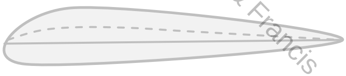
Aerodynamics



Aerodynamics Symbol List

Symbol	Definition	Units
a	speed of sound	$(length)/(time)$
a_s	speed of sound at sea level	$(length)/(time)$
A	area	$(length^2)$
AR	aspect ratio	--------
b	wing span	$(length)$
c	chord length	$(length)$
$\bar{c}$	mean aerodynamic chord	$(length)$
c_p	specific heat at constant pressure of air	$\frac{(energy)}{(mass)\cdot(temperature)}$
c_r	root chord	$(length)$
c_t	tip chord	$(length)$
c_v	specific heat at constant volume of air	$\frac{(energy)}{(mass)\cdot(temperature)}$
$c_{1/4}$	quarter chord	$(length)$
C_d	total drag coefficient	--------
$C_{d,i}$	induced drag coefficient	--------
$C_{d,p}$	parasite drag coefficient	--------
$C_{d,w}$	wave drag coefficient	--------
C_{fx}	local skin friction coefficient	--------
C_l	lift coefficient	--------
$C_{l,c}$	compressible lift coefficient	--------
$C_{m,c}$	compressible moment coefficient	--------
$C_{m,p}$	pitching moment coefficient	--------
$C_{m,r}$	rolling moment coefficient	--------
$C_{m,y}$	yawing moment coefficient	--------

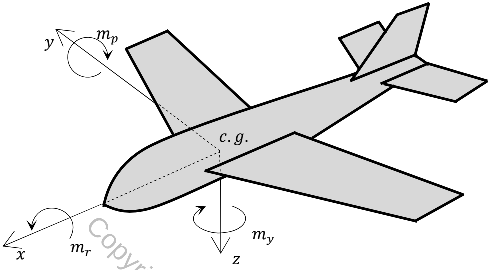
Aerodynamics Symbol List (cont.)

Symbol	Definition	Units
C_p	pressure coefficient	--------
$C_{p,c}$	compressible pressure coefficient	--------
$C_{p,cr}$	critical pressure coefficient	--------
$C_{p,ss}$	supersonic pressure coefficient	--------
D	total drag	$(force)$
D_i	induced drag	$(force)$
D_p	parasite drag	$(force)$
e	span efficiency factor	--------
L	lift	$(force)$
m_p	pitching moment	$(force) \cdot (length)$
m_r	rolling moment	$(force) \cdot (length)$
m_y	yawing moment	$(force) \cdot (length)$
M	mach number	--------
M_{cr}	critical mach number	--------
M_∞	free stream mach number	--------
P	static pressure	$(force)/(length^2)$
P_0	total pressure	$(force)/(length^2)$
P_∞	free stream pressure	$(force)/(length^2)$
q	dynamic pressure	$(force)/(length^2)$
R	specific gas constant of air	$\frac{(energy)}{(mass) \cdot (temperature)}$
Re_x	local Reynolds number	--------
S	wing area	$(length^2)$
T	temperature	$(temperature)$
T_0	total temperature	$(temperature)$
V	speed	$(length)/(time)$

Aerodynamics Symbol List (cont.)

Symbol	Definition	Units
V_c	calibrated airspeed	$(length)/(time)$
V_e	equivalent airspeed	$(length)/(time)$
V_t	true airspeed	$(length)/(time)$
V_∞	free stream velocity	$(length)/(time)$
x	variable distance starting from leading edge	$(length)$
α	angle of attack	Radian
α_i	induced angle of attack	$(angle)$
Λ_{LE}	sweep angle of leading edge	$(angle)$
Λ_{TE}	sweep angle of trailing edge	$(angle)$
$\Lambda_{c/4}$	sweep angle of quarter chord	$(angle)$
β	shock angle	Radian
γ	specific heat ratio	--------
δ	boundary layer thickness	$(length)$
θ	deflection angle	Radian
λ	taper ratio	--------
μ_∞	dynamic viscosity	$\frac{(mass)}{(length)\cdot(time)}$
ρ	density of air	$(mass)/(length^3)$
ρ_0	total density of air	$(mass)/(length^3)$
ρ_s	sea level density of air	$(mass)/(length^3)$
ρ_∞	free stream density of air	$(mass)/(length^3)$

Airplane Axis System



$m_p = positive\ pitching\ moment$
$m_r = positive\ rolling\ moment$
$m_y = positive\ yawing\ moment$

Properties of Air

Specific Heat at Constant Pressure of Air

$$c_p = 1004.5\ \frac{J}{kg \cdot K} = 6006\ \frac{ft \cdot lb}{slug \cdot {}^oR}$$

Specific Heat at Constant Volume of Air

$$c_v = 717.5\ \frac{J}{kg \cdot K} = 4290\ \frac{ft \cdot lb}{slug \cdot {}^oR}$$

Specific Heat Ratio of Air

$$\gamma = \frac{c_p}{c_v} = 1.4$$

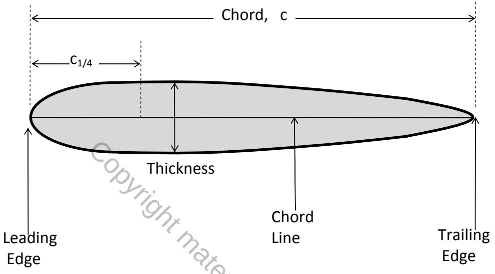
Specific gas constant of air

$$R = 287\ \frac{J}{kg \cdot K} = 1716\ \frac{ft \cdot lb}{slug \cdot {}^oR}$$

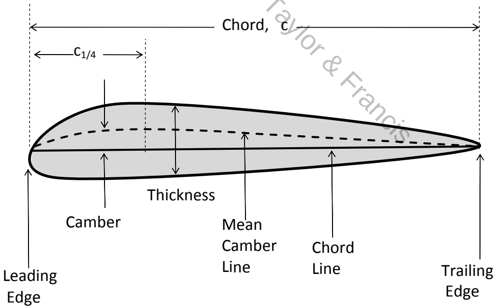
Airfoils

Geometry and Nomenclature

Symmetric Airfoil



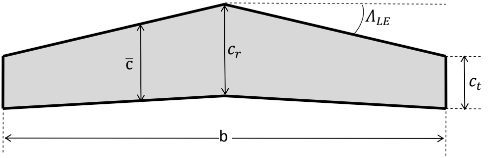
Cambered Airfoil

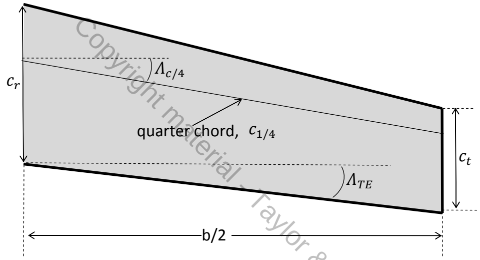


Camber and Thickness are functions of x

Wings

Geometry and Nomenclature of Trapezoidal Planform





Taper Ratio

$$\lambda = \frac{c_t}{c_r}$$

Aspect Ratio

$$AR = \frac{b^2}{S} = \frac{2 \cdot b}{c_r \cdot (1 + \lambda)}$$

Mean Aerodynamic Chord

$$\bar{c} = \frac{2}{3} \cdot \left(c_r + c_t - \frac{c_r \cdot c_t}{c_r + c_t} \right)$$

Area

$$S = \frac{b^2}{AR}$$

Approximate Speed Regimes

Subsonic Incompressible Flow

$M_\infty < 0.3$

Subsonic Compressible Flow

$0.3 < M_\infty < 0.7$

Transonic Flow

$0.7 < M_\infty < 1.2$

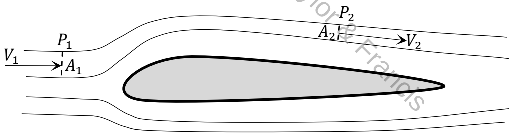
Supersonic Flow

$1.2 < M_\infty < 5$

Hypersonic Flow

$5 < M_\infty$

Subsonic Incompressible Flow



Continuity Equation (Mass Flow Rate)

$A_1 \cdot V_1 = A_2 \cdot V_2$

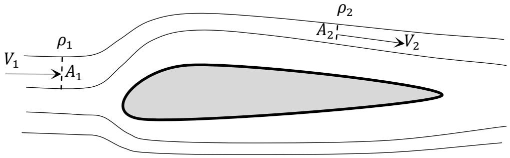
$\rho_1 = \rho_2$

Bernoulli's Equation

$P + \frac{1}{2} \cdot \rho \cdot V^2 = P_0 = \text{constant along stream line}$

$P_1 + \frac{1}{2} \cdot \rho \cdot {V_1}^2 = P_2 + \frac{1}{2} \cdot \rho \cdot {V_2}^2$

Subsonic Compressible Flow



For flow that is isentropic (adiabatic and reversible)

State Equation

$P_1 = \rho_1 \cdot R \cdot T_1$
$P_2 = \rho_2 \cdot R \cdot T_2$

Continuity Equation (Mass Flow Rate)

$\rho_1 \cdot A_1 \cdot V_1 = \rho_2 \cdot A_2 \cdot V_2$

Isentropic Compressible Flow (see Appendix D)

$$\frac{P_{0_1}}{P_1} = \left(\frac{\rho_{0_1}}{\rho_1}\right)^{\gamma} = \left(\frac{T_{0_1}}{T_1}\right)^{\frac{\gamma}{(\gamma-1)}} = \left(1 + \frac{\gamma-1}{2} \cdot {M_1}^2\right)^{\frac{\gamma}{(\gamma-1)}}$$

$$\frac{P_{0_2}}{P_2} = \left(\frac{\rho_{0_2}}{\rho_2}\right)^{\gamma} = \left(\frac{T_{0_2}}{T_2}\right)^{\frac{\gamma}{(\gamma-1)}} = \left(1 + \frac{\gamma-1}{2} \cdot {M_2}^2\right)^{\frac{\gamma}{(\gamma-1)}}$$

Isentropic Conditions along Flow

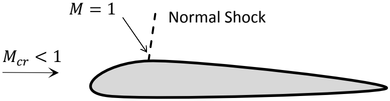
$$\frac{\left(P_{0_2}/P_2\right)^{-1}}{\left(P_{0_1}/P_1\right)^{-1}} = \left(\frac{\left(\rho_{0_2}/\rho_2\right)^{-1}}{\left(\rho_{0_1}/\rho_1\right)^{-1}}\right)^{\gamma} = \left(\frac{\left(T_{0_2}/T_2\right)^{-1}}{\left(T_{0_1}/T_1\right)^{-1}}\right)^{\frac{\gamma}{(\gamma-1)}}$$

$$= \frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{(\gamma-1)}} = \left(\frac{1 + \frac{\gamma-1}{2} \cdot {M_1}^2}{1 + \frac{\gamma-1}{2} \cdot {M_2}^2}\right)^{\frac{\gamma}{(\gamma-1)}}$$

Energy Equation

$$c_p \cdot T_1 + \frac{1}{2} \cdot {V_1}^2 = c_p \cdot T_2 + \frac{1}{2} \cdot {V_2}^2$$

Transonic Flow



Critical Pressure Coefficient

$$C_{p,cr} = \frac{2}{\gamma \cdot (M_{cr})^2} \cdot \left(\left(\frac{2 + (\gamma - 1) \cdot (M_{cr})^2}{\gamma + 1} \right)^{\frac{\gamma}{(\gamma-1)}} - 1 \right)$$

Supersonic Flow

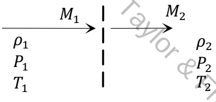
Speed of Sound

$$\mathrm{a} = \sqrt{\gamma \cdot R \cdot T}$$

Mach Number

$$\mathrm{M} = \frac{V}{\mathrm{a}}$$

Normal Shock Waves



Density, Pressure, and Temperature Ratios (see Appendix E)

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1) \cdot {M_1}^2}{(\gamma - 1) \cdot {M_1}^2 + 2}$$

$$\frac{P_2}{P_1} = 1 + \frac{2 \cdot \gamma}{\gamma + 1} \cdot \left({M_1}^2 - 1\right)$$

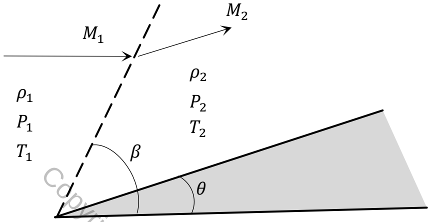
$$\frac{T_2}{T_1} = \frac{\left(2 \cdot \gamma \cdot {M_1}^2 - (\gamma - 1)\right) \cdot \left((\gamma - 1) \cdot {M_1}^2 + 2\right)}{(\gamma + 1)^2 \cdot {M_1}^2}$$

Mach Number after Shockwave (see Appendix E)

$$M_2 = \sqrt{\frac{(\gamma - 1) \cdot {M_1}^2 + 2}{2 \cdot \gamma \cdot {M_1}^2 - (\gamma - 1)}}$$

Supersonic Flow (cont.)

Oblique Shock Waves



Deflection Angle (Radians)

$$\theta = \beta - \arctan\left(\frac{tan(\beta)\cdot(\gamma - 1)\cdot {M_1}^2 \cdot sin^2(\beta) + 2}{(\gamma + 1)\cdot {M_1}^2 \cdot sin^2(\beta)}\right)$$

Density, Pressure, and Temperature Ratios

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)\cdot {M_1}^2 \cdot sin^2(\beta)}{(\gamma - 1)\cdot {M_1}^2 \cdot sin^2(\beta) + 2}$$

$$\frac{P_2}{P_1} = 1 + \frac{2\cdot\gamma}{\gamma + 1}\cdot\left({M_1}^2 \cdot sin^2(\beta) - 1\right)$$

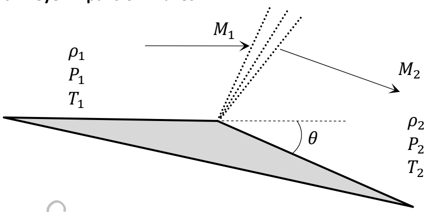
$$\frac{T_2}{T_1} = 1 + \frac{2\cdot(\gamma - 1)\cdot\left({M_1}^2\cdot sin^2(\beta) - 1\right)}{(\gamma + 1)^2\cdot {M_1}^2\cdot sin^2(\beta)}\cdot\left(\gamma\cdot {M_1}^2\cdot sin^2(\beta) + 1\right)$$

Mach Number after Shockwave

$$M_2 = \frac{1}{sin(\beta - \theta)}\cdot\sqrt{\frac{1 + \frac{\gamma - 1}{2}\cdot {M_1}^2 \cdot sin^2(\beta)}{\gamma\cdot {M_1}^2 \cdot sin^2(\beta) - \frac{\gamma - 1}{2}}}$$

Supersonic Flow (cont.)

Prandtl-Meyer Expansion Waves



Isentropic Ratios (see Appendix D)

$$\frac{\rho_2}{\rho_1} = \frac{\left(\rho_{0_2}/\rho_2\right)^{-1}}{\left(\rho_{0_1}/\rho_1\right)^{-1}} = \left(\frac{1+\frac{\gamma-1}{2}\cdot {M_1}^2}{1+\frac{\gamma-1}{2}\cdot {M_2}^2}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{P_2}{P_1} = \frac{\left(P_{0_2}/P_2\right)^{-1}}{\left(P_{0_1}/P_1\right)^{-1}} = \left(\frac{1+\frac{\gamma-1}{2}\cdot {M_1}^2}{1+\frac{\gamma-1}{2}\cdot {M_2}^2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{T_2}{T_1} = \frac{\left(T_{0_2}/T_2\right)^{-1}}{\left(T_{0_1}/T_1\right)^{-1}} = \frac{1+\frac{\gamma-1}{2}\cdot {M_1}^2}{1+\frac{\gamma-1}{2}\cdot {M_2}^2}$$

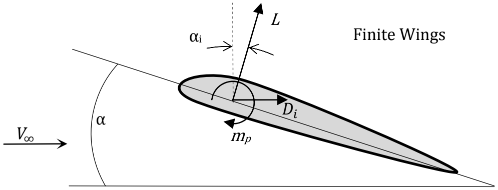
Solving for Mach Number after Expansion Fan

$$\zeta = \sqrt{\frac{\gamma+1}{\gamma-1}}\cdot tan^{-1}\left(\sqrt{\frac{\gamma-1}{\gamma+1}\cdot\left({M_1}^2-1\right)}\right) - tan^{-1}\left(\sqrt{\left({M_1}^2-1\right)}\right) + \theta$$

$$\zeta = \sqrt{\frac{\gamma+1}{\gamma-1}}\cdot tan^{-1}\left(\sqrt{\frac{\gamma-1}{\gamma+1}\cdot\left({M_2}^2-1\right)}\right) - tan^{-1}\left(\sqrt{\left({M_2}^2-1\right)}\right)$$

Determine M_2 by inputting different values of M_2 until both parameters are equal

Aerodynamic Coefficients



Lift Coefficient

$$C_l = \frac{L}{\frac{1}{2} \cdot \rho_\infty \cdot {V_\infty}^2 \cdot S}$$

Parasite Drag Coefficient

$$C_{d,p} = \frac{D_p}{\frac{1}{2} \cdot \rho_\infty \cdot {V_\infty}^2 \cdot S}$$

Induced Drag Coefficient

$$C_{d,i} = \frac{D_i}{\frac{1}{2} \cdot \rho_\infty \cdot {V_\infty}^2 \cdot S} = \frac{L \cdot sin(\alpha_i)}{\frac{1}{2} \cdot \rho_\infty \cdot {V_\infty}^2 \cdot S}$$

$$for\ small\ \alpha_i: \quad C_{d,i} = \frac{{C_l}^2}{\pi \cdot e \cdot AR}$$

Wave Drag Coefficient (approx. for flat-plate and low angles of attack)

$$C_{d,w} = \frac{4 \cdot \alpha^2}{\sqrt{{M_\infty}^2 - 1}}$$

Note: α is in Radians

Total Drag Coefficient

$$C_d = \frac{D}{\frac{1}{2} \cdot \rho_\infty \cdot {V_\infty}^2 \cdot S} = C_{d,p} + C_{d,i} + C_{d,w}$$

For subsonic speeds, wave drag equals 0

Aerodynamic Coefficients (cont.)

Pressure Coefficient

$$C_p = \frac{P - P_\infty}{\frac{1}{2} \cdot \rho_\infty \cdot {V_\infty}^2}$$

Pitching Moment Coefficient

$$C_{m,p} = \frac{m_p}{\frac{1}{2} \cdot \rho_\infty \cdot {V_\infty}^2 \cdot S \cdot \bar{c}}$$

Rolling Moment Coefficient

$$C_{m,r} = \frac{m_r}{\frac{1}{2} \cdot \rho_\infty \cdot {V_\infty}^2 \cdot S \cdot b}$$

Yawing Moment Coefficient

$$C_{m,y} = \frac{m_y}{\frac{1}{2} \cdot \rho_\infty \cdot {V_\infty}^2 \cdot S \cdot b}$$

Subsonic Compressibility Correction to Lift, Pressure, and Moment Coefficients

$$C_{l,c} = \frac{C_l}{\sqrt{1 - {M_\infty}^2}} \qquad C_{p,c} = \frac{C_p}{\sqrt{1 - {M_\infty}^2}}$$

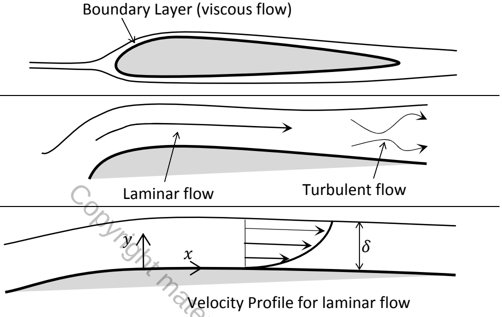
$$C_{m,c} = \frac{C_m}{\sqrt{1 - {M_\infty}^2}}$$

Supersonic Pressure Coefficient

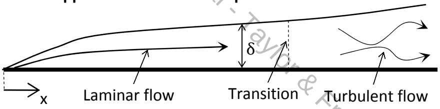
$$C_{p,ss} = \frac{2}{\gamma \cdot {M_1}^2} \cdot \left(\left(\frac{1 + \frac{1}{2} \cdot (\gamma - 1) \cdot {M_1}^2}{1 + \frac{1}{2} \cdot (\gamma - 1) \cdot {M_2}^2} \right)^{\frac{\gamma}{(\gamma - 1)}} - 1 \right)$$

Subsonic Viscous Flow

Boundary Layer over Airfoil



Flat Plate Approximation for Incompressible Flow



Local Reynolds Number

$$Re_x = \frac{\rho_\infty \cdot V_\infty \cdot x}{\mu_\infty}$$

Boundary Layer Thickness for Laminar Flow

$$\delta = \frac{5.2 \cdot x}{\sqrt{Re_x}}$$

Local Skin Friction Coefficient for Laminar Flow

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}}$$

Transition Conditions

$$Re_x \approx 3.5 \cdot 10^5 \rightarrow 4 \cdot 10^6$$

Airspeed Measurements

Subsonic Incompressible Flow

True Airspeed

$$V_t = \sqrt{\frac{2 \cdot (P_0 - P_\infty)}{\rho_\infty}}$$

Equivalent Airspeed

$$V_e = \sqrt{\frac{2 \cdot (P_0 - P_\infty)}{\rho_s}} = V_t \cdot \sqrt{\frac{\rho_\infty}{\rho_s}}$$

Subsonic Compressible Flow

True Airspeed

$$V_t = \sqrt{\frac{2 \cdot \mathrm{a_\infty}^2}{\gamma - 1} \cdot \left(\left(\frac{P_0}{P_\infty} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right)}$$

$$= \sqrt{\frac{2 \cdot \mathrm{a_\infty}^2}{\gamma - 1} \cdot \left(\left(\frac{P_0 - P_\infty}{P_\infty} + 1 \right)^{\frac{\gamma-1}{\gamma}} - 1 \right)}$$

Calibrated Airspeed

$$V_c = \sqrt{\frac{2 \cdot \mathrm{a}_s^2}{\gamma - 1} \cdot \left(\left(\frac{P_0 - P_\infty}{P_s} + 1 \right)^{\frac{\gamma-1}{\gamma}} - 1 \right)}$$

Notes

Notes