# Math 2311
## Review for Test 3
## Key

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1. True or False? Explain.
   a. For a fixed confidence level, when the sample size increases, the length of the confidence interval for a population mean decreases.
      True
   b. The z score corresponding to a 98 percent confidence level is 1.96.
      False, for 98% confidence z = 2.33
   c. The best point estimate for the population mean is the sample mean.
      True
   d. The larger the level of confidence, the shorter the confidence interval.
      False
   e. The margin of error can be computed from $\pm z^{*} \cdot \frac{\sigma}{\sqrt{n}}$
      True
   f. A statement contradicting the claim in the null hypothesis is classified as the power.
      False, the statement contradicting the claim in the null hypothesis is the alternative hypothesis.

   g. If we want to claim that a population parameter is different from a specified value, this situation can be considered as a one-tailed test.
      False, it is two-tail test. Alternative is "not equal to."

   h. In the P-value approach to hypothesis testing, if the P-value is less than a specified significance level, we fail to reject the null hypothesis.
      False, if the p-value is less than or equal to the specified level of significance then we would reject the null hypotheis.

   i. A 90% confidence interval for a population parameter means that if a large number of confidence intervals were constructed from repeated samples, then on average, 90% of these intervals would contain the true parameter.
      True

   j. The point estimate of a population parameter is always at the center of the confidence interval for the parameter.
      True for means and proportions.

2. Suppose that prior to conducting the coin-flipping experiment, we suspect that the coin is fair. How many times would we have to flip the coin in order to obtain a 90% confidence interval of width of at most 0.1 for the probability of flipping a head?

   This is to find the sample size: Use the formula $p*(1-p*)\left(\frac{z}{m}\right)^2 = 0.5(0.5)\left(\frac{1.645}{\frac{.1}{2}}\right)^2 = 270.6$

   round up to the next whole number so we will need at least 271 flips.

3. A certain beverage company is suspected of under filling its cans of soft drink. The company advertises that its cans contain, on the average, 12 ounces of soda with standard deviation 0.4 ounce. Compute the probability that a random sample of 50 cans produces a sample mean fill of 11.9 ounces or less.

   $P(\bar{X} \leq 11.9) = P(\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \leq \frac{11.9-12}{\frac{0.4}{\sqrt{50}}}) = P(Z \leq -1.7677) = 0.0384$ from the Z-table

   Using R:

```
> pnorm(11.9,12,.4/sqrt(50))
[1] 0.03854994
```

4. A Brinell hardness test involves measuring the diameter of the indentation made when a hardened steel ball is pressed into material under a standard test load. Suppose that the Brinell hardness is determined for each specimen in a sample of size 50, resulting in a sample mean hardness of 64.3 and a sample standard deviation of 6.0. Calculate a 99% confidence interval for the true average Brinell hardness for material specimens of this type.

   Point estimate = 64.3
   Confidence level = 99%
   Critical Value = t(df = 49) = 2.68 (In R: qt(1.99/2,49))
   Standard error = $6/\sqrt{50}$ = 0.8485
   Margin of error = 0.8485 * 2.68 = 2.274
   Confidence interval: (64.3 – 2.274, 64.3 + 2.274) = (62.026, 66.574)
   Interpret: We are 99% confident that the Brinell hardness for this type of steel ball is between 62.026 and 66.574.

5. The shear strength of anchor bolts has a standard deviation of 1.30. Assuming that the distribution is normal, how large a sample is needed to determine with a precision of ±0.5 the mean length of the produced anchor bolts to 99% confidence?

σ = 1.30, C = 99%, m = 0.5, z = 2.576

$$n = \left(\frac{z \times \sigma}{m}\right)^2 = \left(\frac{2.576 \times 1.3}{.5}\right)^2 = 44.85$$

We need a sample of **45**

6. The true average tread lives of two competing brands of radial tires (brand X and brand Y) are known to be normally distributed. The standard deviation of brand X tires is known to be 2200, and the standard deviation of brand Y tires is known to be 1900. A sample of 45 brand X tires results in a sample mean of 42,500 and sample standard deviation of 2450. A sample of 45 brand Y tires results in a sample mean of 40,400 and sample standard deviation of 2150. Find a 95% confidence interval for the difference in the true means, mean of X minus mean of Y.

   This is a two-sample z-interval because we are given the population standard deviations.

   Point estimate = $\bar{x}_1 - \bar{x}_2$ = 42,500 – 40,400 = 2100

   Confidence level = 95%

   Critical Value = z* = 1.96 (since both standard deviations of the population are known)

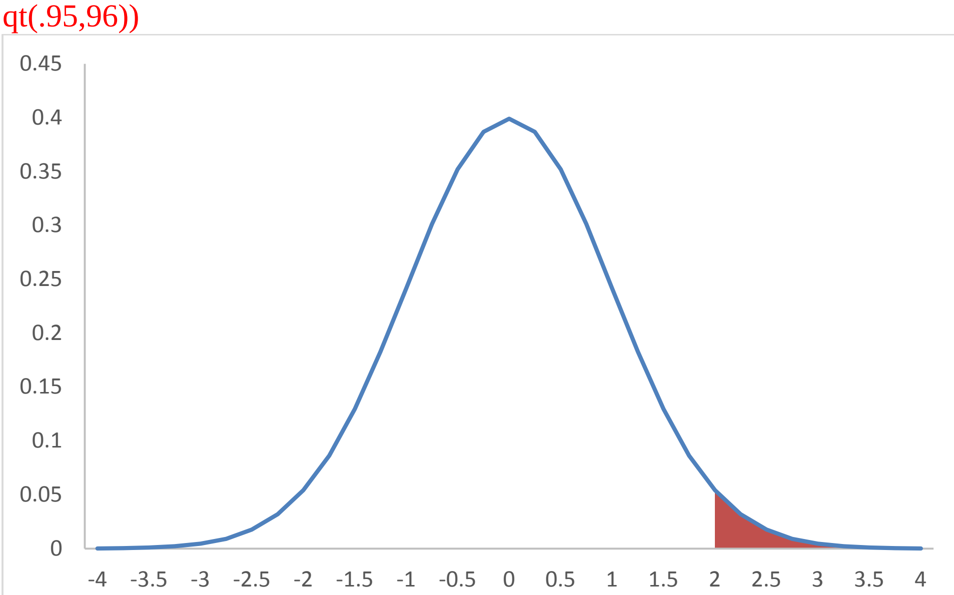
   Standard Error = $\sqrt{\frac{2200^2}{45} + \frac{1900^2}{45}}$ = 433.333

   Margin of error = 1.96(433.333) = 849.333

   Confidence Interval = [2100 – 849.333, 2100 + 849.333] = [1250.667, 2949.333]

7. A sample of 97 Duracell batteries produces a mean lifetime of 10.40 hours and standard deviation 4.83 hours. A sample of 148 Energizer batteries produces a mean lifetime of 9.26 hours and a standard deviation of 4.68 hours. At a 5% significance level, can we assert that the average lifetime of Duracell batteries is greater than the average lifetime of Energizer batteries?

Let population 1 = Duracell batteries and population 2 = Energizer batteries
This is a two-sample t-test because these are two *different* batteries.
Hypotheses: $H_0$: $\mu_1 = \mu_2$ and $H_a$: $\mu_1 > \mu_2$
Rejection Region: Reject if the test statistic is greater than 1.66 (In R use qt(.95,96))



Test statistic: $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{10.40 - 9.26}{\sqrt{\frac{4.83^2}{97} + \frac{4.68^2}{148}}} = 1.829$ RH0

P-value = P(T > 1.829) = 1- pt(1.829, 96) = 0.035 which is less than 0.05 so RH0
Conclusion: The average lifetime of Duracell batteries is significantly greater than the average lifetime of Energizer batteries. At 95% confidence.

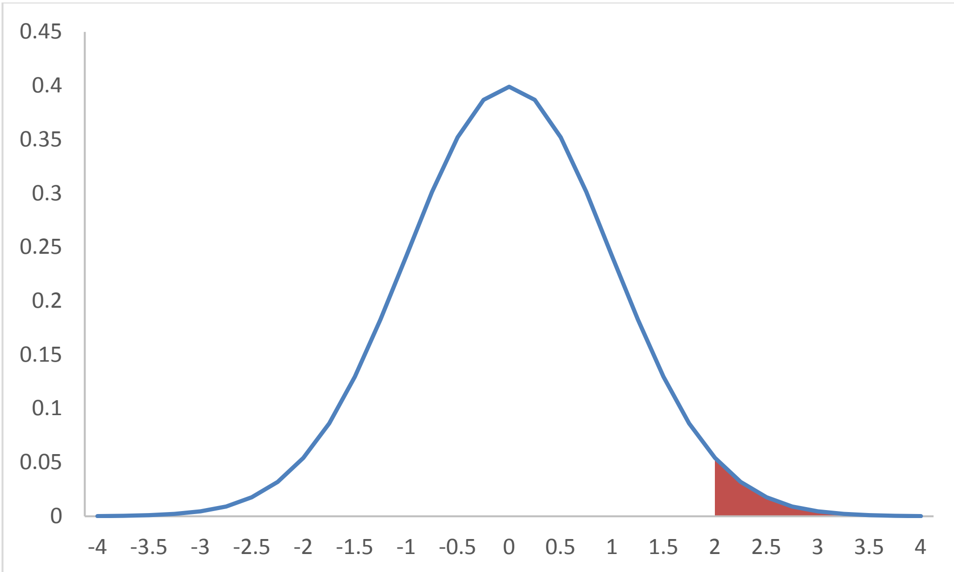
8. In a sample of 539 households from a certain Midwestern city, it was found that 133 of these households owned at least one firearm. Give a 99% confidence interval for the percentage of families in this city who own firearms.

This is a confidence interval for proportions (p).
Point estimate: 133/539 = 0.2468
Confidence level = 99%
Critical value = z = 2.576
Standard Error = $\sqrt{\frac{.2468 \times (1 - .2468)}{539}}$ = 0.01857
Margin of error = 2.576×0.01857 = 0.0478
Confidence Interval: (0.2468 – 0.0478, 0.2468 + 0.0478) = (0.199, 0.2946)
Interpret: We are 99% confident that the percent of families in this city who own firearms is between 20% and 29.5%.

9. In an experiment to study the effects of illumination level on performance, subjects were timed for completion in both a low light level and high light level. The results are below.

| | Subject | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| **Low Light** | 26 | 29 | 32 | 26 | 21 | 41 | 25 | 25 | 27 |
| **High Light** | 18 | 21 | 23 | 20 | 20 | 25 | 16 | 16 | 25 |

Can you say with 95% certainty that the average completion time is lower in high light?
This is a paired t-test because the subjects are the same using both lights.
Hypotheses: $H_0$: $\mu_d = 0$ and $H_a$: $\mu_d > 0$ (where differences = low light – high light)
Rejection Region: Reject the null hypothesis if the test statistic is greater than 1.859 (Used in R qt(0.95,8), if using the T-table it is df = 9 – 1 = 8, upper tail probability is 0.05 = 1 -0.95.




Test statistic: Find the differences of the low light – high light

I did this in R

```
> low = c(26,29,32,26,21,41,25,25,27)
> high = c(18,21,23,20,20,25,16,16,25)
> diff = low - high
> diff
[1]  8  8  9  6  1 16  9  9  2
```

Then you find the mean and standard deviation of the differences.

```
> mean(diff)
[1] 7.555556
> sd(diff)
[1] 4.390647
```

t = $\frac{\bar{x}_d - \mu_d}{s_d/\sqrt{n}} = \frac{7.5556-0}{4.390647/\sqrt{9}} = 5.162519$ RH0 since the test statistic is in the rejection region

P-value = P(T > 5.1625) = 1 – pt(5.1625,8) = 0.0004 RH0 since the p-value is less than 0.05 (the level of significance.

Conclusion: There is extremely strong evidence that the average completion time is lower in high light. At 95% confidence.

I can find the t and p-value using t.test in R:

```
> t.test(low,high,alternative = "greater",paired = TRUE)

  Paired t-test

data:  low and high
t = 5.1625, df = 8, p-value= 0.0004305
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
 4.834016      Inf
sample estimates:
mean of the differences
               7.555556
```

10. A study of the ability of individuals to walk in a straight line reported the accompanying data on cadence (strides per second for a sample of n = 20 randomly selected men).

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| .95 | .85 | .92 | .95 | .93 | .86 | 1.00 | .92 | .85 | .81 |
| .78 | .93 | .93 | 1.05 | .93 | 1.06 | 1.06 | .96 | .81 | .96 |

Assuming the standard deviation of the population is 0.08:

a. Find a 99% confidence interval for the mean cadence of the population.
b. Test the hypothesis that the mean cadence for the population is less than 0.97 at the 5% significance level.

This is a one sample z-test (confidence interval) because we know the population standard deviation.

a. Point estimate = sample mean of the data = 0.9255

Confidence level = 99%
Critical value = z* = 2.567 (In R qnorm(1.99/2))
Standard error = $\frac{\sigma}{\sqrt{n}} = \frac{0.08}{\sqrt{20}} = 0.01789$
Margin of error = 2.567(0.01789) = 0.0461
Confidence Interval : [0.9255 – 0.0461, 0.9255 + 0.0461] = [0.8794, 0.9716]
Interpret: We are 99% confident that the men cadence of the population is between 0.8794 and 0.9716.

b. Hypotheses: $H_0$: μ = 0.97 and $H_a$: μ < 0.97
Rejection region: Reject if the test statistic is less than – 1.645 (In R qnorm(0.05) since α = 0.05)
Test statistic: $z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{0.9255 - 0.97}{\frac{0.08}{\sqrt{20}}} = -2.5$ RH0 since the test statistic is in the rejection region.
P-value = P(Z < -2.5) = 0.0062 (In R pnorm(-2.5)) RH0 since the p-value is less than α = 0.05
Conclusion: There is extremely strong evidence that the mean cadence for the population is less than 0.97 at 95% confidence.

11. Bottles of a popular cola drink are supposed to contain 300 ml of cola. There is some variation from bottle to bottle because the filling machinery is not perfectly precise. The distribution of the contents is normal with standard deviation of 3 ml. A student who suspects that the bottler is under-filling measures the contents of six bottles. The results are:

| | | | | | |
|---|---|---|---|---|---|
| 299.4 | 297.7 | 301.0 | 298.9 | 300.2 | 297.0 |

Is this convincing evidence that the mean contents of cola bottles is less than the advertised 300 ml? Test at the 5% significance level.

This is a one sample z-test because we are given the population standard deviation is 3 ml.
$H_0$: μ = 300 and $H_a$: μ < 300
Test statistic = z = $\frac{299.033 - 300}{3/\sqrt{6}} = -0.7895$ (used z because we know the population standard

deviation)
Rejection region is any z < -1.645 FRH0
P-value = P(Z < -0.7895) = 0.2149
Decision: Since the p-value is greater than 0.05, we fail to reject the null hypothesis.
Conclusion: We say that there is not enough evidence that the null hypothesis is false. Thus there is ***not*** convincing evidence that the mean contents of cola bottle is less than the advertised 300 ml.

12. The guidance office of a school wants to test the claim of an SAT test preparation company that students who complete their course will improve their SAT Math score by at least 50 points. Ten members of the junior class who have had no SAT preparation but have taken the SAT once were selected at random and agreed to participate in the study. All took the course and re-took the SAT at the next opportunity. The results of the testing indicated:

| Student | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Before | 475 | 512 | 492 | 465 | 523 | 560 | 610 | 477 | 501 | 420 |
| After | 500 | 540 | 512 | 530 | 533 | 603 | 691 | 512 | 489 | 458 |

Is there sufficient evidence to support the prep course company's claim that scores will improve at the 1% level of significance?
This is a paired t-test because the students are the same in the before and after
Hypotheses: $H_0$: $\mu_d = 0$ and $H_a$: $\mu_d < 0$ (where differences = before - after)
Rejection Region: Reject the null hypothesis if the test statistic is less than -2.821 (Used in R qt(0.01,9), if using the T-table it is df = 10 – 1 = 9, upper tail probability is 0.01.
Test statistic:
Need to find the differences first:

```
> before = c(475,512,492,465,523,560,610,477,501,420)
> after = c(500,540,512,530,533,603,691,512,489,458)
> diff = before - after
> diff
 [1] -25 -28 -20 -65 -10 -43 -81 -35  12 -38
```

Then find the mean and standard deviation of the differences:

```
> mean(diff)
[1] -33.3
> sd(diff)
[1] 26.39044
```

$t = \frac{\bar{x}_d - \mu_d}{s_d/\sqrt{n}} = \frac{-33.3-0}{26.39044/\sqrt{10}} = -3.99$ RH0 since the test statistic is in the rejection region.

$P-value = P(T < -3.99) = pt(-3.99,9) = 0.0016$ RH0 since the p-value is less than 0.01.

Conclusion: There is strong evidence that the mean test scores before the test is significantly less than after the test, at 99% confidence. This implies that the prep course company's claim is true.

I can use t.test in R to find test statistic and p-value:

```
> t.test(before, after,alternative = "less", conf.level = 0.99, paired = TRUE)

        Paired t-test

data:  before and after
t = -3.9902, df = 9, p-value = 0.001578
alternative hypothesis: true difference in means is less than 0
99 percent confidence interval:
      -Inf -9.753997
```

```
sample estimates:
mean of the differences
                  -33.3
```

13. A random sample of 200 freshmen and 100 seniors at Upper Wabash Tech are asked whether they agree with a plan to limit enrollment in crowded majors as a way of keeping the quality of instruction high. Of the students sampled, 160 freshmen and 20 seniors opposed the plan. We want to determine if there is any difference between the proportion of freshmen who oppose the plan and the proportion of seniors who oppose it.
    a. Formulate the null and alternative hypothesis.
    b. Compute the appropriate test statistic.
    c. Determine the p-value.
    d. Do you reject $H_0$ or fail to reject $H_0$? Explain.
    e. Describe your results for someone who has no training in statistics.
    f. Find a 95% confidence interval for the difference between the population proportions.

This is a two-sample proportions z-test

Let population 1 = freshman and population 2 = seniors

a. $H_0$: $p_1 = p_2$ and $H_a$: $p_1 \neq p_2$

b. $\hat{p}_1 = 160/200 = 0.8$, $\hat{p}_2 = 20/100 = 0.2$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}} = \frac{0.8 - 0.2}{\sqrt{\frac{0.8(1-0.8)}{200} + \frac{0.2(1-0.2)}{100}}} = 12.24745$$

c. P-value = P(Z < - 12.24745 or Z > 12.24745) ≈ 0

d. Since the p-value is very small (approx. 0) we reject the null hypothesis.

e. We have extremely strong evidence that there is a difference in the proportion of freshmen who oppose the plan and the proportion of seniors who oppose it.

f. 95% confidence interval:

$$\hat{p}_1 - \hat{p}_2 \pm z * \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$(0.8 - 0.2) \pm 1.96\sqrt{\frac{0.8(1-0.8)}{200} + \frac{0.2(1-0.2)}{100}}$$

$$0.6 \pm 0.096$$

95% confidence interval is [0.6 – 0.096, 0.6 + 0.096] = [0.504, 0.696]

14. It is fourth down and a yard to go for a first down in an important football game. The football coach must decide whether to go for the first down or punt the ball away. The null hypothesis is that the team will not get the first down if they go for it. The coach will make a Type I error by doing what?

Type I error is rejecting the null hypothesis when in fact it is true. So this means that if the coach makes a Type I error that he went for the first down but did not get the first down.

15. In a recent publication, it was reported that the average highway gas mileage of tested models of a new car was 33.5 mpg and approximately normally distributed. A consumer group conducts its own tests on a simple random sample of 12 cars of this model and finds that the mean gas mileage for their vehicles is 31.6 mpg with a standard deviation of 3.4 mpg.

a. Perform a test to determine if these data support the contention that the true mean gas mileage of this model of car is different from the published value.
$H_0$: μ = 33.5 and $H_a$: μ ≠ 33.5
Test statistic = t = $\frac{31.6-33.5}{3.4/\sqrt{12}} = -1.9358$
P-value = P(t < -1.9358 or t > 1.9358) = 2 * pt(-1.9358,11) = 0.079 FRH0
Conclusion: there is not enough evidence to conclude that the mean gas mileage of the model of car is different from 33.5

b. Perform a test to determine if these data support the contention that the true mean gas mileage of this model of car is less than the published value.
$H_0$: μ = 33.5 and $H_a$: μ < 33.5
Test statistic = t = $\frac{31.6-33.5}{3.4/\sqrt{12}} = -1.9358$
P-value = P(t < -1.9358 ) = pt(-1.9358,11) = 0.0395 RH0
Conclusion: there is enough evidence to conclude that the mean gas mileage of the model of car is significantly less than 33.5

c. Explain why the answers to part a and part b are different.
In part a our alternative is "not equal to" so this is a two-tail test and the p-value will be twice the amount than in a one tailed test like it is in part b.

16. A random sample of size 36 selected from a normal distribution with σ = 4 has $\bar{x} = 75$. A second random sample of size 25 selected from a different normal distribution with σ = 6 has $\bar{x} = 85$. Is there a significant difference between the two population means at the 5% level of significance?

This is a two-sample z-test since we are given the population standard deviations.
Let the first random sample be from population 1 and second random sample be from population 2.

Hypotheses: $H_0$: $\mu_1 = \mu_2$ and $H_a$: $\mu_1 \neq \mu_2$
Rejection Region: Reject the null hypothesis if the test statistic is less than -1.96 or greater than 1.96. (In R qnorm(0.05/2,24) divide by 2 because it is a two tail test).
Test statistic: $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} = \frac{75-85}{\sqrt{\frac{4^2}{36}+\frac{6^2}{25}}} = -7.2846$ RH0 since the test statistic is in the rejection region.
P-value = P(Z < -7.2846 or Z > 7.2846) ≈ 0 RH0 the p-value is extremely small.
Conclusion: There is extremely strong evidence that there is a significant difference between the two population means at 95% confidence.

17. A study was conducted to determine whether remediation in basic mathematics enabled students to be more successful in an elementary statistics course. (Success here means C or better.) Here are the results of the study:

| | Remedial | Non-remedial |
|---|---|---|
| Sample size | 100 | 40 |
| # of successes | 70 | 16 |

Test, at the 5% level, whether the remediation helped the students to be more successful.

This is a two-sample z proportions test
Let population 1 = Remedial students, population 2 = Non-remedial students
Hypotheses: $H_0$: $p_1 = p_2$ and $H_a$: $p_1 > p_2$
Rejection Region: Reject if the test statistic is greater than 1.645
$\hat{p}_1$ = 70/100 = 0.7, $\hat{p}_2$ = 16/40 = 0.4

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}} = \frac{0.7 - 0.4}{\sqrt{\frac{0.7(1-0.7)}{100} + \frac{0.4(1-0.4)}{40}}} = 3.3333$$

Reject H0 because the test statistic is in the rejection region.
P-value = P(Z > 3.3333) = 0.0004 (In R 1 – pnorm(3.3333))
Since the p-value is very small we reject the null hypothesis.
We have extremely strong evidence that the mean test scores for the remedial students are significantly higher than the mean test scores for the non-remedial students.

18. A preacher would like to establish that of people who pray, less than 80% pray for world peace. In a random sample of 110 persons who pray, 77 of them said that when they pray, they pray for world peace. Test at the 10% level.
This is a one sample proportion z – test
$H_0$: p = 0.8 and $H_a$: p < 0.8 $\hat{p} = \frac{77}{110} = 0.7$
Test statistic = z = $\frac{0.7-0.8}{\sqrt{\frac{.8(1-0.8)}{110}}} = -2.622$
Rejection Region: P(Z < c) =0.1 qnorm(0.1) = -1.28, reject the null hypothesis if the test statistic is less than -1.28 (draw the normal curve) RH0
P-value = P(t < -2.622) = pnorm(-2.622) = 0.0044 RH0 this is less than 0.1.
Conclusion: There is strong evidence that the proportion of people who pray for world peace is significantly less than 80%.

**Don't forget to take practice test 3!!!!!**