Magnetic Dipole Moment

Goal

- To determine the strength of Earth's magnetic field in Dallas.
- To determine the magnetic dipole moment of a magnet.

Equipment

Helmholtz Coils, multimeter functioning as an ammeter, DC power supply, ruler, stopwatch, cylindrical magnet, compass, thread, triple beam balance, Vernier caliper, and micrometer.

The Big Picture

It is a curious state of affairs, not fully understood, that Earth behaves in many ways like a giant, spinning bar magnetic. This is shown schematically in figure 2. The particular curvey shape of a bar magnet's magnetic field is referred to as a "dipole" field since such a magnet has 2 magnetic poles, a "north" pole and a "south" pole. The only real difference between the two poles is whether or not the magnetic field lines start from one or end on one. Magnetic field lines start out *from* a magnetic *north* pole but always *end* on a magnetic *south* pole. Earth's magnetic field is not *exactly* like a dipole field, but very close to it. You will also notice from the figure that the axis of Earth's magnetic field is cocked somewhat from Earth's axis of rotation. (This is often true of other astronomical objects like other planets.) Additionally, notice that the *geographic North pole* is a *magnetic south pole.* and a *geographic South pole* is a *magnetic north pole.* This confusing state of affairs is due to the fact that the direction of the geographic north pole was set before people knew much about magnetism. So, just remember that when your compass is pointing to geographic North, it is pointing to where the magnetic field lines are headed.

Although Earth's magnetic field is very weak compared to the magnets you stick on your fridge, it produces interesting effects. The eerie lights seen at far northern and southern latitudes (*aurora borealis* and *aurora australis*) are partly a consequence of Earth's magnetic field. There is strong evidence that the navigational ability of pigeons is due to their ability to sense variations in the direction and magnitude of Earth's magnetic field.

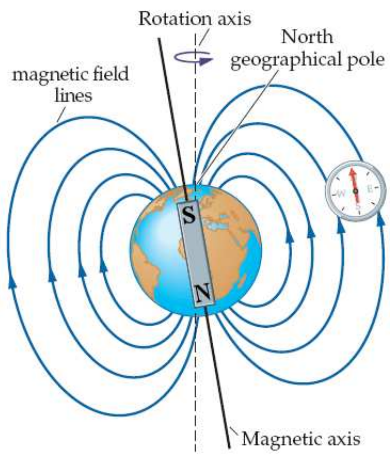

Figure 1: The shape of earth's dipole magnetic field. Note the 11° or so angular difference between the magnetic field axis and the axis the earth spins around. You will also see that Earth's *geographic* north pole is a *magnetic* south pole. Tricky!

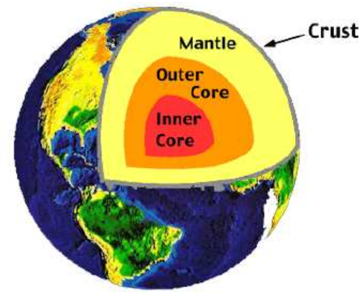

Figure 2: Cut away view of Earth's structure. Its magnetic field is due to a combination of liquid metal motion at the outer core and Earth's rotation. The complete process is not understood.

Magnetic fields are produced by the motion of electrically charged particles. A bar magnet's field results from the motion of negatively charged electrons inside it. Could Earth's magnetic field be due to some kind of a large bar magnet at its center? No. The temperature at Earth's center is too large (5700 °C to support the kind of motion required of electrons in a bar magnet to produce a magnetic field. You can get a bar magnet to lose its magnetism simply by heating it above its "Curie" temperature, typically around 700 °C, or so. The origin of our planet's magnetic field is not understood in detail although geophysicists agree on the basic idea. The field is thought to be due to the motion of the liquid metallic outer core of iron and nickel from Earth's rotation and changes in the core's temperature but the details are murky.

Unlike the magnets you stick on the fridge, the polarity of Earth's magnetic field reverses. That is, the positions of the magnetic north and south pole flip. The North pointing needle end on your handy compass doesn't always point in the same direction. The geologic record shows that over the past 20 million or so years, these field reversals occur every 200 000-300 000 years and when they do occur, it takes several hundreds or thousands of years to complete. The bibliographys has a useful link.

The magnetic dipole moment of a substance (how well it acts as a magnet) can be determined by suspending a sample of the substance on a torsion fiber and measuring the period of the oscillations of the sample in an applied magnetic field. The larger the magnetic dipole moment, the faster the oscillation.

In this experiment, a permanent magnet in the shape of a thin cylindrical rod is suspended from a rigid support by a cotton thread. The cylindrical axis of the magnet is in

Figure 3: Northern lights ("*aurora borealis*") seen in Alaska. Charged particles from the Sun, mostly electrons, are funneled at both geographics poles by earth's magnetic field toward the ground and collide with air molecules in the atmosphere along the way. The molecules eventually de-excite and emit light in doing so. In some sense, the northern lights act like an old-fashioned neon sign. The green light is from oxygen undergoing de-excitation. Nitrogen produces blue or red colors. The equivalent phenomenon at the South pole is the *"aurora australis."*

the horizontal plane, and thus the plane of oscillation is also the horizontal plane. The equilibrium direction of the magnet is determined by the horizontal component of Earth's magnetic field; that is, the magnet acts as a compass and aligns itself north-south.

When the magnet is displaced from its equilibrium direction, it oscillates in simple harmonic motion. The period of the oscillation depends on the magnetic dipole moment of the magnet and on the strength of the magnetic field; thus, if an additional magnetic field is applied the period of oscillation can be changed. By measuring the period as a function of the applied field and plotting a graph of the results, one obtains a straight line from which the magnetic moment of the magnet and the horizontal component of Earth's magnetic field can be calculated.

Theory

The equation of motion for the magnetic torsional oscillator is gven by

$$I\frac{d\boldsymbol{\omega}}{dt} = \boldsymbol{N_F} + \boldsymbol{N_B}, \tag{1}$$

where I is the moment of inertia of the rod,

$$I = \frac{ML^2}{12} \tag{2}$$

$\boldsymbol{\omega}$ is the angular velocity, $\mathbf{N_F}$ is the restoring torque due to the suspension fiber, and $\boldsymbol{N_B}$ is the restoring torque due to the magnetic field.

The magnetic torque $\boldsymbol{N_B}$ applied to a magnet by a magnetic field $\boldsymbol{B}$ is given by

$$\boldsymbol{N_B} = \boldsymbol{\mu} \times \boldsymbol{B}, \tag{3}$$

where $\boldsymbol{\mu}$ is the magnetic dipole moment of the magnet and $\boldsymbol{B}$ is the *total* magnetic field, that is, the vector sum of Earth's magnetic field $\boldsymbol{B_E}$ and the applied magnetic field $\boldsymbol{B_A}$. The restoring torque due to the fiber is very small compared to the the restoring torque due to the magnetic field, $N_F \ll N_B$, so that we will neglect $\boldsymbol{N_F}$ in the following derivation.

The magnitude N_B of the magnetic torque $\boldsymbol{N_B}$ is given by the familiar rule for the cross product

$$N_B = \mu B \sin\theta, \tag{4}$$

where θ is the angle between $\boldsymbol{\mu}$ (which points along the axis of the magnet) and the direction of the magnetic field $\boldsymbol{B}$. The angular velocity magnitude ω is the time rate of change of the angle

$$\omega = \frac{d\theta}{dt} \tag{5}$$

so that after accounting for the direction of the restoring torque the equation of motion of our magnet can be written as

$$I\frac{d^2\theta}{dt^2} = -\mu B \sin\theta. \tag{6}$$

This equation is non-linear and so very difficult to solve. We will make another sensible approximation (the first approximation was neglecting $\boldsymbol{N_F}$). For small angle oscillations ($\theta < 20°$), we can replace $\sin\theta$ by θ with less than a 2% error. We do so. The equation of motion is now

$$I\frac{d^2\theta}{dt^2} = -\mu B\theta. \tag{7}$$

or

$$\frac{d^2\theta}{dt^2} + \frac{\mu B}{I}\theta = 0. \tag{8}$$

Those of you with a well functioning hippocampus may recognize this as the equation for **simple harmonic motion** with frequency of oscillation f given by

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{\mu B}{I}}.} \tag{9}$$

This is a key equation for today's lab so pay attention to it. By the way, if the applied magnetic field is aligned parallel to Earth's magnetic field, then $B = B_A + B_E$.

In this experiment, the applied magnetic field is created by a pair of Helmholtz coils. The magnetic field at the center of the two coils is

$$B_A = Ci, \tag{10}$$

where i is the electrical current flowing through the wires of the coils, and C is given by

$$C = \frac{8N\mu_0}{\sqrt{125}R}. \tag{11}$$

Here, $N = 60$ is the number of turns of wire, R is the average radius of the coils, and $\mu_0 = 4\pi \times 10^{-7}$ henrys/meter is the "permeability of free space." Be careful! This symbol μ_0 has an *entirely different meaning* than the magnetic dipole moment μ – even the units are different.

For practical purposes in today's lab, the expression for the square of the magnet's oscillation frequency is useful

$$\boxed{f^2 = \frac{C\mu}{4\pi^2 I}i + \frac{B_E\mu}{4\pi^2 I}.} \tag{12}$$

Note that as f^2 is a linear function of the current i. This equation looks like the equation for a straight line $y = mx + b$. If f^2 is plotted as a function of i, the slope of the resulting straight line permits a determination of the magnetic moment μ. The y-intercept permits a determination of the horizontal component of Earth's magnetic field B_E.

Procedure

1. Use SI units throughout this lab. That is, convert all length measurements to meters, all mass measurements to kilograms, etc. If SI units are used for the inputs to calculations, the results will automatically come out in SI units as well. The units of magnetic dipole moment are $\mathrm{A\,m^2}$ (amp meter^2) and the units of magnetic field are T (tesla).

2. Use a ruler to measure the average outer radius of the coils. Each coil should be measured several times in different directions and the results for both coils averaged together.

3. Next, we need to calculate the inner radius of the coils. This is where the wood stops and the copper wire begins. The total number of turns of wire on both coils together is $N = 120$. There are 60 turns on each coil. Measure the diameter of the copper wire several times using a micrometer under the platform where the wire is easily accessible. **DO NOT PULL THE WIRE OUT OF THE COILS!** Count the number of turns of wire visible in the top layer. You can now calculate how many layers deep the copper is wound, and knowing the diameter of the wire you can find the inner radius of the coil.

4. Find the average radius R of the coils by averaging the inner and outer radii. Use this average radius to calculate C.

5. Use a triple beam balance to measure the mass M and a digital caliper to measure the length L of the cylindrical magnet. Record these data for use in calculating the moment of inertia I of the magnet.

6. Suspend the magnet by thread so that it hangs in the central region of the coils. Adjust the thread so that the magnet hangs in a horizontal level position.

7. When the freely suspended magnet becomes stationary, it will point in a magnetic north-south direction (by definition). Verify this direction with the compass held far away from the metal tables and coils.

8. Connect the coils, power supply, and ammeter in a series circuit. This allows the coil current (i) to be read as the supply voltage is varied.

9. Supply about 0.30 amps to the coils. Pull the magnet out of the way while holding the compass between the coils. Note the direction of the applied magnetic field. Rotate the coils so that the applied magnetic field aligns with Earth's magnetic field. Replace the cylindrical magnet and make sure that it is level.

10. Supply $i = 0.20$ amps to the coils. Wait about thirty seconds for the coils to reach thermal equilibrium. Readjust the power supply if necessary. Set the magnet into oscillation about an axis along the thread with amplitude no more than about 20°. Time 20 oscillations. Calculate the frequency f.

11. Repeat for currents $i = 0.30\,\text{A}, 0.40\,\text{A}, \ldots 1.20\,\text{A}$.

Analysis

1. Plot f^2 vs. i and fit the best straight line though the data.

2. Find the magnetic dipole moment of the magnet from the slope of the plot. No error estimate is required.

3. Use the magnetic dipole moment of the magnet and the y-intercept of the plot to calculate the horizontal component of Earth's magnetic field. No error estimate is required.

4. Find C numerically with units and an error estimate.

5. Find I numerically with units and an error estimate.

6. Why was it desirable to limit the coil current to 1.2 A? List several reasons.

7. Identify two sources of random error and two sources of systematic error.

8. How would your plot differ if Earth's magnetic field B_E and the applied magnetic field B_A had been in opposite directions rather than in the same direction? (Hint: Would the slope change? Would the y-intercept change?)

Literature

1. Walcott, C. (1996). "Pigeon homing: observations, experiments and confusions". *The Journal of Experimental Biology* **199** (Pt 1): 217. PMID 9317262.

2. "2012: Magnetic Pole Reversal Happens All The (Geologic) Time," `http://www.nasa.gov/topics/earth/features/2012-poleReversal.html`