# LINEAR PROGRAMMING: EXERCISES 

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## PROBLEM 1

A company manufactures 3 products $a$, $b$ and $c$, which sells $€ 14$, $€ 15$ and $€ 22$ per unit respectively. These prices are constant and independent of the market state they are addressed to, and it is also supposed that any produced quantity can be sold. For the manufacturing of these products four types of raw materials are required. The prices of raw materials, the raw material units needed for each product type and the corresponding available quantities within a certain time period are included in the following table.

| Raw <br> material | Unit price <br> $(€)$ | Products |  |  | Available raw <br> material units |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | a | b | c |  |
| 1 | 3 | 0 | 2 | 3 | 50 |
| 2 | 2 | 3 | 2 | 1 | 200 |
| 3 | 0.5 | 4 | 4 | 6 | 200 |
| 4 | 1 | 0 | 0 | 2 | 100 |

The company's goal is to determine the quantities of each product which should be produced in order to achieve the highest profit.

Define in detail the decision variables and form the objective function and all constraints of the problem.

## PROBLEM 2

The management of an industry, in which some machines are under employed, considers the case to produce the products 1, 2 and 3 during the idle time of the machines. This time is estimated at 500, 350 and 150 machine hours per week for machine types A, B and C respectively. The machine hours needed for the production of each product unit are presented in the table below. The sales department estimates that the demand of products 1 and 2 I higher than the production capacity, while the sales of product 3 cannot exceed 20 units per week. This department also predicts that the profit from the sale of each unit of product 1,2 and 3 is $€ 30$, $€ 12$ and $€ 25$ respectively.

| Machines Product | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: |
| A | 9 | 3 | 5 |
| B | 5 | 4 | 0 |
| C | 3 | 0 | 2 |

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Which mathematical model should solve the industry to identify the quantities of products that should be produced, in order to maximize the net profit?

## PROBLEM 3

A company which manufactures canoes employs 120 employees, each of whom working 30 hours per week. Half of them work in the carpenter department, 20 persons in the plastics department, and the rest of them at the completion department. The company manufactures the simple canoes with net unit profit $€ 7$ and the luxury canoes with corresponding profit $€ 10$. A simple canoe requires 4.5 hours in the carpenter department and two hours in each of the other two departments. The working hours for each luxury canoe are 5, 1 and 4 at the carpenter department, plastics department and completion department respectively. Marketing calculations have shown that not less than $1 / 3$ and not more than $2 / 3$ of the total number of the canoes should be luxurious.

How will the company maximize its overall net profit?
Formulate the appropriate LP model.

## PROBLEM 4

A transportation company has signed contracts with a big customer for transporting to him ammunitions, weapons and drugs. The customer has agreed to receive all quantities transferred to him.

|  | Density <br> (kilos/cubic palm) | Profit <br> $(€ / \mathrm{kg})$ |
| :--- | :---: | :---: |
| Ammunitions | 30 | 0.20 |
| Weapons | 40 | 0.30 |
| Drugs | 20 | 0.10 |

The company uses two planes. Plane A cannot transport more than 15 tons neither more than $0.1 \mathrm{~m}^{3}$ of cargo. Plane B cannot transport more than 25 tons and over 0.2 $\mathrm{m}^{3}$ of cargo. There is one more restriction: no more than 100 kg of drugs can be transported in each delivery (the delivery includes two flights, one of plane A and one of plane B).

Formulate - with all the necessary documentation - the appropriate model to solve this problem. Comment also on which unit is appropriate to be represented the decision variables of the problem.

## PROBLEM 5

The two main products of a company are manufactured in a production line of three machines, $M_{1}, M_{2}$ and $M_{3}$. Each of them operates 7 hours daily on a five-day basis. The unit production cost is $€ 160$ and $€ 250$ respectively, while the corresponding profit rates are $20 \%$ and $24 \%$. The durations of the production processes (expressed in seconds) are shown in the following table.

|  | $M_{1}$ | $M_{2}$ | $M_{3}$ | $M_{2}$ or $M_{3}$ |
| :--- | :---: | :---: | :---: | :---: |
| Product A | 25 | 30 | 50 |  |
| Product B | 40 | 15 | 40 | 20 |

The first product is completed in three phases, while the second one is required to pass a fourth phase, which can be performed either by machine $M_{2}$ or machine $M_{3}$. The problem which the company faces is to identify the units that must be produced by each product to maximize the weekly net profit.

Design (variables - function - constraints) the appropriate linear programming model to solve this problem.

## PROBLEM 6

A rural family owns 125 acres and has \$ 40,000 stock for investment. Each member can provide 3500 hours of work during the winter months (mid October - mid April) and 4000 hours during the summer. If any of these hours are not necessary then the younger members of the family can go and work in the nearby farm for $\$ 5$ per hour for the winter months and $\$ 6$ per hour during the summer.

Income in cash can come from the three crops, from cows and from chickens. No stock investment is needed for crops. In contrary an investment of \$ 1200 for each cow and \$ 9 for each chicken is needed.

Each cow needs 1.5 hectares of land, 100 human hours of personal work during the winter months and another 50 hours for the summer. Each cow will give income \$ 1000 each year for the family. The corresponding figures for each chicken is no land, 0.6 hours of personal human work in winter and 0.3 more hours in summer with annual income $\$ 5$ for each chicken. The farm can feed a maximum of 3000 chickens and the existing stable is sufficient for up to 32 cows.

The estimated hours of personal work and the income per cultivated hectare for the three types of crop are the following:

|  | Soya | Corn | Oats |
| :--- | :---: | :---: | :---: |
| Winter hours | 20 | 35 | 10 |
| Summer hours | 50 | 75 | 40 |
| Net annual income (\$) | 500 | 750 | 350 |

The family wants to determine how much land should be cultivated for each crop type and how many chickens and cows should be kept to maximize the annual net profit. Design a linear programming model to solve this problem.

## PROBLEM 7

A farmer has 200 acres of land and wants to cultivate potatoes or pumpkins or a combination of both. He has discovered that there is sufficient demand for these products and does not consider other alternatives. The maximum yield of potatoes is five tons per acre, and if pumpkins will grow only three tons per acre will be produced. The potatoes can be sold at a profit of 50 pounds per ton, while the pumpkins at a profit of 105 pounds per ton. There is a defined demand for both species. A maximum of 750 tons of potatoes and of 300 tons of pumpkins should be produced per year in order to be placed freely in the market.

Both seeds will need fertilizers and the ratio for each growing seed has a limit regarding the available fertilizer. The farmer uses two types of fertilizer, A and B , which are mixed in the right proportion for each seed. He believes that the mix for potatoes should be composed of $40 \%$ of fertilizer A and $60 \%$ of fertilizer B. The mix for the pumpkins should consist of $55 \%$ of fertilizer A and $45 \%$ of fertilizer B. Each acre of potatoes needs 0.4 tons of fertilizer and each acre of pumpkins needs 0.5 tons of fertilizer.

There is a limit to the amount of available fertilizer. The farmer can buy up to 30 tons of fertilizer A and 100 tons of fertilizer B. Fertilizer A is of better quality. The farmer can improve the quality of $B$ by adding enhancing ingredients. If he does so, the improved tons of B can be used as partial or total supplement for $40 \%$ of A which is required in the potatoes mix. However, the farmer estimates that this will cause a decrease of $10 \%$ in yield. Its use is not possible on the pumpkin mix because the result would be disastrous. For every ton of fertilizer $B$ that will be improved in this way 0.1 tons of additional components are required, with an additional cost of 45 pounds.

1) Design (without solving) this problem as a linear programming model in order to maximize the profit.
2) Give arguments for how to strengthen this plan, assuming that the optimal solution has already been calculated.

## PROBLEM 8

A cargo plane has three sections for storing goods. Front, middle and tail. These three parts have capacity limits in weight and space, according to the following table.

| Dept | Storage capacity <br> (tones) | Capacity potential <br> (cubic palm) |
| :--- | :---: | :---: |
| Front | 12 | 7.000 |
| Middle | 18 | 9.000 |
| Tail | 10 | 5.000 |

Also, the weight of the cargo in the corresponding sections must be in the same proportion as the weight limits for each department so the plane has balance. The following four cargoes are given for transfer to a later flight.

| Cargo | Weight <br> (tones) | Volume <br> (cubic palms/ tone) | Profit <br> (\$ / tone) |
| :---: | :---: | :---: | :---: |
| 1 | 20 | 500 | 280 |
| 2 | 16 | 700 | 360 |
| 3 | 25 | 600 | 320 |
| 4 | 13 | 400 | 250 |

Any amount of these cargoes can be accepted for transfer. The goal is to determine what proportion of these cargoes must be transferred and how to be settled in those parts of the plane, so as to maximize the profit of the flight.

Design an appropriate linear programming model to solve this problem.

## PROBLEM 9

An investor has the available profitable investment activities $A$ and $B$ for each year of the next five ones. Every dollar invested at the beginning of the one year in activity $A$ becomes $\$ 1.40$ two years later. Every dollar invested in the activity B for each year becomes $\$ 1.70$ three years later.

Also, investing activities C and D will be available shortly. Every dollar invested in C at the beginning of year 2 will become $\$ 1.90$ at the end of year 5 . Every dollar invested in D at the beginning of year 5 will become $\$ 1.30$ at the end of year 5 .

The investor starts with $\$ 50,000$ and wants to know the way, which will maximize the amount of money he will receive at the beginning of the sixth year.

Design an appropriate linear programming model for this investment problem.

## PROBLEM 10

Solve using the Simplex method, the following linear programming problem:
$\max f(X)=7 / 6 x_{1}+13 / 10 x_{2}$
with structure limitations :
$x_{1} / 30+x_{2} / 40 \leq 1$
$x_{1} / 28+x_{2} / 35 \leq 1$
$x_{1} / 30+x_{2} / 25 \leq 1$
and
$x_{1}, x_{2} \geq 0$

## PROBLEM 11

Solve using the Simplex method, the following linear programming problem:
$\max z(X)=50 x_{1}+120 x_{2}+40 x_{3}+80 x_{4}$
with structure limitations

$$
2 x_{1}+x_{2}+x_{3} \leq 450
$$

$$
3 x_{2}+x_{3}+x_{4} \leq 180
$$

$$
4 x_{1}+x_{3} \leq 400
$$

$$
x_{1}+x_{2}+x_{4} \leq 110
$$

and
$\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}, \mathrm{x}_{4} \geq 0$

If variables $x_{i}$ represent the corresponding quantities of products $i$ that will be produced at a certain time period and the objective function expresses the company's net profit in $€$, what are your conclusions derived from the solution of the problem?

## PROBLEM 12

Consider the following Linear Programming model:

Function maximization $Z=3 x_{1}+2 x_{2}$ with structure constraints
$x_{1} \leq 12$ (Source 1)
$x_{1}+3 x_{2} \leq 45$ (Source 2)
$2 x_{1}+x_{2} \leq 30$ (Source 3)
and
$x_{1} \geq 0, x_{2} \geq 0$
a) Solve the problem with a graphical method.

Recognize all possible corner point feasible solutions for this model.
b) Solve by the algebraic Simplex method.
c) Solve by Simplex method using tables.
d) Identify the'slack' values for the three sources of the final table for Simplex method.

Using the graphical solution method prove that these 'slack' values are right.

## PROBLEM 13

The following calculations represent the design of a production problem in order to maximize the profit of a company.

$$
\begin{aligned}
& F=4 x_{1}+2 x_{2}-x_{3}+x_{4} \\
& \text { and } \\
& x_{1}+x_{2}+x_{3}+x_{4}=100 \text { (A) } \\
& x_{2}+x_{4} \geq 50 \text { (B) } \\
& 6 x_{1}+3 x_{2}-1.5 x_{3}+1.5 x_{4} \leq 220 \text { (C) }
\end{aligned}
$$

Using the Simplex method for the solution of the problem gives the following optimal solution (where $x_{5}$ is the slack variable which cooperates with constraint $C$ and $x_{6}$ the artificial variable that cooperates with the constraint B):

| Base | $\mathrm{x}_{1}$ | $\mathrm{x}_{2}$ | $\mathrm{x}_{3}$ | $\mathrm{x}_{4}$ | $\mathrm{x}_{5}$ | $\mathrm{x}_{6}$ | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{x}_{1}$ | 1 | 0 | 0 | -0.2 | 0.13 | 0.6 | 19.33 |
| $\mathrm{x}_{3}$ | 0 | 0 | 1 | 0.2 | -0.13 | 0.4 | 30.67 |
| $\mathrm{x}_{2}$ | 0 | 1 | 0 | 1 | 0 | -1 | 50 |
| $-f$ | 0 | 0 | 0 | 0 | -0.67 | 0 | -146.67 |

1) From the final Simplex table results there are other optimal solutions.

Explain the reason of this situation and how can this be revealed from the final table.
2) There are two other basic optimal solutions. Beginning from the table given above, determine the final table for each of the other best solutions.
3) The production manager prefers the above optimal solution that contains the variables $x_{1}, x_{2}$ and $x_{3}$ at the base. For this he decided to apply this solution instead of the two alternative ones that were calculated at the question (2). However he would like to achieve a profit near 160. He may be ready to slack constraints $B$ and $C$ in order to succeed his goal, as far as variables $x_{1}, x_{2}$ and $x_{3}$ continue to have non zero values. What would you advise him?

## PROBLEM 14

A company expressed a linear programming model as following:
Function maximization
$f(x)=12 x_{1}+8 x_{2}+10 x_{3}$
with structure limitations

$$
\begin{array}{ll}
3 x_{1}+2 x_{2}+x_{3} & \leq 120 \\
5 x_{1}+4 x_{2}+3 x_{3} & \leq 300 \\
x_{1}+x_{2} & \leq
\end{array}
$$

The final table indicating the optimal solution using Simplex method is the following:

| Base | $\mathrm{x}_{1}$ | $\mathrm{x}_{2}$ | $\mathrm{x}_{3}$ | $\mathrm{x}_{4}$ | $\mathrm{x}_{5}$ | $\mathrm{x}_{6}$ | Right <br> hand side |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{x}_{3}$ | $*$ | 0 | 1 | 0 | 0 | 2 | ${ }^{*}$ |
| $\mathrm{x}_{5}$ | $*$ | 0 | 0 | -3 | 1 | -2 | $*$ |
| $\mathrm{x}_{2}$ | 1 | 1 | 0 | 0 | 0 | -1 | $*$ |
| $-f$ | $*$ | 0 | $*$ | -10 | 0 | $*$ | -600 |

where $x_{4}$ and $x_{5}$ are the slack variables for the first and the second constraint and $x_{6}$ is the artificial variable for the third constraint. Unfortunately, some parts of the table in which there are asterisks are covered with brown spots.

Calculate the 'missed' points and fill the final table.

## PROBLEM 15

Robotix manufactures two domestic robots - Mavis and Charles - each with different capabilities. Both require special circuits, of which only 1000 can be obtained each week. Mavis takes three of them, and Charles two of them.

Work is limited to 400 hours per week. The construction of each Mavis consumes two working hours and Charles one hour. Profits are 500 and 300 pounds respectively for each Mavis and Charles that is sold. The Robotix has signed a contract with a major customer to make and supply 200 Charles each week.

Mathprog computer program was used to produce the following Simplex method for the problem of Robotix:

|  | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{~S}_{1}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Zmax | 0 | 0 | -75 | 0 | -75 | -110000 |
| $\mathrm{X}_{1}$ | 1 | 0 | 0.25 | 0 | 0.75 | 100 |
| $\mathrm{X}_{2}$ | 0 | 1 | 0.00 | 0 | -1.00 | 200 |
| $\mathrm{~S}_{2}$ | 0 | 0 | -0.50 | 1 | -0.50 | 0 |

a) Give a full explanation of the above table.
b) With overtime, the company may increase the working hours to 480 hours. Would you give such an advice?
c) It is foreseen that soon Robotix will have 100 fewer available channels. How will this change affect the production of company products?

## PROBLEM 16

Constructive company Pontlins has recently obtained a range of 40 hectares in Bridleyon Sea where wants to build a new domestic holiday center. Plans have not yet been finalized, but it has been decided that the $70 \%$ of the range will be given for restaurants, social and entertaining operations. From the rest range, an estimated $75 \%$ will be needed for footpaths, streets, sidewalks and grass.
Sections of the wooden houses have three plans. Details are given below:

| Plan | Basic <br> region | Residential <br> units | Constructio <br> n cost <br> (pounds) | Annual income <br> per residential unit <br> (pounds) |
| :--- | :---: | :---: | :---: | :---: |
| Financial | 0.05 | 15 | 200.000 | 3.200 |
| Luxury | 0.075 | 10 | 150.000 | 3.800 |
| Superior | 0.1 | 6 | 100.000 | 5.000 |

The finances are limited and Pontlins cannot spend more than 9 million pounds for the construction of wooden houses. How many homes of each plan the company needs to construct to maximize the total income?

## PROBLEM 17

An English wine merchant introduces two types of wine, A and B, from vineyards that are far away and after the process, puts it in bottles and thus produces his two own brands, the Fein Wein and Party Plonk. Both wines A and B cost 0.80 and 0.20 pounds per liter, respectively, including the processing and bottling. The Fein Wein consists of $60 \%$ wine A and $40 \%$ wine B while the Party Plonk has $20 \%$ wine A and $80 \%$ wine B. The merchant shop sells 2 pounds per liter from Fein Wein and 1.20 pounds per liter from Party Plonk. The processing, bottles and distribution cost 0.5 pounds per liter for both brands.

The merchant has agreed to buy at least 24,000 liters of wine A this year and there are available 120.000 liters most of wine B. It is estimated that sales of Fein Wein during the year will reach 50,000 liters but the demand for the Party Plonk is uncertain. The merchant has this year only 60,000 pounds to buy the wines A and B. How many liters of the two brands must the merchant produce to maximize his profit?

## PROBLEM 18

An industrial company that is situated in the capital conducts its activities in three regional branches (factories) that have enough excess capacity. All three factories have the equipment required and the producing ability of a new specific product and it has already been decided to use part of the extra capacity for this purpose. The product can be manufactured in three sizes - large, medium and small - with a net unit profit of $€ 35$, $€ 30$ and $€ 25$ respectively. The three factories of the company, $\mathrm{X}, \mathrm{Y}$ and $Z$, have the necessary additional manpower and technological equipment to produce 750,900 and 450 units per day of the new product, respectively, regardless of the prevailing conditions.

However, the available storage areas are still limiting the rates of production. The factories $\mathrm{X}, \mathrm{Y}$ and Z store the daily production of the new product 1300, 1200 and 5000 $\mathrm{m}^{2}$ respectively. Each unit produced of the large size requires for its storage $2 \mathrm{~m}^{2}$, each unit requires a medium size of $1.5 \mathrm{~m}^{2}$ and finally each unit's small size requires $1.2 \mathrm{~m}^{2}$.

The sales forecast shows that the quantities can be sold each day from each of the three sizes are 900, 1200 and 750 units respectively. To maintain a consistent workload between factories and to have some flexibility, it has been decided that the additional production will be assigned to each factory must use the same percentage of the existing extra manpower and technological equipment.

The company's management wants to know the quantities of each size that will produce each of the factories in order to maximize the total profit.

## PROBLEM 19

A large multinational company decided to invest a significant part of its surplus by building three new factories, which are intended to produce three innovative products, $A, B$ and $C$ respectively.

Of these, on the one hand product $A$ is used for the production of $B$ and $C$, on the other hand product B is used for the production of C in the following way: To produce two units of product B requires the consumption of one unit of product A . To produce one unit of product $C$ requires the consumption of two units of $B$ and one unit of $A$.

The company's management wants to invest in all three industries the amount of $€$ 5000000 , in order to maximize its profits from the export of the three new products. Profits from the sale of each unit are in the ratio 1:3:11 for products $\mathrm{A}, \mathrm{B}$ and C respectively. The production capacities for each $€ 100000$ invested in each of the three factories are respectively 1000,500 and 300 units annually for the products $\mathrm{A}, \mathrm{B}$ and C .

Which is the best way of distributing the overall amount of investment in the three factories, considering that the demand for the export of products $A$ and $B$ is unlimited and only 1500 units of product C can be exported annually?

## PROBLEM 20

Red Sash Canning company produces cans with anchovies and sardines for super markets across the country. Production is planned on a monthly basis. The decision for next month is under consideration at the moment and the company needs your help. Design an approach you would take in each of the following questions:

1) Formulate the linear programming model with paper and pencil before a computer program is used.
2) Describe each variable and limitation in such way that any solution will be explanatory by itself.

Red Sash operates two canning machines which must give 300 hours canning per month. The company makes frequent checks on the quality of its products. All cans are electronic examined for defects. Next month 640 hours will be available for electronic testing.

As a result of this situation, the company has cash problems and budgeting limits are set by the fish market and canning materials for the next month of $56.000 €$ and $140,000 €$ respectively. More information about the functions of Red Sash are given below:

|  | Canning ratio (cans per hour) | Test ratio (cans per hour) | Fish cost (€ per can) | Canning materia cost <br> ( $€$ per can) | Profit (€ per can) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Anchovies | 1600 | 800 | 0.30 | 0.11 | 0.26 |
| Sardines | 2000 | 800 | 0. 20 | 0.08 | 0. 20 |

a) Before taking any final decision, the company wants your recommendations, your advices and your estimates.

1) Which do you think should be the plan of the enterprise?
2) What total profit will it give?
b) Red Sash wants answers to the following questions too:
3) Will money be given to increase the available time control?
4) What happens if you increase the budget of the canning materials?
5) What happens if you increase the budget of the fish goods?
6) Overtime can do the canning machines with a cost of 150 pounds per hour. Would you advise that it is desirable overtimes to be done?
c) If the budgets of fish goods and canning materials will be combined, can the company make more profit? If so, how?

## PROBLEM 21

Suppose that $x_{1}, x_{2}, x_{3}$ and $x_{4}$ represent the numbers of product units $1,2,3$ and 4 respectively that will be produced the next period. The objective is to maximize the total profit, using the constraints on the three machines $A, B$ and $C$. The problem turned into a linear programming problem under the following model:

```
Maxf \(=4 \times 1+6 \times 2+3 x 3+x 4\)
and
\(1.5 x_{1}+2 x_{2}+4 x_{3}+3 x_{4} \leq 550\) (hours of machine A)
\(4 x_{1}+x_{2}+2 x_{3}+x_{4} \leq 700\) (hours of machine B)
\(2 x_{1}+3 x_{2}+x 3+2 x 4 \leq 200\) (hours of machine C)
\(x 1, x 2, x 3, x 4 \geq 0\)
```

The solution obtained using the LINDO program is as follows:

## : LOOK ALL

MAX $\quad 4 x_{1}+6 x_{2}+3 x_{3}+x_{4}$ SUBJECT TO
2) $1.5 x_{1}+2 x_{2}+4 x_{3}+3 x_{4} \leq 550$
3) $4 x_{1}+x_{2}+2 x_{3}+x_{4} \leq 700$
4) $2 x_{1}+3 x_{2}+x_{3}+2 x_{4} \leq 200$

END
: GO
LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 525.000000

| VARIABLE | VALUE | REDUCED COST |
| :---: | :---: | :---: |
| $\mathrm{X}_{1}$ | .000000 | .050000 |
| $\mathrm{X}_{2}$ | 25.000000 | .000000 |
| $\mathrm{X}_{3}$ | 125.000000 | .000000 |
| $\mathrm{X}_{4}$ | .000000 | 3.500000 |
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| ROW | SLACK OR SURPLUS | DUAL PRICES |  |
| :---: | :---: | :---: | :---: |
| 2) | . 000000 | . 300000 |  |
| 3) | 425.000000 | . 000000 |  |
| 4) | . 000000 | 1.800000 |  |
| NO. ITERATIONS = 2 |  |  |  |
| DO RANGE(SENSITIVITY) ANALYSIS? |  |  |  |
| ?yes |  |  |  |
| RANGES IN WHICH THE BASIS IS UNCHANGED: |  |  |  |
| OBJ COEFFICIENT RANGES |  |  |  |
| VARIABLE | CURRENT | ALLOWABLE | ALLOWABLE |
|  | COEF | INCREASE | DECREASE |
| $\mathrm{X}_{1}$ | 4.000000 | . 050000 | INFINITY |
| $\mathrm{X}_{2}$ | 6.000000 | 3.000000 | . 076923 |
| $\mathrm{X}_{3}$ | 3.000000 | 9.000000 | . 999999 |
| X | 1.000000 | 3.500000 | INFINITY |


| ROW | CURRENT | ALLOWABLE | ALLOWABLE |
| :---: | :---: | :---: | :---: |
|  | RHS | INCREASE | DECREASE |
| 2) | 550.000000 | 250.000000 | 416.666600 |
| $3)$ | 700.000000 | INFINITY | 425.000000 |
| $4)$ | 200.000000 | 625.000000 | 62.500000 |

1) Which is the optimal production plan?
2) Which machines have excess capacity and how much?
3) It is possible to increase the capacity of engines against 100 hours in total costs 1.50 pounds per hour. Is it worthwhile to do this and if so where should the extra hours be used? What change will the total profit have?
4) The administration is thinking of increasing the profit of the products 3 and 4 occasionally with 2 pounds per unit. How will this affect the optimal production plan and how the total profit?
5) The administration decided that does not want the time spent by the production of products with machine A to be more than $50 \%$ of the total production time required for A, B and C. Express this new limitation, so it can be used as input by program LINDO.

## PROBLEM 22

A nursery planted deciduous and evergreen shrubs in an area of $30000 \mathrm{~m}^{2}$. An evergreen shrub requires $1 \mathrm{~m}^{2}$ and a deciduous $2 \mathrm{~m}^{2}$. The two types of shrubs have different climatic requirements, so that the number of the one type not to exceed twice the number of the other type. To be certain that good customers having reasonable orders will not exceed the number of shrubs, the number of deciduous was held between 7000 and 9000 plants, while the evergreen was delimited between 11500 and 14250. In addition, the nursery has long term contracts for a few years later, which require having any time requested 20000 bushes.

Unfortunately, evergreen shrubs require twice the attention the deciduous require while growing, so the nursery can only supply 36000 deciduous and 18000 evergreen shrubs or any possible combination of these two.

Until recently, the profit margin for deciduous shrubs was three times greater than that of evergreen, but some change in the market equated them. What effect will this change cause to the number of shrubs, if the manager of the nursery wants to maximize the total profit?

## PROBLEM 23

Suppose you inherited $6000 €$ and you want to invest them. Once the news are known, two friends of yours made the offer to become a partner in two different economic plans, each designed by each friend. In both cases the investment will work for some time in the summer and you should give some amount. To become a full partner in the first friend's project you need $500 €$ and 400 hours of employment and the profit (ignoring the waste of time) will be $4500 €$. The corresponding figures for the second project is $400 €$ and 500 hours with profit $4500 €$. But your friends are quite flexible and give you the chance to do any cooperation you want. The distributed profit between the partners will be equivalent with the degree of cooperation.

As a busy summer with maximum 600 hours time is expected, you decided to work with both your friends in any way of cooperation will offer the greatest profit. Solve the problem and find the right solution, by successively answering the following questions:
a) Formulate the Linear Programming model for this problem.
b) Solve the problem graphically. What is the total profit assumed?
c) Indicate each of the 4 possible Linear Programming assumptions.

Is any assumption more dubious than others? If so, what has to be done?

## PROBLEM 24

A company intends to maximize its global profits by producing and selling three new products. This problem is formulated as a linear programming model, where $B$, $R$ and $D$ represent the number of units in the budget, normal products and luxury products each week, respectively. There are limitations in the available production time in sections cutting, sewing and packing and marketing study that lead to low production levels in normal and luxury goods. The formulation and the solution given by the linear programming problems' solution package LINDO is as follows:
: LOOK ALL
MAX $3.75 \mathrm{~B}+7.63 \mathrm{R}+8.07 \mathrm{D}$
SUBJECT TO
2) $1.5 \mathrm{~B}+2 \mathrm{R}+\mathrm{D} \leq 9600$ (Cutting department)
3) $4 B+5 R+10 D \leq 38400$ (Sewing department)
4) $B+1.5 R+D \leq 6000$ (Packing department)
5) $R \geq 1000$
6) $D \geq 3000$

END
: GO

## LP OPTIMUM FOUND AT STEP 3

## OBJECTIVE FUCTION VALUE

1) 37028.3984

| VARIABLE | VALUE | REDUCED COST |
| :---: | :---: | :---: |
| B | 0.000000 | 2.354000 |
| R | 1680.000000 | 0.000000 |
| D | 3000.000000 | 0.000000 |

$\left.\begin{array}{crc}\text { ROW } & \begin{array}{rl}\text { SLACK OR }\end{array} & \text { DUAL PRICES } \\ & \text { SURPLUS }\end{array}\right]$

NO OF ITERATIONS =
DO RANGE (SENSITIVITY) ANALYSIS?

## ?YES

RANGES IN WHICH THE BASIS IS UNCHANGED: OBJ COEFFICIENT RANGES

| VARIABLE | CURRENT | ALLOWABLE | ALLOWABLE |
| :---: | :---: | :---: | :---: |
|  | COEFFICIENT | INCREASE | DECREASE |
| B | 3.750000 | 2.354000 | INFINITY |
| R | 7.630000 | INFINITY | 2.942500 |
| D | 8.070000 | 7.190001 | INFINITY |

RIGHTHAND SIDE RANGES

| ROW | CURRENT | ALLOWABLE | ALLOWABLE |
| :---: | :---: | :---: | :---: |
|  | RHS | INCREASE | DECREASE |
| 2 | 9600.000000 | INFINITY | 3240.000000 |
| 3 | 38400.000000 | 1599.999878 | 3400.000000 |
| 4 | 6000.000000 | INFINITY | 480.000000 |
| 5 | 1000.000000 | 680.000000 | INFINITY |
| 6 | 3000.000000 | 340.00000 | 240.000000 |

a) Which are the operation hours (in percentage of total time available) for the sections cutting, sewing and packing to the best solution?
b) Should the administration increase the capacity of the sewing department at 600 cents if the cost of change is $120 €$ ? If so (or not) why exactly?
c) The cost of construction of one product unit is $10 €$. What are the selling prices of the products included in the optimal solution?
d) The profit per unit for each product will increase by $20 \%$. Will it change the optimal production plan? Why exactly?
e) The administration decided that the budget units should be developed in at least one quarter of the total units produced.
Give this extra constraint, so it can be given for entry to LINDO.

