Modeling and Control of dc/dc Boost Converter in FC systems

ME 590 Report to Professor Stefanopoulou from Wei Xi

1. Introduction

1.1 Fuel Cell is one of the future energy resources

Energy and environment problems, such as oil crisis and automobile emission, are always hot topics at present within the context of continued rapid expansion of the world's population. These problems have put and will continue to put tremendous pressure our government and automobile industry to find out some new efficient and clean power source to replace the conventional internal combustion.

Within this context, fuel cell systems are expected to play a major role. First, from efficiency standpoint, the traditional internal combustion has naturally efficiency limitation because of the Second Law of Thermodynamics. For example, the efficiency of the internal combustion (IC) spark ignition is only about 25~35%, whereas the efficiency of PEMFC system is about 40~45%.[1] Here the efficiency is defined as:

$$\eta = \frac{WheelEnergy}{FuelInputEnergy} \tag{1.1}$$

Second, the Fuel Cell system has nearly zero emission. Some people call Fuel Cell techniques as clean energy.

1.2 The importance of dc/dc converter in FC systems

A typical fuel cell system for automobile consists of three parts. As can be seen in figure 1.1, the left hand part is fuel cell stacks, which can provide power source. The right hand part is load part, which include traction motor, air compressor, air conditioner, and other auxiliaries. The boost converter is in the middle of the blocks. From the figure we can see that the boost converter just works as a bridge; it connects the power source and the load.
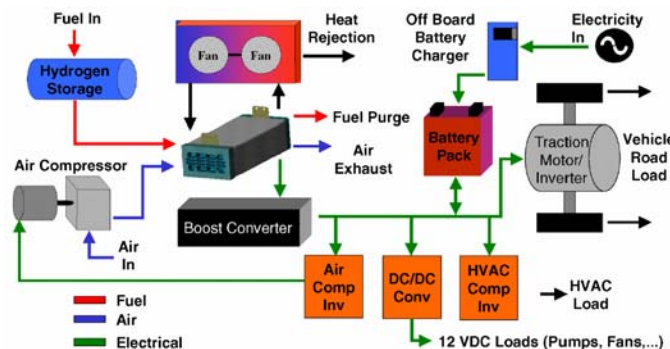

Figure 1.1 Typical Fuel Cell system for automobile[1]

But why we need boost converter. The reason lies in two:

First, the fuel cell stacks have low voltage and high current characteristics (typically the voltage is about 250V and the current is about 200 A, while the motor needs 380V). So we need boost converter to step up the fuel cell stacks output voltage from 250V to about 400V.

Second, the fuel cell stacks are sensitive to load variance. As can be seen in figure 1.2 when the load increases, the fuel cell stacks voltage will drop steeply at first, which will affect the output voltage. So we want to use booster converter to stabilize the output voltage (fig 2.1) and reject the disturbance from load and fuel cells stacks.
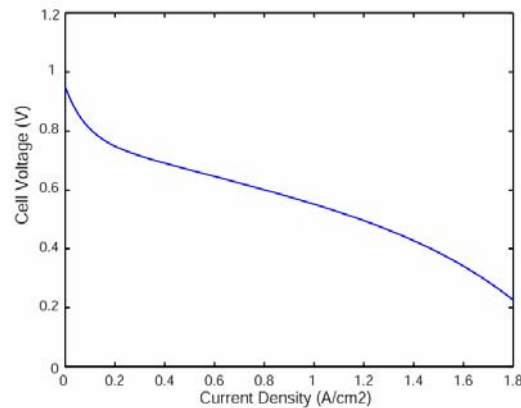

Figure 1.2 Polarization curve of the Fuel Cell

The control issue of the booster converter just to deal with the two problems I have mentioned above. First, operate the system at the desired working condition, which means boost up the output voltage of fuel cell stacks from about 250v to 400v). Second, protect the system from fuel cell stacks and external load fluctuations. (Reverse currents, sudden load variations)

In summary, our control goal for the boost converter is to ensure the output tracks voltage to a desired value, as well as reject FC voltage fluctuations (FC characteristic curve) and external load variations (resistance step) at the boost converter output voltage.

2. Modeling and verification of dc/dc Boost Converter

2.1 Operation of dc/dc converter

The dc/dc converter we address here is a switching converter. Specifically, the dc-dc converter is a power electronics circuit, which uses an inductor, a transformer, or a capacitor as an energy-storage element to convert electrical power from one voltage level into another voltage level by switching action.

There are mainly four types dc-dc converters: buck converter, boost converter, buck-boost converter, and flyback converter. The function of buck converter is to step down the input voltage. The function of boost converter, on the other hand, is to step up the input voltage. The function of buck-boost combines the functions of both buck converter and boost converter and can theoretically achieve any output voltage. The function of flyback converter is the same as the one of the buck-boost converter, but has relatively different circuit.

The advantage of using dc-dc converter is two folds: First is the efficiency. In ideal condition, all the elements in the circuit, such as inductor, capacitor, switch and diode, don't consume any energy. So the ideal efficiency is 100%. In practice, the efficiency of dc/dc converter can exceed 90%, which is very high for power transformation.

The other advantage is the ease to control and integrate. The voltage transformation in dc/dc converter is achieved by changing the duty cycle of the pulse train, which is called PWM (pulse width modulation). The power needed for pulse train is very small and can be neglected. Moreover the pulse train can be easily constructed.

2.2 Basic working principle of dc/dc boost converter

Because of the existence of the switches, the boost converter works in two modes. When switch closed, the inductor stores energy and the capacitor releases energy. When switch open, the inductor releases energy and the capacitor stores energy.

But it is still not clear why boost converter can step up the voltage. Here I just give a simple explanation. As can been seen from circuit of dc/dc boost converter in figure 2.1, since the inductor, capacitor, switch and diode does not consume energy at ideal condition, there must exist two fundamental conservation laws between the output and the input.
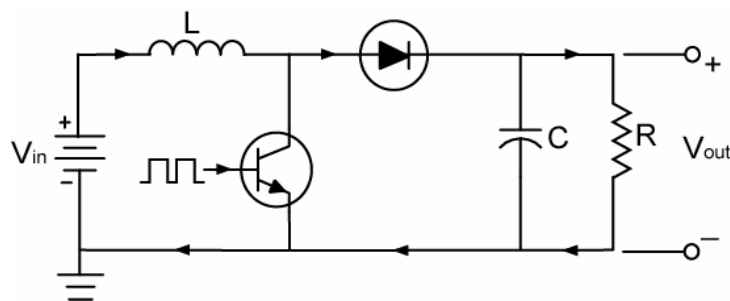

Figure 2.1 Simple dc/dc boost converter circuit

The first law involves the energy balance (2.1), which requires that the input energy equals the output energy:

$$P_{in} = P_{out} \quad \Rightarrow \quad I_{in}V_{in} = I_{out}V_{out} \tag{2.1}$$

The second law is the charge balance (2.2), which means the input charge equal to output charge. Because of the switch, the input current can only provide charge to output side when switch is open, and the time is (1-d)T in one T-period.

$$Q_{in} = Q_{out} \quad \Rightarrow \quad I_{in}(1-d)T = I_{out}T \tag{2.2}$$

Using the two equations we can derive the basic relationship between the input voltage and output voltage.

$$V_{out} = \frac{V_{in}}{1\text{-}d} \tag{2.3}$$

Where d is the duty cycle, which is a positive number less than 1. From the relationship we can see clearly that $V_{out} > V_{in}$.

2.3 Averaging method

Averaging method is a very common method used to analyze and design controller for power electronics circuit.

In theory, the averaging method applies to a system of the form[2]

$$\dot{x} = \varepsilon f(t, x, \varepsilon) \tag{2.4}$$

Where ε is a small positive parameter and $f(t, x, \varepsilon)$ is T-periodic in time: That is.

$$f(t+T, x, \varepsilon) = f(t, x, \varepsilon) \quad , \quad \forall (t, x, \varepsilon) \in [0, \infty) \times D \times [0, \varepsilon_0] \tag{2.5}$$

for some domain $D \subset R^n$. The method approximates the solution of the system by the solution of an "averaged system," obtained by averaging $f(t, x, \varepsilon)$ at ε = 0.

The averaged system can be obtained as an autonomous system

$$\dot{x} = \varepsilon f_{av}(x) \tag{2.6}$$

where

$$f_{av}(x) = \frac{1}{T} \int_0^T f(\tau, x, 0) d\tau \tag{2.7}$$

The idea of averaging method is to use autonomous system behavior approximate the more complicated nonautonomous system. From the system equation we can see, when ε is small, the solution will very "slowly" with relative to the periodic fluctuation of a(t, ε). In other word, if the response of a system is much slower than the excitation, such response will be determined predominately by the average of the excitation. In standpoint of frequency response, if we treat the system as a low pass filter, and the high frequency

excitation is relative small, then it is reasonable to use the first-order approximation to replace the original system.

In a dc/dc converter circuit, the system actually works in two modes: switch open mode and switch close mode. The system is a nonautonomous system, because the system differential equations are discontinous. So if the circuit parameters satisfy some conditions similar as we state above, we can use averaging method to approximate the system behavior. As can be seen in figure 2.1, there are two importance factors in a dc/dc converter that decide whether we can use averaging method. The first is the ripple limitation δ we used to design inductor and capacitor. Here δ is similar like ε. The smaller the ripple limitation δ is, the slower the system response. The other is the switch frequency f $(1/T)$. If switch frequency is very fast and we treat the switch action as excitation, then the response of a dc/dc converter is much slower than switching, and the average response will predominately describe the system behavior.
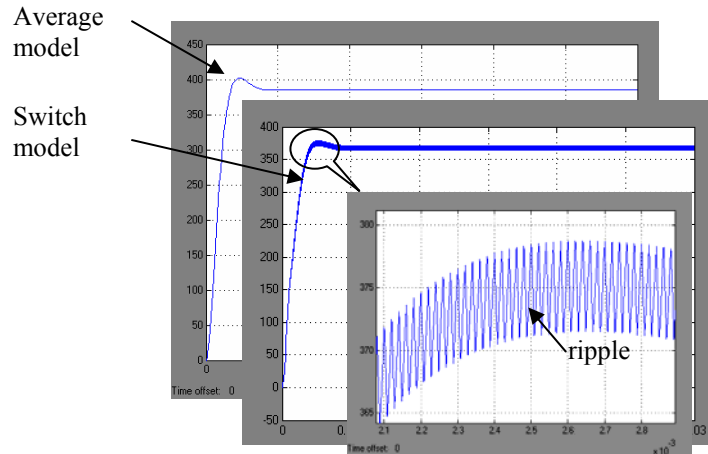

Figure 2.2 Conditions for using averaging method

In summary, if the voltage and current ripples are small enough and the switching frequency is fast enough by choosing proper circuit parameters, then we can use averaging method to obtain an autonomous system.

Because a dc/dc converter only works in two modes, we can simplify the averaging method as following:

$$f_{av}(x) = \frac{1}{T}\int_0^T f(\tau, x, 0)d\tau = d \times f_1(\tau, x, 0) + (1-d) \times f_2(\tau, x, 0) \qquad (2.8)$$

where $f_{1,2}(t, x, \varepsilon)$ refers to state equation of two different modes accordingly, d is the duty cycle of the pulse train.

In state space description, if the state equations of two modes are described as following:

$$\dot{x} = A_1 x + B_1 u \qquad \text{(Switch closed mode)}$$
$$\dot{x} = A_2 x + B_2 u \qquad \text{(Switch open mode)}$$

<div align="right">(2.9)</div>

Then the average state space model is:

$$\dot{x} = \overline{A}x + \overline{B}u \tag{2.10}$$

where $\overline{A} = A_1 d + A_2(1-d) \quad \overline{B} = B_1 d + B_2(1-d)$

A simple proof[3] is given below. Assume the inductor current and capacitor voltage is changing linearly in each working mode. Then in one T period, there exist:

$$
\begin{aligned}
x(dT) &\square\; x(0) + \dot{x} \times dT = x(0) + (A_1 x + B_1 u) \times dT \\
x(T) &\square\; x(dT) + \dot{x} \times (1-d)T = x(dT) + (A_2 x + B_2 u) \times (1-d)T \\
&\square\; x(0) + (A_1 x + B_1 u) \times dT + (A_2 x + B_2 u) \times (1-d)T \\
&\square\; x(0) + [(A_1 d + A_2(1-d))x + (B_1 d + B_2(1-d))u]T \\
\Rightarrow \dot{x} &= \frac{x(T) - x(0)}{T} = (A_1 d + A_2(1-d))x + (B_1 d + B_2(1-d))u
\end{aligned}
\tag{2.11}
$$

Comparing with $\dot{x} = \overline{A}x + \overline{B}u$, we have $\overline{A} = A_1 d + A_2(1-d) \quad \overline{B} = B_1 d + B_2(1-d)$, which is same as the result (2.10).

2.4 dc/dc boost converter design

The key factors that determine the parameters of the circuit include the limit ripple (δ) of I_{in} (I_{fc}) and V_{out}(V_c), the switching frequency and the power. In section 2.4 we have eliminated that the ripple limit and the switching frequency can guarantee the averaged "low" response dominated the whole system behavior. Thus we can get the desired system behavior and use averaging method to analyze the system. The power includes the current and voltage, which determine the size of the circuit components, such as capacitor and inductor.

2.4.1 Inductor design

Let us first look at the inductor voltage and current in steady states as shown in figure 2.3. The inductor current will increase when the switch is closed. During this stage, and the inductor voltage equals to V_{in}. When the switch is open, the inductor current will decrease, and the voltage of inductor will equal to V_{out}-V_{in}.
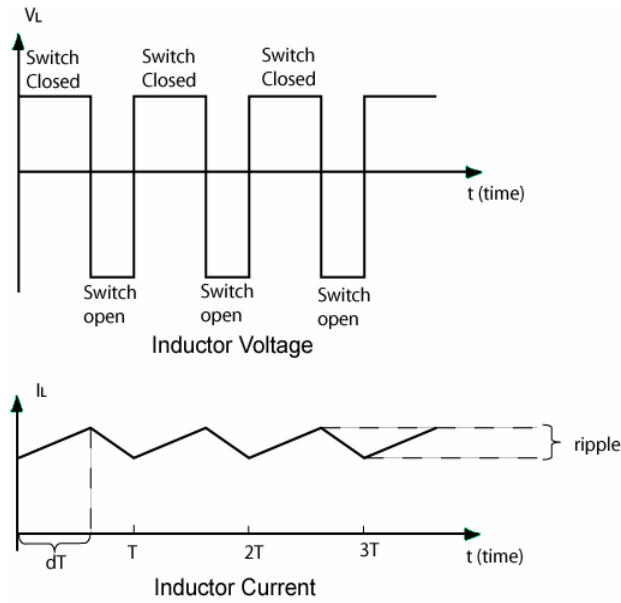
Figure 2.3 The current and voltage changing of inductor

When switch is closed we have

$$L\frac{di_L}{dt} = V_{in} \tag{2.12}$$

To satisfy the ripple limitation we have

$$
\begin{aligned}
&di_L \le I_L \delta \\
&dt = dT \\
&\Rightarrow L \ge \frac{V_{in} dT}{I_{in} \delta}
\end{aligned}
\tag{2.13}
$$

Here we assume the switch is fast enough and the change of inductor current is linear.

2.4.2 Capacitor design

Similar like inductor design, we should look at the capacitor voltage. As can be seen in figure 2.4, when switch is closed, the capacitor voltage decrease, and the capacitor current equals to output current.
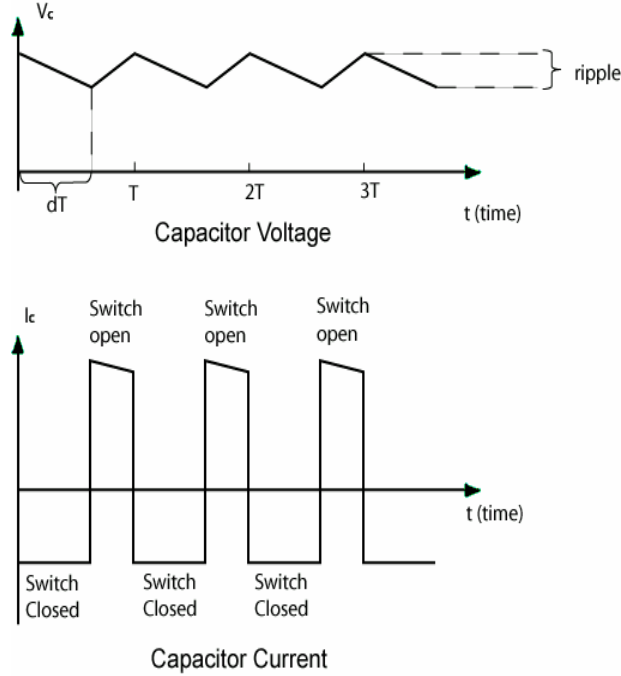
Figure 2.3 The current and voltage changing of capacitor

When switch is closed, the output current is provided by capacitor alone. Assume the voltage changing of the capacitor is constant (In fact, the voltage changing should be decreasing exponentially. But if the switching frequency is fast enough, we can treat the voltage changing as contant), we have

$$C\frac{dv_C}{dt} = -I_{out} \tag{2.14}$$

To satisfy the ripple limitation we have

$$dv_C \le V_{out}\delta$$
$$dt = dT \tag{2.15}$$
$$\Rightarrow C \ge \frac{dTI_{out}}{V_{in}\delta} = \frac{d(1-d)^2 TI_{in}}{V_{in}\delta}$$

where $V_{out} = V_{in}/(1-d)$, $I_{out} = (1-d)I_{in}$

2.4.3 System parameters identification

In our application, the boost converter will connected to FC stacks and provide power to dc traction motor as in figure. So we should first identify the system parameters which will be used for modeling and controller design in the rest parts of the report.
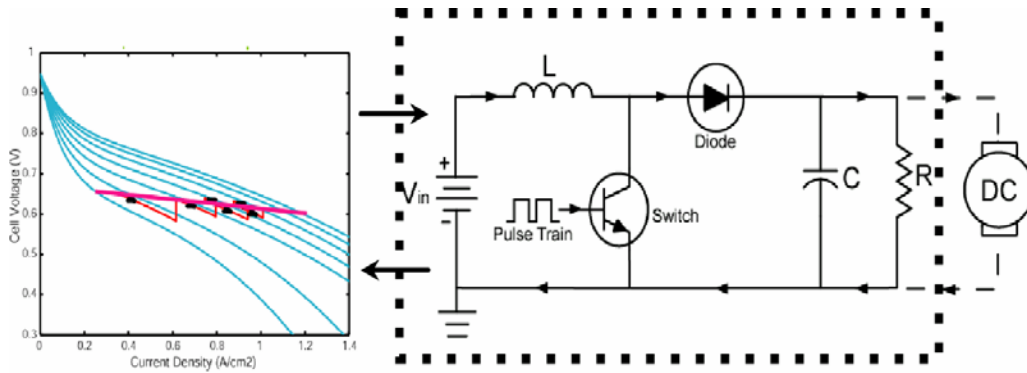
Figure 2.5 Diagram of the FCV

The data of fuel cell stacks are all referred from Jay's paper[4]. From the simulation of the fuel cell stacks model, I read out several data of output voltage and current. On average, output voltage of FC stacks is about 220~250V, and the FC stack current is about 100~300A. The output power from FC stacks range from 20kW to 50kW.

Then I choose Vin= 250V, Iin = 200A as nominal working condition, and the input power is about 50kW.

There are many types of dc traction motor. Each may has different nominal working voltage. Here I just select the motor's nominal working voltage as V_{out}=400V.

The ripple limit here is set to bee 1%. And the switching frequency is 50kHz, which is reasonable for dc/dc boost converter.

Now we can calculate the circuit parameters:

$$V_{out} = \frac{V_{in}}{1-d} \Rightarrow d = 1 - \frac{250}{400} = 0.375$$

$$L \geq \frac{V_{in}dT}{I_{in}\delta} = \frac{250 \times 0.375 \times 2e-5}{200 \times 0.01} = 9.375e-4H$$

$$C \geq \frac{I_{in}d(1-d)^2 T}{V_{in}\delta} = \frac{200 \times 0.375 \times (1-0.375)^2 \times 2e-5}{250 \times 0.01} = 1.172e-4F$$

$$I_{out} = \frac{V_{in}I_{in}}{V_{out}} = 125A$$

$$R_{load} = \frac{V_{out}}{I_{out}} = 3.2\Omega$$

2.5 Modeling and verifications

Using the averaging method, we will derive the average model of the dc/dc boost converter in this section. By comparing the step response with the switch model we can

verify our average model. The switch model is constructed by using power system blocks provide by Matlab.

2.5.1 A simple dc/dc boost converter model

The simple model is based on the circuit shown in fig 2.6 & 2.7. We assume that all components are ideal, that is no internal resistance in the circuit and the circuit components don't consume any energy. We follow the averaging method, and derive the following:
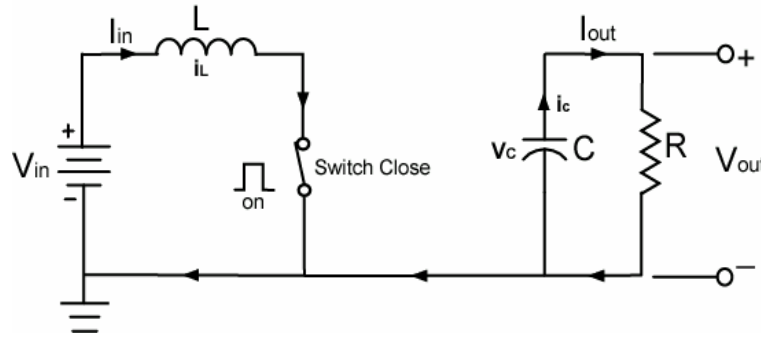
- When the switch is closed, the circuit can be simplified as following:

Figure 2.6 The working mode when switch is closed

The state equations are

$$L\frac{di_L}{dt} = V_{in}$$

$$C\frac{dv_C}{dt} = -\frac{v_C}{R}$$

(2.16)

Where i_L denotes the current of inductor, which equals to I_{in}; v_C denotes the voltage of the capacitor , which equals to V_{out}. Let state variable $x_1 = i_L$ (i_{in}), $x_2 = v_C$ (v_{out}), we can rewrite the state equations in state space

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_{in} \qquad \text{that is: } \dot{x} = A_1 x + B_1 u \qquad (2.17)$$

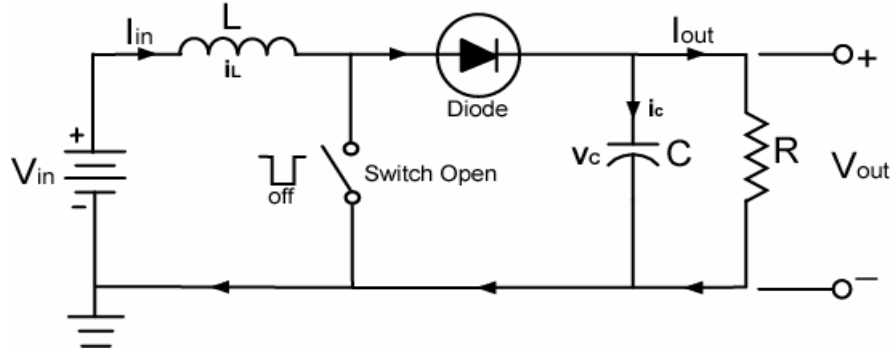- When the switch is open, the simplified circuit is shown below.

Figure 2.7 The working mode when switch is open

The state equations become

$$L\frac{di_L}{dt} = V_{in} - v_C$$

$$C\frac{dv_C}{dt} = i_L - \frac{v_C}{R}$$

(2.18)

In state space, the equations are

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{-1}{L} \\ \dfrac{1}{C} & \dfrac{-1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} V_{in} \qquad \text{that is: } \dot{x} = A_2 x + B_2 u$$

(2.19)

Averaging the state space matrix of two different working modes using (2.10), we get the average model.

$$\dot{x} = \begin{bmatrix} 0 & \dfrac{-(1-d)}{L} \\ \dfrac{(1-d)}{C} & \dfrac{-1}{RC} \end{bmatrix} x + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} V_{in}$$

(2.20)

where $\bar{A} = \begin{bmatrix} 0 & \dfrac{-(1-d)}{L} \\ \dfrac{(1-d)}{C} & \dfrac{-1}{RC} \end{bmatrix}$ $\bar{B} = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix}$

In matlab, we simulate the open loop response by comparing with the switching model. Using the circuit parameter we designed in section 2.4, we initialize the system by setting the duty cycle first at 37.5%, then step up the duty cycle from 37.5% to 50% at 0.01 sec.
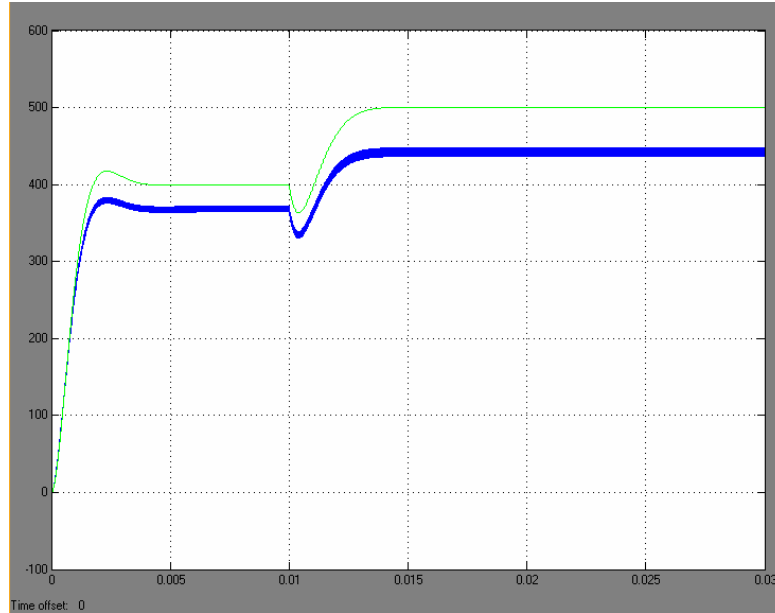
Figure 2.8 Verification of average model for simple boost converter

From the simulation result we can see the response of average model have basically the same transient response. But there is a large steady state error. The reason is that in the circuit there exist a diode drop (a constant voltage about 0.6~0.8V) and some resistance in the components of circuit, such as inductor, capcitor, that we have ignored.

2.5.2 a more accurate model

To achieve more accurate average model for boost converter, I revised the state equations are revised for the two working modes. Here we assume that the resistance in the circuit can be equivalent to the resistance of inductor (R_L). We use V_d to denote the diode drop.

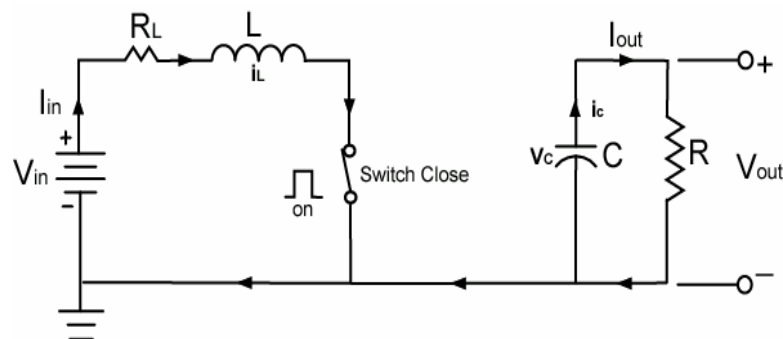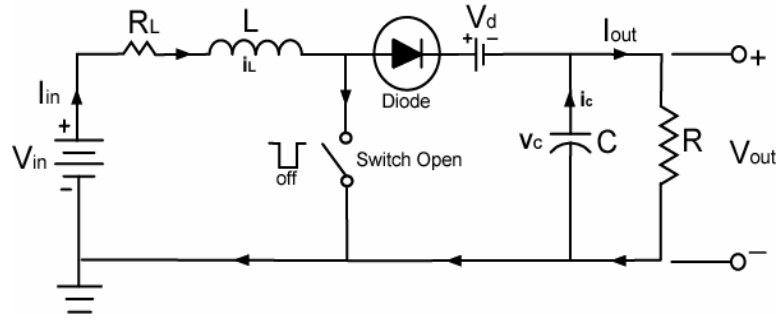- When switch is closed, the circuit is revised as:

Figure 2.9 Switch close mode

The state space equations become

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{R_L}{L} & 0 \\ 0 & \dfrac{-1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} V_{in} \qquad\qquad \dot{x} = A_1 x + B_1 u \qquad (2.21)$$

When switch open, the circuit is revised as:

Figure2.10 Switch open mode

The state space equations become

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{R_L}{L} & \dfrac{-1}{L} \\ \dfrac{1}{C} & \dfrac{-1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} V_{in} + \begin{bmatrix} -\dfrac{1}{L} \\ 0 \end{bmatrix} V_d \qquad\qquad \dot{x} = A_2 x + B_2 u \quad (2.22)$$

Using averaging method we can get the new state space equations

$$\begin{aligned} \overline{A} &= A_1 d + A_2 (1-d) \\ \overline{B} &= B_1 d + B_2 (1-d) \end{aligned} \Longrightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{R_L}{L} & \dfrac{-(1-d)}{L} \\ \dfrac{(1-d)}{C} & \dfrac{-1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} V_{in} + \begin{bmatrix} -\dfrac{(1-d)}{L} \\ 0 \end{bmatrix} V_d \,(2.23)$$

Similarly we simulate the open loop response of both average model and switching model in figure 2.11. The simulations conditions are:

•input voltage change from 250V to 200V at 0.01 sec

•duty cycle change from 37.5% to 50% at 0.02 sec
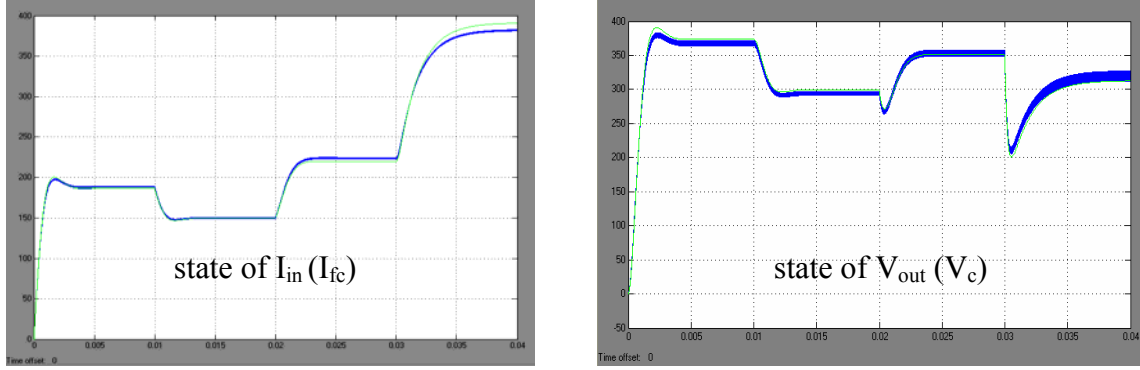
•load resistant change from 3.2 　to 1.6 　at 0.03 sec

Figure 2.11 Verification of revised average model for boost converter

The result shows that the revised average model (consider V_d and R_L) gives excellent approximation of switch model. In the rest part of the report, we would use this model to design and verify the controller performance.

3. Control Design of dc/dc Boost Converter

3.1 Linearization and control problem

As we see the average model is still nonlinear model, because we don't use V_{in} as input, instead we use duty cycle as input. By observing the average model, we can easily find that duty cycle d is just inside the state space matrix. However, to design the controller for boost converter using linear controller design method, we should first linearize the average model at some critical working conditions and get the linear model of the boost converter.

$$\begin{bmatrix} \Delta \dot{x}_1 \\ \Delta \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{R_L}{L} & \dfrac{-(1-d)}{L} \\ \dfrac{(1-d)}{C} & \dfrac{-1}{RC} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} V_{in} + \begin{bmatrix} \dfrac{V_{ss}+V_d}{L} \\ -\dfrac{I_{ss}}{C} \end{bmatrix} \Delta d + \begin{bmatrix} 0 \\ \dfrac{1}{R^2 C} \end{bmatrix} \Delta R \qquad (3.1)$$

Where V_{ss} and I_{ss} denote the steady states of $V_{out}(V_c)$ and $I_{in}(I_{fc})$. R, C, L each denotes the parameters we designed for boost converter. d is the duty cycle in steady state. The output is just one of the states, that is output voltage.

From the transfer function of P_{vd} (3.2) we can see, the boost converter is a NMP system from duty cycle to output voltage. As can been seen in figure 3.1, the linearized system has an ORHP zero at 2009. So there is big undershoot when we step up the duty cycle, which is shown in figure 2.11. When the duty cycle step up from 37.5% to 50%, the undershoot of V_{out} is nearly 50v, about 1/8 of the nominal output voltage.

$$P_{vd} = \frac{582(s/2009 - 1)}{2.715e-7s^2 + 7.369e-4s + 1} \qquad (3.2)$$

- 14 -

Pole-zero map of Pvd
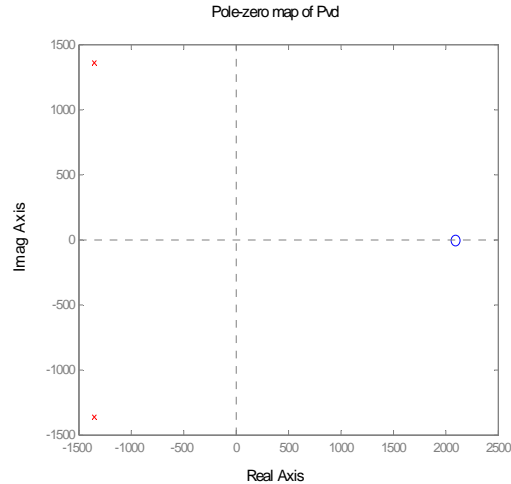
Imag Axis

Real Axis

Figure 3.1 pole zero map of linearized system

For such a NMP system, we cannot use high gain control, because high gain will cause the close loop system to become unstable. On the other hand, to control a NMP system there always exist a trade-off between the transient response and the level of undershoot, because the Bode sensitivity integral imposes such design limitation[5]. So design controller we must compromise between the transient time and the level of the undershoot. Since the open-loop transient response time is about 2~3 ms, which is much smaller than FC system (second level), the transient time is less important than undershoot.

At the beginning of the report, the design goal is stated. The control goal actually address on two aspects: one is tracking the desired the output voltage by control the duty cycle, the other is reject the input source voltage variation and load disturbance. Since we cannot use high gain control, the integral control is necessary.

In the next two sections, I will concentrate on two control algorisms: PI control and LQI control.

3.2 PI controller design

Using Ziegler-Nichols tuning method[6], I designed the PI controller. The final controller I designed is C(s) =0.0005+0.05/s
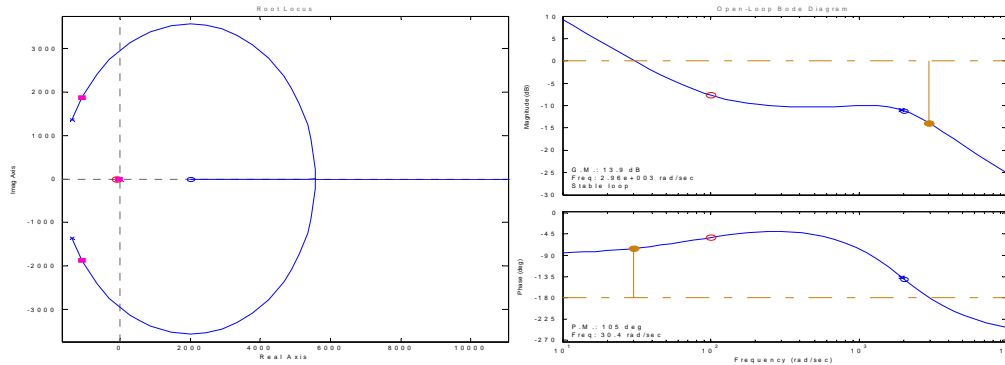
Figure 3.2 PI controller design using SISO tool

As can be seen from the bode plot and root locus plot of the close loop system. The phase margin is about 105 deg and the gain margin is about 14 dB. The cross frequency is about 30 Hz.

The close loop system is simulated with PI controller both in linear model and nonlinear model. The simulation parameters are: L=9.375e-4H, C=1.172e-4F, R=3.2Ω, Vd=0.8V, Rd=0.22Ω, Vin=250V, Vref=450V(step at 0.3sec)
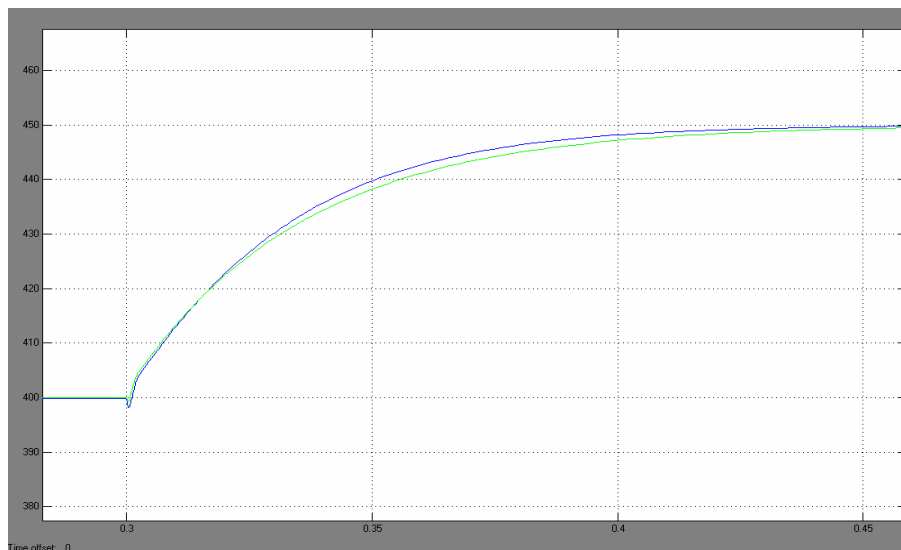

Figure 3.3 Simulation of PI controller

As can be seen from the close loop response, the undershoot is decreased from 50V to about 2V and the transient time is increased from 2~3ms to 100ms, which shows the tradeoff between the transient response and undershoot.

The simulation result shows that the PI controller can get zero tracking error, and is proper for voltage control. Then I simulate the close loop response under source (fuel cell

stack) voltage variance and load disturbance to check if the PI controller can also reject the two disturbances.

I first simulate the load disturbance and assume that input voltage is constant. The simulation result shows that the load disturbance will cause big oscillation in output voltage.
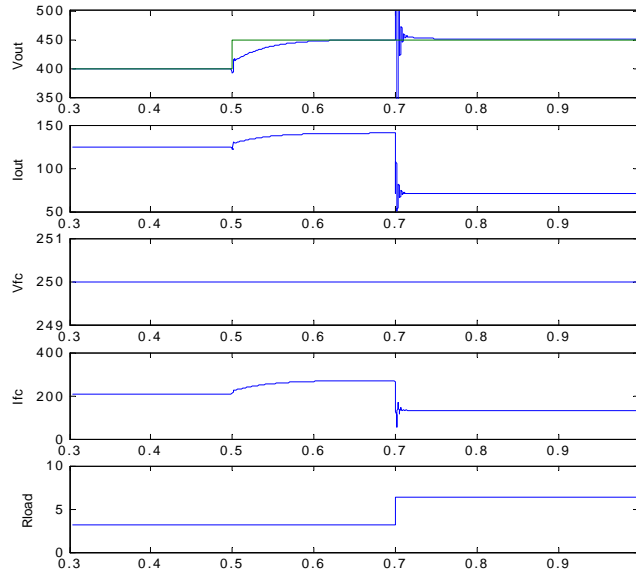

Figure 3.4 Simulation result by introducing step up in the load resistance

Finally I simulate the close loop response under both fuel cell stacks disturbance and load disturbance. As we know, the fuel cell stacks output voltage is sensitive to output current, which can be seen from the polarization curve. So the disturbance from the fuel cell stacks can be modeled as polarization curve. For simplicity, I model the polarization curve as a linear function. Furthermore, I assume the output voltage of the fuel cell stacks is well controlled. By using the data from Jay's paper[4], I use linear regression method to approximate the characteristic curve.
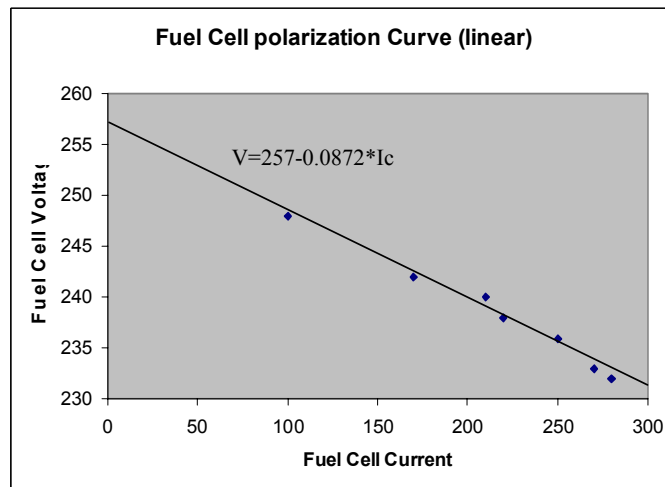

Figure 3.5 Approximation of fuel cell polarization curve
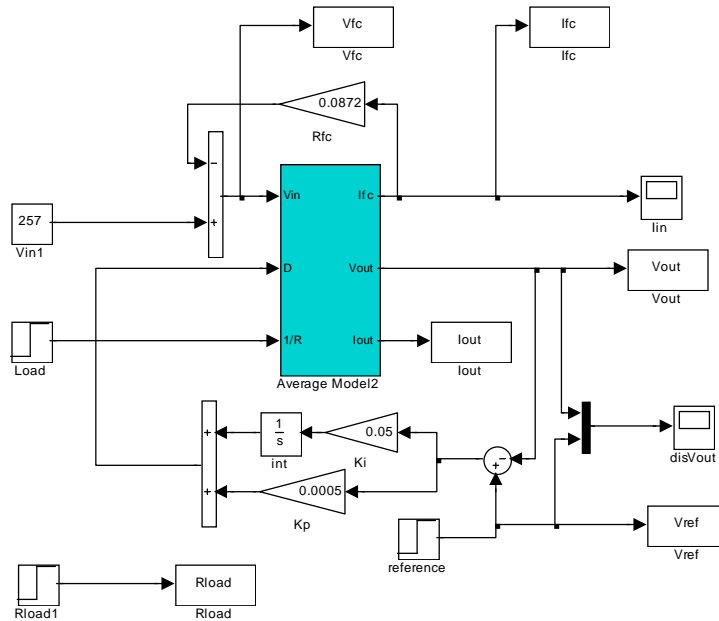
The simulink model is shown below:

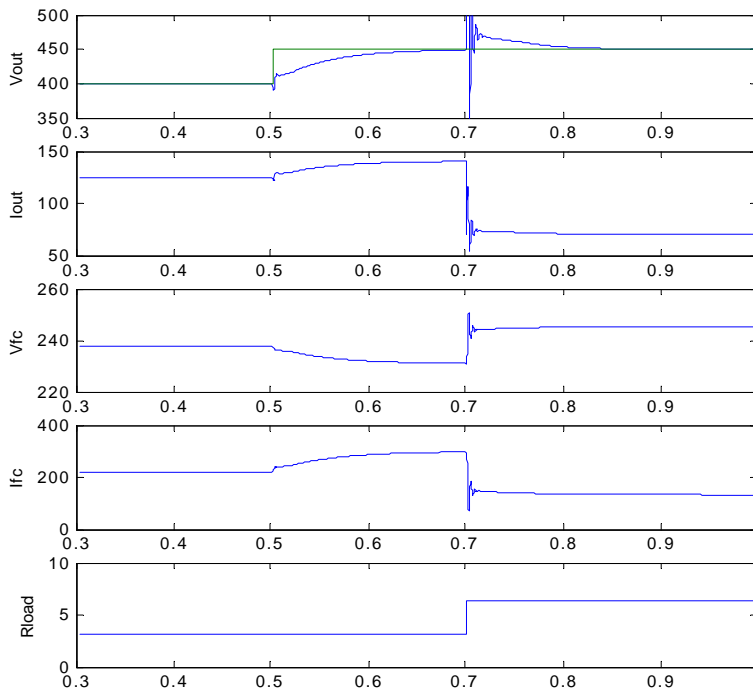**figure3.6 simulink model of average model with PI controller**

The simulation result shows that the fuel cell stacks variation has little affects to the output voltage. The big problem is the load disturbance. This is because I have assumed a static fuel cell behavior (polarization curve).

3.3 LQI controller design

LQI controller design is a MIMO controller design technique, which can directly penalize the states and control input. To design LQI controller we should first augment the system by adding new state variable q.
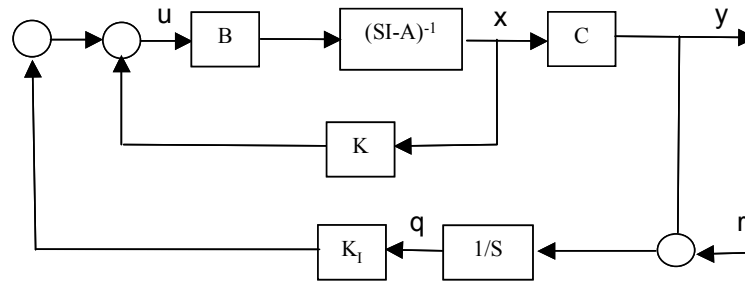

figure 3.8 augmented system for LQI design

The augmented system equation become

$$
A' = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} = \begin{bmatrix} -\dfrac{R_L}{L} & \dfrac{-(1-d)}{L} & 0 \\ \dfrac{(1-d)}{C} & \dfrac{-1}{RC} & 0 \\ 0 & -1 & 0 \end{bmatrix} \qquad B' = \begin{bmatrix} B \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{V_{ss}+V_d}{L} \\ -\dfrac{I_{ss}}{C} \\ 0 \end{bmatrix} \qquad (3.3)
$$

$$
C' = \begin{bmatrix} C & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}
$$

Where u is the duty cycle, y is the output voltage.

By choosing different weighting matrix Q and R in the linear quadratic equation $J = \int x'^T Q x' + u^T R u$, we can get the optimized control.

By comparing the time response of the simulation result with different Q and R, I found the most important factor is the integral error q. Here I just compare with two responses results of different weighting matrix. As can be seen from the simulation result, when I penalize more on the integral error, the transient time become fast.

In simulation, we assume the input voltage is constant. And the other conditions are: Vref step up at 0.1 sec from 400 to 420v, Rload step up from 3.2Ω to 6.4Ω

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1{,}000{,}000 \end{bmatrix} \qquad R = 1$$

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10{,}000 \end{bmatrix} \qquad R = 1$$
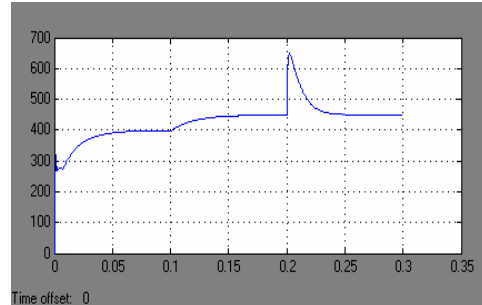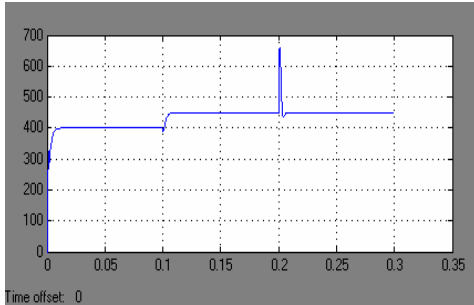

figure 3.9 close loop response of LQI controller

4. Conclusion

In this report we analyze the working principal of the dc/dc boost converter. Using averaging method we derived the continuous average model, and verified in matlab by comparing the open loop time response with switch model. PI controller and LQI controller both are proper for the voltage tacking control. Input source disturbance has little effect on the output voltage. However the load disturbance will have great effect. To decrease the effect we should use feed forward control.

[1] Michael R. von Spakovsky, Douglas J. Nelson, Michael W. Ellis, Benoit Olsommer, Michael J. Ogburn., "A Multi- / Inter-Disciplinary Approach To Fuel Cell System Development: The U.S. DoE GATE Center For Automotive Fuel Cell Systems At Virginia Tech", 2000 Future Car Congress - Paper 00FCC-10

[2] Hassan K. Khalil. Nonlinear Systems (2nd edition Page 330). Prentice Hall 1996.

[3] Philip T. Krein. Elements of Power Electronics (Page 623). Oxford University Press 1998.

[4] Jay T. Pukrushpan, Anna G. Stefanopoulou, Huei Peng. Modeling and Control of PEM Fuel Cell Stack System. Automotive Research Center, Department of Mechanical Engineering, University of Michigan.

[5] Jim Freudenberg. A First Graduate Course in Feedback control (EECS565 course pack Section 6.2.3).

[6] Gene F. Franklin, J. David Powell, Abbas Emami-Naeini. Feedback Control of Dynamic Systems (3rd edition, Page 191). Addison-Wesley Publishing Company 1995.