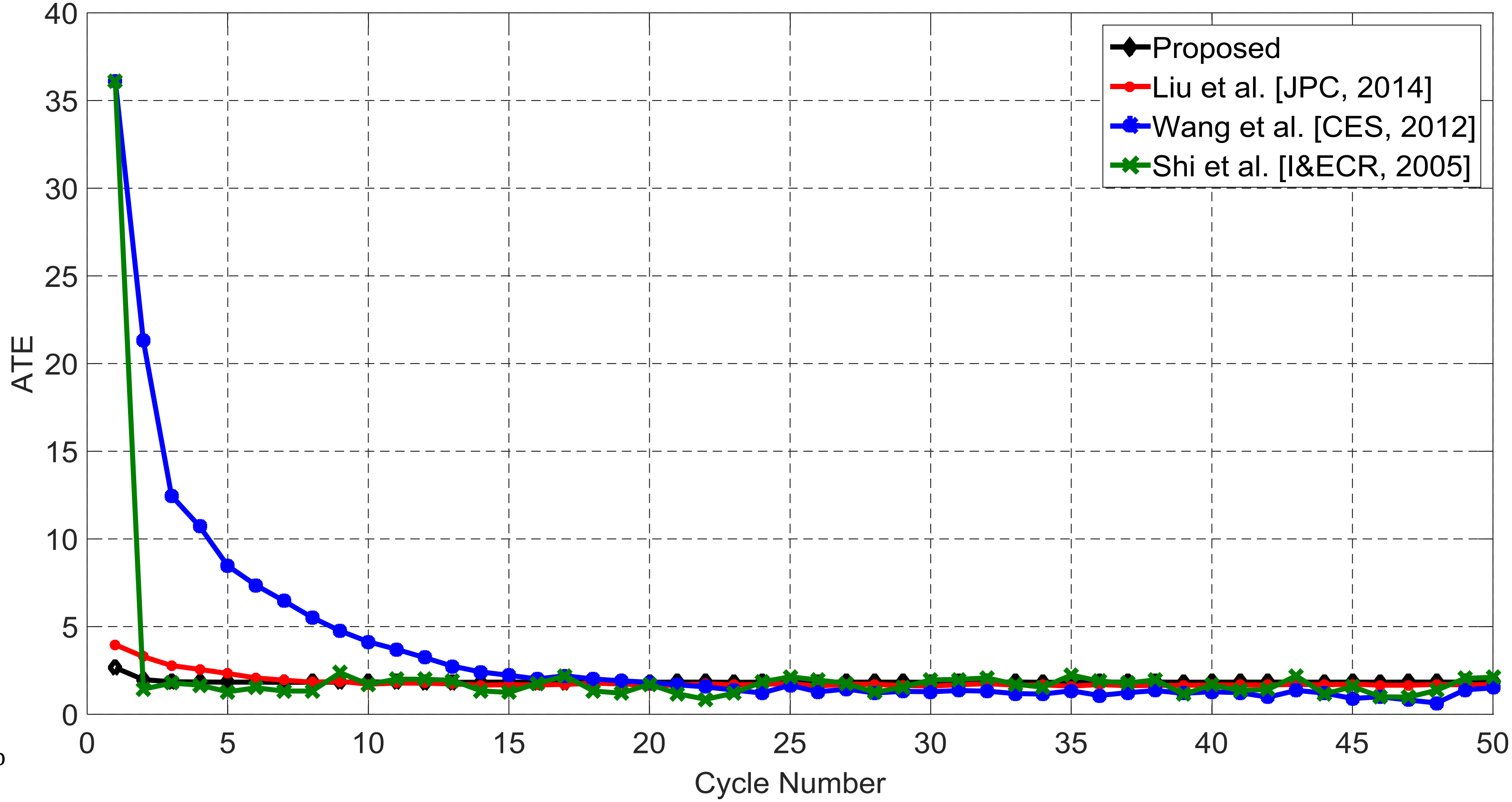
Case 3: Time-varying uncertainties

$$|\delta(t)| \le 0.1$$

$$\omega(t,k+1) = \sin(t+\theta(k))$$

$$\theta(k) \in [0, 2\pi]$$

$$\text{ATE}(k) = \sum_{t=1}^{T_p} |e(t,k)| / T_p$$



Plot of the output error for batch operation

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[1] **Tao Liu**, Xue Z. Wang, Junghui Chen. Robust PID based indirect-type iterative learning control for batch processes with time-varying uncertainties. ***Journal of Process Control***, 24 (12), 95-106, 2014.
[2] Y. Wang, **Tao Liu**, Z. Zhao. Advanced PI control with simple learning set-point design: Application on batch processes and robust stability analysis. ***Chemical Engineering Science***, 71 (1), 153-165, 2012.
[3] Shi, J., Gao, F., Wu, T.-J. Integrated design and structure analysis of robust iterative learning control system based on a two-dimensional model. ***Ind. Eng. Chem. Res.***, 44, 8095-8105, 2005.

Conclusions & Outlook

Main Results:

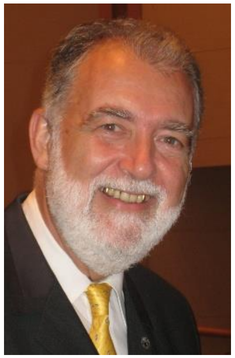
- **Analytical PID design in discrete-time domain for sampled control systems**
- **2DOF control structure based PID design for improving disturbance rejection**
- **Predictor-based PID design for long time delay systems**
- **Robust PID tuning methods with respect to the system uncertainty bounds**
- **PI based indirect type ILC design for batch process optimization**

Outlook:

- **Data-driven PID tuning for sampled control systems**
- **Fractional-order PID design & PID scheduling for nonlinear systems**
- **PID +Memory for learning/intelligent control of industrial batch processes, repetitive systems, and robots etc.**

Acknowledgement

Thanks to my collaborators and students for PID design



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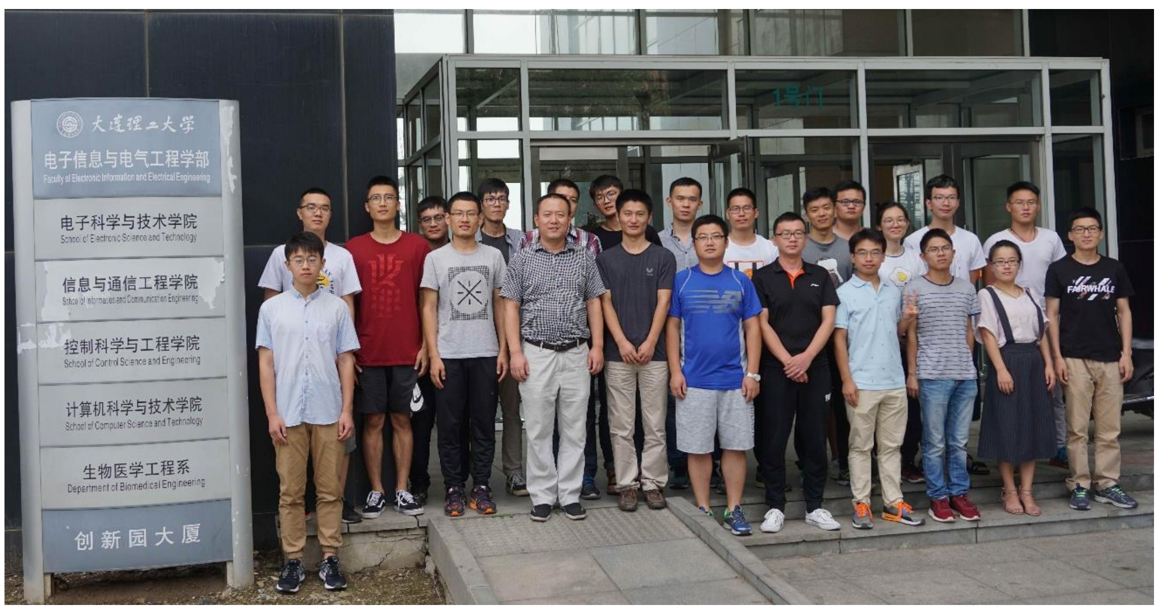
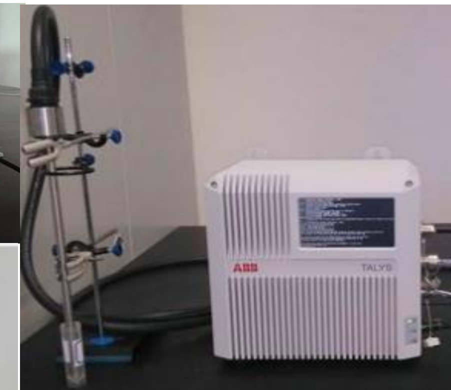


My group photo in 2017

Institute of Advanced Control Technology



Research interests:

- Advanced control system design & system optimization;
- On-line monitoring, control design and control optimization of chemical production batch process;
- Real-time model predictive control and optimization of crystallization & drying processes;
- Design of in-situ measurement and control devices;
- PBM and CFD modeling of crystallization processes.

Thanks for your attention & comments!

Lab website: http://act.dlut.edu.cn/