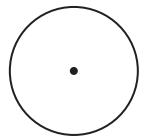
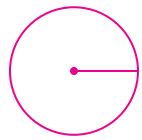
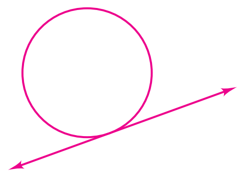
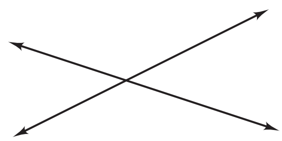
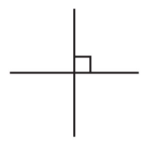
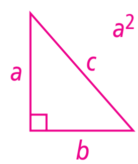
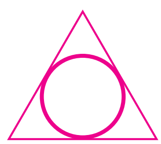
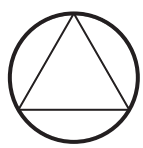
Name ______________________ Class ____________ Date ____________

12-1 Additional Vocabulary Support

Tangent Lines

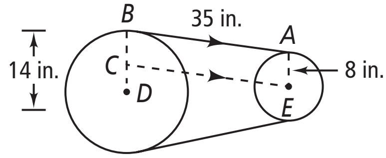
Complete the vocabulary chart by filling in the missing information.

Word or Word Phrase	Definition	Picture or Example
circle	A *circle* is the set of all points that are the same distance from the center point.	
radius	A *radius* is a line segment with one endpoint at the center of a circle and the other endpoint at any point on the circle.	**1.** 
tangent	A line is *tangent* to a circle if it intersects a circle at exactly one point.	**2.** 
intersect	**3.** **Two lines or figures *intersect* if they have one or more points in common.**	
perpendicular	**4.** ***Perpendicular* lines are two lines that intersect each other and form right angles.**	
Pythagorean Theorem	The *Pythagorean Theorem* states that in a right triangle the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.	**5.**  $a^2 + b^2 = c^2$
inscribed	A circle is *inscribed* in a polygon if the sides of the polygon are tangent to the circle.	**6.** 
circumscribed	**7.** **A circle is *circumscribed* in a polygon if the vertices of the polygon are on the circle.**	

Name ______________________ Class ____________ Date ____________

12-1 Think About a Plan

Tangent Lines



a. A belt fits snugly around the two circular pulleys. $\overline{CE}$ is an auxiliary line from E to $\overline{BD}$. $\overline{CE} \parallel \overline{AB}$. What type of quadrilateral is $ABCE$? Explain.

b. What is the length of $\overline{CE}$?

c. What is the distance between the centers of the pulleys to the nearest tenth?

1. What is the definition of a tangent line? **a line that touches a circle at only one point**

2. What is the relationship between a line tangent to a circle and the radius at the point of tangency (Theorem 12-1)? **They are perpendicular.**

3. Where is the point of tangency for $\overline{AB}$ on $\odot D$? On $\odot E$? ***B*; *A***

4. What is the measure of $\angle CBA$? What is the measure of $\angle BAE$? Explain.
90°; 90°; they are formed by perpendicular line segments.

5. How can you use parallel lines to find the measure of $\angle CEA$?
Because $\overline{CE} \parallel \overline{AB}$, $\angle BAE$ and $\angle CEA$ are supplementary angles, so $m\angle CEA = 90°$.

6. How can you use parallel lines or the Polygon Angle-Sum Theorem to find the measure of $\angle BCE$?
Because $\overline{CE} \parallel \overline{AB}$, $\angle CBA$ and $\angle BCE$ are supplementary angles, so $m\angle BCE = 90°$. Or, $\angle BCE$ is the fourth angle in a quadrilateral in which the other angles sum to 270°, so its measure is 90°.

7. What type of quadrilateral has four right angles? **a rectangle**

8. What is the length of $\overline{BA}$? **35 in.**

9. What is the length of $\overline{CE}$? **35 in.**

10. What are the center points of the pulleys? ***D*; *E***

11. How can you use the Segment Addition Postulate to find $\overline{CD}$?
$BD - BC = CD$, $BD = 14$ in., and $BC = 8$ in.

12. What is the measure of $\overline{CD}$? **6 in.**

13. How can you use the Pythagorean Theorem to find the length of $\overline{DE}$?
$a^2 + b^2 = c^2$; if $CD = a$, $CE = b$, and $DE = c$, then $DE = \sqrt{(6^2) + (35^2)} = \sqrt{1261} \approx 35.5$ in.

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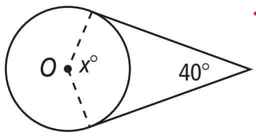
12-1 Practice

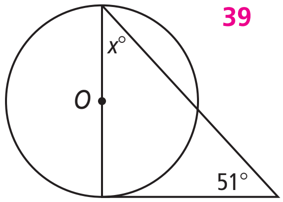
Form G

Tangent Lines

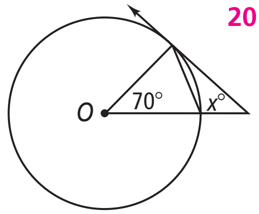
Algebra Assume that lines that appear to be tangent are tangent. *O* is the center of each circle. What is the value of *x*?

1. 

140

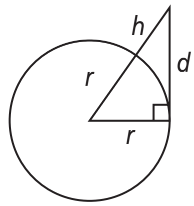
2. 

39

3. 

20

The circle at the right represents Earth. The radius of the Earth is about 6400 km. Find the distance *d* that a person can see on a clear day from each of the following heights *h* above Earth. Round your answer to the nearest tenth of a kilometer.

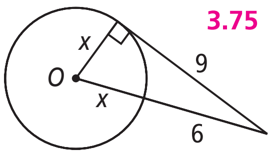


4. 12 km **392.1 km**

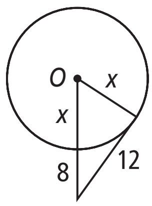
5. 20 km **506.4 km**

6. 1300 km **4281.4 km**

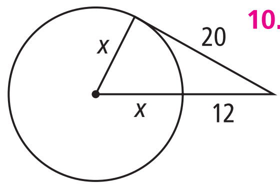
In each circle, what is the value of *x* to the nearest tenth?

7. 

3.75

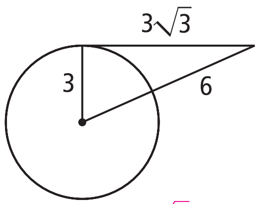
8. 

5

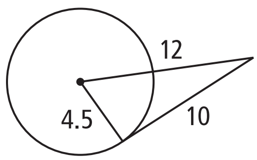
9. 

10.7

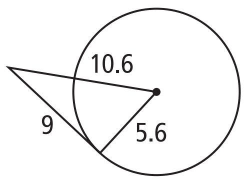
Determine whether a tangent line is shown in each diagram. Explain.

10. 

yes; $3^2 + (3\sqrt{3})^2 = 6^2$

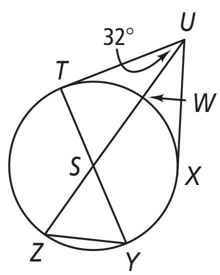
11. 

no; $4.5^2 + 10^2 \neq 12^2$

12. 

yes; $5.6^2 + 9^2 = 10.6^2$

13. $\overline{TY}$ and $\overline{ZW}$ are diameters of $\odot S$. $\overline{TU}$ and $\overline{UX}$ are tangents of $\odot S$. What is $m\angle SYZ$? **61**

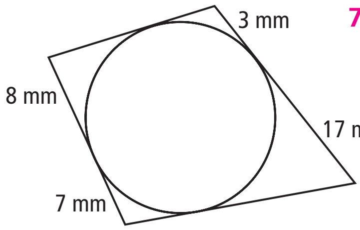


12-1 Practice (continued) *Form G*

Tangent Lines

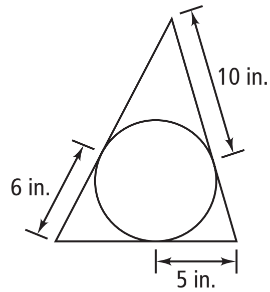
Each polygon circumscribes a circle. What is the perimeter of each polygon?

14.



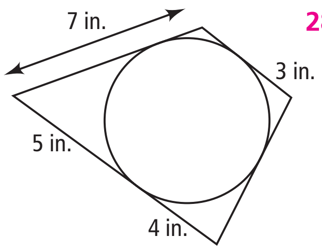
70 mm

15.



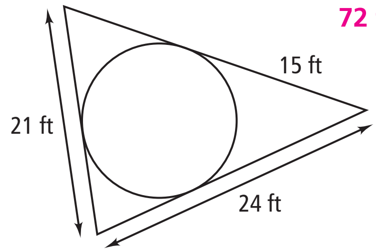
42 in.

16.



28 in.

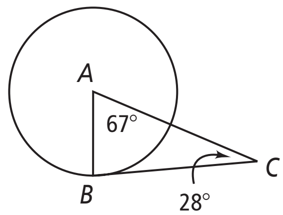
17.



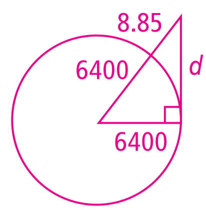
72 ft

18. Error Analysis A classmate states that $\overline{BC}$ is tangent to $\odot A$. Explain how to show that your classmate is wrong.

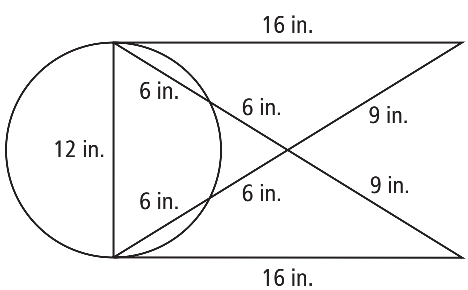
If $\overline{BC}$ is tangent to $\odot A$, then $\overline{AB} \perp \overline{BC}$ and $m\angle B = 90$; this cannot be true because the sum of the three angles would be greater than 180°.



19. The peak of Mt. Everest is about 8850 m above sea level. About how many kilometers is it from the peak of Mt. Everest to the horizon if the Earth's radius is about 6400 km? Draw a diagram to help you solve the problem. **337 km**



20. The design of the banner at the right includes a circle with a 12-in. diameter. Using the measurements given in the diagram, explain whether the lines shown are tangents to the circle. **no; $12^2 + 16^2 \neq 21^2$**



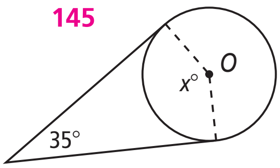
Name ______________________ Class ____________ Date ____________

12-1 Practice — Form K

Tangent Lines

Lines that appear to be tangent are tangent. *O* is the center of each circle. What is the value of *x*?

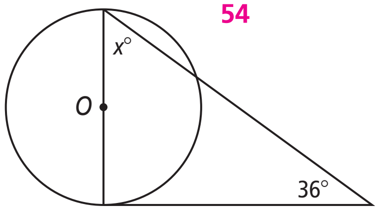
1. 145



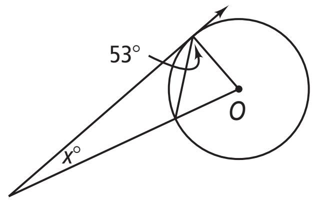
To start, identify the type of geometric figure formed by the tangent lines and radii.

The figure formed is a _?_. quadrilateral

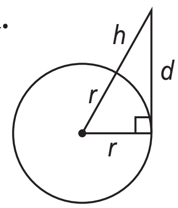
2. 54



3. 16



The circle at the right represents Earth. The radius of Earth is about 6400 km. Find the distance *d* to the horizon that a person can see on a clear day from each of the following heights *h* above Earth. Round your answer to the nearest tenth of a kilometer.



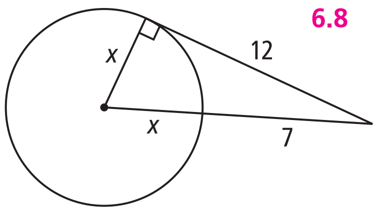
4. 7 km
299.4 km

5. 400 km
2297.8 km

6. 2000 m
160.0 km

Algebra **In each circle, what is the value of *x* to the nearest tenth?**

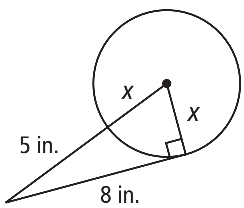
7. 6.8



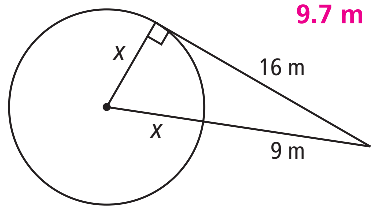
To start, use the Pythagorean Theorem.

$x^2 + 12^2 = (\underline{\ ?\ })^2$ $x + 7$

8. 3.9 in.



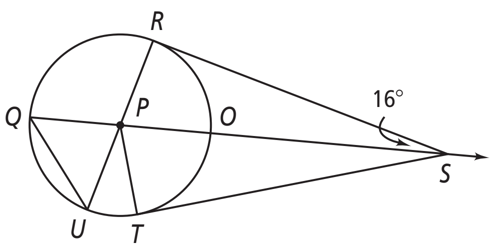
9. 9.7 m



10. $\overline{QO}$ and $\overline{UR}$ are diameters of $\odot P$.

$\overline{RS}$ and $\overline{TS}$ are tangents of $\odot P$.

Find $m\angle UPT$ and $m\angle UQP$. 32; 53



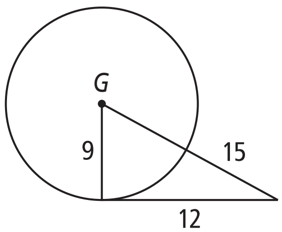
Name ______________________ Class ____________ Date ____________

12-1 Practice (continued) — Form K

Tangent Lines

Determine whether a tangent is shown in each diagram. Explain.

11.

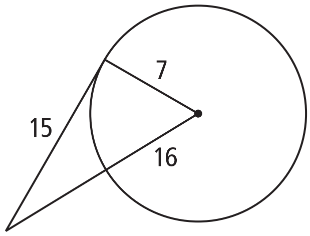


To start, use the Converse of the Pythagorean Theorem to relate the side lengths of the triangle.

$9^2 + 12^2 \stackrel{?}{=} \underline{\ ?\ }^2$ **15**

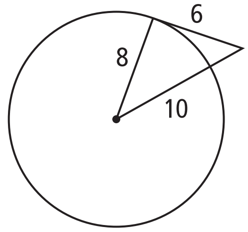
yes; 81 + 144 = 225

12.



no; $7^2 + 15^2 \neq 16^2$

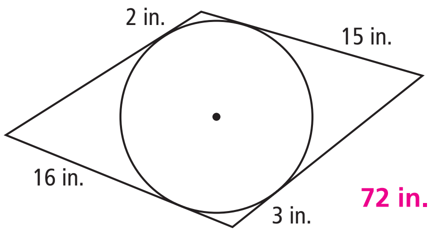
13.



yes; $6^2 + 8^2 = 10^2$

Each polygon circumscribes a circle. What is the perimeter of each polygon?

14.



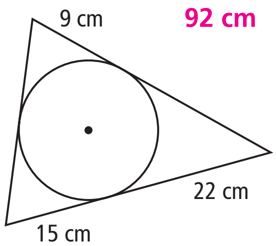
72 in.

To start, find the length of each unknown segment.

$P = 2 + \boxed{2} + 15 + \boxed{15}$

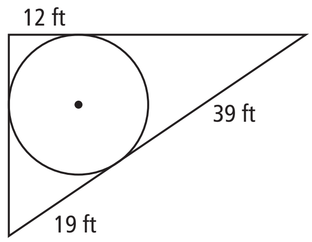
$+ 3 + \boxed{3} + 16 + \boxed{16}$

15.



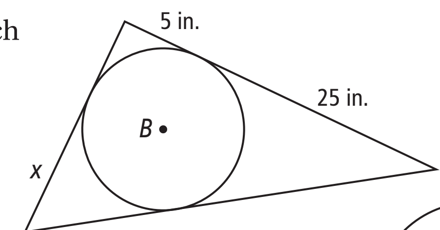
92 cm

16.



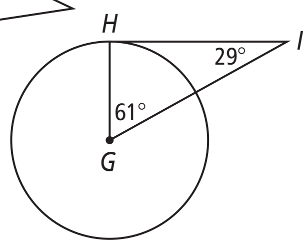
140 ft

17. $\odot B$ is inscribed in a triangle, which has a perimeter of 76 in. What is the value of x? **8 in.**



18. Reasoning GHI is a triangle. How can you prove that $\overline{HI}$ is tangent to $\odot G$?

$m\angle G + m\angle I = 90$. By the Triangle Angle-Sum Thm., $m\angle H = 90$, so $\overline{HI}$ is tangent to $\odot G$ by Thm. 12-2.



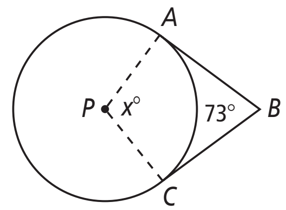
Name ______________________ Class ______________ Date ______________

12-1 Standardized Test Prep

Tangent Lines

Multiple Choice

For Exercises 1–5, choose the correct letter.



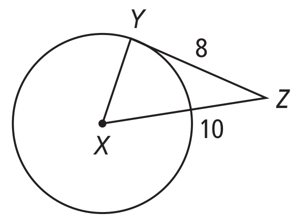
1. $\overline{AB}$ and $\overline{BC}$ are tangents to $\odot P$. What is the value of x? **B**

(A) 73 (C) 117

(B) 107 (D) 146

2. Earth's radius is about 4000 mi. To the nearest mile, what is the distance a person can see on a clear day from an airplane 5 mi above Earth? **G**

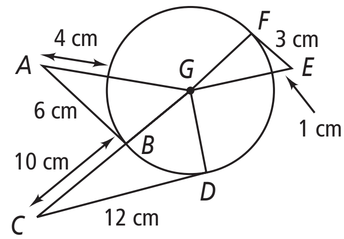
(F) 63 mi (G) 200 mi (H) 4000 mi (I) 5660 mi



3. $\overline{YZ}$ is a tangent to $\odot X$, and X is the center of the circle. What is the length of the radius of the circle? **B**

(A) 4 (C) 12

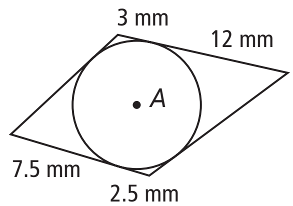
(B) 6 (D) 12.8



4. The radius of $\odot G$ is 4 cm. Which is a tangent of $\odot G$? **I**

(F) $\overline{AB}$ (H) $\overline{BF}$

(G) $\overline{CD}$ (I) $\overline{FE}$



5. $\odot A$ is inscribed in a quadrilateral. What is the perimeter of the quadrilateral? **B**

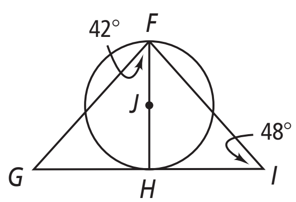
(A) 25 mm (C) 60 mm

(B) 50 mm (D) 150 mm

Short Response

6. Given: $\overline{GI}$ is a tangent to $\odot J$.

Prove: $\triangle FGH \cong \triangle FIH$



[2] Statements: 1) $\overline{GI}$ is a tangent to $\odot J$; 2) $\overline{FH} \perp \overline{GI}$; 3) $\angle FHG$ and $\angle FHI$ are right ∡; 4) $\angle FHG \cong \angle FHI$; 5) $\overline{FH} \cong \overline{FH}$; 6) $m\angle GFH = 42$, $m\angle FIH = 48$; 7) $m\angle HFI = 42$; 8) $\angle HFI \cong \angle HFG$; 9) $\triangle FGH \cong \triangle FIH$; Reasons: 1) Given; 2) Thm. 12-1; 3) Def. of $\perp$; 4) Def. of $\cong$; 5) Refl. Prop.; 6) Given; 7) Triangle Angle-Sum Thm.; 8) Def. of $\cong$; 9) ASA Post. [1] proof missing steps [0] incorrect or missing proof

Name ____________________ Class __________ Date __________

12-1 Enrichment

Tangent Lines

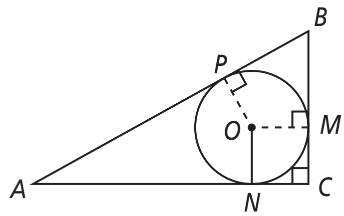
Inscribed Circles and Right Triangles

The following theorem is about a relationship between the radius of the inscribed circle of a right triangle and the lengths of the triangle's sides.

Complete the proof.

Given: Right $\triangle ABC$ with right angle at C. $\odot O$ is inscribed in $\triangle ABC$. M, N, and P are the points of tangency. Radius $\overline{ON}$ is drawn.

Prove: $2ON = AC + BC - AB$

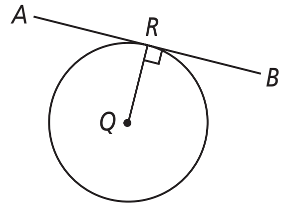


Statements	Reasons
1) In right $\triangle ABC$, $\angle C$ is a right angle. $\odot O$ is inscribed in ABC. M, N, and P are points of tangency. Radius $\overline{ON}$ is drawn.	1) ? Given
2) Draw radii $\overline{OM}$ and $\overline{OP}$.	2) ? Two points determine a line segment.
3) $\overline{BP} \cong \overline{BM}$; $\overline{CM} \cong \overline{CN}$; $\overline{AP} \cong \overline{AN}$	3) ? Theorem 12-3 (Two tangents drawn to a circle from a point outside the circle are congruent.)
4) $\overline{ON} \cong \overline{OM}$	4) ? Radii of a circle are congruent.
5) $\angle OMC$ is a right angle.	5) ? Theorem 12-1 (A radius and a tangent line drawn to the same point of contact form a right angle.)
6) $\angle ONC$ is a right angle.	6) ? Theorem 12-1 (A radius and a tangent line drawn to the same point of contact form a right angle.)
7) Quad. $OMCN$ is a square.	7) ? Definition of a square
8) $\overline{ON} \cong \overline{OM} \cong \overline{CM} \cong \overline{CN}$	8) ? The sides of a square are congruent.
9) $BP + CM = BM + CN$ $AP + CM = AN + CN$	9) ? Addition Property
10) $AN + CN = AC$ $BM + CM = BC$ $BP + AP = AB$	10) ? Segment Addition Postulate
11) $BP + CM = BC$	11) ? Substitution Property
12) $(BP + AP) + 2CM = BC + AC$	12) ? Addition Property and Substitution Property
13) $AB + 2ON = BC + AC$	13) ? Substitution Property
14) $2ON = AC + BC - AB$	14) ? Subtraction Property

Name ______________________ Class ____________ Date ____________

12-1 Reteaching
Tangent Lines

A tangent is a line that touches a circle at exactly one point. In the diagram, $\overline{AB}$ is tangent to $\odot Q$. You can apply theorems about tangents to solve problems.



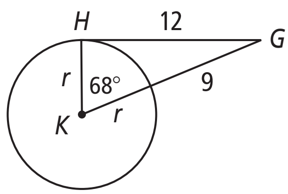
Theorem 12-1

If a line is tangent to a circle, then that line forms a right angle with the radius at the point where the line touches the circle.

Theorem 12-2

If a line in the same plane as a circle is perpendicular to a radius at its endpoint on the circle, then the line is tangent to the circle.

Problem



Use the diagram at the right to solve the problems below. $\overline{GH}$ is tangent to $\odot K$.

What is the measure of $\angle G$?

Because $\overline{GH}$ is tangent to $\odot K$, it forms a right angle with the radius.

The sum of the angles of a triangle is always 180. Write an equation to find $m\angle G$.

$$m\angle G + m\angle H + m\angle K = 180$$
$$m\angle G + 90 + 68 = 180$$
$$m\angle G + 158 = 180$$
$$m\angle G = 22$$

What is the length of the radius?

You can use the Pythagorean Theorem to find missing lengths.

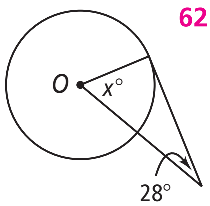
$$HK^2 + HG^2 = GK^2$$
$$r^2 + 12^2 = (9 + r)^2$$
$$r^2 + 144 = (9 + r)(9 + r)$$
$$r^2 + 144 = 81 + 18r + r^2$$
$$63 = 18r$$
$$3.5 = r$$

So, the measure of $\angle G$ is 22 and the length of the radius is 3.5 units.

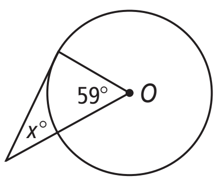
Exercises

In each circle, what is the value of x?

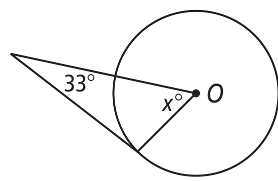
1. 62



2. 31



3. 57

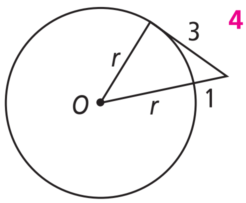


Name ____________________ Class __________ Date __________

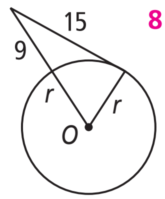
12-1 Reteaching (continued)

Tangent Lines

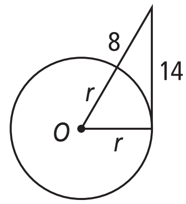
In each circle, what is the value of *r*?

4. 

4

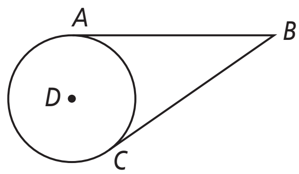
5. 

8

6. 

8.25

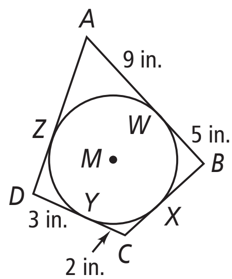
Theorem 12-3



If two segments are tangent to a circle from the same point outside the circle, then the two segments are equal in length.

In the diagram, $\overline{AB}$ and $\overline{BC}$ are both tangent to $\odot D$. So, they are also congruent.

When circles are drawn inside a polygon so that the sides of the polygon are tangents, the circle is inscribed in the figure. You can apply Theorem 12-3 to find the perimeter, or distance around the polygon.



Problem

$\odot M$ is inscribed in quadrilateral *ABCD*.
What is the perimeter of *ABCD?*

$ZA = AW = 9 \qquad WB = BX = 5$

$CY = XC = 2 \qquad YD = DZ = 3$

Now add to find the length of each side:

$AB = AW + WB = 9 + 5 = 14 \qquad BC = BX + CX = 5 + 2 = 7$

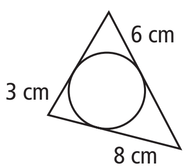
$CD = CY + YD = 2 + 3 = 5 \qquad DA = DZ + ZA = 3 + 9 = 12$

$14 + 7 + 5 + 12 = 38$

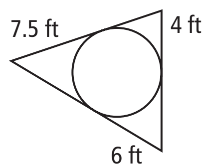
The perimeter is 38 in.

Exercises

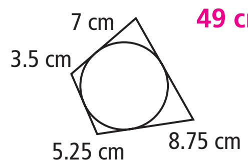
Each polygon circumscribes a circle. What is the perimeter of each polygon?

7. 

34 cm

8. 

35 ft

9. 

49 cm

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12-2 Additional Vocabulary Support

Chords and Arcs

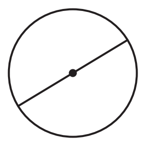
Choose the word from the list below that best matches each sentence.

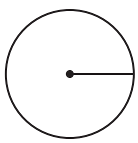
arc	bisect	central angle
chord	diameter	radius

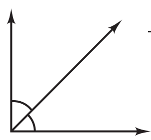
1. A segment with endpoints on a circle. chord
2. To divide exactly in half. bisect
3. A segment from the center of a circle to any point on the circle. radius
4. An angle whose vertex is the center of a circle. central angle
5. A segment with endpoints on a circle that passes through the center. diameter

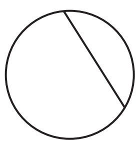
Use a word from the list above that best describes each picture.

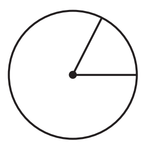
6.  arc

7.  diameter

8.  radius

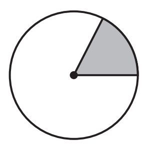
9.  bisect

10.  chord

11.  central angle

Multiple Choice

12. The diagram at the right shows a sector of a circle. Which of the following defines the boundary of a sector? C



(A) two radii and a chord
(B) two diameters
(C) two radii and an arc
(D) two chords

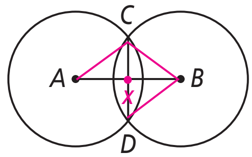
13. The radius of a circle is 5 in. long. How long is the diameter? I

(F) 2.5 in. (G) 5 in. (H) 7.5 in. (I) 10 in.

Name ______________________ Class __________ Date __________

12-2 Think About a Plan

Chords and Arcs



$\odot A$ and $\odot B$ are congruent. $\overline{CD}$ is a chord of both circles.
If $AB = 8$ in. and $CD = 6$ in., how long is a radius?

1. Draw the radius of each circle that includes point C. What is the name of each of the two line segments drawn?
   $\overline{AC}$, $\overline{BC}$
2. Label the intersection of $\overline{CD}$ and $\overline{AB}$ point X.
3. You know that $\overline{CD} \cong \overline{CD}$. How can you use the converse of Theorem 12-7 to show that $AX = XB$?
   Because congruent chords are equidistant from the centers of congruent circles, X is the same distance from A as it is from B. So, $AX = XB$.
4. How long is $\overline{XB}$? 4 in.
5. Draw in radius $\overline{BD}$. What is true about BC and BD? Explain.
   $BC = BD$; all radii of a circle have the same length.
6. Because $AD = AC = BD = BC$, $ACBD$ is a rhombus and its diagonals $\overline{AB}$ and $\overline{CD}$ are perpendicular.
7. What can you say about the diagram using Theorem 12-8? $\overline{AB}$ bisects $\overline{CD}$.
8. How long is $\overline{CX}$? 3 in.
9. How can you use the Pythagorean Theorem to find BC?
   $a^2 + b^2 = c^2$; if $CX = a$, $XB = b$, and $CB = c$, then $CB = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ in.
10. How long is the radius of each circle? 5 in.

Name ______________________ Class ____________ Date ____________

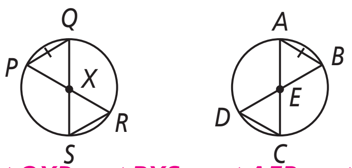
12-2 Practice

Form G

Chords and Arcs

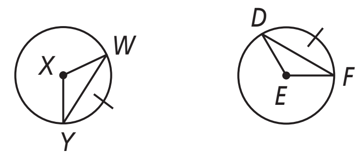
In Exercises 1 and 2, the $\odot X \cong \odot E$. What can you conclude?

1.



$\angle QXP \cong \angle RXS \cong \angle AEB \cong \angle DEC$; all radii are congruent; all chords drawn are congruent.

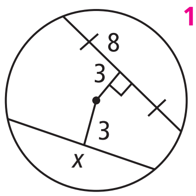
2.



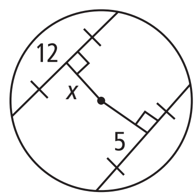
$\angle WXY \cong \angle DEF$; $\overline{WY} \cong \overline{DF}$; all radii are congruent.

Find the value of *x*.

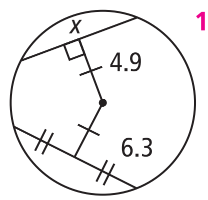
3. **16**



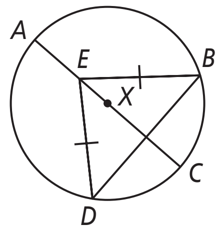
4. **5**



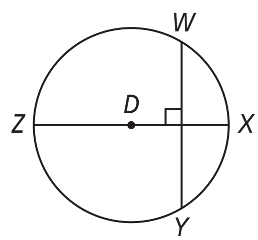
5. **12.6**



6. In $\odot X$, $\overline{AC}$ is a diameter and $\overline{ED} \cong \overline{EB}$. What can you conclude about $\widehat{DC}$ and $\widehat{CB}$? Explain.
$\widehat{DC} \cong \widehat{CB}$; because $\overline{ED} \cong \overline{EB}$ and $\overline{XB} \cong \overline{XD}$, $\overline{AC}$ must be a perpendicular bisector of $\overline{DB}$ by the Converse of the Perpendicular Bisector Theorem. This means $\overline{DC} \cong \overline{CB}$, so by Theorem 12-6, $\widehat{DC} \cong \widehat{CB}$.

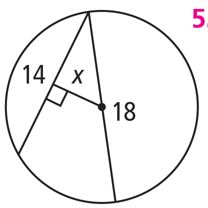


7. In $\odot D$, $\overline{ZX}$ is the diameter of the circle and $\overline{ZX} \perp \overline{WY}$. What conclusions can you make? Justify your answer.
$\overline{WD} \cong \overline{DY}$ because $\overline{ZX}$ is a perpendicular bisector, and $\widehat{WX} \cong \widehat{XY}$ because of Theorem 12-8.

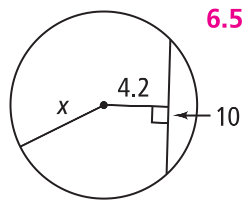


Find the value of *x* to the nearest tenth.

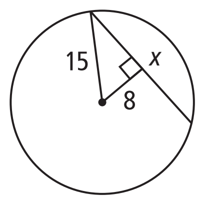
8. **5.7**



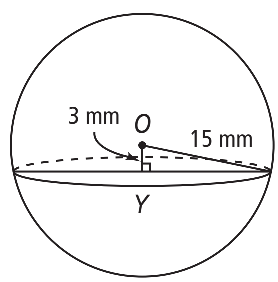
9. **6.5**



10. **25.4**



11. In the figure at the right, sphere *O* with radius 15 mm is intersected by a plane 3 mm from the center. To the nearest tenth, find the radius of the cross section $\odot Y$.
14.7 mm

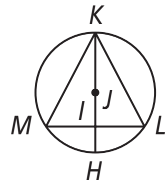


12-2 Practice (continued)

Form G

Chords and Arcs

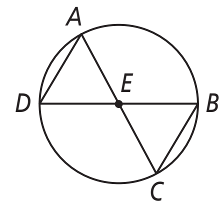
12. Given: $\odot J$ with diameter $\overline{HK}$; $\overset{\frown}{KL} \cong \overset{\frown}{LM} \cong \overset{\frown}{MK}$



Prove: $\triangle KIL \cong \triangle KIM$

Statements: 1) $\overline{KI} \cong \overline{KI}$; 2) $\overset{\frown}{KL} \cong \overset{\frown}{KM}$; 3) $\overline{KM} \cong \overline{KL}$; 4) $\overline{JM} \cong \overline{JL}$; 5) $\overline{KH}$ is the $\perp$ bis. of $\overline{ML}$; 6) $\overline{IM} \cong \overline{IL}$; 7) $\triangle KIL \cong \triangle KLM$; Reasons: 1) Refl. Prop. of $\cong$; 2) Given; 3) Converse Thm. 12-6; 4) All radii in a circle are $\cong$; 5) Converse of $\perp$ Bis. Thm.; 6) Def. of a bis.; 7) SSS

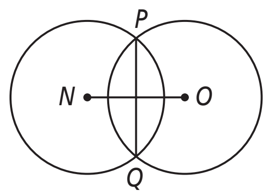
13. Given: $\overline{AC}$ and $\overline{DB}$ are diameters of $\odot E$.



Prove: $\triangle EAD \cong \triangle ECB$

Statements: 1) $\overline{AC}$ and $\overline{DB}$ are diameters of $\odot E$; 2) $\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$; 3) $\angle AED \cong \angle CEB$; 4) $\triangle EAD \cong \triangle ECB$;
Reasons: 1) Given; 2) Def. of radius; 3) Vert. Angles are $\cong$; 4) SAS

$\odot N$ and $\odot O$ are congruent. $\overline{PQ}$ is a chord of both circles.



14. If $NO = 12$ in. and $\overline{PQ} = 8$ in., how long is the radius to the nearest tenth of an inch? **7.2 in.**

15. If $NO = 30$ mm and radius $= 16$ mm, how long is $\overline{PQ}$ to the nearest tenth of a millimeter? **11.1 mm**

16. If radius $= 12$ m and $\overline{PQ} = 9$ m, how long is $\overline{NO}$ to the nearest tenth? **22.2 m**

17. Draw a Diagram A student draws $\odot X$ with a diameter of 12 cm. Inside the circle she inscribes equilateral $\triangle ABC$ so that $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are all chords of the circle. The diameter of $\odot X$ bisects $\overline{AB}$. The section of the diameter from the center of the circle to where it bisects $\overline{AB}$ is 3 cm. To the nearest whole number, what is the perimeter of the equilateral triangle inscribed in $\odot X$? **31 cm**

18. Two concentric circles have radii of 6 mm and 12 mm. A segment tangent to the smaller circle is a chord of the larger circle. What is the length of the segment to the nearest tenth. **20.8 mm**

Name ______________________________ Class ____________ Date ____________

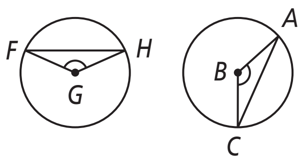
12-2 Practice

Form K

Chords and Arcs

In Exercises 1 and 2, the circles are congruent. What can you conclude?

1.

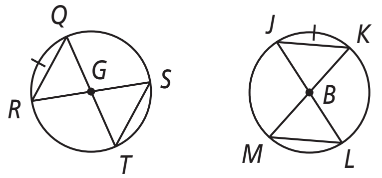


To start, look at the chords. If they are equidistant from the center of the circle, what can be concluded?

The chords must be __?__. **congruent**

$\overline{FH} \cong \overline{AC}$; $\overline{GF} \cong \overline{GH} \cong \overline{BA} \cong \overline{BC}$; $\overset{\frown}{FH} \cong \overset{\frown}{AC}$; $\angle FGH \cong \angle ABC$

2.



$\overline{QR} \cong \overline{TS} \cong$ __?__ $\cong$ __?__

$\overline{KJ}$; $\overline{LM}$

$\angle QGR \cong$ __?__ $\cong$ __?__ $\cong$ __?__

$\angle SGT$; $\angle JBK$; $\angle MBL$

$\overset{\frown}{QR} \cong$ __?__ $\cong$ __?__ $\cong$ __?__

$\overset{\frown}{KJ}$; $\overset{\frown}{ST}$; $\overset{\frown}{LM}$

$\angle QGS \cong$ __?__ $\cong$ __?__ $\cong$ __?__

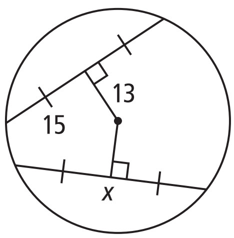
$\angle RGT$; $\angle JBM$; $\angle KBL$

$\overset{\frown}{QS} \cong$ __?__ $\cong$ __?__ $\cong$ __?__

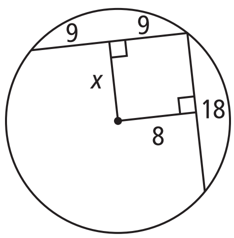
$\overset{\frown}{TR}$; $\overset{\frown}{KL}$; $\overset{\frown}{MJ}$

Find the value of *x*.

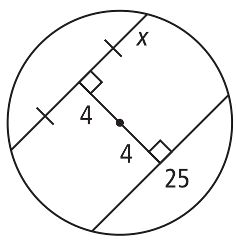
3. **30**



4. **8**



5. **12.5**



6. Reasoning $\angle QRS$ and $\angle TRV$ are vertical angles inscribed in $\odot R$. What must be true of $\overset{\frown}{QS}$ and $\overset{\frown}{TV}$? Explain.
They must be ≅ because vertical ∆ are ≅, and the arcs of ≅ central ∆ in the same circle are ≅.

Draw a Diagram **Tell whether the statement is *always, sometimes,* or *never* true.**

7. $\overset{\frown}{XY}$ and $\overset{\frown}{RS}$ are in congruent circles. Central $\angle XZY$ and central $\angle RTS$ are congruent. **sometimes**

8. $\odot I \cong \odot K$. The length of chord $\overline{GH}$ in $\odot I$ is 3 in. and the length of chord $\overline{LM}$ in $\odot K$ is 3 in. $\angle GIH \cong \angle LKM$. **always**

9. $\angle STU$ and $\angle RMO$ are central angles in congruent circles. $m\angle STU = 50$ and $m\angle RMO = 55$. $\overset{\frown}{SU} \cong \overset{\frown}{RO}$. **never**

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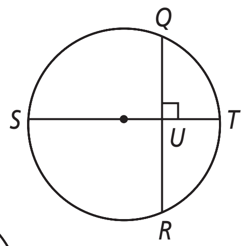
12-2 Practice (continued) — Form K

Chords and Arcs

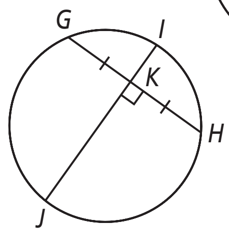
10. In the diagram at the right, $\overline{ST}$ is a diameter of the circle and $\overline{ST} \perp \overline{QR}$. What conclusions can you make?

$\overset{\frown}{QT} \cong \overset{\frown}{TR}$, $\overset{\frown}{SQ} \cong \overset{\frown}{SR}$, and $\overline{QU} \cong \overline{UR}$.

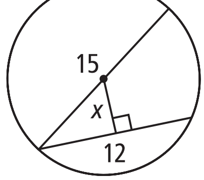


11. In the diagram at the right, $\overline{IJ}$ is a perpendicular bisector of chord $\overline{GH}$. What can you conclude?

Answers may vary. Sample: $\overline{IJ}$ contains the center of the circle.



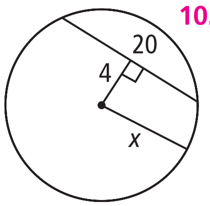
Find the value of *x* to the nearest tenth.

12. 

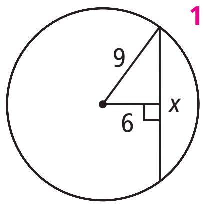
4.5

To start, since the radius is perpendicular to the chord, the chord is bisected.

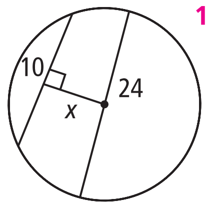
The longer leg of the triangle is $12 \div 2 = \boxed{6}$.

13. 

10.8

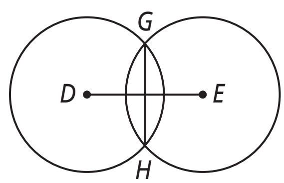
14. 

13.4

15. 

10.9

$\odot D$ and $\odot E$ are congruent. $\overline{GH}$ is a chord of both circles. Round all answers to the nearest tenth.

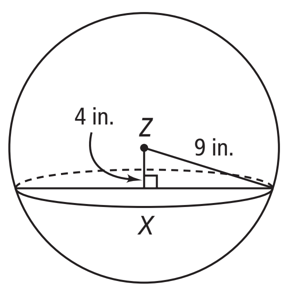


16. If $DE = 10$ in. and $GH = 4$ in., how long is a radius? **5.4 in.**

17. If $DE = 22$ cm and radius $= 14$ cm, how long is $\overline{GH}$? **17.3 cm**

18. If the radius $= 18$ ft and $GH = 32$ ft, how long is $\overline{DE}$? **16.5 ft**

19. In the figure at the right, Sphere Z with radius 9 in. is intersected by a plane 4 in. from center Z. To the nearest tenth, find the radius of the cross section $\odot X$. **8.1 in.**



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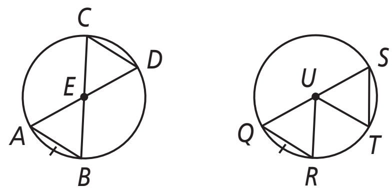
12-2 Standardized Test Prep

Chords and Arcs

Multiple Choice

For Exercises 1–5, choose the correct letter.

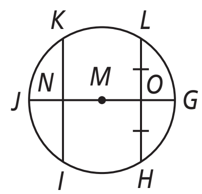
1. The circles at the right are congruent. Which conclusion can you draw? **C**



(A) $\overline{CD} \cong \overline{ST}$ (C) $\angle AEB \cong \angle QUR$

(B) $\angle CED \cong \angle SUT$ (D) $\overset{\frown}{BD} \cong \overset{\frown}{RT}$

2. $\overline{JG}$ is the diameter of $\odot M$. Which conclusion *cannot* be drawn from the diagram? **F**

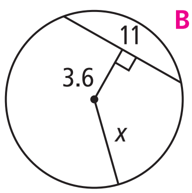


(F) $\overline{KN} \cong \overline{NI}$ (H) $\overline{JG} \perp \overline{HL}$

(G) $\overset{\frown}{LG} \cong \overset{\frown}{GH}$ (I) $\overline{GH} \cong \overline{GL}$

For Exercises 3 and 4, what is the value of x to the nearest tenth?

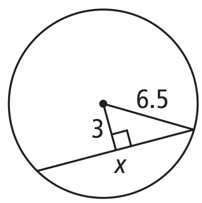
3. **B**



(A) 4.2 (C) 10.4

(B) 6.6 (D) 11.6

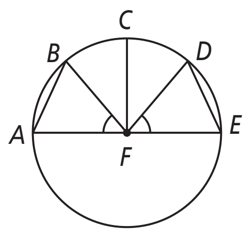
4. **H**



(F) 3.6 (H) 11.5

(G) 5.8 (I) 14.3

5. If $\angle AFB \cong \angle DFE$, what must be true? **A**

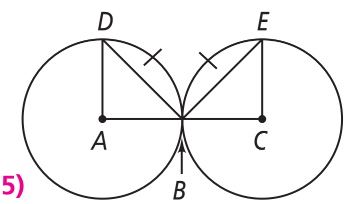


(A) $\overset{\frown}{AB} \cong \overset{\frown}{DE}$ (C) $\overline{CF} \perp \overline{AE}$

(B) $\overset{\frown}{BC} \cong \overset{\frown}{DE}$ (D) $\angle BFC \cong \angle DFC$

Short Response

6. Given: $\odot A \cong \odot C$, $\overset{\frown}{DB} \cong \overset{\frown}{EB}$



Prove: $\triangle ADB \cong \triangle CEB$

[2] Statements: 1) $\odot A \cong \odot C$, $\overset{\frown}{DB} \cong \overset{\frown}{EB}$; 2) $\overline{AB}$, $\overline{CB}$, $\overline{DA}$, and $\overline{CE}$ are all radii; 3) $\overline{AB} \cong \overline{CB} \cong \overline{CE} \cong \overline{AD}$; 4) $\overline{DB} \cong \overline{EB}$; 5) $\triangle ADB \cong \triangle CEB$; Reasons: 1) Given; 2) Def. of radius; 3) Radii of $\cong$ circles are $\cong$; 4) Converse of Thm. 12–6; 5) SSS [1] proof missing steps [0] incorrect or missing proof

Name ______________________ Class ____________ Date ____________

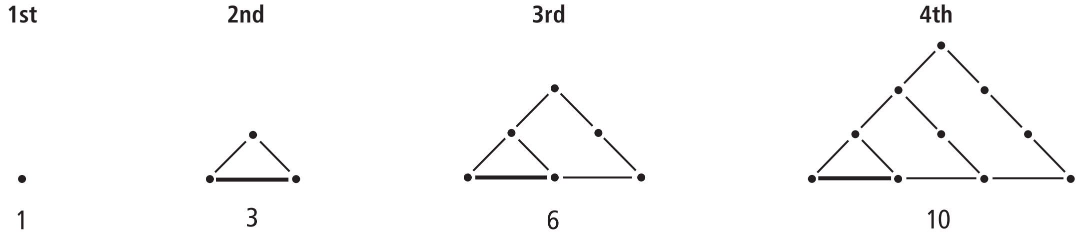
12-2 Enrichment

Chords and Arcs

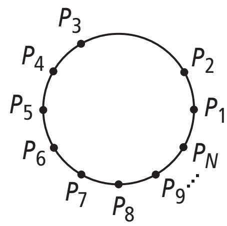
Chords and Triangular Numbers

Number patterns abound in mathematics. The ability to recognize patterns can lead to the discovery of formulas. The ancient Greeks made dot figures for certain numbers. These numbers are called *figurate* (or polygonal) numbers. An example is a sequence of numbers called *triangular numbers*.

Triangular Numbers



Consider *N* points on a given circle. Work in small groups to solve each of the following problems. Draw a diagram where appropriate.



1. How many chords may be drawn between

a. 2 points? **1**
b. 3 points? **3**
c. 4 points? **6**
d. 5 points? **10**
e. 6 points? **15**

2. Write the sequence of numbers for Exercise 1, parts (a)–(e). **1, 3, 6, 10, 15**

3. What type of sequence is this? **triangular numbers**

4. Consider the case of three points on a circle. From each point, how many chords can you draw? Explain.
Two chords can be drawn from each point, one to each of the other points; this is equal to 3 minus 1.

5. How will you find the total number of chords you can draw between these points, using multiplication? Ignore for a moment any chords you will draw twice.
Multiply 3 by the number of chords that can be drawn (3 minus 1, or 2).

6. Now consider how many chords you will draw twice. Is this number equal to, greater than, or less than the total number of chords from Exercise 5?
Each chord (total, 6) will be drawn twice; this number equals the total number of chords from Exercise 5.

7. Use your experience with three points on a circle to find a general formula for the number of chords you can draw between pairs of *N* points. $\frac{N(N-1)}{2}$

8. How many arcs are formed, including major and minor arcs and semicircles? $N(N-1)$

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12-2 Reteaching
Chords and Arcs

Several relationships between chords, arcs, and the central angles of a circle are listed below. The converses of these theorems are also true.

Theorem 12-4 Congruent central angles have congruent arcs.

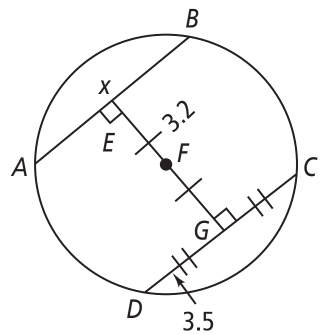
Theorem 12-5 Congruent central angles have congruent chords.

Theorem 12-6 Congruent chords have congruent arcs.

Theorem 12-7 Chords equidistant from the center are congruent.

Problem

What is the value of *x*?



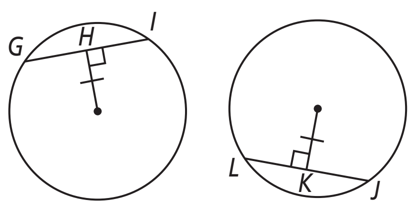
$EF = FG = 3.2$	Given
$\overline{AB} \cong \overline{DC}$	Chords equidistant from the center of a circle are congruent.
$DC = DG + GC$	Segment Addition Postulate
$AB = x + GC$	Substitution
$DG = GC = 3.5$	Given
$x = 3.5 + 3.5 = 7$	Substitution

The values of *x* is 7.

Exercises

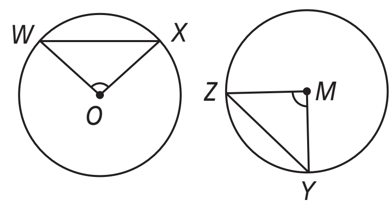
In Exercises 1 and 2, the circles are congruent. What can you conclude? Answers may vary. Samples below:

1.



$\overline{GI} \cong \overline{LJ}$ and $\overset{\frown}{GI} \cong \overset{\frown}{LJ}$

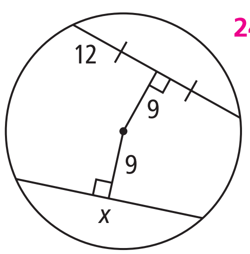
2.



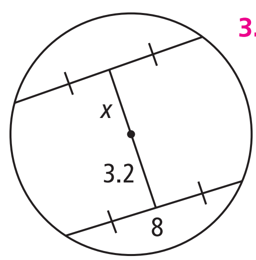
$\overline{WX} \cong \overline{ZY}$ and $\overset{\frown}{WX} \cong \overset{\frown}{ZY}$

Find the value of *x*.

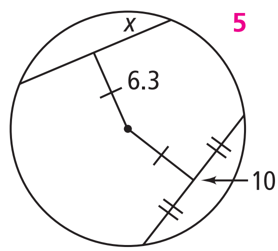
3. 24



4. 3.2



5. 5



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12-2 Reteaching (continued)

Chords and Arcs

Useful relationships between diameters, chords, and arcs are listed below. To bisect a figure means to divide it exactly in half.

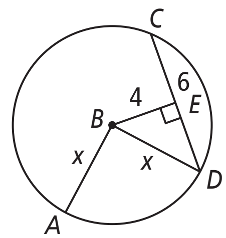
Theorem 12-8 In a circle, if a diameter is perpendicular to a chord, it bisects that chord and its arc.

Theorem 12-9 In a circle, if a diameter bisects a chord that is not a diameter of the circle, it is perpendicular to that chord.

Theorem 12-10 If a point is an equal distance from the endpoints of a line segment, then that point lies on the perpendicular bisector of the segment.

Problem

What is the value of x to the nearest tenth?



In this problem, x is the radius. To find its value draw radius $\overline{BD}$, which becomes the hypotenuse of right $\triangle BED$. Then use the Pythagorean Theorem to solve.

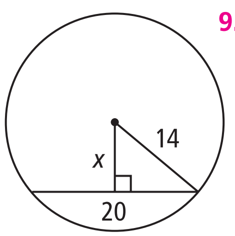
$ED = CE = 3$	A diameter perpendicular to a chord bisects the chord.
$x^2 = 3^2 + 4^2$	Use the Pythagorean Theorem.
$x^2 = 9 + 16 = 25$	Solve for x^2.
$x = 5$	Find the positive square root of each side.

The value of x is 5.

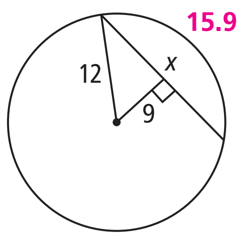
Exercises

Find the value of x to the nearest tenth.

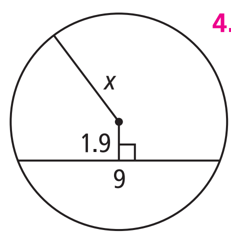
6. 9.8



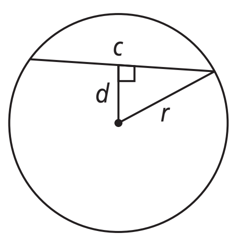
7. 15.9



8. 4.9



Find the measure of each segment to the nearest tenth.



9. Find c when $r = 6$ cm and $d = 1$ cm. **11.8 cm**

10. Find c when $r = 9$ cm and $d = 8$ cm. **8.2 cm**

11. Find d when $r = 10$ in. and $c = 10$ in. **8.7 in.**

12. Find d when $r = 8$ in. and $c = 15$ in. **2.8 in.**

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12-3 Additional Vocabulary Support

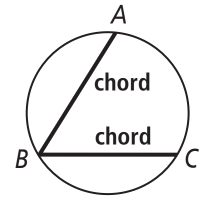
Inscribed Angles

Inscribed Angles and Intercepted Arcs

An *inscribed angle* is made by two *chords* that share an endpoint on the perimeter of a circle.

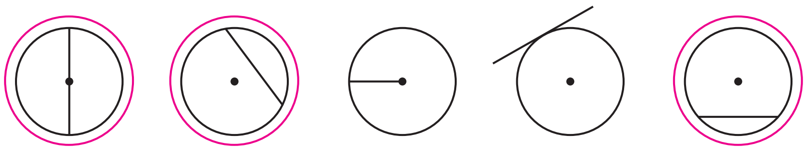
Where they meet is called a *vertex*.

The arc that is between the other endpoints of the chords is called the *intercepted arc*.

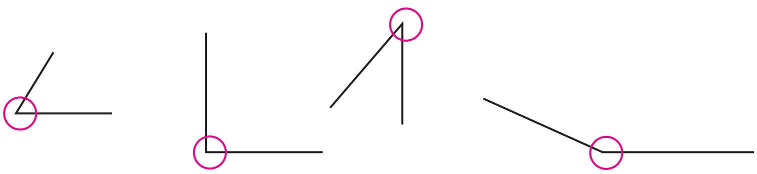


Sample In the diagram at the right, chords $\overline{AB}$ and $\overline{BC}$ meet at vertex B to form inscribed $\angle ABC$ and intercepted $\widehat{AC}$.

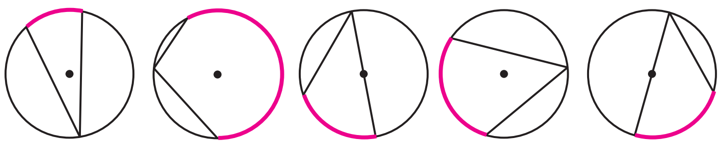
1. Circle each diagram that shows circles with chords.



2. Circle the vertex of each angle.



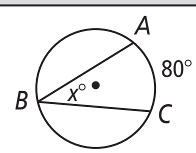
3. Trace the intercepted arc in each diagram.



Measures of Inscribed Angles and Intercepted Arcs

The *measure of an inscribed angle* is half the *measure of its intercepted arc.*

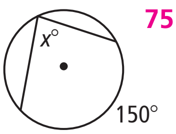
$m\angle B = \frac{1}{2}m\widehat{AC}$



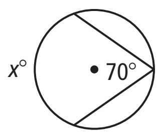
Sample In the diagram at the right, $m\angle B = \frac{1}{2}(80) = 40$

Find the value of *x*.

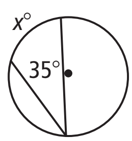
4. 75



5. 140



6. 70



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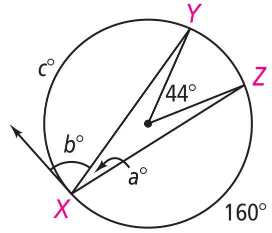
12-3 Think About a Plan

Inscribed Angles

Find the value of each variable. The dot represents the center of the circle.



1. Draw in points X, Y, and Z on the circle so that the measure of $\angle YXZ$ is a, $\overset{\frown}{XY}$ is c, and $\overset{\frown}{XZ}$ is 160.

2. How is the measure of an inscribed angle related to the measure of its intercepted arc?

By Theorem 12-11, the measure of an inscribed angle is half the measure of its intercepted arc.

3. What is the measure of $\overset{\frown}{YZ}$, by the definition of an arc? **44**

4. How can you use Theorem 12-11 to find a?

Because a (the measure of $\angle YXZ$) is half the measure of its intercepted arc ($\overset{\frown}{YZ}$), it is equal to half of 44.

5. What is a? **22**

6. What is the sum of the measures of $\overset{\frown}{XY}$, $\overset{\frown}{YZ}$, and $\overset{\frown}{XZ}$? **360**

7. How can you use the sum of the measures of all these non-overlapping arcs of a circle to find c?

Because c is the measure of $\overset{\frown}{XY}$, and $m\overset{\frown}{XY} + m\overset{\frown}{YZ} + m\overset{\frown}{XZ} = 360$, $m\overset{\frown}{XY} = 360 - (m\overset{\frown}{YZ} + m\overset{\frown}{XZ})$. Then substitute the values for $m\overset{\frown}{YZ}$ and $m\overset{\frown}{XZ}$.

8. What is c? **156**

9. What is the arc that is intercepted by the angle measuring b? What is the measure of this arc? **$\overset{\frown}{XY}$; 156**

10. How can you use Theorem 12-12 to find b?

The measure of an angle that is formed by a tangent and a chord is half the measure of the intercepted arc. So, b is half of 156.

11. What is b? **78**

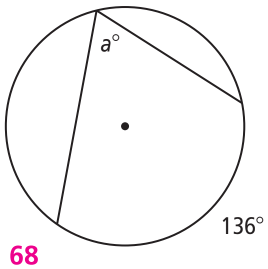
12-3 Practice

Form G

Inscribed Angles

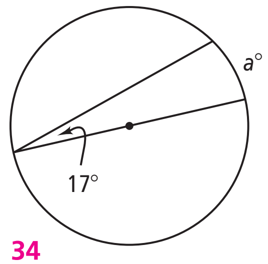
Find the value of each variable. For each circle, the dot represents the center.

1.



68

2.



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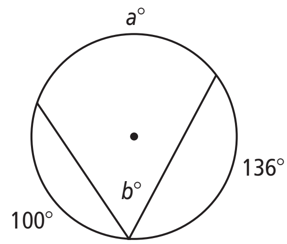
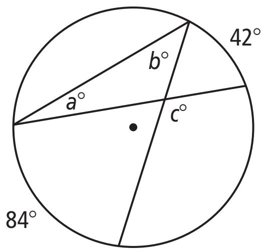
3.



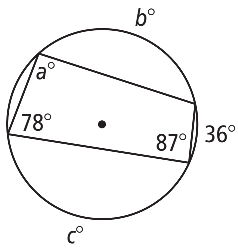
124; 62

4.



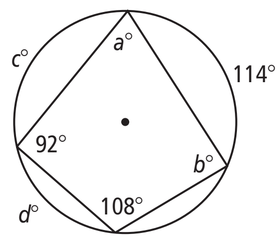
21; 42; 117

5.



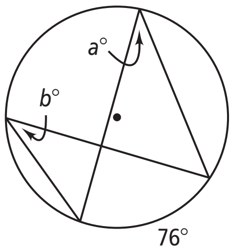
93; 120; 150

6.



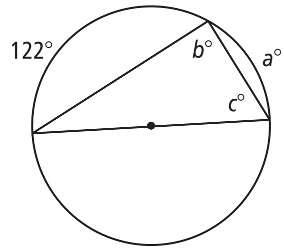
72; 88; 102; 74

7.



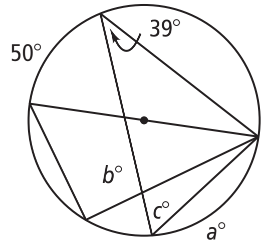
38; 38

8.



58; 90; 61

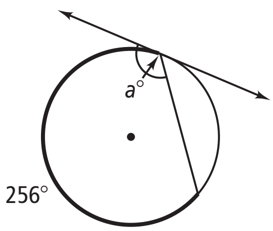
9.



78; 90; 65

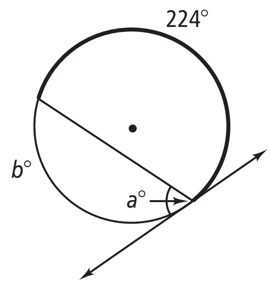
Find the value of each variable. Lines that appear to be tangent are tangent.

10.



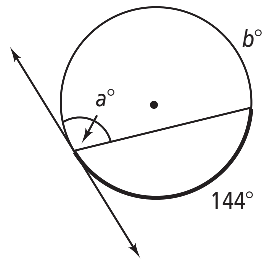
128

11.



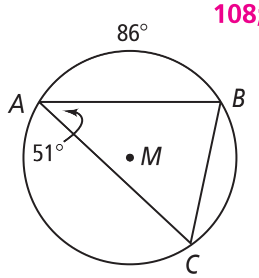
68; 136

12.



108; 216

Find each indicated measure for ⊙M.



13. a. $m\angle B$ 86 **b.** $m\angle C$ 43

c. $m\widehat{BC}$ 102 **d.** $m\widehat{AC}$ 172

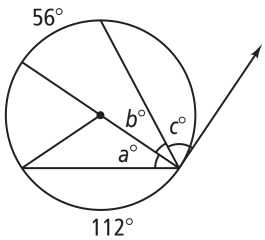
12-3 Practice (continued)

Form G

Inscribed Angles

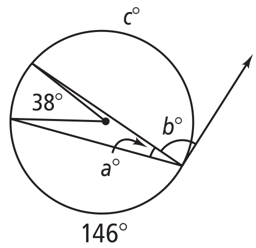
Find the value of each variable. For each circle, the dot represents the center.

14.



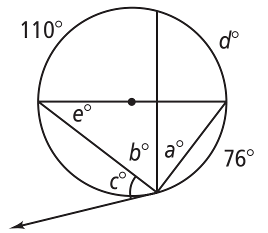
34; 28; 62

15.



19; 88; 176

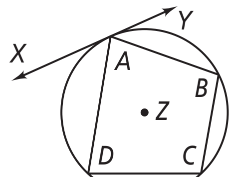
16.



35; 55; 52; 70; 38

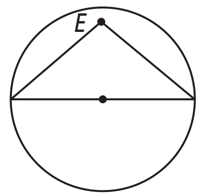
17. Given: Quadrilateral $ABCD$ is inscribed in $\odot Z$.
$\overleftrightarrow{XY}$ is tangent to $\odot Z$.

Prove: $m\angle XAD + m\angle YAB = m\angle C$



Statements: 1) *ABCD* is inscribed in ⊙*Z*; 2) ∠*C* is suppl. to ∠*DAB*; 3) $m\angle C + m\angle DAB = 180$; 4) $m\angle DAB + m\angle XAD + m\angle YAB = 180$; 5) $m\angle DAB + m\angle XAD + m\angle YAB = m\angle C + m\angle DAB$; 6) $m\angle XAD + m\angle YAB = m\angle C$; Reasons: 1) Given; 2) Corollary 3 to Thm. 12-11; 3) Def. of suppl.; 4) ∠ Add. Post.; 5) Subst. Prop.; 6) Subtr. Prop.

18. Error Analysis A classmate says that $m\angle E = 90$. Explain why this is incorrect.



∠*E* is not an inscribed angle because its vertex is not a point on the circle. Only an inscribed angle that intercepts a semicircle has a measure of 90.

19. A student inscribes quadrilateral *ABCD* inside a circle. The measures of angles *A*, *B*, and *C* are given below. Find the measure of each angle of quadrilateral *ABCD*. **$m\angle A = 92$; $m\angle B = 64$; $m\angle C = 88$; $m\angle D = 116$**

$m\angle A = 8x - 4 \quad m\angle B = 5x + 4 \quad m\angle C = 7x + 4$

20. Reasoning Quadrilateral *WXYZ* is inscribed in a circle. If ∠*W* and ∠*Y* are each inscribed in a semicircle, does this mean the quadrilateral is a rectangle? Explain.

No; ∠*W* and ∠*Y* are right angles, but the others do not have to be.

21. Writing A student inscribes an angle inside a semicircle to form a triangle. The measures of the angles that are not the vertex of the inscribed angle are x and $2x - 9$. Find the measures of all three angles of the triangle. Explain how you got your answer.

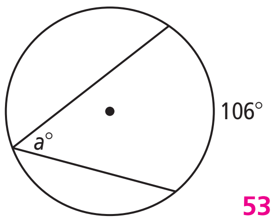
33; 57; 90; if the angle is inscribed in a semicircle it must measure 90. To find the measures of the other angles, set their sum equal to 90: $x + 2x - 9 = 90$.

Name ______________________________ Class ____________ Date ____________

12-3 Practice
Form K

Inscribed Angles

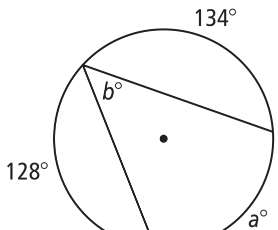
Find the value of each variable. For each circle, the dot represents the center.

1. 

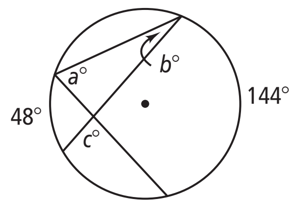
53

To start, describe the relationship between the inscribed angle and the intercepted arc.

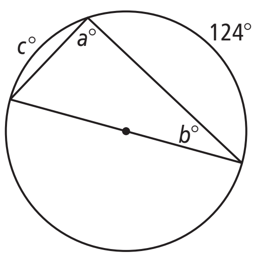
The measure of the inscribed angle is __?__ the measure of the intercepted arc. **half**

2. 

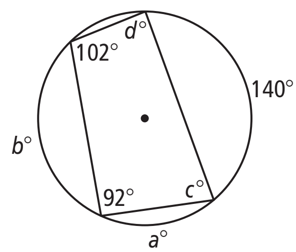
98; 49

3. 

72; 24; 84

4. 

90; 28; 56

5. 

64; 112; 78; 88

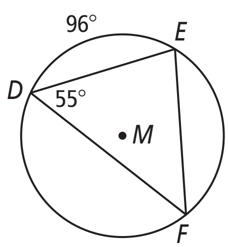
Find each indicated measure for $\odot M$.

6. **a.** $m\overset{\frown}{EF}$ **110**

b. $m\angle E$ **77**

c. $m\angle F$ **48**

d. $m\overset{\frown}{DF}$ **154**

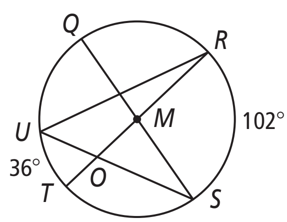


7. **a.** $m\angle S$ **33**

b. $m\overset{\frown}{TS}$ **78**

c. $m\overset{\frown}{QU}$ **66**

d. $m\angle TMS$ **78**



8. Reasoning A quadrilateral that is not a rectangle is inscribed in a circle. What is the least number of arc measures needed to determine the measures of each angle in the quadrilateral? Use drawings to explain.

Two; you can find the other measures using Corollary 3 of Theorem 12-11.

9. Open-Ended Draw a circle. Inscribe two angles in the circle so that the angles are congruent. Explain which corollary to Theorem 12-11 you can use to prove the angles are congruent without measuring them.

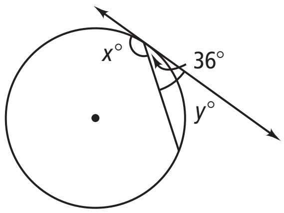
Check students' work. If Corollary 1 is cited, angles should intercept the same arc. If Corollary 2 is cited, angles should intercept a semicircle.

12-3 Practice (continued) — Form K

Inscribed Angles

Find the value of each variable. Lines that appear to be tangent are tangent.

10.



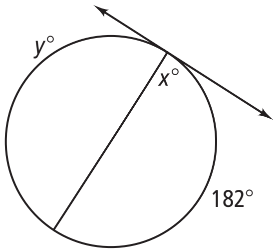
To start, determine the relationship between the inscribed angle formed by a tangent and a chord and the intercepted arc.

The measure of the intercepted arc is *y*.

$36 = \frac{1}{2} \cdot \boxed{y}$

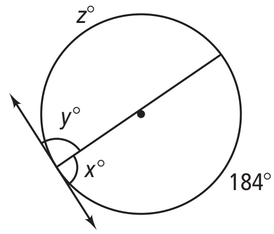
144; 72

11.



91; 178

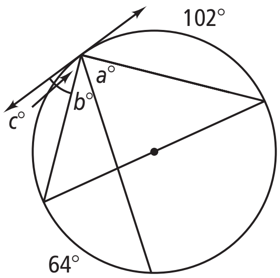
12.



92; 88; 176

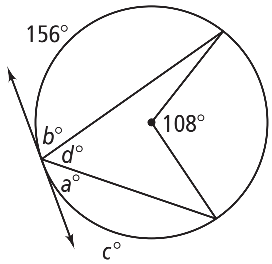
Find the value of each variable. For each circle, the dot represents the center.

13.



58; 32; 39

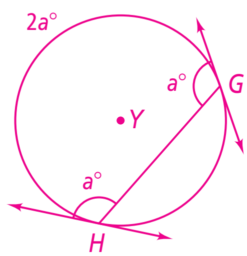
14.



48; 78; 96; 54

15. Reasoning $\angle ABC$ is formed by diameter $\overline{AB}$ and a tangent to $\odot D$ containing point *C*. What is the measure of $\angle ABC$? Explain. **90; the intercepted arc is a semicircle, so the inscribed angle must be a right angle.**

16. Draw a Diagram $\overline{GH}$ is a chord of $\odot Y$. $\overline{GH}$ forms angles with tangents at points *G* and *H*. What is the relationship between the angles formed? Use a drawing in your explanation. **The inscribed angles on the same side of the tangents are congruent because they intercept the same arc.**



17. Writing Explain why the angle formed by a tangent and a chord has the same measure as an inscribed angle that intercepts the same arc. **Answers may vary. Sample: You can move the inscribed angle so that one chord becomes tangent to the circle while keeping it so that the same angle measure still intercepts the same arc.**

Name ______________________ Class ____________ Date ____________

12-3 Standardized Test Prep

Inscribed Angles

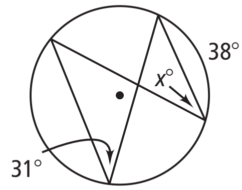
Multiple Choice

For Exercises 1–6, choose the correct letter.

1. What is the value of x? **B**

Ⓐ 19 Ⓒ 38

Ⓑ 31 Ⓓ 62

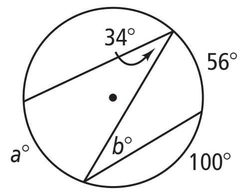


For Exercises 2–3, use the diagram at the right.

2. What is the value of a? **H**

Ⓕ 34 Ⓗ 68

Ⓖ 56 Ⓘ 146



3. What is the value of b? **A**

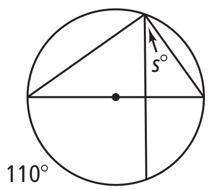
Ⓐ 28 Ⓒ 56

Ⓑ 34 Ⓓ 112

4. What is the value of s? **F**

Ⓕ 35 Ⓗ 70

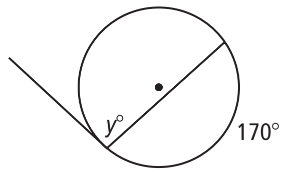
Ⓖ 55 Ⓘ 90



5. What is the value of y if the segment outside the circle is tangent to the circle? **B**

Ⓐ 85 Ⓒ 190

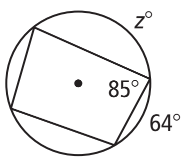
Ⓑ 95 Ⓓ cannot determine



6. What is the value of z? **H**

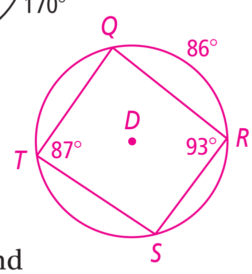
Ⓕ 77 Ⓗ 126

Ⓖ 95 Ⓘ 154



Extended Response

7. A student inscribes quadrilateral $QRST$ in $\odot D$ so that $m\overset{\frown}{QR} = 86$ and $m\angle R = 93$. What is the measure of $\overset{\frown}{RS}$? Draw a diagram and explain the steps you took to find the answer.



[4] 88; Explanations may vary. Sample: $m\angle R = 93$, $\angle T$ and $\angle R$ are suppl. according to Corollary 3 of Inscribed $\angle$ Thm, so $m\angle T = 180 - 93 = 87$. According to Inscribed $\angle$ Thm., $m\overset{\frown}{QRS} = 2m\angle T$, and $m\overset{\frown}{RS} = m\overset{\frown}{QRS} - m\overset{\frown}{QR}$, so $m\overset{\frown}{RS} = 2m\angle T - m\overset{\frown}{QR} = 2(87) - 86 = 88$. [3] appropriate methods, but incorrect answer [2] diagram incorrectly drawn [1] correct answer without work [0] incorrect answer and incomplete work

Name ______________________ Class ____________ Date ____________

12-3 Enrichment

Inscribed Angles

Ptolemy's Theorem

A quadrilateral that can be inscribed in a circle is a *cyclic* quadrilateral. In a cyclic quadrilateral, the sum of the products of the opposite sides is equal to the product of its diagonals. This is *Ptolemy's Theorem.* For quadrilateral $ABCD$, $AD \times BC + AB \times CD = AC \times BD$. (Figure 1)

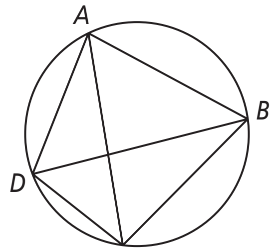


Figure 1

The steps below guide you through a proof of this theorem.

A. Find point E on BD so that $\angle DCA$ and $\angle BCE$ are congruent. (Figure 2)

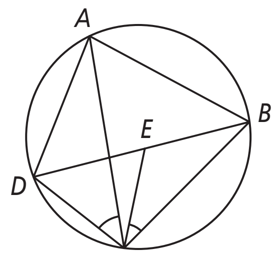


Figure 2

B. $\angle DAC$ and $\angle DBC$ intercept the same arc. What is this arc? What is the relationship between $\angle DAC$ and $\angle DBC$? **$\overset{\frown}{DC}$; $\angle DAC \cong \angle DBC$**

C. $\triangle CDA$ and $\triangle CEB$ are similar, because they have two pairs of congruent angles.

D. $\frac{AD}{BE} = \frac{AC}{BC}$ and $AD \times BC = AC \times BE$, because the triangles are similar.

E. $\angle CAB$ and $\angle CDB$ intercept the same arc. What is this arc? What is the relationship between $\angle CAB$ and $\angle CDB$? **$\overset{\frown}{BC}$; they are congruent.**

F. $m\angle DCA + m\angle ACB = m\angle DCB$ and $m\angle DCE + m\angle ECB = m\angle DCB$. What is the relationship between $\angle ACB$ and $\angle DCE$? **They are congruent.**

G. What is true of $\triangle CBA$ and $\triangle CED$? **They are similar by AA.**

H. $\frac{CD}{AC} = \frac{DE}{AB}$ and $AC \times DE = AB \times CD$.

I. What is the sum of BE and DE? **BD**

J. Add the equations from Steps D and H. What do you find?
$AD \times BC + AB \times CD = AC \times BD$

1. What is the converse of Ptolemy's Theorem?
If the sum of the products of the opposite sides of a quadrilateral is equal to the product of its diagonals, then the quadrilateral is cyclic.

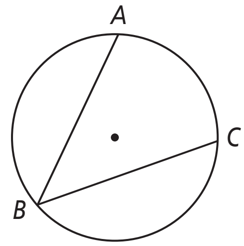
2. How could you use Ptolemy's Theorem to show that a given quadrilateral is *not* cyclic, or cannot be inscribed in a circle?
Show that $AD \times BC + AB \times CD \neq AC \times BD$.

Name ______________________ Class ____________ Date ____________

12-3 Reteaching

Inscribed Angles

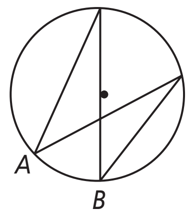
Two chords with a shared endpoint at the vertex of an angle form an inscribed angle. The intercepted arc is formed where the other ends of the chords intersect the circle.



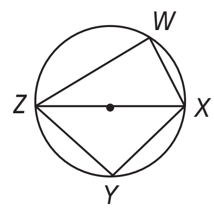
In the diagram at the right, chords $\overline{AB}$ and $\overline{BC}$ form inscribed $\angle ABC$. They also create intercepted arc $\overset{\frown}{AC}$.

The following theorems and corollaries relate to inscribed angles and their intercepted arcs.

Theorem 12-11: The measure of an inscribed angle is half the measure of its intercepted arc.



- *Corollary 1:* If two inscribed angles intercept the same arc, the angles are congruent. So, $m\angle A \cong m\angle B$.
- *Corollary 2:* An angle that is inscribed in a semicircle is always a right angle. So, $m\angle W = m\angle Y = 90$.
- *Corollary 3:* When a quadrilateral is inscribed in a circle, the opposite angles are supplementary. So, $m\angle X + m\angle Z = 180$.



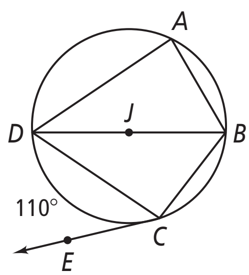
Theorem 12-12: The measure of an angle formed by a tangent and a chord is half the measure of its intercepted arc.

Problem

Quadrilateral $ABCD$ is inscribed in $\odot J$.

$m\angle ADC = 68$; $\overrightarrow{CE}$ is tangent to $\odot J$

What is $m\angle ABC$? What is $m\overset{\frown}{CB}$? What is $m\angle DCE$?



$m\angle ABC + m\angle ADC = 180$	Corollary 3 of Theorem 12-11
$m\angle ABC + 68 = 180$	Substitution
$m\angle ABC = 112$	Subtraction Property
$m\overset{\frown}{DB} = m\overset{\frown}{DC} + m\overset{\frown}{CB}$	Arc Addition Postulate
$180 = 110 + m\overset{\frown}{CB}$	Substitution
$70 = m\overset{\frown}{CB}$	Simplify.
$m\overset{\frown}{CD} = 110$	Given
$m\angle DCE = \frac{1}{2}m\overset{\frown}{CD}$	Theorem 12-12
$m\angle DCE = \frac{1}{2}(110)$	Substitution
$m\angle DCE = 55$	Simplify.

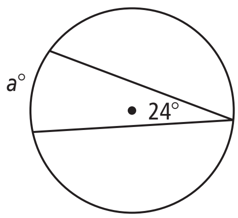
So, $m\angle ABC = 112$, $m\overset{\frown}{CB} = 70$, and $m\angle DCE = 55$.

12-3 Reteaching (continued)

Inscribed Angles

Exercises

In Exercises 1–9, find the value of each variable.

1. 

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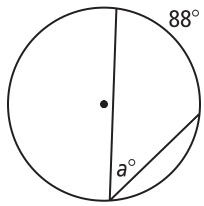
2. 

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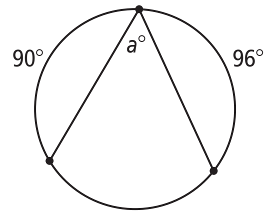
3. 

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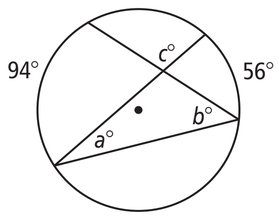
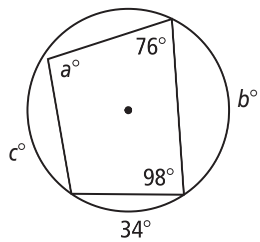
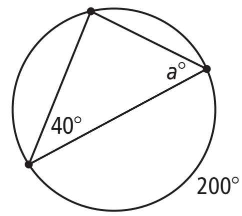
4. 

28; 47; 105

5. 

82; 130; 118

6. 

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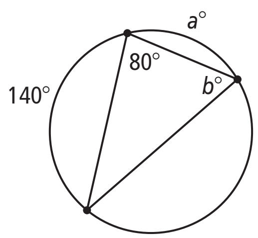
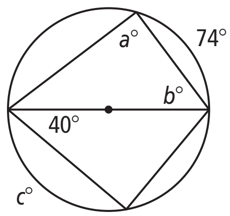
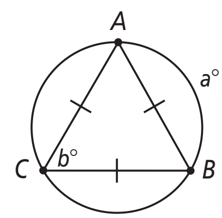
7. 

60; 70

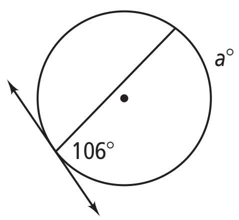
8. 

90; 53; 100

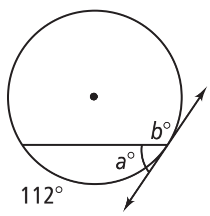
9. 

120; 60

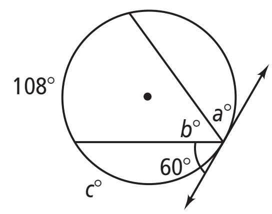
Find the value of each variable. Lines that appear to be tangent are tangent.

10. 

212

11. 

56; 124

12. 

66; 54; 120

Points A, B, and D lie on $\odot C$. $m\angle ACB = 40$. $m\widehat{AB} < m\widehat{AD}$. Find each measure.

13. $m\widehat{AB}$ 40

14. $m\angle ADB$ 20

15. $m\angle BAC$ 70

16. A student inscribes a triangle inside a circle. The triangle divides the circle into arcs with the following measures: 46°, 102°, and 212°. What are the measures of the angles of the triangle? 23; 51; 106

17. A student inscribes $NOPQ$ inside $\odot Y$. The measure of $m\angle N = 68$ and $m\angle O = 94$. Find the measures of the other angles of the quadrilateral.
$m\angle P = 112$; $m\angle Q = 86$

Name ______________________________ Class ____________ Date ____________

12-4 Additional Vocabulary Support

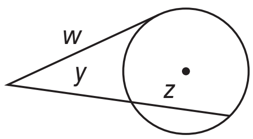
Angle Measures and Segment Lengths

Concept List

chord	diameter	$m\angle 1 = \frac{1}{2}(x + y)$
$m\angle 1 = \frac{1}{2}(x - y)$	secant	tangent
$w \cdot x = y \cdot z$	$(w + x)w = (y + z)y$	$w^2 = (y + z)y$

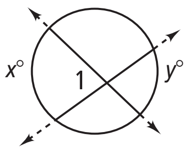
Choose the concept from the list above that best represents the item in each box.

1.



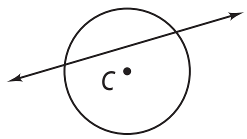
$w^2 = (y + z)y$

2.



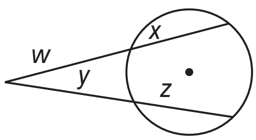
$m\angle 1 = \frac{1}{2}(x + y)$

3.



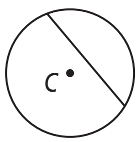
secant

4.



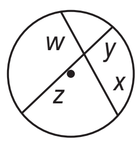
$(w + x)w = (y + z)y$

5.



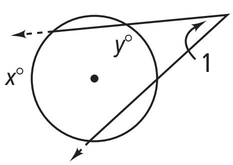
chord

6.



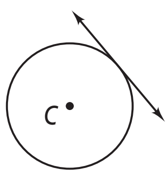
$w \cdot x = y \cdot z$

7.



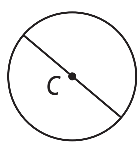
$m\angle 1 = \frac{1}{2}(x - y)$

8.



tangent

9.



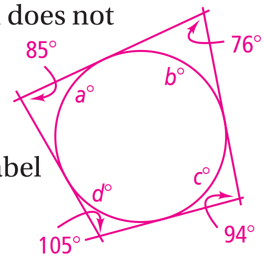
diameter

Name ______________________ Class ____________ Date ____________

12-4 Think About a Plan

Angle Measures and Segment Lengths

A circle is inscribed in a quadrilateral whose four angles have measures 85, 76, 94, and 105. Find the measures of the four arcs between consecutive points of tangency.



1. Draw a diagram of a circle inscribed in a quadrilateral. The diagram does not have to be exact; it is just a visual aid.

2. Label as $a°$ the intercepted arc closest to the angle measuring 85. Label as $b°$ the intercepted arc closest to the angle measuring 76. Label as $c°$ the intercepted arc closest to the angle measuring 94. Finally, label as $d°$ the intercepted arc closest to the angle measuring 105.

3. What is the sum of the measures of the four arcs $(a + b + c + d)$? **360**

4. Write an equation that relates a to the measures of the other arcs. **$a = 360 - (b + c + d)$**

5. Repeat Step 4 to write three more equations that relate the measure of each arc to the measures of the other arcs.
$b = 360 - (a + c + d)$; $c = 360 - (a + b + d)$; $d = 360 - (a + b + c)$

6. How is the measure of each angle in the quadrilateral related to the measures of the intercepted arcs? (*Hint:* Use Theorem 12–14.)
The measure is half the difference of the measures of the intercepted arcs.

7. Write expressions for the measures of the two arcs intercepted by the 85° angle. **a; $b + c + d$**

8. Using Theorem 12-14, write an equation that relates 85 to the measures of the intercepted arcs. **$85 = \frac{1}{2}(b + c + d - a)$**

9. Multiply each side of this equation by 2. **$170 = (b + c + d - a)$**

10. Look at your equation from Step 4. What is the value of $b + c + d$?
$b + c + d = 360 - a$

11. How can you use the equations from Steps 9 and 10 to find a?
Substitute $360 - a$ for $b + c + d$ into $170 = (b + c + d - a)$ to get $170 = 360 - 2a$.
Isolate a and solve.

12. What is the value of a? **95**

13. Repeat this process to find b, c, and d. What are their values? **104; 86; 75**

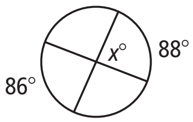
12-4 Practice

Form G

Angle Measures and Segment Lengths

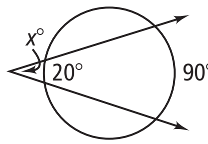
Find the value of *x*.

1.



87

2.



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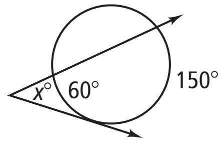
3.



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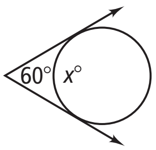
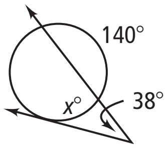
4.



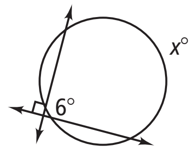
120

5.



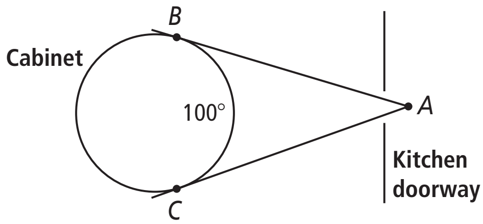
72

6.



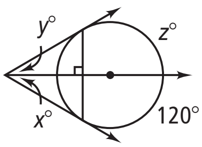
186

7. There is a circular cabinet in the dining room. Looking in from another room at point A, you estimate that you can see an arc of the cabinet of about 100°. What is the measure of $\angle A$ formed by the tangents to the cabinet? 80



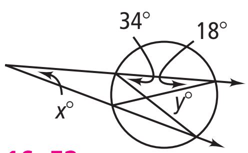
Algebra **Find the value of each variable using the given chord, secant, and tangent lengths. If the answer is not a whole number, round to the nearest tenth.**

8.



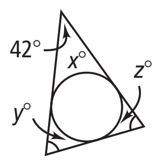
30; 30; 120

9.



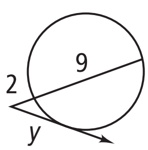
16; 52

10.



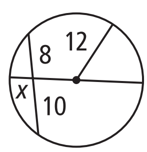
138; 111; 111

11.



4.7

12.



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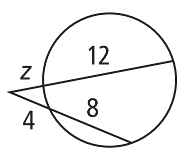
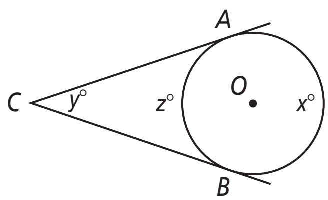
13.



3.2

Algebra **$\overline{CA}$ and $\overline{CB}$ are tangents to $\odot O$. Write an expression for each arc or angle in terms of the given variable.**



14. $m\widehat{AB}$ using x

$360 - x$

15. $m\widehat{AB}$ using y

$180 - y$

16. $m\angle C$ using x

$x - 180$

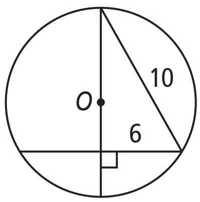
Name ______________________ Class ____________ Date ____________

12-4 Practice (continued)

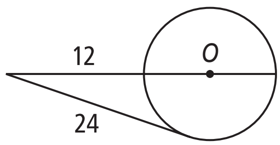
Form G

Angle Measures and Segment Lengths

Find the diameter of $\odot O$. A line that appears to be tangent is tangent. If your answer is not a whole number, round to the nearest tenth.

17. 

12.5

18. 

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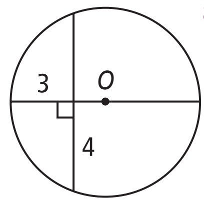
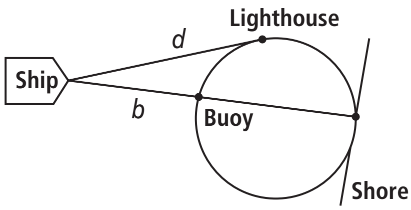
19. 

8.3

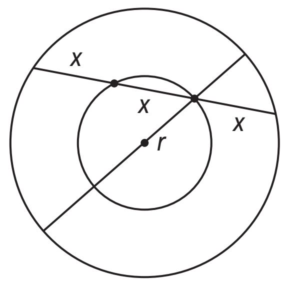
20. The distance from your ship to a lighthouse is *d*, and the distance to the buoy is *b*. Express the distance to the shore in terms of *d* and *b*.

$\frac{d^2}{b} - b$



21. Reasoning The circles at the right are concentric. The radius of the larger circle is twice the radius, *r*, of the smaller circle. Explain how to find the ratio $x : r$, then find it.

Answers may vary. Sample: Use Thm. 12-15, Case I: $x(2x) = r(3r)$, then take the square roots; $\sqrt{3} : \sqrt{2}$ or $\sqrt{6} : 2$.



22. A circle is inscribed in a parallelogram. One angle of the parallelogram measures 60. What are the measures of the four arcs between consecutive points of tangency? Explain.

60, 120, 60, and 120; for arcs *a, b, c,* and *d;* use $a + b + c + d = 360$ and Thm. 12-14 to solve for *a,* then solve for the other arcs.

23. An isosceles triangle with height 10 and base 6 is inscribed in a circle. Create a plan to find the diameter of the circle. Find the diameter.

Answers may vary. Sample: Height is part of diameter, which bisects base, so $d = 10 + x$, and by Thm. 12-15, Case I, $3(3) = 10(x)$; 10.9.

24. If three tangents to a circle form an equilateral triangle, prove that the tangent points form an equilateral triangle inscribed in the circle.

Answers may vary. Sample: Given arcs *a, b, c,* $a + b + c = 360$, and by Thm 12–14, $a = b = c = 120$. By Inscribed Angle Thm., tangents intersect at 60.

25. A circle is inscribed in a quadrilateral whose four angles have measures 86, 78, 99, and 97. Find the measures of the four arcs between consecutive points of tangency.

94, 83, 81, and 102

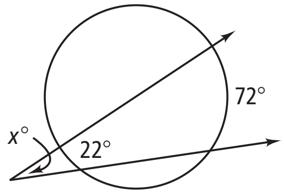
Name ______________________ Class ____________ Date ____________

12-4 Practice

Form K

Angle Measures and Segment Lengths

Algebra **Find the value of each variable.**

1. 

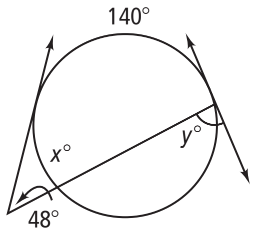
25

To start, identify the type of rays intersecting in the diagram.

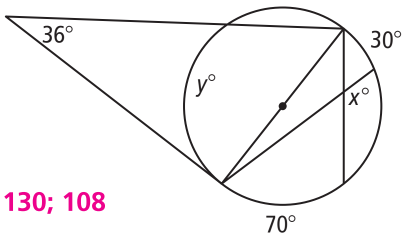
Two __?__ rays intersect outside the circle. **secant**

Then write an equation using Theorem 12-14.

$x = \frac{1}{2} \cdot \left(\boxed{72} - \boxed{22} \right)$

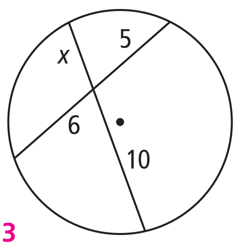
2. 

44; 88

3. 

130; 108

Algebra **Find the value of each variable using the given chord, secant, and tangent lengths. If your answer is not a whole number, round it to the nearest tenth.**

4. 

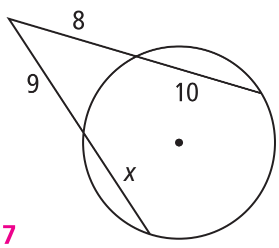
3

To start, identify the type of segments intersecting in the diagram.

Two __?__ intersect inside the circle. **chords**

Then write an equation using Theorem 12-12, Case I.

$\boxed{6} \cdot \boxed{5} = x \cdot \boxed{10}$

5. 

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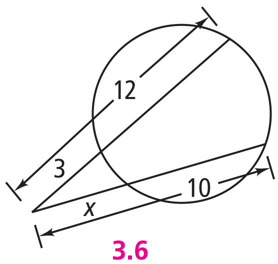
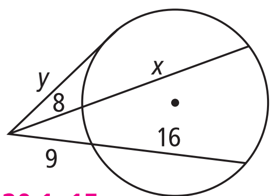
6. 

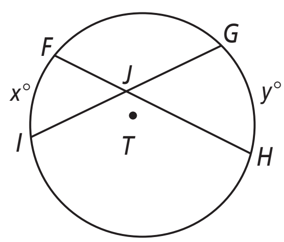
3.6

7. 

20.1; 15

8. Algebra $\overline{FH}$ and $\overline{GI}$ are chords in $\odot T$. Write an expression for $m\angle FJI$ in terms of x and y.

$\frac{1}{2}(x + y)$

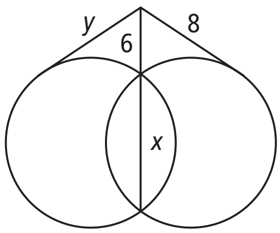


12-4 Practice (continued) Form K

Angle Measures and Segment Lengths

Algebra Find the value of each variable using the given chord, secant, and tangent lengths. If your answer is not a whole number, round it to the nearest tenth.

9.

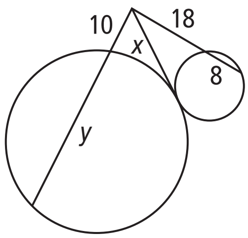


To start, write an equation using Theorem 12-15, Case III.

$\boxed{8} \cdot \boxed{8} = (x + 6) \cdot \boxed{6}$

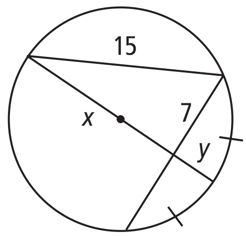
4.7; 8

10.



21.6; 36.8

11.

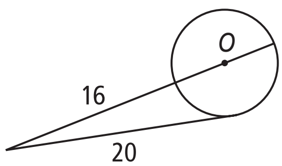


13.3; 3.7

12. You look through binoculars at the circular dome of the Capitol building in Washington, D.C. Your binoculars are at the vertex of the angle formed by tangents to the dome. You estimate that this vertex angle is 70°. What is the measure of the arc of the circular base of the dome that is visible? **110**

Find the diameter of ⊙*O*. A line that appears to be tangent is tangent. If your answer is not a whole number, round it to the nearest tenth.

13.



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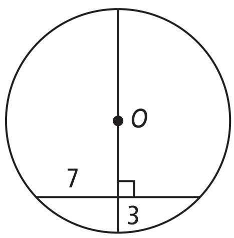
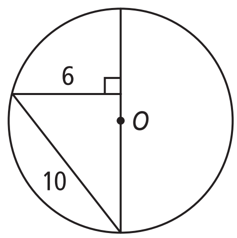
14.



19.3

15.



12.5

16. A circle is inscribed in a quadrilateral whose four angles have measures 74, 96, 81, 109. Find the measures of the four arcs between consecutive points of tangency. **106; 84; 99; 71**

17. $\triangle CED$ is inscribed in a circle with $m\angle C = 40$, $m\angle E = 55$, and $m\angle D = 85$. What are the measures of $\overset{\frown}{CE}$, $\overset{\frown}{ED}$, and $\overset{\frown}{DC}$? Explain how you can check that your answers are correct. **$m\overset{\frown}{CE} = 170$; $m\overset{\frown}{ED} = 80$; $m\overset{\frown}{DC} = 110$; to check, find the sum of the arc measures. The sum should be 360.**

Name ______________________ Class ____________ Date ____________

12-4 Standardized Test Prep

Angle Measures and Segment Lengths

Multiple Choice

For Exercises 1–6, choose the correct letter.

1. Which of the following statements is false? **C**

(A) Every chord is part of a secant.

(C) Every chord is a diameter.

(B) Every diameter is part of a secant.

(D) Every diameter is a chord.

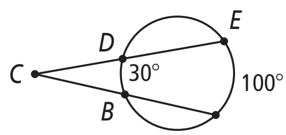
2. In the figure at the right, what is $m\angle C$? **G**

(F) 15

(H) 50

(G) 35

(I) 65



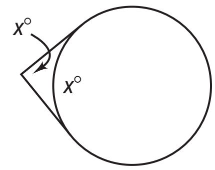
3. In the figure at the right, what is the value of x? **D**

(A) 45

(C) 75

(B) 60

(D) 90



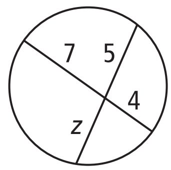
4. In the figure at the right, what is the value of z? **G**

(F) 2.9

(H) 6

(G) 5.6

(I) 8.75



5. An equilateral triangle with sides of length 6 is inscribed in a circle. What is the diameter of the circle? **C**

(A) 5.2

(B) 6

(C) 6.9

(D) 7.5

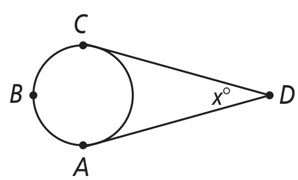
6. In the figure at the right, what is $m\overset{\frown}{ABC}$ in terms of x? **F**

(F) $180 + x$

(H) $2(180 + x)$

(G) $180 - x$

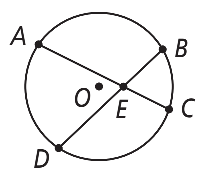
(I) $360 - x$



Short Response

7. Use $\odot O$ to prove that $\triangle AED \sim \triangle BEC$.

By Thm. 12.15, $AE \cdot CE = DE \cdot BE$, and by division, $\frac{AE}{EB} = \frac{DE}{CE}$. $\angle AED \cong \angle CEB$. because they are vert. $\triangle$s. The two $\triangle$s are $\sim$ by SAS similarity. [2] Student sets up proof correctly using SAS similarity. [1] one error in proof [0] proof incorrect or no proof given

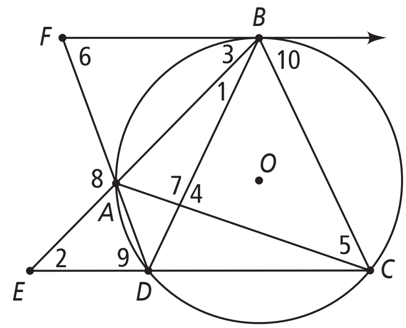


Name ______________________ Class ____________ Date ____________

12-4 Enrichment

Angle Measures and Segment Lengths

Quadrilateral *ABCD* is inscribed in $\odot O$. Chords $\overline{BA}$ and $\overline{CD}$ are extended to intersect at point *E*. A tangent at *B* intersects $\overline{DA}$ where $\overline{DA}$ is extended to point *F*. Diagonals $\overline{BD}$ and $\overline{AC}$ of quadrilateral *ABCD* are drawn.



$m\overset{\frown}{AB} = 2x$

$m\overset{\frown}{BC} = 2x + 8$

$m\overset{\frown}{DC} = x$

$m\overset{\frown}{DA} = x - 32$

For Exercises 1–16 use the figure above.

1. Write an equation that can be used to solve for *x*.
   $2x + 2x + 8 + x + x - 32 = 360$; $6x - 24 = 360$; $6x = 384$
2. Solve for *x*. **64**
3. $m\overset{\frown}{AB} =$ __?__ **128**
4. $m\overset{\frown}{BC} =$ __?__ **136**
5. $m\overset{\frown}{DC} =$ __?__ **64**
6. $m\overset{\frown}{DA} =$ __?__ **32**

Find the measures of angles 1–10. Complete the table below.

	Angle	Secants, chords, or tangents that form angle	Measure
7.	∠1	chords $\overline{AB}$, $\overline{BD}$	16
8.	∠2	secants $\overline{EB}$, $\overline{EC}$	52
9.	∠3	tangent $\overrightarrow{FB}$ and chord $\overline{AB}$	64
10.	∠4	chords $\overline{BD}$, $\overline{AC}$	84
11.	∠5	chords $\overline{AC}$, $\overline{BC}$	64
12.	∠6	tangent $\overrightarrow{FB}$ and secant $\overline{FD}$	36
13.	∠7	chords $\overline{AC}$, $\overline{BD}$	96
14.	∠8	$\overline{AF}$, $\overline{AE}$	100
15.	∠9	$\overline{DE}$, $\overline{DA}$	48
16.	∠10	tangent $\overrightarrow{FB}$ and chord $\overline{BC}$	68

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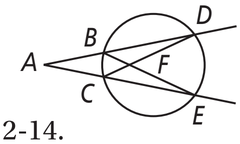
12-4 Reteaching

Angle Measures and Segment Lengths

Problem



In the circle shown, $m\widehat{BC} = 15$ and $m\widehat{DE} = 35$.

What are $m\angle A$ and $m\angle BFC$?

Because $\overleftrightarrow{AD}$ and $\overleftrightarrow{AE}$ are secants, $m\angle A$ can be found using Theorem 12-14.

$$\begin{aligned} m\angle A &= \tfrac{1}{2}(m\widehat{DE} - m\widehat{BC}) \\ &= \tfrac{1}{2}(35 - 15) \\ &= 10 \end{aligned}$$

Because $\overline{BE}$ and $\overline{CD}$ are chords, $m\angle BFC$ can be found using Theorem 12-13.

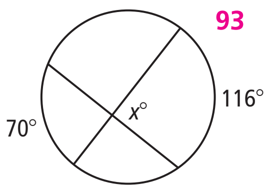
$$\begin{aligned} m\angle BFC &= \tfrac{1}{2}(m\widehat{DE} + m\widehat{BC}) \\ &= \tfrac{1}{2}(35 + 15) \\ &= 25 \end{aligned}$$

So, $m\angle A = 10$ and $m\angle BFC = 25$.

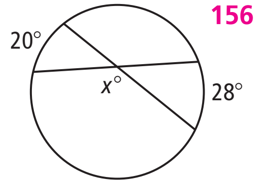
Exercises

Algebra Find the value of each variable.

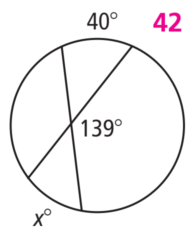
1. **93**



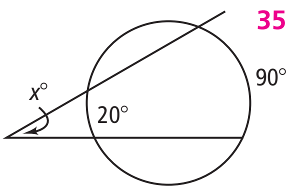
2. **156**



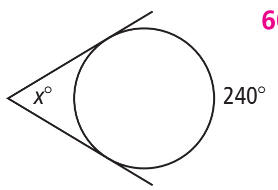
3. **42**



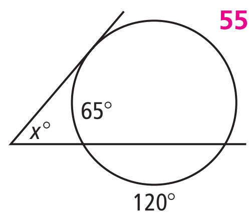
4. **35**



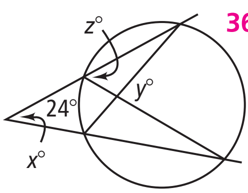
5. **60**



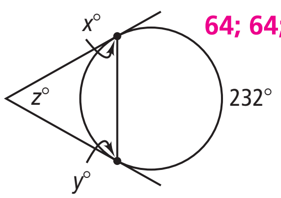
6. **55**



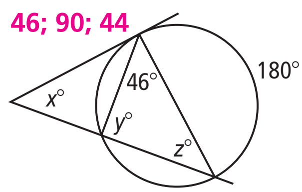
7. **36; 60; 48**



8. **64; 64; 52**



9. **46; 90; 44**



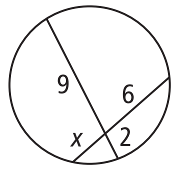
12-4 Reteaching (continued)

Angle Measures and Segment Lengths

Segment Lengths

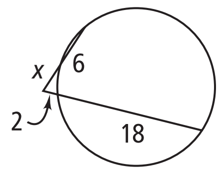
Here are some examples of different cases of Theorem 12-15.

A. Chords intersecting inside a circle:
part · *part* = *part* · *part*



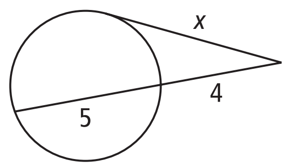
$$6x = 18$$
$$x = \frac{18}{6} = 3$$

B. Secants intersecting outside a circle:
outside · *whole* = *outside* · *whole*



$$x(x + 6) = 2(18 + 2)$$
$$x^2 + 6x = 40$$
$$x^2 + 6x - 40 = 0$$
$$(x + 10)(x - 4) = 0$$
$$x = -10 \text{ or } x = 4$$

C. Tangent and secant intersecting outside a circle:
tangent · *tangent* = *outside* · *whole*

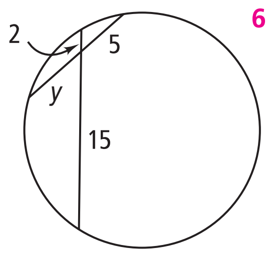


$$x(x) = 4(4 + 5)$$
$$x^2 = 4(9)$$
$$x^2 = 36$$
$$x = -6 \text{ or } x = 6$$

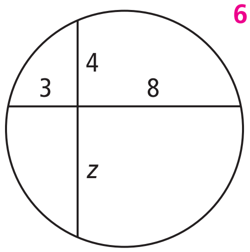
Exercises

Algebra **Find the value of each missing variable.**

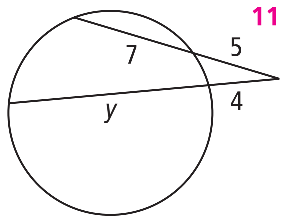
10. 6



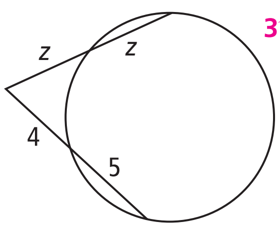
11. 6



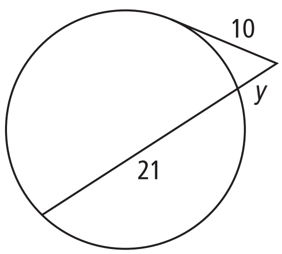
12. 11



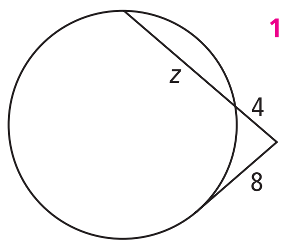
13. $3\sqrt{2}$



14. 4



15. 12



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12-5 Additional Vocabulary Support

Circles in the Coordinate Plane

Problem

What is the equation of the circle with center $(3, -1)$ that passes through the point $(1, 2)$?

Step 1 Use the center and the point on the circle to find the radius.

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Use the Distance Formula to find r.
$r = \sqrt{(1 - 3)^2 + (2 - (-1))^2}$	Substitute $(3, -1)$ for (x_1, y_1) and $(1, 2)$ for (x_2, y_2).
$r = \sqrt{(-2)^2 + (3)^2}$	Simplify.
$r = \sqrt{13}$	Simplify.

Step 2 Use the radius and the center to write an equation.

$(x - h)^2 + (y - k)^2 = r^2$	Use the standard form of an equation of a circle.
$(x - 3)^2 + (y - (-1))^2 = (\sqrt{13})^2$	Substitute $(3, -1)$ for (h, k) and $\sqrt{13}$ for r.
$(x - 3)^2 + (y + 1)^2 = 13$	Simplify.

Exercise

What is the equation of the circle with center $(-2, 5)$ that passes through the point $(4, -1)$?

Step 1 **Use the center and the point on the circle to find the radius.**

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	**Use the Distance Formula to find r.**
$r = \sqrt{(4 - (-2))^2 + (-1 - 5)^2}$	**Substitute $(-2, 5)$ for (x_1, y_1) and $(4, -1)$ for (x_2, y_2).**
$r = \sqrt{(6)^2 + (-6)^2}$	**Simplify.**
$r = \sqrt{72}$	**Simplify.**

Step 2 **Use the radius and the center to write an equation.**

$(x - h)^2 + (y - k)^2 = r^2$	**Use the standard form of an equation of a circle.**
$(x - (-2))^2 + (y - 5)^2 = (\sqrt{72})^2$	**Substitute $(-2, 5)$ for (h, k) and $\sqrt{72}$ for r.**
$(x + 2)^2 + (y - 5)^2 = 72$	**Simplify.**

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12-5 Think About a Plan

Circles in the Coordinate Plane

What are the x- and y-intercepts of the line tangent to the circle $(x - 2)^2 + (y - 2)^2 = 5^2$ at the point $(5, 6)$?

1. What is the relationship between the line tangent to the circle at the point (5, 6) and the radius of the circle containing the point (5, 6)?

They are perpendicular.

2. What is the product of the slopes of two perpendicular lines or line segments? **-1**

3. What is the center of the circle? **(2, 2)**

4. How can you use the slope formula to find the slope of the radius of the circle containing the point (5, 6)? What is this slope?

Use the slope formula with the points (5, 6) and (2, 2); $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{5 - 2} = \frac{4}{3}$.

5. What is the slope of the line tangent to the circle at point (5, 6)? **$-\frac{3}{4}$**

6. What is the slope-intercept equation for a line? **$y = mx + b$**

7. How can you use the slope-intercept equation to find the y-intercept for the line tangent to the circle at point (5, 6)?

Use the known slope, and the x-value and y-value for the point (5, 6) to solve for b; $6 = -\frac{3}{4}(5) + b$; $b = 9\frac{3}{4}$ or 9.75.

8. How can you use this equation to find the x-intercept for the line tangent to the circle at point (5, 6)?

Find the x value of the x-intercept by setting $y = 0$ and substituting for m and b; $0 = -\frac{3}{4}x + 9\frac{3}{4}$; $x = 13$.

9. What are the x- and y-intercepts for the line tangent to the circle at point (5, 6)? **(13, 0) and $(0, 9\frac{3}{4})$**

12-5 Practice

Form G

Circles in the Coordinate Plane

Find the center and radius of each circle.

1. $x^2 + y^2 = 36$ **(0, 0); 6**

2. $(x - 2)^2 + (y - 7)^2 = 49$ **(2, 7); 7**

3. $(x + 1)^2 + (y + 6)^2 = 16$ **(−1, −6); 4**

4. $(x + 3)^2 + (y - 11)^2 = 12$ **(−3, 11); $\sqrt{12}$**

Write the standard equation of each circle.

5. center (0, 0); $r = 7$ **$x^2 + y^2 = 49$**

6. center (4, 3); $r = 8$ **$(x - 4)^2 + (y - 3)^2 = 64$**

7. center (5, 3); $r = 2$ **$(x - 5)^2 + (y - 3)^2 = 4$**

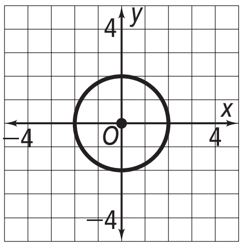
8. center (−5, 4); $r = \frac{1}{2}$ **$(x + 5)^2 + (y - 4)^2 = \frac{1}{4}$**

9. center (−2, −5); $r = \sqrt{2}$ **$(x + 2)^2 + (y + 5)^2 = 2$**

10. center (−1, 6); $r = \sqrt{5}$ **$(x + 1)^2 + (y - 6)^2 = 5$**

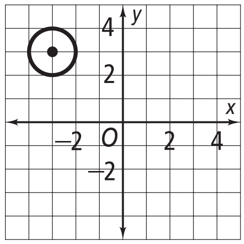
Write the standard equation of each circle.

11.



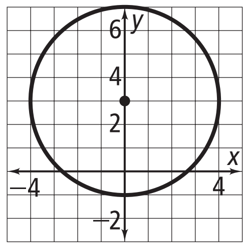
$x^2 + y^2 = 4$

12.



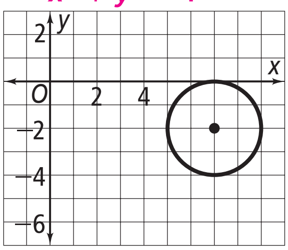
$(x + 3)^2 + (y - 3)^2 = 1$

13.



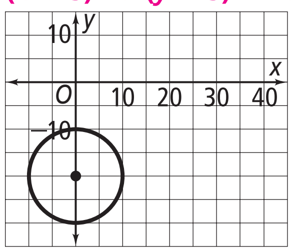
$x^2 + (y - 3)^2 = 16$

14.



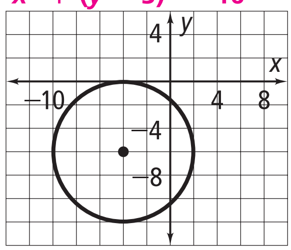
$(x - 7)^2 + (y + 2)^2 = 4$

15.



$x^2 + (y + 20)^2 = 100$

16.



$(x + 4)^2 + (y + 6)^2 = 36$

Find the center and radius of each circle. Then graph the circle.
Check students' graphs.

17. $x^2 + y^2 = 25$
circle with center at (0, 0) and radius 5

18. $(x - 3)^2 + (y - 5)^2 = 9$
circle with center at (3, 5) and radius 3

19. $(x + 2)^2 + (y + 4)^2 = 16$
circle with center at (−2, −4) and radius 4

20. $(x + 1)^2 + (y - 1)^2 = 36$
circle with center at (−1, 1) and radius 6

Write the standard equation of the circle with the given center that passes through the given point.

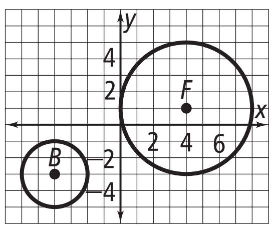
21. center (0, 0); point (3, 4) **$x^2 + y^2 = 25$**

22. center (5, 9); point (2, 9) **$(x - 5)^2 + (y - 9)^2 = 9$**

23. center (−4, −3); point (2, 2)
$(x + 4)^2 + (y + 3)^2 = 61$

24. center (7, −2); point (−1, −6)
$(x - 7)^2 + (y + 2)^2 = 80$

Write the standard equation of each circle in the diagram at the right.



25. $\odot B$ **$(x + 4)^2 + (y + 3)^2 = 4$**

26. $\odot F$ **$(x - 4)^2 + (y - 1)^2 = 16$**

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12-5 Practice (continued)

Form G

Circles in the Coordinate Plane

Write an equation of a circle with diameter $\overline{AB}$.

27. $A(0, 0)$, $B(-6, 8)$
$(x + 3)^2 + (y - 4)^2 = 25$

28. $A(0, -1)$, $B(2, 1)$
$(x - 1)^2 + y^2 = 2$

29. $A(7, 5)$, $B(-1, -1)$
$(x - 3)^2 + (y - 2)^2 = 25$

30. Reasoning Circles in the coordinate plane that have the same center and congruent radii are identical. Circles with congruent radii are congruent. In (a) through (g), circles lie in the coordinate plane.

a. Two circles have equal areas. Are the circles congruent? **yes**

b. Two circles have circumferences that are equal in length. Are the circles congruent? **yes**

c. How many circles have an area of 36π m^2? **infinitely many**

d. How many circles have a center of $(4, 7)$? **infinitely many**

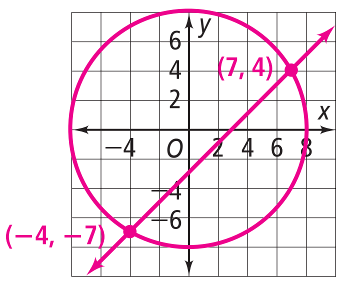
e. How many circles have an area of 36π m^2 and center $(4, 7)$? **1**

f. How many circles have a circumference of 6π in. and center $(4, 7)$? **1**

g. How many circles have a diameter with endpoints $A(0, 0)$ and $B(-6, 8)$? **1**

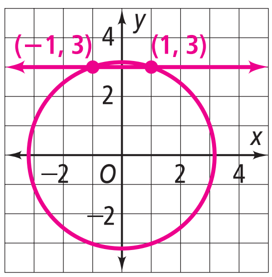
Sketch the graph of each equation. Find all points of intersection of each pair of graphs.

31. $x^2 + y^2 = 65$
$y = x - 3$



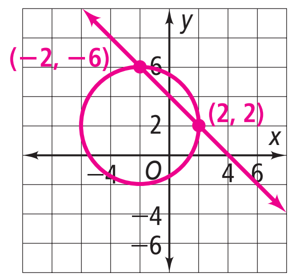
(7, 4); (−4, −7)

32. $x^2 + y^2 = 10$
$y = 3$



(1, 3); (−1, 3)

33. $(x + 2)^2 + (y - 2)^2 = 16$
$y = -x + 4$



(2, 2); (−2, 6)

34. Writing Two circles in the coordinate plane with congruent radii intersect in exactly two points. Why is it not possible for these circles to be concentric?
Answers may vary. Sample: Concentric circles with congruent radii are identical.

35. Find the circumference and area of the circle whose equation is $(x - 5)^2 + (y + 4)^2 = 49$. Leave your answer in terms of π. **14π; 49π sq units**

36. What are the x- and y-intercepts of the line tangent to the circle $(x + 6)^2 + (y - 2)^2 = 100$ at the point $(2, -4)$? **5; $-6\frac{2}{3}$**

12-5 Practice

Form K

Circles in the Coordinate Plane

Write the standard equation of each circle.

1. center $(7, -3)$; $r = 9$ $(x - 7)^2 + (y + 3)^2 = 81$

To start, write the equation of a circle.

$(\boxed{x} - h)^2 + (\boxed{y} - k)^2 = \boxed{r}^2$

Identify the values of h, k, and r.

$h = \boxed{7}$ $k = \boxed{-3}$ $r = \boxed{9}$

2. center $(0, 4)$; $r = 3$
$x^2 + (y - 4)^2 = 9$

3. center $(-2, -8)$; $r = 4$
$(x + 2)^2 + (y + 8)^2 = 16$

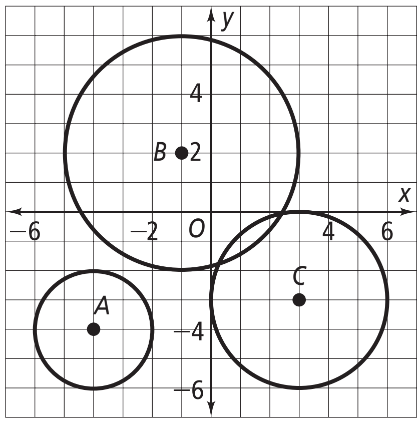
4. center $(2, 6)$; $r = 12$
$(x - 2)^2 + (y - 6)^2 = 144$

5. center $(10, 0)$; $r = 7$
$(x - 10)^2 + y^2 = 49$

6. center $(-5, -4)$; $r = \sqrt{3}$
$(x + 5)^2 + (y + 4)^2 = 3$

7. center $(-3, 2)$; $r = \sqrt{10}$
$(x + 3)^2 + (y - 2)^2 = 10$

Write the standard equation for each circle in the diagram at the right.



8. $\odot A$ $(x + 4)^2 + (y + 4)^2 = 4$

9. $\odot B$ $(x + 1)^2 + (y - 2)^2 = 16$

10. $\odot C$ $(x - 3)^2 + (y + 3)^2 = 9$

Write the standard equation of each circle with the given center that passes through the given point.

11. center $(6, 4)$; point $(9, 12)$
$(x - 6)^2 + (y - 4)^2 = 73$

12. center $(-2, 0)$; point $(5, 8)$
$(x + 2)^2 + y^2 = 113$

13. center $(-4, -1)$; point $(-6, 5)$
$(x + 4)^2 + (y + 1)^2 = 40$

14. center $(0, 6)$; point $(5, -2)$
$x^2 + (y - 6)^2 = 89$

15. center $(3, 0)$; point $(-5, -2)$
$(x - 3)^2 + y^2 = 68$

16. center $(0, 0)$; point $(\sqrt{5}, \sqrt{8})$
$x^2 + y^2 = 13$

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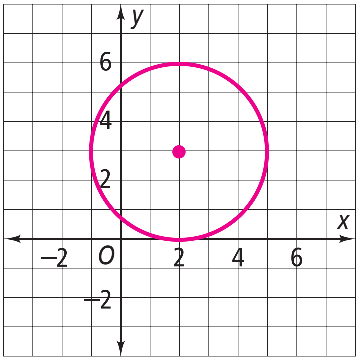
12-5 Practice (continued)

Form K

Circles in the Coordinate Plane

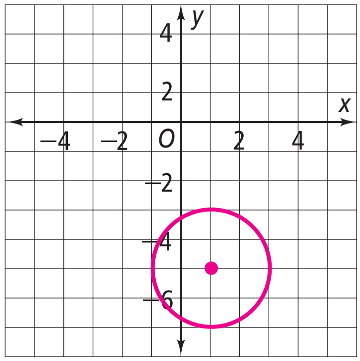
Find the center and radius of each circle. Then graph the circle.

17. $(x - 2)^2 + (y - 3)^2 = 9$



center (2, 3); r = 3

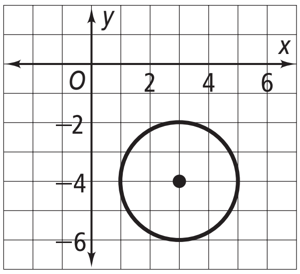
18. $(x - 1)^2 + (y + 5)^2 = 4$



center (1, −5); r = 2

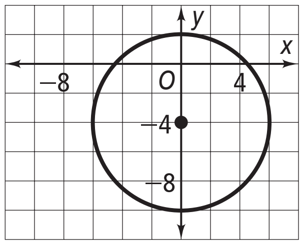
Write the standard equation of each circle.

19.



$(x - 3)^2 + (y + 4)^2 = 4$

20.



$x^2 + (y + 4)^2 = 36$

Write an equation of a circle with diameter $\overline{ST}$.

21. $S(0, 0)$, $T(6, 4)$
$(x - 3)^2 + (y - 2)^2 = 13$

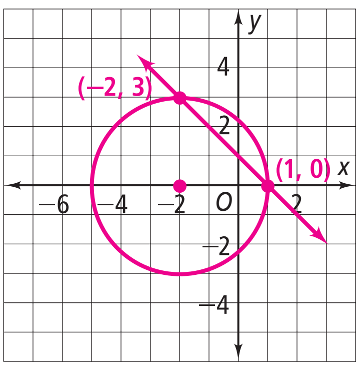
22. $S(0, 2)$, $T(6, 10)$
$(x - 3)^2 + (y - 6)^2 = 25$

23. $S(5, 11)$, $T(9, 3)$
$(x - 7)^2 + (y - 7)^2 = 20$

Sketch the graphs of each equation. Find all points of intersection of each pair of graphs.

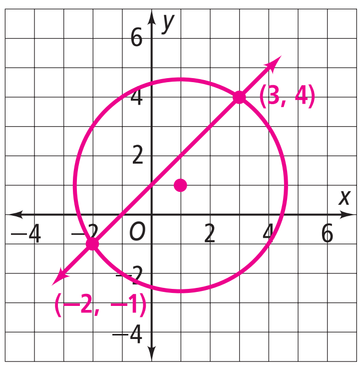
24. $(x + 2)^2 + y^2 = 9$

$y = -x + 1$ (1, 0), (−2, 3)



25. $(x - 1)^2 + (y - 1)^2 = 13$

$y = x + 1$ (3, 4), (−2, −1)



Name ______________________ Class ____________ Date ____________

12-5 Standardized Test Prep

Circles in the Coordinate Plane

Multiple Choice

For Exercises 1–4, choose the correct letter.

1. Which is the equation of a circle with center $(-2, 3)$ and radius $r = 5$? **B**

(A) $(x + 2)^2 + (y - 3)^2 = 10$ (C) $(x - 2)^2 + (y + 3)^2 = 10$

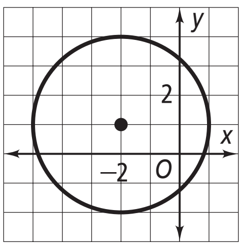
(B) $(x + 2)^2 + (y - 3)^2 = 25$ (D) $(x - 2)^2 + (y + 3)^2 = 25$

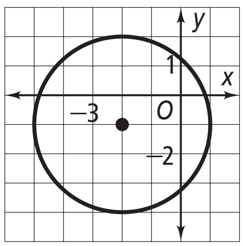
2. A circle with center $(-1, 2)$ passes through point $(2, -2)$. Which is true? **G**

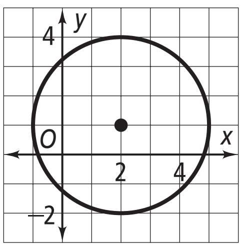
(F) The radius is $\sqrt{5}$. (H) The equation is $(x + 1)^2 + (y - 2)^2 = 10$.

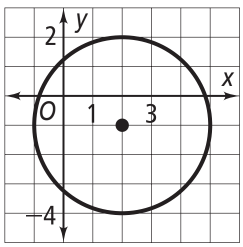
(G) The diameter is 10. (I) The circumference is 25π.

3. Which of the following is the graph of $(x - 2)^2 + (y + 1)^2 = 9$? **D**

(A)


(C)


(B)


(D)


4. Which is the equation of a circle with diameter $\overline{AB}$ with $A(5, 4)$ and $B(-1, -4)$? **H**

(F) $(x - 5)^2 + (y - 4)^2 = 10$ (H) $(x - 2)^2 + y^2 = 25$

(G) $(x + 5)^2 + (y + 4)^2 = 100$ (I) $(x + 2)^2 + y^2 = 5$

Short Response

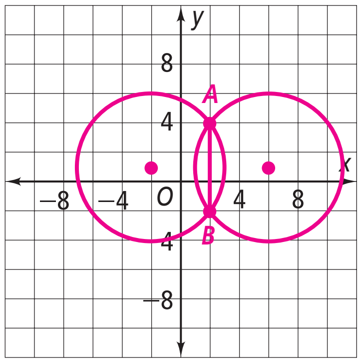
5. Write the standard equation of a circle with a circumference of 14π and center $(4, -1)$. (*Hint:* Use the formula for circumference.)

$(x - 4)^2 + (y + 1)^2 = 49$ [2] correct equation in standard form [1] Student makes one error or equation not in standard form. [0] Equation is incorrect or no equation is given.

12-5 Enrichment

Circles in the Coordinate Plane

Throughout this course you have learned many things about geometric figures on coordinate planes. Use that knowledge with what you have learned about circles to complete the following exercises. Assume all circles are in the coordinate plane.



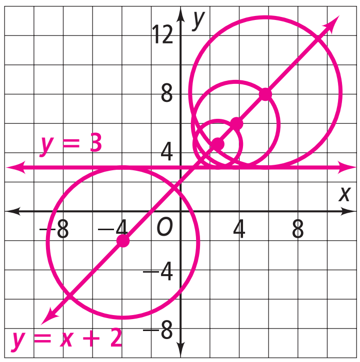
1. $\odot P$ has a chord $\overline{AB}$ with $A(2, 4)$ and $B(2, -2)$ and an area of 25π m^2. Is $\overline{AB}$ a diameter of $\odot P$? Explain.
No; $AB = 6$ and the radius is 5 m^2.

2. How many circles have a chord $\overline{AB}$ with $A(2, 4)$ and $B(2, -2)$ and an area of 25π cm^2? Draw a figure to support your conclusion. **two**

3. Find an example of $\odot R$ with chord $\overline{AB}$ with $A(2, 4)$ and $B(2, -2)$ and an area of 25π ft^2. Write the standard equation of $\odot R$. Show your work.
Answers may vary. Samples: $(x + 2)^2 + (y - 1)^2 = 25$ or $(x - 6)^2 + (y - 1)^2 = 25$

4. How many circles through $(5, -4)$ have a circumference of 10π yd? **infinitely many**

5. Find an example of $\odot S$ through $(5, -4)$ with a circumference of 10π yd. Write the standard equation of $\odot S$. Show your work. Hint: Find r and then find a point that could be the center of $\odot S$ at a distance r from $(5, -4)$.
Answers may vary. Sample: $(x - 1)^2 + (y + 1)^2 = 25$



6. How many circles tangent to the line $y = 3$ have a diameter on the line $y = x + 2$? Draw a figure to support your conclusion. **infinitely many**

7. How many circles tangent to the line $y = 3$ have a diameter on the line $y = x + 2$ and an area of 25π in.^{2}? Add circles to your figure to support your conclusion. **two**

8. Write a standard equation for each circle tangent to the line $y = 3$ with a diameter on the line $y = x + 2$ and an area of 25π in.^{2}. Show your work.
$(x - 6)^2 + (y - 8)^2 = 25$; $(x + 4)^2 + (y + 2)^2 = 25$

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12-5 Reteaching

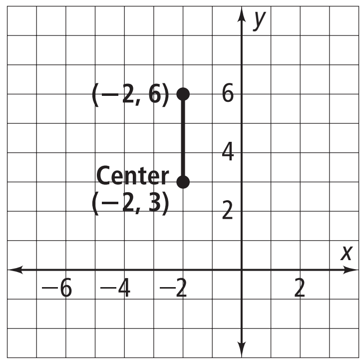
Circles in the Coordinate Plane

Writing the Equation of a Circle from a Description

The standard equation for a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$. The *opposite* of the coordinates of the *center* appear in the equation. The *radius* is *squared* in the equation.

Problem

What is the standard equation of a circle with center $(-2, 3)$ that passes through the point $(-2, 6)$?



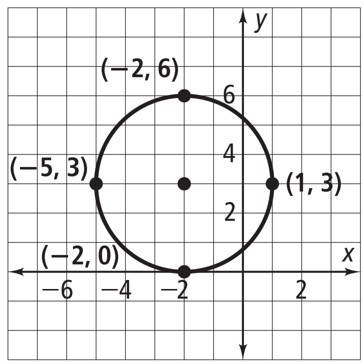
Step 1 Graph the points.

Step 2 Find the radius using both given points. The radius is the *distance* from the center to a point on the circle, so $r = 3$.

Step 3 Use the radius and the coordinates of the center to write the equation.

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - (-2))^2 + (y - 3)^2 = 3^2$$
$$(x + 2)^2 + (y - 3)^2 = 9$$

Step 4 To check the equation, graph the circle. Check several points on the circle.



For $(1, 3)$: $(1 + 2)^2 + (3 - 3)^2 = 3^2 + 0^2 = 9$

For $(-5, 3)$: $(-5 + 2)^2 + (3 - 3)^2 = (-3)^2 + 0^2 = 9$

For $(-2, 0)$: $(-2 + 2)^2 + (0 - 3)^2 = 0^2 + (-3)^2 = 9$

The standard equation of this circle is $(x + 2)^2 + (y - 3)^2 = 9$.

Exercises

Write the standard equation of the circle with the given center that passes through the given point. Check the point using your equation.

1. center $(2, -4)$; point $(6, -4)$
   $(x - 2)^2 + (y + 4)^2 = 16$;
   $(6 - 2)^2 + (-4 + 4)^2 = 16$

2. center $(0, 2)$; point $(3, -2)$
   $x^2 + (y - 2)^2 = 25$;
   $(3 - 0)^2 + (-2 - 2)^2 = 25$

3. center $(-1, 3)$; point $(7, -3)$
   $(x + 1)^2 + (y - 3)^2 = 100$;
   $(7 + 1)^2 + (-3 - 3)^2 = 100$

4. center $(1, 0)$; point $(0, 5)$
   $(x - 1)^2 + y^2 = 26$;
   $(0 - 1)^2 + 5^2 = 26$

5. center $(-4, 1)$; point $(2, -2)$
   $(x + 4)^2 + (y - 1)^2 = 45$;
   $(2 + 4)^2 + (-2 - 1)^2 = 45$

6. center $(8, -2)$; point $(1, 4)$
   $(x - 8)^2 + (y + 2)^2 = 85$;
   $(1 - 8)^2 + (4 + 2)^2 = 85$

12-5 Reteaching (continued)

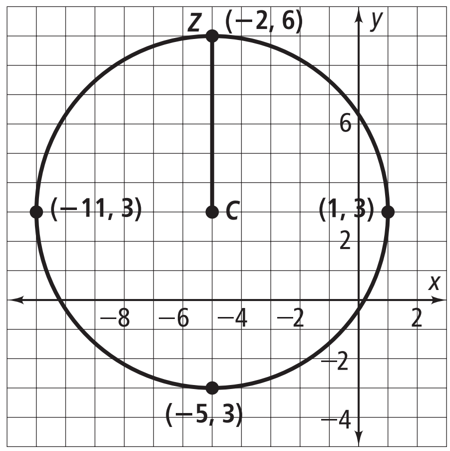
Circles in the Coordinate Plane

Writing the Equation of a Circle from a Graph

You can inspect a graph to find the coordinates of the circle's center. Use the center and a point on the circle to find the radius. It is easier if you use a horizontal or vertical radius.

Problem

What is the standard equation of the circle in the diagram at the right?



Step 1 Write the coordinates of the center.
The center is at $C(-5, 3)$.

Step 2 Find the radius. Choose a vertical radius: $\overline{CZ}$. The length is 6, so the radius is 6.

Step 3 Write the equation using the radius and the coordinates of the center.

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - (-5))^2 + (y - 3)^2 = 6^2$$

$$(x + 5)^2 + (y - 3)^2 = 36$$

Step 4 Check two points on the circle.

For $(1, 3)$: $(1 + 5)^2 + (3 - 3)^2 = 6^2 + 0^2 = 36$

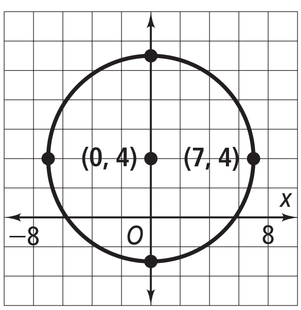
For $(-11, 3)$: $(-11 + 5)^2 + (3 - 3)^2 = 6^2 + 0^2 = 36$

The standard equation of this circle is $(x + 5)^2 + (y - 3)^2 = 36$.

Exercises

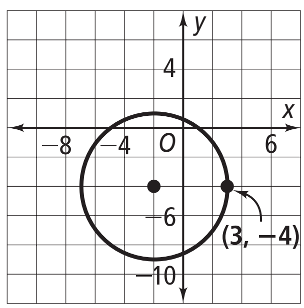
Write the standard equation of each circle. Check two points using your equation.

7.



$x^2 + (y - 4)^2 = 49$
Sample: $0^2 + (-3 - 4)^2 = 49$
$7^2 + (4 - 4)^2 = 49$

8.



$(x + 2)^2 + (y + 4)^2 = 25$
Sample: $(3 + 2)^2 + (-4 + 4)^2 = 25$
$(-7 + 2)^2 + (-4 + 4)^2 = 25$

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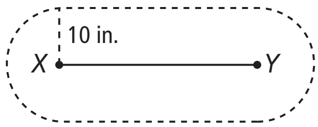
12-6 Additional Vocabulary Support

Locus: A Set of Points

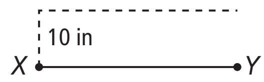
Your friend wants to find the locus of points that is 10 in. from a line segment.

He wrote these steps to solve the problem on the note cards, but they got mixed up.

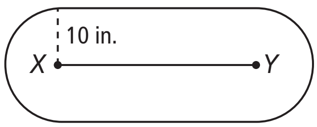
Sketch points 10 in. from Y.



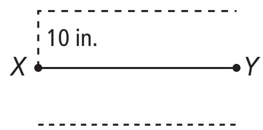
Sketch points 10 in. above $\overline{XY}$.



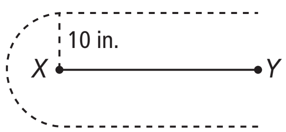
Draw the figure the pattern suggests.



Sketch points 10 in. below $\overline{XY}$.



Sketch points 10 in. from X.



Draw line segment $\overline{XY}$.

Use the note cards to write the steps in order.

1. First, draw line segment $\overline{XY}$.
2. Second, sketch points 10 in. above $\overline{XY}$.
3. Third, sketch points 10 in. below $\overline{XY}$.
4. Next, sketch points 10 in. from X.
5. Then, sketch points 10 in. from Y.
6. Finally, draw the figure the pattern suggests.

Name ______________________ Class ____________ Date ____________

12-6 Think About A Plan

Locus: A Set of Points

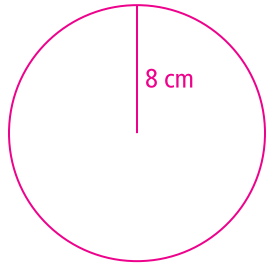
Describe the locus of points in a plane 3 cm from the points on a circle with radius 8 cm.

Know

1. What is a locus of points in a plane? a set of points that meets a certain condition; in this case all of the points in a plane that are 3 cm from the points on a circle with radius 8 cm

Need

2. Make a sketch of a circle with radius 8 cm.

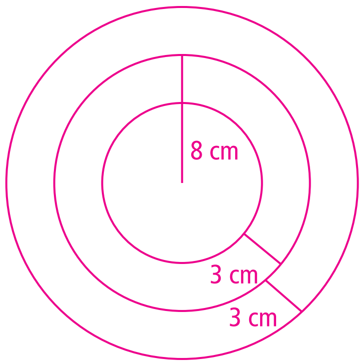


Plan

3. How can you create a sketch of the points *in a plane* that are 3 cm from the points on a circle with radius 8 cm? *Remember these points can be outside or inside the circle.*

Sketch a circle with radius 5 cm inside the circle, with the same center point; sketch a circle with radius 11 cm outside the circle, with the same center point.

4. Make a sketch like the one you described in Step 3.



5. Describe the points you have drawn.

two concentric circles with radii 5 cm and 11 cm

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12-6 Practice

Form G

Locus: A Set of Points

Sketch and describe each locus of points in a plane. **Check students' drawings.**

1. points 1.5 cm from point T
a circle with a radius of 1.5 cm

2. points 1 in. from $\overline{PQ}$ **two line segments that are 1 in. from $\overline{PQ}$ connected by two half circles, each with a radius of 1 in. centered at an endpoint**

3. points that are equidistant from two concentric circles whose radii are 8 in. and 12 in.
a concentric circle with a radius of 10 in.

4. points equidistant from the endpoints of $\overline{AB}$
the perpendicular bisector of $\overline{AB}$

5. points that belong to a given angle or its interior and are equidistant from the sides of the given angle
a ray that bisects the given angle

Describe each locus of points in space.

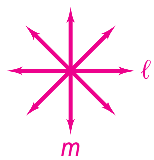
6. the set of points in space a given distance from a point
a sphere whose center is the given point and whose radius is the given distance

7. all points in space 2 cm from a segment **a cylinder with a radius of 2 cm and height the length of the segment, with a half-sphere with radius 2 cm around each end point**

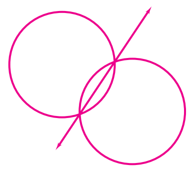
8. all points 5 ft from a given plane P
a pair of parallel planes, one on each side of plane P, and each 5 ft from P

Sketch the locus of points in a plane that satisfy the given conditions.

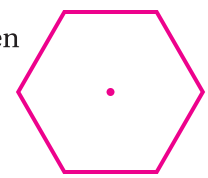
9. points equidistant from two perpendicular lines ℓ and m



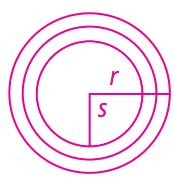
10. all points equidistant from the centers of two given intersecting circles of the same radius



11. all points equidistant from the vertices of a given regular hexagon



12. all points that are equidistant from two concentric circles with larger radius r and smaller radius s



12-6 Practice (continued) *Form G*

Locus: A Set of Points

For Exercises 13–18, describe each locus.

13. What locus contains all the houses that are 1 mi from the library?
a circle whose center is the library and whose radius is 1 mi

14. What locus contains all the bushes that can be placed 20 ft outside of a circular path with a radius of 80 ft?
a circle with a radius of 100 ft that is concentric with the path

15. Identify the locus of points equidistant from two opposite vertices of a cube.
a plane perpendicular to the line containing the opposite vertices of the cube

16. Identify the locus of points equidistant from two concentric circles of radii a and b in a plane.
a circle with the radius $\frac{(a + b)}{2}$

17. Identify the locus of points a units from a circle of radius r in a plane.
two concentric circles, one with radius $r - a$ and one with radius $r + a$

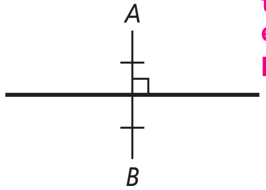
18. Identify the locus of points a units from a given line ℓ in a plane.
two parallel lines a units from line ℓ

For Exercises 19 and 20, sketch and describe each locus in a plane.
Check students' drawings.

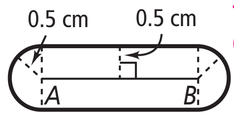
19. all points within n units of point Z
all points within a circle with a radius of n units

20. Given $\angle LMN$ with bisector $\overline{MO}$ in plane P, describe the locus of the centers of circles that are tangent to $\overline{MO}$ and the outside ray of the angle.
the rays that bisect $\angle LMO$ and $\angle OMN$

Describe the locus that each thick line represents.

21.


the locus of points equidistant from points A and B

22.


the locus of points 0.5 cm from $\overline{AB}$

23. Describe the locus of points in a plane 4 cm from the points on a circle with radius 7 cm.
two circles, both concentric with the given circle, one with radius 3 cm and one with radius 11 cm

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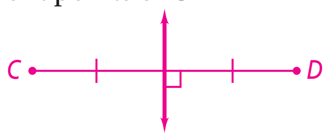
12-6 Practice

Form K

Locus: A Set of Points

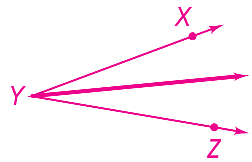
Sketch and describe each locus of points in a plane.

1. points equidistant from the endpoints of $\overline{CD}$



the perpendicular bisector of $\overline{CD}$

2. points in the interior of $\angle XYZ$ and equidistant from the sides of $\angle XYZ$

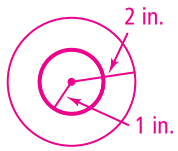


the angle bisector of $\angle XYZ$

3. points between and equidistant from two parallel lines j and k

a ‖ line centered between lines *j* and *k*

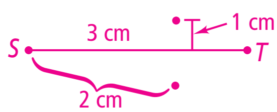
4. midpoints of the radii of a circle that has a radius of 2 in.



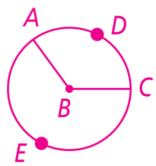
a concentric circle with a radius of 1 in.

For Exercises 5–8, sketch the locus of points in a plane that satisfy the given conditions.

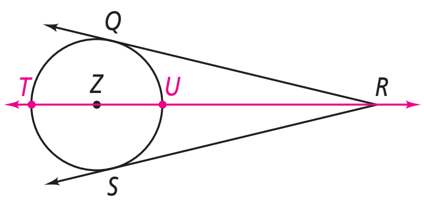
5. 1 cm from $\overline{ST}$ and 2 cm from S, where $ST = 3$ cm



6. equidistant from the sides of $\angle ABC$ and on a circle with center B and radius BC

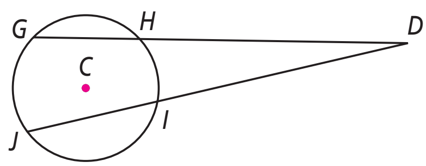


7. equidistant from the sides of $\angle QRS$ and $\odot Z$



***T* and *U*, where $\overleftrightarrow{TU}$ bisects $\angle QRS$**

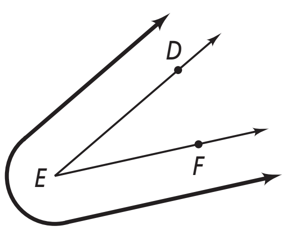
8. equidistant from points G, H, I, and J on $\odot C$



C

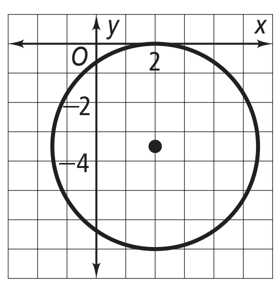
Describe the locus each thick line represents.

9.



points on the exterior of $\angle DEF$ a fixed distance from the sides of $\angle DEF$

10.



points 3.5 units from (2, −3.5)

Name ______________________ Class ____________ Date ____________

12-6 Practice (continued) *Form K*

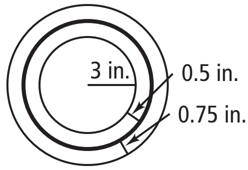
Locus: A Set of Points

Reasoning **For Exercises 11 and 12, describe each locus of points in space.**

11. points 4 cm from a point M **a sphere with radius 4 cm centered on *M***

12. points 5 in. from a segment $\overline{AB}$ **a cylinder with radius 5 in. centered on $\overline{AB}$ with hemispheres of radius 5 in. at *A* and *B***

13. A student made and decorated a plate in ceramics class. Describe the locus represented by the dark ring on the plate. **Answers may vary. Sample: all the points 0.5 in. outside a circle with radius 3 in.**



Coordinate Geometry **Write an equation for the locus of points in a plane equidistant from the two given points.**

14. $E(-3, 2)$, $F(5, -6)$

$y = x - 3$

15. $G(4, 0)$, $H(-4, 2)$

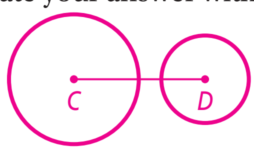
$y = 4x + 1$

16. $R(6, 1)$, $S(-3, -5)$

$y = -\frac{3}{2}x + \frac{1}{4}$

17. Reasoning Points C and D are 6 cm apart. Do the following loci in a plane have any points in common? Explain. Illustrate your answer with a sketch.

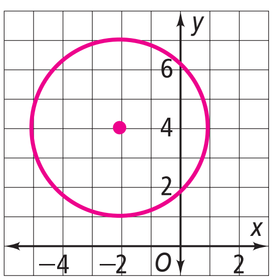
- the points 3 cm from C
- the points 2 cm from D



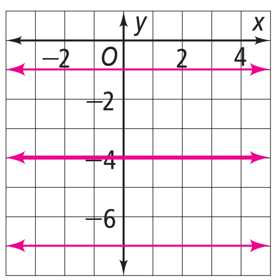
No; the circles would intersect only if the sum of the distances from each point were equal to or greater than the 6-cm distance between the points.

Coordinate Geometry **Draw each locus on the coordinate plane.**

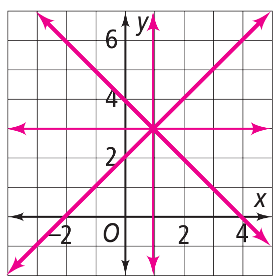
18. all points 3 units from $(-2, 4)$



19. all points equidistant from $y = -1$ and $y = -7$



20. all points equidistant from $x = 1$ and $y = 3$



Name ______________________ Class ____________ Date ____________

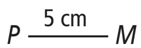
12-6 Standardized Test Prep

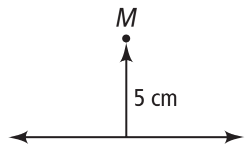
Locus: A Set of Points

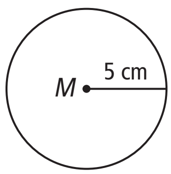
Multiple Choice

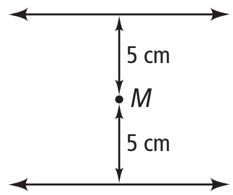
For Exercises 1 and 2 choose the correct letter.

1. Which sketch represents the locus of points in a plane 5 cm from a point *M*? **B**

(A) 

(C) 

(B) 

(D) 

2. Which description best represents the locus of points in a plane equidistant from parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$? **I**

(F) a circle with radius *a*

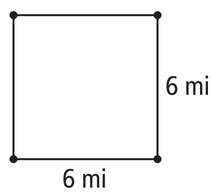
(G) a plane equidistant from $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$

(H) a sphere with chords $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$

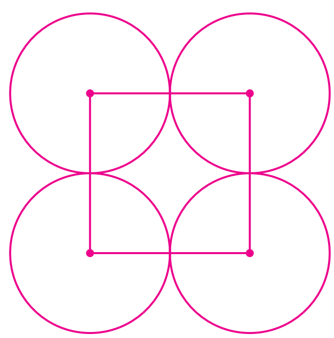
(I) a line equidistant from and between $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$

Extended Response

3. The town's emergency response planning committee wants to place four emergency response centers at the four corners of town. Each would serve the people who live within 3 mi of the response center. Sketch the loci of points for the areas served. What are the problems with this idea? What is one potential solution?



[4] Answers may vary. Sample: The plan leaves people in the center of town without service. Also, three-fourths of the service area is outside of the town. One solution is to move the centers closer together so that everyone is served. There will be overlapping areas, but the people in these areas could simply choose from one of the two centers. [3] Student draws a sketch with minor errors, and answers both questions. [2] Student draws correct sketch, but does not answer one of the two questions. [1] Student only attempts one part of the three-part problem. [0] incorrect or no response



12-6 Enrichment

Locus: A Set of Points

When doing locus problems that involve several steps, it is often helpful to use colored pencils or markers. If possible, use colored markers to do the following exercises.

Draw line ℓ, and label point *P* on ℓ. Using a different marker, draw the locus of points 4 in. from *P*. **Check students' work.**

1. Describe what you have drawn.
$\odot P$ with a radius of 4 in.

With another marker, draw the locus of points 2 in. from line ℓ.

2. Describe what you have drawn.
two lines parallel to line ℓ, one 2 in. above ℓ and the other 2 in. below ℓ

3. Describe the intersection of the loci you have drawn for Exercises 1 and 2.
four points

Using yet another marker, draw the locus of points 4 in. from line ℓ.

4. Describe what you have drawn.
two lines parallel to line ℓ, one 4 in. above ℓ and the other 4 in. below ℓ

5. Describe the intersection of the loci you have drawn for Exercises 1 and 4.
two points

With still another marker, draw the locus of points 6 in. from line ℓ.

6. Describe what you have drawn.
two lines parallel to line ℓ, one 6 in. above ℓ and the other 6 in. below ℓ

7. Describe the intersection of the loci you have drawn for Exercises 1 and 6.
empty set

8. Describe the intersection of the loci you have drawn for Exercises 1, 2, 4, and 6.
empty set

9. Describe the intersection of the loci you have drawn for Exercises 2, 4, and 6.
empty set

Describe the locus of points in space.

10. a. the locus of points 4 in. from *P* **a sphere with radius 4 in. centered at *P***

b. the locus of points 2 in. from ℓ **a cylinder with radius 2 in. centered on ℓ**

c. the locus of points 4 in. from ℓ **a cylinder with radius 4 in. centered on ℓ**

d. The locus of points 6 in. from ℓ **a cylinder with radius 6 in. centered on ℓ**

11. Describe the intersection of the loci in: 10a and 10b, 10a and 10c, 10a and 10d.
two parallel circles with radius 2 in.; one circle with radius 4 in.; empty set

Name ______________________ Class ____________ Date ____________

12-6 Reteaching

Locus: A Set of Points

A *locus* is a set of points that all meet a condition or conditions. Finding a locus is a strategy that can be used to solve a word problem.

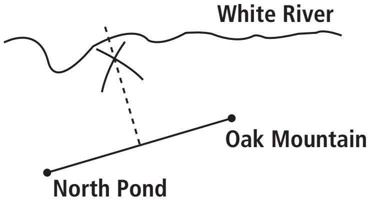
Problem

A family on vacation wants to hike on Oak Mountain and fish at North Pond and along the White River. Where on the river should they fish to be equidistant from North Pond and Oak Mountain?

Draw a line segment joining North Pond and Oak Mountain.

Construct the perpendicular bisector of that segment.

The family should fish where the perpendicular bisector meets the White River.



Exercises

Describe each of the following, and then compare your answers with those of a partner. **Check students' work.**

1. the locus of points equidistant from your desk and your partner's desk

2. the locus of points on the floor equidistant from the two side walls of your classroom

3. the locus of points equidistant from a window and the door of your classroom

4. the locus of points equidistant from the front and back walls of your classroom

5. the locus of points equidistant from the floor and the ceiling of your classroom

Use points A and B to complete the following.

6. Describe the locus of points in a plane equidistant from A and B.
the line perpendicular to $\overline{AB}$ at its midpoint

7. How many points are equidistant from A and B and also lie on $\overleftrightarrow{AB}$? Explain your reasoning.
one, the midpoint of $\overline{AB}$

8. Describe the locus of points in space equidistant from A and B.
the plane perpendicular to $\overline{AB}$ at its midpoint

9. Draw $\overline{AB}$. Describe the locus of points in space 3 mm from $\overline{AB}$.
Check students' drawings; a cylinder with radius 3 mm and central axis $\overline{AB}$ and two hemispheres centered at A and B of radius 3 mm at the bases of the cylinder.

12-6 Reteaching (continued)

Locus: A Set of Points

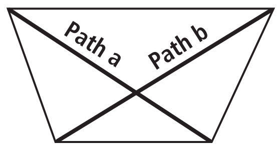
10. Two students meet every Saturday afternoon to go running. Describe how they could use the map to find a variety of locations to meet that are equidistant from their homes.

They could draw a line segment with their homes as the endpoints, and then find its perpendicular bisector through the town. Everywhere the line passes through a street is a possible meeting place.

Use what you know about geometric figures to answer the following questions.

11. Sam tells Tony to meet him in the northeast section of town, 1 mi from the town's center. Tony looks at his map of the town and picks up his cell phone to call Sam for more information. Why?
The locus of points Sam describes is roughly a quarter of a circle, with a radius of 1 mi.

12. How can city planners place the water sprinklers at the park so they are always an equal distance from the two main paths of the park?
The sprinklers can be placed along two lines that bisect the angles formed by the two paths.



Green Park

13. An old pirate scratches the following note into a piece of wood: "The treasure is 50 ft from a cedar tree and 75 ft from an oak." Under what conditions would this give you one point to dig? two? none?
when the trees are 125 ft apart; when the trees are less than 125 ft apart; when the trees are more than 125 ft apart

14. A ski resort has cut a wide path through mountain trees. Skiers will be coming down the hill, but the resort also needs to install the chairlift in the same space. What design allows skiers to ski down the hill with the maximum amount of space between them and the trees and the huge poles that support the chairlift?
The resort should install the chairlift to the far left or right of the open area so that skiers can ski down the center line, equidistant from the line of poles and the line of trees.

15. A telecommunications company is building a new cell phone tower and wants to cover three different villages. What location allows all three villages to get equal reception from the new tower
The tower could be placed at the circumcenter of a triangle formed by the three villages.

Name ______________________ Class ____________ Date ____________

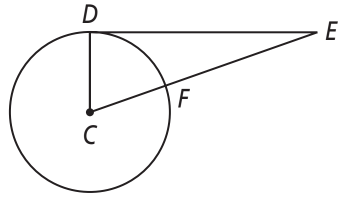
Chapter 12 Quiz 1

Form G

Lessons 12-1 through 12-3

Do you know HOW?

Refer to $\odot C$ for Exercises 1–3. Segment $\overline{DE}$ is tangent to $\odot C$.

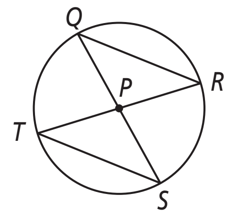


1. If $DE = 4$ and $CE = 8$, what is the radius? **$4\sqrt{3}$**

2. If $DE = 8$ and $EF = 4$, what is the radius? **6**

3. If $m\angle C = 42°$, what is $m\angle E$? **48**

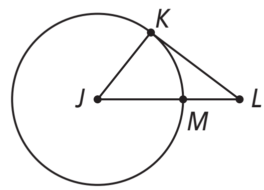
Refer to $\odot P$ for Exercises 4–8, given that $m\overset{\frown}{QR} = 100$.



4. What is $m\overset{\frown}{ST}$? How do you know? **100; $\angle QPR \cong \angle SPT$ because they are vertical angles. So, $\overset{\frown}{ST} \cong \overset{\frown}{QR}$.**
5. What is $m\angle QPT$? How do you know? **80; $\angle QPR$ and $\angle QPT$ are supplementary.**
6. What do you know about the distances from P to $\overline{QR}$ and from P to $\overline{ST}$? How do you know? **The distances are = because $\cong$ arcs have $\cong$ chords, so $\overline{QR} \cong \overline{ST}$. $\cong$ chords are equidistant from the center of the circle.**
7. Which arc does $\angle R$ intercept? **$\overset{\frown}{QT}$**

8. Which two angles intercept $\overset{\frown}{RS}$? **$\angle Q$ and $\angle T$**

Refer to $\odot J$ for Exercises 9–11. Segment $\overline{KL}$ is tangent to $\odot J$.



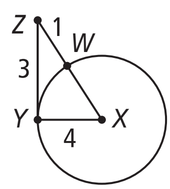
9. If the radius is 3 and $LM = 2$, what is KL? **4**

10. If $\overline{KL} \cong \overline{JK}$, what is $m\angle J$? **45**

11. If the radius is 5 and $\overline{JL}$ is 1 unit longer than $\overline{KL}$, what is KL? (*Hint:* Use x for KL and $x + 1$ for JL.) **12**

Do you UNDERSTAND?

12. **Error Analysis** A classmate insists that $\overline{YZ}$ is not a tangent to $\odot X$. Explain how to show that your classmate is wrong.
Answers may vary. Sample: $3^2 + 4^2 = (1 + 4)^2$



YZ cannot be tangent because $3^2 + 4^2 \neq (1 + 3)^2$

13. **Vocabulary** What is the relationship between the measure of an inscribed angle and the measure of its intercepted arc?
The measure of the inscribed angle equals one-half the measure of its intercepted arc.

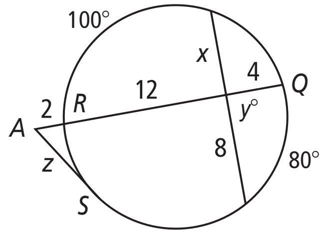
Name ______________________ Class ____________ Date ____________

Chapter 12 Quiz 2

Form G

Lessons 12-4 through 12-6

Do you know HOW?



1. What is the value of x? **6**

2. What is the value of y? **90**

3. What is the value of z? **6**

What is the standard equation of each circle?

4. center (2, 3); radius $= 5$ $\mathbf{(x - 2)^2 + (y - 3)^2 = 25}$

5. center $(0, -1)$; radius $= \sqrt{7}$ $\mathbf{x^2 + (y + 1)^2 = 7}$

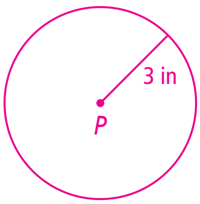
What is the center and radius of each circle?

6. $(x - 4)^2 + (y - 3)^3 = 16$ **(4, 3); 4**

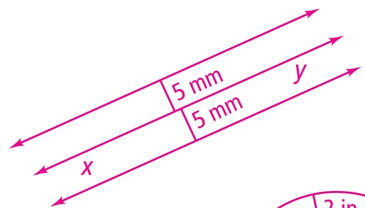
7. $(x + 7)^2 + y^2 = 10$ **(–7, 0);** $\mathbf{\sqrt{10}}$

Provide a sketch and description of each locus of points in a plane.

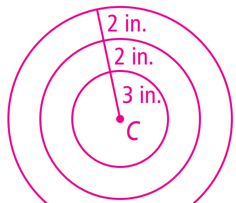
8. points 3 in. from a point P
circle of radius 3 in. centered at *P*



9. points 5 mm from a line $\overleftrightarrow{XY}$
two lines ∥ to $\overleftrightarrow{XY}$, each 5 mm from $\overleftrightarrow{XY}$



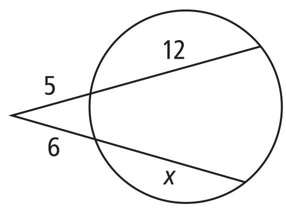
10. points 2 in. from ⊙C of radius 5 in.
two circles concentric with *C*, of radius 7 in. and 3 in.



Do you UNDERSTAND?

11. **Vocabulary** How are the words *circle* and *locus* related?
Answers may vary. Sample: A circle is an example of a locus.

12. **Error Analysis** A classmate insists that the value of x is 10. Write an equation to show that your classmate is wrong.
$\mathbf{5(12 + 5) = 6(x + 6);\ x = \frac{49}{6}}$



13. Suppose you know that $\overline{AB}$ is a diameter of a circle. How do you find the equation of the circle? Describe the process using several steps.
Answers may vary. Sample: Find the distance of the diameter. Divide by 2 to find radius *r*. Find the midpoint of the diameter, which is (*h*, *k*), the center of the circle. Plug *r*, *h*, and *k* into the equation: $(x - h)^2 + (y - k)^2 = r^2$.

Name ______________________ Class ______________ Date ______________

Chapter 12 Test

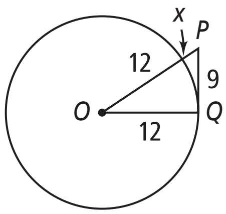
Form G

Circles

Do you know HOW?

For Exercises 1–8, lines that appear tangent are tangent. Find the value of each variable.

1.



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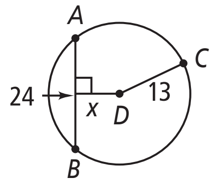
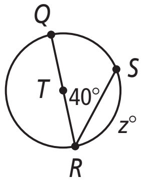
2.



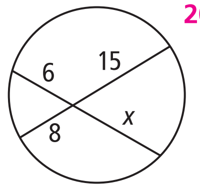
5

3.



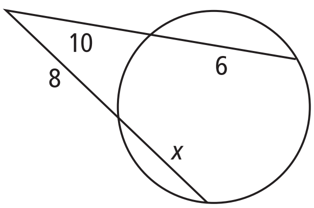
100

4.



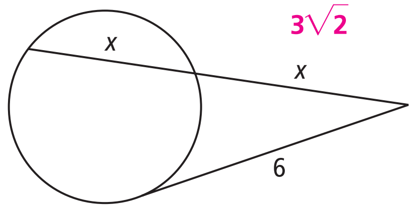
20

5.



12

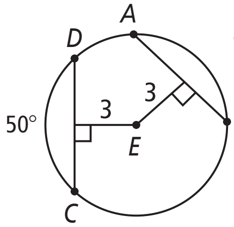
6.



$3\sqrt{2}$

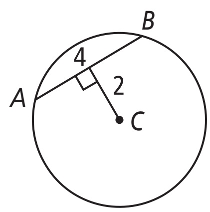
Find $m\widehat{AB}$.

7.



50

8.



90

Graph each circle. Label the center and radius. **Check students' work.**

9. $(x - 5)^2 + (y + 3)^2 = 16$ **(5, –3); 4**

10. $(x - 1)^2 + y^2 = 121$ **(1, 0); 11**

11. Write an equation of the circle with center (0, 2) that passes through (5, –2).
$x^2 + (y - 2)^2 = 41$

12. Describe the graph of $x^2 - 2 = 2 - y^2$. **a circle with center (0, 0) and radius 2**

13. Write an equation for the locus: points in the coordinate plane that are 7 units from the point (3, –1). **$(x - 3)^2 + (y + 1)^2 = 49$**

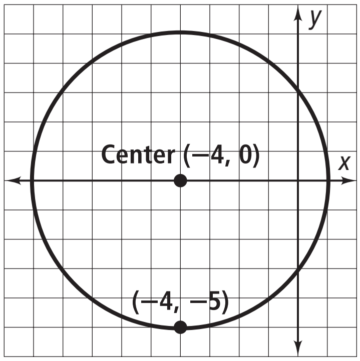
Name ______________________ Class ____________ Date ____________

Chapter 12 Test (continued) *Form G*

Circles

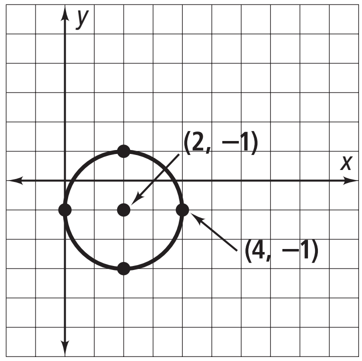
Write the standard equation of each circle.

14.



$(x + 4)^2 + y^2 = 25$

15.



$(x - 2)^2 + (y + 1)^2 = 4$

Sketch each locus on a coordinate plane. **Check students' work.**

16. all points $\sqrt{2}$ units from the line $y = x$ **two lines: $y = x + 1$; $y = x - 1$**

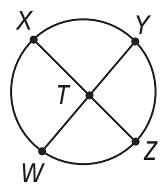
17. all points 3 units from the circle with center (5, 5) and radius 4
two concentric circles with center (5, 5), one with radius 1 and other with radius 7

18 all points equidistant from the points (1, 1) and (−1, −1) **the line $y = -x$**

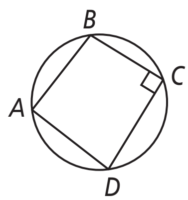
Do you UNDERSTAND?

19. Writing What is special about the number of right angles a quadrilateral inscribed in a circle can have? Explain. **Answers may vary. Sample: A quadrilateral inscribed in a circle can have 0, 2, or 4 right angles. It must have an even number of right angles because its opposite angles are supplementary.**

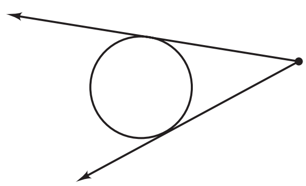
20. Error Analysis A student says that $\angle WTX \cong \angle YTZ$. She concludes that $\widehat{WX} \cong \widehat{YZ}$. What condition would make her conclusion correct?
If *T* is the center of the circle, her conclusion is correct.



21. Reasoning If diagonal $\overline{AC}$ is a diameter, what kind of figure is *ABCD?*
***ABCD* is a rectangle.**



22. Reasoning Two tangents to the same circle intersect utside the circle. What is the locus of points inside the circle equidistant from the two tangents? What is the locus if the two tangents do not intersect?
Answers may vary. Sample: The locus is the diameter of the circle that is collinear with the angle bisector for the angle formed by the two tangents. If the tangents do not intersect, the locus is a diameter equidistant from and parallel to both tangents.



Name ______________________ Class __________ Date __________

Chapter 12 Quiz 1

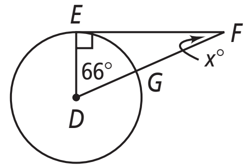
Form K

Lessons 12-1 through 12-3

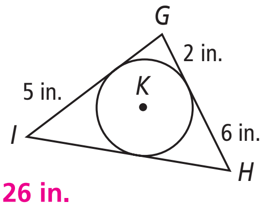
Do you know HOW?

1. $\overline{EF}$ is tangent to $\odot D$. What is the value of x? **24**

2. If $EF = 14$ and $GF = 8$, what is the radius? **8.25**

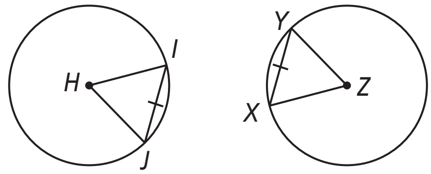


3. $\triangle GHI$ circumscribes $\odot K$. What is the perimeter of $\triangle GHI$?



26 in.

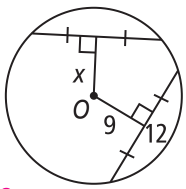
4. The circles below are congruent. What can you conclude?



$\overset{\frown}{IJ} \cong \overset{\frown}{XY}$; $\angle IHJ \cong \angle XZY$; $\triangle IHJ \cong \triangle XZY$

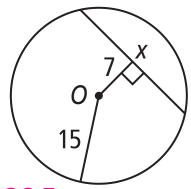
Find the value of x in $\odot O$. Round to the nearest tenth if necessary.

5.



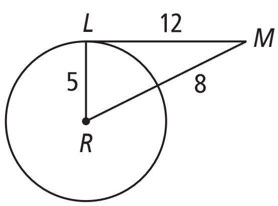
9

6.



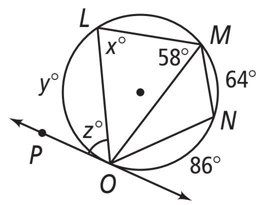
26.5

7. Is $\overline{LM}$ tangent to $\odot R$? Explain.



yes; $5^2 + 12^2 = 13^2$

8. Find the values of x, y, and z.



75; 116; 58

Do you UNDERSTAND?

9. Reasoning Is it possible to draw a triangle with the diameter of the circle as a base and two tangents of the circle as the legs? Explain.
No; the tangents form two 90° ∠s, but a △ can have only one right ∠.

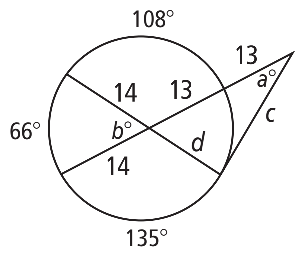
10. Reasoning Explain why opposite angles in a quadrilateral inscribed in a circle are supplementary. **If you put together the two arcs inscribed by the opposite ∠s, they form a complete circle (360°). The sum of the two angle measures is always half of 360, or 180.**

Chapter 12 Quiz 2

Form K

Lessons 12-4 and 12-5

Do you know HOW?



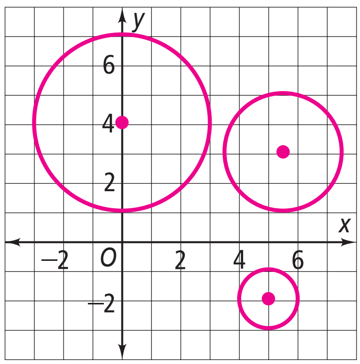
1. What is the value of *a*? **42**
2. What is the value of *b*? **58.5**
3. To the nearest tenth, what is the value of *c*? **22.8**
4. What is the value of *d*? **13**
5. Chords AC and DE intersect at Point R in $\odot Q$. If $AR = 6$, $RC = 7$, and $DR = 3$, what is RE? **14**

What is the standard equation of each circle?

6. center $(3, -8)$; $r = \sqrt{12}$
   $(x - 3)^2 + (y + 8)^2 = 12$
7. center $(-5, 4)$; passes through $(-1, 1)$
   $(x + 5)^2 + (y - 4)^2 = 25$

Find the center and radius of each circle. Then graph each circle.

8. $(x - 5)^2 + (y + 2)^2 = 1$
   center $(5, -2)$; $r = 1$
9. $x^2 + (y - 4)^2 = 9$
   center $(0, 4)$; $r = 3$
10. $(x - 5.5)^2 + (y - 3)^2 = 4$
    center $(5.5, 3)$; $r = 2$



Do you UNDERSTAND?

11. **Compare and Contrast** How is finding the measure of an angle formed by two rays that intersect outside a circle similar to finding the measure of an angle formed by two chords inside a circle? How is it different?
    In both cases you need to know the measures of two intercepted arcs. When the angle is outside the circle, you find half the difference of the arc measures. When the angle is formed by two chords, you find half the sum of the arc measures.
12. **Open-Ended** Write the equation of a circle with a center that is in the third quadrant and with a radius of 6. Find the coordinates of one point on the circle.
    Answers may vary. Check students' responses to make sure that values are added to *x* and *y*, not subtracted, and that the equation is set equal to 36. The easiest points for which to find coordinates are points on the circle 6 units above, below, right, or left of the center.

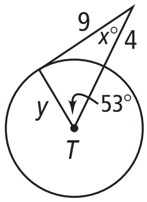
Name ______________________ Class ____________ Date ____________

Chapter 12 Test

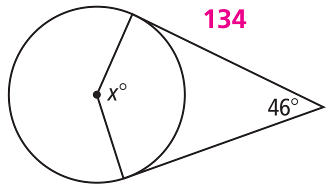
Form K

Do you know HOW?

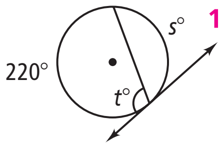
For Exercises 1–4, lines that appear tangent are tangent. Find the value of each variable.

1. 

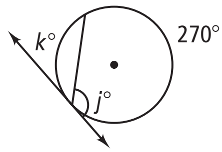
37; 8.125

2. 

134

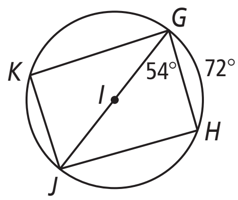
3. 

140; 110

4. 

135; 90

For Exercises 5–10, use the diagram at the right.



5. Which arc does $\angle KGJ$ intercept? $\overset{\frown}{KJ}$

6. Which angle intercepts $\overset{\frown}{GHJ}$? $\angle K$

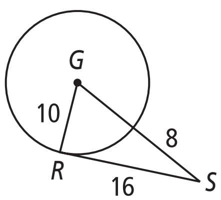
7. What is $m\angle H$? **90**

8. What is $m\angle GJH$? **36**

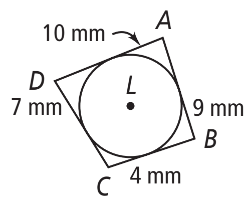
9. What is $m\overset{\frown}{JH}$? **108**

10. Which angles are supplementary?
$\angle K$ and $\angle H$; $\angle KJH$ and $\angle KGH$

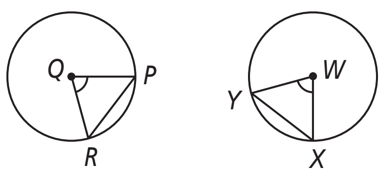
11. Is $\overline{RS}$ tangent to $\odot G$? Explain.
no; $10^2 + 16^2 \neq 18^2$



12. Polygon $ABCD$ circumscribes $\odot L$.
What is the perimeter of $ABCD$? **60 mm**



13. $\odot Q \cong \odot W$ and $\angle PQR \cong \angle XWY$.
What can you conclude?
$\overset{\frown}{PR} \cong \overset{\frown}{YX}$; $\overline{PR} \cong \overline{YX}$; $\triangle QPR \cong \triangle WXY$



What is the center and radius of each circle?

14. $(x - 5)^2 + y^2 = 36$

center (5, 0); $r = 6$

15. $(x + 1)^2 + (y + 6)^2 = 15$

center (−1, −6); $r = \sqrt{15}$

Write the standard equation of each circle.

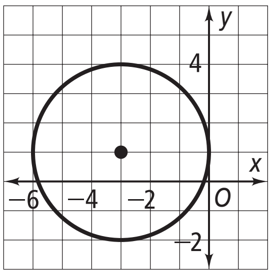
16. center $(0, -9)$; $r = 9$

$x^2 + (y + 9)^2 = 81$

17. center $(4, 7)$; passes through $(10, 15)$

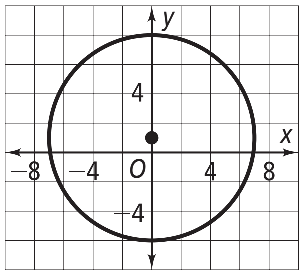
$(x - 4)^2 + (y - 7)^2 = 100$

18.



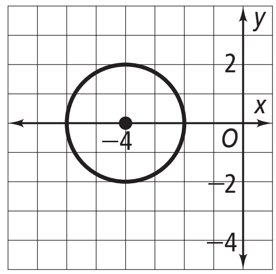
$(x + 3)^2 + (y - 1)^2 = 9$

19.



$x^2 + (y - 1)^2 = 49$

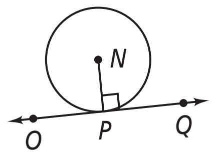
20.



$(x + 4)^2 + y^2 = 4$

Do you UNDERSTAND?

21. Vocabulary Explain the relationship between $\overline{NP}$ and $\overleftrightarrow{OQ}$ in terms of $\odot N$.

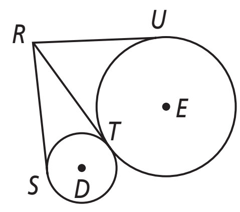


$\overleftrightarrow{OQ}$ must be a tangent to $\odot N$ at point P because $\overline{NP}$ is a radius and $\overleftrightarrow{OQ} \perp \overline{NP}$.

22. Suppose the diameter of $\odot Z$ has endpoints X and Y and you know the coordinates of these endpoints. What would you need to determine to write the standard form of the equation of $\odot Z$?

You need to find the midpoint of the diameter. This is the center of the circle. Then you need to find the distance between the midpoint and one of the endpoints. This is the length of the radius.

23. Error Analysis $\overline{RS}$ and $\overline{RT}$ are tangent to $\odot D$. $\overline{RT}$ and $\overline{RU}$ are tangent to $\odot E$. Your classmate said that $\overline{RU}$ must be longer than $\overline{RS}$ and $\overline{RT}$ because $\odot E$ is larger. Explain your classmate's error.



The segments must all be ≅. $\overline{RS} \cong \overline{RT}$ because both are tangent to $\odot D$ from the same point. $\overline{RT} \cong \overline{RU}$ because both are tangent to $\odot E$ from the same point. $\overline{RS} \cong \overline{RU}$ by the Trans. Prop. of ≅.

24. Reasoning $\odot A$ has a diameter of 10 cm. $\odot L$ has a diameter of 8 cm. In $\odot A$, $m\angle BAC = 45$. In $\odot L$, $m\angle KLM = 45$. Can you conclude that $\overline{BC} \cong \overline{KM}$? Explain.

No; although the central ∠s are ≅, $\odot A$ is larger than $\odot L$. The segments are ≅ only if the circles are ≅.

Name ________________________________ Class ______________ Date ______________

Performance Tasks

Chapter 12

Task 1

Consider the circle $\odot C$ with equation $(x - 3)^2 + (y + 12)^2 = 9$.

a. Explain how to determine the center and radius from the equation.
Answers may vary. Sample: The *x*-coordinate of the center of the circle is the opposite of the number that appears inside the parentheses with *x*. The *y*-coordinate of the center of the circle is the opposite of the number that appears inside the parentheses with *y*. The radius is the square root of the number that is equal to the expression.

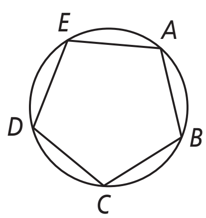
b. Find the center and radius of $\odot C$.
(3, –12); 3

c. Explain why the equation of a circle is similar to the distance formula.
The equation of the circle is similar to the distance formula because a circle is a locus of points that are all the same distance from a fixed point.

[4] Student gives accurate answers and correct explanations. [3] Student gives answers and explanations that may contain minor errors. [2] Student gives answers and explanations that contain some errors. [1] Student gives answers and explanations that contain significant errors. [0] Student makes little or no progress toward the correct answers or explanations.

Task 2

Regular pentagon $ABCDE$ is inscribed in a circle as shown.



a. Find $m\overset{\frown}{AB}$. Explain.
Because all arcs are congruent, $m\overset{\frown}{AB} = \frac{360}{5} = 72$.

b. Describe a method in distinct steps to find the center of the circle.
Answers may vary. Sample: Step 1) Construct the perpendicular bisector of $\overline{AB}$. Step 2) Construct the perpendicular bisector of $\overline{BC}$. The point of intersection of the bisectors is the center of the circle, *P*.

c. If the radius is 6, find AB. Explain.
$\triangle APX$ is a 54°-36°-90° triangle (where X is the midpoint of $\overline{AB}$ and $\overline{AP}$ is a radius).
Therefore, $\sin 36° = \frac{\frac{1}{2}AB}{6} = \frac{AB}{12}$. So, $AB = 12 \sin 36° \approx 7.05$.

[4] Student devises correct methods, gives correct answers and valid explanations. [3] Student devises methods, gives answers, and provides explanations with only minor errors. [2] Student devises methods, gives answers, and provides explanations with some errors. [1] Student methods, answers, and explanations contain significant errors. [0] Student makes little or no progress toward correct methods, answers, or explanations.

Performance Tasks (continued)

Chapter 12

Task 3

Parallelogram $ABCD$ is inscribed in $\odot O$.

a. Show that $\angle A$ and $\angle B$ are right angles. **Answers may vary. Sample: $\angle A$ and $\angle C$ are inscribed angles, so they are supplementary. Thus, $m\angle A + m\angle C = 180$. Because $ABCD$ is a parallelogram, opposite angles are equal, so $m\angle A = m\angle C = 90$. The same is true for $\angle B$ and $\angle D$. So, all angles are right angles.**

b. Show that $AC = BD$.

$\overline{AC}$ and $\overline{BD}$ are diagonals of a rectangle, because all angles are right. Therefore, they must be congruent.

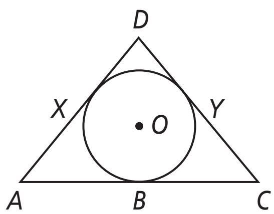
c. Use your findings to draw a conclusion about $ABCD$. Explain.

A parallelogram with two adjacent right angles or two congruent diagonals is a rectangle.

[4] Student gives valid and accurate arguments and draws valid conclusions. [3] Student's arguments and conclusions have only minor errors. [2] Student's arguments and conclusions have some errors. [1] Student's arguments and conclusions have significant errors. [0] Student makes little or no progress toward the correct arguments or conclusions.

Task 4

Given: The sides of $\triangle ACD$ are tangent to $\odot O$ at points B, X, and Y.
Also, $AB = BC = 10$ and $m\overset{\frown}{XY} = 100$.



a. Show that $\triangle ACD$ is isosceles.

Answers may vary. Sample: $AB = XA$, $BC = CY$, and $XD = DY$ because they are tangent to and intersect at a point outside the circle. $XA = AB = BC = CY = 10$ because $AB = BC = 10$; $AD = 10 + XD$; $CD = 10 + YD$; $XD = YD$; so by substitution, $AD = CD$.

b. Find the measure of the angles of $\triangle ACD$.

$m\angle D = 80$; $m\angle A = m\angle C = 50$

c. Find measure of the sides of $\triangle ACD$.

$AC = 10 + 10 = 20$; $\cos 50° = \frac{10}{AD}$; $AD = \frac{10}{\cos 50°} \approx 15.56$; $\triangle ACD$ is isosceles, so $AD = CD = 15.56$.

d. Find the radius of $\odot O$.

Because $m\angle A = 50$ and AB is tangent at B, $\triangle AOB$ is a 25°-65°-90° triangle. So, $\tan 25° = \frac{\text{radius}}{AB}$. Therefore, radius $= 10 \cdot \tan 25° \approx 4.66$.

[4] Student gives accurate answers and explanations. [3] Student's answers and explanations have minor errors. [2] Student's answers and explanations have some errors. [1] Student's answers and explanations have significant errors. [0] Student makes little or no progress toward answers or explanations.

Name ____________________ Class ____________ Date ____________

Cumulative Review

Chapters 1–12

Multiple Choice

1. $\triangle ABC \sim \triangle DEF$. Which of the following is not necessarily true? C

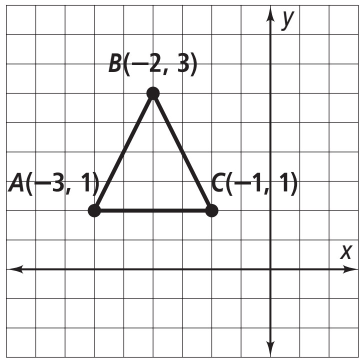
(A) $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

(C) If $AB \leq DE$, then $EF \leq BC$.

(B) $\angle B \cong \angle E$

(D) If $AB \leq DE$, then $BC \leq EF$.

2. In the figure at the right, the vertices of $\triangle ABC$ are $A(-3, 1)$, $B(-2, 3)$, and $C(-1, 1)$. $\triangle ABC$ is reflected over the x-axis and then reflected over the y-axis to $\triangle A'B'C'$. What are the coordinates of the vertices of $\triangle A'B'C'$? G



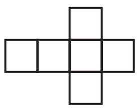
(F) $A'(1, 3)$, $B'(1, 2)$, $C'(3, 3)$

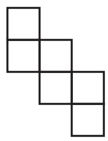
(G) $A'(3, -1)$, $B'(2, -3)$, $C'(1, -1)$

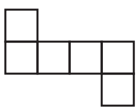
(H) $A'(1, -3)$, $B'(2, -1)$, $C'(3, -3)$

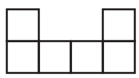
(I) $A'(3, -3)$, $B'(2, -1)$, $C'(1, -3)$

3. Which of the following is *not* the net of a cube? D

(A) 

(C) 

(B) 

(D) 

4. In a coordinate plane, unique lines m and p are both perpendicular to line q. Which of the following must be true? F

(F) Lines m and p are always parallel to each other.

(G) Lines m and p are always perpendicular to each other.

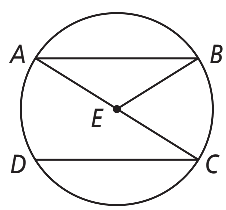
(H) Lines m and p are sometimes but not always parallel to each other.

(I) Lines m and p are sometimes but not always perpendicular to each other.

5. What is the surface area of a cube whose volume is 27? D

(A) 3 (B) 9 (C) 27 (D) 54

6. In $\odot E$, if $m\overset{\frown}{AD} = 80$, what is $m\angle ACD$? F



(F) 40 (H) 100

(G) 80 (I) 160

Cumulative Review (continued)

Chapters 1–12

Short Response

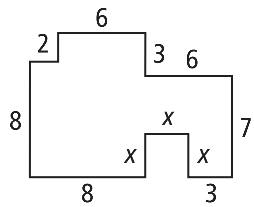
7. Use this statement to answer parts (a) and (b): If a triangle has two congruent angles, then it is isosceles.

a. Write the converse of the statement. **If a triangle is isosceles, then it has two congruent angles.**

b. Is the statement true? Is the converse true? **true; true**

c. If a statement is true, must its converse be true? If so, explain why. If not, write a true statement and its false converse. **No; check students' work.**

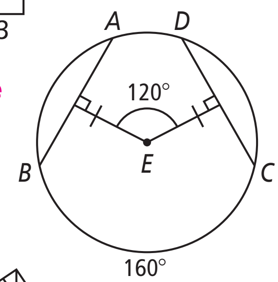
8. Use the figure at the right.



a. What is the value of *x*? **3**

b. What is the area of the figure? **109**
Show your work. **Check students' work.**

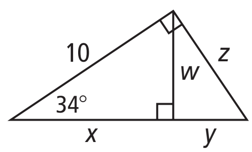
9. In the figure at the right, $\overline{AB}$ and $\overline{CD}$ are chords of circle $\odot E$. Find $m\overset{\frown}{AD}$, $m\overset{\frown}{AB}$, and $m\overset{\frown}{CD}$. Explain.



Explanations may vary. Sample: Because $\overline{AB}$ and $\overline{CD}$ are the same distance from the center, $\overline{AB} \cong \overline{CD}$. So, $m\overset{\frown}{AB} = m\overset{\frown}{CD}$. Therefore, $m\overset{\frown}{AB} + m\overset{\frown}{AD} = 120$ and $m\overset{\frown}{AB} + 160° = 240$. Therefore, $m\overset{\frown}{AD} = 160 - 120 = 40$ and $m\overset{\frown}{AB} = 120 - 40 = 80 = m\overset{\frown}{CD}$.

Extended Response

10. In the diagram to the right, *w, x, y,* and *z* are shown. Write a trigonometric equation to find the values of *w, x, y,* and *z* using the angle of 34°. Find the values of *w, x, y,* and *z* to the nearest hundredth.



$\sin 34° = \frac{w}{10} \Rightarrow w = 10 \cdot \sin 34° \approx 5.59$; $\cos 34° = \frac{x}{10} \Rightarrow x = 10 \cdot \cos 34° \approx 8.29$

$\tan 34° = \frac{y}{w} \Rightarrow y = w \cdot \tan 34° \approx 3.77$; $\cos 34° = \frac{w}{z} \Rightarrow z = \frac{w}{\cos 34°} \approx 6.75$

[4] Student shows four correctly solved trigonometric equations. [3] Student shows three correctly solved trigonometric equations. [2] Student shows two correctly solved trigonometric equations. [1] Student shows one correctly solved trigonometric equation. [0] Student shows no correctly solved trigonometric equations.

11. Prove parallelogram *ABCD* with congruent diagonals is a rectangle.
Let *E* be the point of intersection of the two diagonals.

It is given that *ABCD* is a parallelogram and $\overline{AC} \cong \overline{BD}$. $\overline{AE} \cong \overline{EB} \cong \overline{DE} \cong \overline{EC}$, because the diagonals of parallelograms bisect each other. $\angle AEB \cong \angle CED$ and $\angle BEC \cong \angle AED$ because vertical angles are congruent. So, $\triangle AED \cong \triangle CEB$ and $\triangle AEB \cong \triangle CED$ by SAS. Because each of the triangles are isosceles, $\angle EAB \cong \angle EBA \cong \angle EDC \cong \angle ECD$ and $\angle EBC \cong \angle ECB \cong \angle EAD \cong \angle EDA$. Adding adjacent angles, we find that $m\angle BAE + m\angle DAE = m\angle ABE + m\angle CBE = m\angle BCE + m\angle DCE$. So, in the parallelogram, $m\angle A = m\angle B = m\angle C = m\angle D = 90$, because they are all congruent and there are 360° in a parallelogram.

[4] Student creates a complete proof with correct statements and reasons. [3] Student proof is one correct statement and reason short of a complete proof. [2] Student proof is two correct statements and reasons short of a complete proof. [1] Student proof is three correct statements and reasons short of a complete proof. [0] Student provides incorrect or no proof.

Chapter 12 Project Teacher Notes: Going in Circles

About the Project

Students will explore techniques used for centuries to produce circular art. Then they will apply the techniques to craft their own designs.

Introducing the Project

- Ask students to describe designs, emblems, or logos they have seen that use circles. Students should be familiar with the Olympic rings or emblems on different automobiles made from circles and arcs.
- Have students list other real-world examples of objects that are made of intertwined circles, such as chains, necklaces, and rings.

Activity 1: Doing

Some students may need help drawing circles with radii $5\sqrt{2}$ and $4\sqrt{2}$. Remind them that these are the diagonals of a square with sides 5 and 4, respectively.

Activity 2: Exploring

Students may want to use drawing software to create their own op art (short for optical art). Display students' work on a bulletin board, and ask them to explain how they made their designs.

Activity 3: Constructing

Help students see how they can use diameters of a circle to draw a square and the diagonals of a square to draw a circle. Students may want to use construction tools in geometry software to make this or other designs.

Finishing the Project

You may wish to plan a project day on which students share their completed projects. Encourage students to share their processes as well as their products.

- Have students review both their designs and their instructions for drawing them.
- Ask students to share the techniques they used to craft their designs, any experimentation with other designs, and how they made imporvements in their designs.

Chapter 12 Project: Going in Circles

Beginning the Chapter Project

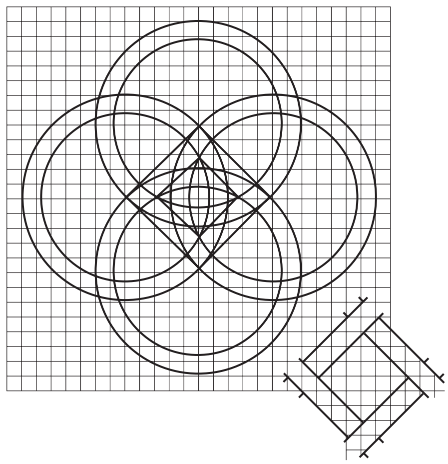
For centuries, artists have used the simple elegance of the circle in their designs. Some have crafted intertwining patterns that, like the circle itself, have no beginning and no end. Some have disturbed the symmetry of the circle to create optical illusions.

In your chapter project, you will explore some of the techniques used through the ages to produce circular art. You then will apply your discoveries to craft a dizzying design. You will see why some artists find that "going in circles" may be the best way to reach their objective.

Activities

Activity 1: Doing

Artists throughout the world have used ropelike patterns, called *knots,* on jewelry, clothing, stone carvings, and other items. You can make a knot design using graph paper and a compass. Use a pencil because you will need to erase portions of your drawing.



- Mark the origin at the center of a sheet of graph paper, but do not draw any axes. Draw four circles with centers (0, 5), (5, 0), (0, −5), (−5, 0), and with radius $5\sqrt{2}$. Using the same centers, draw four circles with radius $4\sqrt{2}$.
- Connect the four centers to form a square.
- Draw segments through the intersections of the smaller and larger circles.
- Erase arcs to make bands that appear to weave in and out. Color your design.

Activity 2: Exploring

Stare at these circular patterns for a few seconds. Notice how they seem to pulsate. In contrast to the ancient art form you explored in Activity 1, op art is a twentieth-century phenomenon.

Figure A

Figure B

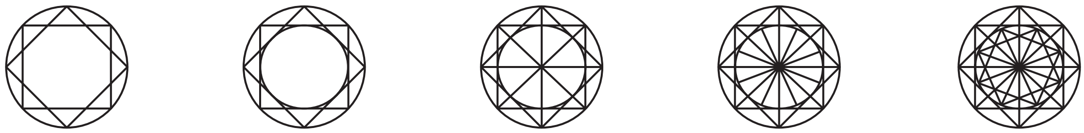
- Use geometric terms to describe how Figures A and B are related.
- Make your own op art by transforming a target design like Figure A. Use geometric terms to describe your design.

Name ______________________ Class ____________ Date ____________

Chapter 12 Project: Going in Circles (continued)

Activity 3: Constructing

Follow the steps below to make a pattern commonly found in fourteenth-century Islamic art and furniture.



Step 1: In a circle, inscribe two squares rotated 45° from each other.

Step 2: Inscribe a circle in the squares.

Step 3: In each square, draw the diagonals.

Step 4: Bisect the central angles of the smaller circle.

Step 5: Inscribe two squares as shown. Then color to make the desired design.

Finishing the Project

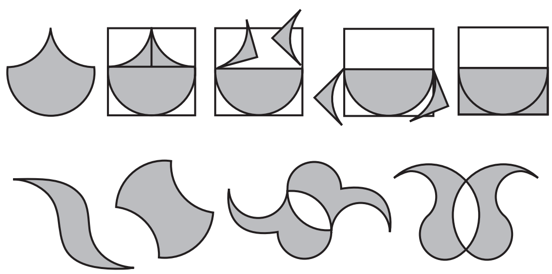
The activities will help you complete your project. Using one or more of the techniques you explored, make your own design. First, decide how you will use the design. Some possibilities are a poster, a school logo, or a tile pattern for a floor. State the purpose of your design, make your design, give instructions for drawing it, and explain the geometric concepts incorporated in it.

Reflect and Revise

Ask a classmate to review your project with you. Together, check that the diagrams and explanations are clear and accurate. Can you improve your design? Can someone follow your instructions? Have you used geometric terms correctly? Revise your work as needed.

Extending the Project

Leonardo da Vinci explored regions bounded by arcs. He showed that the first region at the right could be cut apart and reassembled into a rectangle. Read about da Vinci's efforts, and try to rearrange the other figures to form a rectangle.



Name ______________________________ Class ____________ Date ____________

Chapter 12 Project Manager: Going in Circles

Getting Started

Read about the project. As you work on it, you will need graph paper, compass, straightedge, markers, and, if available, geometry or graphics software. Keep all your work for the project in a folder, along with this Project Manager.

Checklist

☐ Activity 1: Knot design

☐ Activity 2: op art

☐ Activity 3: Islamic art

☐ your own design

Suggestions

☐ To set your compass to $5\sqrt{2}$, set it to the diagonal of a square with side 5.

☐ Notice how sections of Figure B look as though they have been cut out and moved.

☐ For Step 2, recall that a tangent to a circle is perpendicular to the radius at the point of tangency.

☐ Use the methods that you had the most fun with or the ones that will result in the desired effect. Do research to generate ideas.

Scoring Rubric

4 Your design meets the stated purpose and shows much thought and effort. Your instructions and all other diagrams, explanations, and proofs are clear, complete, and accurate. You use geometric language appropriately and correctly. Your display is organized, attractive, and instructional.

3 Your design meets the stated purpose and shows thought and effort. Your instructions and all other diagrams, explanations, and proofs are adequate but may contain some minor errors and omissions. Most of the geometric language is used appropriately and correctly. Your display shows a reasonable attempt to present material in an organized and instructional fashion.

2 Your design shows some thought and effort to meet the stated purpose. Your work is disorganized or has some major errors.

1 Your design shows little effort. Diagrams and explanations are hard to follow or misleading. Geometric terms are not used, used sparsely, or often misused.

0 Major elements of the project are incomplete or missing.

Your Evaluation of Project Evaluate your work, based on the *Scoring Rubric.*

Teacher's Evaluation of Project