



MINKOWSKI FRACTAL GEOMETRY: AN ATTRACTIVE CHOICE OF COMPACT ANTENNA AND FILTER DESIGNS

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ABSTRACT

Various fractal geometries have been successfully applied to design compact bandpass filters and miniaturized multiband antennas for different multi-services wireless applications. In this paper, a thorough investigation of the space-filling characteristics of the classical Minkowski fractal geometry has been presented. Many variants of this geometry can be derived by using generators by varying the standard one-third ratio adopted in the traditional fractal geometry. The use of multi-ratio will result in a large number of variants with fractal dimensions that are larger or smaller than 1.465 of the normal Minkowski fractal geometry. The higher the fractal dimension, the better the fractal curve fills the given area, therefore achieving higher compactness. These variants will provide the antenna and filter designer with many choices to implement his structures.

Keywords: Minkowski fractal geometry, fractal-based antenna, compact BPF, fractal geometry.

1. INTRODUCTION

Among the earliest predictions of the use of fractals in the design and fabrication of filters is that of Yordanov *et al.*, [1]. His predictions were based on their investigation of Cantor fractal geometry. Fractal curves are well known for their unique space-filling properties. Research results showed that, due to the increase of the overall length of the microstrip line on a given substrate area as well as to the specific line geometry, using fractal curves reduces the resonant frequency of microstrip resonators, and gives narrow resonant peaks [2-4].

The various fractal geometries have been applied to produce bandpass filters for a wide variety of applications [5-17]. In this context, miniaturized Sierpinski fractal-based bandpass filters have been presented in [5-7]. Also, Hilbert fractal based resonators have been suggested in [8-13] to design many compact bandpass filters. Peano fractal geometry as applied to the conventional square open-ring resonators has led to producing miniaturized bandpass filters (BPFs) and BSFs, with high performance [14-16]. Furthermore, Moore, Koch, and other fractal geometries have also found their ways to construct compact microstrip BPFs and BSFs [17-18].

On the other hand, the Minkowski fractal geometry and its variants were utilized in the design of small size planar single-band, dual-band, and multiband BPFs as reported in the literature [19-33]. In this respect, many compact size BPFs, with single-band resonant responses, have been successfully designed based on Minkowski fractal geometry [19-25]. Most recently, a small size dual-mode BPF based on the second iteration Minkowski variant has been presented in [24]. The filter has achieved a circuit area reduction of 97.5% as compared with the conventional square dual-mode loop BPF.

Besides, dual-band BPFs with small size have been reported in [26-28]. Furthermore, triple-band and quad-band bandpass filters have been recently proposed based on this geometry [29-30]. More interesting, among

the previously stated fractal geometries, only the Minkowski fractal variants have been employed to design compact bandstop filters [31-34].

Away from the application of fractal geometries in the filter design, different fractal geometries have been used in the design of compact multiband antennas for various communication applications. Fractal geometries, such as Moore, Koch, Peano, and many others have been successfully applied to perform this task [35-41].

In this paper, a thorough investigation is presented to explore the capability of the modified (multi-scale) Minkowski fractal geometry to produce a wide range of resonator variants in antenna and filter design.

2. THE MINKOWSKI FRACTAL GEOMETRY

Figure-1 shows the steps of growth of the standard Minkowski fractal curve up to the 3rd iteration. It is clear that the generator, depicted in Figure-1(b), is composed of five segments; each has the ratio of one-third the initiator as in Figure-1(a). The one-third ratio is common in generating the most well-known fractal curves such as Koch and Cantor geometries [42]. The dimension D can be determined as a logarithmic ratio between the number of self-similar segments obtained from one portion after each iteration k and the number of parts derived from one segment in each iteration r

$$D = \frac{\log k}{\log r} \quad (1)$$

According to Equation (1), the corresponding fractal dimension is 1.465. For comparison purposes, Table-2.1 shows the fractal dimensions of some fractal curves that are widely adopted in the design of microwave antennas and circuits.

Table-1 reveals that the standard Minkowski fractal curve has a fractal dimension larger than that of the Koch fractal curve and lower than that of the Sierpinski triangle. As it has been mentioned before, the fractal



dimension of a fractal curve is an indication of achieving better space-filling of that fractal. On the other hand, not all fractal curves can be used in the filter design applications. Some of the fractal geometries have been successfully employed to the design different antennas for many. It is not conditional that the same geometries could play the same role in the filter design with the same degrees of success. This conclusion is mainly attributed to the input/output coupling requirements in the filter design which is entirely different from the radiation problem in the antenna design.

For this, to enhance the input/output coupling, and to obtain a practical range of the fractal dimension of the standard Minkowski fractal curve, a modified variant is introduced.

Table-1. The fractal dimensions of some fractal geometries.

Fractal curve type	Fractal dimensions
Koch Curve	1.2618
Sierpinski Triangle	1.5848
Sierpinski Carpet	1.8928
Koch Snowflake	1.2618
Cantor Set	0.631
Minkowski Curve	1.465

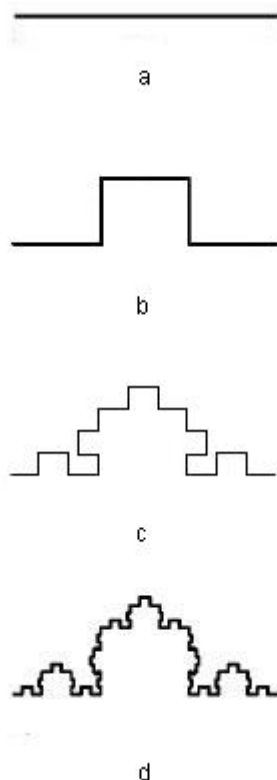


Figure-1. Steps of growth of the conventional Minkowski fractal curve up to the 3rd iteration.

3. THE MODIFIED MINKOWSKI FRACTAL GEOMETRY

The modified version of the Minkowski fractal curve is shown in Figure-2. This figure demonstrates the generation of the modified version up to the 2nd iteration as applied to a square ring. In this version, the typical 1/3 ratio which is the most popular in the generation of the majority of various fractal curves has been replaced by an arbitrary ratio. The generator, shown in Figure-2(a), is composed of five segments. The middle segment has the length w_1 , the second and the third segments have the length w_2 while the first and the fifth segments have the length $L_0(1-w_1)/2$.

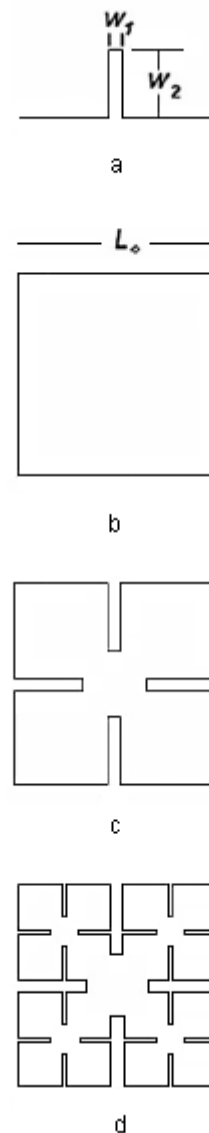


Figure-2. The steps of growth of the modified Minkowski fractal structure: (a) the generator, (b) the square ring, (c) the 1st iteration, and (d) the 2nd iteration.

Consequently, the shape modification of the structure depicted in Figure-2 (c) and (d) are ways to increase the surface current path length compared with that of the conventional square ring resonator. This



increase will result in a reduced resonant frequency or a reduced resonator size if the design frequency is to be maintained. For the nth iteration, the modified Minkowski fractal structures, depicted in Figure-2, have been found to have the perimeters given by:

$$P_n = \left(1 + 2 \frac{w_2}{L_0}\right) P_{n-1} \quad (2)$$

where P_n is the perimeter of the nth iteration fractal structure, w_2 , and L_0 are as depicted in Figure-2. Equation (2) and Figure-2 imply that at particular iteration level, a wide variety of structures with different perimeters can be obtained by varying w_1 , w_2 , or both.

According to Falconer [42], the modified version of the Minkowski fractal geometry is called multi-fractal or fractal geometry with more than one ratio in the generator; a_1 and a_2 . For this case, the fractal dimension, D , can be obtained from the solution of the following equation:

$$2\left(\frac{1}{2}(1-a_1)\right)^D + 2a_2^D + a_1^D = 1 \quad (3)$$

where a_1 and a_2 are the ratios w_1/L_0 and w_2/L_0 respectively. The parameters w_1 , w_2 , and L_0 , are as indicated in Figure-2 (a).

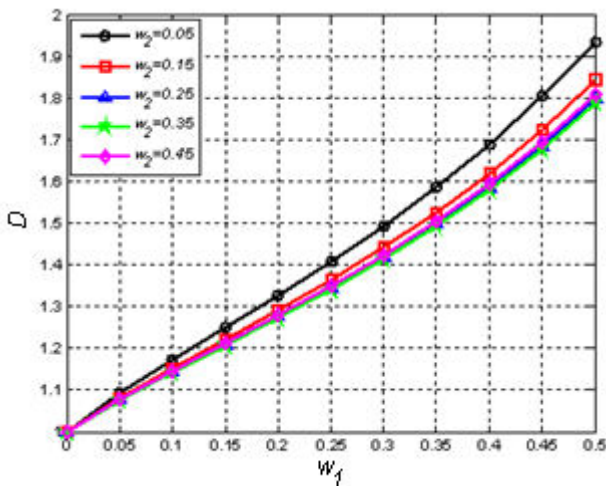


Figure-3. The variation of the fractal dimensions of the modified Minkowski fractal with the parameter w_2 as a parameter.

For demonstration purpose, the effects of different values of w_1 , w_2 on the resulting dimension of the modified Minkowski fractal structure, Equation (3) has been plotted against w_1 , w_2 for as shown in Figures 3 and 4 respectively. The parameter w_1 has been varied from zero to 0.5 in steps of 0.1 of while w_2 has been ranged from 0.05 to 0.45 in steps of 0.1.

Investigating Figures 2, 3 and 4 and Equation (3), it is clear that when w_1 is equal to zero, the fractal dimension will equal to 1, which represents the non-fractal

state. In this case, the resulting structure is not with a fractal shape; it is merely a Euclidean square which has the dimension of 1. Furthermore, when w_1 , w_2 are both equal to one-third, which is the case of the conventional Minkowski fractal curve, the dimension will be 1.465 as depicted in Table-1. Furthermore, as the results of Figure-3 imply the effect of varying the segment w_1 on the resulting the fractal dimension is more different from that of w_2 as shown in Figure-4.

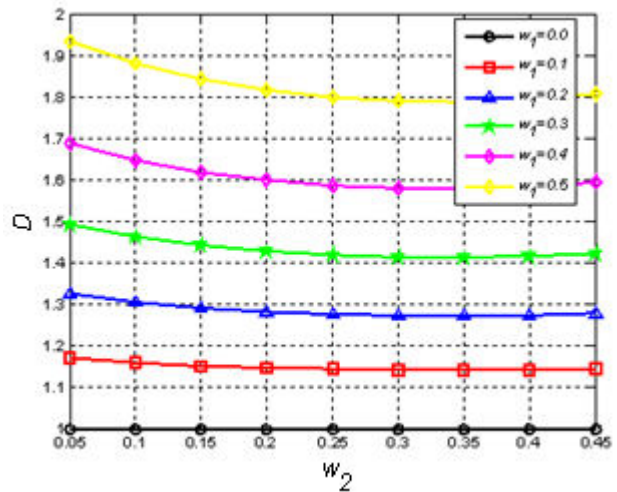


Figure-4. The variation of the fractal dimensions of the modified Minkowski fractal with the parameter w_1 as a parameter.

In Figure-3, the variation of w_1 in the range 0 - 0.05, causes the fractal dimension to change from 1 to about 1.95. The fractal dimension with these values makes the corresponding Minkowski variant better, in space-filling, than many fractal curves depicted in Table-1. While the results of Figure-4 show a different behavior; at any value of w_1 , the fractal dimension varies slowly with w_2 . In summary, theoretically, the variation of both w_1 and w_2 can suppose any values such that the inter-segment structures of the fractal variant have not intersected. In any fractal curve, a point in the space has not to be visited twice [42-44].

4. CONCLUSIONS

The analysis presented in this paper, reveal that the Minkowski fractal has an unlimited number of variants theoretically. The fractal dimensions of the many Minkowski variants are found to be better, in space-filling, than numerous fractal curves depicted in the literature. These variants can be applied to the traditional resonators to produce compact resonators. As a result, small size bandpass filters and antennas can be then designed with suitable sizes meeting the requirements of modern communication applications.



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