CONTENTS

List of Boxes xxvii Preface xxxi Acknowledgments xxxix

PART I FOUNDATIONS 1

1 Newtonian Physics: Geometric Viewpoint 5

- 1.1 Introduction 5
 - 1.1.1 The Geometric Viewpoint on the Laws of Physics 5
 - 1.1.2 Purposes of This Chapter 7
 - 1.1.3 Overview of This Chapter 7
- 1.2 Foundational Concepts 8
- 1.3 Tensor Algebra without a Coordinate System 10
- 1.4 Particle Kinetics and Lorentz Force in Geometric Language 13
- 1.5 Component Representation of Tensor Algebra 16
 - 1.5.1 Slot-Naming Index Notation 17
 - 1.5.2 Particle Kinetics in Index Notation 19
- 1.6 Orthogonal Transformations of Bases 20
- 1.7 Differentiation of Scalars, Vectors, and Tensors; Cross Product and Curl 22
- 1.8 Volumes, Integration, and Integral Conservation Laws 26
 - 1.8.1 Gauss's and Stokes' Theorems 27
- 1.9 The Stress Tensor and Momentum Conservation 29
 - 1.9.1 Examples: Electromagnetic Field and Perfect Fluid 30
 - 1.9.2 Conservation of Momentum 31
- 1.10 Geometrized Units and Relativistic Particles for Newtonian Readers 33
 - 1.10.1 Geometrized Units 33
 - 1.10.2 Energy and Momentum of a Moving Particle 34
 - Bibliographic Note 35
 - Track Two; see page xxxiv
 - Nonrelativistic (Newtonian) kinetic theory; see page 96
 - Relativistic theory; see page 96

- 2 Special Relativity: Geometric Viewpoint 12 37
- 2.1 Overview 37
- 2.2 Foundational Concepts 38
 - 2.2.1 Inertial Frames, Inertial Coordinates, Events, Vectors, and Spacetime Diagrams 38
 - 2.2.2 The Principle of Relativity and Constancy of Light Speed 42
 - 2.2.3 The Interval and Its Invariance 45
- 2.3 Tensor Algebra without a Coordinate System 48
- 2.4 Particle Kinetics and Lorentz Force without a Reference Frame 49
 - 2.4.1 Relativistic Particle Kinetics: World Lines, 4-Velocity, 4-Momentum and Its Conservation, 4-Force 49
 - 2.4.2 Geometric Derivation of the Lorentz Force Law 52
- 2.5 Component Representation of Tensor Algebra 54
 - 2.5.1 Lorentz Coordinates 54
 - 2.5.2 Index Gymnastics 54
 - 2.5.3 Slot-Naming Notation 56
- 2.6 Particle Kinetics in Index Notation and in a Lorentz Frame 57
- 2.7 Lorentz Transformations 63
- 2.8 Spacetime Diagrams for Boosts 65
- 2.9 Time Travel 67
 - 2.9.1 Measurement of Time; Twins Paradox 67
 - 2.9.2 Wormholes 68
 - 2.9.3 Wormhole as Time Machine 69
- 2.10 Directional Derivatives, Gradients, and the Levi-Civita Tensor 70
- 2.11 Nature of Electric and Magnetic Fields; Maxwell's Equations 71
- 2.12 Volumes, Integration, and Conservation Laws 75
 - 2.12.1 Spacetime Volumes and Integration 75
 - 2.12.2 Conservation of Charge in Spacetime 78
 - 2.12.3 Conservation of Particles, Baryon Number, and Rest Mass 79
- 2.13 Stress-Energy Tensor and Conservation of 4-Momentum 82
 - 2.13.1 Stress-Energy Tensor 82
 - 2.13.2 4-Momentum Conservation 84
 - 2.13.3 Stress-Energy Tensors for Perfect Fluids and Electromagnetic Fields 85
 - Bibliographic Note 88

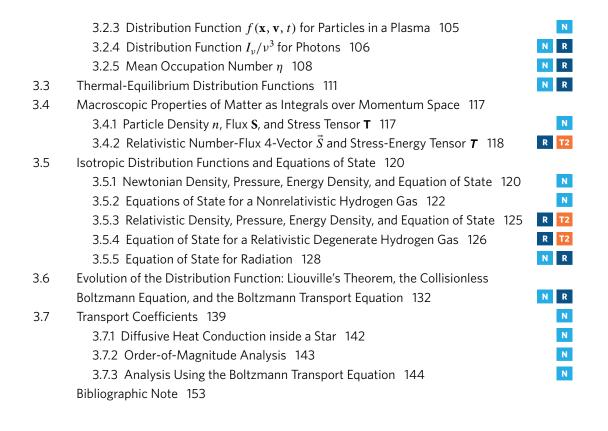
PART II STATISTICAL PHYSICS 91

3 Kinetic Theory 95

- 3.1 Overview 95
- 3.2 Phase Space and Distribution Function 97
 - 3.2.1 Newtonian Number Density in Phase Space, \mathcal{N} 97
 - 3.2.2 Relativistic Number Density in Phase Space, \mathcal{N} 99



viii Contents



4 Statistical Mechanics 155

4.1 Overview 155

- 4.2 Systems, Ensembles, and Distribution Functions 157
 - 4.2.1 Systems 157
 - 4.2.2 Ensembles 160
 - 4.2.3 Distribution Function 161
- 4.3 Liouville's Theorem and the Evolution of the Distribution Function 166
- 4.4 Statistical Equilibrium 168
 - 4.4.1 Canonical Ensemble and Distribution 169
 - 4.4.2 General Equilibrium Ensemble and Distribution; Gibbs Ensemble; Grand Canonical Ensemble 172
 - 4.4.3 Fermi-Dirac and Bose-Einstein Distributions 174
 - 4.4.4 Equipartition Theorem for Quadratic, Classical Degrees of Freedom 177
- 4.5 The Microcanonical Ensemble 178
- 4.6 The Ergodic Hypothesis 180
- 4.7 Entropy and Evolution toward Statistical Equilibrium 181
 - 4.7.1 Entropy and the Second Law of Thermodynamics 181
 - 4.7.2 What Causes the Entropy to Increase? 183
- 4.8 Entropy per Particle 191
- 4.9 Bose-Einstein Condensate 193

4.10	Statistical Mechanics in the Presence of Gravity 201	Т2
	4.10.1 Galaxies 201	Т2
	4.10.2 Black Holes 204	Т2
	4.10.3 The Universe 209	T2
	4.10.4 Structure Formation in the Expanding Universe: Violent Relaxation	
	and Phase Mixing 210	Т2
4.11	Entropy and Information 211	Т2
	4.11.1 Information Gained When Measuring the State of a System	
	in a Microcanonical Ensemble 211	T2
	4.11.2 Information in Communication Theory 212	Т2
	4.11.3 Examples of Information Content 214	Т2
	4.11.4 Some Properties of Information 216	T2
	4.11.5 Capacity of Communication Channels; Erasing Information	
	from Computer Memories 216	T2
	Bibliographic Note 218	

5 Statistical Thermodynamics 219

5.1 Overview 219

5.2 Microcanonical Ensemble and the Energy Representation of Thermodynamics 221

- 5.2.1 Extensive and Intensive Variables; Fundamental Potential 221
- 5.2.2 Energy as a Fundamental Potential 222
- 5.2.3 Intensive Variables Identified Using Measuring Devices; First Law of Thermodynamics 223
- 5.2.4 Euler's Equation and Form of the Fundamental Potential 226
- 5.2.5 Everything Deducible from First Law; Maxwell Relations 227
- 5.2.6 Representations of Thermodynamics 228
- 5.3 Grand Canonical Ensemble and the Grand-Potential Representation of Thermodynamics 229
 - 5.3.1 The Grand-Potential Representation, and Computation of Thermodynamic Properties as a Grand Canonical Sum 229
 - 5.3.2 Nonrelativistic van der Waals Gas 232
- 5.4 Canonical Ensemble and the Physical-Free-Energy Representation
 - of Thermodynamics 239
 - 5.4.1 Experimental Meaning of Physical Free Energy 241
 - 5.4.2 Ideal Gas with Internal Degrees of Freedom 242
- 5.5 Gibbs Ensemble and Representation of Thermodynamics; Phase Transitions and Chemical Reactions 246
 - 5.5.1 Out-of-Equilibrium Ensembles and Their Fundamental Thermodynamic Potentials and Minimum Principles 248
 - 5.5.2 Phase Transitions 251
 - 5.5.3 Chemical Reactions 256
- 5.6 Fluctuations away from Statistical Equilibrium 260
 - **x** Contents

- 5.7 Van der Waals Gas: Volume Fluctuations and Gas-to-Liquid Phase Transition 266
- 5.8 Magnetic Materials 270
 - 5.8.1 Paramagnetism; The Curie Law 271
 - 5.8.2 Ferromagnetism: The Ising Model 272
 - 5.8.3 Renormalization Group Methods for the Ising Model 273
 - 5.8.4 Monte Carlo Methods for the Ising Model 279

Bibliographic Note 282

6 Random Processes 283

- 6.1 Overview 283
- 6.2 Fundamental Concepts 285
 - 6.2.1 Random Variables and Random Processes 285
 - 6.2.2 Probability Distributions 286
 - 6.2.3 Ergodic Hypothesis 288
- 6.3 Markov Processes and Gaussian Processes 289
 - 6.3.1 Markov Processes; Random Walk 289
 - 6.3.2 Gaussian Processes and the Central Limit Theorem; Random Walk 292
 - 6.3.3 Doob's Theorem for Gaussian-Markov Processes, and Brownian Motion 295
- 6.4 Correlation Functions and Spectral Densities 297
 - 6.4.1 Correlation Functions; Proof of Doob's Theorem 297
 - 6.4.2 Spectral Densities 299
 - 6.4.3 Physical Meaning of Spectral Density, Light Spectra, and Noise in a Gravitational Wave Detector 301
 - 6.4.4 The Wiener-Khintchine Theorem; Cosmological Density Fluctuations 303
- 6.5 2-Dimensional Random Processes 306
 - 6.5.1 Cross Correlation and Correlation Matrix 306
 - 6.5.2 Spectral Densities and the Wiener-Khintchine Theorem 307
- 6.6 Noise and Its Types of Spectra 308
 - 6.6.1 Shot Noise, Flicker Noise, and Random-Walk Noise; Cesium Atomic Clock 3086.6.2 Information Missing from Spectral Density 310
- 6.7 Filtering Random Processes 311
 - 6.7.1 Filters, Their Kernels, and the Filtered Spectral Density 311
 - 6.7.2 Brownian Motion and Random Walks 313
 - 6.7.3 Extracting a Weak Signal from Noise: Band-Pass Filter, Wiener's Optimal Filter, Signal-to-Noise Ratio, and Allan Variance of Clock Noise 315
 - 6.7.4 Shot Noise 321
- 6.8 Fluctuation-Dissipation Theorem 323
 - 6.8.1 Elementary Version of the Fluctuation-Dissipation Theorem; Langevin Equation, Johnson Noise in a Resistor, and Relaxation Time for Brownian Motion 323
 - 6.8.2 Generalized Fluctuation-Dissipation Theorem; Thermal Noise in a Laser Beam's Measurement of Mirror Motions; Standard Quantum Limit for Measurement Accuracy and How to Evade It 331

T2

T2

T2

Contents **xi**

T2

T2

T2

T2

Т2

T2

T2

6.9 Fokker-Planck Equation 335

6.9.1 Fokker-Planck for a 1-Dimensional Markov Process 336
6.9.2 Optical Molasses: Doppler Cooling of Atoms 340
6.9.3 Fokker-Planck for a Multidimensional Markov Process; Thermal Noise in an Oscillator 343
Bibliographic Note 345

PART III OPTICS 347

7 Geometric Optics 351 7.1 Overview 351 7.2 Waves in a Homogeneous Medium 352 7.2.1 Monochromatic Plane Waves; Dispersion Relation 352 7.2.2 Wave Packets 354 7.3 Waves in an Inhomogeneous, Time-Varying Medium: The Eikonal Approximation and Geometric Optics 357 7.3.1 Geometric Optics for a Prototypical Wave Equation 358 7.3.2 Connection of Geometric Optics to Quantum Theory 362 7.3.3 Geometric Optics for a General Wave 366 7.3.4 Examples of Geometric-Optics Wave Propagation 368 7.3.5 Relation to Wave Packets; Limitations of the Eikonal Approximation and Geometric Optics 369 7.3.6 Fermat's Principle 371 7.4 Paraxial Optics 375 7.4.1 Axisymmetric, Paraxial Systems: Lenses, Mirrors, Telescopes, Microscopes, and Optical Cavities 377 7.4.2 Converging Magnetic Lens for Charged Particle Beam 381 7.5 Catastrophe Optics 384 7.5.1 Image Formation 384 7.5.2 Aberrations of Optical Instruments 395 7.6 Gravitational Lenses 396 7.6.1 Gravitational Deflection of Light 396 7.6.2 Optical Configuration 397 7.6.3 Microlensing 398 7.6.4 Lensing by Galaxies 401 7.7 Polarization 405 7.7.1 Polarization Vector and Its Geometric-Optics Propagation Law 405 7.7.2 Geometric Phase 406 Bibliographic Note 409

xii Contents

8 Diffraction 411

- 8.1 Overview 411
- 8.2 Helmholtz-Kirchhoff Integral 413
 - 8.2.1 Diffraction by an Aperture 414
 - 8.2.2 Spreading of the Wavefront: Fresnel and Fraunhofer Regions 417
- 8.3 Fraunhofer Diffraction 420
 - 8.3.1 Diffraction Grating 422
 - 8.3.2 Airy Pattern of a Circular Aperture: Hubble Space Telescope 425
 - 8.3.3 Babinet's Principle 428
- 8.4 Fresnel Diffraction 429
 - 8.4.1 Rectangular Aperture, Fresnel Integrals, and the Cornu Spiral 430
 - 8.4.2 Unobscured Plane Wave 432
 - 8.4.3 Fresnel Diffraction by a Straight Edge: Lunar Occultation of a Radio Source 432
 - 8.4.4 Circular Apertures: Fresnel Zones and Zone Plates 434
- 8.5 Paraxial Fourier Optics 436
 - 8.5.1 Coherent Illumination 437
 - 8.5.2 Point-Spread Functions 438
 - 8.5.3 Abbé's Description of Image Formation by a Thin Lens 439
 - 8.5.4 Image Processing by a Spatial Filter in the Focal Plane of a Lens: High-Pass, Low-Pass, and Notch Filters; Phase-Contrast Microscopy 441
 - 8.5.5 Gaussian Beams: Optical Cavities and Interferometric Gravitational-Wave Detectors 445
- 8.6 Diffraction at a Caustic 451 Bibliographic Note 454

9 Interference and Coherence 455

- 9.1 Overview 455
- 9.2 Coherence 456
 - 9.2.1 Young's Slits 456
 - 9.2.2 Interference with an Extended Source: Van Cittert-Zernike Theorem 459
 - 9.2.3 More General Formulation of Spatial Coherence; Lateral Coherence Length 462
 - 9.2.4 Generalization to 2 Dimensions 463
 - 9.2.5 Michelson Stellar Interferometer; Astronomical Seeing 464
 - 9.2.6 Temporal Coherence 472
 - 9.2.7 Michelson Interferometer and Fourier-Transform Spectroscopy 474
 - 9.2.8 Degree of Coherence; Relation to Theory of Random Processes 477
- 9.3 Radio Telescopes 479
 - 9.3.1 Two-Element Radio Interferometer 479
 - 9.3.2 Multiple-Element Radio Interferometers 480
 - 9.3.3 Closure Phase 481
 - 9.3.4 Angular Resolution 482

Contents xiii

- 9.4 Etalons and Fabry-Perot Interferometers 483
 - 9.4.1 Multiple-Beam Interferometry; Etalons 483
 - 9.4.2 Fabry-Perot Interferometer and Modes of a Fabry-Perot Cavity with Spherical Mirrors 490
 - 9.4.3 Fabry-Perot Applications: Spectrometer, Laser, Mode-Cleaning Cavity, Beam-Shaping Cavity, PDH Laser Stabilization, Optical Frequency Comb 496

9.5 Laser Interferometer Gravitational-Wave Detectors 502

9.6 Power Correlations and Photon Statistics: Hanbury Brown and Twiss Intensity
 Interferometer 509
 Bibliographic Note 512

10 Nonlinear Optics 513

- 10.1 Overview 513
- 10.2 Lasers 515
 - 10.2.1 Basic Principles of the Laser 515
 - 10.2.2 Types of Lasers and Their Performances and Applications 519
 - 10.2.3 Ti:Sapphire Mode-Locked Laser 520
 - 10.2.4 Free Electron Laser 521
- 10.3 Holography 521
 - 10.3.1 Recording a Hologram 522
 - 10.3.2 Reconstructing the 3-Dimensional Image from a Hologram 525
 - 10.3.3 Other Types of Holography; Applications 527
- 10.4 Phase-Conjugate Optics 531
- 10.5 Maxwell's Equations in a Nonlinear Medium; Nonlinear Dielectric Susceptibilities; Electro-Optic Effects 536
- 10.6 Three-Wave Mixing in Nonlinear Crystals 540
 - 10.6.1 Resonance Conditions for Three-Wave Mixing 540
 - 10.6.2 Three-Wave-Mixing Evolution Equations in a Medium That Is Dispersion-Free and Isotropic at Linear Order 544
 - 10.6.3 Three-Wave Mixing in a Birefringent Crystal: Phase Matching and Evolution Equations 546

T2

- 10.7 Applications of Three-Wave Mixing: Frequency Doubling, Optical Parametric
 - Amplification, and Squeezed Light 553
 - 10.7.1 Frequency Doubling 553
 - 10.7.2 Optical Parametric Amplification 555
 - 10.7.3 Degenerate Optical Parametric Amplification: Squeezed Light 556
- 10.8 Four-Wave Mixing in Isotropic Media 558
 - 10.8.1 Third-Order Susceptibilities and Field Strengths 558
 - 10.8.2 Phase Conjugation via Four-Wave Mixing in CS_2 Fluid 559
 - 10.8.3 Optical Kerr Effect and Four-Wave Mixing in an Optical Fiber 562
 - Bibliographic Note 564

xiv Contents

PART IV ELASTICITY 565

11	Elastostatics 567
11.1	Overview 567
11.2	Displacement and Strain 570
	11.2.1 Displacement Vector and Its Gradient 570
	11.2.2 Expansion, Rotation, Shear, and Strain 571
11.3	Stress, Elastic Moduli, and Elastostatic Equilibrium 577
	11.3.1 Stress Tensor 577
	11.3.2 Realm of Validity for Hooke's Law 580
	11.3.3 Elastic Moduli and Elastostatic Stress Tensor 580
	11.3.4 Energy of Deformation 582
	11.3.5 Thermoelasticity 584
	11.3.6 Molecular Origin of Elastic Stress; Estimate of Moduli 585
	11.3.7 Elastostatic Equilibrium: Navier-Cauchy Equation 587
11.4	Young's Modulus and Poisson's Ratio for an Isotropic Material: A Simple
	Elastostatics Problem 589
11.5	Reducing the Elastostatic Equations to 1 Dimension for a Bent Beam: Cantilever Bridge,
	Foucault Pendulum, DNA Molecule, Elastica 592
11.6	Buckling and Bifurcation of Equilibria 602
	11.6.1 Elementary Theory of Buckling and Bifurcation 602
	11.6.2 Collapse of the World Trade Center Buildings 605
	11.6.3 Buckling with Lateral Force; Connection to Catastrophe Theory 606
	11.6.4 Other Bifurcations: Venus Fly Trap, Whirling Shaft, Triaxial Stars, and
	Onset of Turbulence 607
11.7	Reducing the Elastostatic Equations to 2 Dimensions for a Deformed Thin Plate: Stress Polishing a Telescope Mirror 609
11.8	Cylindrical and Spherical Coordinates: Connection Coefficients and Components
	of the Gradient of the Displacement Vector 614 T2
11.9	Solving the 3-Dimensional Navier-Cauchy Equation in Cylindrical Coordinates 619
	11.9.1 Simple Methods: Pipe Fracture and Torsion Pendulum 619
	11.9.2 Separation of Variables and Green's Functions: Thermoelastic Noise
	in Mirrors 622
	Bibliographic Note 627
12	Elastodynamics 629

- 12.1 Overview 629
- 12.2 Basic Equations of Elastodynamics; Waves in a Homogeneous Medium 630 12.2.1 Equation of Motion for a Strained Elastic Medium 630
 - 12.2.2 Elastodynamic Waves 636
 - 12.2.3 Longitudinal Sound Waves 637

Contents XV

- 12.2.4 Transverse Shear Waves 638
- 12.2.5 Energy of Elastodynamic Waves 640
- 12.3 Waves in Rods, Strings, and Beams 642
 - 12.3.1 Compression Waves in a Rod 643
 - 12.3.2 Torsion Waves in a Rod 643
 - 12.3.3 Waves on Strings 644
 - 12.3.4 Flexural Waves on a Beam 645
 - 12.3.5 Bifurcation of Equilibria and Buckling (Once More) 647
- 12.4 Body Waves and Surface Waves—Seismology and Ultrasound 648
 - 12.4.1 Body Waves 650
 - 12.4.2 Edge Waves 654
 - 12.4.3 Green's Function for a Homogeneous Half-Space 658
 - 12.4.4 Free Oscillations of Solid Bodies 661
 - 12.4.5 Seismic Tomography 663
 - 12.4.6 Ultrasound; Shock Waves in Solids 663
- 12.5 The Relationship of Classical Waves to Quantum Mechanical Excitations 667 Bibliographic Note 670

T2

PART V FLUID DYNAMICS 671

13 Foundations of Fluid Dynamics 675

- 13.1 Overview 675
- 13.2 The Macroscopic Nature of a Fluid: Density, Pressure, Flow Velocity; Liquids versus Gases 677

13.3 Hydrostatics 681

- 13.3.1 Archimedes' Law 684
- 13.3.2 Nonrotating Stars and Planets 686
- 13.3.3 Rotating Fluids 689
- 13.4 Conservation Laws 691
- 13.5 The Dynamics of an Ideal Fluid 695
 - 13.5.1 Mass Conservation 696
 - 13.5.2 Momentum Conservation 696
 - 13.5.3 Euler Equation 697
 - 13.5.4 Bernoulli's Theorem 697
 - 13.5.5 Conservation of Energy 704
- 13.6 Incompressible Flows 709
- 13.7 Viscous Flows with Heat Conduction 710
 - 13.7.1 Decomposition of the Velocity Gradient into Expansion, Vorticity, and Shear 710
 - 13.7.2 Navier-Stokes Equation 711
 - 13.7.3 Molecular Origin of Viscosity 713
 - 13.7.4 Energy Conservation and Entropy Production 714
 - xvi Contents

13.7.5 Reynolds Number 716 13.7.6 Pipe Flow 716 13.8 Relativistic Dynamics of a Perfect Fluid 719 **T2** 13.8.1 Stress-Energy Tensor and Equations of Relativistic Fluid Mechanics 719 **T2** 13.8.2 Relativistic Bernoulli Equation and Ultrarelativistic Astrophysical Jets 721 13.8.3 Nonrelativistic Limit of the Stress-Energy Tensor 723 **T2** Bibliographic Note 726 14 Vorticity 729 14.1 Overview 729 14.2 Vorticity, Circulation, and Their Evolution 731 14.2.1 Vorticity Evolution 734 14.2.2 Barotropic, Inviscid, Compressible Flows: Vortex Lines Frozen into Fluid 736 14.2.3 Tornados 738 14.2.4 Circulation and Kelvin's Theorem 739 14.2.5 Diffusion of Vortex Lines 741 14.2.6 Sources of Vorticity 744 Low-Reynolds-Number Flow—Stokes Flow and Sedimentation 746 14.3 14.3.1 Motivation: Climate Change 748 14.3.2 Stokes Flow 749 14.3.3 Sedimentation Rate 754 14.4 High-Reynolds-Number Flow—Laminar Boundary Layers 757 14.4.1 Blasius Velocity Profile Near a Flat Plate: Stream Function and Similarity Solution 758 14.4.2 Blasius Vorticity Profile 763 14.4.3 Viscous Drag Force on a Flat Plate 763 14.4.4 Boundary Layer Near a Curved Surface: Separation 764 14.5 Nearly Rigidly Rotating Flows—Earth's Atmosphere and Oceans 766 14.5.1 Equations of Fluid Dynamics in a Rotating Reference Frame 767 14.5.2 Geostrophic Flows 770 14.5.3 Taylor-Proudman Theorem 771 14.5.4 Ekman Boundary Layers 772 14.6 Instabilities of Shear Flows—Billow Clouds and Turbulence in the Stratosphere 778 **T2** 14.6.1 Discontinuous Flow: Kelvin-Helmholtz Instability 778 14.6.2 Discontinuous Flow with Gravity 782 14.6.3 Smoothly Stratified Flows: Rayleigh and Richardson Criteria for Instability 784 **T2** Bibliographic Note 786 15 Turbulence 787 15.1 Overview 787 15.2 The Transition to Turbulence—Flow Past a Cylinder 789

Contents **xvii**

- 15.3 Empirical Description of Turbulence 798
 - 15.3.1 The Role of Vorticity in Turbulence 799
- 15.4 Semiquantitative Analysis of Turbulence 800
 - 15.4.1 Weak-Turbulence Formalism 800
 - 15.4.2 Turbulent Viscosity 804
 - 15.4.3 Turbulent Wakes and Jets; Entrainment; the Coanda Effect 805
 - 15.4.4 Kolmogorov Spectrum for Fully Developed, Homogeneous, Isotropic Turbulence 810
- 15.5 Turbulent Boundary Layers 817
 - 15.5.1 Profile of a Turbulent Boundary Layer 818
 - 15.5.2 Coanda Effect and Separation in a Turbulent Boundary Layer 820
 - 15.5.3 Instability of a Laminar Boundary Layer 822
 - 15.5.4 Flight of a Ball 823
- 15.6 The Route to Turbulence—Onset of Chaos 825
 - 15.6.1 Rotating Couette Flow 825
 - 15.6.2 Feigenbaum Sequence, Poincaré Maps, and the Period-Doubling Route to Turbulence in Convection 828
 - 15.6.3 Other Routes to Turbulent Convection 831
 - 15.6.4 Extreme Sensitivity to Initial Conditions 832
 - Bibliographic Note 834

16 Waves 835

- 16.1 Overview 835
- 16.2 Gravity Waves on and beneath the Surface of a Fluid 837
 - 16.2.1 Deep-Water Waves and Their Excitation and Damping 840
 - 16.2.2 Shallow-Water Waves 840
 - 16.2.3 Capillary Waves and Surface Tension 844
 - 16.2.4 Helioseismology 848
- 16.3 Nonlinear Shallow-Water Waves and Solitons 850
 - 16.3.1 Korteweg-de Vries (KdV) Equation 850
 - 16.3.2 Physical Effects in the KdV Equation 853
 - 16.3.3 Single-Soliton Solution 854
 - 16.3.4 Two-Soliton Solution 855
 - 16.3.5 Solitons in Contemporary Physics 856
- 16.4 Rossby Waves in a Rotating Fluid 858
- 16.5 Sound Waves 862
 - 16.5.1 Wave Energy 863
 - 16.5.2 Sound Generation 865
 - 16.5.3 Radiation Reaction, Runaway Solutions, and Matched Asymptotic Expansions 869

Bibliographic Note 874

xviii Contents

17 Compressible and Supersonic Flow 875

- 17.1 Overview 875
- 17.2 Equations of Compressible Flow 877
- 17.3 Stationary, Irrotational, Quasi-1-Dimensional Flow 880 17.3.1 Basic Equations; Transition from Subsonic to Supersonic Flow 880 17.3.2 Setting up a Stationary, Transonic Flow 883 17.3.3 Rocket Engines 887 17.4 1-Dimensional, Time-Dependent Flow 891 17.4.1 Riemann Invariants 891 17.4.2 Shock Tube 895 17.5 Shock Fronts 897 17.5.1 Junction Conditions across a Shock; Rankine-Hugoniot Relations 898 17.5.2 Junction Conditions for Ideal Gas with Constant γ 904 17.5.3 Internal Structure of a Shock 906 17.5.4 Mach Cone 907 17.6 Self-Similar Solutions—Sedov-Taylor Blast Wave 908 17.6.1 The Sedov-Taylor Solution 909 17.6.2 Atomic Bomb 912 17.6.3 Supernovae 914 Bibliographic Note 916

18 Convection 917

- 18.1 Overview 917
- 18.2 Diffusive Heat Conduction—Cooling a Nuclear Reactor; Thermal Boundary Layers 918
- 18.3 Boussinesq Approximation 923
- 18.4 Rayleigh-Bénard Convection 925
- 18.5 Convection in Stars 933
- 18.6 Double Diffusion—Salt Fingers 937Bibliographic Note 941

19 Magnetohydrodynamics 943

- 19.1 Overview 943
- 19.2 Basic Equations of MHD 944

19.2.1 Maxwell's Equations in the MHD Approximation 946

- 19.2.2 Momentum and Energy Conservation 950
- 19.2.3 Boundary Conditions 953
- 19.2.4 Magnetic Field and Vorticity 957

19.3 Magnetostatic Equilibria 958

19.3.1 Controlled Thermonuclear Fusion 958

19.3.2 Z-Pinch 960

T2

	19.3.3 @-Pinch 962	
	19.3.4 Tokamak 963	
19.4	Hydromagnetic Flows 965	
19.5	Stability of Magnetostatic Equilibria 971	
	19.5.1 Linear Perturbation Theory 971	
	19.5.2 Z-Pinch: Sausage and Kink Instabilities 975	
	19.5.3 The Θ -Pinch and Its Toroidal Analog; Flute Instability; Motivation	
	for Tokamak 978	
	19.5.4 Energy Principle and Virial Theorems 980	T2
19.6	Dynamos and Reconnection of Magnetic Field Lines 984	Т2
	19.6.1 Cowling's Theorem 984	T2
	19.6.2 Kinematic Dynamos 985	Т2
	19.6.3 Magnetic Reconnection 986	T2
19.7	Magnetosonic Waves and the Scattering of Cosmic Rays 988	
	19.7.1 Cosmic Rays 988	
	19.7.2 Magnetosonic Dispersion Relation 989	
	19.7.3 Scattering of Cosmic Rays by Alfvén Waves 992	
	Bibliographic Note 993	

PART VI PLASMA PHYSICS 995

20 The Particle Kinetics of Plasma 997

- 20.1 Overview 997
- 20.2 Examples of Plasmas and Their Density-Temperature Regimes 998
 - 20.2.1 Ionization Boundary 998
 - 20.2.2 Degeneracy Boundary 1000
 - 20.2.3 Relativistic Boundary 1000
 - 20.2.4 Pair-Production Boundary 1001
 - 20.2.5 Examples of Natural and Human-Made Plasmas 1001
- 20.3 Collective Effects in Plasmas—Debye Shielding and Plasma Oscillations 1003
 - 20.3.1 Debye Shielding 1003
 - 20.3.2 Collective Behavior 1004
 - 20.3.3 Plasma Oscillations and Plasma Frequency 1005
- 20.4 Coulomb Collisions 1006
 - 20.4.1 Collision Frequency 1006
 - 20.4.2 The Coulomb Logarithm 1008
 - 20.4.3 Thermal Equilibration Rates in a Plasma 1010
 - 20.4.4 Discussion 1012
- 20.5 Transport Coefficients 1015
 - 20.5.1 Coulomb Collisions 1015
 - 20.5.2 Anomalous Resistivity and Anomalous Equilibration 1016

XX Contents

- 20.6 Magnetic Field 1019
 - 20.6.1 Cyclotron Frequency and Larmor Radius 1019
 - 20.6.2 Validity of the Fluid Approximation 1020

20.6.3 Conductivity Tensor 1022

20.7 Particle Motion and Adiabatic Invariants 1024
20.7.1 Homogeneous, Time-Independent Magnetic Field and No Electric Field 1025
20.7.2 Homogeneous, Time-Independent Electric and Magnetic Fields 1025
20.7.3 Inhomogeneous, Time-Independent Magnetic Field 1026
20.7.4 A Slowly Time-Varying Magnetic Field 1029
20.7.5 Failure of Adiabatic Invariants; Chaotic Orbits 1030
Bibliographic Note 1032

21 Waves in Cold Plasmas: Two-Fluid Formalism 1033

- 21.1 Overview 1033
- 21.2 Dielectric Tensor, Wave Equation, and General Dispersion Relation 1035
- 21.3 Two-Fluid Formalism 1037
- 21.4 Wave Modes in an Unmagnetized Plasma 1040
 - 21.4.1 Dielectric Tensor and Dispersion Relation for a Cold, Unmagnetized Plasma 1040
 - 21.4.2 Plasma Electromagnetic Modes 1042
 - 21.4.3 Langmuir Waves and Ion-Acoustic Waves in Warm Plasmas 1044
 - 21.4.4 Cutoffs and Resonances 1049
- 21.5 Wave Modes in a Cold, Magnetized Plasma 1050
 - 21.5.1 Dielectric Tensor and Dispersion Relation 1050
 - 21.5.2 Parallel Propagation 1052
 - 21.5.3 Perpendicular Propagation 1057
 - 21.5.4 Propagation of Radio Waves in the Ionosphere; Magnetoionic Theory 1058
 - 21.5.5 CMA Diagram for Wave Modes in a Cold, Magnetized Plasma 1062
- 21.6 Two-Stream Instability 1065 Bibliographic Note 1068

22 Kinetic Theory of Warm Plasmas 1069

- 22.1 Overview 1069
- 22.2 Basic Concepts of Kinetic Theory and Its Relationship to Two-Fluid Theory 1070
 - 22.2.1 Distribution Function and Vlasov Equation 1070
 - 22.2.2 Relation of Kinetic Theory to Two-Fluid Theory 1073
 - 22.2.3 Jeans' Theorem 1074
- 22.3 Electrostatic Waves in an Unmagnetized Plasma: Landau Damping 1077
 - 22.3.1 Formal Dispersion Relation 1077
 - 22.3.2 Two-Stream Instability 1079
 - 22.3.3 The Landau Contour 1080
 - 22.3.4 Dispersion Relation for Weakly Damped or Growing Waves 1085

Contents xxi

	22.3.5 Langmuir Waves and Their Landau Damping 1086
	22.3.6 Ion-Acoustic Waves and Conditions for Their Landau Damping to Be Weak 1088
22.4	Stability of Electrostatic Waves in Unmagnetized Plasmas 1090
	22.4.1 Nyquist's Method 1091
	22.4.2 Penrose's Instability Criterion 1091
22.5	Particle Trapping 1098
22.6	N-Particle Distribution Function 1102
	22.6.1 BBGKY Hierarchy 1103 T2
	22.6.2 Two-Point Correlation Function 1104
	22.6.3 Coulomb Correction to Plasma Pressure 1107
	Bibliographic Note 1108
23	Nonlinear Dynamics of Plasmas 1111
23.1	Overview 1111
23.2	Quasilinear Theory in Classical Language 1113
	23.2.1 Classical Derivation of the Theory 1113
	23.2.2 Summary of Quasilinear Theory 1120
	23.2.3 Conservation Laws 1121
	23.2.4 Generalization to 3 Dimensions 1122
23.3	Quasilinear Theory in Quantum Mechanical Language 1123
	23.3.1 Plasmon Occupation Number η 1123
	23.3.2 Evolution of η for Plasmons via Interaction with Electrons 1124
	23.3.3 Evolution of f for Electrons via Interaction with Plasmons 1129
	23.3.4 Emission of Plasmons by Particles in the Presence of a Magnetic Field 1131
	23.3.5 Relationship between Classical and Quantum Mechanical Formalisms 1131
	23.3.6 Evolution of η via Three-Wave Mixing 1132
23.4	Quasilinear Evolution of Unstable Distribution Functions—A Bump in the Tail 1136
	23.4.1 Instability of Streaming Cosmic Rays 1138
23.5	Parametric Instabilities; Laser Fusion 1140
23.6	Solitons and Collisionless Shock Waves 1142
	Bibliographic Note 1149

PART VII GENERAL RELATIVITY 1151

24 From Special to General Relativity 1153

24.1 Overview 1153

- 24.2 Special Relativity Once Again 1153
 - 24.2.1 Geometric, Frame-Independent Formulation 1154
 - 24.2.2 Inertial Frames and Components of Vectors, Tensors, and Physical Laws 1156
 - 24.2.3 Light Speed, the Interval, and Spacetime Diagrams 1159
- 24.3 Differential Geometry in General Bases and in Curved Manifolds 1160 24.3.1 Nonorthonormal Bases 1161
 - xxii Contents

- 24.3.2 Vectors as Directional Derivatives; Tangent Space; Commutators 1165
- 24.3.3 Differentiation of Vectors and Tensors; Connection Coefficients 1169

24.3.4 Integration 1174

- 24.4 The Stress-Energy Tensor Revisited 1176
- 24.5 The Proper Reference Frame of an Accelerated Observer 1180
 - 24.5.1 Relation to Inertial Coordinates; Metric in Proper Reference Frame; Transport Law for Rotating Vectors 1183
 - 24.5.2 Geodesic Equation for a Freely Falling Particle 1184
 - 24.5.3 Uniformly Accelerated Observer 1186
 - 24.5.4 Rindler Coordinates for Minkowski Spacetime 1187

Bibliographic Note 1190

25 Fundamental Concepts of General Relativity 1191

- 25.1 History and Overview 1191
- 25.2 Local Lorentz Frames, the Principle of Relativity, and Einstein's Equivalence Principle 1195
- 25.3 The Spacetime Metric, and Gravity as a Curvature of Spacetime 1196
- 25.4 Free-Fall Motion and Geodesics of Spacetime 1200
- 25.5 Relative Acceleration, Tidal Gravity, and Spacetime Curvature 1206
 - 25.5.1 Newtonian Description of Tidal Gravity 1207
 - 25.5.2 Relativistic Description of Tidal Gravity 1208
 - 25.5.3 Comparison of Newtonian and Relativistic Descriptions 1210
- 25.6 Properties of the Riemann Curvature Tensor 1213
- 25.7 Delicacies in the Equivalence Principle, and Some Nongravitational Laws of Physics in Curved Spacetime 1217
 - 25.7.1 Curvature Coupling in the Nongravitational Laws 1218
- 25.8 The Einstein Field Equation 1221
 - 25.8.1 Geometrized Units 1224
- 25.9 Weak Gravitational Fields 1224
 - 25.9.1 Newtonian Limit of General Relativity 1225
 - 25.9.2 Linearized Theory 1227
 - 25.9.3 Gravitational Field outside a Stationary, Linearized Source of Gravity 1231 12
 - 25.9.4 Conservation Laws for Mass, Momentum, and Angular Momentum in Linearized Theory 1237
 - 25.9.5 Conservation Laws for a Strong-Gravity Source 1238

Bibliographic Note 1239

26 Relativistic Stars and Black Holes 1241

- 26.1 Overview 1241
- 26.2 Schwarzschild's Spacetime Geometry 1242
 - 26.2.1 The Schwarzschild Metric, Its Connection Coefficients, and Its Curvature Tensors 1242

Contents **xxiii**

T2

- 26.2.2 The Nature of Schwarzschild's Coordinate System, and Symmetries of the Schwarzschild Spacetime 1244
- 26.2.3 Schwarzschild Spacetime at Radii $r \gg M$: The Asymptotically Flat Region 1245
- 26.2.4 Schwarzschild Spacetime at $r \sim M$ 1248
- 26.3 Static Stars 1250
 - 26.3.1 Birkhoff's Theorem 1250
 - 26.3.2 Stellar Interior 1252
 - 26.3.3 Local Conservation of Energy and Momentum 1255
 - 26.3.4 The Einstein Field Equation 1257
 - 26.3.5 Stellar Models and Their Properties 1259
 - 26.3.6 Embedding Diagrams 1261
- 26.4 Gravitational Implosion of a Star to Form a Black Hole 1264
 - 26.4.1 The Implosion Analyzed in Schwarzschild Coordinates 1264
 - 26.4.2 Tidal Forces at the Gravitational Radius 1266
 - 26.4.3 Stellar Implosion in Eddington-Finkelstein Coordinates 1267
 - 26.4.4 Tidal Forces at r = 0—The Central Singularity 1271
 - 26.4.5 Schwarzschild Black Hole 1272
- 26.5 Spinning Black Holes: The Kerr Spacetime 1277
 - 26.5.1 The Kerr Metric for a Spinning Black Hole 1277
 - 26.5.2 Dragging of Inertial Frames 1279
 - 26.5.3 The Light-Cone Structure, and the Horizon 1279
 - 26.5.4 Evolution of Black Holes—Rotational Energy and Its Extraction 1282
- 26.6 The Many-Fingered Nature of Time 1293 Bibliographic Note 1297

27 Gravitational Waves and Experimental Tests of General Relativity 1299

- 27.1 Overview 1299
- 27.2 Experimental Tests of General Relativity 1300
 - 27.2.1 Equivalence Principle, Gravitational Redshift, and Global Positioning System 1300
 - 27.2.2 Perihelion Advance of Mercury 1302
 - 27.2.3 Gravitational Deflection of Light, Fermat's Principle, and Gravitational Lenses 1305
 - 27.2.4 Shapiro Time Delay 1308
 - 27.2.5 Geodetic and Lense-Thirring Precession 1309
 - 27.2.6 Gravitational Radiation Reaction 1310
- 27.3 Gravitational Waves Propagating through Flat Spacetime 1311
 - 27.3.1 Weak, Plane Waves in Linearized Theory 1311
 - 27.3.2 Measuring a Gravitational Wave by Its Tidal Forces 1315
 - 27.3.3 Gravitons and Their Spin and Rest Mass 1319

xxiv Contents

T2

T2

T2

27.4	Gravitational Waves Propagating through Curved Spacetime 1320 27.4.1 Gravitational Wave Equation in Curved Spacetime 1321	
	27.4.2 Geometric-Optics Propagation of Gravitational Waves 1322	
	27.4.3 Energy and Momentum in Gravitational Waves 1324	
27.5	The Generation of Gravitational Waves 1327	
27.5	27.5.1 Multipole-Moment Expansion 1328	
	27.5.2 Quadrupole-Moment Formalism 1330	
	27.5.3 Quadrupolar Wave Strength, Energy, Angular Momentum, and Radiation	
	Reaction 1332	
	27.5.4 Gravitational Waves from a Binary Star System 1335	
	27.5.5 Gravitational Waves from Binaries Made of Black Holes, Neutron Stars,	
	or Both: Numerical Relativity 1341	Т2
27.6	The Detection of Gravitational Waves 1345	
	27.6.1 Frequency Bands and Detection Techniques 1345	
	27.6.2 Gravitational-Wave Interferometers: Overview and Elementary	
	Treatment 1347	
	27.6.3 Interferometer Analyzed in TT Gauge 1349	T2
	27.6.4 Interferometer Analyzed in the Proper Reference Frame of the	
	Beam Splitter 1352	T2
	27.6.5 Realistic Interferometers 1355	T2
	27.6.6 Pulsar Timing Arrays 1355	Т2
	Bibliographic Note 1358	
28	Cosmology 1361	
28.1	Overview 1361	
28.2	General Relativistic Cosmology 1364	
	28.2.1 Isotropy and Homogeneity 1364	
	28.2.2 Geometry 1366	
	28.2.3 Kinematics 1373	
	28.2.4 Dynamics 1376	
28.3	The Universe Today 1379	
	28.3.1 Baryons 1379	
	28.3.2 Dark Matter 1380	
	28.3.3 Photons 1381	
	28.3.4 Neutrinos 1382	
	28.3.5 Cosmological Constant 1382	
	28.3.6 Standard Cosmology 1383	
28.4	Seven Ages of the Universe 1383	
	28.4.1 Particle Age 1384	
	28.4.2 Nuclear Age 1387	
	28.4.3 Photon Age 1392	

	28.4.4 Plasma Age 1393	
	28.4.5 Atomic Age 1397	
	28.4.6 Gravitational Age 1397	
	28.4.7 Cosmological Age 1400	
28.5	Galaxy Formation 1401	T2
	28.5.1 Linear Perturbations 1401	T2
	28.5.2 Individual Constituents 1406	T2
	28.5.3 Solution of the Perturbation Equations 1410	T2
	28.5.4 Galaxies 1412	T2
28.6	Cosmological Optics 1415	T2
	28.6.1 Cosmic Microwave Background 1415	T2
	28.6.2 Weak Gravitational Lensing 1422	T2
	28.6.3 Sunyaev-Zel'dovich Effect 1428	T2
28.7	Three Mysteries 1431	T2
	28.7.1 Inflation and the Origin of the Universe 1431	T2
	28.7.2 Dark Matter and the Growth of Structure 1440	T2
	28.7.3 The Cosmological Constant and the Fate of the Universe 1444	T2
	Bibliographic Note 1447	

References 1449

Name Index 1473

Subject Index 1477