10th STD.

MARCH - 2017

Public Exam Question Paper with Answers

MATHEMATICS

[Time Allowed : 2½ Hrs.] **[Maximum Marks : 100]**

SECTION - I

Note: (*i*) Answer **all** the **15** questions.

(*ii*) Choose the **correct** answer from the given **four** alternatives and write the option code and the corresponding answer. **[15 × 1 = 15]**

1. If $f(x) = x^2 + 5$, than $f(-4) =$

   (a) 26 (b) 21
   (c) 20 (d) –20

2. If $k + 2, 4k - 6, 3k - 2$ are the three consecutive terms of an A.P., then the value of k is :

   (a) 2 (b) 3 (c) 4 (d) 5

3. If the product of the first four consecutive terms of a G.P. is 256 and if the common ratio is 4 and the first term is positive, then its 3rd term is :

   (a) 8 (b) $\frac{1}{16}$ (c) $\frac{1}{32}$ (d) 16

4. The remainder when $x^2 - 2x + 7$ is divided by $x + 4$ is :

   (a) 28 (b) 29 (c) 30 (d) 31

5. The common root of the equations $x^2 - bx + c = 0$ and $x^2 + bx - a = 0$ is :

   (a) $\frac{c+a}{2b}$ (b) $\frac{c-a}{2b}$
   (c) $\frac{c+b}{2a}$ (d) $\frac{a+b}{2c}$

6. If $A = \begin{pmatrix} 7 & 2 \\ 1 & 3 \end{pmatrix}$ and $A + B = \begin{pmatrix} -1 & 0 \\ 2 & -4 \end{pmatrix}$ then the matrix B =

   (a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 6 & 2 \\ 3 & -1 \end{pmatrix}$
   (c) $\begin{pmatrix} -8 & -2 \\ 1 & -7 \end{pmatrix}$ (d) $\begin{pmatrix} 8 & 2 \\ -1 & 7 \end{pmatrix}$

7. Slope of the straight line which is perpendicular to the straight line joining the points (–2, 6) and (4, 8) is equal to :

   (a) $\frac{1}{3}$ (b) 3 (c) –3 (d) $-\frac{1}{3}$

8. If the points (2, 5), (4, 6) and (a, a) are collinear, then the value of 'a' is equal to :

   (a) –8 (b) 4
   (c) – 4 (d) 8

9. The perimeters of two similar triangles are 24 cm and 18 cm respectively. If one side of the first triangle is 8 cm, then corresponding side of the other triangle is :

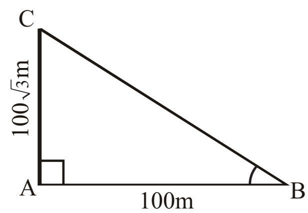
   (a) 4 cm (b) 3 cm
   (c) 9 cm (d) 6 cm

10. ΔABC is a right angled triangle where $\angle B = 90^\circ$ and $BD \perp AC$. If BD = 8 cm, AD = 4 cm, then CD is :

    (a) 24 cm (b) 16 cm
    (c) 32 cm (d) 8 cm

11. In the adjoining figure $\angle ABC =$

    (a) 45º
    (b) 30º
    (c) 60º
    (d) 50º



12. $9 \tan^2\theta - 9 \sec^2\theta =$

    (a) 1 (b) 0
    (c) 9 (d) –9

13. If the surface area of a sphere is 100 π cm^2, then its radius is equal to :

    (a) 25 cm (b) 100 cm
    (c) 5 cm (d) 10 cm

14. Standard deviation of a collection of a data is $2\sqrt{2}$. If each value is multiplied by 3, then the standard deviation of the new data is :

(a) $\sqrt{12}$ (b) $4\sqrt{2}$

(c) $6\sqrt{2}$ (d) $9\sqrt{2}$

15. A card is drawn from a pack of 52 cards at random. The probability of getting neither an ace nor a king card is:

(a) $\frac{2}{13}$ (b) $\frac{11}{13}$

(c) $\frac{4}{13}$ (d) $\frac{8}{13}$

SECTION - II

Note: (*i*) Answer **10** questions

(*ii*) Question number **30** is **compulsory**. Select **any 9** questions from the first **14** questions. **[10 × 2 = 20]**

16. Given, A = {1, 2, 3, 4, 5}, B = {3, 4, 5, 6} and C = {5, 6, 7, 8}, show that $A\cup(B\cup C) = (A\cup B)\cup C$

17. The following table represents a function from A = {5, 6, 8, 10} to B = {19, 15, 9, 11} where $f(x) = 2x - 1$. Find the values of a and b.

x	5	6	8	10
$f(x)$	a	11	b	19

18. If $-\frac{2}{7}, m, -\frac{7}{2}(m+2)$ are in G.P. find the values of m.

19. Solve by elimination method : $13x + 11y = 70$, $11x + 13y = 74$.

20. Simplify : $\frac{6x^2 + 9x}{3x^2 - 12x}$.

21. Construct a 2×2 matrix A = $[a_{ij}]$ whose elements are given by $a_{ij} = 2i - j$.

22. Let A = $\begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix}$ and B = $\begin{pmatrix} 8 & -1 \\ 4 & 3 \end{pmatrix}$. Find the matrix C, if C = 2A + B.

23. Find the coordinates of the point which divides the line segment joining (–3, 5) and (4, –9) in the ratio 1 : 6 internally.

24. "The points $(0, a)$, $a > 0$ lie on x - axis for all a". Justify the truthness of the statement.

25. In ΔPQR, AB || QR. If AB is 3 cm, PB is 2 cm and PR is 6 cm, then find the length of QR.

26. The angle of elevation of the top of a tower as seen by an observer is 30º. The observer is at a distance of $30\sqrt{3}$ m from the tower. If the eye level of the observer is 1.5 m above the ground level, then find the height of the tower.

27. The total surface area of a solid right circular cylinder is 1540 cm^2. If the height is four times the radius of the base, then find the height of the cylinder.

28. The smallest value of a collection of data is 12 and the range is 59. Find the largest value of the collection of data.

29. In tossing a fair coin twice, find the probability of getting :

(i) Two heads (ii) Exactly one tail

30. (a) If the volume of a solid sphere is $7241\frac{1}{7}$ cu.cm, then find its radius. $\left(\text{Take } \pi = \frac{22}{7}\right)$

[OR]

(b) If $x = a \sec\theta + b \tan\theta$ and $y = a \tan\theta + b \sec\theta$, then prove that $x^2 - y^2 = a^2 - b^2$.

SECTION - III

Note: (*i*) Answer **9** questions.

(*ii*) Question No. **45** is **Compulsory**. Select **any 8** questions from the **14** questions. **[9 × 5 = 45]**

31. Let A = $\{a, b, c, d, e, f, g, x, y, z\}$, B = $\{1, 2, c, d, e\}$ and C = $\{d, e, f, g, 2, y\}$. Verify $A \setminus (B\cup C) = (A \setminus B) \cap (A \setminus C)$.

32. Let A = {6, 9, 15, 18, 21}; B = {1, 2, 4, 5, 6} and $f : A \to B$ be defined by $f(x) = \dfrac{x-3}{3}$.
Represent f by :
(i) an arrow diagram
(ii) a set of ordered pairs
(iii) a table
(iv) a graph

33. Find the sum of the first $2n$ terms of the series $1^2 - 2^2 + 3^2 - 4^2 + ...$

34. Find the sum of first n terms of the series $7 + 77 + 777 + ...$

35. The speed of a boat in still water is 15 km/hr. It goes 30 km upstream and return downstream to the original point in 4 hrs. 30 minutes. Find the speed of the stream.

36. Find the values of a and b if $16x^4 - 24x^3 + (a-1)x^2 + (b+1)x + 49$ is a perfect square.

37. If $A = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$ verify that $(AB)^T = B^T A^T$.

38. Find the area of the quadrilateral formed by the points (–4, –2), (–3, –5), (3, –2) and (2, 3).

39. State and prove Pythagoras theorem.

40. A flag post stands on the top of a building. From a point on the ground, the angles of elevation of the top and bottom of the flag post are 60° and 45° respectively. If the height of the flag post is 10m, find the height of the building. $\left(\sqrt{3} = 1.732\right)$

41. The perimeter of the ends of a frustum of a cone are 44 cm and 8.4 π cm. If the depth is 14 cm, then find its volume.

42. The length, breadth and height of a solid metallic cuboid are 44 cm, 21 cm and 12 cm respectively. It is melted and a solid cone is made out of it. If the height of the cone is 24 cm, then find the diameter of its base.

43. Find the coefficient of variation of the following data. 18, 20, 15, 12, 25

44. If a die is rolled twice, find the probability of getting an even number in the first time or a total of 8.

45. (a) Find the GCD of the following polynomials $3x^4 + 6x^3 - 12x^2 - 24x$ and $4x^4 + 14x^3 + 8x^2 - 8x$.

[OR]

(b) A straight line cuts the coordinate axes at A and B. If the mid point of AB is (3, 2), then find the equation of AB.

SECTION - IV

Note: Answer **both** the questions choosing either of the alternatives. **[2 × 10 = 20]**

46. (a) Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 6 cm. Also, measure the lengths of the tangents.

[OR]

(b) Construct a cyclic quadrilateral ABCD, given AB = 6 cm, ∠ABC = 70°, BC = 5 cm and ∠ACD = 30°.

47. (a) Solve graphically $2x^2+x-6=0$.

[OR]

(b) Draw the graph of $xy = 20$, $x, y > 0$. Use the graph to find y when $x = 5$, and to find x when $y = 10$.

★★★

ANSWERS

SECTION – I

1. (b)	2. (b)	3. (a)	4. (d)	5. (a)
6. (c)	7. (c)	8. (d)	9. (d)	10. (b)
11. (c)	12. (d)	13. (c)	14. (c)	15. (b)

SECTION – II

16. ***Solution:***

Now, $B \cup C = \{3, 4, 5, 6\} \cup \{5, 6, 7, 8\}$
$= \{3, 4, 5, 6, 7, 8\}$

$\therefore A \cup (B \cup C)$ $\{1, 2, 3, 4, 5\} \cup \{3, 4, 5, 6, 7, 8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\}$ ...(1)

Now, $A \cup B = \{1, 2, 3, 4, 5\} \cup \{3, 4, 5, 6\}$
$= \{1, 2, 3, 4, 5, 6\}$

$\therefore (A \cup B) \cup C = \{1, 2, 3, 4, 5, 6\} \cup \{5, 6, 7, 8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\}$ ...(2)

From (1) and (2), we have $A \cup (B \cup C) = (A \cup B) \cup C$.

17. ***Solution:***

$y = f(x) = 2x - 1$

$a = f(5) = 2 \times 5 - 1 = 9$

$b = f(8) = 2 \times 8 - 1 = 15$

18. ***Solution:***

If $-\frac{2}{7}$, m, $-\frac{7}{2}$ $(m + 2)$ are in G.P., find the values of m. Since the given terms are in G.P. their Common ratio is the same.

$$\therefore \frac{m}{-\frac{2}{7}} = \frac{-\frac{7}{2}(m+2)}{m}$$

$$\Rightarrow m^2 = -\frac{\cancel{2}}{\cancel{7}} \times -\frac{\cancel{7}}{\cancel{2}}(m+2)$$

$\Rightarrow m^2 = m + 2 \Rightarrow m^2 - m - 2 = 0$

$\Rightarrow (m - 2) + (m + 1) = 0 \Rightarrow m = 2$ or $- 1$.

19. ***Solution :***

$13x + 11y = 70,\ 11x + 13y = 74$

$13x + 11y = 70$ ...(1)

$11x + 13y = 74$ ...(2)

Adding (1) & (2) $\Rightarrow 24x + 24y = 144$

$\div 24 \Rightarrow x + y = 6$ ...(3)

Subtracting (1) & (2) $2x - 2y = -4$

$\div 2 \Rightarrow x - y = -2.$ ...(4)

Solving (3) & (4) $\Rightarrow$

$$\begin{array}{rcll} x - \cancel{y} & = & -2 & \text{...(4)} \\ x + \cancel{y} & = & 6 & \text{...(3)} \\ \hline 2x & = & 4 & \Rightarrow x = 2 \end{array}$$

Substituting in (3) $\Rightarrow 2 + y = 6 \Rightarrow y = 4$

$\therefore$ The required the solution is (2, 4)

20. ***Solution :***

$$\frac{6x^2 + 9x}{3x^2 - 12x} = \frac{\cancel{3x}(2x+3)}{\cancel{3x}(x-4)} = \frac{2x+3}{x-4}$$

Hint :
Only products can be cancelled.

21. ***Solution :***

$aij = 2i - j$

$$\left.\begin{array}{l} a_{11} = 2(1) - 1 = 2 - 1 = 1 \\ a_{12} = 2(1) - 2 = 2 - 2 = 0 \\ a_{21} = 2(2) - 1 = 4 - 1 = 3 \\ a_{22} = 2(2) - 2 = 4 - 2 = 2 \end{array}\right\}$$

$$\Rightarrow A = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$$

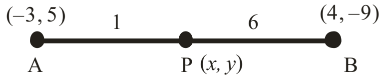
22. ***Solution:***

$$C = 2A + B = 2\begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix} + \begin{pmatrix} 8 & -1 \\ 4 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 4 \\ 10 & 2 \end{pmatrix} + \begin{pmatrix} 8 & -1 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 14 & 3 \\ 14 & 5 \end{pmatrix}$$

23. ***Solution:***

(x_1, y_1) is $(-3, 5)$ (x_2, y_2) is $(4, -9)$; $m = 1$, $m = 6$



$$P_{(x,y)} = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$P_{(x,y)} = \left(\frac{4-18}{1+6}, \frac{-9+30}{1+6}\right) = \left(\frac{-14}{7}, \frac{21}{7}\right)$$

$$= \left(\frac{-14}{7}, \frac{21}{7}\right) = (-2, 3)$$

24. ***Solution:***

For all $a>0$ $(0, a)$ kind of points will be on the positive side of y-axis.

Hence, the given statement is false.

25. ***Solution:***

Given AB is 3 cm, PB is 2 cm PR is 6 cm and AB || QR In ΔPAB and ΔPQR

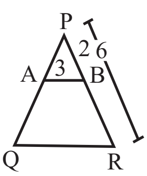
∠PAB = ∠PQR (corresponding angles)

and ∠P is common

∴ ΔPAB ~ ΔPQR (AA similarity criterion)

Since corresponding sides are proportional,

$$\frac{AB}{QR} = \frac{PB}{PR}$$



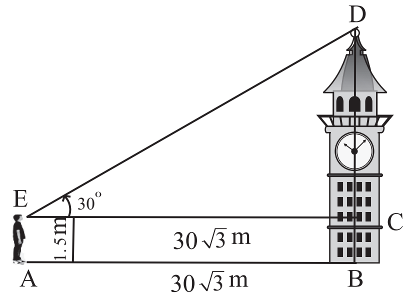
$$QR = \frac{AB \times PR}{PB}$$

$$= \frac{3 \times 6}{2}$$

Thus, QR = 9 cm.

26. ***Solution:***

Let BD be the height of the tower and AE be the distance of the eye level of the observer from the ground level.



Draw EC parallel to AB such that AB = EC.

Given AB = EC $= 30\sqrt{3}$ m and

AE = BC = 1.5 m

In right angled ΔDEC

$$\tan 30^\circ = \frac{CD}{EC}$$

$$\Rightarrow CD = EC \tan 30^\circ = \frac{30\sqrt{3}}{\sqrt{3}}$$

∴ CD = 30 m

Thus, the height of the tower,

BD = BC + CD

= 1.5 + 30 = 31.5 m

27. ***Solution:***

Height = 4(radius) ⇒ $h = 4r$

T.S.A. of a circular cylinder = 1540 cm^2

(i.e.) $2\pi r\,(h + r) = 1540$ cm^2

$$\Rightarrow \cancel{2} \times \frac{\overset{2}{\cancel{22}}}{7} \times r[4r + r] = \overset{\overset{35}{\cancel{70}}}{\cancel{140}}\!\!\!\!\!\!\cancel{1540}$$

$$\Rightarrow \frac{r}{7}\left(\cancel{5}r\right) = \cancel{35}^{\,7}$$

$r^2 = 7 \times 7$

$r = 7$ cm.

∴ $h = 4r = 4 \times 7 = 28$ cm

28. ***Solution:***

Given, S = 12; R = 59

R = L – S ⇒ 59 = L – 12 ⇒ L = 59 + 12 = 71

29. ***Solution:***

In tossing a coin twice, the sample space

S = {HH, HT, TH, TT}

∴ n(S) = 4.

(i) Let A be the event of getting two heads. Then A = {HH}.

Thus, $n(A) = 1$.

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$$

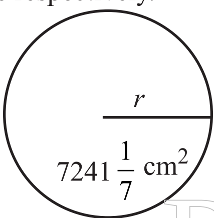
(ii) Let C be the event of getting exactly one tail. Then C = {HT, TH}

Thus, $n(C) = 2$.

$$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

30. ***Solution:***

(a) Let r and V be the radius and volume of the solid sphere respectively.



Given that $V = 7241\frac{1}{7}$ cu. cm

$$\Rightarrow \quad \frac{4}{3}\pi r^2 = \frac{50688}{7}$$

$$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{50688}{7}$$

$$r^3 = \frac{50688}{7} \times \frac{3 \times 7}{4 \times 22}$$

$$= 1728 = 4^3 \times 3^3$$

Thus, the radius of the sphere, $r = 12$ cm.

(or)

(b) L.H.S $= x^2 - y^2$

Hint : $(a+b)^2 = a^2 + 2ab + b^2$

$= (a \sec\theta + b\tan\theta)^2 - (a\tan\theta + \sec\theta)^2$

$= a^2\sec^2\theta + \cancel{2ab\sec\theta\tan\theta} + b^2\tan^2\theta -$

$(a^2\tan^2\theta + \cancel{2ab\sec\theta\tan\theta} + b^2\sec^2\theta)$

$= a^2(\sec^2\theta - \tan^2\theta) - b^2(\sec^2\theta - \tan^2\theta)$

$= a^2(1) - b^2(1) = a^2 - b^2 =$ R.H.S.

SECTION – III

31. ***Solution:***

First, we find $B \cup C = \{1, 2, c, d, e\} \cup \{d, e, f, g, 2, y\}$

$= \{1, 2, c, d, e, f, g, y\}$.

Then $A \setminus (B \cup C) = \{a, b, c, d, e, f, g, x, y, z\} \setminus \{1, 2, c, d, e, f, g, y\}$

$= \{a, b, x, z\}$. ...(1)

Next, we have $A \setminus B = \{a, b, f, g, x, y, z\}$ and

$A \setminus C = a, b, c, x, z\}$

and so $(A \setminus B) \cap (A \setminus C) = \{a, b, x, z\}$. ...(2)

Hence, from (1) and (2) it follows that

$A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.

32. ***Solution:***

$$f(x) = \frac{x-3}{3}$$

$$f(6) = \frac{6-3}{3} = \frac{3}{3} = 1$$

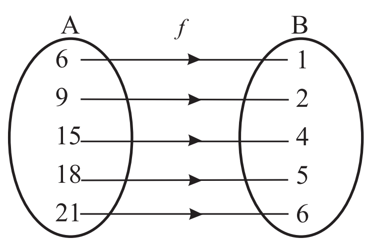
$$f(9) = \frac{9-3}{3} = \frac{6}{3} = 2$$

$$f(15) = \frac{15-3}{3} = \frac{12}{3} = 4$$

$$f(18) = \frac{18-3}{3} = \frac{15}{3} = 5$$

$$f(21) = \frac{21-3}{3} = \frac{18}{3} = 6$$

(i)

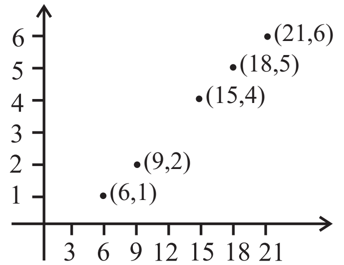


(ii) $f = \{(6, 1), (9, 2), (15, 4), (18, 5), (21, 6)\}$

(iii)

f	6	9	15	18	21
$f(x)$	1	2	4	5	6

(iv)



33. ***Solution:***

We want to find $1^2 - 2^2 + 3^2 - 4^2 + \ldots$ to $2n$ terms

$= 1 - 4 + 9 - 16 + 25 - \ldots$ to $2n$ terms

$= (1 - 4) + (9 - 16) + (25 - 36) + \ldots$ to n terms. (after grouping)

$= -3 + (-7) + (-11) + \ldots\; n$ terms

Now, the above series is in an A.P. with first term $a = -3$ and common difference $d = -4$

Therefore, the required sum

$$= \frac{n}{2}\,[2a + (n-1)\,d]$$

$$= \frac{n}{2}\,[2(-3) + (n-1)(-4)]$$

$$= \frac{n}{2}\,[-6 - 4n + 4] = \frac{n}{2}\,[-4n - 2]$$

$$= \frac{-2n}{2}\,(2n+1) = -\,n\,(2n+1)$$

34. ***Solution:***

$Sn = 7 + 77 + 777 + \ldots + (n \text{ terms})$

$Sn = 7(1 + 11 + 111 + \ldots + n \text{ terms})$

$$= \frac{7}{9}\,(9 + 99 + 999 + \ldots + n \text{ terms})$$

$$= \frac{7}{9}\,[(10-1) + (100-1) + (1000-1) + \ldots + (10^n - 1)]$$

$$= \frac{7}{9}\,[(10 + 10^2 + 10^3 + \ldots + 10^n) - (1 + 1 + 1 \ldots n \text{ terms})]$$

$$= \frac{7}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$$

$$= \frac{7}{9}\left[\frac{10(10^n - 1)}{9} - n\right] = \frac{70}{81}\left(10^n - 1\right) - \frac{7n}{9}$$

35. ***Solution:***

Speed in still water $= 15$km/hr.

Total Time taken = 4 hrs 30 m $= 4\frac{1}{2} = \frac{9}{2}$ hrs.

Let speed of the stream $= x$ km/hr

Speed of the boat upstream $= 15 - x$

Speed of the boat down stream $= 15 + x$

Time taken for upstream $T_1 = \dfrac{30}{15 - x}$

Time taken for downstream $T_2 = \dfrac{30}{15 + x}$

$$T_1 + T_2 = \frac{9}{2}$$

$$\Rightarrow \frac{30}{15 - x} + \frac{30}{15 + x} = \frac{9}{2}$$

$$\Rightarrow \overset{10}{\cancel{30}}\left(\frac{(15 + \cancel{x}) + (15 - \cancel{x})}{(15 - x)(15 + x)}\right) = \overset{3}{\cancel{9}}/2$$

$\Rightarrow 3\,(225 - x^2) = 20 \times 30$

$\Rightarrow \quad 675 - 3x^2 = 600$

$3x^2 = 75$

$x^2 = \dfrac{75}{3} = 25$

$\therefore x = 5$ km/hr.

Aliter :

$225 - x^2 = 200$

$-x^2 = -25$

$x^2 = 25$

$x = 5$

(i.e.) Speed of the stream = 5 km/hr.

36. ***Solution:***

$$
\begin{array}{r|l}
 & 4x^2 - 3x + \left(\frac{a-10}{8}\right) \\
\hline
4x^2 & \cancel{16x^4} - 24x^3 + (a-1)\,x^2 + (b+1)\,x + 49 \\
 & \cancel{16x^4} \\
\hline
8x^2 - 3x & \cancel{-24x^3} + (a-1)\,x^2 \\
 & (+) \qquad (-) \\
 & \cancel{-24x^3} + \ 9\,x^2 \\
\hline
 & (a-10)\,x^2 + (b+1)\,x + 49 \\
 & (+) \qquad (-) \qquad (-) \\
 & (a-10)x^2 + \left(\frac{a-10}{8}\right)6x + \left(\frac{a-10}{8}\right)\left(\frac{a-10}{8}\right) \\
\hline
8x^2 - 6x + \left(\frac{a-10}{8}\right) & \left[b+1+\frac{a-10}{8}\times 6\right]x + 49 - \left(\frac{a-10}{8}\right)^2
\end{array}
$$

Since the given expression is a perfect square, the remainder is zero.

$$\therefore\ b + 1 + \frac{a-10}{8} \times 6 = 0 \qquad \text{...(1)}$$

$$49 - \left(\frac{a-10}{8}\right)^2 = 0 \qquad \text{...(2)}$$

$$\text{From (2)}\ \left(\frac{a-10}{8}\right)^2 = 49 = 7^2$$

$$\Rightarrow \quad \frac{a-10}{8} = 7 \Rightarrow a - 10 = 56$$

$$\Rightarrow \boxed{a = 66}$$

Substituting a = 66 in (1) we get,

$$b + 1 + \frac{56}{8} \times 6 = 0$$

$$b + 1 + 42 = 0$$

$$\Rightarrow \boxed{b = -43}$$

37. ***Solution:***

$$A = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \Rightarrow B^T = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 10-2 & -5+2 \\ 14-3 & -7+3 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & -3 \\ 11 & -4 \end{pmatrix}$$

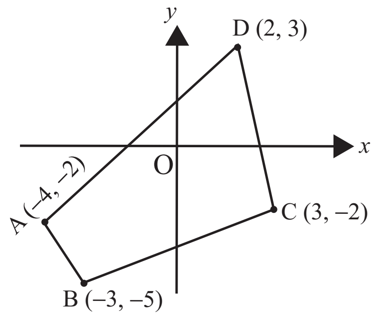
$$\therefore (AB)^T = \begin{pmatrix} 8 & 11 \\ -3 & -4 \end{pmatrix} \qquad \text{... (1)}$$

$$B^T A^T = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 10-2 & 14-3 \\ -5+2 & -7+3 \end{pmatrix} = \begin{pmatrix} 8 & 11 \\ -3 & -4 \end{pmatrix} \qquad \text{... (2)}$$

$$\therefore (1) = (2) \Rightarrow (AB)^T = B^T A^T$$

38. ***Solution:***



Let us plot the points roughly and take the vertices in counter clock-wise direction.

Let the vertices be

A(–4, –2), B(–3, –5), C(3, –2) and D(2, 3)

Area of the quadrilateral ABCD

$$= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_2y_1 + x_3y_2 + x_4y_3 + x_1y_4)$$

$$= \frac{1}{2}\{20 + 6 + 9 - 4) - (6 - 15 - 4 - 12)\}$$

$$\frac{1}{2}\begin{Bmatrix} -4 & -3 & 3 & 2 & -4 \\ -2 & -5 & 2 & 3 & -2 \end{Bmatrix}$$

$$= \frac{1}{2}\{31 + 25\} = 28 \text{ sq. units.}$$

39. ***Pythagoras theorem:***

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given : In a right angled ΔABC, ∠A = 90º.

To prove : $BC^2 = AB^2 + AC^2$

Construction : Draw AD ⊥ BC

Proof :

In triangles ABC and DBA , ∠B is the common angle.

Also, we have ∠BAC = ∠ADB = 90º.

∴ΔABC ~ ΔDBA (AA similarity criterion)

Thus, their corresponding sides are proportional.

Hence, $\frac{AB}{DB} = \frac{BC}{BA}$

∴ $AB^2 = DB \times BC$ ...(1)

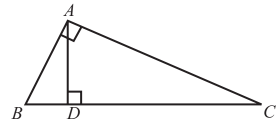
Similarly, we have ΔABC ~ ΔDAC.

Thus, $\frac{BC}{AC} = \frac{AC}{DC}$

∴ $AC^2 = BC \times DC$ ...(2)

Adding (1) and (2) we get,

$AB^2 + AC^2 = BD \times BC + BC \times DC$



= BC (BD + DC)

$= BC \times BC = BC^2$

Thus, $BC^2 = AB^2 + AC^2$

Hence the Pythagoras theorem.

40. ***Solution:***

Let A be the point of observation and B be the foot of the building.

Let BC denote the height of the building and CD denote height of the flag post.

Given that ∠CAB = 45º, ∠DAB = 60º and CD = 10 m

Let BC = h metres and AB = x metres.

Now, in the right angled ΔCAB,

$\tan 45º = \frac{BC}{AB}$

Thus, AB = BC i.e., $x = h$ ...(1)

Also, in the right angled ΔDAB,

$\tan 60º = \frac{BD}{AB}$

$\Rightarrow AB = \frac{h+10}{\tan 60º} \Rightarrow x = \frac{h+10}{\sqrt{3}}$ .....(2)

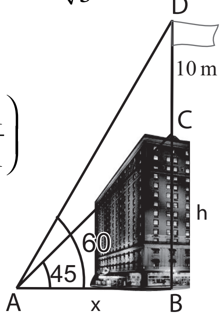
From (1) and (2), we get $h = \frac{h+10}{\sqrt{3}}$

$\Rightarrow \sqrt{3}h - h = 10$

$\Rightarrow h = \left(\frac{10}{\sqrt{3}-1}\right)\left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$

$= \frac{10(\sqrt{3}+1)}{3-1}$

= 5 (2.732) = 13.66 m.



Hence, the height of the building is 13.66 m.

41. ***Solution:***

Depth, h $= 14$ cm.

Given, $2\pi R = 44$ cm

$$\Rightarrow 2 \times \frac{22}{7} \times R = 44 \Rightarrow R = \frac{\cancel{44}^{22} \times 7}{\cancel{2} \times \cancel{22}}$$

$\therefore R = 7$ cm

Also given $\cancel{2}\cancel{\pi}r = \cancel{8.4}^{4.2}\cancel{\pi} \Rightarrow r = 4.2$ cm

$$\text{Volume of the frustum} = \frac{1}{3}\pi h[R^2 + r^2 + Rr]$$

$$= \frac{1}{3} \times \frac{22}{\cancel{7}} \times \cancel{14}^{2}[7^2 + (4.2)^2 + 7 \times 4.2]$$

$$= \frac{44}{3}[49 + 17.64 + 29.4]$$

$$= \frac{44}{3}(96.04) = 1408.58 \text{ cm}^3$$

$$= 1408.6 \text{ cm}^3$$

42. ***Solution:***

Cuboid	Cone
$l = 44$ cm	radius $r = ?$
$b = 21$ cm	height $h = 24$ cm
$h = 12$ cm	

Given,

Volume of the Cuboid = Volume of the Cone

$$l \times b \times h = \frac{1}{3}\pi r^2 h$$

$$44 \times 21 \times 12 = \frac{1}{3} \times \frac{22}{7} \times r^2 \times 24$$

$$\frac{\cancel{44}^{2} \times 21 \times \cancel{12} \times 3 \times 7}{\cancel{22} \times \cancel{24}_{2}} = r^2$$

$\Rightarrow r^2 = 7 \times 3 \times 3 \times 7$

$\Rightarrow r^2 = 21^2$

$\Rightarrow r = 21$ cm

Diameter of its base $= 42$ cm.

43. ***Solution:***

Let us calculate the A.M of the given data.

$$\text{A.M. } \bar{x} = \frac{12 + 15 + 18 + 20 + 25}{5}$$

$$= \frac{90}{5} = 18.$$

x	$d = x - 18$	d^2
12	-6	36
15	-3	9
$\boxed{18}$	0	0
20	2	4
25	7	49
	$\sum d = 0$	$\sum d^2 = 98$

$$\sigma = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{98}{5}}$$

$$= \sqrt{19.6} \simeq 4.428$$

$\therefore$ The coefficient of variation $= \frac{\sigma}{\bar{x}} \times 100$

$$= \frac{4.428}{18} \times 100 = \frac{442.8}{18}$$

$\therefore$ The coefficient of variation is $= 24.6$

44. ***Solution:***

Sample space =

{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\therefore$ n(S) = 36

Let A be the event of getting an even number in the first die and B be the event of getting a total of 8.

$\therefore$ (A) =

{ (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\therefore n(A) = 18 \Rightarrow P(A) = \dfrac{n(A)}{n(S)} = \dfrac{18}{36}$

B = { (2, 6), (6, 2), (5, 3), (3, 5), (4, 4)}

$\therefore n(B) = 5 \Rightarrow P(B) = \dfrac{n(B)}{n(S)} = \dfrac{5}{36}$

$A \cap B = \{(2, 6), (4, 4), (6, 2)\}$

$\Rightarrow n(A \cap B) = 3$

$\therefore P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{3}{36}$

By using the addition theorem on probabilities:

$P(A \cup B) = P(A) + P(B) - (A \cap B)$

$= \dfrac{18}{36} + \dfrac{5}{36} - \dfrac{3}{36} = \dfrac{20}{36} = \dfrac{5}{9}$

45.(a) ***Solution:***

Let $f(x) = 3x^4 + 6x^3 - 12x^2 - 24x$
$= 3x(x^3 + 2x^2 - 4x - 8)$

Let $g(x) = 4x^4 + 14x^3 + 8x^2 - 8x$
$= 2x(2x^3 + 7x^2 + 4x - 4)$

Let us find the GCD for the polynomials $x^3 + 2x^2 - 4x - 8$ and $2x^3 + 7x^2 + 4x - 4$

We choose the divisor to be $x^3 + 2x^2 - 4x - 8$

$$\begin{array}{r|l} & \quad 2 \\ \hline x^3 + 2x^2 - 4x - 8 & 2x^3 + 7x^2 + 4x - 4 \\ & 2x^3 + 4x^2 - 8x - 16 \\ \hline & \quad 3x^2 + 12x + 12 \\ & \Longrightarrow \quad x^2 + 4x + 4 \\ & \qquad \downarrow \\ & \qquad \text{remainder} (\neq 0) \end{array}$$

$$\begin{array}{r|l} & \quad x - 2 \\ \hline x^2 + 4x + 4 & x^3 + 2x^2 - 4x - 8 \\ & x^3 + 4x^2 + 4x \\ \hline & -2x^2 - 8x - 8 \\ & -2x^2 - 8x - 8 \\ \hline & \quad 0 \rightarrow \text{remainder} \end{array}$$

Common factor of $x^3 + 2x^2 - 4x - 8$ and $2x^3 + 7x^2 + 4x - 4$ is $x^2 + 4x + 4$

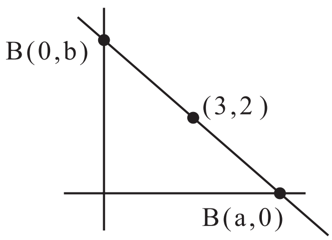
Also common factor of $3x$ and $2x$ is x.

Thus, GCD $(f(x), g(x)) = x(x^2 + 4x + 4)$.

(OR)

(b) Let the x and y intercepts be a and b

$\therefore$ A(a, 0) and B (0, b) are the points on x and y axes



$\because$M is the mid-pt. of AB, (3, 2)

$$= \left(\frac{a+0}{2}, \frac{0+b}{2}\right)$$

$$\Rightarrow (3, 2) = \left(\frac{a}{2}, \frac{b}{2}\right)$$

Equating x and y co-ordinates on both sides

$$3 = \frac{a}{2} \Rightarrow a = 6$$

$$2 = \frac{b}{2} \Rightarrow b = 4$$

Hint:
Any point on x-axis will have its y co-ordinate 0, and any point on y-axis, the x coordinate 0.

$\therefore$ The required equation is $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \frac{x}{6} + \frac{y}{4} = 1 \Rightarrow \frac{2x+3y}{12} = 1$$

Multiplying by 12 $\Rightarrow$ $2x + 3y = 12$

The required equation of AB is

$$2x + 3y - 12 = 0.$$

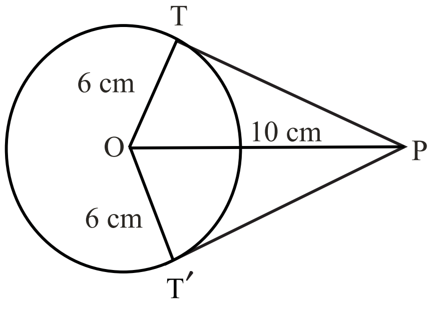
SECTION - IV

46. (a) ***Solution:***

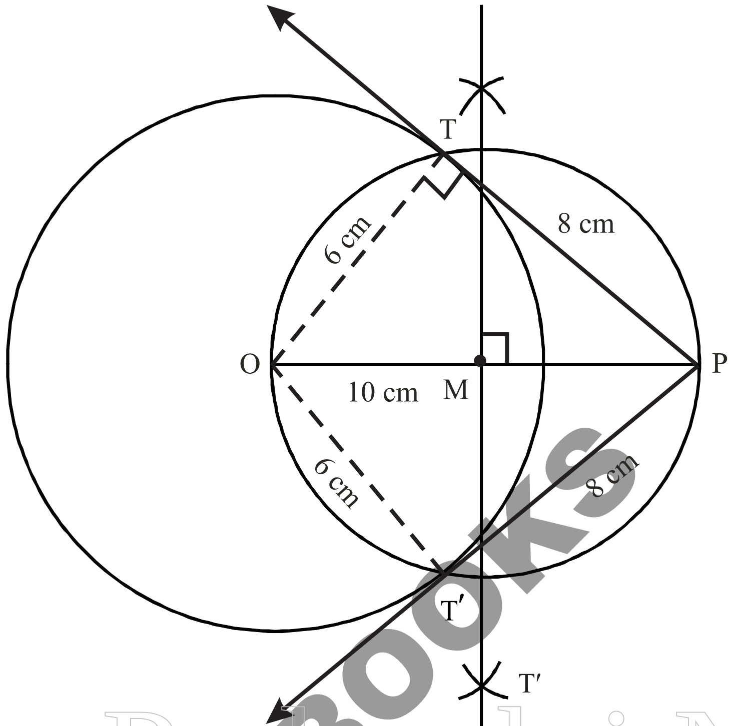
Given:

Radius of the circle = 6 cm. OP = 10 cm

Rough Diagram



Fair Diagram



Steps of Construction:

(i) With O as the centre draw a circle of radius 6 cm.

(ii) Mark a point P at a distance of 10 cm from O and join OP.

(iii) Draw the perpendicular bisector of OP. Let it meet OP at M.

(iv) With M as centre and MO as radius, draw another circle.

(v) Let the two circles intersect at T and T′.

(iv) Join PT and P T′. They are the required tangents.

Length of the tangent, PT = 8 cm.

Verification:

In the right angled triangle OPT,

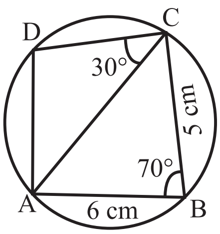
$$PT = \sqrt{OP^2 - OT^2} = \sqrt{10^2 - 6^2}$$
$$= \sqrt{100 - 36} = \sqrt{64} = 8 \text{ cm}$$
$$\therefore PT = PT' = 8 \text{ cm}$$

(OR)

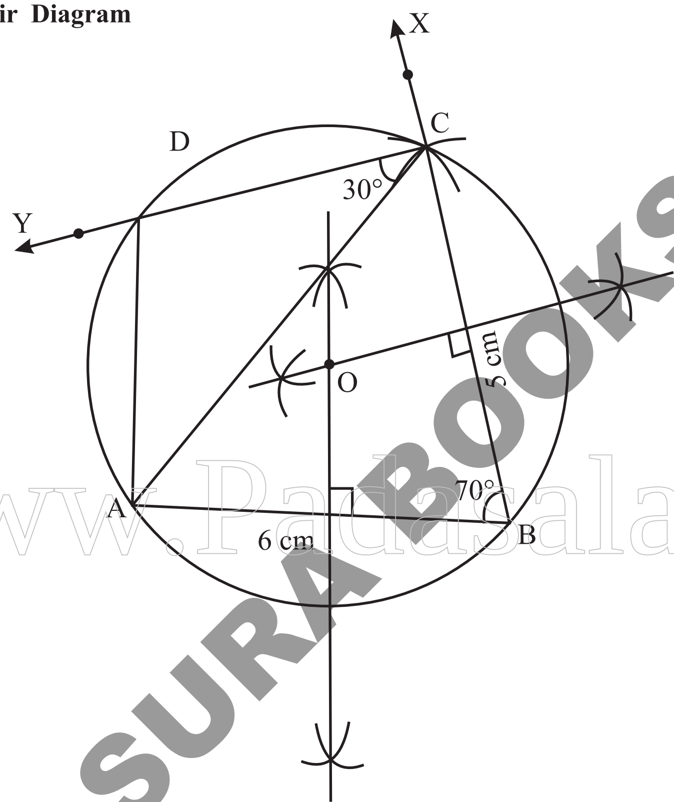
(b)

Given: In the cyclic quadrilateral ABCD, AB = 6 cm, ∠ABC = 70º, BC = 5 cm and ∠ACD = 30º

Rough Diagram



Fair Diagram



Steps of Construction:

(i) Draw a rough diagram and mark the measurements.
Draw a line segment AB = 6 cm.

(ii) From B draw BX such that ∠ ABX = 70º

(iii) With B as centre and radius 5 cm, draw an arc intersecting BX at C.

(iv) Join BC.

(v) Draw the perpendicular bisectors of AB and BC intersecting each other at O.

(vi) With O as centre and OA = OB = OC as radius, draw the circumcircle of ΔABC.

(vii) From C, draw CY such that ∠ACD = 30º which intersects the circle at D.

(viii) Join AD and CD.
Now, ABCD is the required cyclic quadrilateral.

47. (a) ***Solution:***

$2x^2+x-6=0 \Rightarrow y=2x^2+x-6$

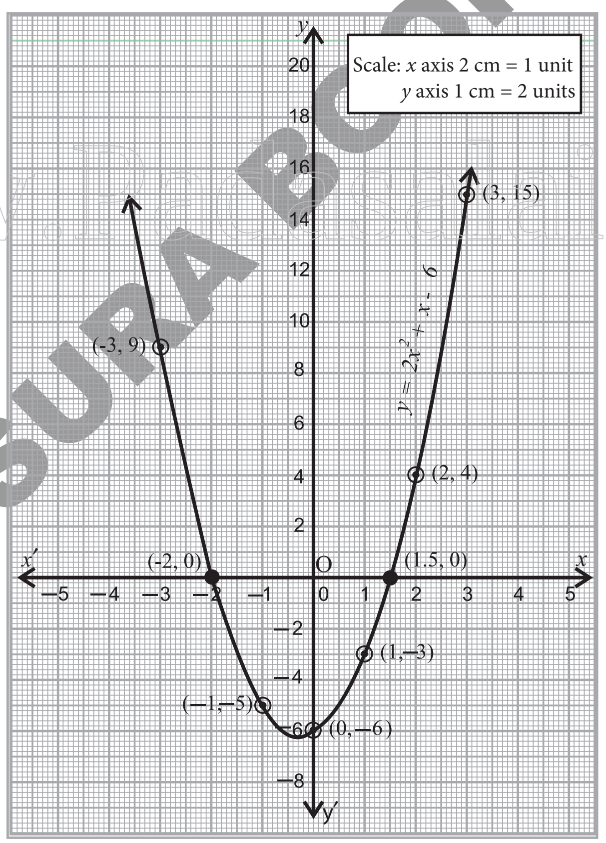
First, let us form the following table by assigning integer values for x from –3 to 3 and finding the corresponding values of $y=2x^2+x-6$

Table

x	–3	–2	–1	0	1	2	3
x^2	9	4	1	0	1	4	9
$2x^2$	18	8	2	0	2	8	18
x	–3	–2	–1	0	1	2	3
–6	–6	–6	–6	–6	–6	–6	–6
y	9	0	–5	–6	–3	4	15

Plot the points (–3, 9), (–2, 0), (–1, –5), (0, –6), (1, –3), (2, 4) and (3, 15) on the graph.

Draw the graph by joining the points by a smooth curve.



Join the points by a smooth curve. The curve, thus obtained, is the graph of $y=2x^2+x-6$.

The curve cuts the x-axis at the points (–2 , 0) and (1.5 , 0).

The x-coordinates of the above points are –2 and 1.5.

Hence, the solution set is {–2 , 1.5 }.

47. (b) ***Solution:***

x	1	2	4	5
y	20	10	5	4

From the table we observe that as x increases y decreases. This type of variation is called indirect variation.

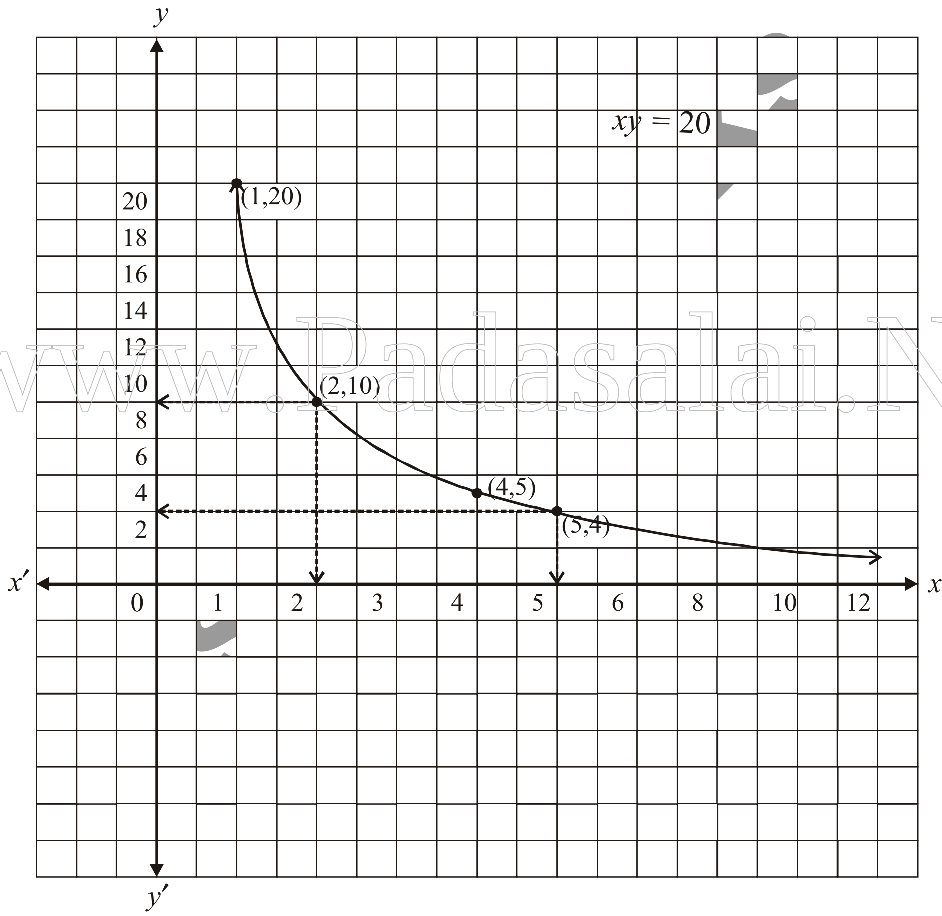
$y \ \alpha\ 1/x$ or $xy = k$ where k is a constant of proportionality. Also from the table we find that,

$1 \times 20 = 2 \times 10 = 4 \times 5 = 20 = k$

$\therefore$ We get $k = 20$

Plot the points (1, 20), (2, 10), (4, 5) and (5, 4) and join them.

Scale: x axis 2 cm = 1 unit
y axis 1 cm = 2 units



$\therefore$ The relation $xy = 20$ is a rectangular hyperbola as exhibited in the graph.

From the graph we find,

(i) When $x = 5, y = 4$

(ii) When $y = 10, x = 2$.

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