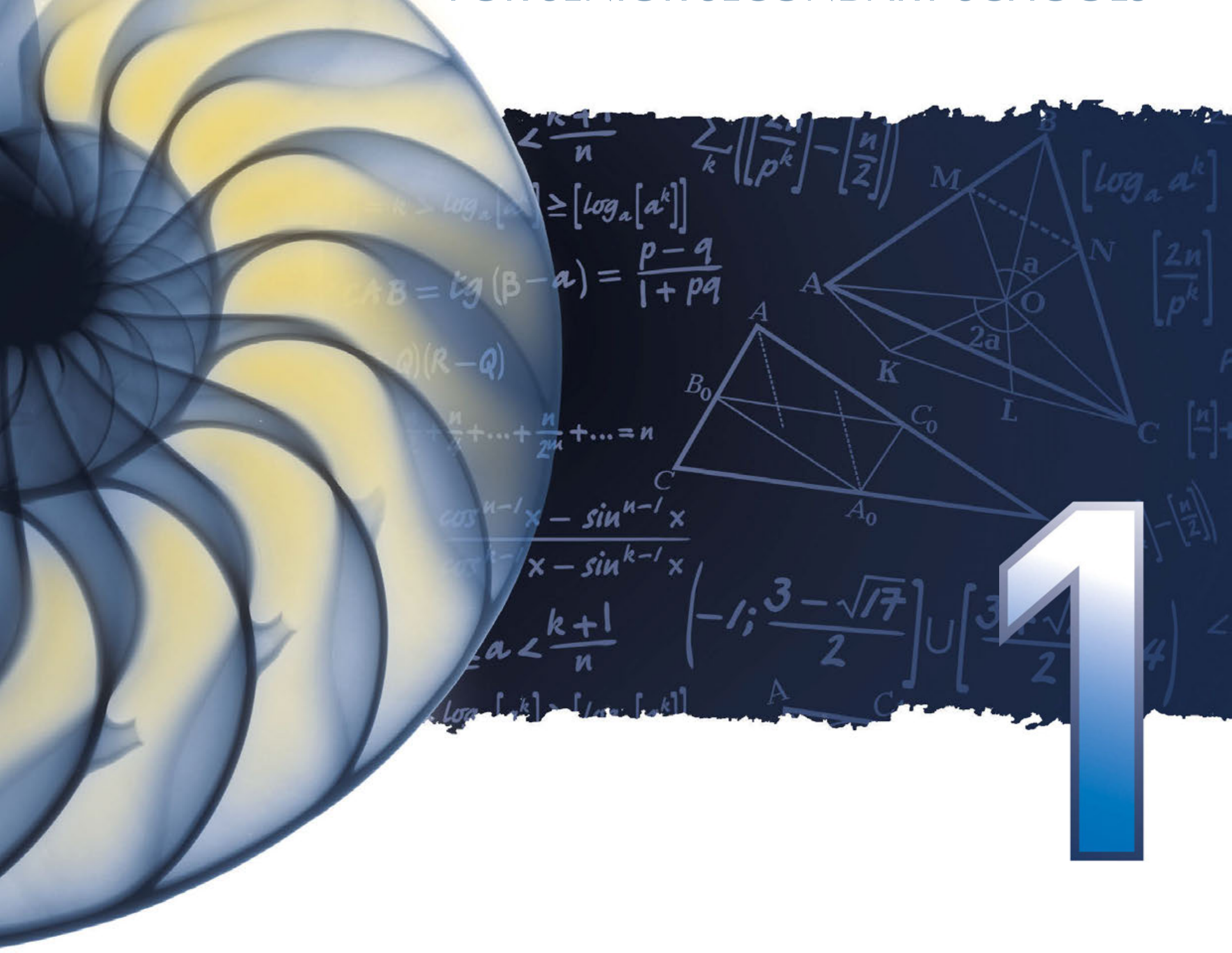
New General Mathematics

FOR SENIOR SECONDARY SCHOOLS



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TEACHER'S GUIDE

ALWAYS LEARNING PEARSON

New General Mathematics

for Secondary Senior Schools 1

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Contents

Review of Junior Secondary School course

1. Learning objectives

1. Number and numeration
2. Algebraic processes
4. Geometry and mensuration
5. Statistics and probability

2. Teaching and learning materials

Teachers should have the Mathematics textbook of the Junior Secondary School Course and Book 1 of the Senior Secondary School Course.

Students should have:

1. Book 1
2. An Exercise book
3. Graph paper
4. A scientific calculator, if possible.

3. Glossary of terms

Algebraic expression A mathematical phrase that can contains ordinary numbers, variables (such as x or y) and operators (such as add, subtract, multiply, and divide). For example, $3x^2y - 3y^2 + 4$.

Algebraic sentence is another word for an algebraic equation where two algebraic expressions are equal to each other.

Angle A measure of rotation or turning and we use a protractor to measure the size of an angle.

Angle of depression The angle through which the eyes must look downward from the horizontal to see a point below.

Angle of elevation The angle through which the eyes must look upward from the horizontal to see a point above.

Bimodal means that the data has two modes.

Cartesian plane A **coordinate** system that specifies each point in a plane uniquely by a pair of numerical coordinates, which are the perpendicular distances of the point from two fixed perpendicular directed lines or axes, measured in the same unit of length. The word Cartesian comes from the inventor of this plane namely René Descartes, a French mathematician.

Coefficient a numerical or constant or quantity ≠ 0 placed before and multiplying the variable in an algebraic expression (for example, 4 in $4x^y$).

Common fraction (also called a vulgar fraction or simple fraction) Any number written as $\frac{a}{b}$ where a and b are both whole numbers and where $a < b$.

Coordinates of point A, for example, (1, 2) gives its position on a Cartesian plane. The first coordinate (x-coordinate) always gives the distance along the x-axis and the second coordinate (y-coordinate) gives the distance along the y-axis.

Data Distinct pieces of information that can exist in a variety of forms, such as numbers. Strictly speaking, data is the plural of *datum*, a single piece of information. In practice, however, people use *data* as both the singular and plural form of the word.

Decimal place values A positional system of notation in which the position of a number with respect to the decimal point determines its value. In the decimal (base 10) system, the value of each digit is based on the number 10. Each position in a decimal number has a value that is a power of 10.

Denominator The part of the fraction that is written below the line. The 4 in $\frac{3}{4}$, for example, is the denominator of the fraction. It also tells you what kind of fraction it is. In this case, the kind of fraction is quarters.

Directed numbers Positive and negative numbers are called directed numbers and are shown on a number line. These numbers have a certain direction with respect to zero.

- If a number is positive, it is on the right-hand side of 0 on the number line.
- If a number is negative, it is on the left-hand side of the 0 on the number line.

Direct proportion The relationship between quantities of which the ratio remains constant. If a and b are directly proportional, then $\frac{a}{b}$ = a constant value (for example, k).

Direct variation Two quantities a and b vary directly if, when a changes, then b changes in the same ratio. That means that:

- If a doubles in value, b will also double in value.
- If a increases by a factor of 3, then b will also increase by a factor of 3.

Edge A line segment that joins two vertices of a solid.

Elimination is the process of solving a system of simultaneous equations by using various techniques to successively remove the variables.

Equivalent fractions Fractions that are multiples of each other, for example, $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{3 \times 3}{4 \times 3} \ldots =$ and so on.

Expansion of an algebraic expression means that brackets are removed by multiplication

Faces of a solid A flat (planar) surface that forms part of the boundary of the **solid** object; a three-dimensional solid bounded exclusively by flat faces is a polyhedron.

Factorisation of an algebraic expression means that we write an algebraic expression as the product of its factors.

Graphical method used to solve simultaneous linear equations means that the graphs of the equations are drawn. The solution is where the two graphs intersect (cut) each other.

Highest Common Factor (HCF) of a set of numbers is the highest factor that all those numbers have in common or the highest number that can *divide into* all the numbers in the set. The HCF of 18, 24 and 30, for example, is 6.

Inverse proportion The relationship between two variables in which their product is a constant. When one variable increases, the other decreases in proportion so that the product is unchanged. If b is inversely proportional to a, the equation is in the form $b = \frac{k}{a}$ (where k is a constant).

Inverse variation: Two quantities a and b vary inversely if, when a changes, then b changes by the same ratio inversely. That means that:

- If a doubles, then b halves in value.
- If a increases by a factor of 3, then b decreases by a factor of $\frac{1}{3}$.

Joint variation of three quantities x, y and z means that x and y are directly proportional, for example, and x and z are inversely proportional, for example. So $x \propto \frac{y}{z}$ or $x = k\frac{y}{z}$, where k is a constant.

Like terms contain identical letter symbols with the same exponents. For example, $-3x^2y^3$ and $5x^2y^3$ are like terms but $3x^2y^3$ and $3xy$ are not like terms. They are **unlike** terms.

Lowest Common Multiple (LCM) of a set of numbers is the smallest multiple that a set of numbers have in common or the smallest number *into* which all the numbers of the set can *divide* without leaving a remainder. The LCM of 18, 24 and 30, for example, is 360.

Median The median is a measure of central tendency. To find the median, we arrange the data from the smallest to largest value.

- If there is an odd number of data, the median is the middle value.
- If there is an even number of data, the median is the average of the two middle data points.

Mode The value (data point) that occurs the most in a set of values (data) or is the data point with the largest frequency.

Multiple The multiple of a certain number is that number *multiplied by* any other whole number. Multiples of 3, for example, are 6, 9, 12, 15, and so on.

Net A plane shape that can be folded to make the solid.

Numerator The part of the fraction that is written above the line. The 3 in $\frac{3}{8}$, for example, is the numerator of the fraction. It also tells how many of that kind of fraction you have. In this case, you have 3 of them (eighths)

Origin is where the x-axis and the y-axis intersect and is the point (0, 0).

Orthogonal projection A system of making engineering drawings showing several different views (for example, its plan and elevations) of an object at right angles to each other on a single drawing.

Parallel projection Lines that are parallel in reality are also parallel on the drawing

Pictogram (or pictograph) Represents the frequency of data as pictures or symbols. Each picture or symbol may represent one or more units of the data.

Pie chart A circular chart divided into sectors, where each sector shows the relative size of each value. In a pie chart, the angle of the each sector is in the same ratio as the quantity the sector represents.

Place value Numbers are represented by an ordered sequence of digits where both the digit and its place value have to be known to determine its value. The 3 in 36, for example, indicates 3 tens and 6 is the number of units.

Rational numbers are all the numbers which can be written as $\frac{a}{b}$, where $a \in \mathbb{Z}$ (integers), $b \in \mathbb{Z}$ (integers) and $b \neq 0$.

Reciprocal or multiplicative inverse, is simply one of a pair of numbers that, when multiplied together, will give an answer of 1. If you have a fraction and want to find the reciprocal, you swop the numerator and the denominator to get the reciprocal of that specific fraction. To find the reciprocal of a whole number, just turn it into a fraction in which the original number is the denominator and the numerator is 1.

Satisfy an equation, means that there is a certain value(s) that will make the equation true. In the equation $4x + 3 = -9$, $x = -3$ satisfies the equation because $4(-3) + 3 = -9$.

Simplify means that you are writing an algebraic expression in a form that is easier to use if you want to do something else with the expression. If you, for example, want to work out the value of an algebraic expression $3x^2 - 2x - 4x^2 + 5x$, if $x = -2$, you would not substitute the value of x in the expression before you have not written it in a simpler form as $-x^2 + 3x$.

Simultaneous linear equations are equations that you solve by finding the solution that will make them simultaneously true. In $2x - 5y = 16$ and $x + 4y = -5$, $x = 3$ and $y = -2$ satisfy both equations simultaneously.

SI units The international system of units of expressing the magnitudes or quantities of important natural phenomena such as length in metres, mass in kilograms and so on.

Terms in an algebraic expression are numbers and variables which are separated by + or – signs.

Variable In algebra, variables are represented by letter symbols and are called variables because the values represented by the letter symbols may vary or change and therefore are not constant.

Vertex (plural vertices) A point where two or more edges meet.

***x*-axis** The horizontal axis on a Cartesian plane.

***y*-axis** The vertical axis on a Cartesian plane.

Teaching notes

You should be aware of what your class knows about the work of previous years. It would be good if you could analyse their answer papers of the previous end of year examination to find out where they lack the necessary knowledge and ability in previous work. You can then analyse their answers to find out where they experience difficulties with the work and then use this chapter to concentrate on those areas.

A good idea could also be that you review previous work by means of the summary given in each section. Then you let the students do Review test 1 of that section and you discuss the answers when they finished it. You then let the students write Review test 2 as a test, and you let them mark it under your supervision.

Chapter 1 Numerical processes 1: Indices and logarithms

Learning objectives

By the end of this chapter, the students should be able to:

1. Recall and use the laws of indices (multiplication, division, zero, reciprocal).
2. Simplify expressions that contain products of indices and fractional indices.
3. Solve simple equations containing indices.
4. Express and interpret numbers in standard form.
5. Find the logarithms and antilogarithms of numbers greater than 1.
6. Use logarithms to solve problems.

Teaching and learning materials

Students: Copy of textbook with logarithm and antilogarithm tables (pp. 245 and 246), exercise book and writing materials.

Teacher: Index and logarithm charts, graph chalkboard; books of four figure tables (as used in public examinations) and a copy of the textbook, an overhead projector (if available), transparencies of the relevant tables and transparencies of graph paper.

Teaching notes

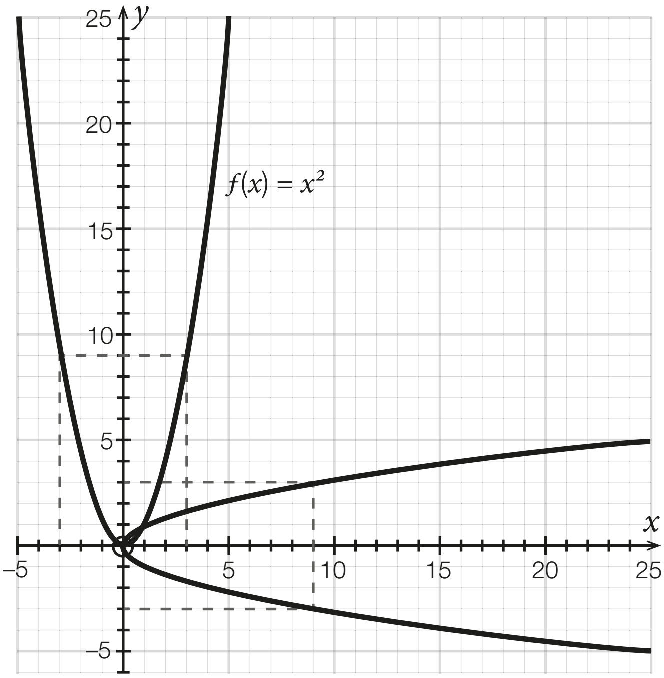
Laws of indices

- When revising the first four laws given on p. 15, it is very important that you illustrate each one with a numerical example as shown in Example 1.
- You could also explain the negative exponent like this:
  $\frac{2^3}{2^5} = 2^{3-5} = 2^{-2}$
  Usually when we divide, we subtract the exponents of the equal bases where the biggest exponent is: $\frac{2^3}{2^5} = \frac{1}{2^{5-3}} = \frac{1}{2^2}$. From this we can deduce that $2^{-2} = \frac{1}{2^2}$ or $\frac{2^5}{2^2} = 2^{5-2} = 2^3$.
  But if we forget that we always subtract exponents of equal bases where the biggest exponent is, the sum can be done like this:
  $\frac{2^5}{2^2} = 2^{2-5} = 2^{-3}$. So, $2^3 = \frac{1}{2^{-3}}$.
  Therefore, to write numbers with positive indices, we write the power of the base with a negative exponent, on the opposite side of the division line for example: $\frac{1}{x^{-3}} = \frac{x^3}{1}$ or $\frac{x^{-3}}{1} = \frac{1}{x^3}$.
- $x^0 = 1$, where $x \neq 0$: Students may ask why x is not equal to 0. You can explain it as follows.
  - If $x = 0$, we may have that x^0 resulted from $\frac{0^m}{0^m} = \frac{0}{0}$. Here we divided by 0 which is not defined. Then you can explain why division by 0 is not defined like this:
    - Say we take $\frac{8}{2} = 4$. This is because $2 \times 4 = 8$. Also $\frac{0}{2} = 0$, because $2 \times 0 = 0$.
    - Now, if we take $\frac{8}{0}$ = any number, then 'that number' $\times$ 0 must be equal to 8.
    - That, however, is impossible, because there is no number that we can multiply by 0 that will give 8. So, division by zero is not defined.
- In this book, $9^{\frac{1}{2}} = \sqrt{9}$ is given as ±3. This can be explained as follows:
  - If we draw the graph of $f(x) = x^2$, we see that the y-values are found by squaring all the x-values. We can show this diagrammatically by means of a flow diagram:

    $x \rightarrow \boxed{\times x^2} \rightarrow y$
  - In the flow diagram:
    - The x-values are the input values or x is the independent variable.
    - The y-values are the output values and y is the dependent variable, because its values depend on the values of x.
  - Now, if we invert this operation, it means that we make the y-values the input values and it becomes the independent variable, x. Instead of squaring the x-values, we now find their square roots. We can show this diagrammatically by means of a flow diagram:

    $x \rightarrow \boxed{\pm\sqrt{x}} \rightarrow y$

- This is called the inverse of f, and the y-values are found by taking ± the square root of x. We say that $f^{-1}(x) = \pm\sqrt{x}$. Although f is a function, its inverse is not a function.
- In a function, the value of each independent variable is associated with only one value of the dependent variable and this is not the case with f^{-1}. There each x-value is mapped onto the two y-values.
- All this is illustrated in the graph below.

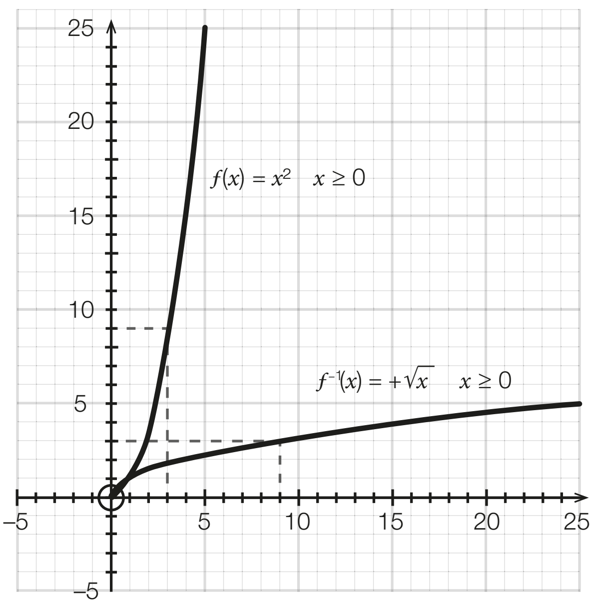


- It is also reasoned that any number has two square roots. This is because if we want to find the square root of a number, we are looking for that unique number that, if multiplied by itself, will give the original number.
- So, if, for example, we want to find the square root of 16 or per definition $16^{\frac{1}{2}}$, we say that the answer is 4 or –4, because $(4) \times (4) = 16$ and $(-4) \times (-4) = 16$.
  - There is, however, another viewpoint, where you can define f in such a way that its inverse is also a function:
  - Let us say that $f(x) = x^2$, where $x \geq 0$. Then, if we invert the operation, we again get the inverse of f, which now also is a function:

    $y^2 = x$ (swop x and y)

    $y = f^{-1}(x) = +\sqrt{x}, x \geq 0$

    (find the square root of both sides of the equation)
  - (For the function (f), $x \geq 0$, so, for its inverse, $y \geq 0$. We, therefore, take the positive square root of x and $x \geq 0$, because you cannot get the square root of a negative number.)
- The graph shows that we now only have the positive square root for any number. On the graph of f, for example, you can see that $3^2 = 9$; and on the graph of f^{-1}, you can see that $\sqrt{9} = 9^{\frac{1}{2}} = +3$.

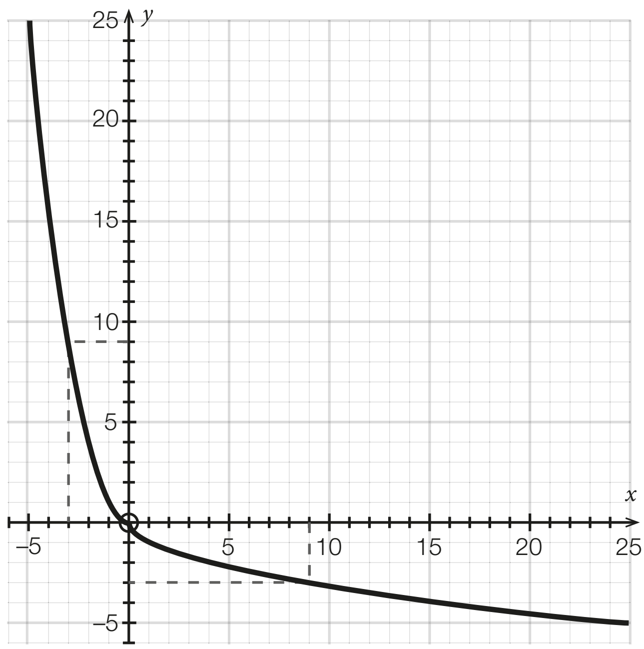


  - Let us say that $f(x) = x^2$, where $x \leq 0$. Then, if we invert the operation again, we again get the inverse of f, which now also is a function:

    $y^2 = x$ (swop x and y)

    $y = f^{-1}(x) = -\sqrt{x}, x \geq 0$

    (find the square root of both sides of the equation)
  - (For the function (f), $x \leq 0$, so, for its inverse, $y \leq 0$. We, therefore, take the negative square root of x and $x \geq 0$, because you cannot get the square root of a negative number.)
- The graph shows that we now only have the negative square root for any number. On the graph of f, for example, you can see that $(-3)^2 = 9$; and on the graph of f^{-1}, you can see that $-\sqrt{9} = -9^{\frac{1}{2}} = -3$.



- Many mathematicians, therefore, define $x^{\frac{1}{n}}$ as the $+\sqrt[n]{x}$, where $x \geq 0$, where n is an even number and $-x^{\frac{1}{n}}$ as the $-\sqrt[n]{x}$, where $x \geq 0$, and n is an even number. So, the students regard $16^{\frac{1}{2}} = +\sqrt{16} = +4$ and $-16^{\frac{1}{2}} = -\sqrt{16} = -4$, for example.
- They reason like this, because they want $x^{\frac{1}{n}}$ ($16^{\frac{1}{2}}$, for example) and $\sqrt[n]{x}$ ($+\sqrt{16}$, for example) to stand for one number only and $-x^{\frac{1}{n}}$ ($16^{\frac{1}{2}}$, for example) and $-\sqrt[n]{x}$ ($-\sqrt{16}$, for example) to also stand for one number only, **but if $x^2 = 9$, then $x = +3$ or $x = -3$.**

- When students revise the standard form, emphasise that:
  - If $A \times 10^n$ is the standard form then $1 < A < 10$.
  - If A does not satisfy this condition, the number is not written in standard form.
- When you explain Example 25, emphasise that students write down the steps to work out the answer (the method) and their working out apart from the table with logarithms.

Areas of difficulty and common mistakes

- Students do not understand the difference between, for example, $(-3)^2$ and -3^2.
  - You can read $(-3)^2$, as negative 3 squared and it means $(-3) \times (-3) = 9$.
  - You can read -3^2, as the negative of 3 squared and it means $-3 \times 3 = -9$.
- Students tend to forget what the word logarithm really means. Emphasise the following:
  - In numbers: if $10^{2.301} = 200$, then $\log_{10} 200 = 2.301$.
  - In words: log base 10 of 200 is the exponent to which 10 must be raised to give 200.
- Students tend to forget what antilog means. If, for example $10^{2.301} = 200$, the antilog means that we want to know what the answer of $10^{2.301}$ is.
- Students tend to forget why they add logarithms of numbers, if they multiply the numbers and why they subtract logarithms of numbers, if they divide these numbers by each other.
- Emphasise that logarithms are exponents (of 10 in this case) and that the first two exponential laws are:

  Law I: $a^x \times b^y = a^{x+y}$. For example:
  $a^3 \times a^4 = (a \times a \times a) \times (a \times a \times a \times a)$
  $= a \times a \times a \times a \times a \times a \times a = a^7$
  Law II: $a^x \div a^y = a^{x-y}$, where $x > y$.
  For example:
  $\frac{a^6}{a^2} = \frac{a \times a \times a \times a \times a \times a}{a \times a} = a^4 = a^{6-2}$
- So, since logarithms to base 10 are the exponents of 10:
  - We add the logs of the numbers, if we multiply the numbers.
  - We subtract the logs of the numbers, if we divide the numbers by each other.
- When working out a number to a certain power, students tend to forget why they multiply the log of the number with the power. Again, emphasise this exponential law since logs to the base 10 are the same as the exponents of 10, which will give the number $(a^m)^n = a^{mn}$. For example:
  - $(a^4)^2 = (a \times a \times a \times a) \times (a \times a \times a \times a)$
    $= a \times a \times a \times a \times a \times a \times a \times a = a^{4 \times 2}$
    $= a^8$
  - $\sqrt{a^6} = (a^6)^{\frac{1}{2}} = a^{6 \times \frac{1}{2}} = a^3$
- In the beginning, students may find it difficult to write down their calculations with logarithms in table form. Give them a lot of guidance and emphasise that it is essential that they write all their calculations out in table form to prevent mistakes.

Supplementary worked examples

Evaluate the following:

1. -4^2	2. $(-4)^2$	3. 4^{-2}
4. -4^{-2}	5. $(-4)^{-2}$	6. $4^{\frac{1}{2}}$

7. $-4^{\frac{1}{2}}$ 8. $4^{-\frac{1}{2}}$ 9. $-4^{-\frac{1}{2}}$

10. $(-4)^{-\frac{1}{2}}$ 11. $\sqrt{(25-9)}$ 12. $\sqrt{(16+9)}$

13. $(10^2-6^2)^{\frac{1}{2}}$ 14. $(10^2-6^2)^{-\frac{1}{2}}$ 15. $(5^2+12^2)^{\frac{1}{2}}$

16. $(25 \times 144)^{\frac{1}{2}}$ 17. $\left(\frac{25}{144}\right)^{\frac{1}{2}}$ 18. $\left(2\frac{1}{4}\right)^{\frac{1}{2}}$

19. $(a^2+b^2)^{\frac{1}{2}}$ 20. $[(a+b)^2]^{\frac{1}{2}}$ 21. $(x^2+y^2)^0$

22. -4^0 23. $(-4)^0$ 24. $\frac{1}{-4^0}$

Answers

1. -16 2. 16 3. $\frac{1}{16}$

4. $-\frac{1}{16}$ 5. $\frac{1}{16}$ 6. 2

7. -2 8. $\frac{1}{2}$ 9. $-\frac{1}{2}$

10. not a real number

11. 4 12. 5 13. 8

14. $\frac{1}{8}$ 15. 13 16. 60

17. $\frac{5}{12}$ 18. $1\frac{1}{2}$ 19. $\sqrt{(a^2+b^2)}$

20. $a+b$ 21. 1 22. -1

23. 1 24. -1

Chapter 2 Geometry 1: Formal geometry: Triangles and polygons

Learning objectives

By the end of this chapter, the students should be able to:

1. Recall and prove elementary theorems in plane geometry:
   - The angle sum of a triangle is 180°.
   - The exterior angle of a triangle is equal to the sum of the opposite interior angles.
   - The angle sum of an n-sided convex polygon is $(2n - 4)$ right angles.
2. Recall and apply the conditions for triangles to be congruent:
   - Two sides and the included angle.
   - Two angles and a corresponding side.
   - Three sides.
   - Right angle, hypotenuse and side.
3. Use the elementary theorems and properties of congruent triangles to prove theorems and riders.
4. Identify properties of parallelograms and related quadrilaterals, and isosceles and equilateral triangles.
5. Use and apply the equal intercept theorem.
6. Apply theorems and riders to solve geometrical problems.

Teaching and learning materials

Students: Copy of textbook, exercise book and writing materials, paper, scissors or blade, geometrical instruments (especially protractors).
Teacher: A copy of the textbook, cardboard, paper scissors; chalkboard instruments (especially a protractor).

Glossary of terms

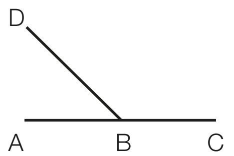
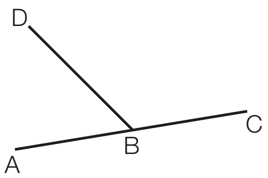
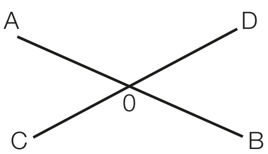
Rider is a problem based on a certain theorem or theorems. We say the problem rides on this/these specific theorem(s).
Re-entrant polygon (or a **concave polygon**) is a polygon that contains one or two reflex angles.

Teaching notes

- When you start doing geometry you could briefly mention the history of geometry.
  - Mention that the father of plane geometry is regarded as Euclid. He lived more than 3 centuries before the birth of Christ and was of Greek decent, but was born in Alexandria in Egypt. He also taught Mathematics there.
  - Euclid's book called "The Elements" consists of 13 volumes of which only one contains plane geometry.
  - Euclid did not create this geometry. Most of the geometry originated from earlier mathematicians or was practically used in navigation or surveying the land.
  - Euclid's main achievement is that he presented this geometry in a logically coherent framework. His work on plane geometry is one of the most influential works in mathematics, because it served as the main textbook for teaching geometry until late in the 19^{th} century/early 20^{th} century.
  - The geometry that is taught in schools today is merely an adaptation of how Euclid taught it. Some people also call plane geometry Euclidian Geometry.
- The logic used in Geometry is based on the principle that we assume certain facts, like the fact that the angles around a point add up to 360° or the fact that the sum of the angles on a straight line is equal to 180°.
  - Then we use these assumptions, called axioms, to prove theorems like the sum of the angles of a triangle is equal to 180°, and so on.

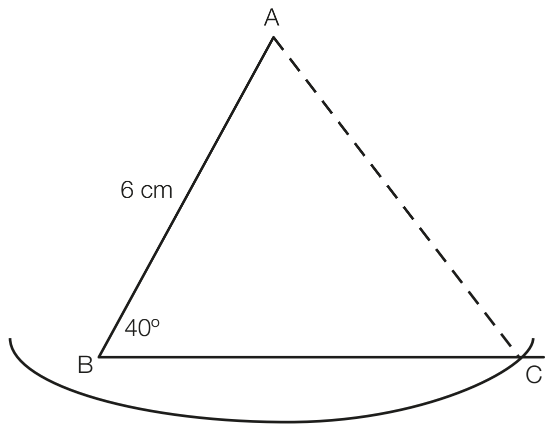
- These theorems are then used to solve problems in geometry or to prove riders.
- It is obvious that, if our assumptions are not true, that the whole logical system would collapse.

- When teaching the proof of the theorems, emphasise that students learn these proofs, because then they could be sure of those marks at least when answering a geometry question paper.
- When you do the examples, emphasise that students must give a reason for each statement they make if the statement is the result of a theorem or an axiom.
  - To help them you could give them summaries of the theorems and suggestions of the reasons they can use.
  - Below are suggestions of how these summaries could look:

Sketch	**Theorem/Axiom**	**Reason you must give, if you use this Theorem/ Axiom**
	If ABC is a straight line, ∠ABD + ∠DBC = 180°.	Sum ∠'s on a str. line = 180°
	If ∠ABD + ∠DBC = 180°, then AB and BC lie in a straight line.	The sum of adjacent angles = 180°
	If two straight lines AB and CD intersect at O, ∠AOD = ∠BOC and ∠AOC = ∠DOB.	Vert. opp. ∠'s =

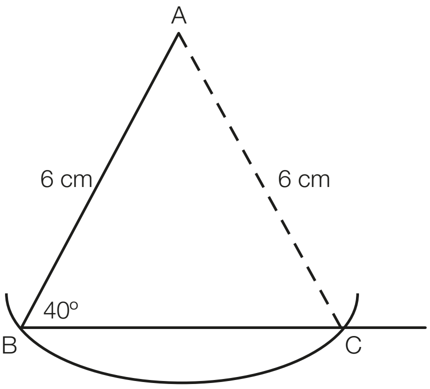
- Teach students to write out the solution of a geometrical rider as if they are explaining it to somebody else who does not understand easily.
  - So every step of the reasoning as well as the reasons for statements must be written down.
  - You can emphasise this when you ask the students to help you to write out the examples.
- **Congruency**
  - When you start your lesson about congruent triangles, you may mention that a triangle has 6 elements, namely 3 angles and 3 sides.
    - If we want to construct a triangle, however, we need only three of the six elements.
    - Then you can go through all the combinations of these three elements to determine which of them will have the result that the whole class would always get identical triangles if they use that particular combination to construct a triangle.
    - It would be a good idea if you could illustrate how the triangles are constructed so that the class can see that no other triangle is possible.
  - When you do the ambiguous case, you can illustrate why and when you can get 1 triangle, 2 triangles or no triangles as follows:
  - Tell the class that if you have a △ABC, with ∠B = 40° and AB = 6 cm, for example, that depending on the length of AC, you could get 1 or 2 triangles or no triangle:
    - To find C, you measure the required length on a ruler with your compass. Then you put the sharp, metal point of the compass on A and chop off the required length through the line by drawing an arc with the pencil of your compass.
    - Illustrate this by drawing a line BC on the board, measuring ∠B = 40° and drawing a line from B through the 40° point.
    - Then you chop off a length that represents the length of BA.
    - After this, you stretch your compass to an arc wider/longer or equal or shorter than BA and draw the arc from A.

a) If AC > AB, it will look something like this:



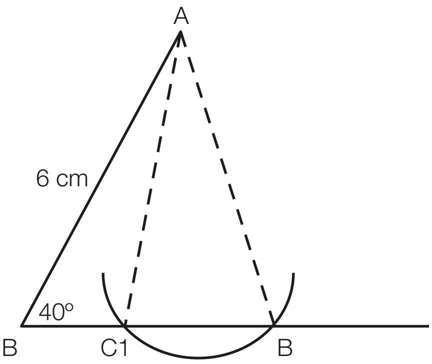
The arc drawn will intersect the line in one point only. So only one 1 triangle is possible.

b) If AC = AB, it will look something like this:



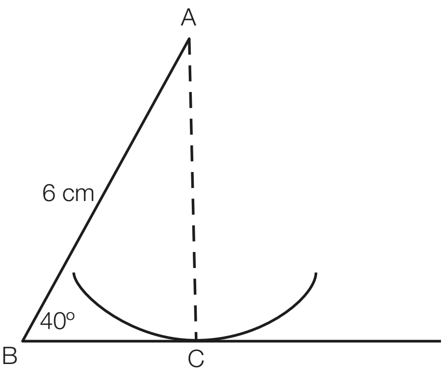
The arc drawn will intersect the line in one point only. So only one 1 triangle is possible.

c) If AC < 6 cm (and not so short that the arc will not intersect the base line), it will look something like this:

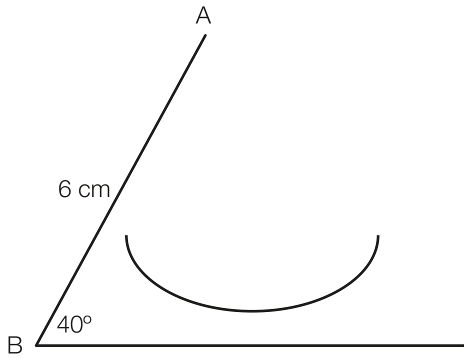


The arc drawn will intersect the line in two places. So, two triangles are possible. ($\triangle ABC_1$ and $\triangle ABC_2$)

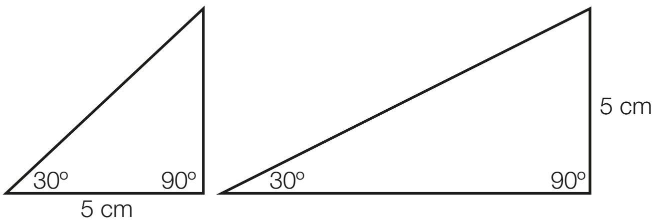
d) If AC is just long enough, the arc will just touch the line through B and only one triangle is possible. In this case, $\angle ACB = 90°$. That is where the fourth case of congruency comes from.



e) If AC is too short, no triangle is possible:



- In congruency problems, the lengths of sides are not given. So, if we do not know the length of the side opposite the given angle and the length of the side adjacent to that angle, we cannot say with certainty whether the triangles are congruent, because it could be any of the cases illustrated above.
- When you do the case of congruency where two angles and a side is used to construct a triangle, illustrate on the chalk board why the given side must be in the same position for the triangles to be congruent.
  - Draw any triangle with the given side between the two given angles and another triangle with the given side opposite one of the given angles, for example.
  - Below is a possible result.



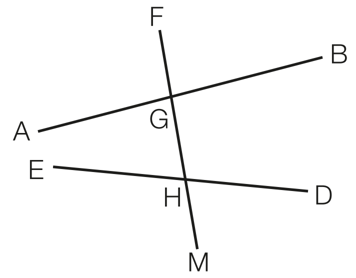
- When you do parallelograms and related quadrilaterals, it is important that you realise that students can be on different levels of understanding geometry. It is useful to know the Van Hiele levels of understanding geometry, especially if you want your students to be able to define a rectangle, or a square or a rhombus.
  - At level 0, a student can just recognise the quadrilateral.
  - At level 1, students recognise the properties of the quadrilateral. They would, for example, realise that a square has equal sides and that all its angles are right angles.

- At level 2, students can see that all squares are also rectangles and they can also write down definitions of figures, but they are unable to write down formal geometrical proofs.
- At level 3, they start to understand the meaning of deduction from simple proofs and can construct simple geometrical proofs.

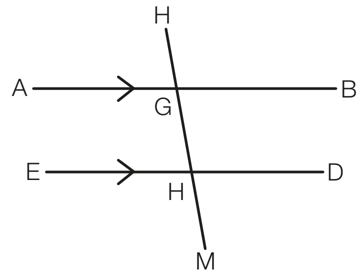
- When students have to prove that a quadrilateral with one pair of opposite sides equal and parallel, is a parallelogram, for example, they have to prove that both pairs of opposite sides of the quadrilateral are parallel (See Exercise 2d).
  - In other words, they have to go back to how the parallelogram was defined.
  - We could, of course, use any of the properties of the parallelogram (except the fact that a diagonal bisects its area) as the definition of the parallelogram and prove the other properties from that definition.
- Remember to tell students that a **good definition** must satisfy the following two properties:
  - It must be **economical**, which means that it is a short as possible while still very clear.
  - It must be **unique**, which means that there must be no confusion of what it defines.
- When you set geometry questions for a test or an examination, always make sure that your solution is the shortest possible path to follow in order to solve the problem.
  - In that way, you make sure that the problem does not count too many marks.
  - You have to, however, accept any answers from your students that are logical and correctly set out, even if it is very long.

Areas of difficulty and common mistakes

- Students do not write reasons for their statements. Make a summary of reasons for them as suggested above and insist on reasons.
- Students tend to leave out steps of their reasoning. Tell them that they are actually explaining the solution to somebody else and want to convince this person of their solution.
- When students state that alternate or corresponding angles are equal or that co-interior angles are supplementary, they omit to say that the lines through which the transversal passes are parallel. In the figure below:



  - ∠AGF and ∠EHG are corresponding angles and ∠AGH and ∠GHD are alternate angles, but these corresponding and alternate angles are not equal, because AB is not parallel to ED.
  - Similarly: ∠BGH and ∠DHG are co-interior angles, but they are not supplementary, because AB is not parallel to ED.
  - So, not all alternate angles and not all corresponding angles are equal and neither are all co-interior angles supplementary. The lines through which the transversal passes, must be parallel to each other.
  - So, it is essential that students say the following, for example:



$\angle AGF = \angle EHG$ (corresponding $\angle$s, AB || ED)
Or
$\angle AGH = \angle MHD$ (alternate $\angle$s, AB || ED)
Or
$\angle AGH + \angle EHG = 180°$ (co-interior $\angle$s, AB || ED)

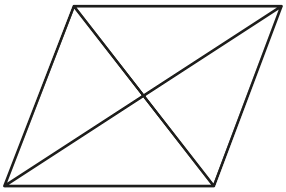
- Students find it very difficult to define the different kinds of quadrilaterals.
  - You could give them the following summary and each time emphasise that the figure is a special parallelogram or rectangle or rhombus, which has all these properties in common with the parallelogram or rectangle or rhombus, but has special properties.

- Then let them try to define a rhombus, rectangle or square themselves.
- An example of a summary that you can use is shown below:

Parallelograms

1. Both pairs of opposite sides parallel (definition).
2. Both pairs of opposite angles equal.
3. Diagonals bisect each other.
4. One pair of opposite sides is equal and parallel.

Rhombuses

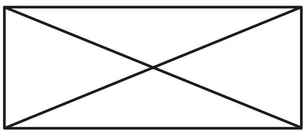


1. Both pairs of opposite sides parallel.
2. Both pairs of opposite angles equal.
3. Diagonals bisect each other.
4. One pair of opposite sides is equal and parallel.

Special properties

1. All sides are equal.
2. Diagonals are perpendicular on each other.
3. Diagonals bisect the angles.

Rectangles

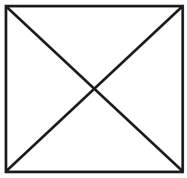


1. Both pairs of opposite sides parallel.
2. Both pairs of opposite angles equal.
3. Diagonals bisect each other.
4. One pair of opposite sides is equal and parallel.

Special properties

1. All the angles are equal to 90°.
2. Diagonals are equal.

Squares



1. Both pairs of opposite sides parallel.
2. Both pairs of opposite angles equal.
3. Diagonals bisect each other.
4. One pair of opposite sides is equal and parallel.

Special properties

1. All the angles are equal to 90°.
2. Diagonals are equal.
3. Diagonals bisect the angles.
4. Diagonals are perpendicular on each other.
5. All the sides are equal.

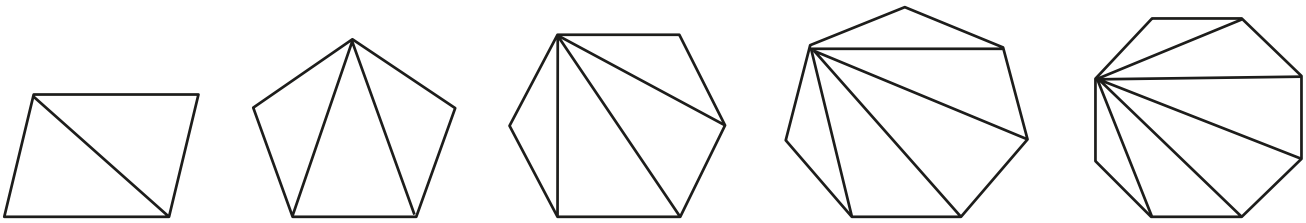
- Students tend to write the letters of two congruent triangles in the wrong order.
  - Emphasise that they have to write the letters according to the corresponding sides and angles that are equal.
  - Let them trace the letters according to the equal parts with their fingers and say the letters out loud.
- Students tend to say that two triangles are congruent if AAS of one triangle are equal to AAS of the other triangle when the equal sides are not in the same position. If necessary, let them construct a pair of triangles where the equal side is not in the same position.
- Students find it difficult to write out a proof or a how they calculate something in a geometrical figure.
  - The only cure for this is practise and explanation and more practise and explanation.
  - It also helps to let the student first verbally explain the solution or proof to you and then write out the whole proof or calculation like an essay.
  - After this, the students can “translate” their essay in the formal form of a proof as illustrated in the textbook.

Supplementary worked examples

- There is an alternate method of proving Theorem 3:

Let the class copy and complete this table, by using the polygons below the table:

Name of figure	Number of sides	Number of $\triangle$s	Number of right angles	Total degrees
Triangle				
Quadrilateral				
Pentagon				
Hexagon				
Heptagon				
Octagon				
n-gon				



Quadrilatera Pentagon Hexagon Heptagon Octagon

Name of figure	Number of sides	Number of $\triangle$s	Number of right angles	Total degrees
Triangle	3	1	2	$2 \times 90° = 180°$
Quadrilateral	4	2	4	$4 \times 90° = 360°$
Pentagon	5	3	6	$6 \times 90° = 540°$
Hexagon	6	4	8	$8 \times 90° = 720°$
Heptagon	7	5	10	$10 \times 90° = 900°$
Octagon	8	6	12	$12 \times 90° = 1\,080°$
n-gon	n	$n - 2$	$2(n - 2) = 2n - 4$	$2 \times 90°(n - 2) = 180°(n - 2)$

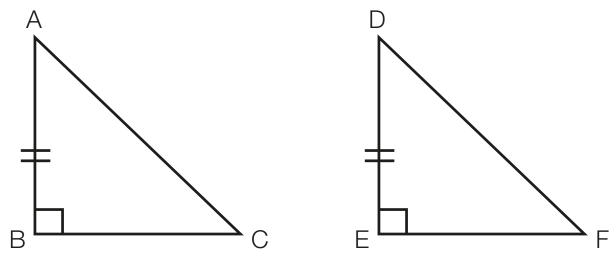
Answer

There is an alternate method of proving Theorem 4:
Each interior angle + each exterior angle = 180°.
The sum of n interior angles + the sum of n exterior angles = $180°n$.
$180°(n - 2)$ + the sum of n exterior angles $= 180°n$.

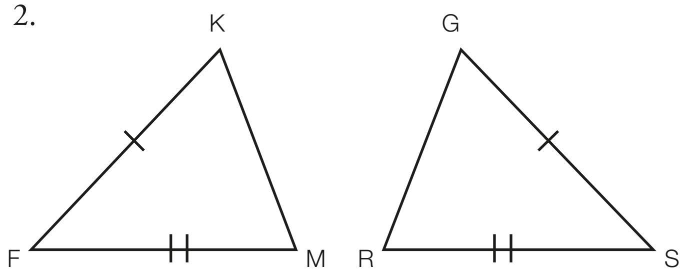
The sum of n exterior angles $= 180°n - 180°(n - 2)$
$= 180°n - 180n + 360°$
$= 360°$

- You could also add these kinds of problems to Exercise 2b:
  - Two triangles with two elements equal, and the student has to add another pair of elements for the two triangles to be congruent.
  - Here are some examples:
    - In each of these pairs of triangles, two pairs of equal elements are marked.
    - In each case, write down another pair of equal elements for the triangles to be congruent.
    - Give the congruency case, which you used and also give all the possibilities without repeating any congruency test you already used:

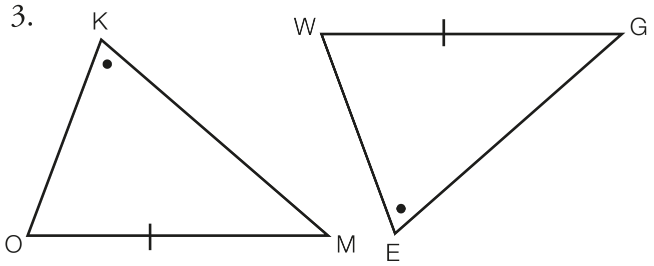
1.



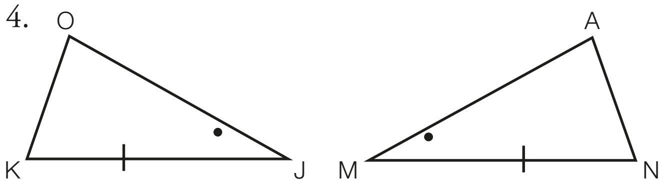
2.



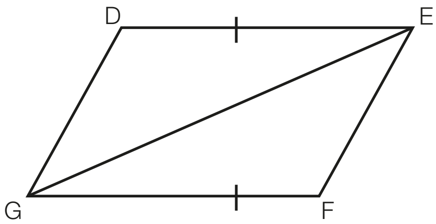
3.



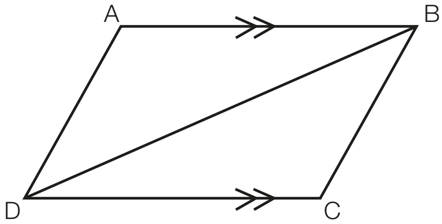
4.



5.



6.

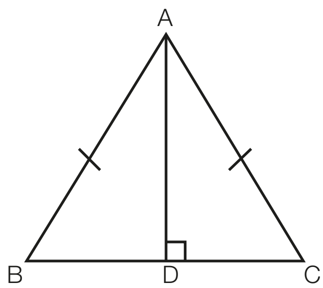


Solutions

1. $|AC| = |DF|$ (RHS), $|BC| = |EF|$ (SAS), $\angle A = \angle D$ (AAS = AA corr S)
2. $|KM| = |GR|$ (SSS), $\angle F = \angle R$ (SAS)
3. $\angle M = \angle W$ (AAS = AA corr S)
4. $|OJ| = |MA|$ (SAS), $\angle O = \angle A$ (AAS = AA corr S)
5. $\angle DEG = \angle FGE$ (SAS), $|DG| = |EF|$ (SSS)
6. $|AB| = |DC|$ (SAS), $\angle A = \angle C$ (AAS = AA corr S)

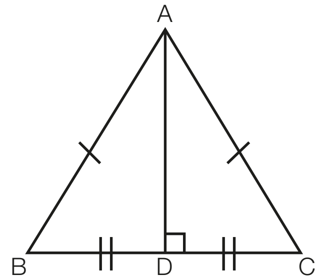
There are two other constructions that can be used to prove Theorem 5:

Construction: Draw $AD \perp BC$ so that D is on BC



Proof: In $\triangle ABD$ and $\triangle ACD$:
(1) $|AB| = |AC|$ (given)
(2) $|AD| = |AD|$ (same side)
(3) $\angle BDA = \angle CDA$ (construction)
$\therefore \triangle ABD \equiv \triangle ACD$ (RHS)
$\therefore \angle B = \angle C$ (corr angles in $\triangle$s ABD and ACD)

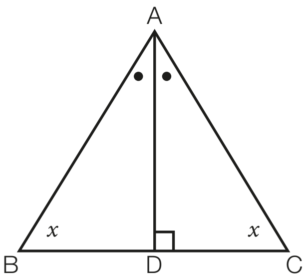
Construction: Draw AD with D on BC so that BD = DC



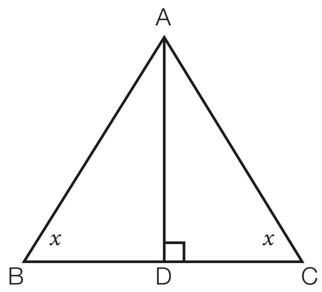
Proof: In $\triangle ABD$ and $\triangle ACD$:
1) $|AB| = |AC|$ (given)
2) $|BD| = |DC|$ (construction)
3) $|AD| = |AD|$ (same side)
$\therefore \triangle ABD \equiv \triangle ACD$ (SSS)
$\therefore \angle B = \angle C$ (corr angles in $\triangle$s ABD and ACD)

Theorem 5 states: The base angles of an isosceles triangle are equal. The converse is also true, however:

If two angles of triangle are equal, the triangle is isosceles.
Given: $\triangle ABC$ with $\angle B = \angle C$.
To prove: AB = AC.
Construction: Draw the bisector of $\angle A$ to meet BC at D.



Proof: : In $\triangle ABD$ and $\triangle ACD$:
1) $\angle B = \angle C$ (given)
2) $|AD| = |AD|$ (same side)
3) $\angle BAD = \angle DAC$ (construction)
$\therefore \triangle ABD \equiv \triangle ACD$ (AAS = AA corr S)
$\therefore AB = AC$ (corr sides in $\triangle$'s ABD and ACD)
Or:
Given: $\triangle ABC$ with $\angle B = \angle C$.
To prove: AB = AC.
Construction: Draw $AD \perp BC$ with D on BC.



Proof: In $\triangle ABD$ and $\triangle ACD$:

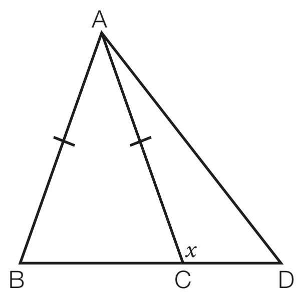
1) $\angle B = \angle C$	(given)
2) $\lvert AD\rvert = \lvert AD\rvert$	(same side)
3) $\angle BDA = \angle ADC$	(construction)
$\therefore \triangle ABD \equiv \triangle ACD$	(AAS = AA corr S)

$\therefore$ AB = AC (corr sides in $\triangle$s ABD and ACD)

If you draw AD so that BD = DC, you would get SSA which is the ambiguous case and not a congruency case. So, there are only two alternate proofs for this theorem.

An alternate problem for questions 9 and 17 of Exercise 2c is the following. (It is a rather difficult problem and can be given to the more talented students in your class.):

In the figure, AB = AC = BD. Prove, with reasons, that $\angle ACD = 2\angle ADB$.



Let $\angle ACD = x$.

$\therefore \angle ACB = 180° - x$ (sum $\angle$s on a str. line = 180°)

$= \angle B$ (AB = AC)

$\angle BAD = \angle ADB$ (AB = BD)

$= \dfrac{180° - (180° - x)}{2} = \dfrac{x}{2}$

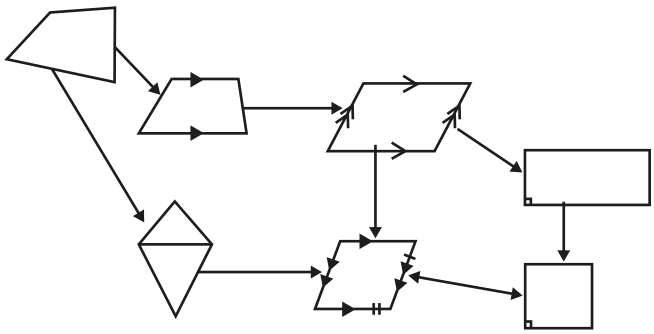
$\therefore \angle ACD = x = 2\angle ADB$

Class activity

You could let the class work in groups to draw representations of family trees of how all the quadrilaterals are related.

They then have to write down why their family tree looks a certain way and they then must use their family tree to write down definitions for each of the quadrilaterals.

Below is an example of a representation:



Chapter 3 Numerical processes 2: Fractions, decimals, percentages, number bases

Learning objectives

By the end of this chapter, the students should be able to:

1. Add, subtract, multiply, divide fractions, decimals, percentages as appropriate.
2. Solve real-life problems using fractions, decimals and percentages as appropriate.
3. Convert whole numbers from base ten to other bases and vice versa.
4. Add, subtract, multiply and divide numbers in various bases.
5. Operate with base two fractions (bicimals).

Teaching and learning materials

Students: Textbook, exercise book and writing materials as well as counters (bottle tops, pebbles).
Teacher: Counters (bottle tops, matchsticks, pebbles); number charts (fractions, decimals, conversions between bases).

Teaching notes

- Explain to students that we can only add or subtract fractions if they are of the same kind. So, we have to write them with the same denominator. For example:
  $\frac{3}{5} + \frac{3}{4} = \frac{3 \times 4}{5 \times 4} + \frac{3 \times 5}{4 \times 5} = \frac{12}{20} + \frac{15}{20} = \frac{27}{20}$
  (The LCM of 5 and 4 is 20: $5 \times 4 = 20$ (and $4 \times 5 = 20$).
  So, we multiply below and above the line by 4 (5) and, therefore, do not change anything, because we multiplied by $\frac{4}{4} = 1$ or $\frac{5}{5} = 1$).
- Example 3(a) on p. 44 of the textbook can also be done like this:
  $\frac{3}{5} + \frac{3}{4} = \frac{12 + 15}{20} = \frac{27}{20}$ (we can leave the answer as an improper fraction)
  So, there are $\frac{7}{20}$ of the students who study both history and geography.
  $\therefore \frac{7}{20} \times 40 = 14$
- Keep the order of operations in mind. (BODMAS: which means Brackets, Of, Division and Multiplication, Add and Subtract)
- Note that, if students multiply by:
  - 10, the decimal point moves 1 place to the right
  - 100, the decimal point moves 2 places to the right
  - 1 000, the decimal point moves 3 places to the right
  - So, if there are *n* zero's, the decimal point moves *n* places to the right.
- Note that, if students divide by:
  - 10, the decimal point moves 1 place to the left
  - 100, the decimal point moves 2 places to the left
  - 1 000, the decimal point moves 3 places to the left
  - So, if there are *n* zeros, the decimal point moves *n* places to the left.
- If we divide by a fraction, we multiply by the reciprocal of the fraction. The reason why we do this can be explained as follows:
  Say we have $0.66 \div \frac{2}{3}$.
  We multiply by 0.66 by 3 because we want to know how many thirds there are in 0.66:
  $0.66 \times 3 = 1.98$.
  Then we divide this answer by 2, because we want to know how many 2's there are in 1.98:
  $1.98 \div 2 = 0.99$
  In one step: $0.66 \div \frac{2}{3} = 0.66 \times \frac{3}{2} = 0.33 \times 3 = 0.99$.
- When decimal numbers with decimal fractions are multiplied, the answer has as many digits after the decimal point as there are in total.
  Say we have $1.2 \times 0.06 \times 0.003 = 0.000216$ (the answer has 6 digits after the decimal point).
  The reason is that we can write:
  $\frac{12}{10} \times \frac{6}{100} \times \frac{3}{1\ 000} = \frac{12 \times 6 \times 3}{10 \times 100 \times 1\ 000} = \frac{216}{1\ 000\ 000}$
  $= 0.000216$
- If we want a certain percentage of a number or an amount, the percentage is always written as a 100^{th}. So, 22% of 24 $= \frac{22}{100} \times 24 = 5.28$.

Students can make their work much easier, if they can remember the following:
$25\% = \frac{1}{4}$; $50\% = \frac{1}{2}$; $75\% = \frac{3}{4}$
$12.5\% = \frac{1}{8}$; $37.5\% = \frac{3}{8}$; $62.5\% = \frac{5}{8}$; $87.5\% = \frac{7}{8}$
$33\frac{1}{3}\% = \frac{1}{3}$; $66\frac{2}{3}\% = \frac{2}{3}$

Areas of difficulty and common mistakes

- When multiplying numbers like 3.4 and 6.2 by long multiplication, care should be taken that students understand that the answer should now have 2 places after the comma. You can explain this by writing the numbers in ordinary fraction form: $\frac{4}{10} \times \frac{2}{10} = \frac{8}{100} = 0.08$.
- Students could make horrible mistakes if they just follow a recipe or method.
  - This can especially happen when they use the shorter method to convert a number from a base 10 number to another base.
  - Students should always fall back to common sense and use the method where the number is written by expanding into powers.
- When students convert a number from the base ten to another base, for example to the base 8, and they use reading the remainders upwards, they can in the end do this by rote without understanding what they are doing.
- Example 16 (b) can also be explained as follows:

8	2077	
8	259 + rem. 5	$8^1 \times 259 + \mathbf{5} \times \mathbf{8^0}$
8	32 + rem. 3	$8^2 \times 32 + \mathbf{3} \times \mathbf{8^1} + \mathbf{5} \times \mathbf{8^0}$
8	4 + rem. 0	$8^3 \times 4 + \mathbf{0} \times \mathbf{8^2} + \mathbf{3} \times \mathbf{8^1} + \mathbf{5} \times \mathbf{8^0}$
	Number to the base 8:	4035

- Students may find Example 18(c) difficult. The full explanation is:

314	$4 \times 4 = 16 = 3 \times 5 + 1$. So write 1 and carry 3.
$\times$ 24	$4 \times 1 + 3$ (carried) $= 7 = 1 \times 5 + 2$. So write 2 and carry 1.
2321	$4 \times 3 + 1$ (carried) $= 13 = 2 \times 5 + 3$. So write 23.
<u>11330</u>	$2 \times 4 = 8 = 1 \times 5 + 3$. So write 3 and carry 1.
<u>14201</u>	$2 \times 1 + 1$ (carried) $= 3$. So write 3. $2 \times 3 = 6 = 1 \times 5 + 1$. So write 11.

Supplementary worked examples

- $0.99 \div \frac{11}{12} = 0.99 \times \frac{12}{11} = 0.09 \times 12 = 1.08$

  $0.056 \div \frac{8}{9} = 0.056 \times \frac{9}{8} = 0.007 \times 9 = 0.063$

Chapter 4 Algebraic processes 1: Simplification and substitution

Learning objectives

By the end of this chapter, the students should be able to:

1. Use letters of the alphabet to represent numbers.
2. Simplify algebraic expressions by grouping like terms, by removing brackets, by expanding brackets.
3. Add and subtract algebraic fractions.
4. Factorise algebraic expressions by determining common factors and by grouping terms.
5. Substitute values into given algebraic expressions.

Teaching and learning materials

Students: Textbook, exercise book and writing materials.

Teacher: Factorisation charts.

Glossary of terms

Terms are separated by + and – signs. An expression, such as $-3x + 7x \div 2 + (2x + 3)$ has three terms, namely, $-3x$, $7x \div 2$ and $(2x + 3)$. Brackets group terms as one term.
Terms are alike, if they have precisely the same letters with the same exponents. If this is not the case, the terms are seen as unlike.
Example of like terms: $3a$ and $4a$, $3x^2y^3$and $2y^3x^2$ and unlike terms are $3x^2y^3$ and $3x^3y^2$, $3x$ and $3y$.

Perfect square in algebra means that an algebraic expression can be written as a bracket squared, for example $a^2 + 2ab + b^2$ can be written as $(a + b)^2$ or $a^2 - 2ab + b^2$. can be written as $(a - b)^2$.

Evaluate is another word for "work out the number value"

Teaching notes

Simplification

- You as the teacher should realise that the word "simplify" means that an algebraic expression is written in a different form.
  This other form is simpler or easier to use in certain other situations.
  If you want to substitute x with –2, for example, you would choose to write
  $9x - 5x - 8x + 10x$ as $6x$.
  It is much easier to substitute x with –2 in the expression $6x$, than in the expression $9x - 5x - 8x + 10x$.
  So, this expression is simpler for the purpose of substitution.
- You could teach students to add and subtract like terms by grouping terms of the same kind (like terms) together. Some students would not need to group terms like this. This method actually makes the work easier for those students who find algebra difficult.
- When you teach students to add like terms and that unlike terms cannot be added, beware of using words for the letter symbols.
  An example would be where a teacher looks at the expression $3a + 2b$ and says that you cannot add 3 apples to 2 bananas.
  It is absolutely wrong to reason like this, because these letter symbols are variables, which means that they represent numbers and not objects.
  It would be correct, however, if you say that we do not know whether a stands for 2 and b for 3. If we knew that we could work out the answer, but as it is written here it is impossible to write the expression $3a + 2b$ in another form.
- When brackets are removed, emphasise that students multiply each term inside the bracket by the number or expression directly in front of the bracket (there is no + or – sign between the number and the bracket) and that the sign of this

number is included when the bracket is removed by multiplication. Brackets are also removed from the inside brackets to the outside brackets.

- Expansion of brackets means that each term in the one bracket is multiplied by each term in the other bracket. This again is writing an algebraic expression in another form.
  Sometimes the factorised form of an expression is its simplest form especially, if we want to find a common denominator.
  Sometimes the expanded form is the simpler form especially, if we want to find where graph intersects the y-axis.
- Emphasise that we can write fractions on the same denominator by multiplying the numerator and the denominator of each fraction by the same number, namely, the number of times that the denominator divides into the LCM of the denominators. If we multiply the top and the bottom of a fraction by the same number, we do not change the fraction because we actually multiply by 1.
- Make sure that the students understand that the line of a fraction with a numerator with more than 1 term, acts like a bracket and that they use brackets to avoid mistakes as shown in Example 12. Insist that they use brackets.
- Remember in a fraction like $\frac{-x+18}{12x}$, $x \neq 0$, because one cannot divide by 0. You can explain this to your students as follows:
  We know that $\frac{8}{4} = 2$, because $4 \times 2 = 8$.
  We know that $\frac{0}{4} = 0$, because $4 \times 0 = 0$.
  Now if $\frac{8}{0}$ = a number, then 0 × 'that number' must be equal to 8 and that is impossible.
  So, division by 0 is not defined.
- Before any other factorisation can take place, the common factor must be taken out. Emphasise that the common factor is the HCF of each term of the algebraic expression.
- When you have to group terms together to find a common factor that is a bracket, do not put brackets around the terms of which you want to take out a common factor, because the + or negative signs in the brackets may then be wrong. Emphasise that, if the negative of the common factor is taken out, the signs of the terms in brackets also change to the opposite signs.
- When taking out −1 to change signs, remember that: $(x - y)^2 = (y - x)^2$.
  Let $x = 5$ and $y = 2$:
  $(5 - 2)^2 = (3)^2 = 9$ and $(2 - 5)^2 = (-3)^2 = 9$
  $-a + b = -(a - b)$.
  Let $a = 7$ and $b = 5$:
  $-7 + 5 = -2$ and $-(7 - 2) = -(5) = -5$
  $-a - b = -(a + b)$.
  Let $a = 10$ and $b = 7$:
  $-10 - 7 = -17$ and $-(10 + 7) = -(17) = -17$

Common mistakes and Areas of difficulty

- Remember that:
  If you have a length of $7 - \frac{1}{2}x$ and the length is measured in cm, you cannot write $7 - \frac{1}{2}x$ cm.
  You have to use brackets, because the whole expression gives a length in cm.
  You, therefore, write $(7 - \frac{1}{2}x)$ cm.
  If the answer was $\frac{14-x}{2}$, then, because this is one term, we can write $\frac{14-x}{2}$ cm.
- When removing brackets by multiplication, it should be remembered that one multiplies all the terms inside the brackets with the number **and** its negative sign, for example.
  If there is only a negative sign in front of the brackets, one multiplies all the terms by −1.
  For example:
  $3 - a(a - \overline{5 - 6a})$
  $= 3 - a[a + (-1)(5) + (-1)(-6a)]$
  $= 3 - a(a - 5 + 6a)$
  $= 3 - a(a) - 5(-a) + (-a)(6a)$
  $= 3 - a^2 + 5a - 6a^2$
  $= 3 + 5a - 7a^2$
- $(2x - 5y)^2 \neq 4x^2 + 25y^2$. To avoid this mistake, we always write $(2x - 5y)^2$ as $(2x - 5y)(2x - 5y)$ and then multiply each term in the first bracket with each term in the second bracket.
- When adding fractions, students should remember that the line underneath a numerator with more than one term also acts like a bracket:
  $\frac{4x+3}{4x} - \frac{x-3}{12x} - \frac{3x-2}{3x}$
  $\frac{3(4x+3)}{3 \times 4x} - \frac{1(x-3)}{12x \times 1} - \frac{4(3x-2)}{4 \times 3x}$
  (The principle here is that, if one multiplies the numerator and denominator by the same number, one actually multiplies by 1 and then the fraction stays unchanged)
  $= \frac{3(4x+3) - 1(x-3) - 4(3x-2)}{12x}$ (brackets are used to avoid mistakes)
  $= \frac{12x + 9 - x + 3 - 12x + 8}{12x}$
  $= \frac{-x+20}{12x}$, where $x \neq 0$.

- When grouping terms to take out a common factor of each pair of terms, it is wise **not to use** brackets to show which two terms you are grouping together.
  You can make mistakes with signs, for example: $cd - ce - d^2 + de \neq (cd - ce) - (d^2 + de)$.
  The final bracket is not correct, because $(-1)(de) = -de$, and in the original expression we had $a + de$.
  To avoid this kind of mistake, brackets should only be used when the common factor is actually taken out: $c(d - e) - d(d - e) = (d - e)(c - d)$.
- When substituting letter symbols by numbers, use brackets.
  If it is stated that $p = -2$ and that the value of $p^2 - 2p - 3$ must be evaluated, it should be written as $(-2)^2 - 2(-2) - 3$.
  The common mistake here is that students tend to write $-2^2 + 4 - 3 = 4 + 4 - 3 = 5$.
  This actually is the correct answer, but -2^2 is equal to the negative of 2 squared, and $(-2)^2$ is equal to negative 2 squared.
  The first answer is -4 and the second answer is 4.
- Note that the use of brackets for substitution is essential to avoid unnecessary errors.
  A good habit when doing substitution is to immediately write brackets where the letters were, for example: $\frac{y^2 - 1}{x - z} = \frac{(-5)^2 - 1}{(4) - (-2)} = \frac{25 - 1}{4 + 2} = \frac{24}{6} = 4$.
  Sometimes the brackets are strictly spoken not necessary, but let the students still use brackets to get them in the habit of using brackets when doing substitution. It prevents a lot of mistakes later.

Supplementary worked examples

Examples like these could precede the examples, where terms have been grouped to get brackets that are common terms.

Factorise these:

1. $x(a - b) + (a - b)$
2. $k(a - b) + n(b - a)$
3. $(a - b)^2 + a - b$
4. $(a - b)^2 - a + b$
5. $(b - a)^2 + a - b$
6. $x(a + b)^2 - b - a$
7. $x(a - b)^2 + 2b - 2a$
8. $x(a - b)^2 - 2b + 2a$
9. $2x(2k - 1) + 4k - 2$
10. $5t(3p - k) - 9p + 3\text{k}$

Solutions

1. $(a - b)(x + 1)$
2. $k(a - b) - n(a - b) = (a - b)(k - n)$
3. $(a - b)(a - b) + (a - b) = (a - b)(a - b + 1)$
4. $(a - b)(a - b) - (a - b) = (a - b)(a - b - 1)$
5. $(a - b)^2 + (a - b) = (a - b)(a - b + 1)$
6. $x(a + b)^2 - (a + b) = (a + b)(x(a + b) - 1)$
   $= (a + b)(ax + bx - 1)$
7. $x(a - b)^2 - 2(a - b) = (a - b)(x(a - b) - 2)$
   $= (a - b)(ax - bx - 2)$
8. $x(a - b)^2 - 2b + 2a = x(a - b)^2 + 2a - 2b$
   $= x(a - b)^2 + 2(a - b)$
   $= (a - b)(ax - bx + 2)$
9. $2x(2k - 1) + 2(2k - 1) = (2k - 1)(2x + 2)$
   $= 2(2k - 1)(x + 1)$
10. $5t(3p - k) - 3(3p - k) = (3p - k)(5t - 3)$

Chapter 5 Sets 1

Learning objectives

By the end of this chapter, the students should be able to:

1. Use various ways of writing and describing sets in terms of their members or elements.
2. Identify types of sets (including equal sets, the universal set, the empty or null set, finite, infinite, subsets, disjoint).
3. Draw simple Venn diagrams to illustrate sets.
4. Determine and write the number of elements in a set.
5. Find the union and intersection of sets.

Teaching and learning materials

Students: Textbook, exercise book, writing materials and mathematical sets.

Teacher: Instruments, furniture, books, clothes, loops of string to make Venn diagrams, charts showing Venn diagrams.

Glossary of terms

Elements of a set are the members of the set. The symbol $\in$ is used for element. The number of elements of a set, A, for example, is called the cardinal number of A and written as $n(A)$.

Subset: If A is a **subset** of B, then A is part of B. A contains some or all of the elements of B and no elements that do not appear in B. We write **A ⊆ B**.

Proper subset: If A is a **proper subset** of B, then A is a subset that consists of at least one, but not all, of the elements of B. We write $A \subset B$.

Union of sets The elements of the sets are put together without repeating any element. If A and B are united, we write $A \cup B$. Then **A ∪ B** will contain all the elements that are in **A or B**.

Intersection of sets: The intersection of sets is the elements that these sets have in common. If the intersection of A and B is determined, we write **A ∩ B** and this is the set that contains all the elements that are in **A and B**.

Disjoint: If $A \cap B = \varnothing$ (that is they have no elements in common), we say that sets A and B are disjoint.

Finite set is a set with a last element.

Infinite set is a set with an infinite number of elements and, therefore, has no last element.

Teaching notes

- A well-defined set clearly indicates what is an element of the set and what is not.
  A set identified as the set of all people taller than 2 m in Freetown is well defined. The reason is that its description cannot be interpreted differently.
  On the other hand, if a set is identified as the set of tall people in Freetown, it is open to interpretation because different people could have different ideas of when a person is tall.
- Two sets are **equal**, if they have exactly the same elements. We use the symbol = .
- A set that contains no elements is called an empty or null set, and is written as { } or $\varnothing$.
- An empty set is always a subset of any set, because a subset has no element that is not an element of any other set.

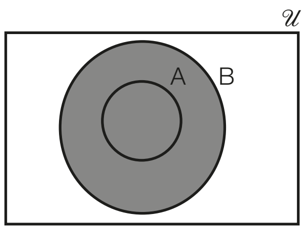
- The number of subsets of any set is 2^n.
  Exercise 5d 4(a):

Number of elements in a set (n)	Example of set	Subsets	Number of subsets
0	{ }	{ }	$1 = 2^0$
1	{a}	{ }, {a}	$2 = 2^1$
2	{a, b}	{ }, {a}, {b}, {a, b}	$4 = 2^2$
3	{a, b, c}	{ }, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}	$8 = 2^3$
4	{a, b, c, d}	{ }, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, c, d}, {a, b, d}, {b, c, d}, {a, b, c, d}	$16 = 2^4$
n			2^n

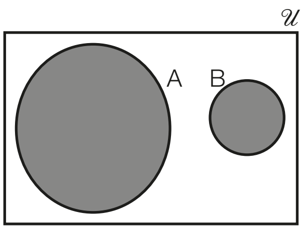
The number of proper subsets is $2^n - 1$, because the set itself must be left out.

- There are three cases where we can find the union and intersections of sets. They can be represented as follows:
  - In the figures below, the shaded part shows the union of the two sets:

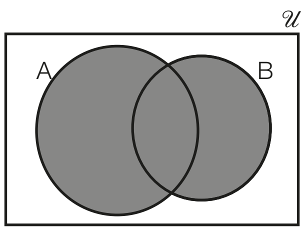
A ⊂ B:



A and B are disjoint or
$A \cap B = \varnothing$

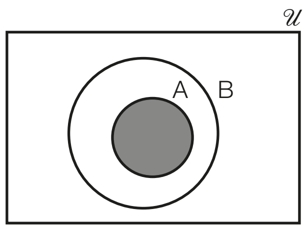


A and B are not disjoint or
$A \cap B \neq \varnothing$

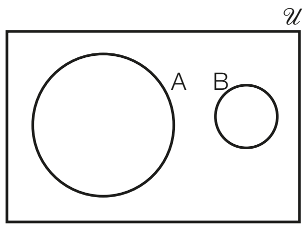


  - In the figures below, the shaded part shows the intersection of the two sets.

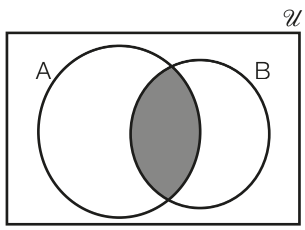
A ⊂ B:



A and B are disjoint or
$A \cap B = \varnothing$



A and B are not disjoint or
$A \cap B \neq \varnothing$



Areas of difficulty

- Students sometimes struggle to give a common defining property for a set. This only improves with practice and good guidance.
- Students sometimes struggle to identify sets that are not well defined. Teach them that, if there is some property that can be debated or interpreted differently, then the set is not well defined.
- Sometimes students do not know where to start if they have to represent sets in Venn diagrams.
  - Teach them to first list the elements of the sets.
  - Then they should look whether sets have elements in common and draw the Venn diagrams to overlap.
  - Or if the whole of one set is part of the other set, they should draw the one set inside the other one.

Chapter 6 Algebraic processes 2: Equations and formulae

Learning objectives

By the end of this chapter, the students should be able to:

1. Solve linear equations.
2. Solve linear equations that contain brackets and/or fractions.
3. Create and solve equations from word problems.
4. Substitute given values into a formula.
5. Change the subject of a formula.

Teaching and learning materials

Students: Textbook, exercise book and writing materials.

Teacher: Wall charts of equations.

Glossary of terms

Solve an equation means that the value of the unknown is found that will make the two sides of the equation equal to each other. This solution is also called the root of the equation.

Teaching notes

- Explain to the class what an equation is and what it means to solve an equation as explained in the textbook. You could also first start with equations which are solved by inspection. That means that you do not have to use the balance method. You deduce the answer by logical reasoning.
- It is very important that the balance method is again consciously taught as shown in the textbook. Do not tell the students that terms are "taken over".
  An equals sign (=) does not work like magic to change all the signs to opposite signs on the other side of the equals (=) sign.
  There is a logical reason why the signs change. If students do not understand this, they may later experience problems when working with letter symbols only when they change the subject of a formula.
- When testing whether the solution of an equation is correct, emphasise that students treat the two sides of the equations separately:
  LHS = …
  RHS = …
  ∴ LHS = RHS
  ∴ $x = 3$ (for example) is correct/is the root/is the solution.
- When students solve equations with brackets, emphasise that they multiply with the number and its sign that is written directly in front of the brackets, and that they have to multiply this number (with its sign) with all the terms inside the bracket.
- Explain how to solve equations with fractions by doing Examples 3–5 strictly using the three steps as set out in the textbook.
  It is very important that you emphasise the use of brackets to clear fractions where the numerators consist of more than one term.
  If there was a negative sign in front of the term and brackets are not used, the negative sign only influences the first term of the numerator.
- These steps are followed when solving equations:
  a) Clear all fractions by multiplying each term (even if it is not necessary) or both sides of the equation, by the LCM of the denominators of the fraction.
  b) Remove all brackets by multiplication.
  c) Add all the like terms on both sides of the equation.
  d) Add a term with a sign opposite to the term that you want on the other side of the equation. (You always want the terms with the variable on the left-hand side and all the other terms on the right-hand side of the equals (=) sign.)

e) Add all the like terms on both sides of the equation
f) Multiply both sides by the reciprocal of the coefficient of the unknown.

- When doing **Example 6** (which is about the speed, distance and time for walking or cycling), you could use an alternative method by using a table to summarise all the facts given. If you complete the table, it will look like this:

	Distance	Time	Speed
Walking	x	2 h	$2x$
Cycling	$3x$	4 h	$12x$

$2x + 12x = 77$, and so on.

- When doing **Example 7** (which is about the ages of Jumoke and Amina now and in 11 years' time), you could also use a table to fill in the facts:

	Age now	Age in 5 years
Jumoke	$x + 11$	$x + 16$
Amina	x	$x + 5$

$x + 16 = 2(x + 5)$, and so on.

- When doing **Example 8** (which has to do with the amount to buy the oranges and the amount at which they are sold), you could also use a table to fill in the facts:

	Number of oranges	Ratio	Total amount
Buys	n	$\frac{n}{5}$	$\frac{n}{5}(90) = 18n$
Sells	$n - 8$	$\frac{n-8}{4}$	$\frac{n-8}{4}(120) = 30n - 240$

$30n - 240 - 18n = 900$, and so on.
In all these problems, two situations are compared and that gives the equation to solve. The unknown in the equation is always that which they ask you to determine.

- We use formulae daily to calculate certain values. Mistakes can be prevented, when brackets first replace letter symbols and then the number values of the letter symbols are filled in.
- These steps can be followed, if we want to change the subject of the formula:
  1. Clear fractions by multiplying by the LCM of the denominators.
  2. If there are square roots, square both sides of the equation.
  3. Remove brackets by multiplication.
  4. Add and subtract the same quantities both sides of the formula to get the subject of the formula to the left-hand side of the formula.
  5. Multiply both sides of the formula by the reciprocal of the coefficient of the subject to get 1 × 'the subject of the formula'.
  6. If more than one term has the letter that you want to make the subject of the formula, take the letter out as a common factor and then divide both sides of the equation by the bracket.

Here students must be very sure of how to use the balance method. If they are used to "taking something over", they will now make the most terrible mistakes, because they will become totally confused when faced with letter symbols only.

Areas of difficulty and common mistakes

- When removing brackets as in $3(4c - 7) - (4c - 1) = 0$, students tend to forget to multiply the second term in the bracket, also by the -1 in front of the bracket. Emphasise that what is directly in front of the brackets, applies to all the terms inside the bracket.
- When solving equations with fractions where the numerator consists of more than one term, you must emphasise that this step **must not be skipped**:
  $15\left(\frac{7x+2}{3}\right) - 15\left(\frac{9x-2}{5}\right) = 2(15)$
  $\mathbf{5(7x + 2) - 3(9x - 2) = 30}$ **(Do not leave this step out!)**
  The reason is that in the 2nd term (in this example), the student multiplies the $9x$ by -3 and -2 either stays the same, or they write -6 instead of $+6$, because -2 is multiplied by 3 only and not $-\mathbf{3}$.
- When there is a whole number and both sides of the equations are multiplied by the LCM of the denominators, some students forget that this whole number (although it is not a fraction) must also be multiplied by the LCM of the denominators.
  Emphasise that each term both sides of the equation must be multiplied by the LCM of the denominators.
  Each term is treated the same.
- When testing whether some answer is the root of the equation, some students tend to work out the two sides of the equation simultaneously. The correct way is to start with one side for example the LHS and work it out.

Then the RHS is worked out.
Afterwards, a conclusion is drawn when one says ∴ LHS = RHS and ∴ 3 is the root of the equation (for example).

- Prevent mistakes by using brackets when substituting the letter symbols of a formula by numbers. For example:
If $y = 2x^2 - 5x - 3$, find the value of y if $x = -1$.
$y = 2(-1)^2 - 5(-1) - 3$
$= 2(1) + 5 - 3$
$= 2 + 5 - 3$
$= 4$.
If you write $y = 2 \times -1^2$ here, the answer should be $2 \times -1 = -2$. Remember the difference between -3^2 and $(-3)^2$. If you work it out:
$-3^2 = -(3 \times 3) = -9$ and $(-3)^2 = (-3) \times (-3) = 9$.
Sometimes it does not seem necessary to use brackets, but students. In this case, then you should teach them to always use brackets even if using brackets is not necessary.
- Students tend to treat the addition and subtraction of algebraic fractions the same as when solving equations with fractions.
It is very important that they realise that when they add or subtract algebraic fractions, they actually work with algebraic expressions.
When they solve equations with fractions it is a completely different matter. The differences are shown below:

Adding algebraic fractions	**Solving equations**
$\frac{3x-4}{4} - \frac{3x-2}{2} = \frac{1(3x-4) - 2(3x-2)}{4}$ $= \frac{3x - 4 - 6x + 4}{4}$ $= \frac{-3x}{4}$	$\frac{3x-4}{4} - \frac{3x-2}{2} = 2$ $1(3x-4) - 2(3x-2) = 4 \times 2$ $3x - 4 - 6x + 4 = 8$ $-3x = 8$ $x = -\frac{8}{3}$

Algebraic fraction (algebraic expression)	**Equation**
Has only one side.	Has two sides (left-hand side and right-hand side)
Is added by putting the whole expression on a common denominator, which is the LCM of all the denominators	Both sides can be multiplied by the LCM of the denominators to remove the fraction and to make it easier to solve the equation.
Ends with one fraction, for example, $\frac{-3x}{4}$.	Ends with a solution, for example, $x = 2$.

- When students test whether their solution(s) of an equation is correct, they should substitute these root(s) in the original equation.
They then tend to work with the two sides of the equation simultaneously. This can lead to mathematical errors.
Emphasise and insist that they substitute the root of the equation in its left-hand side and work out the answer.
Then they should substitute the root in the right-hand side of the equation and work out the answer.
If the two answers are equal, they should then write: ∴ LHS = RHS ∴ $x = 3$ (for example), is the root of the equation. For example, the equation $\frac{x-5}{3} - \frac{9-x}{2} = \frac{x+4}{5}$ was solved and $x = 11$. To test whether this answer is correct students should write:
$\text{LHS} = \frac{11-5}{3} - \frac{9-11}{2} = \frac{6}{3} - \frac{(-2)}{2} = 2 + 1 = 3$
$\text{RHS} = \frac{x+4}{5} = \frac{11+4}{5} = \frac{15}{5} = 3$
∴ LHS = RHS
∴ 11 is a root of the equation.
- When students are asked to change the subject of the formula $P = aW + B$ to W, for example, they tend to do the following:
$aW = P - B$
∴ $W = P - B - a$. The students do not know how the balance method works.
They were "taking over" without knowing what they were doing. *If this happens, go back and again explain how the balance method works.*

Supplementary worked examples

Without showing any written work, write down the values of x that will make these equations true:

1. $5 - x = 7$
2. $4x - 3 = -5$
3. $x^2 = 16$
4. $x + 3 = x + 2$
5. $2x + 6 = 2(x + 3)$
6. $\frac{x-1}{x-3} = 0$
7. $\frac{x-1}{x-3} = 1$
8. $5 + x^2 = 9$
9. $\frac{2x+6}{x+3} = 2$
10. $2^x = 16$
11. $x \times x = x + x$
12. $3x = 5x$
13. $(x + 1)^2 = 9$
14. $5 - 3x = 11$

Solutions

1. $5 - (-2) = 7$
   $\therefore x = -2$
2. $-2 - 3 = -5$
   $\therefore 4x = -2$
   $4(-\frac{1}{2}) = -2$
   $\therefore x = -\frac{1}{2}$
3. $x = 4$ or $x = -4$, because $(4)^2 = 16$ and $(-4)^2 = 16$.
4. No solution, because 'a number' + 3 can never be equal to 'that same number' + 2.
5. $2x + 6 = 2x + 6$. This is true for all $x \in \mathbb{R}$. This kind of equation is called an identity and its solution is always $x \in \mathbb{R}$.
6. We know that $\frac{0}{\text{any number}} = 0$.
   $\therefore x - 1 = 0$
   $\therefore x = 1$
7. We know that $\frac{\text{a number}}{\text{the same number}} = 1$.
   $\therefore x - 1 = x - 3$.
   This is impossible. So, there is no solution.
8. $5 + 4 = 9$
   $x^2 = 4$
   $x = 2$ or $x = -2$
9. $\frac{2x+6}{x+3} = 2$
   $\therefore 2 = 2$.
   So, x can be any value except a value (-3) that causes us to divide by 0.
   $\therefore x \in \mathbb{R}, x \neq -3$
10. $2^4 = 16$
    $\therefore x = 4$
11. $0 \times 0 = 0 + 0$ and $2 \times 2 = 2 + 2$
    $\therefore x = 0$ or $x = 2$
12. $3(0) = 5(0)$, because $0 = 0$
    $\therefore x = 0$
13. $x + 1 = 3$ or $x + 1 = -3$
    $x = 2$ or $x = -4$
14. $5 - (-6) = 11$
    $\therefore 3x = -6$
    $\therefore x = -2$

Chapter 7 Algebraic processes (3): Linear and quadratic graphs

Learning objectives

By the end of this chapter, the students should be able to:

1. Make tables of values for given linear functions and use them to draw straight-line graphs on the Cartesian plane.
2. Read off values from straight-line graphs.
3. Construct tables of values for given quadratic functions.
4. Use tables of values to draw the graphs of quadratic functions on the Cartesian plane.
5. Draw a smooth parabolic curve through the plotted points.
6. Read off values from the graphs of quadratic functions (including maximum and minimum values).

Teaching and learning materials

Students: Textbook, exercise book, pencil and ruler, graph paper or graph exercise book.
Teacher: Graph board, graph paper for class distribution. If an overhead projector is available: transparencies of graph paper and transparency pens.

Teaching notes

- Explain to the class what a function is by telling them that there are the following elements in a function:
  - An independent variable (usually x) and that its value is the input value.
  - A dependent variable (usually y), because its value depends on the value of the independent variable. Its value is also called the output value.
  - A formula, such as $y = 2x + 3$ that changes all the values of the independent variable (x), into the y-values that depend on the values of x.
  - For every value of the independent variable (usually x), there is only one value of the dependent variable (usually y).
  - Examples of formulae which define functions: $y = 2x + 3$ or $y = 3x^2 - 2x - 5$ because for each value of the independent variable (x), there is only one value of the dependent variable (y).
  - An example of a formula which does not represent a function is $y = \pm\sqrt{x}$, because, if $x = 9$ (for example), then $y = 3$ or -3. So, for each value of the independent variable, there are two values of the dependent variable.
- Make the scale large enough when graphs are drawn, because it makes it easier to read accurate values from the graph.
  Take into consideration what values of x and y are given.
  If you have to read the value of y, if $x = 5.5$ (for example), it would be advisable that the scale of the graph is such that it is easy to find the position of $x = 5.5$.
- A function can also be written as $f(x) = 2x + 3$. Then the function is called f, and $f(x)$ represents all the y-values for certain x-values.
  So, $f(2)$ means that x is substituted by 2.
  Therefore, $f(2) = 2(2) + 3 = 7$.
  So, $f(2)$ can also be read as the y-value, if $x = 2$.
- A graph is a mathematical picture of the function and shows all the values of the independent and dependent variables that satisfy the specific formula that defines the graph.

Areas of difficulty and common mistakes

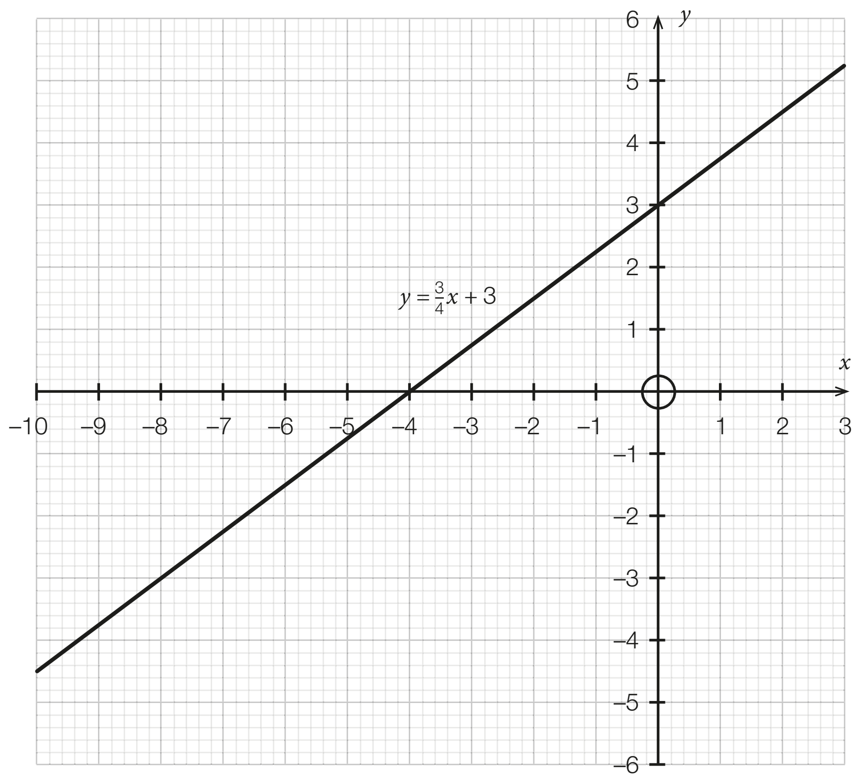
- Some students find it difficult to read answers from graphs. If it is, however, explained very carefully, it should not be a problem.
  The equation that defines the graph is $y = 2x^2 + 5x + 2$, for example:
  - If you want to find the **value of *x*,** when y is a certain value, you draw a horizontal line from that value on the y-axis until it intersects the graph. Then you draw a vertical line from that point on the graph until the line intersects the x-axis and you read the **answer** from the ***x*-axis**

- If you want to find the value of $2x^2 + 5x + 2$ in any way, you look towards the y-axis for your answers.
- If want to find for which x-values $2x^2 + 5x + 2 > 0$, for example, you read off the x-values of the graph for which the y-axis is positive.
- The same is true for $2x^2 + 5x + 2 < 0$, where you read the x-values of the graph for which the y-axis is negative.
- If $2x^2 + 5x + 2 = 0$, you read the x-values of the graph for which $y = 0$ (that is, where the graph crosses the x-axis).

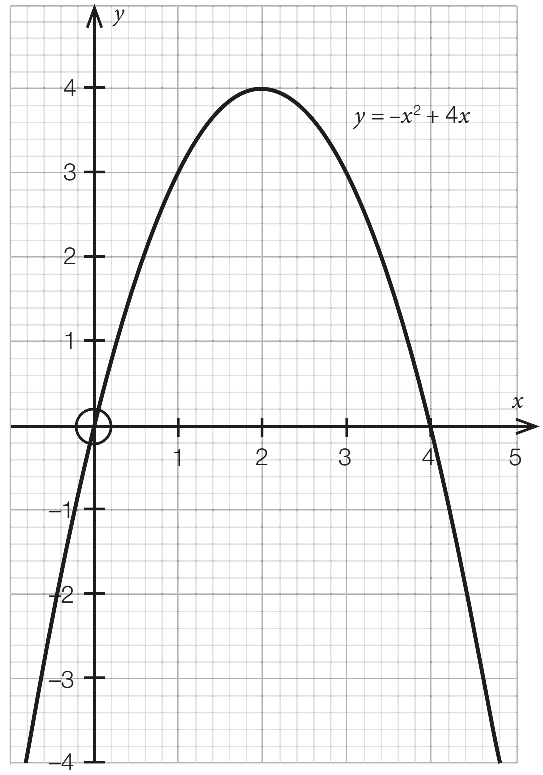
- Some students may find it difficult to work out the correct y-values for their tables. Teach them to write their work out in their exercise books to prevent mistakes.

Supplementary worked examples

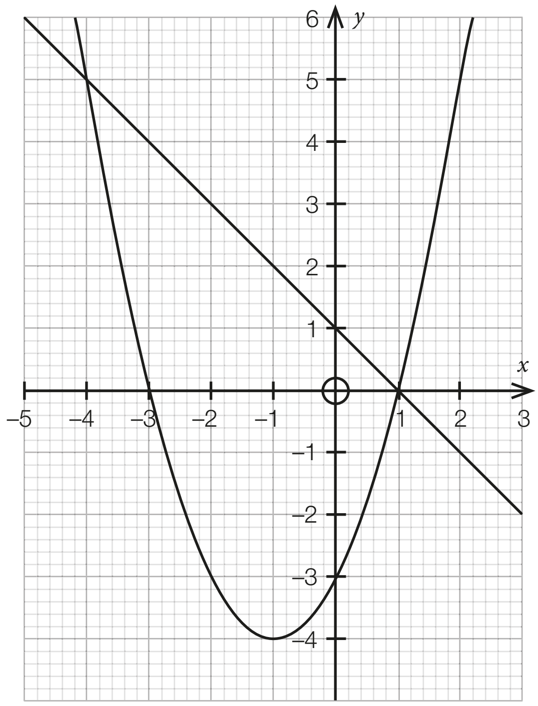
1. a) Write down values of x, if:
   i) $\frac{3}{4}x + 3 = 0$
   ii) $\frac{3}{4}x + 3 < 0$
   iii) $\frac{3}{4}x + 3 > 0$
   b) Write down the value of y, if $x = 0$.



2. a) Write down the values of x, if:
   i) $-x^2 + 4x = 0$
   ii) $-x^2 + 4x \geq 0$
   iii) $-x^2 + 4x < 0$
   b) Write down the value of y, if $x = 0$.



3. This parabola is defined by $y = x^2 + 2x - 3$, and the straight-line graph is defined by $y = -x + 1$.
For which values of x is $x^2 + 2x - 3 = -x + 1$?



Solutions

1. a) i) $\frac{3}{4}x + 3 = 0$, where $y = 0$ (where the graph intersects the x-axis).
Answer: $x = -4$.
You will get the same answer if you solve the equation algebraically.
ii) $\frac{3}{4}x + 3 < 0$, where the graph lies below the x-axis. The reason is that below the x-axis, all the y-values are negative.
Answer: $x < -4$
iii) $\frac{3}{4}x + 3 > 0$, where the graph lies above the x-axis. The reason is that above the x-axis, all the y-values are positive.
Answer: $x > -4$
b) On the y-axis, all the x-values are 0. The graph intersects the y-axis at 3.
Answer: $y = 3$.
2. a) i) $-x^2 + 4x = 0$, where $y = 0$ (where the graph intersects the x-axis).
Answer: $x = 0$ or $x = 4$
ii) $-x^2 + 4x \geq 0$, where the graph lies above the x-axis, because that is where $y > 0$.
Answer: $0 \leq x \leq 4$
iii) $-x^2 + 4x < 0$, where the graph lies below the x-axis, because that is where $y < 0$.
Answer: $x < 0$ or $x > 4$
b) $x = 0$, where the graph intersects the y-axis, because on the y-axis all the x-coordinates are equal to 0.
Answer: $y = 0$
3. $x^2 + 2x - 3 = -x + 1$, where the two graphs intersect each other, because the two y-values of the two graphs are equal at the points where they intersect.
Since the equation has x as variable, we read off the values of x at the points where the two graphs intersect.
Answer: $x = -4$ or $x = 1$.

Chapter 8 Sets 2: Practical applications

Learning objectives

By the end of this chapter, the students should be able to:

1. Recall and use set notation and language.
2. Identify the complement of a set.
3. Write and interpret sets of values using set-builder notation.
4. Use universal sets, complements and disjoint sets to solve practical problems.
5. Use Venn diagrams, with up to three intersecting sets, to solve practical problems.

Teaching and learning materials

Students: Textbook, exercise book, writing utensils.
Teacher: Loops of string to make Venn diagrams, charts showing Venn diagrams.

Glossary of terms

Complement of a set contains all the elements of the universal set that are not in the set. We write the complement of A as A'.

Teaching notes

- The set of rational numbers can also be written as $Q = \{\frac{a}{b}: a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0\}$, because, as explained earlier, division by 0 is not defined.
- When students solve practical problems by using Venn diagrams, let them make the quantity they want to find, equal to x. Then let them fill in the remainder of the Venn diagrams in terms of x.
- If there is not a specific quantity that has to be found (Example 9), let the students start with the quantity which is the intersection of all the sets and then complete the remainder of the Venn diagrams according to this value.

Areas of difficulty and common mistakes

- Some students may find it difficult to correctly draw Venn diagrams. Let them first list the elements if it is possible.
- Students find it difficult to write a set in set-builder notation. Let them practise this over and over if they find it difficult.

Chapter 9 Logical reasoning: Simple and compound statements

Learning objectives

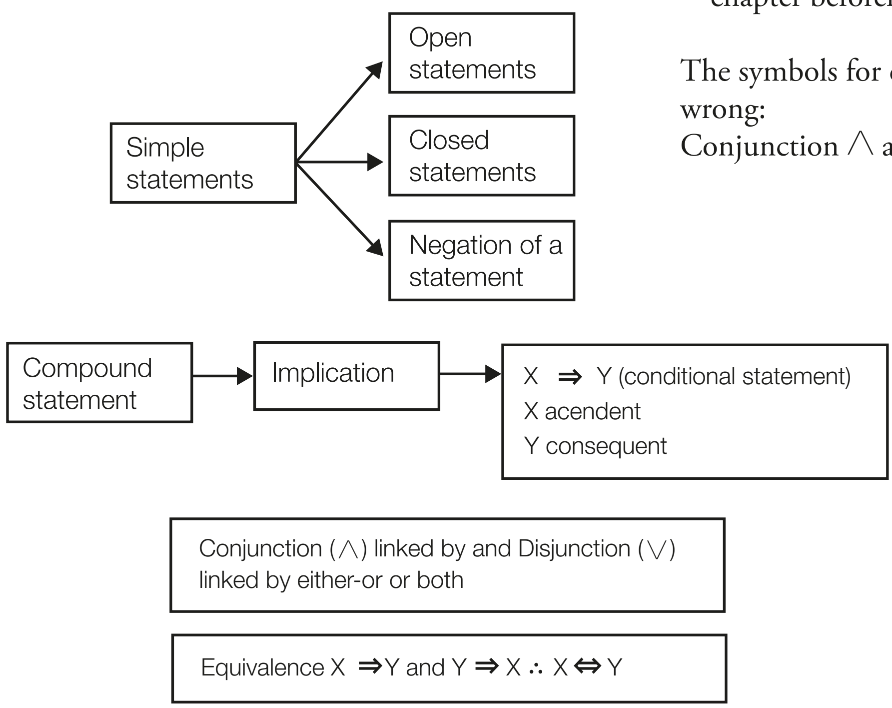
By the end of this chapter, the students should be able to:
1. Identify and form open and closed simple statements.
2. Deduce the truth or otherwise of simple statements.
3. Form the negation of a simple statement.
4. Distinguish between simple and compound statements.
5. Draw conclusions from a given implication.
6. Distinguish between conjunction and disjunction, representing them on truth tables.
7. Recognise equivalent statements and apply them to arguments.
8. Recognise and use the symbols for negation, conjunction, disjunction, implication and equivalence.
9. Use Venn diagrams to demonstrate connections between statements.

Teaching and learning materials

Students: Textbook, exercise book, writing utensils.
Teacher: Wall charts showing truth tables, loops of string for Venn diagrams.

Teaching notes

The structure of this chapter is:



Areas of difficulty

- This whole chapter has the potential to be difficult for the students. They may tend to do the work by rote by just following the examples.
- Let them work in groups and encourage them to argue about the problems.
- You could also show them the structure of the chapter beforehand.

The symbols for conjunction and disjunction are wrong:
Conjunction $\wedge$ and Disjunction $\vee$

Chapter 10 Algebraic processes 4: Quadratic equations

Learning objectives

By the end of this chapter, the students should be able to:

1. Factorise quadratic expressions.
2. Solve quadratic equations using the principle that, if $a \times b = 0$, then either $a = 0$ or $b = 0$ or both a and b are 0.
3. Use factorisation to solve quadratic equations.
4. Form a quadratic equation, given its roots.
5. Use graphical method to solve quadratic equations.
6. For a quadratic equation from a given graph.

Teaching and learning materials

Students: Textbook, exercise book, pencil and ruler, graph paper or graph exercise book.
Teacher: Graph board, graph paper for class distribution. If an overhead projector is available: transparencies of graph paper and transparency pens.

Teaching notes

- Teach the students that before they factorise a quadratic expression, they must always first determine whether there is a common factor and take it out. Sometimes this can make factorisation much easier.
- If you explain how quadratic expressions are factorised when the coefficient of the x^2-term is 1, you should mention:
  - If the sign of the last term is positive, the two brackets have the same sign, namely, the sign in front of the middle term:
    - $x^2 + bx + c$ (both brackets have positive signs)
    - $x^2 - bx + c$ (both brackets have negative signs)
  - If the sign of the last term is negative, the two brackets have opposite signs. The largest product will have the sign of the middle term
    - $x^2 + bx - c$ (the largest product will be positive)
    - $x^2 - bx - c$ (the largest product will be negative)
- When solving quadratic equations by factorisation, emphasise that we use this principle:
  If $a \times b = 0$, it means that $a = 0$ or $b = 0$ or both are equal to 0.
  So, the right-hand side of a quadratic equation must always be equal to 0 and must be written in factor form, which in this case is the simplest form for solving quadratic equations.
- If one of the roots of a quadratic equation is given, and you want to find the other root, you can (instead of long division) also use division by inspection:
  Let's say we know that one of the factors of an expression $x^2 - 17x + 42$, is $x - 3$.
  Then $x^2 - 17x + 42 = (x - 3)(x - 14)$
  You know $x \times x = x^2$.
  You know that $-3 \times (-14) = +42$.
  Now you have to test for the sum of the two factors in x:
  $-3 \times x = -3x$ and $-14 \times x = -14x$ and
  $(-3x) + (-14x) = -17x$.
  This method is perhaps easier for the students, since they tend to struggle with long division.
- When we want to find the equation of a given graph, there must always be three facts because in the equation $y = ax^2 + bx + c$, we want to find the values of a, b and c. In this chapter, the two roots and one other point is given.
- An alternate method to determine the equation of a graph:
  Question: Find the equation of a graph that intersects the x-axis at $\frac{1}{2}$ and 2, and the y-axis at -2.

Solution:

By using the x-intercepts:

$y = a(x - (-\frac{1}{2}))(x - 2) = a(x + \frac{1}{2})(x - 2)$

The y-intercept is $(0, -2)$. Substitute this point into $y = a(x + \frac{1}{2})(x - 2)$:

$$-2 = a(0 + \tfrac{1}{2})(0 - 2))$$

$$\therefore -a = -2$$

$$\therefore a = 2$$

$$\therefore y = 2(x + \tfrac{1}{2})(x - 2)$$

$$= 2(x^2 - 1\tfrac{1}{2}x - 1)$$

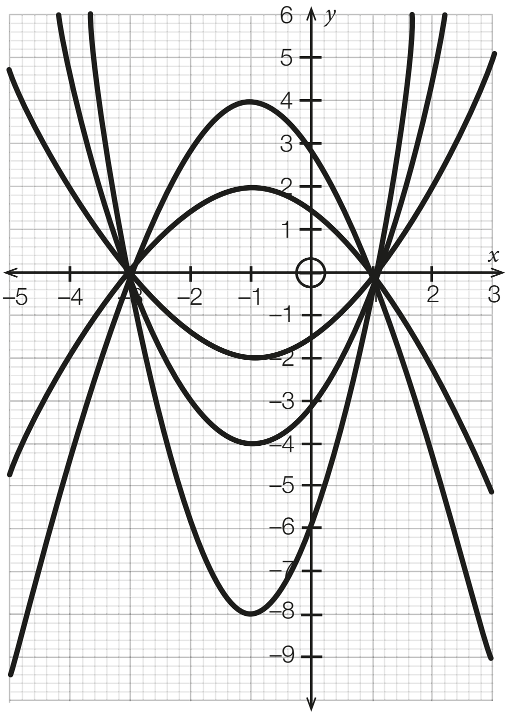
$$= 2x^2 - 3x - 2$$

- When finding a (not the) **quadratic equation** that has certain roots, it is important that you stress that there are an infinite number of quadratic equations with these specific roots.
  - Let's say that you want to find a quadratic equation with roots -2 and 3.
  - Then you know that the factors of such an equation are $(x + 2)$ and $(x - 3)$. So, some equations could be:
    $(x + 2)(x - 3) = 0$, which gives $x^2 - x - 6 = 0$
    $2(x + 2)(x - 3)$, which gives $2x^2 - 2x - 12 = 0$
    $\frac{1}{2}(x + 2)(x - 3)$, which gives $\frac{1}{2}x^2 - \frac{1}{2}x - 3 = 0$
    And so on.

  In general, the equation is $a(x + 2)(x - 3) = 0$, where a can be any value not equal to 0. The value of a does not play a role in the value(s) of the root(s), since one can always divide both sides of the equation by the value of a.

  It is, therefore, very important that you stress: When we want to find **a** quadratic equation with certain roots, we can never say that we found **the** equation since there are an infinite number of quadratic equations with the same roots.

  Below you can see that there are several different quadratic graphs that intersect the x-axis at the same points.



When we want to find the equation of a quadratic graph, there is only one equation. So, we find **the** equation of the graph.

Areas of difficulty and common mistakes

- When solving quadratic equations, such as $x^2 + 4x = 21$ or $x(6x - 5) = 6$ or $x(x - 1) = 6$ by factorisation, students sometimes do not realise that the right-hand side of the equation must always be equal to 0.
  You must emphasise that we use the zero product principle that states that if $a \times$ b $= 0$, then $a = 0$ or $b = 0$ or both of them are equal to 0.
  If students solve the equation $x(6x - 5) = 6$ and write $x = 6$ or $6x - 5 = 6$, it is totally wrong because, then $x(6x - 5) = 6 \times 6 = 36 \neq 6$.
  If students say that $x = 1$ and $6x - 5 = 6$, that is also incorrect, because there are an infinite number of products which will give 6. For example, 3×2, $12 \times \frac{1}{2}$, 3×4, $\frac{1}{4} \times 24$, and so on.
  If the right-hand side is equal to zero, the possibilities are limited to 0 or 0.
- Emphasise that they have to follow these steps, if they want to solve the equation $(2x + 3)(x - 1) = 12$.
  1. Subtract 12 from both sides of the equation: $(2x + 3)(x - 1) - 12 = 0$
  2. Remove the brackets by multiplication: $2x^2 + x - 3 - 12 = 0$

3. Add like terms:
   $2x^2 + x - 15 = 0$
4. Factorise:
   $(2x - 5)(x + 3) = 0$
5. Use the Zero product principle to solve for x:
   $2x - 5 = 0$ or $x + 3 = 0$
   $\therefore x = 2\frac{1}{2}$ or $x = -3$

Supplementary worked examples

1. Factorise the following:
   a) $6x^2 + 13x + 6$ b) $2x^2 - 13x + 15$
   c) $2x^2 + 3x - 5$ d) $6x^2 - 19x - 7$
2. Solve for x:
   a) $(6x + 5)(x + 1) = 15$
   b) $2x(4x - 11) = -15$
   c) $(x - 3)(x - 2) = 12$
   d) $(x - 8)(x + 1) = -18$
   e) $ax^2 - ax = 0$
   f) $px^2 - px = 2px$
   g) $(x - 3)(x + 2) = x - 3$
   h) $2(x - 1)(x + 1) = 7(x + 2) - 1$
3. If $(x - 3)(y + 4) = 0$, solve for x and y, if:
   a) $x = 3$ b) $x \neq 3$
   c) $y = -4$ d) $y \neq -4$
4. Find the equation of the graph of quadratic function $y = ax^2 + bx + c$, if the graph intersects the x-axis at 2 and 4 and passes through the point (6; 4).

Solutions

1. a) Factors of 36:
   6×6 sum = 12
   12×3 sum = 15
   18×2 sum = 20
   9×4 sum = 13 ✓

   $$6x^2 + 13x + 6 = 6x^2 + 9x + 4x + 6$$
   $$= 3x(2x + 3) + 2(2x + 3)$$
   $$= (2x + 3)(3x + 2)$$

   b) Factors of 30:
   2×15 sum = 17
   10×3 sum = 13 ✓

   $$2x^2 - 13x + 15 = 2x^2 - 10x - 3x + 15$$
   $$= 2x(x - 5) - 3(x - 5)$$
   $$= (x - 5)(2x - 3)$$

   c) Factors of −10:
   $2 \times (-5)$ sum = −3
   $(-2) \times 5$ sum = 3 ✓
   $$2x^2 + 3x - 5 = 2x^2 - 2x + 5x - 5$$
   $$= 2x(x - 1) + 5(x - 1)$$
   $$= (x - 1)(2x + 5)$$

   d) Factors of −42:
   $6 \times (-7)$ sum = −1
   $(-6) \times 7$ sum = 1
   $(-21) \times 2$ sum = −19 ✓

   $$6x^2 - 19x - 7 = 6x^2 - 21x + 2x - 7$$
   $$= 3x(2x - 7) + (2x - 7)$$
   $$= (2x - 7)(3x + 1)$$

2. a) $6x^2 + 11x + 5 = 15$
   $6x^2 + 11x - 10 = 0$
   $(2x + 5)(3x - 2) = 0$
   $\therefore x = -2\frac{1}{2}$ or $x = \frac{2}{3}$
   b) $8x^2 - 22x + 15 = 0$
   $(2x - 3)(4x - 5) = 0$
   $\therefore x = 1\frac{1}{2}$ or $x = 1\frac{1}{4}$
   c) $x^2 - 5x + 6 = 12$
   $x^2 - 5x - 6 = 0$
   $(x - 6)(x + 1) = 0$
   $\therefore x = 6$ or $x = -1$
   d) $x^2 - 7x - 8 = -18$
   $x^2 - 7x + 10 = 0$
   $(x - 5)(x - 2) = 0$
   $\therefore x = 5$ or $x = 2$
   e) $ax(x - 1) = 0$
   $x = 0$, if $a \neq 0$ or $x \in \mathbb{R}$, if $a = 0$ or $a = 1$
   f) $px^2 - 3px = 0$
   $px(x - 3) = 0$
   $\therefore x = 0$, if $p \neq 0$ or $x \in \mathbb{R}$, if $p = 0$ or $x = 3$
   g) $x^2 - x - 6 = x - 3$
   $x^2 - 2x - 3 = 0$
   $(x - 3)(x + 1) = 0$
   $\therefore x = 3$ or $x = -1$
   OR
   $(x - 3)(x + 2) - (x - 3) = 0$
   $(x - 3)(x + 2 - 1) = 0$
   (take $(x - 3)$ out as a common factor)
   $(x - 3)(x + 1) = 0$
   $\therefore x = 3$ or $x = -1$
   h) $2(x^2 - 1) = 7x + 14 - 1$
   $2x^2 - 2 - 7x - 13 = 0$
   $2x^2 - 7x - 15 = 0$
   $(2x + 3)(x - 5) = 0$
   $\therefore x = -1\frac{1}{2}$ or $x = 5$.

3. a) $y \in \mathbb{R}$
   b) $y = -4$
   c) $x \in \mathbb{R}$
   d) $x = 3$
4. $y = a(x - 2)(x - 4)$
   Substitute the point (6; 4):
   $$\begin{aligned} a(6 - 2)(6 - 4) &= 4 \\ a(4)(2) &= 4 \\ 8a &= 4 \\ \therefore a &= \tfrac{1}{2} \\ y &= \tfrac{1}{2}(x^2 - 6x + 8) \\ &= \tfrac{1}{2}x^2 - 3x + 4 \end{aligned}$$

Chapter 11 Trigonometry 1: Solving right-angled triangles

Learning objectives

By the end of this chapter, the students should be able to:

1. Apply the trigonometrical ratios tangent, sine and cosine to solve right-angled triangles.
2. Combine trigonometrical ratios with Pythagoras' theorem to solve right-angled triangles.
3. Use trigonometrical tables and tables of squares and square roots and/or a scientific calculator when solving right angled triangles.
4. Derive the trigonometrical ratios of special angles (45°, 60°, 30°) and use them to solve related problems.
5. Apply trigonometry and Pythagoras' theorem to solve problems involving bearings and distances, angels of elevation and depression, angles and lengths in shapes.

Teaching and learning materials

Students: Textbook, exercise book, pencil and ruler, Mathematical instruments; sets of 4 figure trigonometrical tables and tables of squares and square roots (see pages 254 to 256 of the textbook). Teacher: Posters on tangent, sine, cosine, Pythagoras, sets of 4-figure tables as provided by Examination Boards; chalkboard, board mathematical instruments.

Glossary of terms

Solve a triangle means to calculate the sizes of all its sides and angles.

Hypotenuse is the side opposite the 90° angle in a right angled triangle.

Right angle is an angle of size 90°.

Pythagoras was a Greek philosopher who lived about 2 500 years ago.

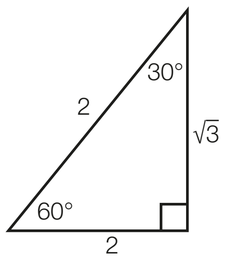
Pythagoras' theorem states that in any right angled triangle the square of the side opposite the right angle is equal to the squares of the other two sides added.

Pythagorean triple is a set of *three whole numbers* that gives the lengths of the sides of a right-angled triangle.

Teaching notes

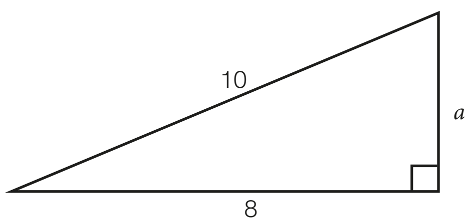
- The side opposite the 90° angle in a right-angled triangle is always the hypotenuse even if the triangle is in a different orientation from what is expected.
- Students should by now know what the sine, cosine and tangent ratios are. They should also know that these are just names for specific ratios of sides in a right-angled triangle and that there are no specific mystery attached to the names of these ratios.
- When students solve the problem where they have to work out the sides of the triangle, they should reason as follows:
  - Which side do I want to calculate? Which side do I have?
  - Write down the ratio as $\frac{\text{the side that I want}}{\text{the side that I have}}$. ❶
  - Ask yourself what trigonometric ratio of the angle in the problem does this give?
  - It could be a trigonometric ratio or $\frac{1}{\text{the trig ratio}}$. ❷
  - Write ❶ = ❷ and solve for the side that you want.
  - Do not work out the answer, if it is not the final answer. Use the expression as it is, to find the final answer.
- Note that $\sin^{-1}$ on the calculator does not mean $\frac{1}{\sin}$ (which makes no sense in any case, because one must have the sin of a specific angle), but it means that this gives us the angle of which that ratio is the sin ratio. The same applies for the cos ratio of an angle.
- When students have to solve a problem where they have to work out angles, they must look at the sides given in the right-angled triangle and ask themselves what trigonometric ratio it gives of the angle they want to calculate. If the angle is θ, they should then write (trig. ratio) $\theta = \frac{\text{side}}{\text{other side}}$.

- If the answer for the next section of the problem depends on the answer of the previous section, it is the most accurate if you use the expression that gave the answer of the previous section.
  - In Example 1 (p. 129), x was calculated in question a).
  - In question b), the value of x is used to calculate y. Your answer would be the most accurate, if you use 6tan 50° for x to work out y: $y = x \cdot \sin 28° = 6\tan 50° \cdot \sin 28° = 3.36$
- Use the diagrams in the textbook to explain the sin, cos and tan ratios of these three angles.
  - Students should remember the 45° triangle.
  - For 60° and 30° they only have to remember 1 triangle:



Areas of difficulty and common mistakes

- If you use a scientific calculator for trigonometrical calculations, make sure that your calculator is set on degrees (deg) and not rad or grad.
- Students do not realise the difference between working out the hypotenuse and one of the other two sides of a right-angled triangle using Pythagoras.
  - To prevent this the lengths of the sides should be substituted in the theorem and then you solve for the unknown side.
  - For example:



$8^2 + a^2 = 10^2$
$a^2 = 10^2 - 8^2$
$= 100 - 64$
$= 36$
$= 6$ is wrong, because it says that $a^2 = 6$.

The final answer should be $\therefore a = 6$.
Only when students know the work very well could they immediately write $a^2 = 100 - 64$ (always the square of the longest side minus the square of the other given side).

- Writing the sine, cosine and tangent ratios with capital letters: They should write sin A and **not** Sin A; cos A and **not** Cos A; tan A and **not** Tan A.
- Students tend to write sin or cos or tan without associating these ratios with an angle. Emphasise that writing sin, cos or tan only, has no meaning.
- In a problem, the student may write $\sin\theta = 8{,}866 = 60°$, instead of $\therefore \theta = 60°$. You can only prevent a problem like this, if you look at the work of the students and tell them repeatedly what is correct.
- Students do not always see sin of an angle as a ratio.
  They would give senseless answers like this: $\frac{\sin 32}{\sin 64} = \frac{1}{2}$ or cos 2A = 2cos A.
  The only way you can correct this thinking mistake is to let them work out the values using tables or scientific calculators
- Students still get the square roots of separate terms and do not realise that they first have to add the terms and then get the square root of the answer. For example: $\sqrt{100 - 36} = \sqrt{64} = 8$ **and not** $\sqrt{100} - \sqrt{36} = 10 - 6 = 4$.

Supplementary worked examples

Give all answers correct to two decimal places.

1. Use tables or a scientific calculator to determine the values of these pairs of expressions to test whether they are equal:
   a) cos 70°
      2 cos 35°
   b) tan (40° + 40°)
      2 tan 40°
   c) sin 60°
      sin 30° + sin 30°
   d) 2 cos 45°
      cos 90°
   e) cos 80°
      2 cos 40°
   f) cos 35°
      $\frac{1}{2}$ cos 70°

2. Use a scientific calculator or tables to determine the values of each of the following:
   a) $\frac{3}{7}$ tan 47° 8'
   b) tan (71.2° – 28.9°)
   c) 2 sin (33.3° + 39° 48')
   d) –3 cos 33.7° + 5.1
3. If θ = 18.3° and ϕ = 24.5°, calculate each of the following by using tables or a scientific calculator:

a) sin θ + sin ϕ	b) sin 2ϕ
c) cos (ϕ – θ)	d) cos θ – tan ϕ
e) sin (θ + ϕ)	f) 2 cos θ
g) 2 tan (2θ – ϕ)	h) $\frac{\sin\theta + \cos\phi}{\tan(\theta + \phi)}$

Solutions

1. a) cos 70° = 0.34
   2 cos 35° = 1.64
   b) tan (40° + 40°) = tan 80° = 5.67
   2tan 40° = 1.68
   c) sin 60° = 0.87
   sin 30° + sin 30° = $\frac{1}{2} + \frac{1}{2} = 1$
   d) 2 cos 45° = 1.41
   cos 90° = 0
   e) cos 80° = 0.17
   2 cos 40° = 1.53
   f) cos 35° = 0.82
   $\frac{1}{2}$ cos 79° = 0.17

2. a) 0.46	b) 0,91
c) 1.91	d) 2.6
3. a) 0.73	b) 0.75
c) 0.99	d) 0.49
e) 0.68	f) 1.9
g) 0.43	h) 1.32

Chapter 12 Mensuration 1: Plane shapes

Learning objectives

By the end of this chapter, the students should be able to:

1. Recall and use appropriate formulae to calculate the perimeter of plane shapes.
2. Recall and use appropriate formulae to calculate the area of plane shapes.
3. Solve problems relating to parallelograms and triangles drawn between parallels.
4. Calculate the lengths of arcs and perimeters of shapes in circles.
5. Calculate the area of sectors and segments or a circle.

Teaching and learning materials

Students: Textbook, exercise book, pencil and ruler and Mathematical instruments and coloured pencils or highlighters.

Teacher: Posters on perimeter and area of shapes (Fig. 12.1), parts of a circle (Fig. P18 on p. 10), cardboard cut-out shapes; string to demonstrate perimeter.

Teaching notes

- When proving that the areas of plane shapes are equal in area, let students use coloured pencils or highlighters to shade figures with equal areas. Let them always concentrate seeing triangles or parallelograms on the same base and between the same two parallel lines.
- Students have to remember that the diagonal of a parallelogram bisects its area into two triangles with equal areas.
- Make sure that students always give a reason for each statement that they make.
- When letting your students do the problems of this chapter, choose problems according to their ability.
  - It does not help them to pass Mathematics, if you choose all the most difficult problems and they cannot do any of them. This would only serve to give them a lower self-esteem where Mathematics is concerned.
  - Rather choose problems that let students practise the basic principles.
- In the JSS course, a formula was derived to determine the area of a triangle where the altitude of the triangle was not used (See Fig. 12.1).

You can remind the students that the formula was derived as follows:

$\frac{AD}{c} = \sin B, \therefore AD = c{\cdot}\sin B$

$\frac{AD}{b} = \sin C, \therefore AD = b{\cdot}\sin C$

In the same way, it can be proven that:

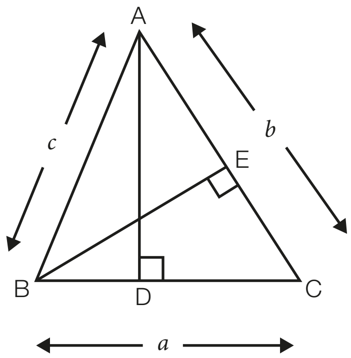
BE = $c{\cdot}\sin A$ and BE = $a{\cdot}\sin C$

So,

$$\begin{aligned}\text{Area } \triangle ABC &= \tfrac{1}{2}BC \times AD\\ &= \tfrac{1}{2}ac{\cdot}\sin B\\ &= \tfrac{1}{2}ab{\cdot}\sin C\end{aligned}$$

And

$$\begin{aligned}\text{Area } \triangle ABC &= \tfrac{1}{2}AC \times BE\\ &= \tfrac{1}{2}bc{\cdot}\sin A\\ &= \tfrac{1}{2}ab{\cdot}\sin C\end{aligned}$$



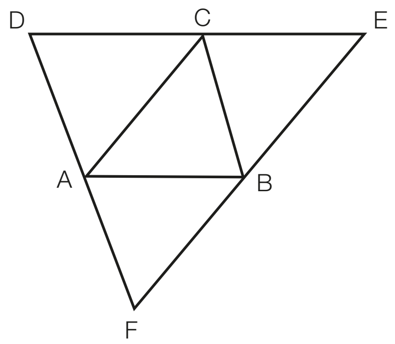
This formula is used when you have two sides, and the angle between the two sides.

Areas of difficulty and common mistakes

- Students cannot see what side of a triangle or parallelogram to use as base when they have a certain altitude. Teach them, that if they are in doubt about this, to draw a line parallel to the base of the triangle or parallelogram and to take the perpendicular distance between those parallel lines as the altitude of the figure.

- Students find it difficult to prove that areas of plane shapes are equal by adding or subtracting shapes with equal areas.
  - It could help if the students shade certain of the shapes.
  - It could also help, if the students let a certain area be equal to x (say), and then write all the areas of the other figures in terms of x.

Supplementary worked examples



In the figure, ABCD and ABEC are parallelograms; and EBF and DAF are straight lines. Prove that:

a) $\triangle BAF = \triangle ADC$
b) Area of quad FACE = area of quad ADEB.

Solution

a) $BE \parallel AC$ (opp sides $\parallel^m$)
   $FB \parallel AC$ (FBE is a straight line given)
   In the same way, $CB \parallel AF$.
   AFBC is a parallelogram (both pairs opp sides $\parallel$)
   Let $\triangle BAF = x$
   $\therefore$ Area $\triangle ABC = x$ (diagonals bisect area $\parallel^m$ AFBC)
   $\therefore$ Area $\triangle ACD = x$ (diagonals bisect area $\parallel^m$ ADCB)
   $\therefore$ Area $\triangle BAF = \triangle ADC$
b) Area $\triangle BEC = x$ (diagonal $\parallel^m$ ABEC bisects its area)
   Area quad FACE = $3x$ = Area quad ADEB.

Chapter 13 Numerical processes (3): Ratio, rate and proportion

Learning objectives

By the end of this chapter, the students should be able to:

1. Express ratios in their simplest terms.
2. Increase and decease quantities in a given ratio.
3. Compare and simplify ratios.
4. Use rates to connect quantities of different kinds (for example, wage rates, ₦/hour, km/hour, population per km^2).
5. Divide quantities in given proportions.

Teaching and learning materials

Students: Textbook, exercise book, writing materials.
Teacher: Newspaper articles that refer to rates.

Glossary of terms

Ratio is a **relationship** between two numbers of the **same kind** sometimes expressed as a quotient with no units that explicitly indicates how many times the first number contains the second (not necessarily an integer).

In layman's terms, a ratio represents for every amount of one thing, how much there is of another thing. For example, if one has 8 oranges and 6 lemons in a bowl of fruit, the ratio of oranges to lemons would be 8 : 6 = 4 : 3.

Notice that the order in which oranges and lemons is written determines the order of the numbers of the ratio.

The 4 : 7 ratio can also be written as $\frac{4}{7}$ to represent how much of the fruit is oranges.

Rate is the **ratio** between two **related quantities but not necessarily of the same kind**.

It is often a rate of change, such as a quantity per unit of time. For example, the rate of change can also be specified as per unit of length or mass.

The most common type of rate is per unit of time, like speed or the rate of a person's heartbeat.

Other rates could be exchange rates, growth rate, and so on.

In describing the units of a rate, the word "per" is used to separate the units of the two measurements used to calculate the rate (for example, a heart rate is expressed as beats per minute).

A rate can also use two numbers of the same units (such as tax rates).

Proportion: Two variables are **proportional**, if a change in one is always accompanied by a change in the other, and if the changes are always related by the use of a constant. The constant is called the coefficient of proportionality or **proportionality constant**.

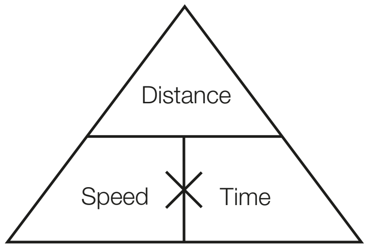
Teaching notes

- Ratios can be written as fractions. If in Example 1 fractions are cleared:
  $\frac{57}{95} = \frac{12}{x}$
  Both sides are multiplied by $95x$:
  $95x \times \frac{57}{95} = \frac{12}{x} \times 95x$
  $\therefore 57 \times x = 12 \times 95$.
  This is the same as if you cross-multiply:
  $\frac{57}{95} \times \frac{12}{x}$
- When reducing a quantity in a certain ratio (Example 2), the same result is obtained:
  - If the quantity is decreased in a certain ratio, it is multiplied by smallest number and divided by the largest number in the ratio:
    $273\ 000 \times \frac{11}{13}$.
  - If the quantity is increased in a certain ratio, it is multiplied by the largest number and divided by the smallest number in the ratio:
    $273\ 000 \times \frac{13}{11}$.

- Remember that:
  - A ratio is always without units, provided that the units are the same.
  - A ratio is written in its simplest form.
- When comparing ratios, the ratio can either be expressed in the form $n : 1$, which comes down to expressing the ratio as a decimal fraction.
  We can also write the ratios as fractions with the same denominators by taking the LCM of the denominators.
  This method is easier to use to compare ratios, if calculators are not available.
- The scale of a plan or a map is always $\frac{\text{length on plan or map}}{\text{real length in the same units}}$. Remember to tell the students that these lengths must always be in the same units before they can be compared in a scale.
- Emphasise the different kinds of rates as described in the textbook. Also add some other examples like the exchange rate Nigerian ₦ per €, growth rate of a population of elephants, the rate at which the level of a dam increases/decreases, and so on.

Areas of difficulty and common mistakes

- Students do not take the word order into account when writing down a ratio. For example, the amount of \$100 was divided between Anne and Gift in the ratio 2 : 3. This means that Anne gets $\frac{2}{5} \times 100 = \$40$ and Gift gets $\frac{3}{5} \times 100 = \$60$.
- Students make mistakes with problems involving speed, time and distance. Let them remember this triangle:



  This tells us that: $\frac{\text{distance}}{\text{time}}$ = speed; $\frac{\text{distance}}{\text{speed}}$ = time, and and that distance = speed × time.
- Students tend to forget that they can only have the correct ratio, if the quantities that are compared are in the same unit.

Chapter 14 Statistics: Data presentation

Learning objectives

By the end of this chapter, the students should be able to:

1. Present data in frequency tables, bar charts, pie charts.
2. Interpret data presented in frequency tables, bar charts and pie charts.
3. Estimate the mode and median from bar and pie charts.
4. Where appropriate, group data into class intervals.
5. Present grouped data in frequency tables, histograms and frequency polygons.
6. Interpret grouped data presented in frequency tables, histograms and frequency polygons.
7. Find the modal and median classes of grouped data.

Teaching and learning materials

Students: Textbook, exercise book, writing materials, graph paper or graph exercise book, a protractor, compass and a ruler.

Teacher: Graph board and a chalkboard compass and protractor or transparencies of graphs shown in the book and transparencies of graph paper to draw graphs as well as transparency pens, a ruler, a compass and a protractor, if an overhead projector is available. Newspapers and magazine articles with data presented in tabular and graphical form.

Teaching notes

- When data are **not grouped**, **a frequency table** merely list the number of times that a certain data point, for example a specific number or an error, occurs.
- Ungrouped data are usually graphically illustrated by bar charts and pie charts. There are spaces between the columns of the bar chart.
- The size of an angle in a pie chart is proportional to the frequency and the height of a column of a bar chart also is proportional to the frequency of the data.
- Explain that the mode is the age, or shoe size or anything that occur most of the time or has the highest frequency.
- Explain the median as follows:
  - First numbers or measurements for example, are arranged from the lowest to the highest.
    - If the number of measurements or numbers is *odd*, the middle measurement or number is the median.
    - If the number of measurements or numbers are *even*, then the median is the average of the two middle measurement or numbers.
- When there are a large number of values, these values are grouped in a frequency distribution table. In this table, the values are arranged in class intervals. If a class interval is **100–199**, then:
  - **100** and **199** are called the **class limits**
  - Each class interval starts 0.5 below the lower class limit and ends 0.5 above the upper class limit and so **99.5** and **199.5** are called the **class boundaries**.
  - The **class width** is the **difference between the upper and lower class boundaries**, that is, 199.5 – 99.5 = 100, for example.
  - The **class mid-value** or class mid-point is $\frac{1}{2}$ **the sum of the lower and upper class limits,** or in this example it is $\frac{1}{2}(100 + 199) = 149.5$.
- Grouped data are graphically illustrated by a:
  - Histogram that consists of rectangles with no spaces between them and of which the widths represent the class width.
  - Frequency polygon where the top midpoints of each of the rectangles of the histogram are joined, and that starts with the midpoints of adjacent intervals on the horizontal axis.
- Explain to the students that the modal class is the class with the highest frequency.
- Explain that the approximate mode is obtained by using the explanation on p. 179 under b).

- Explain to the class that the median class is found by finding $\frac{1}{2}$ of the number of data values; and then finding the class in which this data point falls (top of p. 180).

Areas of difficulty and common mistakes

- The frequency polygon does not start on the horizontal axis and end on the horizontal axis. Emphasise that any polygon is a closed figure and, in this case, the horizontal axis serves as one of the sides of a polygon.
- Students cannot see what the scale of the graph is and their readings from the graphs are, therefore, wrong.
  - Teach them to carefully find the scale by counting. For example, 10 little squares represent 5. Therefore, 2 little squares represent 1.
  - Always try to find out how many measurements represent 1 unit.
- The scale of the graph is too small for accurate readings. Teach the students to make the scale of their graphs as big as possible.

Chapter 15 Mensuration 2: Solid shapes

Learning objectives

By the end of this chapter, the students should be able to:

1. Recall and use appropriate formulae to calculate the volume and surface area of cubes, cuboids, prisms and cylinders.
2. Recall and use appropriate formulae to calculate the surface area and volume of cones and pyramids.
3. Use addition and subtraction methods for the mensuration of composite solids and hollow shapes.
4. Use similar triangles and subtraction to calculate the volume of the frustum of a cone or a pyramid.

Teaching and learning materials

Students: Textbook, exercise book, writing materials.
Teacher: Collection of solid shapes (use familiar objects such as boxes, tin cans, tyres, buckets and lampshades) and stiff paper (the kind found in new shirts) or cardboard (the kind find in shoe boxes).

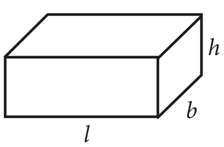
Glossary of terms

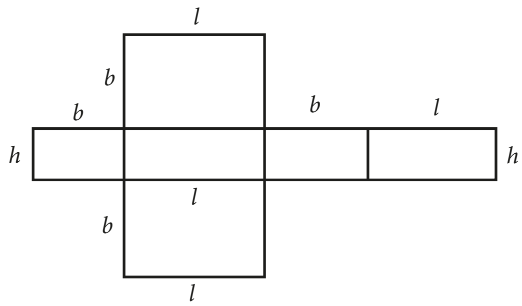
Prism: If you cut through a solid parallel to its base and the cross section is identical or congruent to the base, the solid is a prism.
Or
A solid object with two identical ends and flat sides: The sides are parallelograms (4-sided shapes with opposites sides parallel). The cross section is the same all along its length. The shape of the ends gives the prism its name, such as “triangular prism”.

Teaching notes

- If we look at the net of the figure, the total surface area can also be calculated as follows:

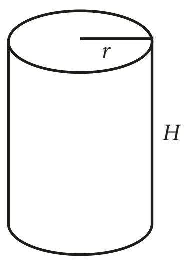
A cuboid

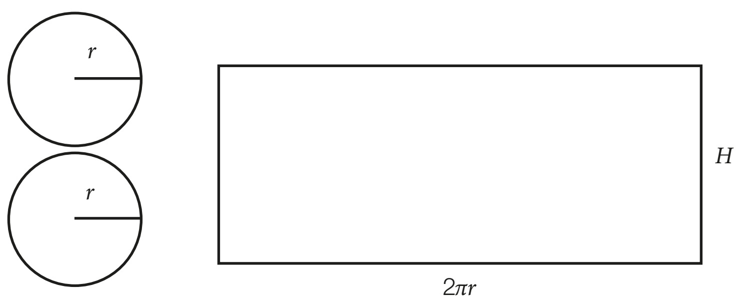




Total surface area = $2(l \times b) + 2(l + b) \times h$

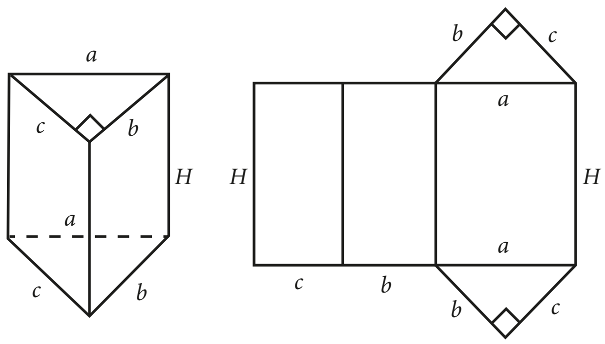
A cylinder





Total surface area = $2\pi r^2 + 2\pi rH$

A triangular prism



Total surface area = $2(\frac{1}{2}bc) + (c + b + a)H$
The total surface area in general for a prism
= 2 × base area + perimeter base × height of prism
The students would understand this method better, if the nets above were cut out and shown to them. You can then fold the nets to form the 3D prism.

- If you explain how the formula for the surface area of the cone is found, you could illustrate by cutting out a sector of a circle as shown in Fig. 15.4 and then folding this sector to form the required cone.
- Remind the students to always use the same units when they calculate the volume or total surface area. To avoid working with fractions they should always convert all units to the smallest unit. For example, if the units are given as cm and m, they should convert all units to cm.
- If the volume is required to be in m^3 or the total surface area is required to be in m^2, for example, convert the lengths to m from the beginning. In this way errors are avoided, because it is much more difficult to convert cm^3 or cm^2 to m^3 or m^2, than it is to convert cm to m.

Areas of difficulty and common mistakes

- The students do not always convert all the units to the same unit. Emphasise that they can only work with length units that are the same.
- If the volume is required to be in m^3 or the total surface area is required to be in m^2, for example, convert the lengths to m from the beginning. In this way errors are avoided, because it is much more difficult to convert cm^3 or cm^2 to m^3 or m^2, than it is to convert cm to m. So, to make the work easier and to avoid errors, the length units should always be converted to same length unit as the area or volume units.
- Students may find it difficult to work out the total surface area or volume of a prism, if it is of another form than the four basic prisms shown on p. 182 in Fig. 15.1.
  The reason for this is that they cannot identify the bases of the prisms. They could overcome this difficulty if they imagine that they have a knife and that they cut through the solid, like they would cut through bread to obtain identical slices.
  In this way, they could identify the base and apply the general formula for working out the volume or the total surface area of the solid.
- When working with composite solids, students may find it difficult to visualise the 3D forms. Their work can be made less complicated by drawing the separate solids and then working out the required volume or area and adding them.
- When calculating the volume of the frustum of a cone or a pyramid, students may find it difficult to identify the two similar triangles that are necessary to calculate the altitude of the remainder of the cone or pyramid.
  Teach them to always start with the length they want to work out.
  This length is part of a right-angled triangle of which the length of another side is known.
  Then they must look for another right-angled triangle of which the lengths of the corresponding sides are both known.

Chapter 16 Geometry (2): Constructions and loci

Learning objectives

By the end of this chapter, the students should be able to:

1. Use ruler and compasses to:
   - Bisect a given line segment.
   - Bisect any angle.
   - Construct 'special' angles (90°, 45°, 60°, 30°, 15°) and combinations of these angles (75°, 105°, 135°, and so on).
   - Copy a given angle.
2. Construct triangles and quadrilaterals involving the above constructions.
3. Define locus.
4. Construct the locus of points that are:
   - A given distance from a fixed point.
   - A given distance from a straight line.
   - Equidistant from two given points.
   - Equidistant from two given straight lines.
5. Apply the above constructions to solve locus problems.

Teaching and learning materials

Students: Textbook, exercise book, writing materials, Mathematical set (especially a ruler, compass, sharp pencil), plain paper.

Teacher: Chalkboard instruments (especially ruler or straight edge and compasses), plain paper for class distribution, thread for class distribution.

Glossary of terms

Locus (plural loci) is a set of points whose location satisfies, or is determined by, one or more specified conditions; or can be defined as a set of points that share a common property or satisfy a certain condition. A circle is the best example. It is the locus of points equidistant from a fixed point.

Teaching notes

- Use your board compass and straight edge to illustrate all the constructions on pp. 193–194. Let the students do the same constructions on plain paper while you are illustrating them on the board.
- All constructions should be illustrated on the chalkboard by using board instruments. The students should copy all constructions on their books while you illustrate them. You should also walk around and make sure that they do the constructions correctly.
- When you do Example 1 and 2 on the chalkboard:
  - Emphasise that it is essential that the sketch with all the information must be made before the construction is attempted.
  - Construct the angles separately and then copy the angles by construction onto your construction.
  - Insist that everybody has a sharp pencil.
  - Insist that all construction arcs are shown or visible.
  - Let students measure the lengths of lines with a compass (ideally a measuring compass) and then putting the compass on a ruler and reading off the length on the ruler, because measuring with a ruler only may be inaccurate if a parallax error is made.
  - Insist that every student work with you and do the same constructions while you illustrate them on the chalkboard.
- When you explain specific loci, describe the condition of the locus and then first ask the

students to describe what the locus would look like.
- To be able to do combinations of loci as illustrated in Examples 4–6, the students should know the basic loci by heart. They should for example know that that the locus of points equidistant from two fixed points A and B is the perpendicular bisector of AB.

Areas of difficulty and common mistakes

- Students use blunt pencils.
  Insist that they sharpen their pencils. If necessary, provide a sharpener.
  Also insist that the pencil has a point that is not to soft and also not the hard. An HB point is the best.
- Students do not show construction arcs. Insist that they do.
- Students generally find loci difficult.
  Great care should be taken when Exercise 16c is done.
  The whole exercise should ideally be done in class in groups under your supervision and guidance.
- The construction of loci can be very difficult as the concept may be very difficult to understand.
  This can be overcome if:
  - Students know all the basic loci.
  - Make a rough sketch before attempting to make the accurate construction.
  - If you teach students to read very carefully through the instructions of the problem and to make the rough sketch step by step as the reading progresses.

Chapter 17 Trigonometry (2): Angles between 0° and 360°

Learning objectives

By the end of this chapter, the students should be able to:

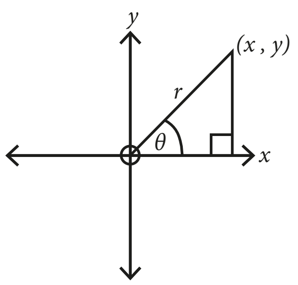
1. Extend sine, cosine and tangent ratios of acute angles (in first quadrant) to obtuse and reflex angles (in the Quadrants 2, 3 and 4).
2. Express a positive or negative angle of any size in terms of an equivalent positive angle between 0° and 360°.
3. Use trigonometric tables to determine the sine, cosine and tangent of any angle θ, where $0° \leq \theta \leq 360°$.
4. Define sin θ and cos θ as ratios within a unit circle.
5. Use unit circles to draw the graphs of sin θ and cos θ for $0° \leq \theta \leq 360°$ and use the graphs to solve related trigonometric problems.

Teaching and learning materials

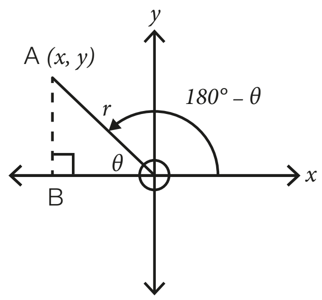
Students: Textbook, exercise book, writing materials 4-figure trigonometric tables (see pp. 247 to 252 of the textbook); graph paper.
Teacher: Graph chalkboard; sets of 4-figure tables as provided by Examinations Boards.

Teaching notes

- Make sure that the students understand the fact that $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$ and $\tan\theta = \frac{y}{x}$ and that the size of θ determines the values of the *x*- and *y*-coordinates:

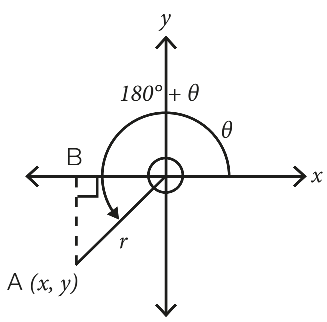


- In Quadrant 2, *x* is negative so all the trigonometric ratios that contain an *x*-value will be negative. They are the cos and tan ratios.
  - The sin ratio contains a *y*-value and is, therefore, positive in Quadrant 2.
  - The angles in Quadrant 2 can all be written as 180° – θ, where θ is an acute angle.



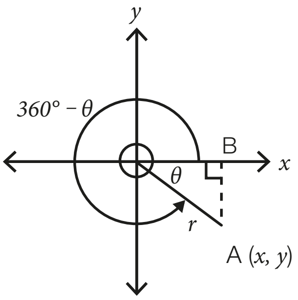
So, sin 120° = sin (180° – 60°) = sin 60°
cos 120° = cos (180° – 60°) = –cos 60°
tan 120° = tan (180° – 60°) = –tan 60°

- In Quadrant 3 both the *x*- and *y*-coordinates are negative, so all the trigonometric ratios that contain either an *x*-value or a *y*-value will be negative. They are the sin and cos ratios.
  The tan ratio contains both an *x*-value and a *y*-value, and since a negative number divided by a negative number gives a positive answer, the tan ratio is positive in Quadrant 3.
  The angles in Quadrant 3 can all be written as 180° + θ, where θ is an acute angle.



So, $\sin 240° = \sin (180° + 60°) = -\sin 60°$
$\cos 240° = \cos (180° + 60°) = -\cos 60°$
$\tan 240° = \tan (180° + 60°) = \tan 60°$

- In Quadrant 4, the y-coordinates are negative, so all the trigonometric ratios that contain a y-value will be negative. They are the sin and tan ratios.
  - The cos ratio contains an x-coordinate and will be positive in Quadrant 4.
  - The angles in Quadrant 4 can all be written as $360° - \theta$, where θ is an acute angle.



So, $\sin 300° = \sin (360° - 60°) = -\sin 60°$
$\cos 300° = \cos (360° - 60°) = \cos 60°$
$\tan 300° = \tan (360° - 60°) = -\tan 60°$

- When the sine and cosine graphs are seen as waves, you can also regard:
  - Their maximum deviation from the x-axis, as the amplitude of the wave (which in these cases) is 1.
  - One repetition of the graph is its period, and for these two graphs the period is 360°.
- The sign of the trigonometric ratio depends entirely on the quadrant in which the radius of the circle falls.
- The Greek letter θ (theta) is often used to indicate the size of an unknown angle. Other Greek letters often used to indicate sizes of angles in trigonometry are a (alpha), β (beta) and φ (phi).

Areas of difficulty and common mistakes

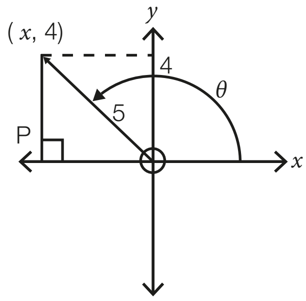
This chapter does not normally present any areas of difficulty or common mistakes, if it is explained very carefully.

Supplementary worked examples

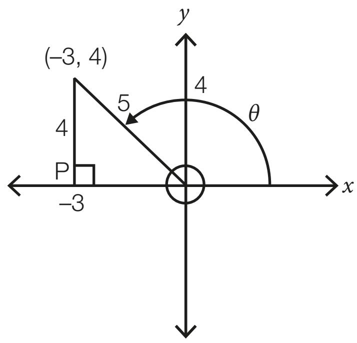
1. $5\sin\theta - 4 = 0$, where $90° \le \theta \le 360°$.
   a) Write down $\sin\theta$?
   b) Draw a sketch showing θ in the correct quadrant.
   c) Determine the following by using your sketch of question a):
      i) the x-coordinate
      ii) $5\cos\theta - 3\tan\theta$
      iii) $\sin\theta \tan\theta$
      iv) $1 - \cos^2\theta$
2. $13\cos\theta - 5 = 0$, where $180° \le \theta \le 360°$.
   a) What is $\cos\theta$?
   b) Draw a sketch with θ in the correct quadrant. Show all relevant information in your sketch.
   c) Use your sketch to determine the following:
      i) $13\sin\theta + 10\tan\theta$
      ii) $1 - \cos^2\theta$
      iii) $\sin\theta \div \tan\theta$
      iv) $\cos\theta + \sin\theta$

Solutions

1. a) $5\sin\theta = 4, \therefore \sin\theta = \frac{4}{5}$



   b) θ could be in Quadrants 2, 3 and 4, but of these three quadrants, $\sin\theta$ is positive in Quadrant 2 only.
   c) i) $OP^2 = 5^2 - 4^2 = 25 - 16 = 9 \therefore OP = 3$



Note: P is the point (–3; 0), but the length of OP is 3. x-coordinate = –3.

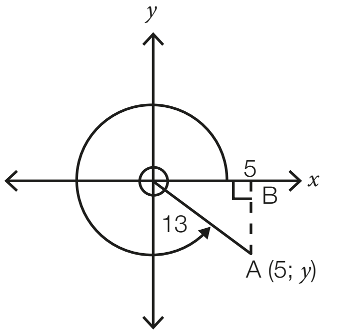
ii) $5\left(\frac{-3}{5}\right) - 3\left(\frac{4}{-3}\right) = -3 + 4 = 1$

iii) $\left(\frac{4}{5}\right)\left(\frac{-3}{4}\right) = \frac{-3}{5}$

iv) $1 - \left(\frac{-3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{25}{25} - \frac{9}{25} = \frac{16}{25}$

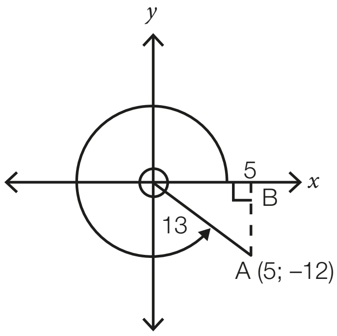
2. a) $\cos \theta = \frac{5}{13}$

b) θ could be in the, Quadrants 3 or 4, but of these two quadrants, $\cos \theta$ is positive in Quadrant 4 only.



$AB^2 = 13^2 - 5^2 = 169 - 25 = 144$

$\therefore AB = 12$



c) i) $13\left(\frac{-12}{13}\right) + 10\left(\frac{-12}{5}\right) = -12 - 24 = -36$

ii) $1 - \left(\frac{5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{169 - 25}{169} = \frac{144}{169}$

iii) $\left(\frac{-12}{13}\right) \div \left(\frac{-12}{5}\right) = \left(\frac{-12}{13}\right) \times \left(\frac{5}{-12}\right) = \frac{5}{13}$

iv) $\frac{5}{13} + \left(\frac{-12}{13}\right) = \frac{-7}{13}$

Chapter 18 Algebraic processes (5): Variation

Learning objectives

By the end of this chapter, the students should be able to:

1. Solve numerical and word problems involving direct and inverse variation.
2. Solve numerical and word problems involving joint and partial variation.

Teaching and learning materials

Students: Textbook, exercise book, writing materials.
Teacher: Wall charts with examples of the four kinds of variation.

Teaching notes

- If x is the independent variable and y is the dependent variable, they **vary directly** if there is a constant number $k \neq 0$ such that $y = kx$, where k is the **constant of variation**. If two quantities vary directly, we say that they have a direct variation.
  - This means that if x is multiplied by a number, then y is also multiplied by that same number.
  - It also means that if the value of one quantity increases, the value of the other quantity increases by the same ratio.
- The variables x and y **vary inversely** for a constant $k \neq 0$, $y = \frac{k}{x}$ **or** $xy = k$, where k **is the constant of variation** and we say that the two quantities have an inverse variation.
  This also means that if a value of the independent variable (x) is multiplied by 2, for example, then the corresponding value of y is multiplied by the multiplicative inverse of 2, namely, $\frac{1}{2}$.
  So, if the value of one of the variables is multiplied by a number, the value of the other variable is multiplied by the multiplicative inverse of that number.
  If two variables vary inversely, it means that if the value of one increases, the value of the other decreases in the same ratio or increases inversely.
- When we say c **varies jointly** to a set of **variables,** it means that c **varies directly and/ or inversely to each variable one at a time**.
  If c varies directly to a and inversely to b, the equation will be of the form $c = k\frac{a}{b}$, where k is the **constant of variation** and $k \neq$ **0**.
  - The area, A, of a triangle, for example, varies directly to the length of its base, b, and to the length, h, of its perpendicular height.
  - The equation is $A = kbh$ and $k = \frac{1}{2}$, therefore, the final equation is $A = \frac{1}{2}bh$.
- When L varies partially to F, then L, is the sum of a constant number and a constant multiple of F. This is called partial variation.
  The formula is then of the form $L = kF + c$, where k and c are constants. We can also say that k is the constant of variation and c is the initial value of L.
  We say L varies partially to F because in the equation $L = kF$, L varies directly to F, but now something is added.
  If a force is applied to a spring, we know that the increase in length varies directly to the force applied to the spring.
  If we want to get the length of the spring, we have to add the increase in length of the spring to its original length.

Areas of difficulty and common mistakes

There are no specific areas of difficulty and common mistakes in this work.

Chapter 19 Numerical processes 4: Tax and monetary exchange

Learning objectives

By the end of this chapter, the students should be able to:

1. Calculate the income tax (or PAYE) on given incomes.
2. Determine the VAT (value-added tax) that is paid on certain goods and services.
3. Use exchange rates for buying and selling currencies across borders.

Teaching and learning materials

Students: Textbook, exercise book, writing materials, logarithm tables (see pp. 245–256) and a calculator.
Teacher: Newspaper articles that refer to taxation; VAT and exchange rates (for example, up-to-date FOREX tables); computer with an Internet connection.

Teaching notes

An alternate method to do Example 4 is:

105% of the bill = ₦94 500

100% of the bill = ₦94 500 × $\frac{100}{105}$ = ₦90 000

VAT = ₦94 500 − ₦90 000 = ₦4 500

Areas of difficulty and common mistakes

- Calculating tax (PAYE) by using the taxation bands. This can be explained as follows:
  - The first ₦400 000 falls in the first band and 10% of this amount is taxed.
  - The second ₦400 000 falls in the second taxation band and 20% of this amount is taxed.
  - After taxes were paid, the remainder falls in the third taxation band and 40% of this amount is taxed.
  - If there is still an amount that exceeds ₦2 000 000, 60% tax is paid.
  - So, say a man earns a taxable amount of ₦3 000 000 per year. Then his taxes are calculated as follows:

Tax on ₦400 000 @ 10%	=	₦40 000
Tax on ₦400 000 @ 20%	=	₦80 000
Tax on ₦2 000 000 @ 40%	=	₦800 000
Tax on ₦200 000 @ 60%	=	₦120 000
Total: Tax on ₦3 000 000	=	₦1 040 000

- Calculating the value of money from a certain exchange rate.
  - Students tend to multiply or divide blindly without thinking.
  - Teach them to always work to 1 of the currency unit they want to convert to another currency. For example:

What is the value of ₦1 000 in €?

₦150 = 1US$ = 0.80€

1 ₦ = $\frac{0.80}{150}$€

₦1 000 = $\frac{0.80}{150}$ × 1 000€ = €5.33

Chapter 20 Numerical processes 5: Modular arithmetic

Learning objectives

By the end of this chapter, the students should be able to:

1. Describe and interpret cyclic events.
2. Reduce numbers to their simplest form with a given modulus.
3. Add, subtract, multiply, divide numbers in various moduli.
4. Apply modular arithmetic to real-life situations.

Teaching and learning materials

Students: Textbook, exercise book, writing materials.
Teacher: Charts from newspapers showing cyclic events (weekly, monthly, duty rosters, market day announcements), clock with rotation hands.

Teaching notes

- The sine and cosine functions are also examples of cyclic events. The modulus is 360° because these functions repeat themselves every 360°, therefore, 420° = 60°(mod 360°) and 480° = 120° (mod 360°), and so on.
- If students do not understand adding and subtracting go back to a basic cycle like the one illustrated on p. 227, explain addition and subtraction by using the cycle as illustrated in Example 4 on p. 228.

Areas of difficulty and common mistakes

The whole chapter would seem rather strange to the students, but if you explain every concept carefully and let the students work continuously under your guidance, there should not be any serious problems. The students would, however, need a lot of practice and guidance to master this work.

Revision exercises and tests Chapters 1–6 and 7–13

At the end of the term, there is time allocated for revision.

- Each revision exercise could be used as a group activity to revise the work under your supervision.
- The Revision Test that follows can then be given as a tutorial and must be done individually.
- Give the class a time limit to complete the revision test.
- You can then mark the test from a memorandum that you worked out. Then hand back the revision test with memorandum as soon as possible.