Mech 302 Heat Transfer **HW5 Solution**

1. (**Problem 5.5 in the Book except for part (e))**

For each of the following cases, determine an appropriate characteristic length Lc and the corresponding Biot number Bi that is associated with the transient thermal response of the solid object. State whether the lumped capacitance approximation is valid. If the temperature information is not provided, evaluate properties at T = 300 K. a. A toroidal shape of diameter D = 65 mm and cross-sectional area $Ac = 7mm^2$ is of thermal conductivity k = 2.3 W/mK. The surface of the torus is exposed to a coolant corresponding to a convection coefficient of h = 50 W/m2K.

b. A long, hot AISI 302 stainless steel bar of rectangular cross-section has dimensions w = 5 mm, W = 7 mm, and L = 150 mm. The bar is subjected to a coolant that provides a heat transfer coefficient of h = 10 W/m2K at all exposed surfaces.

c. A long extruded aluminum (Alloy 2024) tube of diameter of inner and outer dimensions w = 25 mm and W = 30 mm, respectively, is suddenly submerged in water, resulting in a convection coefficient of h = 40 W/m2K at the four exterior tube surfaces. The tube is plugged at both ends, trapping stagnant air inside the tube.

d. An L = 300-mm-long solid stainless steel rod of diameter D = 13 mm and mass M = 0.328 kg is exposed to a convection coefficient of h = 30 W/m2K.

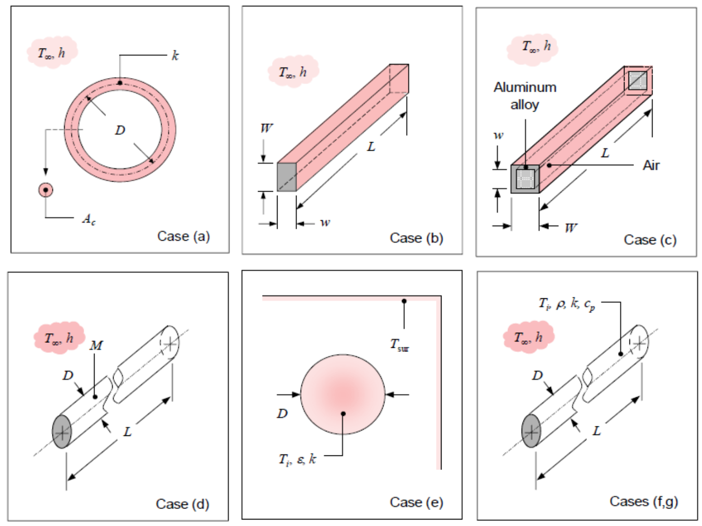
f. A long cylindrical rod of diameter D = 20 mm, density, specific heat cp = 1750 J/kgK, and thermal conductivity k = 16 W/mK is suddenly exposed to convective conditions with. The rod is initially at a uniform temperature of T = 100℃ at t = 225 s.

g. Repeat part (f) but now consider a rod diameter of D = 200 mm.

KNOWN: Geometries of various objects. Material and/or properties. Cases (a) through (d): Convection heat transfer coefficient between object and surrounding fluid. Case (e): Emissivity of sphere, initial temperature, and temperature of surroundings. Cases (f) and (g): Initial temperature, spatially averaged temperature at a later time, and surrounding fluid temperature.

FIND: Characteristic length and Biot number. Validity of lumped capacitance approximation.

SCHEMATIC:



Case (a): D = 65 mm, $Ac = 7mm^2$, k = 2.3 W/m·K, $h = 50$ W/m^2 ·K.
Case (b): W = 7 mm, w = 5 mm, L = 150 mm, h = 10 W/m^2.K, AISI 302 stainless steel.
Case (c): w = 25 mm, W = 30 mm, h = 40 W/m^2·K (L not specified), 2024 aluminum.
Case (d): L = 300 mm, D = 13 mm, M = 0.328 kg, h = 30 W/m^2·K. stainless steel.
Cases (f, g): D = 20 mm or 200 mm, ρ = 2300 kg/m^3, cp = 1750 J/kg·K, k = 16 W/m·K, $T\infty$ = 20°C, Ti =200°C, T = 100°C at t = 225 s.

ASSUMPTIONS: (1) Constant properties

PROPERTIES: *Table A.1*, Stainless steel, AISI 302 (T = 300 K): k = 15.1 W/m·K. Aluminum 2024 (T = 300 K): k = 177 W/m·K.

		Properties at 300 K				k (W/m · K)/c_p (J/kg · K)									
Composition	Melting Point (K)	ρ (kg/m^3)	c_p (J/kg · K)	k (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	100	200	400	600	800	1000	1200	1500	2000	2500
Aluminum															
Pure	933	2702	903	237	97.1	302	237	240	231	218					
Stainless steels															
AISI 302		8055	480	15.1	3.91				17.3 512	20.0 559	22.8 585	25.4 606			
AISI 304	1670	7900	477	14.9	3.95		9.2 272	12.6 402	16.6 515	19.8 557	22.6 582	25.4 611	28.0 640	31.7 682	
AISI 316		8238	468	13.4	3.48				15.2 504	18.3 550	21.3 576	24.2 602			
AISI 347		7978	480	14.2	3.71				15.8 513	18.9 559	21.9 585	24.7 606			

ANALYSIS: Characteristic lengths can be calculated as $Lc1 = V/As$, or they can be taken conservatively as the dimension corresponding to the maximum spatial temperature difference, $Lc2$. The former definition is more convenient for complex geometries. The lumped capacitance approximation is valid for $Bi = hLc/k < 0.1$.

(a) The radius of the torus, ro, can be found from 2
$Ac = \pi r_0^2$. The characteristic lengths are

$$L_{c1} = \frac{V}{A_s} = \frac{A_c \pi D}{2\pi\sqrt{A_c/\pi} \times \pi D} = \frac{1}{2}\sqrt{\frac{A_c}{\pi}} = \frac{1}{2}\sqrt{\frac{7\text{ mm}^2}{\pi}} = 0.75\text{ mm}$$

L_{c2} = Maximum center to surface distance = radius = $\sqrt{A_c/\pi} = \sqrt{7mm^2/\pi} = 1.49mm$

The corresponding Biot numbers are

$$Bi_1 = \frac{hL_{c1}}{k} = \frac{50 \text{ W/m}^2 \cdot \text{K} \times 0.00075 \text{ m}}{2.3 \text{ W/m} \cdot \text{K}} = 0.016$$

$$Bi_2 = \frac{hL_{c2}}{k} = \frac{50 \text{ W/m}^2 \cdot \text{K} \times 0.00149 \text{ m}}{2.3 \text{ W/m} \cdot \text{K}} = 0.032$$

The lumped capacitance approximation is valid according to either definition.

(b) For this complex shape, we will calculate only $Lc1$.

$$L_{c1} = \frac{V}{A_s} = \frac{WwL}{2(W+w)L + 2Ww} = \frac{7 \text{ mm} \times 5 \text{ mm} \times 150 \text{ mm}}{2(7 \text{ mm} + 5 \text{ mm}) \times 150 \text{ mm} + 14 \text{ mm} \times 5 \text{ mm}} = 1.43 \text{ mm}$$

Notice that the surface area of the ends has been included, and does have a small effect on the result – 1.43 mm versus 1.46 mm if the ends are neglected. The corresponding Biot number is

$$Bi_1 = \frac{hL_{c1}}{k} = \frac{10 \text{ W/m}^2 \cdot \text{K} \times 0.00143 \text{ m}}{15.1 \text{ W/m} \cdot \text{K}} = 0.00095$$

The lumped capacitance approximation is valid.

Furthermore, since the Biot number is very small, the lumped capacitance approximation would certainly still be valid using a more conservative length estimate.

(c) Again, we will only calculate $Lc1$. There will be very little heat transfer to the stagnant air inside the tube, therefore in determining the surface area for convection heat transfer, A_s only the outer surface area should be included. Thus,

$$L_{c1} = \frac{V}{A_s} = \frac{(W^2 - w^2)L}{4WL} = \frac{(30 \text{ mm})^2 - (25 \text{ mm})^2}{4 \times 30 \text{ mm}} = 2.29 \text{ mm}$$

The corresponding Biot number is

$$Bi_1 = \frac{hL_{c1}}{k} = \frac{40 \text{ W/m}^2 \cdot \text{K} \times 0.00229 \text{ m}}{177 \text{ W/m} \cdot \text{K}} = 5.18 \times 10^{-4}$$

The lumped capacitance approximation is valid.

Furthermore, since the Biot number is very small, the lumped capacitance approximation would certainly still be valid using a more conservative length estimate.

(d) We are not told which type of stainless steel this is, but we are told its mass, from which we can find its density:

$$\rho = \frac{M}{V} = \frac{M}{\pi D^2 L / 4} = \frac{0.328 \text{ kg}}{\pi (0.013 \text{ m})^2 \times 0.3 \text{ m} / 4} = 8237 \text{ kg/m}^3$$

This appears to be AISI 316 stainless steel, with a thermal conductivity of k = 13.4 W/m·K at T = 300 K.

The characteristic lengths are

$$L_{c1} = \frac{V}{A_s} = \frac{\pi D^2 L / 4}{\pi DL + 2\pi D^2 / 4} = \frac{DL/4}{L + D/2} = \frac{13 \text{ mm} \times 300 \text{ mm} / 4}{300 \text{ mm} + 13 \text{ mm} / 2} = 3.18 \text{ mm}$$

$$L_{c2} = \text{Maximum center to surface distance} = \frac{\text{D}}{2} = 6.5\text{mm}$$

Notice that the surface area of the ends has been included in $Lc1$, and does have a small effect on the result. The corresponding Biot numbers are

$$Bi_1 = \frac{hL_{c1}}{k} = \frac{15 \text{ W/m}^2 \cdot \text{K} \times 0.00318 \text{ m}}{13.4 \text{ W/m} \cdot \text{K}} = 0.0036$$

$$Bi_2 = \frac{hL_{c2}}{k} = \frac{15 \text{ W/m}^2 \cdot \text{K} \times 0.0065 \text{ m}}{13.4 \text{ W/m} \cdot \text{K}} = 0.0073$$

The lumped capacitance approximation is valid according to either definition.

(f) The characteristic lengths are

$$L_{c1} = \frac{V}{A_s} = \frac{\pi D^2 L / 4}{\pi DL} = \frac{D}{4} = 5 \text{ mm}$$

L_{c2} =Maximum center to surface distance = D/2 = 10mm

We are not told the convection heat transfer coefficient, but we do know the fluid temperature and the temperature of the rod initially and at t = 225 s. If we assume that the lumped capacitance approximation is valid, we can determine the heat transfer coefficient from Equation 5.5:

$$h = \frac{\rho Vc}{A_s t} \ln\left(\frac{\theta_i}{\theta}\right) = \frac{\rho Dc}{4t} \ln\left(\frac{T_i - T_\infty}{T - T_\infty}\right)$$

$$= \frac{2300 \text{ kg/m}^3 \times 0.020 \text{ m} \times 1750 \text{ J/kg} \cdot \text{K}}{4 \times 225 \text{ s}} \ln\left(\frac{200°\text{C} - 20°\text{C}}{100°\text{C} - 20°\text{C}}\right) = 72.5 \text{ W/m}^2 \cdot \text{K}$$

The resulting Biot numbers are:

$$Bi_1 = \frac{hL_{c1}}{k} = \frac{72.5 \text{ W/m}^2 \cdot \text{K} \times 0.005 \text{ m}}{16 \text{ W/m} \cdot \text{K}} = 0.023$$

$$Bi_2 = \frac{hL_{c2}}{k} = \frac{72.5 \text{ W/m}^2 \cdot \text{K} \times 0.01 \text{ m}}{16 \text{ W/m} \cdot \text{K}} = 0.045$$

The lumped capacitance approximation is valid according to either definition.
This also means that it was appropriate to use the lumped capacitance approximation to calculate h.

(g) With the diameter increased by a factor of ten, so are the characteristic lengths:

$$L_{c1} = \frac{V}{A_s} = \frac{\pi D^2 L / 4}{\pi DL} = \frac{D}{4} = 50 \text{ mm}$$

$$L_{c2} = \text{Maximum center to surface distance} = \frac{\text{D}}{2} = 100\text{mm}$$

Once again, we assume that the lumped capacitance approximation is valid to calculate the heat transfer coefficient according to

$$h = \frac{\rho Vc}{A_s t} \ln\left(\frac{\theta_i}{\theta}\right) = \frac{\rho Dc}{4t} \ln\left(\frac{T_i - T_\infty}{T - T_\infty}\right)$$

$$= \frac{2300 \text{ kg/m}^3 \times 0.20 \text{ m} \times 1750 \text{ J/kg} \cdot \text{K}}{4 \times 225 \text{ s}} \ln\left(\frac{200°\text{C} - 20°\text{C}}{100°\text{C} - 20°\text{C}}\right) = 725 \text{ W/m}^2 \cdot \text{K}$$

The resulting Biot numbers are:

$$Bi_1 = \frac{hL_{c1}}{k} = \frac{725 \text{ W/m}^2 \cdot \text{K} \times 0.05 \text{ m}}{16 \text{ W/m} \cdot \text{K}} = 2.3$$

$$Bi_2 = \frac{hL_{c2}}{k} = \frac{725 \text{ W/m}^2 \cdot \text{K} \times 0.1 \text{ m}}{16 \text{ W/m} \cdot \text{K}} = 4.5$$

The lumped capacitance approximation is *not* valid according to either definition.

This means that the calculated value of h is incorrect; therefore the above values of the Biot number are incorrect. However, we can still conclude that the Bi number is too large for lumped capacitance to be valid by the following reasoning. If the lumped capacitance approximation were valid, then the calculated h would be correct, and its value would be small enough to result in $Bi < 0.1$. Since the calculated Biot number does not satisfy the criterion to use the lumped capacitance approximation, the initial assumption that the lumped capacitance method is valid must have been false.

2. (**Problem 5.22 in the book**)

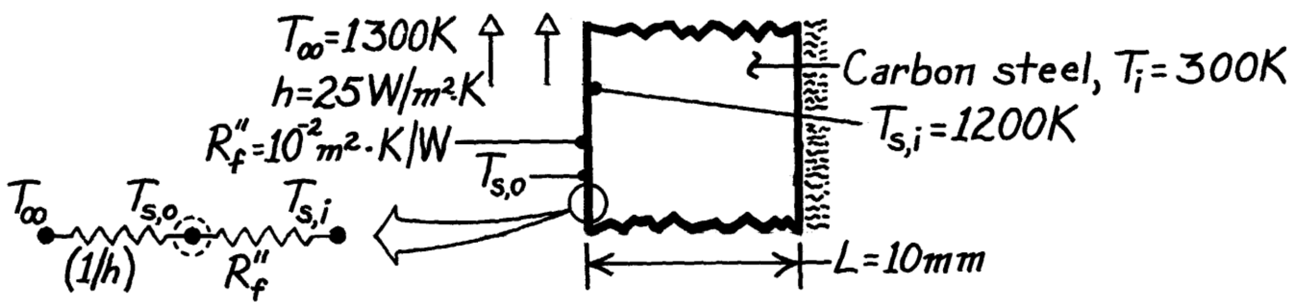
A plane wall of a furnace is fabricated from plain carbon steel (k = 60 W/mK,ρ=7850kg/m^3, c = 430 J/kgK) and is of thickness L = 10 mm. To protect it from the corrosive effects of the furnace combustion gases, one surface of the wall is coated with a thin ceramic film that, for a unit surface area, has a thermal resistance of $R''_{t,f}$ = 0.01 m2K/W. The opposite surface is well insulated from the surroundings.

At furnace start-up the wall is at initial temperature of Ti = 300 K, and combustion gases at $T_\infty = 1300K$ enter the furnace, providing a convection coefficient of h = 25 W/m2K at the ceramic film. Assuming the film to have negligible thermal capacitance, how long will it take for the inner surface of the steel to achieve a temperature of Ts,i = 1200 K? What is the temperature Ts,o of the exposed surface of the ceramic film at this time?

KNOWN: Thickness and properties of furnace wall. Thermal resistance of film on surface of wall exposed to furnace gases. Initial wall temperature.

FIND: (a) Time required for surface of wall to reach a prescribed temperature, (b) Corresponding value of film surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible film thermal capacitance, (3) Negligible radiation.

PROPERTIES: Carbon steel (given): ρ = 7850 kg/m^3, c = 430 J/kg□K, k = 60 W/m.K.

ANALYSIS: The overall coefficient for heat transfer from the surface of the steel to the gas is

$$U = (R''_{tot})^{-1} = \left(\frac{1}{h} + R''_f\right)^{-1} = \left(\frac{1}{25\ \text{W/m}^2\cdot\text{K}} + 10^{-2}\text{m}^2\cdot\text{K/W}\right)^{-1} = 20\ \text{W/m}^2\cdot\text{K}.$$

Hence,

$$Bi = \frac{UL}{k} = \frac{20\ \text{W/m}^2\cdot\text{K} \times 0.01\ \text{m}}{60\ \text{W/m}\cdot\text{K}} = 0.0033$$

And the lumped capacitance method can be used.

(a) It follows that

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp(-t/\tau_t) = \exp(-t/RC) = \exp(-Ut/\rho Lc)$$

$$t = -\frac{\rho Lc}{U}\ln\frac{T - T_\infty}{T_i - T_\infty} = -\frac{7850\ \text{kg/m}^3\,(0.01\ \text{m})\,430\ \text{J/kg}\cdot\text{K}}{20\ \text{W/m}^2\cdot\text{K}}\ln\frac{1200-1300}{300-1300}$$

$t = 3886\text{s} = 1.08\text{h}.$

(b) Performing an energy balance at the outer surface (s,o),

$$h\left(T_\infty - T_{s,o}\right) = \left(T_{s,o} - T_{s,i}\right)/R''_f$$

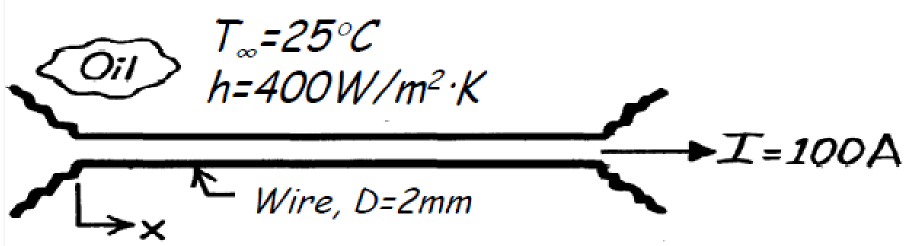
$$T_{s,o} = \frac{hT_\infty + T_{s,i}/R''_f}{h + \left(1/R''_f\right)} = \frac{25\ \text{W/m}^2\cdot\text{K}\times 1300\ \text{K} + 1200\ \text{K}/10^{-2}\text{m}^2\cdot\text{K/W}}{(25+100)\ \text{W/m}^2\cdot\text{K}}$$

$T_{s,o} = 1220\ \text{K}.$

3. **(Problem 5.29 in the book)**

A long wire of diameter D = 2 mm is submerged in an oil bath of temperature t_∞ = 25°C. The wire has a electrical resistance per unit length of R'_e. If a current of I = 100 A flows through the wire and the convection coefficient h = 400 W/m2K, what is the steady-state temperature of the wire? From the time the current is applied, how long does it take for the wire to reach a temperature that is within 1°C of the steady-state value? The properties of the wire are, c = 500 J/kgK, and k = 20 W/mK.

KNOWN: Diameter, resistance and current flow for a wire. Convection coefficient and temperature of surrounding oil.
FIND: Steady-state temperature of the wire. Time for the wire temperature to come within 1°C of it's steady-state value.
SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Wire temperature is independent of x.
PROPERTIES: Wire (given): ρ= 8000 kg/m^3, c_p = 500 J/kg·K, k = 20 W/m·K,R'_e = 0.01 Ω/m.
ANALYSIS: Since

$$\mathrm{Bi} = \frac{h(r_o/2)}{k} = \frac{400\ \mathrm{W/m^2\cdot K}\left(5.0\times10^{-4}\mathrm{m}\right)}{20\ \mathrm{W/m\cdot K}} = 0.01 < 0.1$$

The lumped capacitance method can be used. The problem has been analyzed in Example 1.4, and without radiation the steady-state temperature is given by

$$\pi\, Dh\left(T - T_\infty\right) = I^2 R'_e.$$

Hence

$$T = T_\infty + \frac{I^2 R'_e}{\pi\, Dh} = 25^\circ\mathrm{C} + \frac{(100\mathrm{A})^2\, 0.01\Omega/\mathrm{m}}{\pi(0.002\ \mathrm{m})400\ \mathrm{W/m^2\cdot K}} = 64.8^\circ\mathrm{C}.$$

With no radiation, the transient thermal response of the wire is governed by the expression (Example 1.4)

$$\frac{dT}{dt} = \frac{I^2 R'_e}{\rho c_p\left(\pi D^2/4\right)} - \frac{4h}{\rho\, c_p D}\left(T - T_\infty\right).$$

With T = Ti = 25°C at t = 0, the solution is

$$\frac{T - T_\infty - \left(I^2 R'_e/\pi\, Dh\right)}{T_i - T_\infty - \left(I^2 R'_e/\pi\, Dh\right)} = \exp\left(-\frac{4h}{\rho\, c_p D}t\right).$$

Substituting numerical values, find

$$\frac{63.8-25-39.8}{25-25-39.8}=\exp\left(-\frac{4\times 400\ \mathrm{W/m^2\cdot K}}{8000\ \mathrm{kg/m^3}\times 500\ \mathrm{J/kg\cdot K}\times 0.002\ \mathrm{m}}t\right)$$

$t = 18.4\mathrm{s}.$

4. **(Problem 5.87 in the book)** A tile-iron consists of a massive plate maintained at 150oC by an Embedded electrical heater. The iron is placed in contact with a tile to soften the adhesive, allowing the tile to be easily lifted from the subflooring. The adhesive will soften sufficiently if heated above 50oC for at least 2 min, but its temperature shouldn't exceed 120oC to avoid deterioration of the adhesive. Assume the tile and subfloor to have an initial temperature of 25oC and to have equivalent thermophysical Properties of k = 0.15 W/mK and $\rho c_p = 1.5 \times 10^6 J/m^3.K$

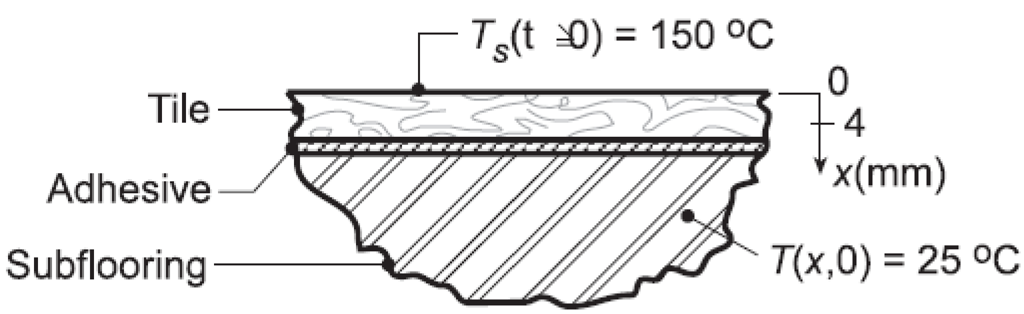
a. How long will it take a worker using the tile-iron to lift a tile? Will the adhesive temperature exceed 120°C?

b. If the tile-iron has a square surface area 254 mm to the side, how much energy has been removed from it during the time it has taken to lift tile?

KNOWN: Tile-iron, 254 mm to a side, at 150°C is suddenly brought into contact with tile over a subflooring material initially at Ti = 25°C with prescribed thermophysical properties. Tile adhesive softens in 2 minutes at 50°C, but deteriorates above 120°C.

FIND: (a) Time required to lift a tile after being heated by the tile-iron and whether adhesive temperature exceeds 120°C, (2) How much energy has been removed from the tile-iron during the time it has taken to lift the tile.

SCHEMATIC:



ASSUMPTIONS: (1) Tile and subflooring have same thermophysical properties, (2) Thickness of adhesive is negligible compared to that of tile, (3) Tile-subflooring behaves as semi-infinite solid experiencing one-dimensional transient conduction.

PROPERTIES: Tile-subflooring (given): k = 0.15 W/m·K, ρc_p = 1.5 × 10^6 J/m3·K, α = k/ρc_p = 1.00 ×
$10^{-7} m^2$/s.

ANALYSIS: (a) The tile-subflooring can be approximated as a semi-infinite solid, initially at a uniform temperature Ti = 25°C, experiencing a sudden change in surface temperature Ts = T(0,t) = 150°C. This corresponds to Case 1, Figure 5.7. The time required to heat the adhesive (xo = 4 mm) to 50°C follows from Eq. 5.60

$$\frac{T(x_o,t_o)-T_s}{T_i-T_s}=\operatorname{erf}\left(\frac{x_o}{2(\alpha t_o)^{1/2}}\right)$$

$$\frac{50-150}{25-150}=\operatorname{erf}\left(\frac{0.004\,\text{m}}{2\left(1.00\times10^{-7}\ \text{m}^2/\text{s}\times t_o\right)^{1/2}}\right)$$

$$0.80=\operatorname{erf}\left(6.325t_o^{-1/2}\right)$$

$t_o = 48.7\text{s} = 0.81$ min

Using error function values from Table B.2. Since the softening time, Δts, for the adhesive is 2 minutes, the time to lift the tile is

$$t_\ell=t_o+\Delta t_s=(0.81+2.0)\,\text{min}=2.81\,\text{min}$$

To determine whether the adhesive temperature has exceeded 120°C, calculate its temperature at t_ℓ = 2.81 min; that is, find T (x_0, t_ℓ)

$$\frac{T(x_o,t_\ell)-150}{25-150}=\operatorname{erf}\left(\frac{0.004\,\text{m}}{2\left(1.0\times10^{-7}\ \text{m}^2/\text{s}\times2.81\times60\text{s}\right)^{1/2}}\right)$$

$$T(x_o,t_\ell)-150=-125\operatorname{erf}(0.4880)=-125\times0.5098$$

$$T(x_o,t_\ell)=86^\circ\text{C}$$

Since T (x_0, t_ℓ) < 120°C, the adhesive will not deteriorate.

(c) The energy required to heat a tile to the lift-off condition is

$$Q=\int_0^{t_\ell}q''_x(0,t)\cdot A_s dt\,.$$

Using Eq. 5.61 for the surface heat flux q''_x(t) = $q''_x(0,t)$, find

$$Q=\int_0^{t_\ell}\frac{k(T_s-T_i)}{(\pi\alpha)^{1/2}}A_s\frac{dt}{t^{1/2}}=\frac{2k(T_s-T_i)}{(\pi\alpha)^{1/2}}A_s t_\ell^{1/2}$$

$$Q=\frac{2\times0.15\,\text{W/m}\cdot\text{K}(150-25)^\circ\text{C}}{\left(\pi\times1.00\times10^{-7}\ \text{m}^2/\text{s}\right)^{1/2}}\times(0.254\,\text{m})^2\times(2.81\times60\text{s})^{1/2}=56\,\text{kJ}$$