12. Spur Gear Design and selection

Objectives

- Apply principles learned in Chapter 11 to actual design and selection of spur gear systems.
- Calculate forces on teeth of spur gears, including impact forces associated with velocity and clearances.
- Determine allowable force on gear teeth, including the factors necessary due to angle of involute of tooth shape and materials selected for gears.
- Design actual gear systems, including specifying materials, manufacturing accuracy, and other factors necessary for complete spur gear design.
- Understand and determine necessary surface hardness of gears to minimize or prevent surface wear.
- Understand how lubrication can cushion the impact on gearing systems and cool them.
- Select standard gears available from stocking manufacturers or distributors.

Standard proportions

- American Standard Association (ASA)
- American Gear Manufacturers Association (AGMA)
- Brown and Sharp
- 14 ½ deg; 20 deg; 25 deg pressure angle
- Full depth and stub tooth systems

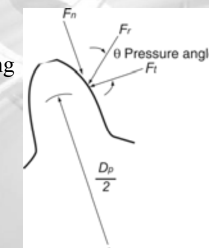
Specifications for standard gear teeth

Item	Full depth & pitches coarser than 20		Full depth & pitches finer than 20	14½° full depth
Pressure angle	20°	25°	20°	14½°
Addendum (in.)	1.0/P	1.0/P	1.0/P	1/P
Dedendum (in.)	1.250/P	1.250/P	1.2/P + 0.002	1.157/P

Forces on spur gear teeth

- F_t = Transmitted force
- F_n = Normal force or separating force
- F_r = Resultant force
- θ = pressure angle
- $F_n = F_t \tan\theta$

$$F_r = \frac{F_t}{\cos\theta}$$



Forces on spur gear teeth

- Power, P; $P = \frac{T\,n}{63{,}000}$ or $T = \frac{63{,}000\ P}{n}$
- Torque, $T = F_t\,r$ and $r = D_p/2$
- Combining the above we can write

$$F_t = \frac{2\,T}{D_p}$$

Example Problem 12-1: Forces on Spur Gear Teeth

- 20-tooth, 8 pitch, 1-inch-wide, 20° pinion transmits 5 hp at 1725 rpm to a 60-tooth gear.
- Determine driving force, separating force, and maximum force that would act on mounting shafts.

Example Problem 12-1: Forces on Spur Gear Teeth

- 20-tooth, 8 pitch, 1-inch-wide, 20° pinion transmits 5 hp at 1725 rpm to a 60-tooth gear.
- Determine driving force, separating force, and maximum force that would act on mounting shafts.

(2-6)

$$P = \frac{Tn}{63{,}000}$$

$$T = \frac{63{,}000P}{n}$$

$$T = \frac{(63{,}000)5}{1725} = 183 \text{ in-lb}$$

– Find pitch circle:

(11-4)

$$D_p = \frac{N_p}{P_d}$$

$$D_p = \frac{20 \text{ teeth}}{8 \text{ teeth/in diameter}} = 2.5 \text{ in}$$

Example Problem 12-1: Forces on Spur Gear Teeth (cont'd.)

– Find transmitted force:

(12-3)

$$F_t = \frac{2T}{D_p}$$

$$F_t = \frac{(2)183 \text{ in-lb}}{2.5 \text{ in}} = 146 \text{ lb}$$

– Find separating force:

(12-1)

$$F_n = F_t \tan\theta$$

$$F_n = 146 \text{ lb} \tan 20°$$

$$F_n = 53 \text{ lb}$$

– Find maximum force:

(12-2)

$$F_r = \frac{F_t}{\cos\theta}$$

$$F_r = \frac{146 \text{ lb}}{\cos 20°}$$

$$F_r = 155 \text{ lb}$$

Surface Speed

- Surface speed (V_m) is often referred to as pitch-line speed
- $V_m = \dfrac{\pi\, D_p\, n}{12}$ ft/min
- $V_m = \dfrac{\pi\, D_p\, n}{1000}$ m/min -- Metric units

Example Problem 12-2: Surface Speed

- In previous problem, determine the surface speed:

(12-7)

$$V_m = \pi D n$$

or

(12-5)

$$V_m = \frac{\pi D_p n}{12}$$

$$V_m = \pi 2.5 \text{ in } 1725 \text{ rpm } \frac{\text{ft}}{12 \text{ in}}$$

$$V_m = 1129 \text{ ft/min}$$

Strength of Gear Teeth

- Lewis form factor method

Forces on Gear Tooth

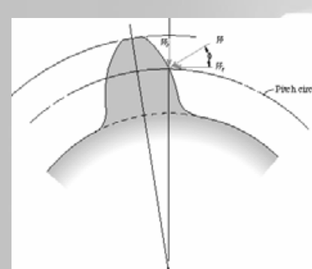


Figure 14.20 Forces acting on individual gear tooth.

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Lewis equation

$$F_s = \frac{S_n \, Y \, b}{P_d}$$

- F_s = Allowable dynamic bending force (lb)
- S_n = Allowable stress (lb/in^2). Use endurance limit and account for the fillet as the stress concentration factor
- b = Face width (in.)
- Y = Lewis form factor (Table 12.1)
- P_d = Diametral pitch

Table 12.1 Lewis form factors (Y)

Number of Teeth	14½° Full-depth Involute	20° Full-depth Involute	20° Stub Involute
12	0.210	0.245	0.311
13	0.223	0.261	0.324
14	0.236	0.277	0.339
15	0.245	0.290	0.346
16	0.254	0.296	0.361
17	0.264	0.302	0.368
18	0.270	0.308	0.377
19	0.276	0.314	0.386
20	0.283	0.321	0.393
21	0.289	0.327	0.399
22	0.292	0.330	0.405
24	0.298	0.337	0.415
26	0.308	0.346	0.424
28	0.314	0.352	0.430
30	0.317	0.359	0.437
34	0.327	0.371	0.447
38	0.333	0.384	0.455
43	0.339	0.397	0.462
50	0.346	0.410	0.474
60	0.355	0.421	0.484
75	0.361	0.434	0.496
100	0.368	0.447	0.505
150	0.374	0.460	0.518
300	0.383	0.472	0.534
rack	0.390	0.484	0.550

Example Problem 12-3: Strength of Gear Teeth

- In Example Problem 12-1, determine the force allowable (F_s) on these teeth if the pinion is made from an AISI 4140 annealed steel, the mating gear is made from AISI 1137 hot-rolled steel, and long life is desired.
  - Pinion:

$$S_n = .5\,S_u = .5\,(95\text{ ksi}) = 47.5\text{ ksi}$$

(12-9)

$$F_s = \frac{S_n\, b\, Y}{P_d}$$

  - Find Lewis form factor (*Y*) from Table 12-1, assuming full-depth teeth:

$$Y = .320$$

$$F_s = \frac{47{,}500\ (1)\ .320}{8}$$

$$F_s = 1900\text{ lb}$$

Example Problem 12-3: Strength of Gear Teeth (cont'd.)

- Gear:

$$S_n = .5\,(88\text{ ksi}) = 44\text{ ksi}$$

(Table 12-1)

$$Y = .421$$

$$F_s = \frac{44{,}000\ (1)\ .421}{8}$$

$$F_s = 2316\text{ lb}$$

- Use F_s = 1900 lb for design purposes.

Classes of Gears

- Transmitted load depends on the accuracy of the gears
- Gear Manufacture
  - Casting
  - Machining
    - Forming
    - Hobbing
    - Shaping and Planing
  - Forming
  - stamping

Force Transmitted

- Transmitted load depends on the accuracy of the gears
- A dynamic load factor is added to take care of this.
- F_t = Transmitted force
- F_d = Dynamic force
- Commercial

$$F_d = \frac{600 + V_m}{600}\,F_t$$

Classes of Gears

- Carefully cut $F_d = \dfrac{1200 + V_m}{1200} F_t$
- Precision $F_d = \dfrac{78 + V_m^{0.5}}{78} F_t$
- Hobbed or shaved $F_d = \dfrac{50 + V_m^{0.5}}{50} F_t$

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Design Methods

- Strength of gear tooth should be greater than the dynamic force; $F_s \geq F_d$
- You should also include the factor of safety, N_{sf}

$$\frac{F_s}{N_{sf}} \geq F_d$$

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Table 12.2 Service Factors

$1 < N_{sf} < 1.25$	Uniform load without shock
$1.25 < N_{sf} < 1.5$	Medium shock, frequent starts
$1.5 < N_{sf} < 1.75$	Moderately heavy shock
$1.75 < N_{sf} < 2$	Heavy shock

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Face width of Gears

- Relation between the width of gears and the diametral pitch

$$\frac{8}{P_d} < b < \frac{12.5}{P_d}$$

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Example Problem 12-4: Design Methods

- If, in Example Problem 12-1, the gears are commercial grade, determine dynamic load and, based on force allowable from Example Problem 12-3, would this be an acceptable design if a factor of safety of 2 were desired?
- Use surface speed and force transmitted from Example Problems 12-2 and 12-3.
  - Dynamic load:

$$F_d = \frac{600 + V_m}{600} F_t \qquad (12\text{-}10)$$

$$F_d = \frac{600 + 1129}{600}(146\ lb)$$

$$F_d = 421\ lb$$

  - Comparing to force allowable:

$$\frac{F_s}{N_{sf}} \geq F_d$$

$$\frac{1900\ lb}{2} \geq 421$$

$$950\ lb > 421\ lb$$

- This design meets the criteria.

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Example Problem 12-5: Design Methods

- Spur gears from the catalog page shown in Figure 12-3 are made from a .2% carbon steel with no special heat treatment.
- What factor of safety do they appear to use in this catalog?
- Try a 24-tooth at 1800 rpm gear for example purposes.
- From Appendix 4, an AISI 1020 hot-rolled steel would have .2% carbon with an S_y = 30 ksi and S_u = 55 ksi.
  - Therefore:

$$S_n = .5\ S_u$$

$$S_n = .5\ (55\ ksi) = 27.5\ ksi$$

  - Find D_p:

(11-4)

$$D_p = \frac{N_p}{P_d}$$

$$D_p = \frac{24}{12} = 2\ in$$

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Example Problem 12-5: Design Methods (cont'd)

– Find V_m:

(12-5)

$$V_m = \frac{\pi D_p n}{12}$$

$$V_m = \pi 2 \text{ in } (1800 \text{ rpm}) \frac{ft}{12 \text{ in}}$$

$$V_m = 942 \text{ ft/min}$$

– Find F_s:

(12-9)

$$F_s = \frac{S_n b Y}{P_d}$$

(from Table 12-1)

$$Y = .302$$

$$F_s = \frac{27{,}500 (¾) .302}{12}$$

$$F_s = 519 \text{ lb}$$

P N RAO 25

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Example Problem 12-5: Design Methods (cont'd)

– Set $F_s = F_d$ and solve for F_t:

$$F_d = F_s = \left(\frac{600 + V_m}{600}\right) F_t$$

$$519 \text{ lb} = \left(\frac{600 + 942}{600}\right) F_t$$

$$F_t = 202 \text{ lb}$$

(12-3)

$$T = F\left(\frac{D_p}{2}\right)$$

$$T = 202 \text{ lb} \left(\frac{2 \text{ in}}{2}\right)$$

$$T = 202 \text{ in-lb}$$

(2-6)

$$P = \frac{Tn}{63{,}000} = \frac{202\,(1800)}{63{,}000} = 5.8 \text{ hp}$$

or

(12-6)

$$P = \frac{F_t V_m}{33{,}000} = \frac{202\,(942)}{33{,}000} = 5.8 \text{ hp}$$

P N RAO 26

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Example Problem 12-5: Design Methods (cont'd)

– Compared to catalog:

$$N_{sf} = \frac{\text{hp - calculated}}{\text{hp - catalog}}$$

$$N_{sf} = \frac{5.8}{4.14} = 1.4$$

• Appears to be reasonable value.

• Manufacturer may also have reduced its rating for wear purposes as these are not hardened gears.

P N RAO 27

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Example Problem 12-6: Design Methods

• Pair of commercial-grade spur gears is to transmit 2 hp at a speed of 900 rpm of the pinion and 300 rpm for gear.
• If class 30 cast iron is to be used, specify a possible design for this problem.
• The following variables are unknown: P_d, D_p, b, N_t, N_{sf}, θ.
• As it is impossible to solve for all simultaneously, try the following:
  – $P_d = 12$, $N_t = 48$, $\theta = 14½°$, $N_{sf} = 2$
  – Solve for face width b.

– Miscellaneous properties:

(11-4)

$$D_p = \frac{N_p}{P_d} = \frac{48}{12} = 4 \text{ in}$$

$$N_g = N_p V_r = 48(3) = 144 \text{ teeth}$$

P N RAO 28

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Example Problem 12-6: Design Methods (cont'd.)

– Surface speed:

(12-5)

$$V_m = \frac{\pi D_p n}{12}$$

$$V_m = \pi \frac{4 \text{ in } 900 \text{ rpm}}{12 \text{ in/ft}}$$

$$V_m = 943 \text{ ft/min}$$

– Finding force on teeth:

(12-6)

$$F_t = \frac{33{,}000 \text{ hp}}{V_m}$$

$$F_t = \frac{33{,}000\,(2)}{943}$$

$$F_t = 70 \text{ lb}$$

P N RAO 29

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Example Problem 12-6: Design Methods (cont'd.)

– Dynamic force

$$F_d = \left(\frac{600 + V_m}{600}\right) F_t \quad (12\text{-}10)$$

$$F_d = \left(\frac{600 + 943}{600}\right) 70$$

$$F_d = 180 \text{ lb}$$

– Since width b is the unkown:

$$\frac{F_s}{N_{sf}} \geq F_d$$

and

$$F_s = \frac{S_n\ b\ Y}{P_d} \quad (12\text{-}8)$$

$$\frac{S_n\ b\ Y}{N_{sf}\ P_d} = F_d$$

–Class 30 CI; S_u = 30 ksi; S_n = .4 S_u (.4 is used because cast iron):
–S_n = 12 ksi
–Y = .344 (Table 12-1)

P N RAO 30

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Example Problem 12-6: Design Methods (cont'd.)

– Substituting:

(12-9)

$$\frac{S_n \; b \; Y}{N_{sf} \; P_d} = F_d$$

$$\frac{12{,}000 \; b \; .344}{2 \; (12)} = 180$$

$$b = 1.0 \text{ inches}$$

– Check ratio of width to pitch:

(12-14)

$$\frac{8}{P_d} < b < \frac{12.5}{P_d}$$

$$\frac{8}{12} < 1 < \frac{12.5}{12}$$

$$.66 < 1 < 1.04$$

- This is an acceptable design.
- Many other designs are also possible.

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- To increase the dynamic beam strength of the gear
  - Increase tooth size by decreasing the diametral pitch
  - Increase face width upto the pitch diameter of the pinion
  - Select material of greater endurance limit
  - Machine tooth profiles more precisely
  - Mount gears more precisely
  - Use proper lubricant and reduce contamination

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Buckingham Method of Gear Design

- It offers greater flexibility
- Expected error is based on different-pitch teeth
- More conservative design

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Fig. 12.4 Expected error in tooth profiles

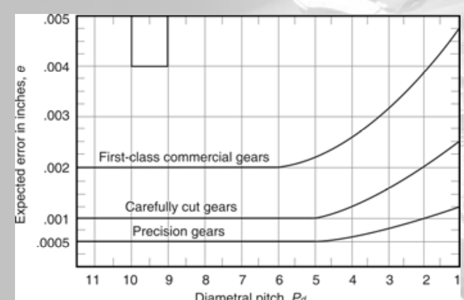

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Table 12.3 Values of C for e = 0.001 inch

Material	14 ½ deg	20 deg
Gray iron and Gray iron	800	830
Gray iron and steel	1,100	1,140
Steel and steel	1,600	1,660

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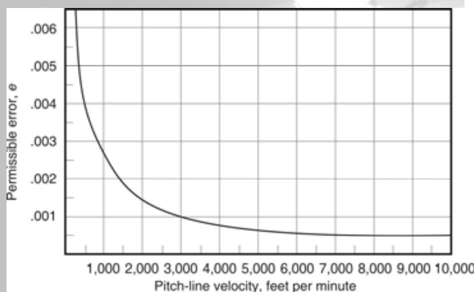
Buckingham Method of Gear Design

$$F_d = F_t + \frac{0.05 \; V_m \; (bC + F_t)}{0.05 \; V_m + (bC + F_t)^{0.5}}$$

Fig. 12.5 Recommended maximum error in gear teeth



Example Problem 12-7: Buckingham Method of Gear Design and Expected Error

- A pair of steel gears is made from annealed AISI 3140.
- Each gear has a surface hardness of BHN = 350.
- The pinion is a precision, 16 pitch, 20° involute, with 24 teeth 1 inch wide.
- The gear has 42 teeth.
- To transmit 3 hp at a speed of 3450 rpm for a safety factor of 1.4, is this a suitable design?

(Appendix 4)

$$S_u = 95\ ksi$$

$$S_n = .5\ S_u = .5(95\ ksi) = 47.5\ ksi$$

(11-4)

$$D_p = \frac{N_p}{P_d} = \frac{24}{16} = 1.5\ in$$

Example Problem 12-7: Buckingham Method of Gear Design and Expected Error (cont'd.)

– Find torque:

(2-6)

$$P = \frac{Tn}{63{,}000}$$

$$T = \frac{P\ (63{,}000)}{n}$$

$$T = \frac{3\ (63{,}000)}{3450}$$

$$T = 55\ in\text{-}lb$$

– Find force transmitted:

(12-3)

$$F_t = \frac{2T}{D_p}$$

$$F_t = \frac{2(55\ in\text{-}lb)}{1.5\ in}$$

$$F_t = 73\ lb$$

Example Problem 12-7: Buckingham Method of Gear Design and Expected Error (cont'd.)

– Find surface speed:

(12-5)

$$V_m = \frac{\pi\ D_p\ n}{12}$$

$$V_m = \frac{\pi\ 1.5\ in\ \ 3450\ rpm}{12\ ft/in}$$

$$V_m = 1355\ ft/min$$

– Find force allowable (F_s):

(12-9)

$$F_s = \frac{S_n\ b\ Y}{P_d}$$

(from Table 12-1)

$$Y = .337$$

$$F_s = \frac{47{,}500\ \ (1)\ \ .337}{16}$$

$$F_s = 1000$$

Example Problem 12-7: Buckingham Method of Gear Design and Expected Error (cont'd.)

- Expected error from Figure 12-4:
  $e = .0005$
- The value of C from Table 12-3 for steel and steel and 20° involute angle gears is 1660:
  C = .0005 (1000) 1660
  $C = 830$
- Solve for dynamic load using Buckingham's equation:

$$F_d = F_t + \frac{.05V_m\ (bC + F_t)}{.05\ V_m + (bC + F_t)^{1/2}} \quad (12\text{-}15)$$

$$F_d = 73 + \frac{.05\ (1355)\ (1 \bullet 830 + 73)}{.05\ (1355) + (1 \bullet 830 + 73)^{1/2}}$$

$$F_d = 699\ lb$$

$$\frac{F_s}{N_{sf}} \geq F_d$$

$$\frac{1000}{1.4} \geq 699$$

$$714 > 699$$

- This meets the criteria.

Wear strength (Buckingham)

$$F_W = D_P\ b\ K_g \left(\frac{2\ D_g}{D_p + D_g} \right)$$

F_w = tooth wear strength (lb)

D_p = diametral pitch of pinion (in.)

D_g = diametral pitch of gear (in.)

b = face width (in.)

K_g = load stress factor (Appendix 13)

Example Problem 12-8: Wear of Gears

• In prior example problem, verify the surface is suitable for wear considerations. For wear use $N_{ss} = 1.2$

– Wear formula:

$$F_w = D_p \; b \; Q \; K_g \qquad (12\text{-}16)$$

– Find Q :

$$Q = \frac{2\,N_g}{N_g + N_p} \qquad (12\text{-}17)$$

$$Q = \frac{2\,(42)}{42 + 24}$$

$$Q = 1.27$$

$$K_g = 270 \qquad \text{(from Appendix 13)}$$

Example Problem 12-8: Wear of Gears (cont'd.)

– Substituting into equation 12-16:

$$F_w = 1.5\,(1)\,1.27\,(270)$$

$$F_w = 514$$

• This would not be suitable. Try if surfaces each had a BHN = 450.

$$K_g = 470 \qquad \text{(from Appendix 13)}$$

$$F_w = 1.5\,(1)(1.27)\,(470)$$

$$F_w = 895$$

$$\frac{F_w}{N_{sf}} \geq F_d$$

$$\frac{895}{1.2} > 699$$

$$746 > 699$$

• This would now be acceptable if the gear teeth were hardened to a BHN of 450.