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Quality-assured setup planning based on the stream-of-variation model for multi-stage machining processes

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Setup planning is a set of activities used to arrange manufacturing features into an appropriate sequence for processing. It has significant impact on the product quality, which is often measured in terms of dimensional variation in key product characteristics. Current approaches to setup planning are experience-based and tend to be conservative due to the selection of unnecessarily precise machines and fixtures to ensure final product quality. This is especially true in multi-stage machining processes (MMPs) since it is difficult to predict variation propagation and its impact on the quality of the final product. In this paper, a methodology is proposed to realize cost-effective, quality-assured setup planning for MMPs. Setup planning is formulated as an optimization problem based on quantitative evaluation of variation propagations. The optimal setup plan minimizes the cost related to process precision and satisfies the quality specifications. The proposed approach can significantly improve the effectiveness as well as the efficiency of the setup planning for MMPs.

Keywords: Setup planning, multi-stage machining processes, variation propagation

1. Introduction

Process planning is the systematic determination of the steps by which a product is manufactured. It is a key element that bridges activities in design and manufacturing. In the past decades, process planning and its automation enablers have been extensively studied and significant progress has been reported (Maropoulos, 1995). Many approaches to process planning have been suggested including conceptual process planning, setup planning and detailed process planning; see Fig. 1. Conceptual process planning includes engineering feature recognition, process selection and machine/tooling selection. Detailed process planning includes fixture design, quality-assurance-strategy selection and cost analysis.

Setup planning constitutes a critical component that connects conceptual process planning and detailed process planning. Conceptual process planning provides qualitative information to setup planning, including designated features, selected processes and datum scheme constraints. The purpose of setup planning is to arrange manufacturing features into an appropriate sequence of setups in order to

ensure product quality and productivity (Huang and Liu, 2003). A setup plan is comprised of setup formation, datum scheme selection and setup sequencing (Huang, 1998). It defines a series of datum/fixturing schemes for a Multi-Stage Machining Process (MMP), as shown in Fig. 1. However, the setup plan obtained from those traditional methods provides limited detailed information to subsequent planning activities in process planning.

Product quality is one of the main concerns of setup planning. A well-defined setup plan should be able to satisfy quality specifications under normal manufacturing conditions. Product quality is affected by the outcome of setup planning since the series of datum and fixtures defined by a specific setup plan may introduce errors which will propagate along the machining stages and accumulate in the final product. Different setup plans specify different datum/fixturing schemes, lead to different variation propagation scenarios, and result in different product quality. Thus, one of the major tasks in setup planning is to identify the optimal setup from multiple alternatives to ensure product quality.

Some research has been conducted in quality-assured setup planning, addressing issues in setup formation, datum scheme selection and setup sequencing. Zhang *et al.* (1996) proposed principles for achieving tolerance control

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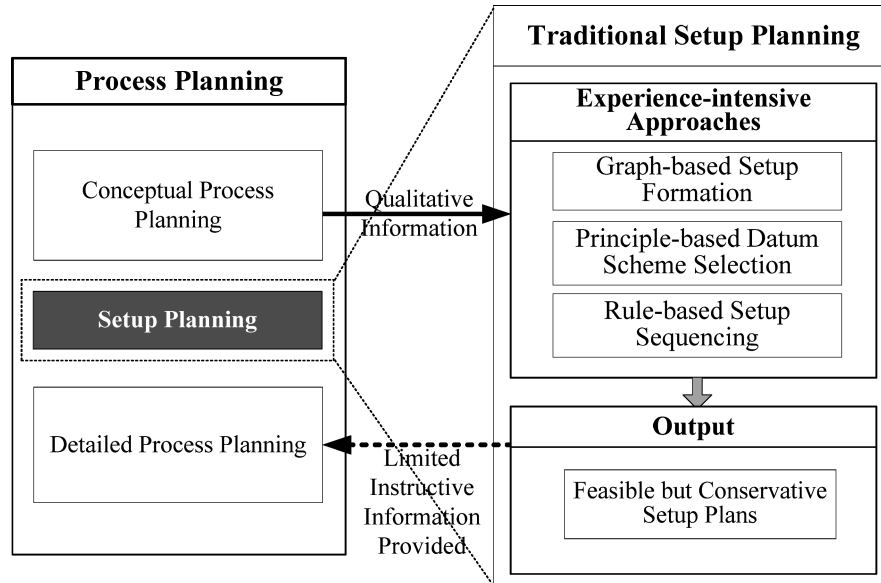


Fig. 1. The existing commonly used setup planning approaches.

proactively via appropriately grouping and sequencing features according to their tolerance relationships. Mantripragada and Whitney (1998) presented the “datum flow chain” concept to relate datum logic explicitly with Key Product Characteristics (KPCs) tolerances and assembly sequences. Quantitative approaches were also developed to evaluate variation stack-up associated with different process design. Rong and Bai (1996) presented a method to verify machining accuracy corresponding to fixture design. Song *et al.* (2005) developed a Monte Carlo simulation-based method to analyze the quality impact of production planning. Xu and Huang (2006) modeled the simulated quality distributions in multiple attribute utility functions. In addition to the simulation-based approaches, analytical methods have also been used to investigate the interactions between product quality and process variability. For a given setup plan, Stream of Variation (SoV) methodologies (Shi, 2006) and state space modeling techniques have been developed to model the dimensional variation propagation along different setups (Hu, 1997; Jin and Shi, 1999; Ding *et al.*, 2002; Zhou *et al.*, 2003; Huang *et al.*, 2007a; Huang *et al.*, 2007b).

Cost-effectiveness is another critical concern in setup planning. It can be evaluated in terms of Cost Related to Process Precision (CRPP), such as the cost to achieve necessary fixture precision to satisfy product quality requirements. The precision refers to the inherent variability in an MMP and CRPP is the cost to achieve a required precision level to ensure product quality requirements. The CRPP is assumed to be inversely proportional to the necessary process precision. Corresponding to different setup plans, different process precisions are required and thus different costs are incurred. Therefore, setup planning should be a discrete constrained optimization procedure.

Ong *et al.* (2002) considered various cost factors in the optimization index, including the cost of machines and fixtures. However, these cost factors are not directly linked with process precision.

It is desirable that the optimal setup plan is the one that satisfies the product quality specification using relatively imprecise fixtures and machines to minimize the CRPP. However, setup plans developed solely based on principles and experience can be very conservative. Although they are generally feasible with respect to the quality consideration, cost-effectiveness may not be optimal. For instance, in order to ensure the final product quality, engineers tend to conservatively select unnecessarily precise fixtures and thus cause unnecessary CRPP. This is especially true for the upstream stages of an MMP where there are no techniques to evaluate variation propagation. Furthermore, in order to automate process planning, it should be easy to integrate the outcomes of the setup planning procedure with other detailed process planning activities, e.g., fixture design. Fixture layout design for a particular setup is critical input data for setup planning, whereas the setup planning results determine an MMP whose fixture system should be optimized at the process level. However, although the fixture layout design has been successfully investigated at both the single-stage level (Cai *et al.*, 1997) and process level (Kim and Ding, 2004), effective setup/fixture planning studies are still required. This is because qualitative-principle-based setup planning provides limited potential for specifying quantitative precision requirements of fixture design. In addition, conservative process precision requirements will make the designed fixture unnecessarily expensive. This functional limitation of conventional setup planning significantly hinders the implementation of process planning automation.

Table 1. Approaches to setup planning

	<i>Evaluation-oriented setup planning</i>	<i>Optimization-oriented setup planning</i>
Qualitative quality evaluation	Zhang <i>et al.</i> (1996), Mantripragada and Whitney (1998)	Ong <i>et al.</i> (2002)
Simulation-based quality evaluation	Song <i>et al.</i> (2005)	Xu and Huang (2006)
Analytical quality evaluation	Hu (1997), Ding <i>et al.</i> (2002), Zhou <i>et al.</i> (2003), etc.	To be studied in this paper

Existing setup planning approaches are summarized in Table 1. As can be seen, most reported research has focused on the evaluation of setup plan alternatives. Some work exists that uses qualitative or simulation-based evaluation of product quality to perform optimal setup planning. Although simulation provides an effective strategy to compare alternative setup plans in terms of their output product quality, it consumes a substantial amount of time and computational resources.

This paper adopts an integrated setup/fixture planning strategy to process planning. It focuses on the systematic development of a cost-effective, quality-assured setup planning, which is a fundamental enabler to integrated setup/fixture planning. Because of the complexity of the integrated problem and the overwhelming computational requirements, an iterative approach is appropriate. As illustrated in Fig. 2, the stage/setup level optimal fixture layouts for all candidate datum schemes are first determined and fixed. In each stage, different datum scheme options may

be assigned with different fixture layouts. These stage/setup level fixture layouts are the inputs to the setup planning, together with information on the feature representation, design specification, constraints on datum scheme and setup sequence. As shown in Fig. 2, the development of the proposed setup planning consists of three steps.

1. Candidate setup formations and datum schemes are formulated based on input information. Their potential variation stack-up can be analytically predicted by the SoV model.
2. Based on those candidate setups defined in step 1, the setup planning is formulated as a sequential decision-making process on an optimal series of setups that cost-effectively satisfies product quality specifications. A cost criterion is defined to evaluate the optimality of candidate setup plans under the constraints of product quality specifications.
3. Dynamic Programming (DP) is used to solve the optimal sequential decision-making problem and generate the optimal setup plan, which provides setup information for subsequent activities in process planning. Based on an analytical quality evaluation strategy, the proposed optimal setup planning methodology will be effective and efficient. When the optimal setup plan is determined, the approach of Kim and Ding (2004) can be applied to achieve a process level optimal fixture layout, which will be used to update the stage/setup level fixture layouts for repeating the iterative optimization procedure.

The remainder of this paper is organized as follows. The SoV-based optimal setup planning methodology is introduced in Section 2. Section 3 presents a case study in which the proposed approach is used to generate a setup plan for MMPs. Conclusions are drawn and areas of future work are discussed in Section 4.

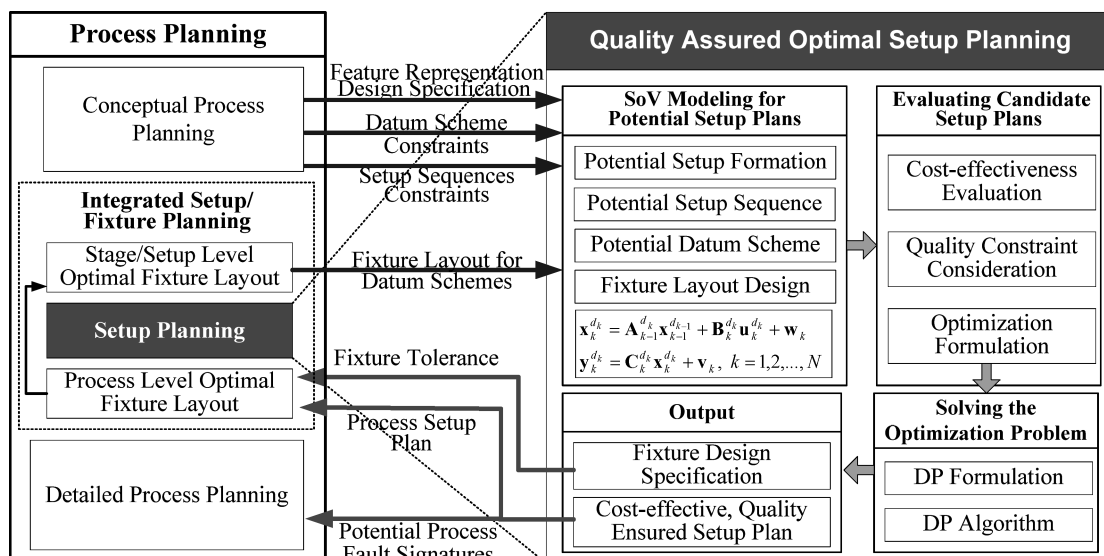


Fig. 2. Overview of the proposed “SoV-based, quality-assured setup planning.”

2. Quality-assured cost-effective setup planning

The design specifications of a machined product are often satisfied by machining operations performed in a series of stages. In each stage, a set of features are generated with a specific setup. The dimensional precision of the final product is affected by three major variation sources in the machining operations.

1. *Machine and cutting tool*, which refers to the random deviation of the cutting tools from their nominal paths.
2. *Fixture*, which refers to the random deviation of the fixture locators from their nominal positions.
3. *Datum*, which refers to the random deviation of the datum features, generated in previous stages, from their nominal positions and/or dimensions.

Both Sources 1 and 2 are treated as random process deviations. The third source exists because some features generated in the upstream stages are used as the datum features in the downstream stages according to the setup plan. Thus, the dimensional variation, which is introduced by fixtures and/or machine and cutting tools in the upstream stages, is propagated through datum features and accumulated in the features generated in the downstream stages. Different setup plans, i.e., different datum schemes and different setup sequences, lead to different variation propagation scenarios, and thus result in different final product quality. In order to compare candidate setup plans, an effective method is needed to evaluate the impact of potential datum schemes and setup sequences on the quality of the final product.

2.1. Variation propagation model for setup planning

One effective tool to model the variation propagation in MMPs is the state space modeling technique (Shi, 2006). Zhou *et al.* (2003) presented a detailed derivation and validation of a model with given process/product design, including information on the setup formation, datum scheme selection and setup sequence. However, some additions are necessary due to the following unique characteristics in setup planning.

1. *Multiple datum scheme options*: In setup planning, every stage has a set of candidate datum schemes. Different datum schemes support different operations that generate different features, which further constrain the pool of candidate datum schemes for downstream stages. Also, datum scheme selection is directly related to the fixture design and thus significantly affects the CRPP. Thus, there is a need for explicit representation of the selected datum scheme for every stage.
2. *Setup precedence requirements*: According to the design specifications, some features must be fabricated in a stage with comparatively precise datum features, which may be machined in an upstream stage. This kind of precedence

relationships is not often straightforward to determine, especially when the tolerance interdependences among features are complicated. Therefore, a capability to explicitly represent the sequence of setups and the chain of datum schemes is needed to evaluate different setup precedence options.

3. *Tracing the setup chain*: Since the CRPP is inversely proportional to the precision of fixtures, process planners tend to select less precise fixtures to reduce the cost. However, this will increase the dimensional variation of the generated features and increase the datum variation if some of them are used as datum in the downstream stages. As a result, datum features with a large variation force the downstream fixtures to be very precise to satisfy quality specifications. In other words, due to the complex variation propagation, relaxing the upstream process precision may result in the need for tighter tolerances in the downstream processes and thus increase the total CRPP. Therefore, to achieve an overall cost-effectiveness, the variation propagation of the setup chain must be traced and explicitly modeled.

Figure 3 illustrates the variation propagation scenario of the setup plan of an MMP. The nomenclature is explained as follows.

1. The datum scheme of stage k ($k = 1, 2, \dots, N$) is denoted as DS d_k ($d_k = 1, 2, \dots, D_k$, where D_k is the total number of feasible datum scheme options for stage k). A datum scheme refers to the coordinate system specified by a group of datum surfaces, within which the machining process can be performed. A datum scheme is very important to the variation propagation modeling since all three aforementioned variation sources affect the quality of newly generated features through datum, as shown in Fig. 3.
2. Corresponding to a selected datum scheme d_k in stage k , the quality of all features are denoted by a state vector $\mathbf{x}_k^{d_k}$, with each element representing the dimensional deviation from its nominal value.
3. The random deviation of process variables associated with a selected datum scheme d_k in stage k is denoted as $\mathbf{u}_k^{d_k}$. Corresponding to the major variation sources, $\mathbf{u}_k^{d_k}$ models the random process deviations of both machine/cutting tools and fixture locators, as defined in Zhou *et al.* (2003). Represented as deviations of the tool path from its nominal path, $\mathbf{u}_k^{d_k}$ models many types of sources, including geometric and kinematic errors, thermal errors, cutting-force-induced errors and tool-wear-induced errors (Zhou *et al.*, 2003). The elements in $\mathbf{u}_k^{d_k}$ are called process variables and are treated as independent system input data that follow a multivariate normal distribution.
4. The unmodeled system noises due to the model linearization are represented by \mathbf{w}_k . Compared to the deviations modeled in $\mathbf{u}_k^{d_k}$ and $\mathbf{x}_k^{d_k}$, the elements in \mathbf{w}_k are higher

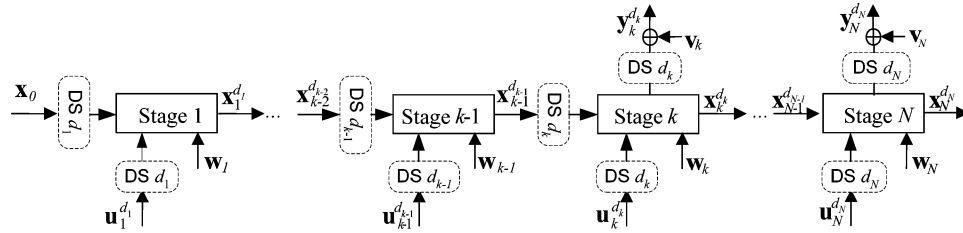


Fig. 3. Variation propagation in a setup plan.

- order small values. The \mathbf{w}_k are assumed to be independent of any component of $\mathbf{u}_k^{d_k}$ ($k = 1, 2, \dots, N$; $d_k = 1, 2, \dots, D_k$). Also, the elements of \mathbf{w}_k are assumed to be independent of each other and to have a mean of zero.
5. Since the features are measured in the coordinate system defined by the selected datum scheme d_k , the measurements of quality are denoted as $\mathbf{y}_k^{d_k}$. In this paper, the measurements are assumed to be multivariate normal.
 6. The measurement noise is denoted by a random vector \mathbf{v}_k , which is independent of $\mathbf{x}_k^{d_k}$, $\mathbf{u}_k^{d_k}$ and \mathbf{w}_k ($k = 1, 2, \dots, N$; $d_k = 1, 2, \dots, D_k$). The components of \mathbf{v}_k are assumed to be independent of each other and to have a mean of zero. Also, the magnitudes of the components of \mathbf{v}_k are determined by the accuracy/precision of the measurement device, which is usually at the level of $1 \mu\text{m}$.

Adopting the assumptions of rigid parts and small errors, a linear state space model can be constructed to associate the product quality with a sequence of setups according to the setup plan, as shown in Equation (1):

$$\begin{aligned} \mathbf{x}_k^{d_k} &= \mathbf{A}_{k-1}^{d_k} \mathbf{x}_{k-1}^{d_{k-1}} + \mathbf{B}_k^{d_k} \mathbf{u}_k^{d_k} + \mathbf{w}_k, \\ \mathbf{y}_k^{d_k} &= \mathbf{C}_k^{d_k} \mathbf{x}_k^{d_k} + \mathbf{v}_k, \quad k = 1, 2, \dots, N, \end{aligned} \quad (1)$$

where $\mathbf{A}_{k-1}^{d_k} \mathbf{x}_{k-1}^{d_{k-1}}$ represents the datum-induced random deviation corresponding to the selected datum scheme d_k in stage k , and $\mathbf{x}_{k-1}^{d_{k-1}}$ is the quality, in terms of dimensional deviation, transmitted from upstream stages. $\mathbf{B}_k^{d_k} \mathbf{u}_k^{d_k}$ describes the impact of deviation from the process variables, corresponding to the selected datum scheme d_k , in the quality of features generated in stage k . $\mathbf{C}_k^{d_k}$ is the observation matrix

mapping features' quality to the measurements. A validation of this SoV modeling in Zhou *et al.* (2003) demonstrates that the SoV model can adequately represent the process errors and their propagations in MMPs. Ren *et al.* (2006) further demonstrated that the model linearization is valid when number of stages is moderate.

As previously mentioned, setup planning is a series of decisions based on alternative datum schemes for multiple stages, as illustrated in Fig. 4. For the optimal datum scheme selected for stage k , Equation (1) can be reformulated as

$$\begin{aligned} \mathbf{x}_k^{d_k^*} &= \mathbf{A}_{k-1}^{d_k^*} \mathbf{x}_{k-1}^{d_{k-1}^*} + \mathbf{B}_k^{d_k^*} \mathbf{u}_k^{d_k^*} + \mathbf{w}_k, \\ \mathbf{y}_k^{d_k^*} &= \mathbf{C}_k^{d_k^*} \mathbf{x}_k^{d_k^*} + \mathbf{v}_k, \end{aligned} \quad (2)$$

where $d_k^* \in \{d_k | d_k = 1, 2, \dots, D_k\}$, for $k = 1, 2, \dots, N$, represents the index of the selected optimal datum scheme in stage k . Please note that d_k^* is one link of the optimal datum scheme chain ($d_1^* d_2^* \dots d_N^*$) that is determined through considering all the stages in the entire processes. Thus, d_k^* may not necessarily be optimal for a single-stage k .

The state space model in Equation (1) can be transformed into a linear input–output model as

$$\mathbf{y}_k^{d_k} = \sum_{i=1}^k \mathbf{C}_k^{d_k} \Phi_{k,i}^{(\bullet)} \mathbf{B}_i^{d_i} \mathbf{u}_i^{d_i} + \mathbf{C}_k^{d_k} \Phi_{k,0}^{(\bullet)} \mathbf{x}_0 + \sum_{i=1}^k \mathbf{C}_k^{d_k} \Phi_{k,i}^{(\bullet)} \mathbf{w}_i + \mathbf{v}_k, \quad (3)$$

where $\Phi_{k,i}^{(\bullet)}$ is the state transition matrix tracing the datum schemes transformation from stage i to $k - 1$; and $\Phi_{k,i}^{(\bullet)} = \mathbf{A}_{k-1}^{d_{k-1}} \mathbf{A}_{k-2}^{d_{k-2}} \dots \mathbf{A}_i^{d_i}$ for $i < k$, and $\Phi_{k,k}^{(\bullet)} = \mathbf{I}$. The initial state vector \mathbf{x}_0 represents the original quality of the part that enters the first stage of the process. Without loss

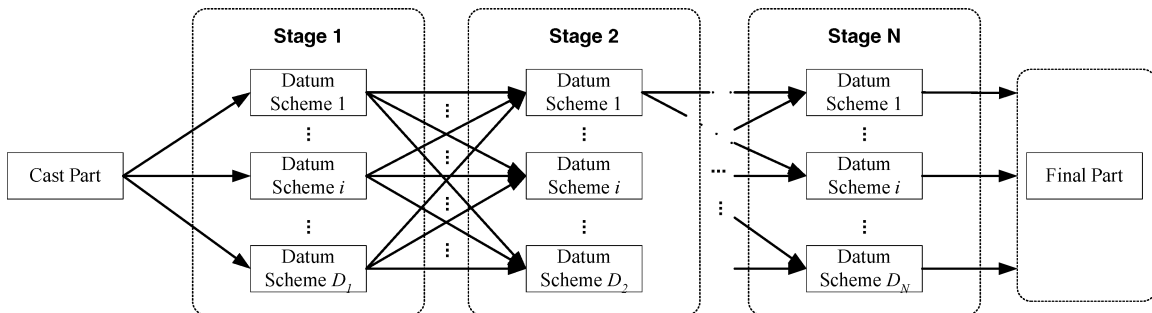


Fig. 4. Datum scheme alternatives for sequential decision making.

of generality, \mathbf{x}_0 is set to zero. Then Equation (3) changes to

$$\mathbf{y}_k^{d_k} = \sum_{i=1}^k \mathbf{C}_k^{d_k} \Phi_{k,i}^{(\bullet)} \mathbf{B}_i^{d_i} \mathbf{u}_i^{d_i} + \sum_{i=1}^k \mathbf{C}_k^{d_k} \Phi_{k,i}^{(\bullet)} \mathbf{w}_i + \mathbf{v}_k. \quad (4)$$

For a selected datum scheme, d_k , and the decisions on datum schemes for upstream stages $\{d_1, d_2, \dots, d_{k-1}\}$, the coefficient matrices, $\mathbf{A}_k^{d_k}$, $\mathbf{B}_k^{d_k}$, $\mathbf{C}_k^{d_k}$ and $\Phi_{k,i}^{(\bullet)}$ ($i = 1, 2, \dots, k$), can be derived following the same procedure as presented in Zhou *et al.* (2003). This variation propagation modeling technique provides the setup planner with a tool to predict the product quality of candidate datum schemes and alternative setup sequences of an MMP. Compared to the method, proposed by Xu and Huang (2006), that can only assess the quality after the whole setup plan is defined, state space modeling provides the capability to assess product quality for each intermediate setup. This modeling technique can be effectively incorporated into the decision-making process for optimal setup plan determination.

2.2. Setup plan evaluation strategy

Different setup plans will result in different product qualities in terms of KPC variation and incur different CRPPs. From the optimization point of view, setup planning can be formulated as a discrete constrained optimization problem.

2.2.1. Optimization of the setup planning

In this paper, the objective of setup planning is to minimize the CRPP while satisfying the KPC quality constraints. The mathematical representation is defined as

$$\min_{\mathbf{T}_u} \{C_{\mathbf{T}_u}(\mathbf{T}_u)\},$$

subject to

$$\frac{USL_i - LSL_i}{\sigma_{y_i}} \geq \tau_i, \quad i = 1, 2, \dots, M, \quad (5)$$

where $\mathbf{T}_u = [T_{u_1}, T_{u_2}, \dots, T_{u_p}]^T$ is a $P \times 1$ vector with each element T_{u_j} representing the tolerance of a corresponding process variable u_j defined in \mathbf{u} , and $\mathbf{u} = [\mathbf{u}_1^T, \mathbf{u}_2^T, \dots, \mathbf{u}_N^T]^T$, with \mathbf{u}_k ($k = 1, 2, \dots, N$) as a $p_k \times 1$ vector representing the process variables (i.e., fixture locator deviations) in stage k . Please note that $P = \sum_{k=1}^N p_k$. M is the total number of KPCs and P is the total number of process variables. USL_i and LSL_i are the predefined upper specification limit and lower specification limit of KPC y_i , respectively. σ_{y_i} is the standard deviation of KPC y_i and τ_i is a constant, $i = 1, 2, \dots, M$. $C_{\mathbf{T}_u}(\mathbf{T}_u)$ is the CRPP function of process tolerance. Various cost functions have been proposed for different tolerance syntheses. Considering the structural simplicity, a reciprocal function is adopted in this paper:

$$C_{\mathbf{T}_u} = \sum_{j=1}^P \frac{w_j}{T_{u_j}}, \quad (6)$$

where the w_j , $j = 1, 2, \dots, P$, are weighting coefficients. These weighting coefficients should be determined according to the practical situation. For instance, coefficients assigned to the fixtures used in the same stage can be equal to each other; fixtures or machine tools manufactured by the same supplier or used in the same stage may be assigned with the same value. More discussion on the selection of these weighting coefficients are provided in the case studies in Section 3.

For a complicated MMP, there always exist multiple quality characteristics. It is desirable to define a multivariate process capability index for process quality control. However, at the setup planning stage, there is no *a priori* information on the correlations between quality characteristics. A scalar multivariate process capability index may be misleading if it is defined without appropriate consideration of correlations between quality characteristics. Thus, in industrial applications, for the sake of convenience, most of the tolerance regions are specified as a collection of individual specifications for each variable, as defined in Equation (5). The intersection of these specifications would form a rectangular solid zone (Jackson, 1991). Chen (1994) proposed a multivariate process capability index over a rectangular solid tolerance zone $V = \{y \in R^M: \max(|y_i - \mu_i|/r_i, i = 1, 2, \dots, M) \leq 1\}$. Based on this definition, a necessary condition for a process to be capable over a rectangular solid zone is that each individual univariate process is capable with respect to the corresponding specification limits. In addition, according to the discussion of Chen (1994), correlations between quality characteristics make the process more capable over a rectangular tolerance zone. Therefore, in this paper, individual process capability constraints are adopted to conservatively ensure that the setup plan is able to satisfy the specifications on all quality characteristics.

Ding *et al.* (2005) studied the relationship between tolerance and variation of process variables by examining the clearance of the pin-hole locating pair. In this paper, the process capability ratios, $\eta_j = T_{u_j}/\sigma_{u_j}$, are assumed to be constants. Therefore, the tolerance of a process variable can be replaced by its standard deviation. Recall that the elements in $\mathbf{u}_k^{d_k}$ are defined as the deviations of fixture locators with a mean of zero, thus their variances $\sigma_{u_j}^2 = E(u_j^2)$, $j = 1, 2, \dots, P$. Let $\Xi_u = [\sigma_{u_1}, \sigma_{u_2}, \dots, \sigma_{u_p}]^T$, then the tolerance of the process variables can be defined by $\mathbf{T}_u = [T_{u_1}, T_{u_2}, \dots, T_{u_p}]^T = \text{diag}\{\eta_1, \eta_2, \dots, \eta_P\} \cdot \Xi_u$. Then, the objective function $C_{\mathbf{T}_u}(\mathbf{T}_u)$ in Equation (5) can be transformed to

$$C_u(\mathbf{u}) = \sum_{j=1}^P \frac{w_j}{\eta_j \times \sigma_{u_j}}. \quad (7)$$

2.2.2. DP formulation

Previous sections present the techniques that enable: (i) the description of the impacts of datum scheme selection and setup sequencing on the variation of product quality; (ii) the

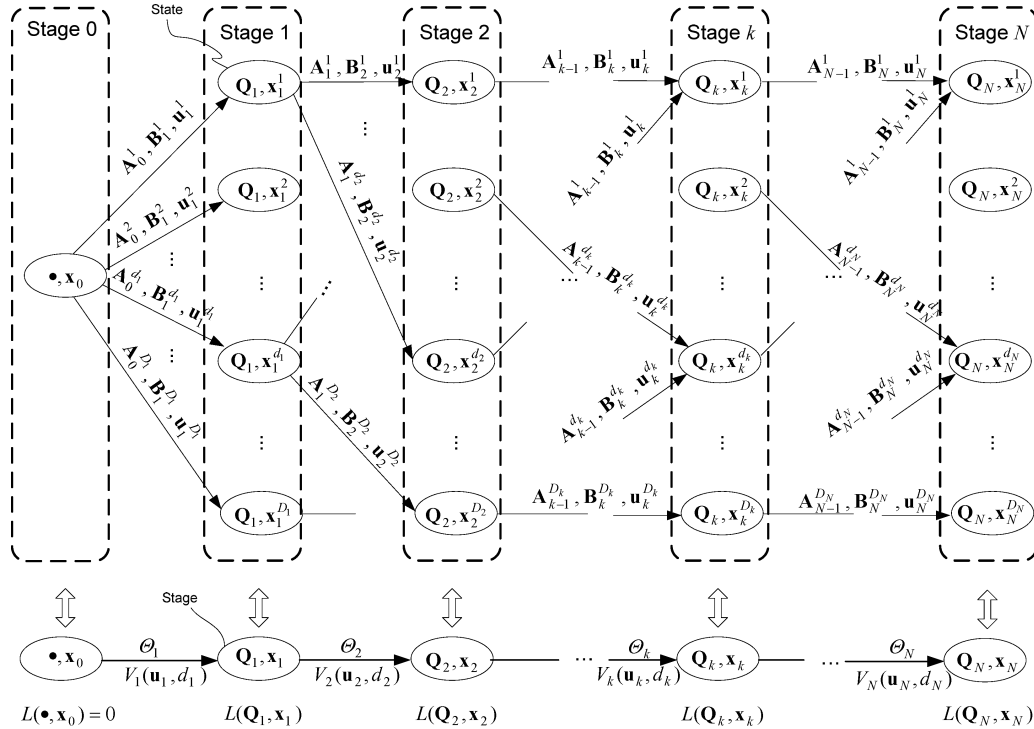


Fig. 5. DP network of a setup planning decision sequence.

modeling of the variation propagation; and (iii) the quantitative evaluation of the candidate setup plans. Based on these enablers, setup planning can be formulated as a sequential decision-making process for the selection of datum schemes in all stages to satisfy quality specifications with overall cost-effectiveness. In this sequential decision-making problem, the datum scheme selected for stage k is affected by that selected for the upstream stages and will then affect the one selected for the downstream stages. This characteristic is identical to that of a DP problem. Therefore, DP methodology is adopted to solve the optimization problem. Figure 5 illustrates a sequential decision process for a chain of datum scheme selection.

In Fig. 5, there are $N + 1$ columns in the diagram, representing the N stages of the machining processes, and an initial DP state (\bullet, \mathbf{x}_0) . Each column k ($k = 1, 2, \dots, N$) consists of D_k nodes, corresponding to D_k feasible datum schemes. A node $(\mathbf{Q}_k, \mathbf{x}_k^{d_k})$, $d_k = 1, 2, \dots, D_k$, in Fig. 5 is a DP state that represents the datum scheme selection in stage k , where \mathbf{Q}_k defines the in-process quality specifications for the features generated from stage 1 to stage k . Since the quality specification for the incoming part is not related to the quality consideration of the machining process, it is set to “•” in the initial DP state, i.e., not specified. According to Equation (5), \mathbf{Q}_k is an $M \times M$ matrix with the diagonal elements $q_{k,i,i} = [(USL_{k,i} - LSL_{k,i})/\tau_i]^2$, $i = 1, 2, \dots, M$; $k = 1, 2, \dots, N$. $USL_{k,i}$ and $LSL_{k,i}$ are the given in-process specification limits for KPC i in stage k . The off-diagonal elements of \mathbf{Q}_k can also be specified in terms of the covariance matrix structure of $\mathbf{y}_k^{d_k}$ for a given d_k . The connections link-

ing nodes in column $k - 1$ to those in column k reflect state transitions. Given the datum scheme and setup sequence selected for upstream stages, different nodes from two neighboring stages are connected or disconnected, according to the predefined datum scheme constraints. Although there are D_k potential DP states for each stage, the process planner observes only the one that is finally selected. Therefore, the concept of “DP-stage” $(\mathbf{Q}_k, \mathbf{x}_k)$ is defined to “contain” all the possible states, $(\mathbf{Q}_k, \mathbf{x}_k^{d_k})$, $d_k = 1, 2, \dots, D_k$, in a column k (Denardo, 2003). As shown in the bottom portion of Fig. 5, the \mathbf{u}_k “contains” all the possible $\mathbf{u}_k^{d_k}$ s, $d_k = 1, 2, \dots, D_k$. Associated with each DP-stage is a set of decisions Θ_k on datum scheme selection.

Selecting datum scheme d_k incurs cost $V_k(\mathbf{u}_k, d_k)$ and implements transition from DP-stage $(\mathbf{Q}_{k-1}, \mathbf{x}_{k-1})$ to DP-stage $(\mathbf{Q}_k, \mathbf{x}_k)$. Let $\mathbf{q}_k(\mathbf{u}_k, d_k)$ be the constraints on the KPC variations generated in stage k if datum scheme d_k is selected. In other words, $\mathbf{q}_k(\mathbf{u}_k, d_k)$ is the maximum KPC variations that can be allowed after the fabrication performed in stages 1 through k . Also let $t((\mathbf{Q}_k, \mathbf{x}_k), d_k, d_{k-1})$ be the state transition function linking $\mathbf{x}_{k-1}^{d_{k-1}}$ and $\mathbf{x}_k^{d_k}$, then Equation (1) can be of the form $\mathbf{x}_k^{d_k} = t((\mathbf{Q}_k, \mathbf{x}_k), d_k, d_{k-1}) = \mathbf{A}_{k-1}^{d_k} \mathbf{x}_{k-1}^{d_{k-1}} + \mathbf{B}_k^{d_k} \mathbf{u}_k^{d_k} + \mathbf{w}_k$. The decision making on the d_k s, ($k = 1, 2, \dots, N$) repeats itself for all stages, following $t((\mathbf{Q}_k, \mathbf{x}_k), d_k, d_{k-1})$. The cost of decision d_k in stage k is defined as

$$V_k(\mathbf{u}_k, d_k) = C_{\mathbf{u}_k^{d_k}}(\mathbf{u}_k^{d_k}) = \sum_{j=1}^{p_k} \frac{w_j}{\eta_j \cdot \sigma_{u_{k,j}^{d_k}}}, \quad (8)$$

where $p_k (k = 1, 2, \dots, N)$ is the dimension of $\mathbf{u}_k^{d_k}$ and $P = \sum_{k=1}^N p_k$. Also, $\sigma_{u_{k,j}^{d_k}}$ is the standard deviation of the j th element of $\mathbf{u}_k^{d_k}, j = 1, 2, \dots, p_k$. This cost can be interpreted as the cost consumed to provide enough process precision for stage k , corresponding to the selected datum scheme d_k . Let $L(\mathbf{Q}_k, \mathbf{x}_k)$ be the minimum CRPP that is consumed from stage 1 to stage k by selecting datum schemes d_1, d_2, \dots, d_k , and generating quality variation at most \mathbf{Q}_k , the DP function can be defined as

$$L(\mathbf{Q}_k, \mathbf{x}_k) = \begin{cases} \min_{\substack{d_k \in \Theta_k, d_{k-1} \in \Theta_{k-1} \\ q_k(\mathbf{u}_k, d_k) \leq \mathbf{Q}_k}} \{L(\mathbf{Q}_k - \mathbf{q}_k(\mathbf{u}_k, d_k), t((\mathbf{Q}_k, \mathbf{x}_k), d_k, d_{k-1})) \\ + V_k(\mathbf{u}_k, d_k)\} & \text{for } k = 1, \dots, N, \\ 0 & \text{for } k = 0, \end{cases} \quad (9)$$

where \mathbf{Q}_k is predefined, $\mathbf{q}_k(\mathbf{u}_k, d_k)$ and $V_k(\mathbf{u}_k, d_k)$ can be derived based on the state space model (1). According to Equation (4), the covariance matrix of $\mathbf{y}_k^{d_k}$ is

$$\begin{aligned} \Sigma_{\mathbf{y}_k^{d_k}} &= \sum_{i=1}^{k-1} (\mathbf{C}_k^{d_k} \Phi_{k,i}^{(\bullet)} \mathbf{B}_i^{d_i}) \Sigma_{\mathbf{u}_i^{d_i}} (\mathbf{C}_k^{d_k} \Phi_{k,i}^{(\bullet)} \mathbf{B}_i^{d_i})^T \\ &+ (\mathbf{C}_k^{d_k} \Phi_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k}) \Sigma_{\mathbf{u}_k^{d_k}} (\mathbf{C}_k^{d_k} \Phi_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k})^T \\ &+ \sum_{i=1}^k (\mathbf{C}_k^{d_k} \Phi_{k,i}^{(\bullet)}) \Sigma_{\mathbf{w}_i} (\mathbf{C}_k^{d_k} \Phi_{k,i}^{(\bullet)})^T + \Sigma_{\mathbf{v}_k}, \end{aligned} \quad (10)$$

where Σ_{\bullet} is the covariance matrix for variable “ \bullet ”. Equation (10) shows that the KPC covariance can be treated as the accumulated covariance of all process variables used from stage 1 to stage k , plus the covariance of the unmodeled process variations and the variance of measurement noise. In order to ensure that the product quality generated from stage 1 to stage k satisfies the specifications, $\Sigma_{\mathbf{y}_k^{d_k}}$ should satisfy the specification

$$\sigma_{y_{k,i}^{d_k}}^2 \leq s \times q_{k,i,i}, \quad i = 1, 2, \dots, M, \quad (11)$$

where $\sigma_{y_{k,i}^{d_k}}^2$ is the i th diagonal element of matrix $\Sigma_{\mathbf{y}_k^{d_k}}$, $q_{k,i,i}$ is the i th diagonal element of matrix \mathbf{Q}_k and the scalar s is the safety factor ($0 \leq s \leq 1$). Since \mathbf{w}_k and \mathbf{v}_k contain second or higher order terms whose values are much smaller than that of $\mathbf{x}_k^{d_k}$ and $\mathbf{u}_k^{d_k}$, their contribution to $\Sigma_{\mathbf{y}_k^{d_k}}$ can be ignored. Thus, by eliminating the third and the fourth terms on the right-hand-side of Equation (10), $\Sigma_{\mathbf{y}_k^{d_k}}$ can be approximated by

$$\begin{aligned} \tilde{\Sigma}_{\mathbf{y}_k^{d_k}} &= \sum_{i=1}^{k-1} (\mathbf{C}_k^{d_k} \Phi_{k,i}^{(\bullet)} \mathbf{B}_i^{d_i}) \Sigma_{\mathbf{u}_i^{d_i}} (\mathbf{C}_k^{d_k} \Phi_{k,i}^{(\bullet)} \mathbf{B}_i^{d_i})^T \\ &+ (\mathbf{C}_k^{d_k} \Phi_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k}) \Sigma_{\mathbf{u}_k^{d_k}} (\mathbf{C}_k^{d_k} \Phi_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k})^T. \end{aligned} \quad (12)$$

The first term on the right-hand-side of Equation (12) stands for the quality covariance (measured based on datum scheme d_k) accumulated from stage 1 to stage k , whereas the second term stands for the quality covariance generated in stage k by selecting datum scheme d_k . Let

$$\tilde{\Sigma}_{\mathbf{y}_{k-1}^{\bullet}} = \sum_{i=1}^{k-1} (\mathbf{C}_k^{d_k} \Phi_{k,i}^{(\bullet)} \mathbf{B}_i^{d_i}) \Sigma_{\mathbf{u}_i^{d_i}} (\mathbf{C}_k^{d_k} \Phi_{k,i}^{(\bullet)} \mathbf{B}_i^{d_i})^T, \quad (13)$$

be the quality covariance accumulated from stage 1 to stage $k-1$, then the amount of newly generated quality covariance can be derived as

$$\Sigma_{d_k} = \tilde{\Sigma}_{\mathbf{y}_k^{d_k}} - \tilde{\Sigma}_{\mathbf{y}_{k-1}^{\bullet}} = (\mathbf{C}_k^{d_k} \Phi_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k}) \Sigma_{\mathbf{u}_k^{d_k}} (\mathbf{C}_k^{d_k} \Phi_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k})^T. \quad (14)$$

since the process cost modeled in Equation (7) is inversely proportional to the process variations. In order to minimize the process cost, the process variations, the diagonal elements in $\Sigma_{\mathbf{u}_k^{d_k}}, k = 1, 2, \dots, N$, should be relaxed as much as possible. This will lead to the increase of the KPC variations defined by the diagonal elements in $\tilde{\Sigma}_{\mathbf{y}_k^{d_k}}$. Considering the quality constraints specified by \mathbf{Q}_k , $\tilde{\Sigma}_{\mathbf{y}_k^{d_k}}$ should satisfy

$$\tilde{\Sigma}_{\mathbf{y}_k^{d_k}} = s \mathbf{Q}_k, \quad (15)$$

where s is the same as that defined in Equation (11). Given the \mathbf{Q}_k values, $k = 1, 2, \dots, N$, the constraints $\mathbf{q}_k(\mathbf{u}_k, d_k)$ have the form:

$$\begin{aligned} \mathbf{q}_k(\mathbf{u}_k, d_k) &= (s \mathbf{Q}_k - \tilde{\Sigma}_{\mathbf{y}_{k-1}^{\bullet}}) \\ &= (\mathbf{C}_k^{d_k} \Phi_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k}) \Sigma_{\mathbf{u}_k^{d_k}} (\mathbf{C}_k^{d_k} \Phi_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k})^T. \end{aligned} \quad (16)$$

From Equation (16), the covariance matrix of $\mathbf{u}_k^{d_k}$ can be derived as

$$\Sigma_{\mathbf{u}_k^{d_k}} = (\Gamma_k^{(\bullet)})^{-1} (s \mathbf{Q}_k - \tilde{\Sigma}_{\mathbf{y}_{k-1}^{\bullet}}) [(\Gamma_k^{(\bullet)})^T]^{-1}, \quad (17)$$

where $\Gamma_k^{(\bullet)} = \mathbf{C}_k^{d_k} \Phi_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k} = \mathbf{C}_k^{d_k} \mathbf{B}_k^{d_k}$, and $(\Delta)^{-}$ denotes the Moore–Penrose inverse of the rectangular matrix Δ . $\tilde{\Sigma}_{\mathbf{y}_{k-1}^{\bullet}}$ contains the variation propagation information and is determined by the datum scheme selection and sequencing decisions made for upstream stages. When $\Gamma_k^{(\bullet)}$ is column-wise full rank, Equation (17) can give a real solution of $\Sigma_{\mathbf{u}_k^{d_k}}$. Assuming that the process variables are mutually independent, the tolerance specification for $\mathbf{u}_k^{d_k}$ can be obtained by a $p_k \times 1$ vector:

$$\mathbf{T}_{\mathbf{u}_k^{d_k}} = [\eta_k \sigma_1^{d_k} \quad \eta_k \sigma_2^{d_k} \quad \dots \quad \eta_k \sigma_{p_k}^{d_k}]^T, \quad (18)$$

where $(\sigma_j^{d_k})^2$ is the j th diagonal element of $\Sigma_{\mathbf{u}_k^{d_k}}, j = 1, 2, \dots, p_k, k = 1, 2, \dots, N$ and $d_k = 1, 2, \dots, D_k$. According to the definition of $\mathbf{u}_k^{d_k}$, $\mathbf{T}_{\mathbf{u}_k^{d_k}}$ contains the tolerance of machining/cutting tools and fixture locators. In order to increase the exchangeability of fixture locators, improve maintainability of the fixture system and reduce the “long-run

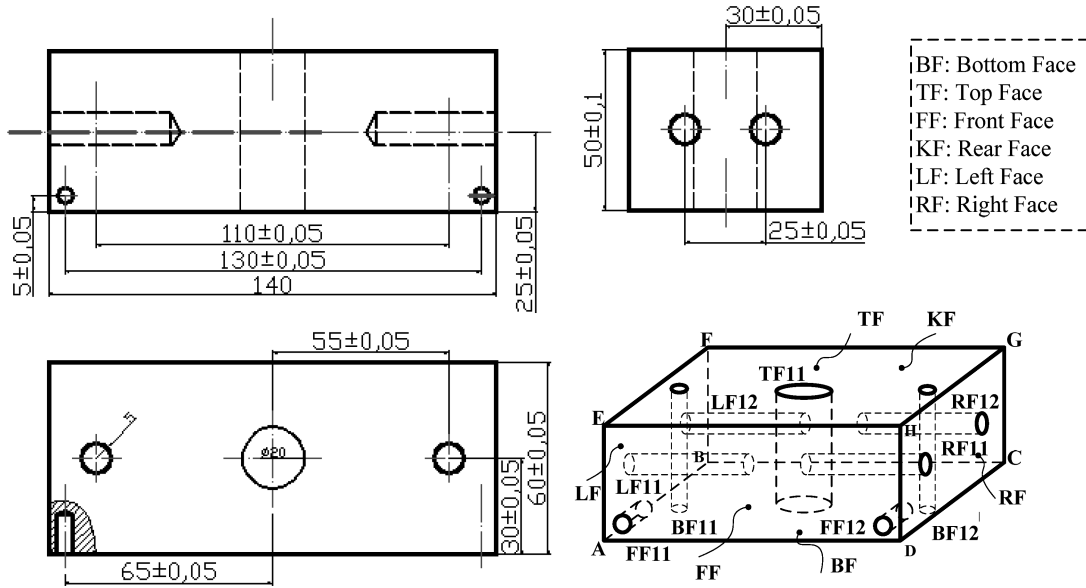


Fig. 6. Part drawing and KPC specifications.

overall production cost,” different locators on the same fixture are assigned with the same tolerance, as discussed by Chen *et al.* (2006). Therefore, fixture locators’ tolerances can also be specified as $\eta_k \sigma_*^{d_k}$, where $\sigma_*^{d_k} = \min_{j \in J_f} \{\sigma_j^{d_k}\}$ and J_f is a set containing all the index of fixture locators in $\mathbf{u}_k^{d_k}$. Using Equations (8) to (18) it is possible to formulate setup planning by solving a series of DP functional equations.

2.2.3. Optimization algorithm

The reaching algorithm (Denardo, 2003) is used to solve the DP problem defined in Equation (9). According to Fig. 5, the value of each DP-state node $(\mathbf{Q}_k, \mathbf{x}_k^{d_k})$ is denoted as s_{k,d_k} , which represents the minimum process precision cost incurred so far from stage 1 to stage k by selecting datum scheme d_k in stage k . Let $v_k^{d_{k-1}}$ denote the corresponding cost incurred in stage k according to datum selections of the upstream stage $k - 1$ and that of the stage k , and $v_k^{d_{k-1}} = V_k(\mathbf{u}_k, d_k)$. The pseudo code of the reaching algorithm is as follows.

Set $s_{0,\bullet} = 0$ and $s_{k,d_k} = +\infty$ for $k = 1, 2, \dots, N$; $d_k = 1, 2, \dots, D_k$,

DO for $k = 1, 2, \dots, N$

DO for $d_k = 1, 2, \dots, D_k$

$$s_{k,d_k} \leftarrow \min \left\{ s_{k,d_k}, \inf_{d_{k-1} \in \Theta_{k-1}} \left\{ s_{k-1,d_{k-1}} + v_k^{d_{k-1}} \right\} \right\}.$$

In this algorithm, $v_k^{d_{k-1}}$ will be set to ∞ for an infeasible datum scheme selection. This value indicates that, given the variation accumulated in upstream stages, the selected datum scheme for the current stage cannot meet the quality specification. The final results include: (i) the minimized total CRPP, $L(\mathbf{Q}_N, \mathbf{x}_N)$; (ii) a sequence of decisions $(d_1^* d_2^* \dots d_N^*)$ on datum schemes for a sequence of stages, which is the optimal setup plan; and (iii) the tolerance specifications, \mathbf{T}_u , of the fixtures used in all stages.

3. Case study

A case study is conducted to demonstrate the SoV-model-based quality assured optimal setup planning for an MMP.

Table 2. Setup options for the three-stage machining process

Index	Stage		
	1	2	3
1	DS: TF-FF-RF SF: BF, BF11, BF12	DS: BF-BF11-BF12 SF: FF, FF11, FF12, KF	DS: BF-BF11-BF12 SF: TF, TF11, LF, LF11, LF12, RF, RF11, RF12
2	DS: FF-TF-RF SF: BF, BF11, BF12	DS: BF-BF11-BF12 SF: TF, TF11	DS: FF-FF11-FF12 SF: FF, FF11, FF12, KF, LF, LF11, LF12, RF, RF11, F12
3	DS: BF-FF-LF SF: TF, TF11	DS: TF-FF-RF SF: BF, BF11, BF12	DS: BF-BF11-BF12 SF: TF, TF11, LF, LF11, LF12, RF, RF11, RF12, KF

DS = datum scheme, SF = setup formation.

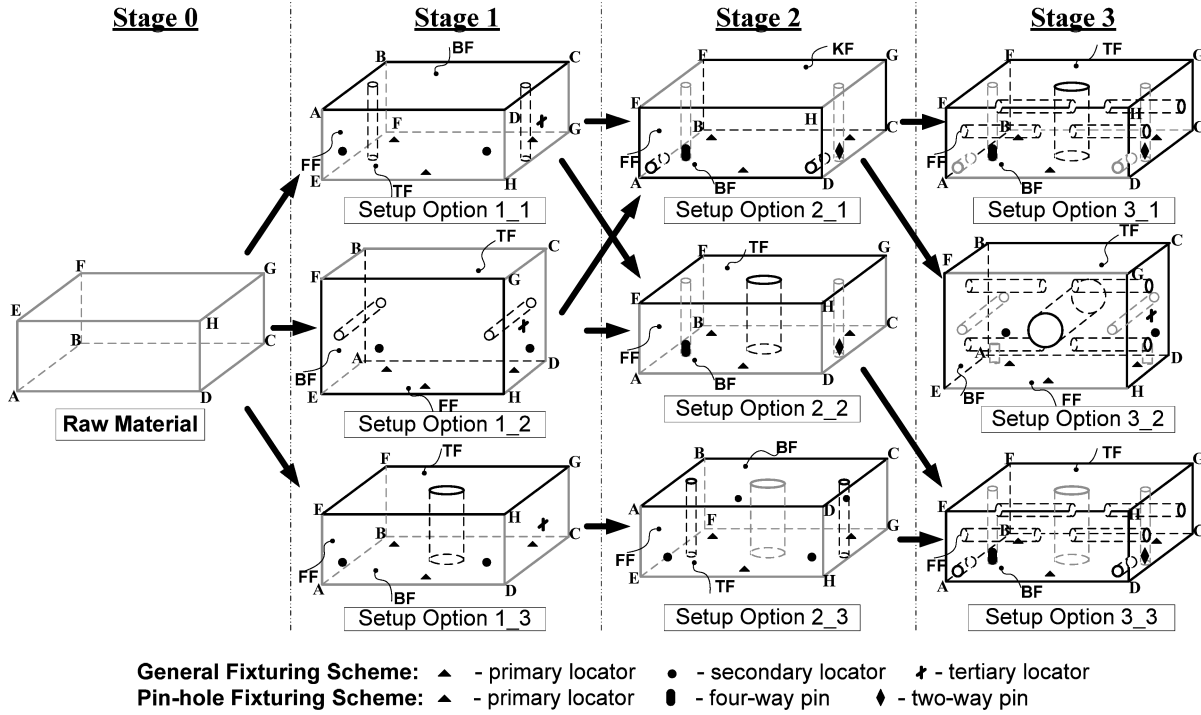


Fig. 7. Setup options for a three-stage machining process.

The product KPCs and their associated design specifications are defined in Fig. 6. Based on the analysis of feature locations and tooling approach directions, a three-stage machining process is proposed. The candidate datum schemes for each stage are proposed and shown in Fig. 7. Correspondingly, stage/setup level fixture layouts are assumed as given. These include general 3-2-1 fixturing schemes (e.g., Setup Option 1_1) and pin-hole fixturing schemes (e.g., Setup Option 2_1), as discussed by Zhou *et al.* (2003).

Table 2 summarizes the alternative datum schemes and setup formations for each stage. Corresponding to these datum scheme candidates d_k ($k = 1, 2, 3$), the coefficient matrices in state space models, $A_k^{d_k}$, $B_k^{d_k}$ and $C_k^{d_k}$, are gener-

ated. According to the constraints on datum scheme and datum sequence, the DP network is established, as shown in Fig. 8.

In this case study, without loss of generality, only the variation of fixture locators are included in the process variable vectors $u_k^{d_k}$. Thus, each one of the three $u_k^{d_k}$ s ($k = 1, 2, 3$) contains six process variables, corresponding to the six locators. The total number of process variables, P , is 18. The safety factor s in Equation (15) is set to 1.5 to account for the potential quality impacts of process variations contributed by machine/cutting tools. Coefficients η_j ($j = 1, 2, \dots, 18$) in Equation (7) are set six. The weighting coefficients w_j are set to 1/18, which means all fixture locators are treated

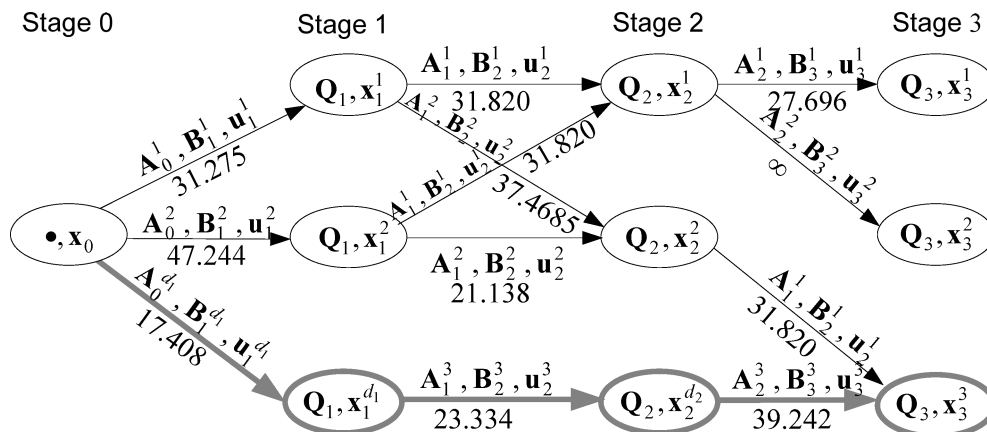


Fig. 8. DP network for the three-stage machining process.

Table 3. Intermediate results of the reaching algorithm

s_{k,d_k}	Stage		
	1	2	3
Setup option d_k			
1	31.275	60.095	87.791
2	47.244	68.744	∞
3	17.408	40.742	79.983*

equally with respect to their CRPP. The costs for each state transition are also shown in Fig. 8. The notation “ ∞ ” indicates that the variation accumulated from upstream stages has made it impossible to satisfy the quality specifications in the third stage.

The intermediate results are summarized in Table 3. The optimal setup plan is identified and highlighted, in Fig. 8, as the bold path. The optimal setup plan is: (i) in the first stage, the part is fixed with datum features BF, FF and LF, and features TF, TF11 are generated; (ii) in the second stage, the part is fixed with datum features TF, FF and TF, and features BF, BF11 and BF12 are generated; (iii) the remaining features are generated in the third stage with the part being fixed on datum features BF, BF11 and BF12. This optimal setup plan can be denoted as a DFC: $\{\{\text{BF-FF-LF}\}, \{\text{TF-FF-TF}\}, \{\text{BF-BF11-BF12}\}\}$, with the total CRPP of 79.983.

One of the by-products of the SoV-based setup planning methodology is the tolerance specifications for the fixture design. In this case study, based on the $\sigma_{u_i^*}$ s, $\mathbf{T}_{u_i^*}$ ($i = 1, 2, 3$) are given as $\mathbf{T}_{u_1^*} = [0.086 \ 0.086 \ 0.086 \ 0.086 \ 0.086 \ 0.086]^T$, $\mathbf{T}_{u_2^*} = [0.037 \ 0.037 \ 0.037 \ 0.037 \ 0.037 \ 0.037]^T$, and $\mathbf{T}_{u_3^*} = [0.019 \ 0.019 \ 0.019 \ 0.019 \ 0.019 \ 0.019]^T$. The fixture design that meets these specifications will be cost-effective and sufficiently precise to ensure the product quality. The results show that the fixtures for upstream stages, i.e., stage 1 and stage 2, are not required to be as precise as that for the downstream operations, i.e., the optimal setup plan is not conservative.

A sensitivity analysis was also conducted to examine the impact of the assignments the values to the weighting coefficients on the optimization results. It is assumed that: (i) the weighting coefficients assigned to locators belonging to the same fixture are the same; (ii) fixtures used at stage 1 are assigned different weighting coefficients to those assigned to fixtures used at stage 2 and stage 3; and (iii) the weighting coefficients assigned to fixtures used in stage 2 and stage 3 are the same. For instance, if a sum of weighting coefficients, 0.1 (0.1/6 for each locator) is assigned to a stage 1 fixture, fixtures in stage 2 and stage 3 will be 0.45 (0.45/6 for each locator).

Table 4 shows the optimization results associated with different combinations of the coefficients assignments. The optimal setup plans are consistent, except for case 1, where the fixture in stage 1 is significantly under-weighted with

Table 4. Impact of the sum weighing coefficients on optimization results

Case number	Sum weighting coefficients for stage 1	Optimal setup plan
1	0.1	$\{\{\text{FF-TF-RF}\}, \{\text{BF-BF11-BF12}\}, \{\text{BF-BF11-BF12}\}\}$
2	0.2	$\{\{\text{BF-FF-LF}\}, \{\text{TF-FF-TF}\}, \{\text{BF-BF11-BF12}\}\}$
3	0.3	$\{\{\text{BF-FF-LF}\}, \{\text{TF-FF-TF}\}, \{\text{BF-BF11-BF12}\}\}$
4	0.4	$\{\{\text{BF-FF-LF}\}, \{\text{TF-FF-TF}\}, \{\text{BF-BF11-BF12}\}\}$
5	0.5	$\{\{\text{BF-FF-LF}\}, \{\text{TF-FF-TF}\}, \{\text{BF-BF11-BF12}\}\}$
6	0.6	$\{\{\text{BF-FF-LF}\}, \{\text{TF-FF-TF}\}, \{\text{BF-BF11-BF12}\}\}$
7	0.7	$\{\{\text{BF-FF-LF}\}, \{\text{TF-FF-TF}\}, \{\text{BF-BF11-BF12}\}\}$
8	0.8	$\{\{\text{BF-FF-LF}\}, \{\text{TF-FF-TF}\}, \{\text{BF-BF11-BF12}\}\}$
9	0.9	$\{\{\text{BF-FF-LF}\}, \{\text{TF-FF-TF}\}, \{\text{BF-BF11-BF12}\}\}$

a weighting coefficient of 0.1. This indicates that the optimization result for this case study is not sensitive to the value of the weighting coefficient. This is because the datum scheme option 3 for stage 1 significantly outperforms the other two options in terms of CRPP. The differences among those three options dominate the whole optimization of the three stages, as shown in Fig. 8.

4. Conclusions

This paper proposed a methodology for optimal setup planning for MMPs. Based on the SoV concept, a state space modeling technique is expanded to be applicable to datum selection and setup sequencing decisions. The SoV model provides the basis for quantitative, analytical evaluation of the quality impacts of candidate setup plans. This evaluation capability enables the formulation of the setup planning as an optimization problem that minimizes the CRPP with the final product quality satisfying constraints. DP is employed to solve this sequential optimal decision-making problem.

In the proposed method, setup planning is formulated as a DP problem, which provides a clear representation of the sequential decision-making procedure. However, one disadvantage of DP is that it needs intensive computational resources. When the number of stages and the number of alternative datum schemes becomes large, the cost to obtain an optimal solution becomes unaffordable. Potential solutions include: (i) using a different formulation, such as reinforcement learning, neurodynamic programming or approximate DP; and (ii) incorporating engineering domain

knowledge to decouple an MMP into smaller segments of subprocesses and/or add more constraints to reduce the number of alternative datum schemes. These topics will be investigated in our future work.

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