glossary

Worksheet 8-01

Brainstarters 8

Start up

1 Express each of these ratios as a fraction.

a 5 : 13 **b** 13 : 5 **c** 25 : 7 **d** 7 : 25

e 12 : 13 **f** 13 : 12 **g** 1 : 4 **h** 4 : 1

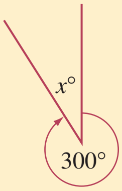
2 Express each of these ratios as a fraction in simplest form.

a 20 cm : 1 m **b** 20 min : 1 h **c** 15 s : 1 min **d** 1 min : 15 s

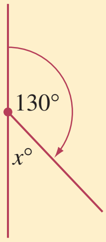
e 20 cm : 2 mm **f** 2 mm : 20 cm **g** 40 s : 1 min **h** 1 min : 40 s

3 Calculate the values of the pronumerals in these diagrams.

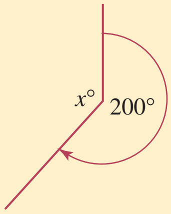
a



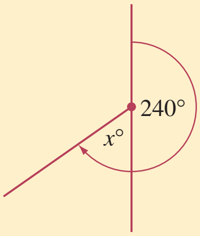
b



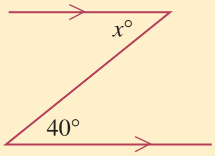
c



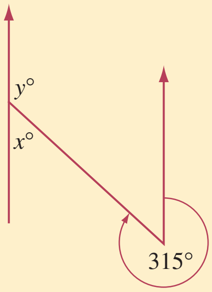
d



e

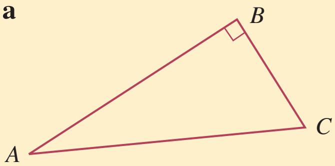


f

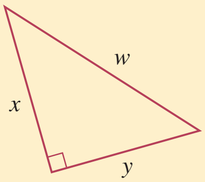


4 Name the hypotenuse in each of the following right-angled triangles.

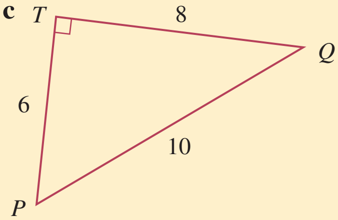
a



b

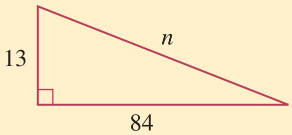


c

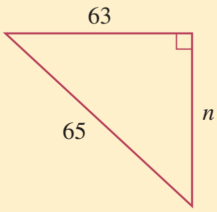


5 Use Pythagoras' theorem to find the values of n in each of these right-angled triangles.

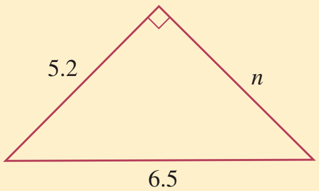
a



b



c



6 Solve each of the following:

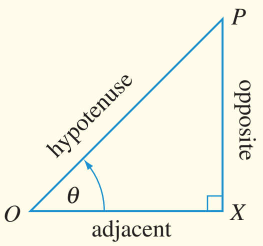
a $\frac{x}{5} = 7$ **b** $\frac{k}{4} = 8.3$ **c** $\frac{d}{9.7} = 8$ **d** $\frac{m}{12.8} = 4.9$

Naming the sides of a right-angled triangle

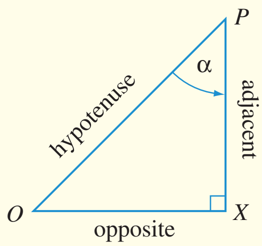
Special names are given to the sides of a right-angled triangle. These names depend on the position of the sides in the triangle relative to a given angle.

- The **hypotenuse** is the longest side and is always opposite the right angle.
- The **opposite** side is directly opposite the given angle.
- The **adjacent** side is next to, or an arm of, the given angle. It runs from the angle to the right angle.

For angle θ below, the hypotenuse is OP, the opposite side is XP, the adjacent side is OX.

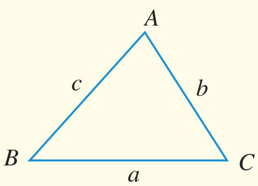


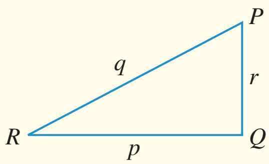
For angle α below, the hypotenuse is OP, the opposite side is OX, the adjacent side is XP.



Note: In both triangles, the position of the hypotenuse is fixed while the positions of the opposite and adjacent sides depend on the position of the angle.

In geometry and trigonometry, we use capital letters to label the angles (or vertices) of a triangle, and lower-case letters to label the sides. For example, in ΔABC we use a to label the side opposite $\angle A$, b to label the side opposite $\angle B$, and so on.

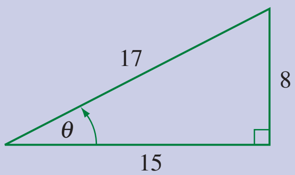




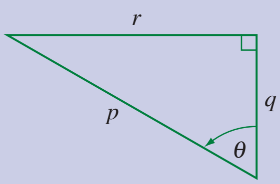
Example 1

Name the hypotenuse, opposite side and adjacent side for the angle θ in each of these triangles.

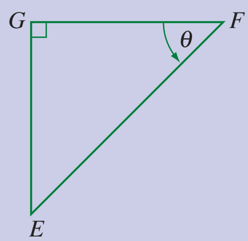
a



b



c



Solution

a Hypotenuse is 17.
Opposite side is 8.
Adjacent side is 15.

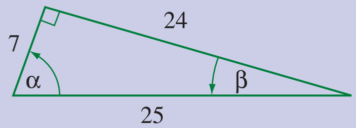
b Hypotenuse is p.
Opposite side is r.
Adjacent side is q.

c Hypotenuse is EF.
Opposite side is EG.
Adjacent side is FG.

glossary

Example 2

For both angles α and β in this triangle, name the hypotenuse, opposite side and adjacent side.



Solution

For angle α:
- hypotenuse is 25
- opposite side is 24
- adjacent side is 7.

For angle β:
- hypotenuse is 25
- opposite side is 7
- adjacent side is 24.

Just for the record

The ABC of Greek

Below are six of the letters from the Greek alphabet. (The lower case and capital letters are both shown.)

α, A alpha β, B beta γ, Γ gamma θ, Θ theta ϕ, Φ phi ω, Ω omega

Because a great deal of our mathematics has come from the ancient Greeks, it is traditional to use Greek letters as pronumerals, particularly in geometry and trigonometry.

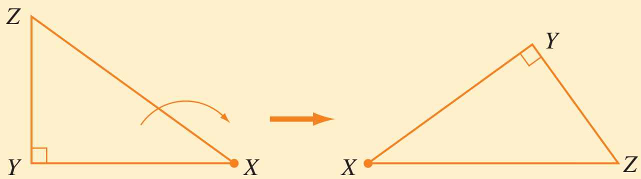
1 a How many letters are there in the Greek alphabet?
b List them and name each one, comparing them to our Roman alphabet.

2 Where did the word ‘alphabet’ come from? Explain how it was derived.

3 What other alphabets are there?

Exercise 8-01

1 ΔXYZ is rotated about vertex X as shown.



State whether the following statements are true (T) or false (F).

a The hypotenuse has remained the same.

b The side opposite angle X is the same.

c The side adjacent to $\angle X$ is different.

d The side adjacent to $\angle Z$ is the same.

e The side opposite $\angle Z$ is different.

f In either triangle:

i the sides opposite $\angle X$ and $\angle Z$ are different

ii the sides adjacent to $\angle X$ and $\angle Z$ are the same.

2 Explain why the position of the hypotenuse is fixed in every right-angled triangle.

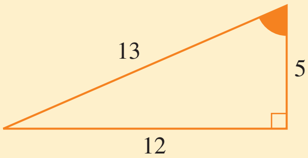
3 Explain how the positions of the opposite and adjacent sides can change in a right-angled triangle.

glossary

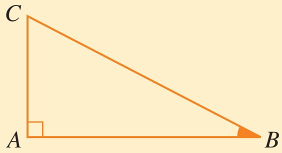
4 In the following triangles, find:
  i the hypotenuse (H)
  ii the side opposite (O) the marked angle
  iii the side adjacent (A) to the marked angle.

Example 1

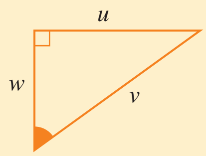
a



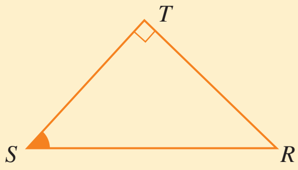
b



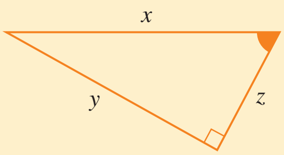
c



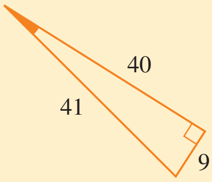
d



e



f

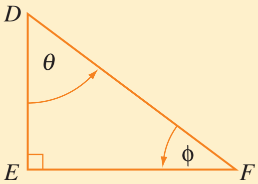


5 In each of the following triangles, find the hypotenuse (H), opposite side (O) and adjacent side (A) for:
  i angle θ **ii** angle ϕ

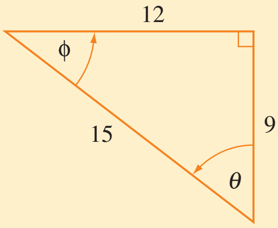
Example 2

SkillBuilder 22-01 Introduction to trigonometry

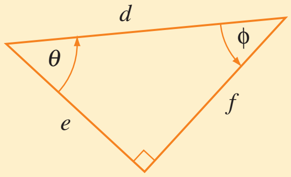
a



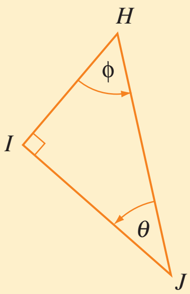
b



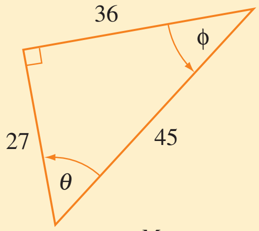
c



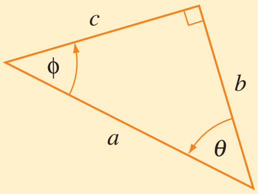
d



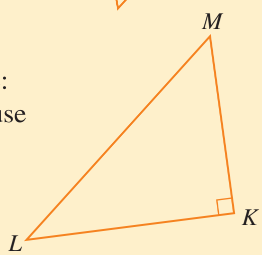
e



f



6 **a** For ΔLKM, find the angle:
  i opposite the hypotenuse
  ii opposite side *KM*
  iii opposite side *LK*
  iv adjacent to side *KM*
  v adjacent to side *LK*.



b Copy and complete these sentences for ΔSTU:
  i Side *SU* is _________ angle *T* and _________ to angle *S*.
  ii Side *TU* is _________ angle *S* and _________ to angle *T*.

glossary

7 **a** Sketch a right-angled triangle *ABC* with hypotenuse *AB*, and side *AC* opposite angle θ.
b Sketch a right-angled triangle *XYZ* with hypotenuse *YZ*, and side *XZ* adjacent to angle α.
c Sketch a right-angled triangle *PRQ* with side *RQ* opposite angle *RPQ* and adjacent to angle *QRP*. Name the hypotenuse.
d Sketch a right-angled triangle *DEF*, right-angled at *E* and with the sides opposite and adjacent to angle *D* equal. What type of triangle is ΔDEF?

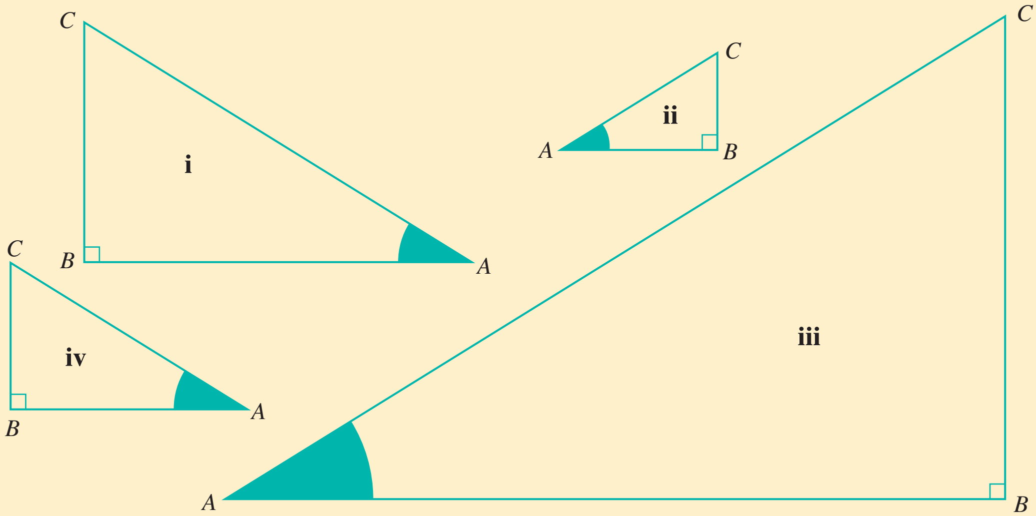
Worksheet 8-02 Investigating the tangent rate

Comparing ratios of sides in similar right-angled triangles

Working mathematically

Questioning and reasoning: Ratios in similar right-angled triangles

1 Four similar right-angled triangles are shown below. The shaded angle, *A*, is 32° (constant) in every triangle.



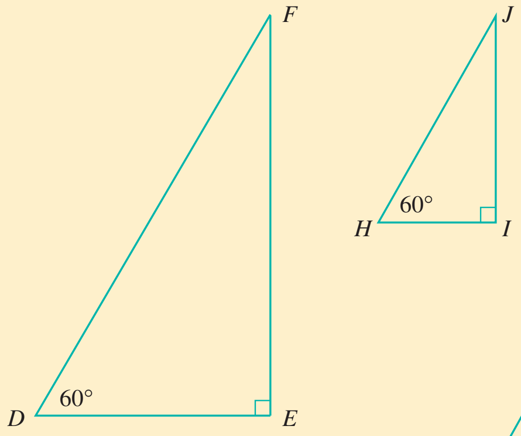
a Copy and complete this table, evaluating the ratios (with respect to $\angle A$) correct to two decimal places.

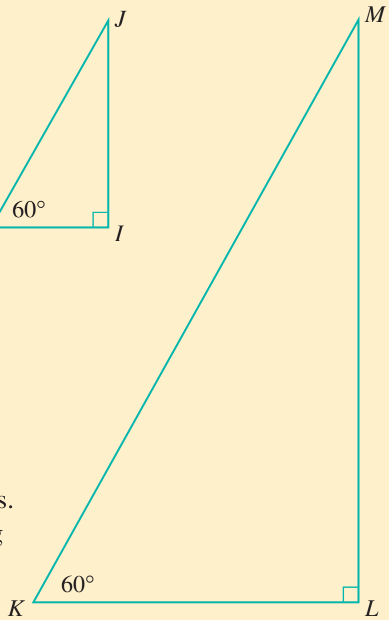
	Side length (mm)			Ratios (with respect to $\angle A$)		
	BC	*AB*	*AC*	$\frac{BC}{AC} = \frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{AB}{AC} = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\frac{BC}{AB} = \frac{\text{opposite}}{\text{adjacent}}$
i						
ii						
iii						
iv						

b What do you notice about the values of the three ratios for the triangles?

glossary

2 a Construct three similar right-angled triangles each with an angle of 60°.





b Measure the length of each side, in millimetres.

c Copy and complete the table below, evaluating the ratios (with respect to 60°) correct to two decimal places.

Triangle	Ratios (with respect to 60°)		
	$\frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{\text{adjacent}}{\text{hypotenuse}}$	$\frac{\text{opposite}}{\text{adjacent}}$
ΔDEF			
ΔHIJ			
ΔKLM			

d What can you say about the ratios of sides for any given angle? Give reasons.

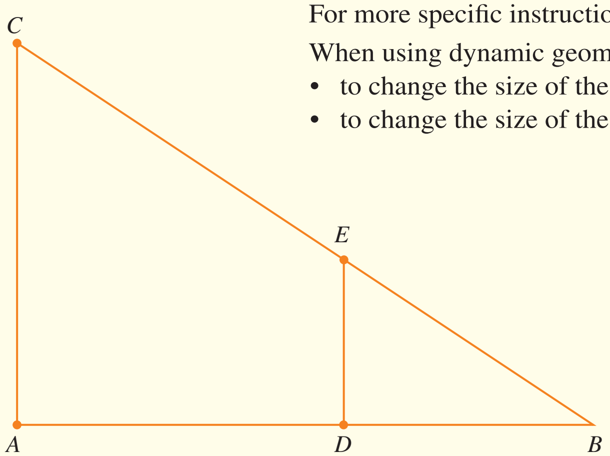
Using technology

Comparing ratios of sides in similar right-angled triangles

For more specific instructions click on the technology link.

When using dynamic geometry software:

- to change the size of the smaller triangle, use point D
- to change the size of the larger triangle, move C or B.



Geometry
8-01
Ratios of sides in right-angled triangles

Instructions

Step 1: Construct the right-angled triangle *ACB* as shown on the previous page.

Step 2: Construct the right-angled triangle *DEB* inside *ACB*.

Step 3: Measure the ratios $\frac{AC}{AB}, \frac{AC}{BC}, \frac{AB}{BC}$ and compare these to $\frac{DE}{DB}, \frac{DE}{BE}, \frac{DB}{BE}$.

Step 4: Resize each of the triangles and compare the ratios again.

1 What do you notice about the ratios?

2 Write a statement about the relationship between the corresponding ratios of the sides of the two triangles.

The trigonometric ratios

The ratios of two sides of a right-angled triangle are known as the **trigonometric ratios**. The following three trigonometric ratios are the most often used:

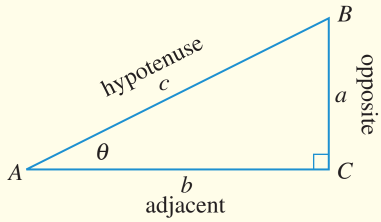
- the **sine** ratio, which is abbreviated to **sin** (but still pronounced as *sign*)
- the **cosine** ratio, which is abbreviated to **cos**
- the **tangent** ratio, which is abbreviated to **tan**.

This is how the three trigonometric ratios are defined for a given angle (θ) in a right-angled triangle:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AB} \text{ or } \sin\theta = \frac{a}{c}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AC}{AB} \text{ or } \cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AC} \text{ or } \tan\theta = \frac{a}{b}$$

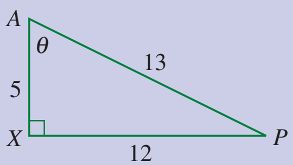


A useful mnemonic (memory aid) for remembering the three ratios is **SOH CAH TOA** (pronounced '*so-car-towa*'), where:

- SOH means **s**ine is **o**pposite over **h**ypotenuse
- CAH means **c**osine is **a**djacent over **h**ypotenuse
- TOA means **t**angent is **o**pposite over **a**djacent

Example 3

In ΔAPX, find $\sin\theta$, $\cos\theta$ and $\tan\theta$.



Solution

For angle θ, the hypotenuse is 13, the opposite side is 12 and the adjacent side is 5.

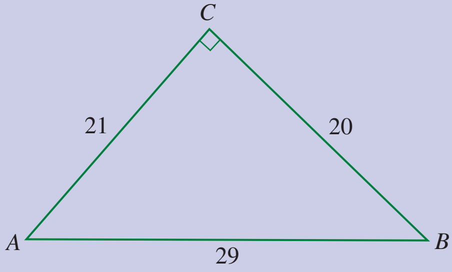
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\therefore \sin\theta = \frac{12}{13} \qquad \therefore \cos\theta = \frac{5}{13} \qquad \therefore \tan\theta = \frac{12}{5}$$

glossary

Example 4

a For angle A, find sin A, cos A and tan A.
b For angle B, find sin B, cos B, and tan B.



Solution

a For angle A:
H is 29
O is 20
A is 21

$$\sin = \frac{\text{O}}{\text{H}},\ \cos = \frac{\text{A}}{\text{H}},\ \tan = \frac{\text{O}}{\text{A}}$$

$$\therefore\ \sin A = \frac{20}{29}$$

$$\cos A = \frac{21}{29}$$

$$\tan A = \frac{20}{21}$$

b For angle B:
H is 29
O is 21
A is 20

$$\therefore\ \sin B = \frac{21}{29}$$

$$\cos B = \frac{20}{29}$$

$$\tan B = \frac{21}{20}$$

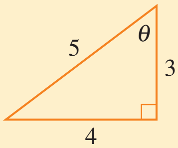
Exercise 8-02

Example 3

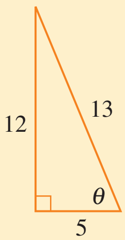
Spreadsheet 8-01
Trigonometric ratios

1 Find sin θ, cos θ and tan θ for each of the following triangles:

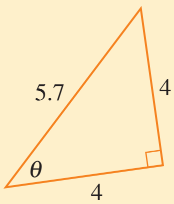
a



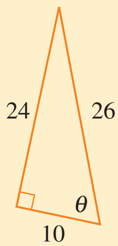
b



c

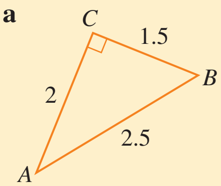


d

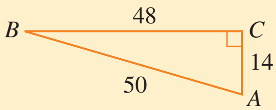


2 Find sin A, cos A and tan A for each of the following triangles:

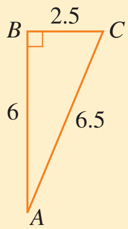
a



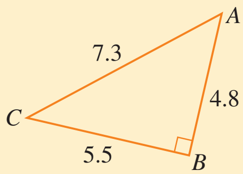
b



c



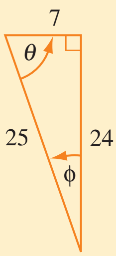
d



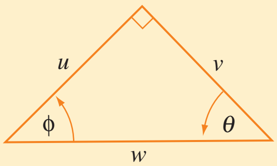
Example 4

3 In each of these triangles find the sine, cosine and tangent ratios for:
i angle θ **ii** angle ϕ

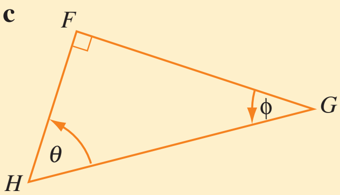
a



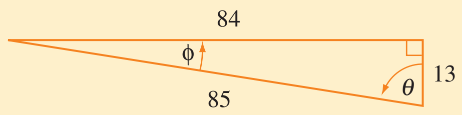
b



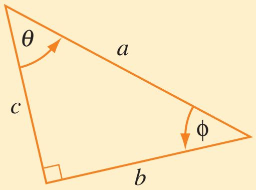
c



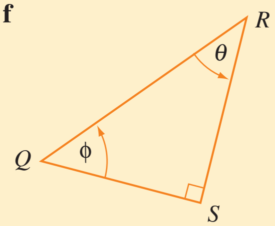
d



e



f



4 Sketch a right-angled triangle, *ABC*, for each of the following trigonometric ratios. Then find the length of the third side and write the other two ratios.

a $\tan A = \frac{5}{12}$ **b** $\sin B = \frac{3}{5}$ **c** $\cos C = \frac{8}{17}$

d $\sin A = \frac{4}{5}$ **e** $\cos C = \frac{1}{3}$ **f** $\tan B = 1$

Worksheet 8-03
Trigonometric calculations

The calculator in trigonometry

A scientific calculator can be used to find the trigonometric ratios of angles. We can use this information to find any unknown lengths and angles in a right-angled triangle. The order in which keys are used depends on the brand and model of your calculator. Consult your calculator manual or ask your teacher for help.

STAGE 5.2

Using the calculator

In trigonometry, angles are usually measured in degrees, minutes and seconds. The key relationships are:

1 degree = 60 minutes	and	1 minute = 60 seconds
$1° = 60'$		$1' = 60''$

An angle of $27°36'15''$ is an acute angle of 27 degrees, 36 minutes and 15 seconds. (In size it lies about halfway between $27°$ and $28°$.) To enter angle sizes involving degrees and minutes into a calculator, use the ° ′ ″ or DMS (degrees–minutes–seconds) key.

Example 5

Convert:

a $46°45'$ to a decimal **b** $61.87°$ to degrees, minutes and seconds.

Solution

a Enter $46°45'$ as follows: — **Display**

46 ° ′ ″ 45 ° ′ ″ = — 46°45'00"

Press: ° ′ ″ — 46.75

$\therefore 46°45' = 46.75°$

b Enter 61.87: — **Display**

61.87 = — 61.87

Press: ° ′ ″ — 61°52'12"

$\therefore 61.87° = 61°52'12''$

Example 6

Round these angles, correct to the nearest degree.

a $48°24'$ **b** $64°30'$ **c** $15°42'$

Solution

$1° = 60'$, so angles with 30 minutes or more are rounded up to the nearest degree.

a $48°24' \approx 48°$ **b** $64°30' \approx 65°$ **c** $15°42' \approx 16°$

Example 7

Round these angles, correct to the nearest minute.

a $35°42'18''$ **b** $72°35'30''$ **c** $39°12'49''$

Solution

$1' = 60''$, so angles with 30 seconds or more are rounded up to the nearest minute.

a $35°42'18'' \approx 35°42'$ **b** $70°35'30'' \approx 70'36'$ **c** $39°12'49'' \approx 39°13'$

Example 8

Calculate, correct to four decimal places:

a $\cos 76°$ **b** $\tan 57.4°$ **c** $\sin 46°27'$

Solution

Make sure that your calculator is in the degree mode (DEG) or your answer will be incorrect.

a cos 76 = — **Display** 0.241921...

$\therefore \cos 76° \approx 0.2419$

b tan 57.4 = — **Display** 1.563656...

$\therefore \tan 57.4° \approx 1.5637$

c sin 46 ° ′ ″ 27 ° ′ ″ = — **Display** 0.724773...

$\therefore \sin 46°27' \approx 0.7248$

Example 9

Evaluate, correct to two decimal places:

a $37 \times \tan 23°$ **b** $68.3 \times \sin 38°25'$ **c** $\dfrac{23}{\cos 18°50'}$

Solution

a 37 × tan 23 =

$$\therefore 37 \times \tan 23° = 15.705\,568\ldots$$
$$\approx 15.71$$

b 68.3 × sin 38 ° ′ ″ 25 ° ′ ″ =

$$\therefore 68.3 \times \sin 38°25' = 42.439\,961\ldots$$
$$\approx 42.44$$

c 23 ÷ cos 18 ° ′ ″ 50 ° ′ ″ =

$$\therefore \frac{23}{\cos 18°50'} = 24.301\,038\ldots$$
$$\approx 24.301$$

STAGE 5.2

Just for the record

Degrees, minutes and seconds

The development of trigonometry began with Hipparchus in Greece around 150 BC, when early astronomers tried to solve problems dealing with the positions and apparent movements of the stars and planets. Hipparchus made a table of chords, which today is known as a table of sines.

Measuring angles

In 2000 BC, the Babylonians lived where Iraq is today. The Babylonians invented the units for measuring angles and time. They believed the Earth took 360 days to travel around the Sun and so a complete revolution was divided into 360 equal parts called **degrees**. As measuring devices and calculations required greater precision, each degree was subdivided into 60 equal parts called **minutes**, and these were further divided into 60 parts called **seconds**.

The word 'minute' has different meanings. When pronounced '*my-newt*', it means tiny, but this meaning is still related to the minute as a unit of measurement. A minute is a tiny fraction of a degree or hour, and comes from the Latin *pars minuta prima*, meaning the first (*prima*) division of a degree or an hour.

The word 'second' means 'coming after first', and this meaning is also related to the second as a unit of measurement. Find out how.

Exercise 8-03

Example 5

1 Use your calculator to express the following as decimals.

a 57°55′ **b** 84°12′ **c** 16°41′ **d** 24°30′
e 50°53′ **f** 75°8′ **g** 30°30′25″ **h** 82°40′15″

2 Use your calculator to express the following in degrees, minutes and seconds.

a 33.76° **b** 14.1° **c** 78.15° **d** 55.5°
e 16.7° **f** 79.23° **g** 43.9° **h** 27.07°

Example 6

3 Round each of the following, correct to the nearest degree.

a 56.18° **b** 56.5° **c** 56.75°
d 27°18′ **e** 27°30′ **f** 27°54′
g 33°41′55″ **h** 33°7′5″ **i** 33°14′35″

Example 7

4 Round each of the following, correct to the nearest minute:

a 68°39′42″ **b** 68°39′21″ **c** 68°39′30″
d 18°30′27.2″ **e** 18°30′55.3″ **f** 18°30′9.5″

5 Express each of the following in degrees correct to one decimal place:

a 38°14′ **b** 66°7′ **c** 27°11′38″
d 45°50′37″ **e** 8°25′11.3″ **f** 81°3′22.5″

6 Round each of the following correct to the nearest minute:

a 34.45° **b** 71.087° **c** 5.4829°
d 69.4545° **e** 41.31° **f** 50.213 67°

Example 8

7 Evaluate each of the following, correct to two decimal places:

a cos 68° **b** tan 54° **c** sin 84°
d sin 15° **e** cos 45° **f** tan 38°

8 Evaluate each of the following, correct to four decimal places:

a $\sin 23°$ **b** $\cos 60.1°$ **c** $\tan 39.55°$
d $\cos 18°24'$ **e** $\tan 75°57'$ **f** $\sin 56°13'$

9 Calculate each of the following, correct to two decimal places:

Example 9

a $15 \times \sin 46°$ **b** $27 \times \tan 37°$ **c** $13.5 \times \cos 15°$
d $14 \div \tan 58$ **e** $\dfrac{34.5}{\sin 80°}$ **f** $6.7 \times \cos 35$
g $\dfrac{120}{\sin 72°}$ **h** $28 \times \cos 17°$ **i** $\dfrac{84.8}{\tan 68°}$

10 Evaluate, correct to two decimal places:

a $12 \times \tan 8°25'$ **b** $66.2 \times \cos 81°42'$ **c** $18.53 \times \sin 11.813°$
d $\dfrac{27}{\cos 38.35°}$ **e** $\dfrac{44.5}{\tan 65°58'}$ **f** $\dfrac{200}{\sin 54°45'}$
g $24 \div \tan 36°42'$ **h** $\dfrac{50}{\sin 70°}$ **i** $\dfrac{15.7}{\sin 30°}$
j $\dfrac{12.8}{\cos 46°22'}$ **k** $5.3 \sin 46.7°$ **l** $\dfrac{75.8}{\tan 23°32'}$
m $19.7 \div \sin 38°45'$ **n** $8.9 \times \tan 72.3°$ **o** $35.8 \times \cos 87°24'$

Working mathematically

Applying strategies and reasoning: Finding an angle, given a trigonometric ratio

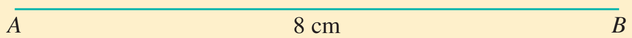
You will need: a ruler, a protractor and a calculator.

1 a Given that $\tan A = \frac{3}{8}$, measure the size of $\angle A$ to the nearest degree.

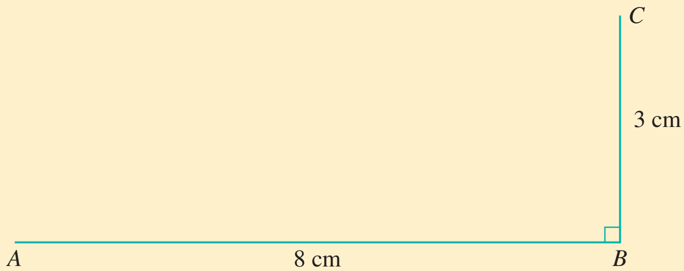
Method 1: Use a scale drawing.

Since $\tan A = \frac{3}{8} = \dfrac{\text{opposite}}{\text{adjacent}}$, construct a right-angled triangle ABC with the opposite side to $\angle A$ measuring 3 cm and the adjacent side measuring 8 cm.

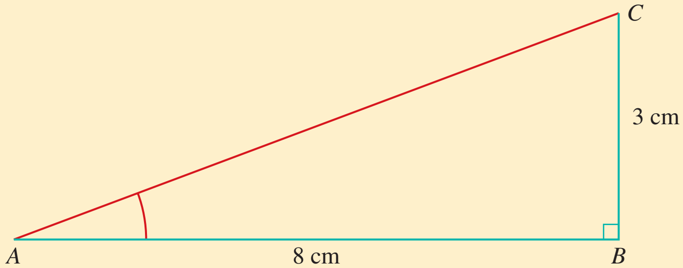
i Draw a horizontal interval AB of length 8 cm.



ii At B, draw a perpendicular interval BC of length 3 cm.



iii Join A to C.



iv Measure the size of $\angle A$ (to the nearest degree).

Method 2: Use 'guess and check' with a calculator.

i Express $\tan A = \frac{3}{8}$ as a decimal.

$\tan A = 0.375$

ii Copy and complete the table to find $\angle A$.

Guess	tan *A*
$A = 40°$	
$A = 50°$	
$A = \ldots$	

b Repeat the process in part **a** to find $\angle A$, given that:

i $\cos A = \frac{2}{5}$ **ii** $\tan A = \frac{7}{10}$ **iii** $\sin A = 0.9$

c Compare your results with those of other students.

2 a Copy the table below and complete it using your calculator (correct to four decimal places where necessary).

A	sin *A*	cos *A*	tan *A*
0°			
45°			
90°			

b The display for tan 90° is $\boxed{\text{ERROR}}$.

i Give an explanation for this.

ii Find the value of tan 89.9° instead of tan 90°.

c Use your table to state whether each of the following is true (T) or false (F).

i $\sin A = 0.74$
$\therefore A$ is less than 45°

ii $\tan \theta = 0.95$
$\therefore \theta$ is less than 45°

iii $\cos Y = 0.4183$
$\therefore Y$ is greater than 45°

iv $\sin \alpha = 0.93$
$\therefore \alpha$ is greater than 45°

3 Use the 'guess and check' method to find B in each case, correct to the nearest degree.

a $\tan B = 1.356$ **b** $\sin B = 0.2715$ **c** $\tan B = 0.3$

d $\cos B = \frac{7}{11}$ **e** $\sin B = \frac{11}{20}$ **f** $\cos B = 0.0813$

glossary

Using the calculator to find angles

A calculator can be used to find an angle if we are given the trigonometric ratio of the angle. For example, if $\sin A = 0.76$, then angle A can be found by using the **inverse sin** or $\mathbf{sin^{-1}}$ function.
The $\sin^{-1}$ function is activated by keying **shift** **sin** or **INV** **sin**.

Worksheet 8-03 Trigonometric calculations

Example 10

If $\sin\theta = 0.65$, find the angle θ, correct to the nearest degree.

Solution

shift **sin** 0.65 **=**

$\theta = 40.541\ \ldots$

$\therefore\ \theta \approx 41°$

Example 11

If $\tan X = 3.754$, find the angle X, correct to the nearest minute.

Solution

shift **tan** 3.754 **=** **° ′ ″**

$X = 75°5'1.62''\ \ldots$

$\therefore\ X \approx 75°5'$

Example 12

If $\cos\theta = \frac{5}{7}$, find the angle θ, correct to one decimal place.

Solution

shift **cos** 5 **a b/c** 7 **=**

$\theta = 44.415\ \ldots$

$\therefore\ \theta \approx 44.4°$

STAGE 5.2

Exercise 8-04

Example 10

1 Find the angle θ in each case, correct to the nearest degree.

a $\cos\theta = 0.76$ **b** $\tan\theta = 2.0532$ **c** $\sin\theta = \frac{\sqrt{3}}{2}$

d $\tan\theta = 6$ **e** $\sin\theta = \frac{7}{8}$ **f** $\cos\theta = \frac{13}{15}$

g $\sin\theta = \frac{1}{10}$ **h** $\cos\theta = \frac{1}{\sqrt{3}}$ **i** $\tan\theta = \sqrt{3}$

j $\cos\theta = 0.1352$ **k** $\tan\theta = 8.836$ **l** $\sin\theta = 0.4987$

SkillBuilders 22-15 to 22-16 Finding the angle

Example 11

2 For each of the following, find the angle A, correct to the nearest minute.

a $\tan A = \frac{15}{7}$ **b** $\sin A = 0.815$ **c** $\cos A = \frac{4}{5}$

d $\cos A = 0.9387$ **e** $\tan A = 4.8$ **f** $\cos A = \frac{\sqrt{5}-1}{4}$

g $\sin A = 5 : 11$ **h** $\sin A = 0.88$ **i** $\tan A = 15.07$

j $\cos A = 0.\dot{3}$ **k** $\tan A = \frac{\sqrt{2}}{2}$ **l** $\sin A = \frac{7}{9}$

STAGE 5.2

Example 12

3 Find the angle X, correct to one decimal place.

a $\sin X = 0.1$ **b** $\cos X = \frac{5}{11}$ **c** $\sin X = 0.71$ **d** $\tan X = 4:3$

e $\sin X = 0.4044$ **f** $\tan X = 1.369$ **g** $\cos X = \frac{19}{20}$ **h** $\tan X = 0.45$

i $\cos X = \frac{2}{7}$ **j** $\tan X = 0.502$ **k** $\cos X = 3:10$ **l** $\sin X = 0.\dot{6}$

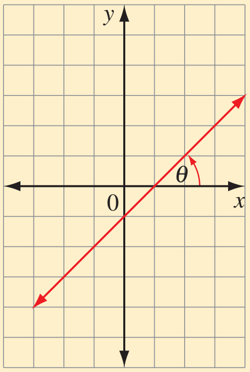
Working mathematically

Reasoning and reflecting: The tangent ratio and the gradient of a line

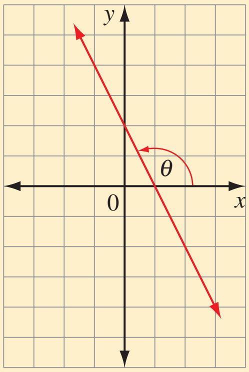
1 a The gradient, m, of a line is given by: $m = \dfrac{\text{rise}}{\text{run}}$

Find the gradient of each of the lines below, expressing your answers as decimals.

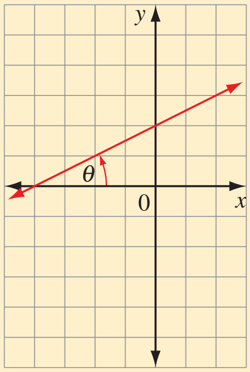
i



ii



iii



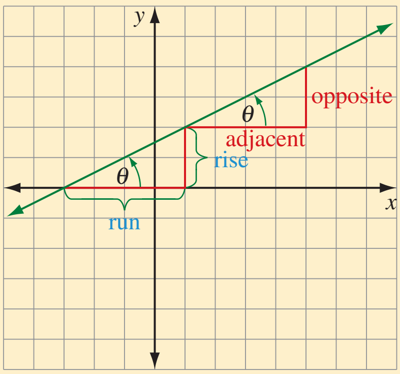
b In each of the diagrams above, θ is the angle the graph line makes with the positive direction of the x-axis. (This is also called the **angle of inclination**.) For each line in part **a** use a protractor to find the size of θ and then use a calculator to calculate $\tan\theta$ (correct to three decimal places).

c What do you notice about your results for parts **a** and **b**?

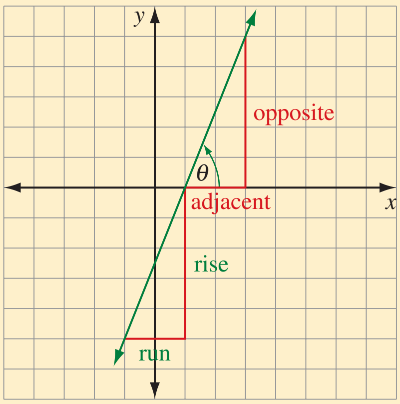
2 For each of the following lines find (as a fraction):

i m (*Remember*: $m = \dfrac{\text{rise}}{\text{run}}$) **ii** $\tan\theta$ (*Remember*: $\tan = \dfrac{\text{opposite}}{\text{adjacent}}$)

a



b



What do you notice about your results?

3 Copy and complete:

If θ is the angle between a line and the positive direction of the x-axis, then the gradient, $m =$ _________.

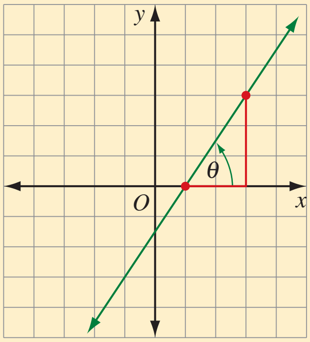
4 a Copy and complete the following to find θ:

$m =$ _____

$\therefore\ m = \tan\,\theta =$ _____

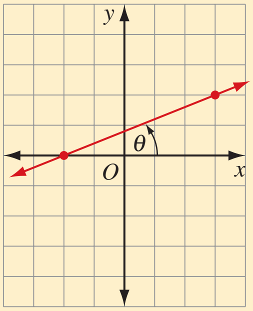
$\therefore\ \theta =$ _____ (to the nearest degree)

(Use **shift** **tan** to find θ.)

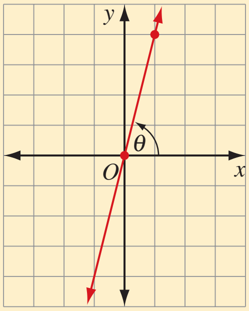


b By first finding the gradient, m, use $m = \tan\,\theta$ to determine θ for each of the following. Check your answers with those of other students.

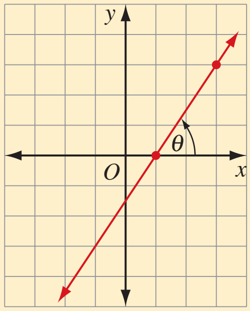
i



ii



iii



5 a Use $m = \tan\,\theta$ to calculate θ if:

i $m = 3$ **ii** $m = \frac{2}{3}$ **iii** $m = 0.84$ **iv** $m = 12.6$

b Use $m = \tan\,\theta$ to calculate m if:

i $\theta = 60°$ **ii** $\theta = 45°$ **iii** $\theta = 36°$ **iv** $\theta = 135°$

Just for the record

$\sin^{-1}$ notation

It may be easy to say 'find the angle whose sine is 0.3' but it is long-winded and inconvenient to write. (Try putting it on a calculator key!) Mathematicians, as always, looked for a shorter way of writing this phrase. They settled on the symbols $\sin^{-1} 0.3$.

The $^{-1}$ in $\sin^{-1} 0.3$ means the inverse or opposite and is *not* a power. (Unfortunately mathematics is not always consistent.)

You need to remember:

- $\sin^{-1} x$ means the angle whose sine is x.
- $\cos^{-1} x$ means the angle whose cosine is x.
- $\tan^{-1} x$ means the angle whose tangent is x.

So $\sin^{-1} 0.3$ is 17.458° or 17°27′27″ because the sine of 17.458° is 0.3. Similarly, $\tan^{-1} 0.3$ is 16°42′, correct to the nearest minute.

Write in words what is meant by $\sin^{-1} 0.5$, $\tan^{-1} 0.5$ and $\cos^{-1} 0.5$.

glossary

Skillbank 8A

SkillTest 8-01
Finding 15%, 2.5%, 25% or 12.5%

Finding 15%, $2\frac{1}{2}$%, 25% and $12\frac{1}{2}$%

We already now that:

- to find 10% or $\frac{1}{10}$ of a number, we simply divide that number by 10
- to find 5% of a number, we find 10% first, then halve it (since 5% is half of 10%).

So, to find 15% of a number, we can find 10% and 5% of the number separately, then add the answers together (since 15% = 10% + 5%).

1 Examine these examples:

a $15\% \times 80 = (10\% \times 80) + (5\% \times 80) = 8 + 4 = 12$

b $15\% \times \$170 = (10\% \times \$170) + (5\% \times \$170) = \$17 + \$8.50 = \$25.50$

c $15\% \times 3600 = (10\% \times 3600) + (5\% \times 3600) = 360 + 180 = 540$

d $15\% \times \$28 = (10\% \times \$28) + (5\% \times \$28) = \$2.80 + \$1.40 = \$4.20$

2 Now find 15% of each of these amounts:

a 120	**b** \$840	**c** 260	**d** \$202	**e** \$50	**f** 72
g \$180	**h** 400	**i** \$1600	**j** \$22	**k** 6000	**l** \$350

$2\frac{1}{2}$% is half of 5% so, to find $2\frac{1}{2}$% of a number, we first find 5%, then halve it.

3 Examine these examples:

a $2\frac{1}{2}\% \times 600$

$10\% \times 600 = 60,\ 5\% \times 600 = \frac{1}{2} \times 60 = 30$

$\therefore\ 2\frac{1}{2}\% \times 600 = \frac{1}{2} \times 30 = 15$

b $2\frac{1}{2} \times \$820$

$10\% \times \$820 = \$82,\ 5\% \times \$820 = \frac{1}{2} \times \$82 = \$41$

$\therefore\ 2\frac{1}{2}\% \times \$820 = \frac{1}{2} \times \$41 = \$20.50$

c $2\frac{1}{2}\% \times \$116$

$10\% \times \$116 = \$11.60,\ 5\% \times \$116 = \frac{1}{2} \times \$11.60 = \$5.80$

$\therefore\ 2\frac{1}{2}\% \times \$116 = \frac{1}{2} \times \$5.80 = \$2.90$

4 Now find $2\frac{1}{2}$% of each of these amounts:

a 400	**b** 6640	**c** \$2000	**d** \$880	**e** 1500	**f** \$232
g 5400	**h** \$904	**i** 3520	**j** \$700	**k** \$1160	**l** 2800

To find 25% of a number, we find 50% (or $\frac{1}{2}$) first, then halve it (since 25% is half of 50%). In other words, we halve the number twice to find 25%. (Alternatively, we find $\frac{1}{4}$ of the number because 25% = $\frac{1}{4}$).

5 Examine these examples:

a $25\% \times 700$

$50\% \times 700 = 350$

$\therefore\ 25\% \times 700 = \frac{1}{2} \times 350 = 175$

b $25\% \times \$86$

$50\% \times \$86 = \$43$

$\therefore\ 25\% \times \$86 = \frac{1}{2} \times \$43 = \$21.50$

c $25\% \times 204$

$50\% \times 204 = 102$

$\therefore\ 25\% \times 204 = \frac{1}{2} \times 102 = 51$

6 Now find 25% of each of these amounts:

a 2000	**b** \$80	**c** 18	**d** \$25	**e** \$324	**f** \$140
g 66	**h** 298	**i** \$780	**j** \$1700	**k** \$126	**l** 1160

To find $12\frac{1}{2}\%$ of a number, we find 25% first, then halve it (since $12\frac{1}{2}\%$ is half of 25%). In other words, we halve the number three times to find $12\frac{1}{2}\%$. (Alternatively, we find $\frac{1}{8}$ of the number because $12\frac{1}{2}\% = \frac{1}{8}$).

7 Examine these examples:

a $12\frac{1}{2}\% \times 400$

$50\% \times 400 = 200,\ 25\% \times 400 = 100$

$\therefore\ 2\frac{1}{2}\% \times 400 = \frac{1}{2} \times 100 = 50$

b $12\frac{1}{2}\% \times \$144$

$50\% \times \$144 = \$72,\ 25\% \times \$144 = \$36$

$\therefore\ 2\frac{1}{2}\% \times \$144 = \frac{1}{2} \times \$36 = \$18$

c $12\frac{1}{2}\% \times \$58$

$50\% \times \$58 = \$29,\ 25\% \times \$58 = \$14.50$

$\therefore\ 12\frac{1}{2}\% \times \$58 = \frac{1}{2} \times \$14.50 = \$7.25$

8 Now find $12\frac{1}{2}\%$ of each of these amounts:

a 1280	**b** \$12	**c** 60	**d** \$260	**e** \$540	**f** \$250
g 304	**h** 1360	**i** \$76	**j** \$480	**k** \$104	**l** 600

Using technology

Tables of trigonometric values and graphs

Spreadsheet 8-02 Tables of trigonometric values

A spreadsheet can be used to calculate the sine and cosine values for any angle. The graph tool (Chart Wizard or Graph Wizard) linked to the spreadsheet allows a graph or picture of the sine and cosine relationship to be drawn.

1 Set up your spreadsheet as shown below.

	A	B	C	D	E	F
5			**Trigonometry tables**			
6			**Angle (*x*)**	**sin (*x*)**	**cos (*x*)**	**tan (*x*)**
7			0			
8			=C7+5			
9						
10						
11						
12						

glossary

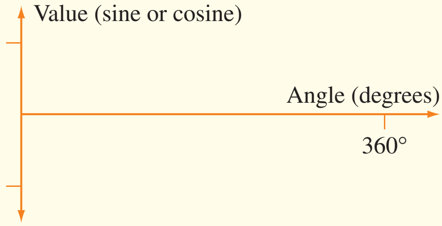
2 Enter the formula =sin(C7*PI()/180) in cell D7. What formulas would be used in cells E7 and F7?

The *PI()/180 part of the formula means 'multiply by $\frac{\pi}{180}$'. This converts angle degrees to **radians** (another way of measuring angles). The spreadsheet works in radians, not degrees.

3 What is the formula =C7+5 doing?

4 Use 'fill down' to produce a table of values of angles from 0° to 90° for the sine, cosine and tangent functions.

5 Use the graph tool linked to the spreadsheet to produce a line graph of the sine, cosine and tangent functions. Set the axes as shown.



6 Print your spreadsheet and graph and keep them.

Extension

7 Extend your table of angles from 90° to 360°.
Draw the sine, cosine and tangent graphs using the graph tool.

Geometry 8-02 Graph of sine

Geometry 8-03 Graphs of sine, cosine and tangent

Worksheet 8-04 Finding a missing side

Finding the length of a side

The length of a side of a right-angled triangle can be found using trigonometric ratios.

Step 1: Mark the hypotenuse (H), opposite side (O) and adjacent side (A) of the triangle.

Step 2: Decide whether sin, cos or tan should be used.

Step 3: Write an equation using the correct ratio.

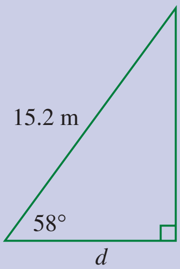
Step 4: Make the pronumeral the subject.

Step 5: Use a calculator to evaluate the answer.

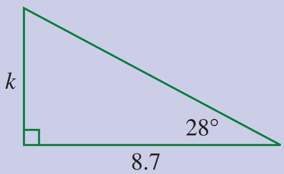
Example 13

In each of these triangles, find the value of the pronumeral, correct to two decimal places.

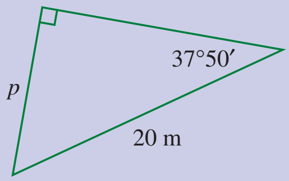
a



b



c



Solution

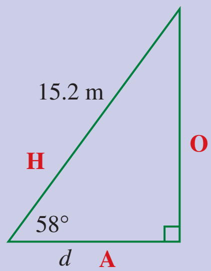
a SOH, CAH or TOA?

Use cos because A (adjacent side) and H (hypotenuse) are marked.

$$\cos 58° = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 58° = \frac{d}{15.2}$$

$$\cos 58° \times 15.2 = \frac{d}{15.2} \times 15.2 \qquad \text{(multiply both sides by 15.2)}$$



$15.2 \cos 58° = d$

$d = 15.2 \cos 58°$

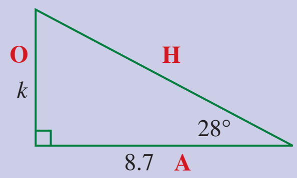
$= 8.0547 \ldots$

$\therefore d = 8.05$ m

The answer 8.05 m seems reasonable from the diagram since it is less than 15.2 m (the hypotenuse).

b SOH, CAH or TOA?

Use tan because O (opposite) and A (adjacent) are marked.



$$\tan 28° = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 28° = \frac{k}{8.7}$$

$$\tan 28° \times 8.7 = \frac{k}{8.7} \times 8.7 \qquad \text{(multiply both sides by 20)}$$

$8.7 \tan 28° = k$

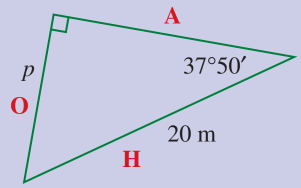
$k = 8.7 \tan 28°$

$= 4.6258 \ldots$

$\therefore k = 4.63$

c SOH, CAH or TOA?

Use sin because O (opposite) and H (hypotenuse) are marked.



$$\sin 37°50' = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 37°50' = \frac{p}{20}$$

$$\sin 37°50' \times 20 = \frac{p}{20} \times 20 \qquad \text{(multiply both sides by 20)}$$

$20 \sin 37°50' = p$

$p = 20 \sin 37°50'$

$= 12.267\,3 \ldots$

$\therefore p \approx 12.27$ m

Example 14

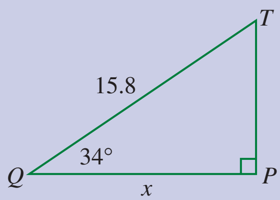
1 ΔPQT is right-angled at P, side $QT = 15.8$ m and $\angle Q = 34°$. Find the length of side QP, correct to one decimal place.

Solution

Draw a diagram. Let x be the length of side QP.

SOH, CAH or TOA?

Use cos because A (adjacent) and H (hypotenuse) are marked.



$$\cos 34° = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 34° = \frac{x}{15.8}$$

$$\cos 34° \times 15.8 = \frac{x}{15.8} \times 15.8 \qquad \text{(multiply both sides by 15.8)}$$

$15.8 \cos 34° = x$

$x = 15.8 \cos 34°$

$= 13.0987 \ldots$

≈ 13.1

$\therefore QP \approx 13.1$ m

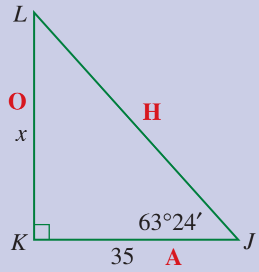
STAGE 5.2

glossary

STAGE 5.2

2 ΔJKL is right-angled at K, side $JK = 35$ m and $\angle J = 63°24'$. Find side LK, correct to the nearest metre.

Solution



Draw a diagram. Let x be the length of side LK.
SOH, CAH or TOA?
Use tan because O and A are marked.

$$\tan 63°24' = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 63°24' = \frac{x}{35}$$

$$\tan 63°24' \times 35 = \frac{x}{35} \times 35 \qquad \text{(multiply both sides by 35)}$$

$$35 \tan 63°24' = x$$

$$x = 35 \tan 63°24'$$

$$= 69.893\ \ldots$$

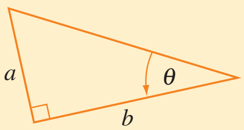
$$\approx 70 \text{ m}$$

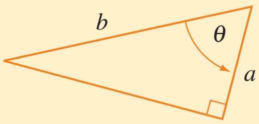
$$\therefore LK \approx 70 \text{ m}$$

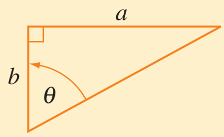
Exercise 8-05

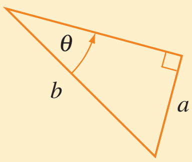
Spreadsheet 8-03 sine, cosine or tangent?

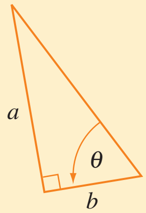
1 Which trigonometric ratio ($\sin\theta$, $\cos\theta$ or $\tan\theta$) is equal to $\frac{a}{b}$ in each case?

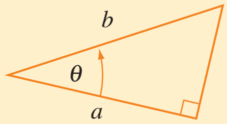
a


b


c


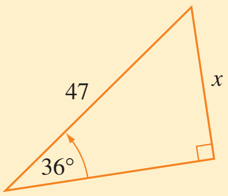
d


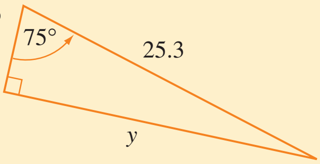
e


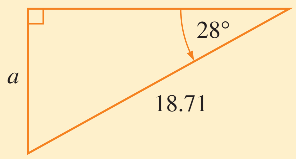
f


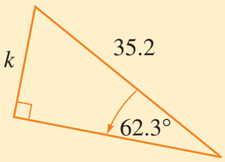
Example 13

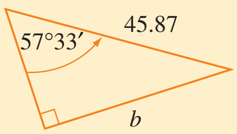
2 Calculate the value of each pronumeral, correct to two decimal places.

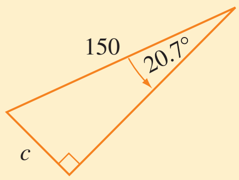
a


b


c


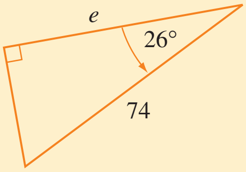
d


e


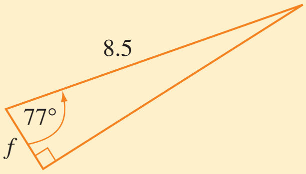
f


3 Calculate the value of each pronumeral, correct to one decimal place.

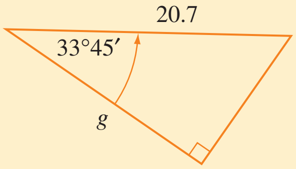
a



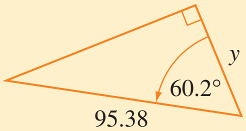
b



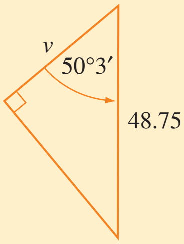
c



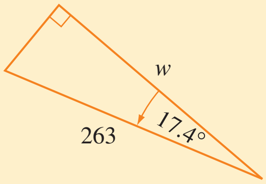
d



e

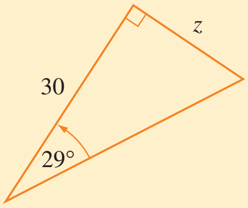


f

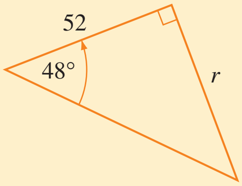


4 Calculate the value of each pronumeral, correct to the nearest whole number.

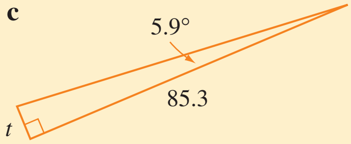
a



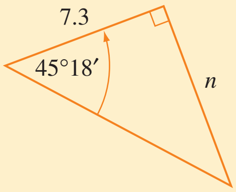
b



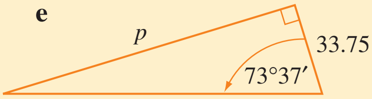
c



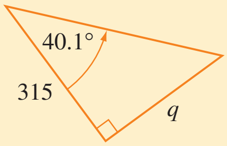
d



e

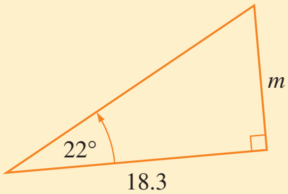


f

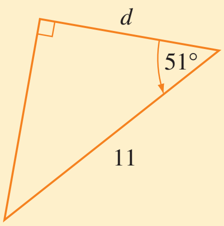


5 Calculate the value of each pronumeral, correct to one decimal place.

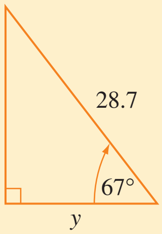
a



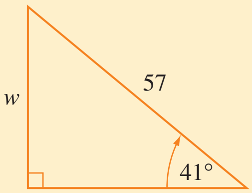
b



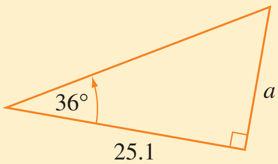
c



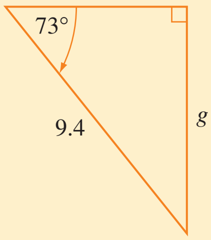
d



e



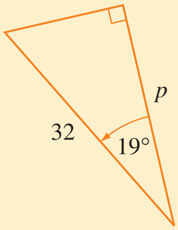
f



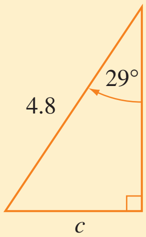
glossary

STAGE 5.2

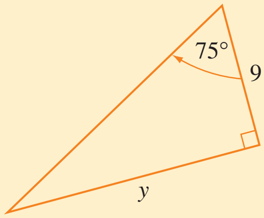
g



h

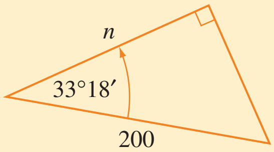


i

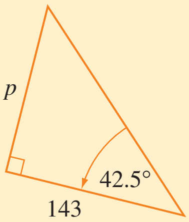


6 Use either the sine, cosine or tangent ratio to find each pronumeral, correct to one decimal place.

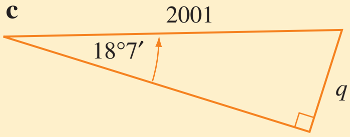
a



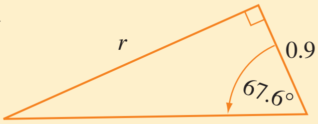
b



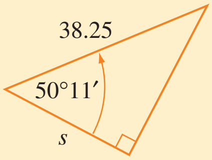
c



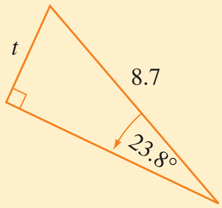
d



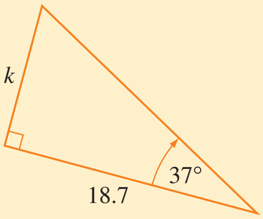
e



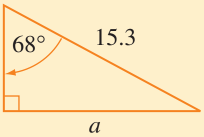
f



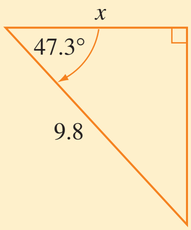
g



h

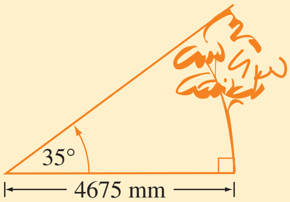


i

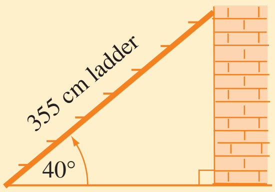


7 Use sine, cosine or tangent to find the required lengths, correct to the nearest unit.

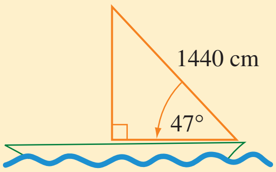
a How high is the tree?



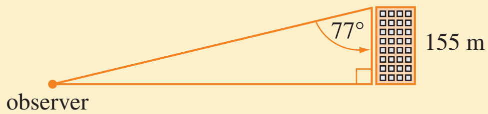
b How high is the wall?



c How high is the mast?

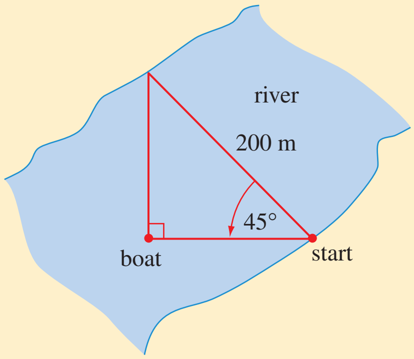


d How far is the observer from the base of the building?

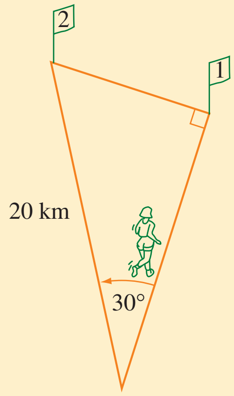


glossary

e How far is the boat from the start?



f How far is it to the first checkpoint from the start?



Example 14

8 For each of the following, mark the given information on a diagram.

a ΔABC is right-angled at B, side $AC = 14.8$ m and $\angle C = 56°$. Find the length of side AB, correct to one decimal place.

b ΔMNR is right-angled at M, side $MN = 19$ cm and $\angle N = 27°14'$. Find the length of MR, correct to the nearest centimetre.

c In ΔPQT, $\angle P = 90°$, $\angle T = 55.86°$ and $PT = 43.5$ m. Find the length of PQ, correct to two decimal places.

d In ΔXYW, $\angle X = 90°$, $\angle Y = 43.7°$ and $WY = 8.34$ m. Find the length of XW, correct to two decimal places.

e ΔAHK is right-angled at K, $\angle H = 76°$ and $AH = 13.9$ m. Find the length of HK, correct to one decimal place.

f In ΔBET, $\angle B = 90°$, $\angle T = 83°15'$ and $BT = 3.5$ m. Find the length of side BE, correct to the nearest metre.

9 Draw a diagram for each of the following.

a A tree casts a shadow 20 m long. If the Sun's rays meet the ground at 25°, find the height of the tree, correct to the nearest centimetre.

b A 6 m ladder is placed against a pole. If the ladder makes an angle of 17° with the pole, how high up the pole does the ladder reach? (Answer to the nearest millimetre.)

c A golfer is 180 m (in a straight line) from the eighth hole. The ball is hit 15° to the right of the hole but still ends up level with the hole. How far is the ball from the hole? (Answer to the nearest metre.)

d A car travels 3 km along a road that rises at an angle of 12°. What is the vertical height gained by the car? (Answer to the nearest metre.)

e A park is in the shape of a rectangle. A path 450 m in length crosses the park diagonally. If the path makes an angle of 36° with the longer side, find the dimensions of the park. (Answer to the nearest metre.)

f A truck travels at 80 km/h on a straight country road. In the distance, at an angle of 38° on the right, the driver sees a bushfire. Exactly 1.5 min later the car is directly opposite the fire. Calculate how close the car comes to the fire. (Answer to the nearest 0.1 kilometre.)

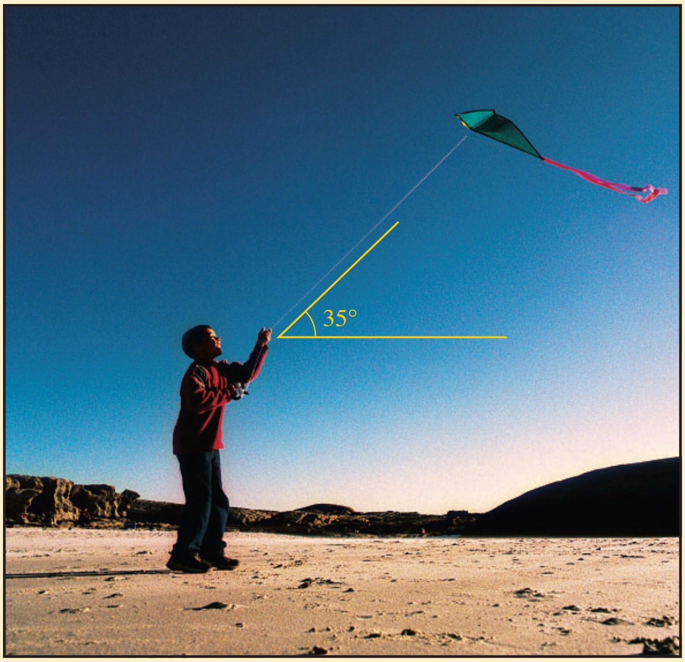
g A wheelchair ramp is 6 m long and makes an angle of 4.5° with the ground. How high is the top of the ramp above the ground (in metres, correct to two decimal places)?

h A boat is anchored by a rope 5.5 m long attached to the boat at the water line. If the rope makes an angle of 23° with the vertical, calculate the depth of the water (in metres, correct to one decimal place).

STAGE 5.2

i A kite is attached to a string 155 m long. The string makes an angle of 35° with the ground (as shown). Calculate, correct to the nearest metre, the height of the kite if the string is held 1 m above the ground. (Remember to add the 1 m to your answer.)

j A rectangular gate has a diagonal brace that makes an angle of 60° with the bottom of the gate. The length of the diagonal brace is 1860 mm. Calculate the width of the gate.



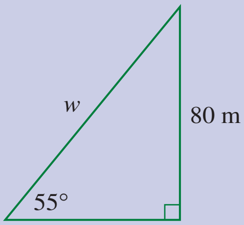
Worksheet 8-04 Finding a missing side

More about finding the length of a side

In the following examples, the missing side appears in the denominator of the equation.

Example 15

Find w, correct to two decimal places.

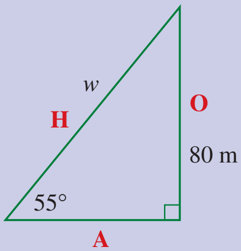


Solution

SOH, CAH or TOA?

Use sin because O and H are marked.

$$\sin 55° = \frac{80}{w}$$



Note that the missing side, w appears in the denominator (bottom) of the equation.

$$\sin 55° \times w = \frac{80}{w} \times w \quad \text{(multiplying both sides by } w\text{)}$$

$$w \sin 55° = 80$$

$$\frac{w \sin 55°}{\sin 55°} = \frac{80}{\sin 55°} \quad \text{(dividing both sides by } \sin 55°\text{)}$$

$$w = \frac{80}{\sin 55°}$$

$$= 97.6619\ldots$$

$$\approx 97.66 \text{ m}$$

glossary

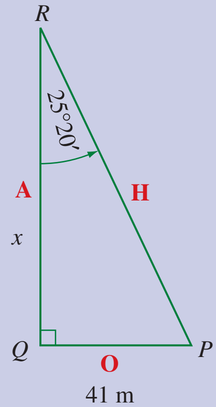
STAGE 5.2

Example 16

ΔPQR is right-angled at Q, $PQ = 41$ m and $\angle R = 25°20'$. Find RQ, correct to one decimal place.

Solution

Let x be the length of RQ.



Method 1:
SOH, CAH or TOA?

$$\tan 25°20' = \frac{41}{x}$$

$$\tan 25°20' \times x = \frac{41}{x} \times x \qquad \text{(multiply both sides by } x\text{)}$$

$$x \tan 25°20' = 41$$

$$\frac{x \tan 25°20'}{\tan 25°20'} = \frac{41}{\tan 25°20'} \qquad \text{(dividing both sides by } \tan 25°20'\text{)}$$

$$x = \frac{41}{\tan 25°20'}$$

$$= 86.605 \ldots$$

$$\therefore RQ = 86.6 \text{ m}$$

Method 2:
The length of RQ could also have been found using angle P. (Angles P and R are complementary.)
$\angle P = 90° - 25°20' = 64°40'$

Use $\tan 64°40' = \dfrac{x}{41}$ and, hence, find RQ. Compare the results found using the two methods.

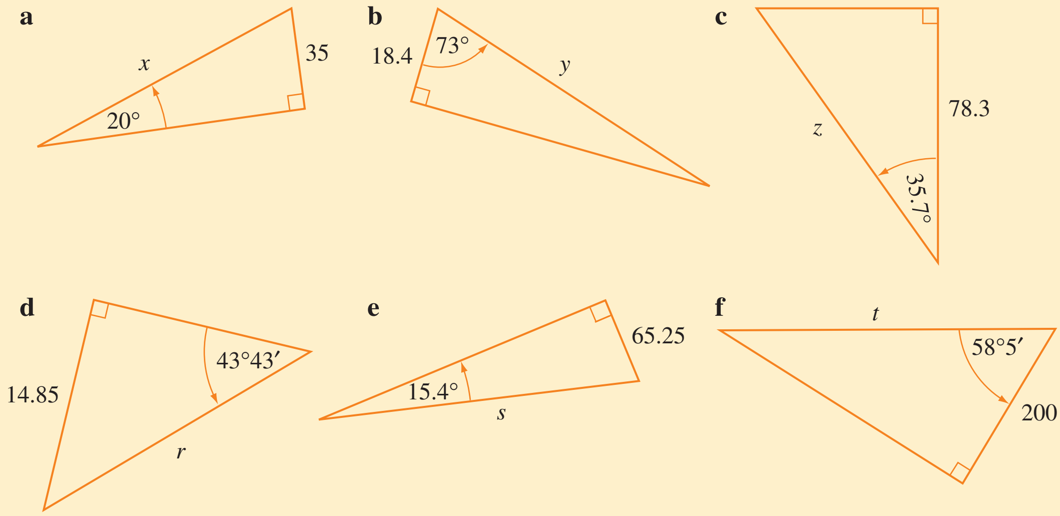
Exercise 8-06

Example 15

SkillBuilders 22-05 to 22-09 Using sine

SkillBuilders 22-10 to 22-13 Using sin–cos–tan

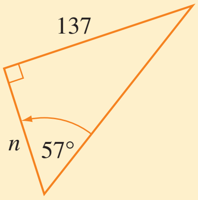
1 Find the value of each pronumeral, correct to one decimal place.



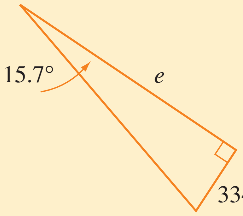
STAGE 5.2

2 Evaluate each unknown side, correct to the nearest whole number.

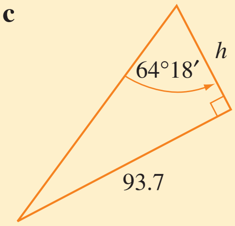
a



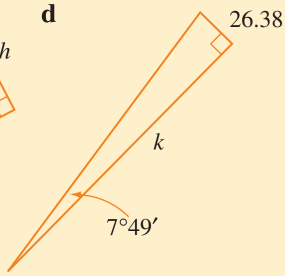
b



c

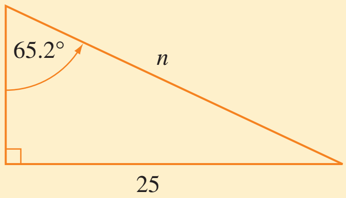


d

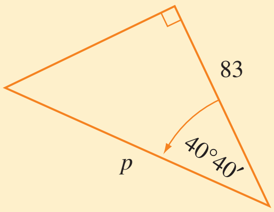


3 Use the sine, cosine or tangent ratio to find each pronumeral, correct to one decimal place.

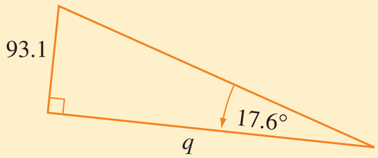
a



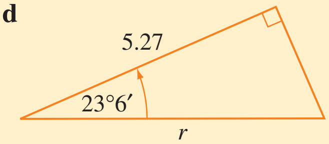
b



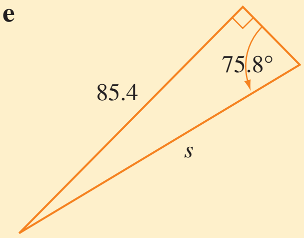
c



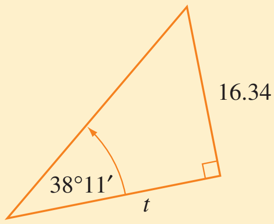
d



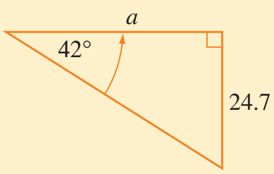
e



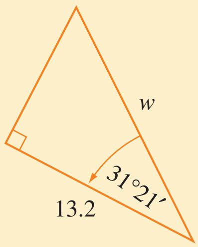
f



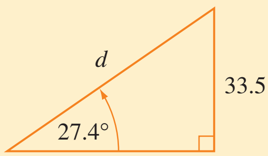
g



h

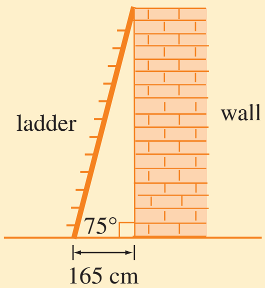


i

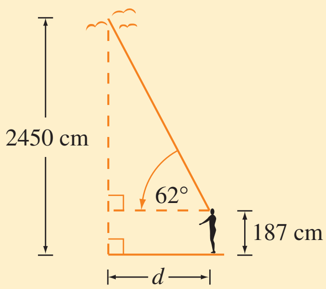


4 Use the sine, cosine or tangent ratio to find the required length in each case. Answer correct to the nearest unit.

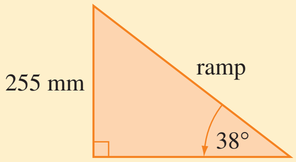
a How long is the ladder?



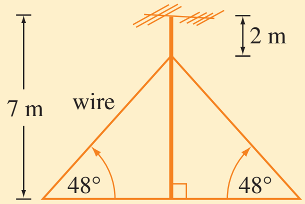
b How far is the observer from being directly under the birds?



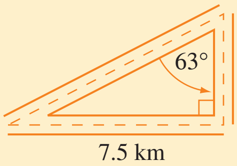
c How long is the ramp?



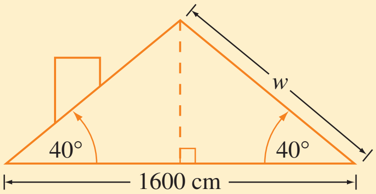
d How long is the support wire?



e How long is the shortest road?



f How long is the roof?



5 For each of the following, mark the given information on a diagram. Example 16

a In ΔKLW, $\angle L = 90°$, $KL = 12$ m and $\angle W = 75.2°$. Find KW, correct to the nearest metre.

b ΔCDE is right-angled at D, $\angle E = 36°$ and $DE = 4.95$ m. Find the length of side CD, correct to two decimal places.

c ΔLPW is right-angled at W, $LW = 230$ mm and $\angle L = 45°35'$. Find the length of LP, correct to the nearest millimetre.

d In ΔHMT, $\angle T = 90°$, $\angle M = 19°47'$ and side $HT = 18.4$ cm. Find the length of side HM, correct to one decimal place.

e ΔFGW is right-angled at F, $\angle W = 84°$ and $WF = 42.1$ m. Find the length of WG, correct to one decimal place.

f In ΔDNR, $\angle N = 90°$, $\angle D = 18.3°$ and $RN = 25.34$ m. Find the length of DR, correct to two decimal places.

6 **a** A ladder rests against a wall. The foot of the ladder is 355 cm from the wall and makes an angle of 63° with the ground. How long (to the nearest centimetre) is the ladder?

b A supporting wire is attached to the top of a flagpole. The wire meets the ground at an angle of 51° and the flagpole is 15 m high. How far from the base of the flagpole is the wire anchored to the ground? (Give your answer to the nearest 0.1 m.)

c A glider is flying at an altitude of 1.5 km. To land, the glider is pointed directly at the airstrip and begins descending at an angle of 18°. How far must the glider travel along this flight path before landing? (Give your answer to the nearest 0.1 km.)

d The entrance to the school library is 60 cm above ground level and is reached by stairs. A ramp for wheelchairs will be built to the entrance at an angle of 5° with the ground. How long (to the nearest 0.01 m) will the ramp be?

e At a shooting gallery, a shooter aims at the target directly ahead. Just before firing, the rifle is lifted 1° off the horizontal. The shot misses the target by 67 mm. How far is the shooter standing from the target? (Give your answer to the nearest 0.01 m.)

f A hot air balloon is anchored to the ground by a rope. When the balloon has drifted 20 m downwind, the rope makes an angle of 75° with the ground. How long is the rope (to the nearest 0.1 m)?

g A rectangular gate is 900 mm wide. The diagonal brace makes an angle of 58° with the horizontal. Calculate the length of the diagonal brace. (Give your answer correct to the nearest millimetre.)

h Dale is flying a kite at a height of 45 m.The string attached to the kite makes an angle of 72° with the vertical. Calculate the length of the string, correct to the nearest metre.

Worksheet 8-05
Finding a missing angle

Finding an angle

The size of an angle in a right-angled triangle can be found by following these steps.

Step 1: Mark the hypotenuse (H), opposite side (O) and adjacent side (A).

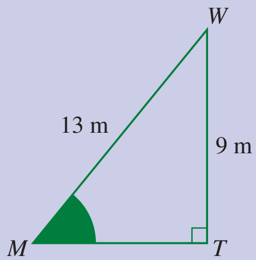
Step 2: Decide whether sin, cos or tan should be used.

Step 3: Write an equation using the correct ratio.

Step 4: Use a calculator to evaluate the angle.

Example 17

Find the size of angle M, correct to one decimal place.



Solution

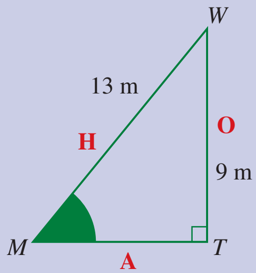
SOH, CAH or TOA?

Use sin because O and H are known.

$\sin M = \frac{9}{13}$ (enter **shift** **sin** 9 **a b/c** 13 **=**)

$\therefore M = 43.813\ldots$

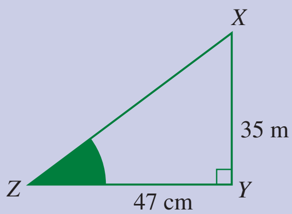
$\therefore M \approx 43.8°$ (which seems reasonable from the diagram)



STAGE 5.2

Example 18

ΔXYZ is right-angled at Y, side $XY = 35$ cm and side $YZ = 47$ cm.
Find $\angle Z$, correct to the nearest minute.



Solution

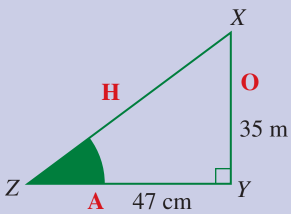
SOH, CAH or TOA?

Use tan because O and A are known.

$\tan Z = \frac{35}{47}$ (enter **shift** **tan** 35 **a b/c** 47 **=** **° ′ ″**)

$\therefore Z = 36°40'27.66''$

$\therefore Z \approx 36°40'$



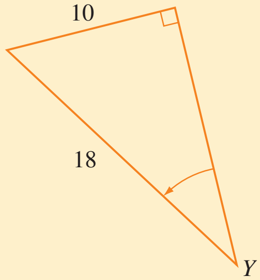
glossary

Exercise 8-07

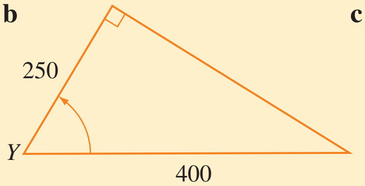
Example 17

1 In each of these right-angled triangles find $\angle Y$, correct to the nearest degree.

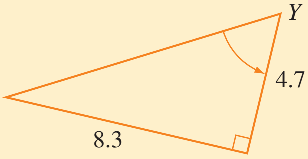
a



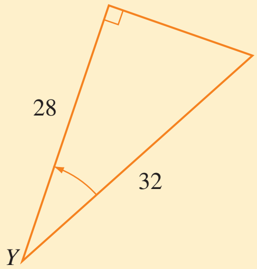
b



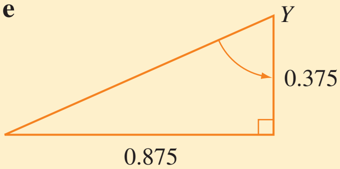
c



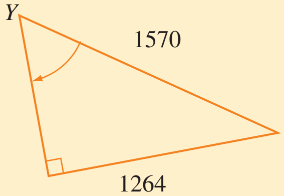
d



e

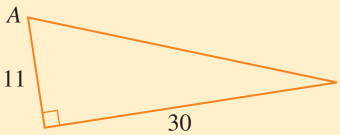


f

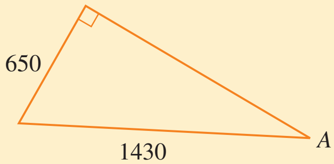


2 Find the size of angle A, correct to the nearest minute each time:

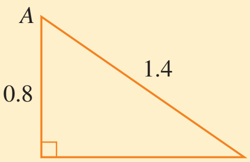
a



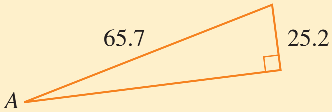
b



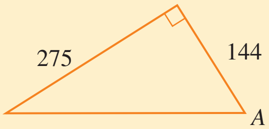
c



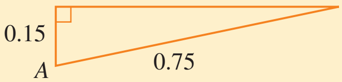
d



e

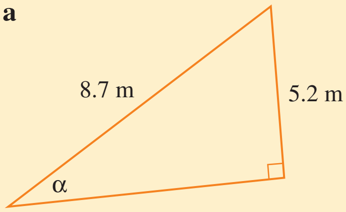


f

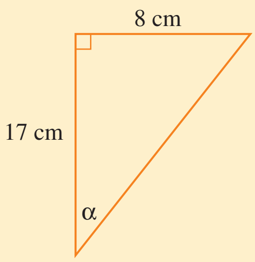


3 Find α, correct to one decimal place, in each of the following:

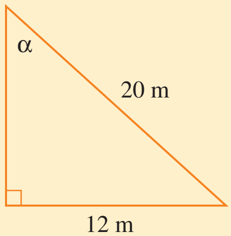
a



b



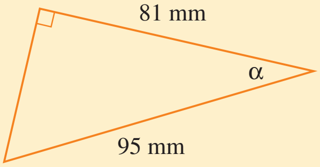
c



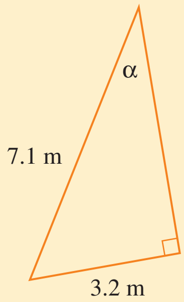
glossary

STAGE 5.2

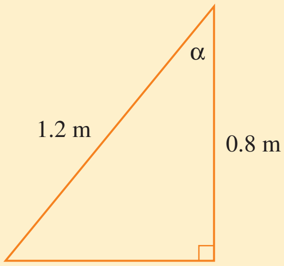
d



e

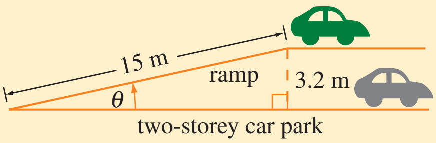


f

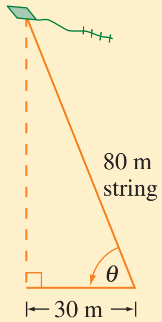


4 Find angle θ, correct to the nearest 0.1 degree, in each of the following:

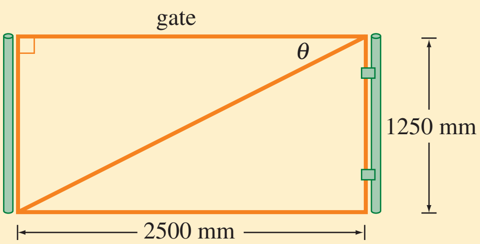
a



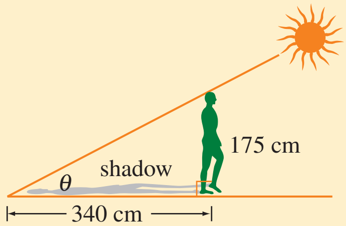
b



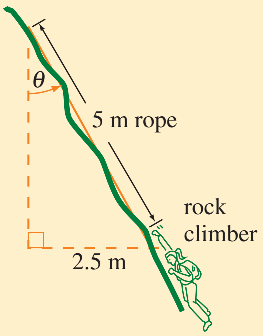
c



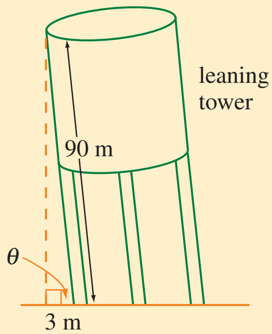
d



e



f



Example 18

5 For each of the following, first mark the given information on a diagram, then find the answer.

a In ΔXYW, $\angle X = 90°$, $XY = 8$ cm and $XW = 10$ cm. Find $\angle W$, correct to the nearest degree.

b In ΔFGH, $\angle G = 90°$, $GH = 3.7$ m and $FH = 19.5$ m. Find the size of angle F, correct to the nearest minute.

c ΔHTM is right-angled at T, $HM = 45$ m and $MT = 35$ m. Find $\angle M$, correct to one decimal place.

d ΔTSV is right-angled at S, $TV = 9.5$ cm, and $ST = 8.4$ cm. Find $\angle V$, correct to the nearest degree.

e In ΔBKW, $\angle K = 90°$, $KW = 5$ m and $BK = 23$ m. Find the size of angle W, correct to one decimal place.

f In ΔAEC, $\angle C = 90°$, $CE = 3.9$ m and $AE = 4.2$ m. Find $\angle A$, correct to the nearest minute.

6 For each of the following, first draw a diagram and mark the given information. Give your answers to the nearest degree.

a A stretch of freeway rises 55 m for every 300 m travelled along the road. Find the angle at which the road is inclined to the horizontal.

b A ladder 20 m long is placed against a building. If the ladder reaches 16 m up the building, find the angle of the inclination of the ladder to the building.

c The slope of a train track is 1 in 10 (where 10 represents the horizontal distance and 1 the vertical distance). Find the angle the train track makes with the horizontal.

d An aircraft is descending in a straight line to an airport. At a height of 1270 m, it is 1500 m horizontally from the airport. Find the angle of descent.

e A tree 8.5 m high casts a shadow 3 m long. At what angle does a ray from the Sun strike the Earth?

f A roof truss is in the shape of an isosceles triangle with a base span of 9600 mm and a height of 3300 mm. Find the pitch (angle) of the roof on either side.

g At a resort, an artificial beach slopes down at a steady angle inclined to the horizontal. After walking 8.5 m down the slope from the water's edge, the water has a depth of 1.6 m. At what angle is the beach inclined to the horizontal?

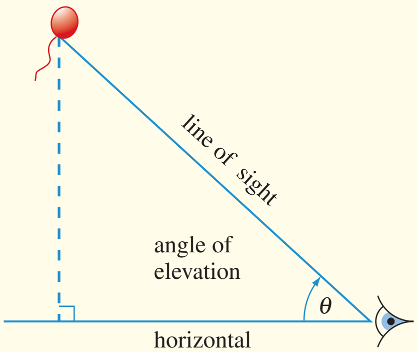
h A pile of wheat is in the shape of a cone that has a diameter of 35 m and measures 27 m up the slope to the apex. Calculate the angle of repose of the wheat (the angle the sloping side makes with the horizontal base).

i A 2.8 m vertical tent pole is supported by a 3.1 m rope. What angle does the rope make with the vertical pole?

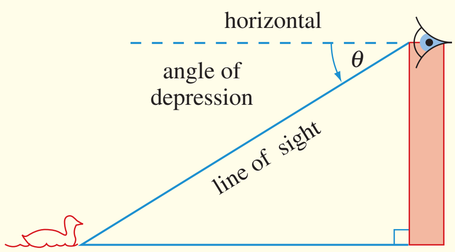
j A ship is anchored in water 40 m deep by a 65 m anchor chain (5 m of the chain is above the water). Find the angle at which the chain is inclined to the horizontal.

Angles of elevation and depression

Problems that involve measurements that cannot be made directly often use an angle of elevation or an angle of depression.



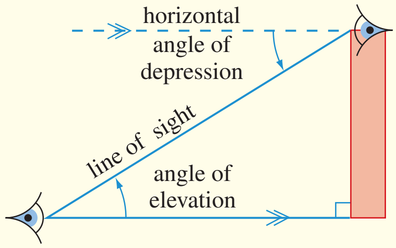
Angle of elevation is the angle of looking up, measured from the horizontal.



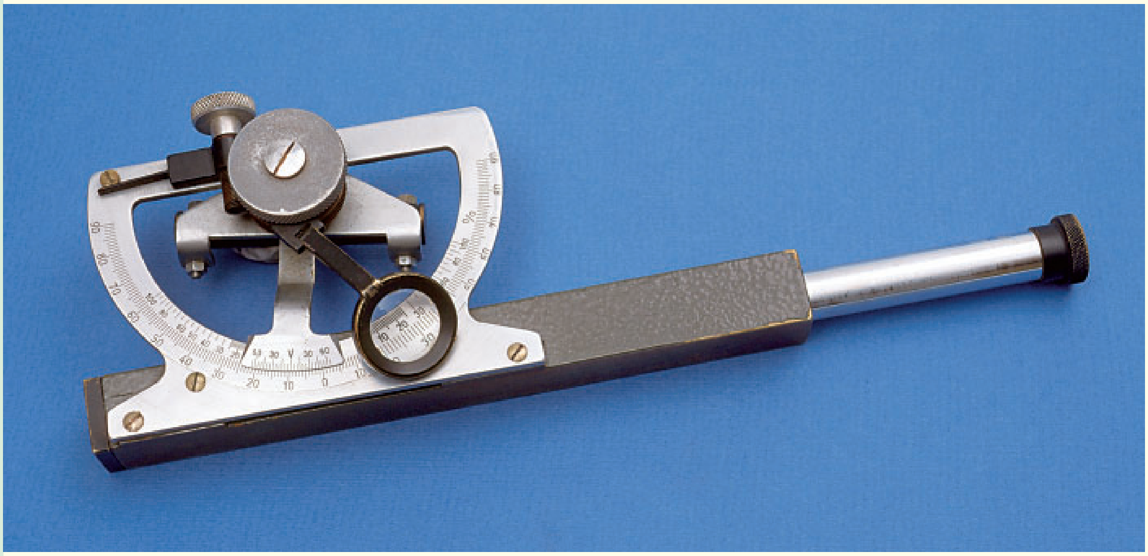
Angle of depression is the angle of looking down, measured from the horizontal.

Note: If there are two people looking at each other, where one is looking up (angle of elevation) and one is looking down (angle of depression), then:

Angle of elevation = angle of depression
(equal alternate angles)

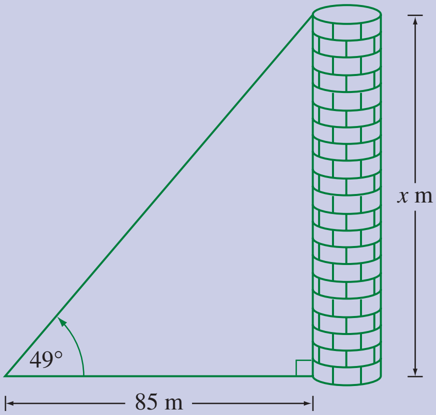


An instrument for measuring an angle of elevation or depression is a **clinometer**. It is like a 180° protractor with a sighting tube attached (see below).



Example 19

The angle of elevation of the top of a chimney is 49° at a distance of 85 m from its base. Find the height of the chimney, to the nearest metre.



Solution

Let the height be x metres.

$$\tan 49° = \frac{x}{85}$$
$$x = 85 \tan 49°$$
$$x = 97.781\,314\,61\ldots$$
$$\approx 98$$

$\therefore$ Height of chimney is 98 m.

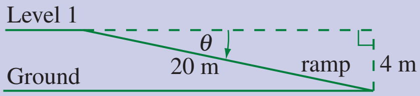
Example 20

1 The ramp from one level to the next in a car park is 20 m long and drops 4 m. Find the angle of depression of the ramp, to the nearest degree.

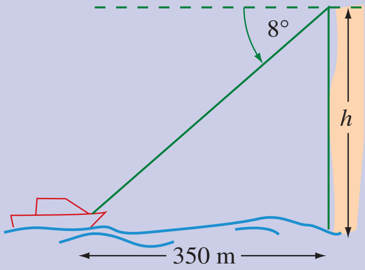
Solution

$$\sin \theta = \frac{4}{20}$$
$$\theta = 11.536\,959\,03\ldots°$$
$$\theta \approx 12°$$

$\therefore$ Angle of depression of the ramp is 12°.



2 The angle of depression of a boat from the top of a cliff is 8°. If the boat is 350 m from the base of the cliff, calculate the height of the cliff, correct to the nearest metre.



Solution

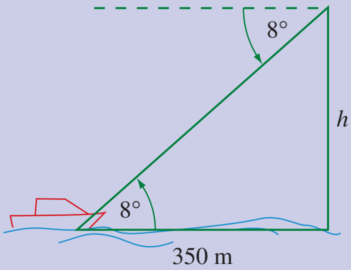
By alternate angles, the angle of elevation of the top of the cliff from the boat is also 8°.

$$\tan 8° = \frac{h}{350}$$

$$h = 350 \tan 8°$$

$$= 49.189 \ldots$$

$\therefore$ The height of the cliff is 49 m.



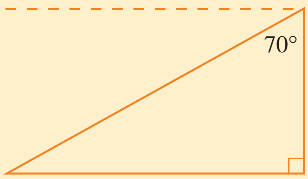
Exercise 8-08

1 Copy each of the following diagrams into your book.

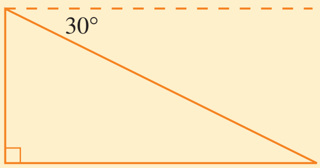
i Use θ to mark the angle of elevation.

ii Find the size of the angle of elevation.

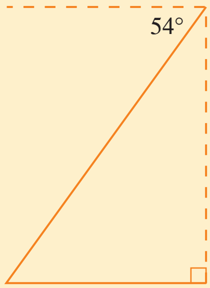
a



b



c

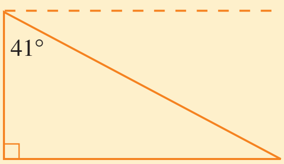


2 Copy each of the following diagrams into your book.

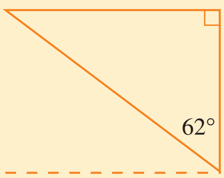
i Use θ to mark the angle of depression.

ii Find the size of the angle of depression.

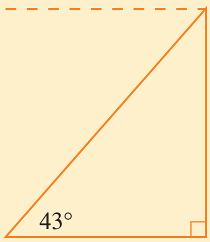
a



b

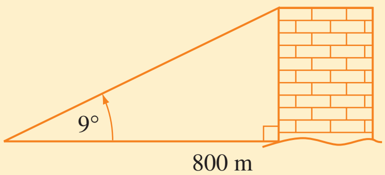


c

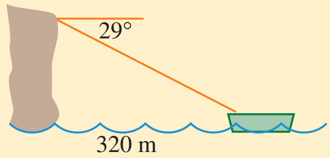


3 a A girl stands 800 m from the base of a building. Using a clinometer, she finds that the angle of elevation of the top is 9°. Find the height of the building, to the nearest centimetre.

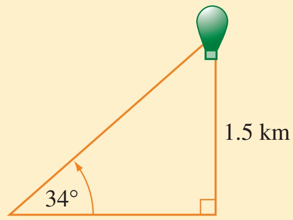
Example 19



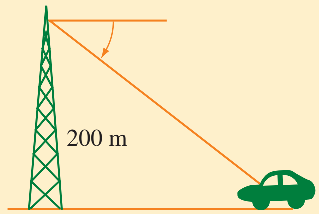
b A raft is 320 m from the base of a cliff. The angle of depression of the raft from the top of the cliff is 29°. Find the height of the cliff, to the nearest metre.



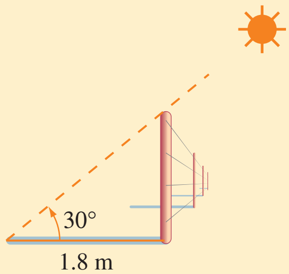
c The angle of elevation of a weather balloon at a height of 1.5 km is 34°. How far (to the nearest metre) from being directly under the balloon is the observer?



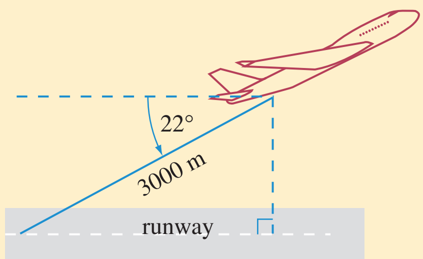
d From the top of a 200 m tower, the angle of depression of a car is 48°. How far is the car from the foot of the tower?



e When the angle of elevation of the Sun is 30°, a fence post casts a shadow of length 1.8 m. Calculate the height of the fence post (correct to two decimal places).



f The angle of depression of a plane's flight path is 22°. If the distance of the plane from the runway is 3000 m, calculate the altitude of the plane (correct to the nearest metre).



Example 20 STAGE 5.2

4 Draw a diagram to help you answer each of the following.

a In a concert hall, Bill is sitting 20 m from the stage by line of sight. He is also 5 m above the level of the stage. At what angle of depression is the stage?

b A 275 m radio mast is 1.7 km from a school. Find the angle of elevation of the top of the mast from the school.

c A monument 24 m high casts a shadow 20 m long. Calculate the angle of elevation of the Sun at this time of day.

d A plane is 340 m directly above one end of a 1000 m runway. Find the angle of depression to the far end of the runway.

5 a A flagpole is mounted on top of a tall building. At a distance of 250 m from the base of the building, the angles of elevation of the bottom and top of the flagpole are 38° and 40° respectively. Calculate the height of the flagpole, correct to one decimal place.

b A news helicopter hovers at a height of 500 m. The angles of depression of a fire moving in the direction of the helicopter are first 10° and then 15°. How far (to the nearest metre) has the fire moved between the two observations?

c An observer 174 cm tall is standing 11.6 m from the base of a flagpole. The angle of elevation to the top of the flagpole is 43°. How high is the flagpole, to the nearest centimetre?

d From an observer 1.8 km away from the base of a hill the angle of elevation to the bottom of a transmission tower on the hill is 5°. The angle of elevation from the observer to the top of the tower is 6.8°. Find the height of the tower to the nearest metre.

e The New Century Blimp is hovering 240 m above the finish line of a car race. The angle of depression to a car on the home straight is 19° initially, and 5 seconds later it is 37°.

i Calculate the distance (to the nearest metre) travelled by the car in that 5 seconds.

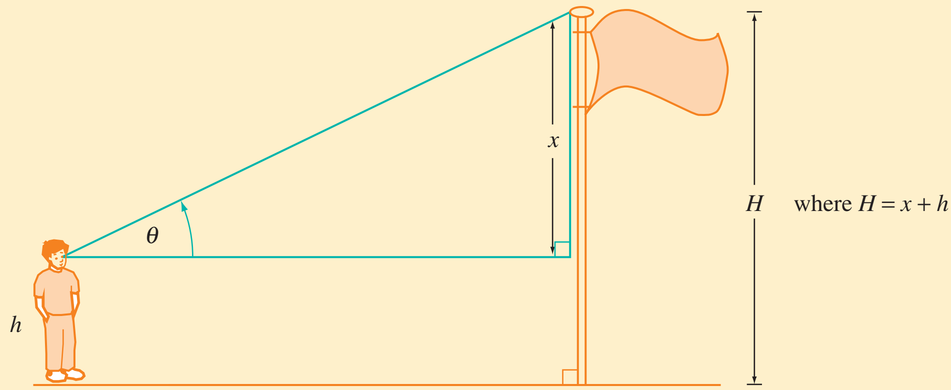
ii Find the speed of the car in km/h.

Working mathematically

Applying strategies and reasoning: Calculating the height of an object

Trigonometry can be used to find the heights of buildings, flagpoles and trees without actually measuring the objects. This can be done by measuring the distance along the ground from the base of the object to a person who then measures the angle to the top of the object.

For instance, the diagram on the next page shows the key lengths and angles involved in finding the height of a flagpole.



Note that distance h is not the height of the person, but the height of the person's **eye** above the ground (the 'eye height'). It is important to find the 'eye height' of the person who measures the angle to the top of the flagpole. Also, remember all practical measurements, of both lengths and angles, involve an error due to the accuracy of the measuring tools and the local conditions that affect the measurements. To overcome the errors made in measurement, take measurements from different positions around the object, calculate values for the height of the object, then take the average of these results.

You will need: measuring tape and a clinometer (or protractor) to measure the angle.

Step 1: Go outside and select an object whose height you would like to measure.

Step 2: Work with a partner to measure the distance along the ground, the 'eye height' of the person measuring the angle, and the angle to the top of the object. Record your information in a table.

Measurement	Length along ground, L	Angle, θ	'Eye height' of person, h
1			
2			
3			
4			

Step 3: Repeat Step 2 from different positions, with different persons measuring the angle.

1 For each set of measurements, calculate the height of the object, using trigonometry.

2 Calculate the average height of the object.

3 How accurate is your answer?

Skillbank 8B

SkillTest 8-02 Estimating answers

Estimating answers

A quick way of estimating the answer to a calculation is to round each number in the calculation to the nearest whole number or multiple of 10.

1 Examine these examples:

a $631 + 280 + 51 + 43 + 96 \approx 600 + 300 + 50 + 40 + 100$
$= (600 + 300 + 100) + (50 + 40)$
$= 1000 + 90$
$= 1090$ (Exact answer = 1101)

b $55 + 132 - 34 + 17 - 78 \approx 60 + 130 - 30 + 20 - 80$
$= (60 + 20 - 80) + (130 - 30)$
$= 0 + 100$
$= 100$ (Exact answer = 92)

c $78 \times 7 \approx 80 \times 7$
$= 560$ (Exact answer = 546)

d $510 \div 24 \approx 500 \div 20$
$= 25$ (Exact answer = 21.25)

2 Now estimate the answers to these:

a $27 + 11 + 87 + 142 + 64$ **b** $55 + 34 - 22 - 46 + 136$
c $684 + 903$ **d** $35 + 81 + 110 + 22 + 7$
e $517 - 96$ **f** $210 - 38 - 71 + 151 - 49$
g $766 - 353$ **h** 367×2
i 83×81 **j** 984×16
k $828 \div 3$ **l** $507 \div 7$

3 Examine these decimal examples:

a $4.78 \times 19.2 \approx 5 \times 20$
$= 100$ (Exact answer = 91.776)

b $75.13 \div 8.4 \approx 75 \div 8$
$< 80 \div 8$
< 10
≈ 9 (Exact answer = 8.944 …)

c $\dfrac{37.6 + 8.4}{41.2 - 12.7} \approx \dfrac{38 + 9}{40 - 13}$
$\approx \frac{47}{27}$
$\approx \frac{50}{30}$
≈ 1.6 (Exact answer = 1.614 …)

d $(2.79)^2 \times 17.67 \approx 3^2 \times 20$
$= 9 \times 20$
$= 180$ (Exact answer = 127.545 047)

Note: Estimates involving powers, such as squares and cubes, may be less accurate.

4 Now estimate the answers to these:

a 18.47×9.61 **b** 4.27×97.6
c $\dfrac{11.07 \times 18.4}{12.2}$ **d** $\dfrac{38.18}{17.2 - 9.6}$
e $34.75 + 18.01 - 14.4$ **f** $\dfrac{18.46 \times 4.97}{39.72 - 18.9}$
g $62.13 \div 10.7$ **h** $(5.02 \times 1.9)^2$
i $\dfrac{38.79 \times 19.6}{92.719}$

STAGE 5.2

Worksheet 8-06
A page of bearings

Worksheet 8-07
NSW map bearings

Worksheet 8-08
16 points of the compass

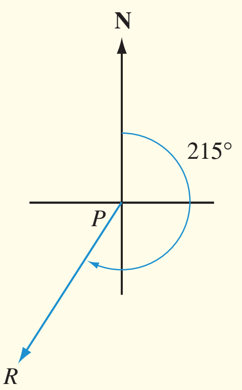
Bearings

Bearings are used in navigation. A **bearing** is an angle measurement used to describe precisely the direction of one location from another.

Three-figure bearings

Three-figure bearings, also called **true bearings**, use angles from 000° to 360° to show the amount of turning measured clockwise from north.

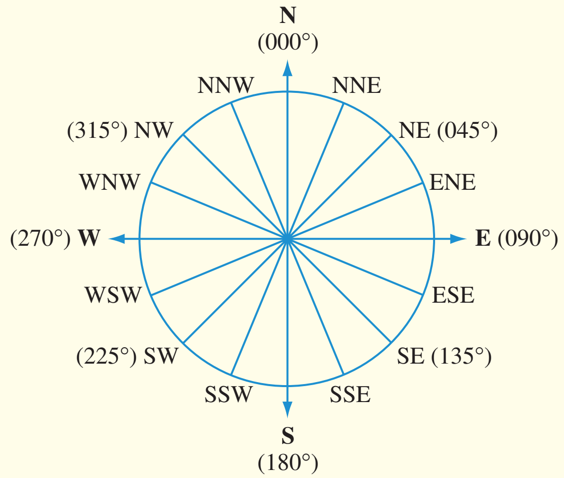
For example, if the bearing of a location *R* from *P* is 215°, this means that the direction of *R* from *P* is 215° measured clockwise from north.



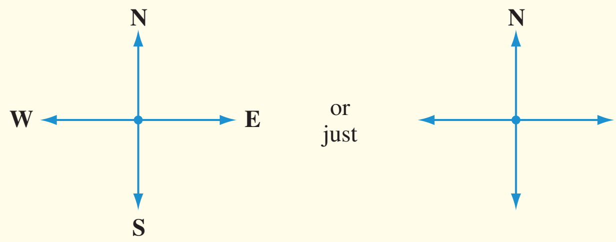
Compass bearings

Compass bearings, which use directions such as NW (north-west) or ESE (east-south-east), are still in use today (for example in weather reports).

The diagram on the right shows the sixteen points of a mariner's (a sailor's) compass. The angles between the sixteen points of a mariner's compass are each 22.5°.

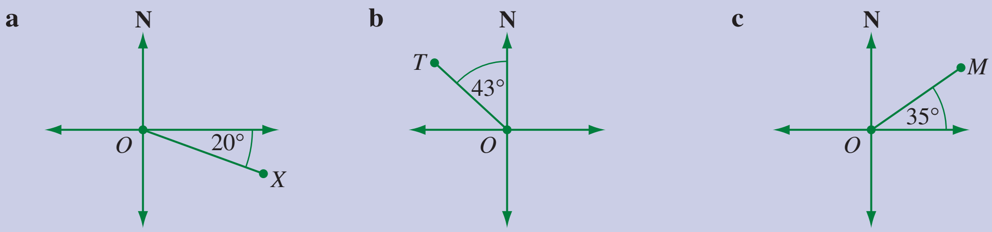


A simplified compass rose, showing north, south, east and west, is usually drawn when working with bearings.



Example 21

Write the bearing of each of the given points from *O*.



Solution

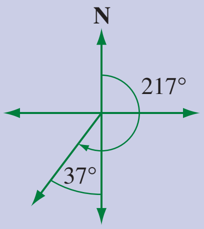
a Bearing of X from O is $90° + 20° = 110°$
b Bearing of T from O is $360° - 43° = 317°$
c Bearing of M from O is $90° - 35° = 055°$ (must be written as a three-digit number)

Example 22

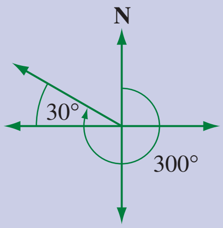
Sketch the following bearings on a compass rose.
a 217° **b** 300°

Solution

a



b

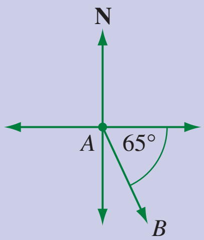


Example 23

Sketch point B if B has a bearing of 155° from A.

Solution

Draw the compass rose on the point where the bearing is to be measured *from*.

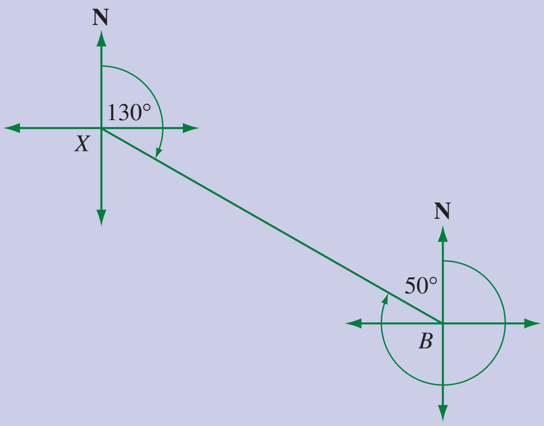


Example 24

The bearing of B from X is 130°. What is the bearing of X from B?

Solution

Sketch the bearing of B *from* X.
On the same diagram, draw a compass rose at B and find $\angle NBX$.
$\angle NBX = 50°$ (co-interior with $\angle NXB$)
$\therefore$ Bearing of X from B is $360° - 50° = 310°$.



glossary

STAGE 5.2

Exercise 8-09

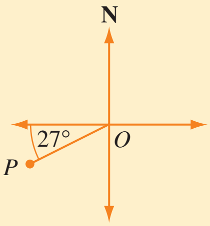
Example 21

Spreadsheet 8-04 Bearings

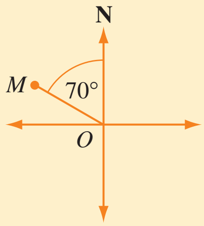
SkillBuilder 22-17 Applications in navigation

1 Write the bearing of each point from O.

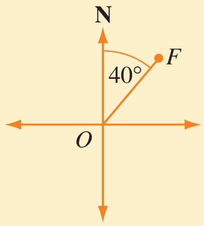
a



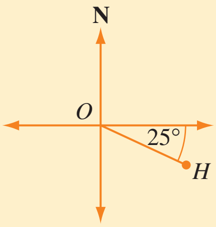
b



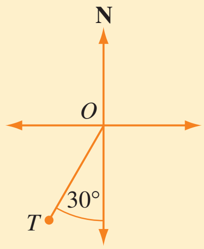
c



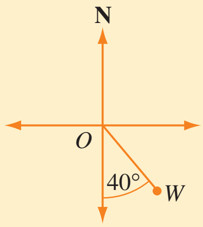
d



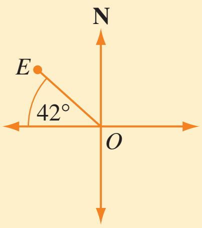
e



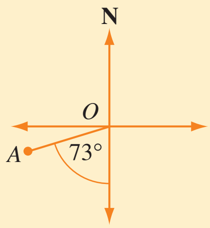
f



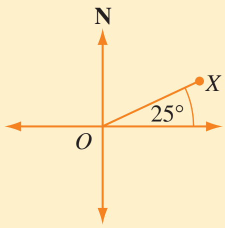
g



h

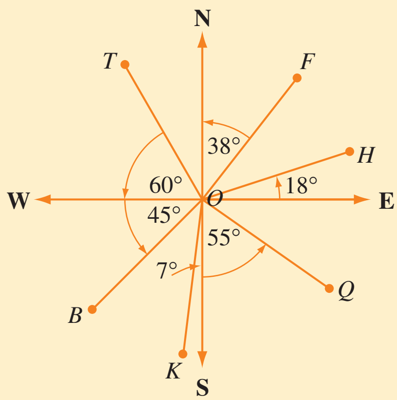


i



2 What is the bearing of each of these points from O?

a N **b** E
c S **d** W
e F **f** Q
g T **h** B
i H **j** K



3 What is the bearing of each of the following:

a NW **b** SE **c** NNE **d** ESE
e SW **f** NNW **g** ENE **h** WSW

Example 22

4 Sketch the following bearings on a compass rose.

a 220° **b** 060° **c** 260° **d** 125°
e 350° **f** 267° **g** 171° **h** 032°

Example 23

5 Sketch the point P if P has a bearing of:

a 132° from T **b** 260° from M **c** 335° from X
d 010° from K **e** 215° from R **f** 095° from Q

glossary

Example 24

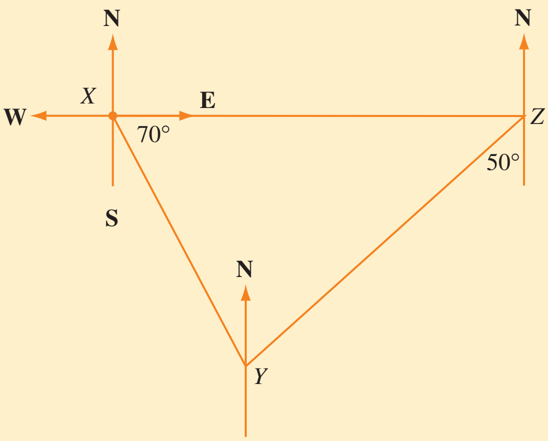
6 **a** If the bearing of P from A is 060°, what is the bearing of A from P?
b The bearing of T from Y is 100°. What is the bearing of Y from T?
c The bearing of H from M is 320°. Find the bearing of M from H.
d The bearing of X from Q is 212°. What is the bearing of Q from X?
e The bearing of A from W is 175°. Find the bearing of W from A.

7 What is the angle from:
a S to SW? **b** NE to SE? **c** E to NW? **d** SW to NNW?
e ESE to NW? **f** E to SSW? **g** SSE to NNW? **h** NNE to WSW?

8 A plane flies in the direction SW from point A to a point B. When it reaches B, the plane changes direction and flies due south. Show this information on a diagram.

9 Draw a diagram for each of the following situations.
a A boat sails on a bearing of 180° for 40 km and then on a bearing of 060° for 50 km.
b A plane flies on a bearing of 280° for 150 km and then another 250 km on a bearing of 080°.
c A cyclist travels 15 km due east and then 20 km on a bearing of 210°.

10 Find the bearing of:
a Y from Z
b X from Z
c Y from X
d X from Y
e Z from Y
f Z from X

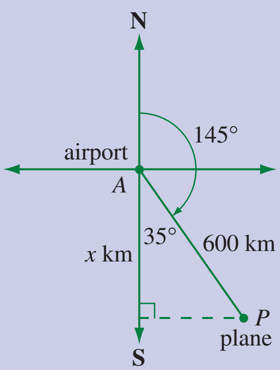


Problems involving bearings

Worksheet 8-09 Elevations and bearings

Example 25

A plane leaves an airport and flies 600 km on a bearing of 145°.
a How far south of the airport is the plane (to the nearest kilometre)?
b What is the three-figure bearing of the airport from the plane?



Solution

a Let x = distance south and note that $\angle SAP = 180° - 145° = 35°$.

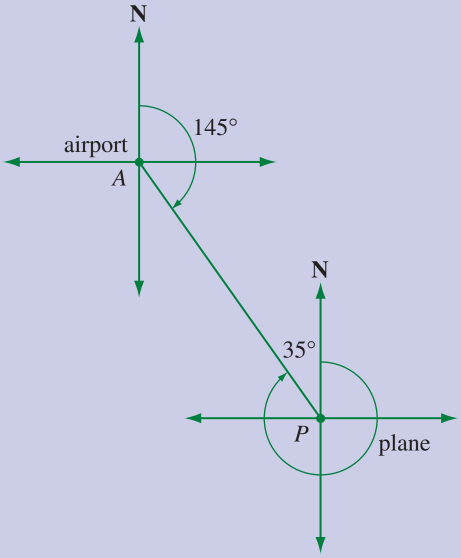
$$\cos 35° = \frac{x}{600}$$
$$x = 600 \cos 35°$$
$$x = 491.491\,226\ldots$$
$$x \approx 491$$

$\therefore$ The plane is 491 km south of the airport.

b A compass rose is drawn at the position of the plane.

$\angle \mathbf{N}PA = 35°$ (co-interior with 145°)

∴ The bearing of the airport from the plane is $360° - 35° = 325°$.

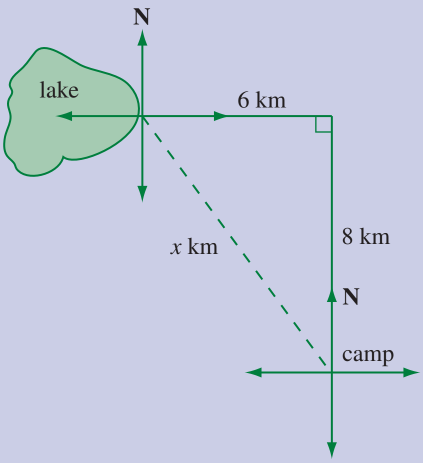


Example 26

From camp, a hiker walks due north for 8 km, then 6 km due west to a lake.

a How far is the hiker from the camp?

b What is the bearing of the camp from the lake (to the nearest degree)?



Solution

a Let x = distance from camp.

$x^2 = 6^2 + 8^2$

$x^2 = 36 + 64$

$x^2 = 100$

$x = \sqrt{100}$

$x = 10$

∴ The hiker is 10 km from the camp.

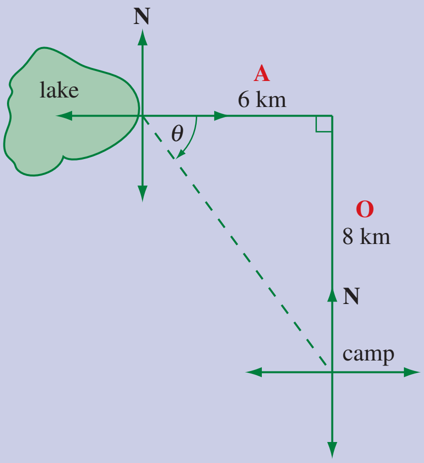
b Let θ be the angle shown in the diagram.

$\tan \theta = \frac{8}{6}$

$\theta = 53.130\,102\ldots$

$\theta \approx 53°$

∴ The bearing of the camp from the lake is $90° + 53° = 143°$.



glossary

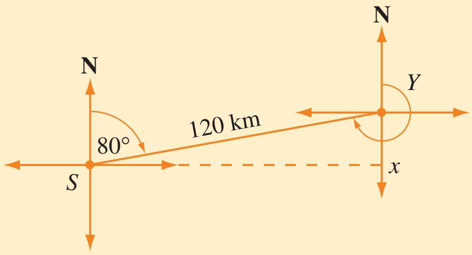
Exercise 8-10

Example 25

1 A yacht leaves Sydney and sails 120 km on a bearing of 080°.

a How far north of Sydney is the yacht (to the nearest kilometre)?

b What is the bearing of Sydney from the yacht?

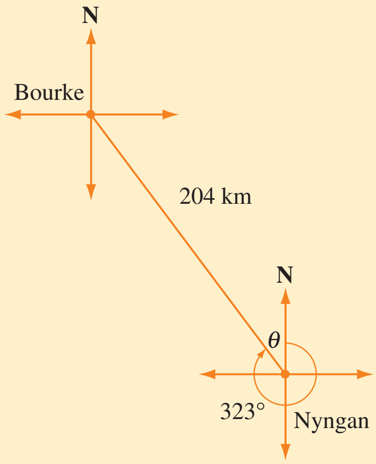


2 Colin leaves Nyngan and drives 204 km to Bourke. The bearing of Bourke from Nyngan is 323°.

a Find the value of θ.

b How far north of Nyngan is Bourke (to the nearest kilometre)?

c What is the bearing of Nyngan from Bourke?



3 The distance 'as the crow flies' from Sydney to Wollongong is 69 km. If the bearing of Wollongong from Sydney is 205°, calculate:

a how far south Wollongong is from Sydney (to the nearest kilometre)

b how far east Sydney is from Wollongong (to the nearest kilometre)

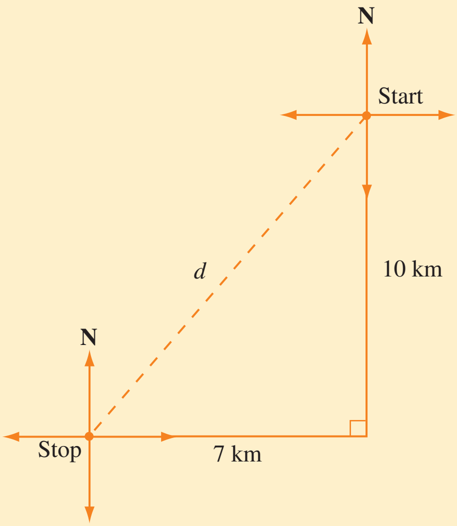
c the bearing of Sydney from Wollongong (to the nearest degree).

Example 26

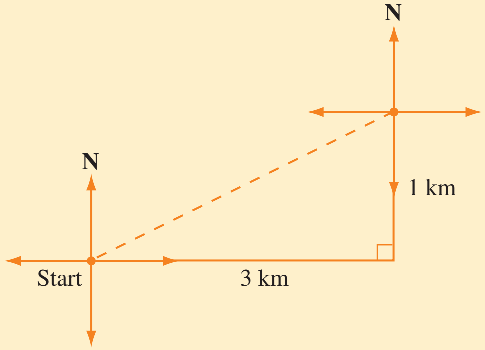
4 Jana cycles 10 km due south, then 7 km due west.

a How far is Jana from her starting point?

b What is the bearing of the starting point from where she stops?



5 Nelson runs every morning. He first runs 3 km due east, then 1 km due north and then directly back to his starting point.
   a Calculate correct to two decimal places, how far Nelson runs every morning.
   b On what bearing does Nelson run on his way back to the starting point?



6 A triathlete cycles 20 km from the end of the swim leg on a bearing of 200° to the finish line.
   a How far (to the nearest kilometre) has the triathlete travelled in a southerly direction?
   b What is the bearing of the end of the swim leg from the finish line?

7 A hiking group walks from Sandy Flats to Black Ridge (a distance of 20.9 km) at a bearing of 078°. They then turn and hike due south to Rivers End, then due west back to Sandy Flats. How far have they hiked altogether (to the nearest 0.1 km)?

8 A triangular orienteering run starts at Alpha and passes through the checkpoints of Bravo and Charlie before finishing at Alpha. Bravo is 8.5 km due east of Alpha, and Charlie is 10.5 km due south of Bravo.
   a Calculate the distance from checkpoint Charlie to the finish. Give your answer to the nearest kilometre and also to the nearest metre.
   b Find the bearing of checkpoint Alpha from checkpoint Charlie, to the nearest degree.

9 A fitness class starting from the gym walks 3.5 km west, then 1.2 km south to the pool.
   a How far (to the nearest tenth of a kilometre) are they from the gym?
   b What is the bearing of the gym from the pool (to the nearest tenth of a degree)?

10 A plane takes off at 10:15am and flies on a bearing of 150° at 700 km/h.
   a How far (to the nearest kilometre) due south of the airport is the plane at 1:45pm?
   b What is the bearing of the airport from the plane?

11 A fishing trawler sails 30 km from port on a bearing of 120° until it reaches a submerged reef. How far (to the nearest kilometre) is the port:

a north of the reef? **b** west of the reef?

12 Two racing pigeons are set free at the same time. The first bird flies on a course with a bearing of 040° while the second bird flies on a course with a bearing of 130°.

a The first bird flies 200 km until it is due north of the second bird. Find their distance apart (to the nearest kilometre).

b How far has the second bird flown?

13 Two horse riders start from the same stable. The rider of the black horse goes due west for 5.5 km and stops. The rider of the chestnut horse travels in a direction with a bearing of 303° until he is due north of the black horse. How far did the rider of the chestnut horse travel? Give your answer in kilometres, correct to three decimal places.

14 Two ships leave from the same port. One ship travels on a bearing of 157° at 20 knots. The second ship travels on a bearing of 247° at 35 knots. (1 knot is a speed of 1 nautical mile per hour.)

a How far apart are the ships after 8 hours, to the nearest nautical mile?

b Calculate the bearing of the second ship from the first, to the nearest minute.

Power plus

1 a Copy the following and use a calculator to complete it:

i sin 20° = _____, cos 70° = _____ **ii** sin 47° = _____, cos 43° = _____

iii sin 55° = _____, cos 35° = _____ **iv** sin 85° = _____, cos 5° = _____

v cos 60° = _____, sin 30° = _____ **vi** cos 38° = _____, sin 52° = _____

b What do you notice about your answers in part **a**?

c What do you notice about the pairs of angles in part **a**?

d Copy and complete: If cos 30° = 0.8660, then sin 60° = _____

e Write true (T) or false (F) for each of the following.

i sin 75° = cos 15° **ii** sin 80° = cos 20°

iii cos 8° = sin 72° **iv** sin 30° = cos 60°

v cos 65° = sin 25° **vi** sin 12° = cos 78°

2 A plane is flying at an angle of 15° inclined to the horizontal.

a How far will the plane need to fly along its line of flight to increase its altitude (height above the ground) by 500 m?

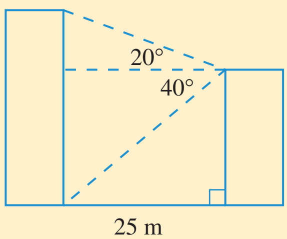
b At what angle must the plane climb to achieve an increase in altitude of 500 m in half the distance needed at an angle of 15°?

3 A wheelchair ramp has a gradient of 1 in 12.

a Calculate the angle of inclination of the ramp.

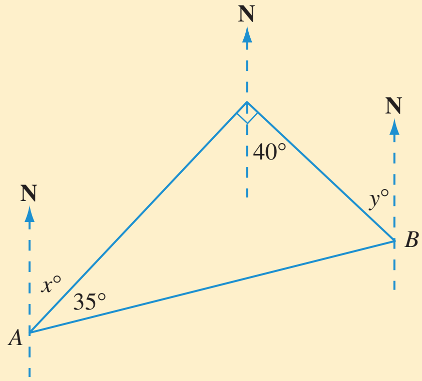
b If the ramp is 8 m long, how many metres does it rise vertically?

4 Two buildings are 25 m apart. From the top of the smaller building, the angle of depression of the base of the taller building is 40° and the angle of elevation of the roof is 20°. Find the heights of both buildings.



glossary

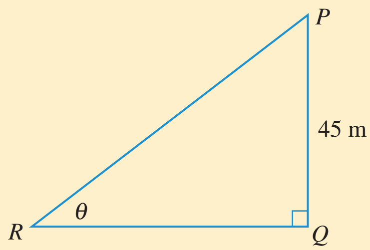
5 **a** Calculate angles $x°$ and $y°$ in the diagram on the right.
b Find the bearings from:
i A to B
ii B to A.



6 A lighthouse is 2.5 km from a ship on a bearing of 200°. How far is the ship:
a east of the lighthouse? **b** north of the lighthouse?

7 An orienteering course consists of three legs. The first leg is 2.4 km from A to B on a bearing of 110°. The second leg is 3.2 km on a bearing of 200°. The third leg is from C to A. Find:
a the distance from C to A **b** the bearing of A from C.

8 In ΔPQR, $PQ = 45$ m and $\sin \theta = \frac{3}{5}$. What are the lengths of RP and RQ?



Worksheet 8-10 Trigonometry crossword

abc Language of maths

adjacent *alternate angle* *angle of depression* *angle of elevation*
bearing *clinometer* *co-interior angles* *compass bearing*
cosine (cos) *degree* *denominator* *east*
horizontal *hypotenuse* *inverse* *magnetic compass*
mariner's compass *minute* *mnemonic* *north*
north-east *north-west* *opposite* *ratio*
right-angled *second* *similar triangles* *sine (sin)*
south *south-east* *south-west* *tangent (tan)*
theta (θ) *three-figure bearing* *trigonometric* *trigonometry*
west

1 Explain where the **hypotenuse** in a right-angled triangle is positioned.

2 What does the word 'trigonometry' mean? Which language does it come from?

3 What are the full names of the three trigonometric ratios?

4 What is the difference between the bearing 'of A from B' and the bearing 'of B from A'?

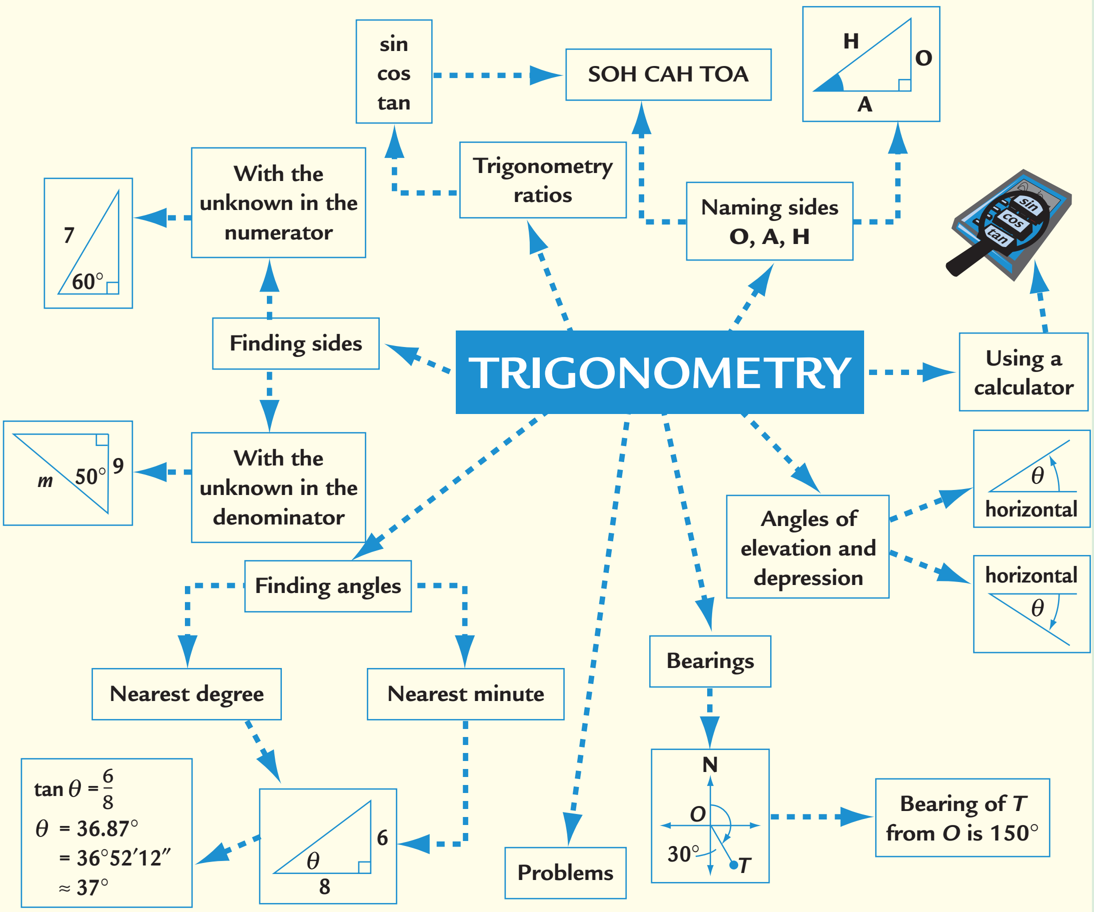
5 Which compass bearing is halfway between **west** and **north-west**?

Topic overview

- Rate your understanding of and ability with the work in this chapter by copying and completing the following scales. Draw an arrow on each scale to indicate your rating.

Understand the meaning of the sine, cosine and tangent ratios.	Low 0 1 2 3 4 5 High
Able to use the trigonometry ratios to find sides.	Low 0 1 2 3 4 5 High
Able to use the trigonometry ratios to find angles.	Low 0 1 2 3 4 5 High
Able to solve problems involving the trigonometry ratios and bearings.	Low 0 1 2 3 4 5 High
Able to solve problems involving the trigonometry ratios and angles of elevation and depression.	Low 0 1 2 3 4 5 High

- Copy and complete the overview below. If necessary, refer to the double-page opening of the chapter and the 'Language of maths' section for key words. Ask your teacher to check your overview to make sure nothing is missing or incorrect.



glossary

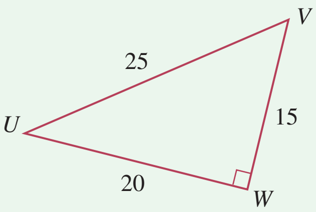
Chapter 8 Review

Topic test Chapter 8

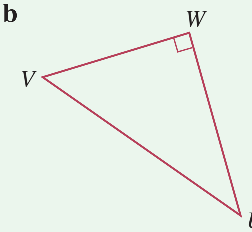
Ex 8-01

1 For both angles, U and V, find the opposite side (O), adjacent side (A) and the hypotenuse (H).

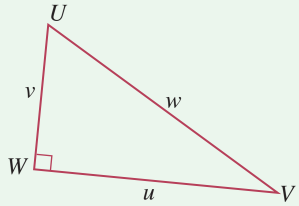
a



b



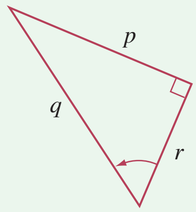
c



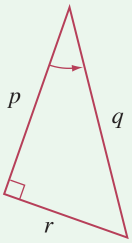
Ex 8-02

2 **a** Which trigonometric ratio (sin, cos or tan) is equal to $\frac{p}{q}$ in each of these triangles?

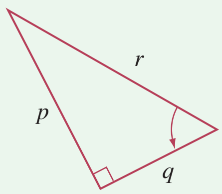
i



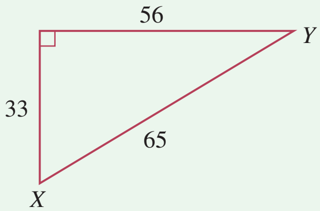
ii



iii



b Find the sine, cosine and tangent of both angles, X and Y in the triangle on the right. (Give your answers as fractions.)



c Given that $\sin \beta = \frac{36}{85}$ and β is acute, find $\cos \beta$ and $\tan \beta$. (Draw a diagram.)

Ex 8-03

3 **a** Round to the nearest degree:

i 38.9° **ii** 45.475° **iii** 65°43′ **iv** 12°8′50″ **v** 27°30′

b Round to the nearest minute:

i 12°8′50″ **ii** 12°8′5″ **iii** 12°8′30″ **iv** 12°30′55″ **v** 12°40′29″

c Express as a decimal:

i 73°30′ **ii** 46°24′ **iii** 50°33′

d Express in degrees and minutes:

i 18.25° **ii** 14.35°

e Evaluate p, correct to one decimal place.

i $p = 200 \tan 18°$ **ii** $p = 200 \sin 18.7°$ **iii** $p = \dfrac{200}{\cos 18°27'}$

Ex 8-04

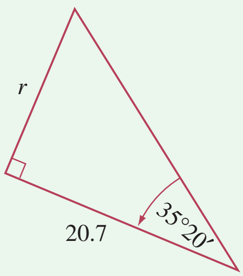
4 Evaluate angle θ, correct to the nearest degree.

a $\tan \theta = 1.75$ **b** $\sin \theta = \frac{2}{3}$ **c** $\cos \theta = 73 : 100$

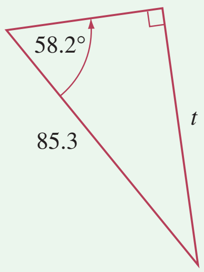
Ex 8-05

5 Find the value of each pronumeral below, correct to two decimal places.

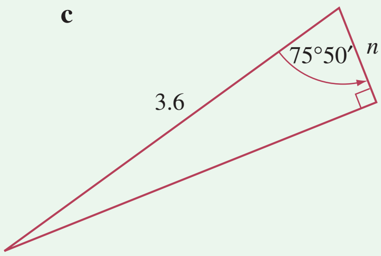
a



b



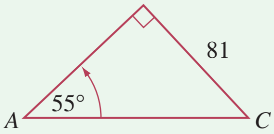
c



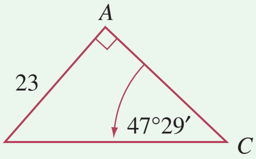
glossary

6 Evaluate side AC, correct to the nearest whole number: Ex 8-06

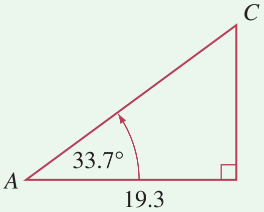
a



b

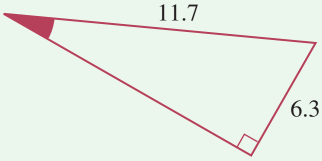


c

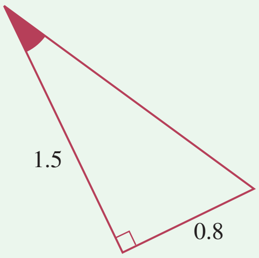


7 Find the shaded angle each time, correct to the nearest minute: Ex 8-07

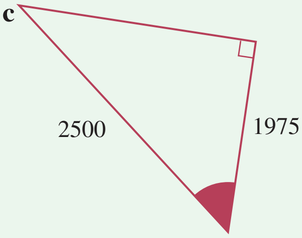
a



b



c



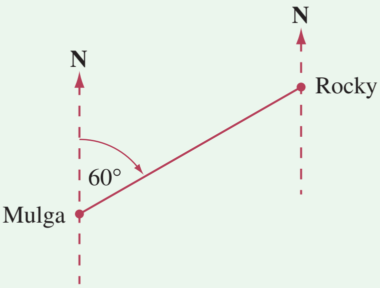
8 Draw a diagram and mark the given information to help you answer each of these questions. Ex 8-08

- **a** The angle of elevation of a tower roof is 26° at a point 400 m from its base. Find the height of the tower in metres.
- **b** Find the angle of depression of a boat that is 100 m from the base of a 55 m cliff.

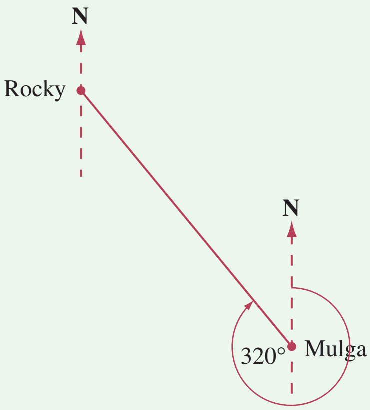
9 In each diagram below, what is the three-figure bearing of: Ex 8-09

i Rocky from Mulga? **ii** Mulga from Rocky?

a



b



10 **a** A runner leaves home and jogs to the pool on a bearing of 240° for 6 km. How far due west of home is the runner after 6 km? (Answer to one decimal place.) Ex 8-10

b Two planes leave an airport at the same time. The first travels on a bearing of 063° at 500 km/h. The second travels on a bearing of 153° at 400 km/h.

i How far apart are the planes after 2 hours?

ii Calculate the bearing of the first plane from the second plane.