THE INSTITUTE OF EDUCATION

Maths

Louise Boylan

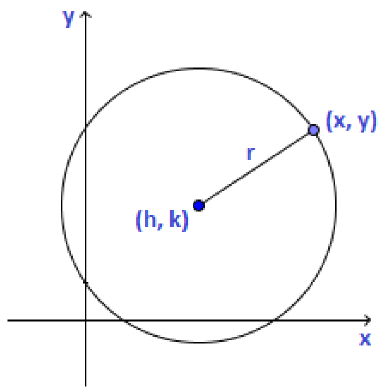
Higher Level

2020-21

Coordinate Geometry – The Circle

Section A – Equation of a circle

The diagram shows a circle of centre (h, k) and radius r.
The point (x, y) is on the circle.



The radius, r, can be found using the formula for the distance between two points: (h, k) = (x_1, y_1) and (x, y) = (x_2, y_2)

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Radius} = \sqrt{(x - h)^2 + (y - k)^2}$$

$$\text{Radius}^2 = (x - h)^2 + (y - k)^2$$

Equation of a circle of centre (h, k) and radius r: $(x - h)^2 + (y - k)^2 = r^2$

Equation of a circle of centre (0, 0) and radius r: $x^2 + y^2 = r^2$

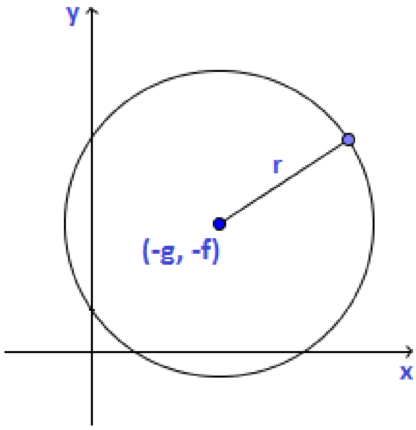
Practice Q: Find the equation of the circle with centre (-1, 4) and radius 6.

Practice Q: *P* (-3, 4) and *Q* (3, -4) are two points on the coordinated plane.
Find the equation of the circle which has [*PQ*] as its diameter.

Practice Q: Find the equation of the circle with centre (1, -2) and having the line 2x + 3y – 9 = 0 as a tangent. [Hint: draw a sketch]

Answer: $(x-1)^2+(y+2)^2=13$ or $x^2+y^2-2x+4y-8=0$

General equation of a circle



The general equation of a circle is written as:

$$x^2+y^2+2gx+2fy+c=0$$

To find the centre and the length of the radius, follow the steps:

1. Ensure that the coefficients of x^2 and y^2 are equal to 1.

2. Centre $=(-g,-f)$
   To find the centre:
   $\left(-\frac{1}{2}(\text{coefficient of the } x), -\frac{1}{2}(\text{coefficient of the } y)\right)$

3. Radius $=\sqrt{g^2+f^2-c}$

Practice Q: Find the centre and radius of each of the following circles:

(i) $x^2 + y^2 + 6x - 4y - 3 = 0$

(ii) $x^2 + y^2 - 8x + 2y - 10 = 0$

(iii) $x^2 + y^2 - 6y + 8 = 0$

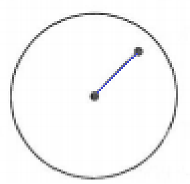
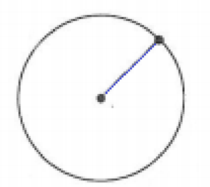
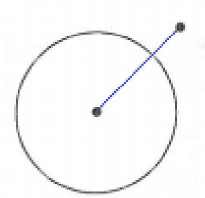
(iv) $3x^2 + 3y^2 - 3x + 9y - 21 = 0$

Practice Q: The circle $x^2 + y^2 - 2x + 4y + k = 0$ has a radius of length $3\sqrt{5}$. Find the value of *k*.

Section B – Position of a point in relation to a circle

In relation to a circle, a point can either be inside the circle, on the circle or outside the circle.

Method 1:
To determine position of a point, we calculate the distance between the centre of the circle and the given point. Then compare this distance to the radius of the circle.

Inside the circle	**On the circle**	**Outside the circle**
		
The distance from the centre of the circle to the given point is **less** than the radius.	The distance from the centre of the circle to the given point is **equal** to the radius.	The distance from the centre of the circle to the given point is **greater** than the radius.

Method 2:

A quicker method for determining the position of a point in relation to a circle is to substitute the coordinates of the given point in for x and y into the equation of the circle. Then if

Left hand side **<** Right hand side, then the point is **inside** the circle

Left hand side **=** Right hand side, then the point is **on** the circle

Left hand side **>** Right hand side, then the point is **outside** the circle

Practice Q: Determine whether each of the following points are inside, on or outside of the circle $x^2 + y^2 + 4x - 2y - 32 = 0$.

(i) $(4, 0)$

(ii) $(2, 7)$

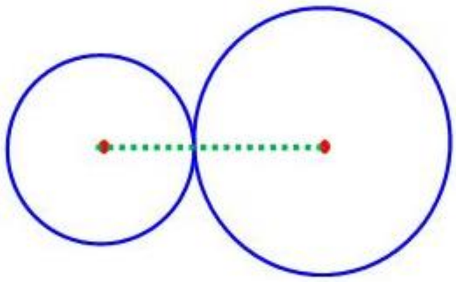
(iii) $(-8, 2)$

(iv) $(-6, -3)$

Section C – Touching circles

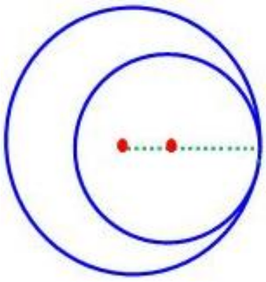
When circles intersect at one point only, they are said to be touching.

Touching externally



Distance between centres
= Sum of radii

Touching internally



Distance between centres
= Difference between radii

Finding the point of contact between two touching circles

To find the point of contact between the two circles we must use the skills learned in finding the point which divides a line segment in a given ratio.

We can do this by using translations, or using the formula:

$$P = \left(\frac{bx_1 + ax_2}{b + a}, \frac{by_1 + ay_2}{b + a}\right)$$

Practice Q: Prove that the circles $c: x^2 + y^2 = 25$ and $s: x^2 + y^2 + 8x + 6y - 75 = 0$ touch internally.

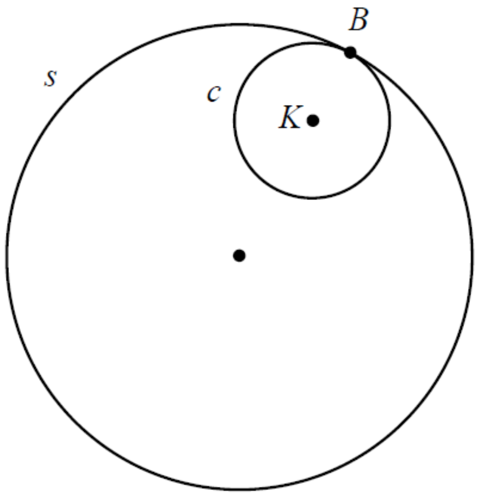
Practice Q: Prove that the circles c: $x^2 + y^2 + 8x + 6y - 15 = 0$ and s: $x^2 + y^2 - 10x + 15 = 0$ touch externally and find their point of contact.

Answer: (2, -1)

Practice Q: The circles $x^2 + y^2 + 16x + 14y + k = 0$ and $x^2 + y^2 - 14x + 2y + 21 = 0$ touch externally. Find two possible values for k.

Answers: k = -3, -351

2015 – Paper 2 – Q.4



Two circles s and c touch internally at B, as shown.

(a) The equation of the circle s is
$(x-1)^2 + (y+6)^2 = 360.$
Write down the co-ordinates of the centre of s.

Centre:________________

Write down the radius of s in the form $a\sqrt{10}$, where $a \in \mathbb{N}$.

Radius:______________________

(b) **(i)** The point K is the centre of circle c.
The radius of c is one-third the radius of s.
The co-ordinates of B are (7, 12).
Find the co-ordinates of K.

(ii) Find the equation of c.

(c) Find the equation of the common tangent at B.
Give your answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$.

Section D – Intersection between a line and a circle

To find the points of intersection between a line and a circle we use solve the equations simultaneously. The line will be a linear equation and the circle will be a quadratic equation.

Substitute the linear equation into the quadratic one and solve.

If there is only one point of intersection, the line is a tangent to the circle.

Practice Q:

(a) Find the points of intersection between the line $x - y + 7 = 0$ and the circle $x^2 + y^2 + 2x - 6y - 19 = 0$.

Answers: (-6, 1) and (1, 8)

(b) Find the length of the chord the line makes in the circle.

Practice Q:

(a) Find the coordinates of the points *A* and *B*, the points of intersection between the line $2x - y - 3 = 0$ and the circle $x^2 + y^2 - 2x - 8y - 8 = 0$.

Answers: (1, -1) and (5, 7)

(b) Hence, investigate if [*AB*] is a diameter of the circle.