LESSON 26

Multiplying and Dividing Mixed Numbers

Power Up

Building Power

facts Power Up F

mental math

a. **Number Sense:** $8.56 + 98¢

b. **Decimals:** 30¢ × 100

c. **Number Sense:** $1.00 − 7¢

d. **Calculation:** 3 × 74

e. **Calculation:** $\frac{2}{3} + \frac{2}{3}$

f. **Fractional Parts:** $\frac{2}{3}$ of 24

g. **Patterns:** What number comes next in the pattern: 5, 11, 15, 21, ______

h. **Calculation:** 7 × 7, + 1, × 2, ÷ 5, + 5, ÷ 5, − 5, × 5

problem solving

The sum of two whole numbers is 17 and their product is 60. Find the two numbers.

New Concept

Increasing Knowledge

Math Language
An **improper fraction** is a fraction whose numerator is equal to or greater than its denominator.

One way to multiply or divide mixed numbers is to first rewrite the mixed numbers as improper fractions. Then we multiply or divide the improper fractions as indicated.

Example 1

Sergio used three lengths of ribbon $2\frac{1}{2}$ feet long to wrap packages. How many feet of ribbon did he use?

Solution

This is an equal groups problem. We want to find the total.

$$3 \times 2\frac{1}{2} = T$$

We will show two ways to find the answer. One way is to recognize that $3 \times 2\frac{1}{2}$ equals three $2\frac{1}{2}$s, which we add.

$$3 \times 2\frac{1}{2} = 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} = \mathbf{7\frac{1}{2}}$$

Another way to find the product is to write 3 and $2\frac{1}{2}$ as improper fractions and multiply.

Explain How can we write 3 as an improper fraction?

$$3 \times 2\frac{1}{2}$$
$$\downarrow \quad \downarrow$$
$$\frac{3}{1} \times \frac{5}{2} = \frac{15}{2} = 7\frac{1}{2}$$

Sergio used $7\frac{1}{2}$ **feet of ribbon.**

Example 2

Simplify:

a. $3\frac{2}{3} \times 1\frac{1}{2}$ **b.** $\left(1\frac{1}{2}\right)^2$

Solution

Thinking Skill

Predict

If we multiply example **a** without canceling, will we get the same answer? If so, will the answer be in simplest form?

a. We first rewrite $3\frac{2}{3}$ as $\frac{11}{3}$ and $1\frac{1}{2}$ as $\frac{3}{2}$. Then we multiply and simplify.

$$\frac{11}{\cancel{3}_1} \times \frac{\cancel{3}^1}{2} = \frac{11}{2} = \mathbf{5\frac{1}{2}}$$

b. The expression $\left(1\frac{1}{2}\right)^2$ means $1\frac{1}{2} \times 1\frac{1}{2}$. We write each factor as an improper fraction and multiply.

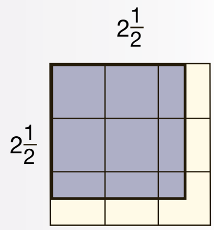
$$1\frac{1}{2} \times 1\frac{1}{2}$$
$$\downarrow \quad \downarrow$$
$$\frac{3}{2} \times \frac{3}{2} = \frac{9}{4} = \mathbf{2\frac{1}{4}}$$

Example 3

Find the area of a square with sides $2\frac{1}{2}$ inches long.

Solution

If we draw the square on a grid, we see a physical representation of the area of the square. We see four whole square inches, four half square inches, and one quarter square inch within the shaded figure. We can calculate the area by adding.



$$4 \text{ in.}^2 + \frac{4}{2} \text{ in.}^2 + \frac{1}{4} \text{ in.}^2 = \mathbf{6\frac{1}{4} \text{ in.}^2}$$

If we multiply $2\frac{1}{2}$ inches by $2\frac{1}{2}$ inches, we obtain the same result.

$$2\frac{1}{2} \text{ in.} \times 2\frac{1}{2} \text{ in.}$$
$$= \frac{5}{2} \text{ in.} \times \frac{5}{2} \text{ in.}$$
$$= \frac{25}{4} \text{ in.}^2 = \mathbf{6\frac{1}{4} \text{ in.}^2}$$

Formulate What multiplication expression can we write to show the perimeter of the same square? What is the perimeter?

Example 4

The biscuit recipe called for $3\frac{2}{3}$ cups of flour. To make half a batch, Greg divided the amount of each ingredient by 2. How many cups of flour should he use?

Solution

As we think about the problem, we see that by dividing $3\frac{2}{3}$ by 2, we will be finding *half of* $3\frac{2}{3}$. We can find half of a number either by dividing by 2 or by multiplying by $\frac{1}{2}$. In other words, the following are equivalent expressions:

$$3\frac{2}{3} \div 2 \qquad 3\frac{2}{3} \times \frac{1}{2}$$

Notice that multiplying by $\frac{1}{2}$ can be thought of as multiplying by the *reciprocal* of 2. We will write $3\frac{2}{3}$ as an improper fraction and multiply by $\frac{1}{2}$.

$$3\frac{2}{3} \times \frac{1}{2}$$

$$\downarrow$$

$$\frac{11}{3} \times \frac{1}{2} = \frac{11}{6} = \mathbf{1\frac{5}{6}}$$

Greg should use **$1\frac{5}{6}$ cups of flour.**

Example 5

Simplify: $3\frac{1}{3} \div 2\frac{1}{2}$

Solution

First we write $3\frac{1}{3}$ and $2\frac{1}{2}$ as improper fractions. Then we multiply by the reciprocal of the divisor and simplify.

$3\frac{1}{3} \div 2\frac{1}{2}$	original problem
$\downarrow \quad \downarrow$	
$\frac{10}{3} \div \frac{5}{2}$	changed mixed numbers to improper fractions
$\downarrow \quad \downarrow$	
$\frac{\overset{2}{\cancel{10}}}{3} \times \frac{2}{\underset{1}{\cancel{5}}} = \frac{4}{3}$	multiplied by reciprocal of the divisor
$= \mathbf{1\frac{1}{3}}$	simplified

Practice Set

a. **Model** Find the area of a rectangle that is $1\frac{1}{2}$ in. wide and $2\frac{1}{2}$ in. long. Illustrate the problem by drawing a 2 by 3 grid and sketching a $1\frac{1}{2}$ by $2\frac{1}{2}$ unit rectangle on the grid. Explain how the area of the rectangle can be found by using the sketch.

Evaluate Simplify:

b. $6\frac{2}{3} \times \frac{3}{5}$

c. $2\frac{1}{3} \times 3\frac{1}{2}$

d. $3 \times 3\frac{3}{4}$

e. $1\frac{2}{3} \div 3$

f. $2\frac{1}{2} \div 3\frac{1}{3}$

g. $5 \div \frac{2}{3}$

h. $2\frac{2}{3} \div 1\frac{1}{3}$

i. $1\frac{1}{3} \div 2\frac{2}{3}$

j. $4\frac{1}{2} \times 1\frac{2}{3}$

Written Practice

Strengthening Concepts

1. (11) After the first hour of the monsoon, 23 millimeters of precipitation had fallen. After the second hour a total of 61 millimeters of precipitation had fallen. How many millimeters of precipitation fell during the second hour?

2. (13) Each photograph enlargement cost 85¢ and Willie needed 26 enlargements. What was the total cost of the enlargements Willie needed?

3. (12) **Connect** The Byzantine Empire can be said to have begun in 330 when the city of Byzantium was renamed Constantinople and became the capital of the Roman Empire. The Byzantine Empire came to an end in 1453 when the city of Constantinople was renamed Istanbul and became the capital of the Ottoman Empire. About how many years did the Byzantine Empire last?

4. (11) At the movie theater, Dolores gave $20 to the ticket seller and got $10.25 back in change. How much did her movie ticket cost?

5. (13) A gross is a dozen dozens. A gross of pencils is how many pencils?

* 6. (22) **Model** Diagram this statement and answer the questions that follow. Begin by changing the percent to a reduced fraction.

   Forty percent of the 60 marbles in the bag were blue.

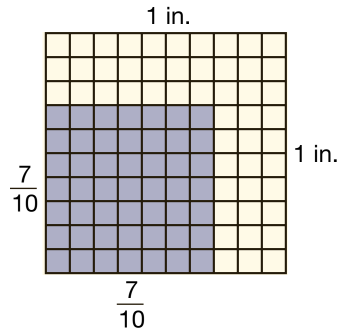
   a. How many of the marbles in the bag were blue?

   b. How many of the marbles in the bag were not blue?

* 7. (16) a. Roan estimated that the weight of the water in a full bathtub is a quarter ton. How many pounds is a quarter of a ton?

   b. **Explain** Describe how you got your answer.

8. *(8, 9)* The figure shows a one-inch square. A smaller square that is $\frac{7}{10}$ of an inch on each side is shaded.



a. What fraction of the square inch is shaded?

b. What percent of the square is not shaded?

*** 9.** *(24)* **a.** Write 210 and 252 as products of prime numbers. Then reduce $\frac{210}{252}$.

b. Find the GCF of 210 and 252.

10. *(9, 10)* Write the reciprocal of each number:

a. $\frac{5}{9}$ **b.** $5\frac{3}{4}$ **c.** 7

11. *(15)* Find the number that makes the two fractions equivalent.

a. $\frac{5}{8} = \frac{?}{24}$ **b.** $\frac{5}{12} = \frac{?}{24}$

c. Add the fractions you found in **a** and **b.**

*** 12.** *(18)* **Represent** Draw a 2-by-2 grid. On the grid sketch a $1\frac{1}{2}$ by $1\frac{1}{2}$ square. Assume that the sketch illustrates a square with sides $1\frac{1}{2}$ inches long. What is the area of the square? Explain how the sketch illustrates the area of the square.

13. *(8)* Draw $\overline{AB}$ 2 in. long. Then draw $\overline{BC}$ $1\frac{1}{2}$ in. long perpendicular to $\overline{AB}$. Complete $\triangle ABC$ by drawing $\overline{AC}$. How long is $\overline{AC}$?

14. *(4, 10)* **a.** Arrange these numbers in order from least to greatest:

$$1, -3, \frac{5}{6}, 0, \frac{4}{3}$$

b. Which of these numbers are whole numbers?

Solve:

15. *(10, 15)* $x - 8\frac{11}{12} = 6\frac{5}{12}$

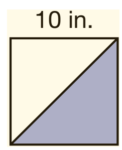
16. *(3)* $180 - y = 75$

17. *(3)* $12w = 360°$

*** 18.** *(23)* $w + 58\frac{1}{3} = 100$

19. *(20)* **a.** Find the area of the square.

b. Find the area of the shaded part of the square.



Simplify:

*** 20.** *(23)* $9\frac{1}{9} - 4\frac{4}{9}$

*** 21.** *(24)* $\frac{5}{8} \cdot \frac{3}{10} \cdot \frac{1}{6}$

*** 22.** *(20, 26)* $\left(2\frac{1}{2}\right)^2$

*** 23.** *(26)* $1\frac{3}{5} \div 2\frac{2}{3}$

*** 24.** *(26)* $3\frac{1}{3} \div 4$

*** 25.** *(26)* $5 \cdot 1\frac{3}{4}$

* **26.** (2, 9) **Justify** Name each property used to simplify this equation.

$$\frac{3}{2}\left(\frac{2}{3}x\right) = \frac{5}{6} \quad \text{Given}$$

$$\left(\frac{3}{2} \cdot \frac{2}{3}\right)x = \frac{5}{6} \quad \textbf{a.}\ \_\_\_\_\_\_\_\_$$

$$1x = \frac{5}{6} \quad \textbf{b.}\ \_\_\_\_\_\_\_\_$$

$$x = \frac{5}{6} \quad \textbf{c.}\ \_\_\_\_\_\_\_\_$$

27. (14, 15) Max is thinking of a counting number from 1 to 10. Deb guesses 7. What is the probability Deb's guess is correct?

28. (1, 9) **Analyze** Evaluate the following expressions for $x = 3$ and $y = 6$:

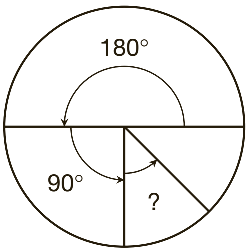
a. $x - \frac{y}{x}$ **b.** $\frac{xy}{y}$ **c.** $\frac{x}{y} \cdot \frac{y}{x}$

d. Which property is illustrated by **c**?

29. (2) **Predict** The rule of the following sequence is $k = 3n - 2$. Find the ninth term.

1, 4, 7, 10, ...

* **30.** (Inv. 2) **Conclude** The central angle of a half circle is 180°. The central angle of a quarter circle is 90°. How many degrees is the central angle of an eighth of a circle?



Early Finishers

Real-World Application

You and some friends volunteered to paint the concession stand for your local ball team. The back and side walls (both inside and outside) need painting.

The two side walls measure $9\frac{3}{5}$ by $8\frac{1}{3}$ feet each, while the back wall measures 39 feet by $8\frac{1}{3}$ feet. Find the total area that must be painted (in square feet). Show each step of your work.

LESSON 27

• Multiples
• Least Common Multiple
• Equivalent Division Problems

Power Up

Building Power

facts

Power Up E

mental math

a. Number Sense: $3.75 + $1.98

b. Decimals: $125.00 ÷ 10

c. Number Sense: 10×42

d. Calculation: 5×42

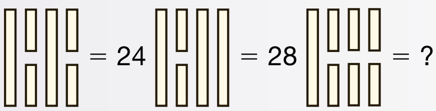
e. Calculation: $\frac{3}{4} + \frac{3}{4}$

f. Fractional Parts: $\frac{3}{4}$ of 24

g. Algebra: If $m = 9$, what does $3m$ equal?

h. Measurement: Start with a score. Add a dozen; then add the number of feet in a yard. Divide by half the number of years in a decade; then subtract the number of days in a week. What is the answer?

problem solving



Each bar shown above has a value. All long bars are worth the same amount, and all small bars are worth the same amount. How much is one long bar worth? How much is one short bar worth? What is the value of the third arrangement?

New Concepts

Increasing Knowledge

multiples

The **multiples** of a number are produced by multiplying the number by 1, by 2, by 3, by 4, and so on. Thus the multiples of 4 are

4, 8, 12, 16, 20, 24, 28, 32, 36, …

The multiples of 6 are

6, 12, 18, 24, 30, 36, 42, 48, 54, …

If we inspect these two lists, we see that some of the numbers in both lists are the same. A number appearing in both of these lists is a **common multiple** of 4 and 6. Below we have circled some of the common multiples of 4 and 6.

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, …

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, …

We see that 12, 24, and 36 are common multiples of 4 and 6. If we continued both lists, we would find many more common multiples.

least common multiple

Of particular interest is the least (smallest) of the common multiples. The **least common multiple** of 4 and 6 is 12. Twelve is the smallest number that is a multiple of both 4 and 6. The term *least common multiple* is often abbreviated **LCM.**

Example 1

Find the least common multiple of 6 and 8.

Solution

We will list some multiples of 6 and of 8 and circle common multiples.

Multiples of 6: 6, 12, 18, (24), 30, 36, 42, (48), …

Multiples of 8: 8, 16, (24), 32, 40, (48), 56, 64, …

We find that the least common multiple of 6 and 8 is **24.**

It is unnecessary to list multiples each time. Often the search for the least common multiple can be conducted mentally.

Example 2

Find the LCM of 3, 4, and 6.

Solution

To find the least common multiple of 3, 4, and 6, we can mentally search for the smallest number divisible by 3, 4, and 6. We can conduct the search by first thinking of multiples of the largest number, 6.

6, 12, 18, 24, …

Then we mentally test these multiples for divisibility by 3 and by 4. We find that 6 is divisible by 3 but not by 4, while 12 is divisible by both 3 and 4. Thus the LCM of 3, 4, and 6 is **12.**

We can use prime factorization to help us find the least common multiple of a set of numbers. The LCM of a set of numbers is the product of *all the prime factors necessary to form any number in the set.*

Example 3

Math Language

A **prime factorization** is the expression of a composite number as a product of its prime factors.

Use prime factorization to help you find the LCM of 18 and 24.

Solution

We write the prime factorization of 18 and of 24.

$$18 = 2 \cdot 3 \cdot 3 \qquad 24 = 2 \cdot 2 \cdot 2 \cdot 3$$

The prime factors of 18 and 24 are 2's and 3's. From a pool of three 2's and two 3's, we can form either 18 or 24. So the LCM of 18 and 24 is the product of three 2's and two 3's.

$$\text{LCM of 18 and 24} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$$
$$= \mathbf{72}$$

equivalent division problems

Tricia's teacher asked this question:

If sixteen health snacks cost $4.00, what was the price for each health snack?

Tricia quickly gave the correct answer, 25¢, and then explained how she found the answer.

I knew I had to divide $4.00 by 16, but I did not know the answer. So I mentally found half of each number, which made the problem $2.00 ÷ 8. I still couldn't think of the answer, so I found half of each of those numbers. That made the problem $1.00 ÷ 4, and I knew the answer was 25¢.

How did Tricia's mental technique work? She used the identity property of multiplication. Recall from Lesson 15 that we can form equivalent fractions by multiplying or dividing a fraction by a fraction equal to 1.

$$\frac{3}{4} \times \frac{10}{10} = \frac{30}{40} \qquad \frac{6}{9} \div \frac{3}{3} = \frac{2}{3}$$

We can form equivalent division problems in a similar way. We multiply (or divide) the dividend and divisor by the same number to form a new division problem that is easier to calculate mentally. The new division problem will produce the same quotient, as we show below.

$$\frac{\$4.00 \div 2}{16 \div 2} = \frac{\$2.00}{8} = \frac{\$2.00 \div 2}{8 \div 2} = \frac{\$1.00}{4} = \$0.25$$

Example 4

Thinking Skill

Explain

How does doubling the number and dividing by 10 make this problem easier?

Instead of dividing 220 by 5, double both numbers and mentally calculate the quotient.

Solution

We double the two numbers in 220 ÷ 5 and get 440 ÷ 10. We mentally calculate the new quotient to be **44.** Since 220 ÷ 5 and 440 ÷ 10 are equivalent division problems, we know that 44 is the quotient for both problems.

Example 5

Instead of dividing 6000 by 200, divide both numbers by 100, and then mentally calculate the quotient.

Solution

We mentally divide by 100 by removing two places (two zeros) from each number. This forms the equivalent division problem 60 ÷ 2. We mentally calculate the quotient as **30.**

Represent Show how the equivalent division problem was formed.

Practice Set

Find the least common multiple (LCM) of each pair or group of numbers:

a. 8 and 10 **b.** 4, 6, and 10

Use prime factorization to help you find the LCM of these pairs of numbers:

c. 24 and 40 **d.** 30 and 75

e. Instead of dividing $7\frac{1}{2}$ by $1\frac{1}{2}$, double each number and mentally calculate the quotient.

Mentally calculate each quotient by finding an equivalent division problem. What strategy did you use and why?

f. 24,000 ÷ 400 **g.** $6.00 ÷ 12 **h.** 140 ÷ 5

Written Practice *Strengthening Concepts*

1. (11) Octavio was writing a report on New Hampshire. He found that, in 2002, the population of Hanover, NH was 11,123. The population of Hollis, NH was 7416. The population of Newmarket, NH was 8449. What was the total population of these three places?

2. (13, 16) Rebecca and her mother built a shelf that was six feet long. How many inches long is this shelf?

*** 3.** (27) **Generalize** If the cost of one dozen eggs was $1.80, what was the cost per egg? Write an equivalent division problem that is easy to calculate mentally. Then find the quotient.

4. (5, 20) Which of the following equals one billion?

A 10^3 **B** 10^6 **C** 10^9 **D** 10^{12}

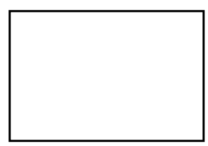
*** 5.** (22) Read this statement and answer the questions that follow.

Three eighths of the 712 students bought their lunch.

a. How many students bought their lunch?

b. How many students did not buy their lunch?

6. (19, 20) The width of this rectangle is 6 inches and its perimeter is 30 inches.



6 in.

a. What is the length of the rectangle?

b. What is the area of the rectangle?

*** 7.** (27) Use prime factorization to find the least common multiple of 25 and 45.

8. (4) What number is halfway between 3000 and 4000?

*** 9.** (15, 24) **a.** Write 24% as a reduced fraction.

b. Use prime factorization to reduce $\frac{36}{180}$.

10. (16) It was a very hot day. The temperature was 102°F in the shade.

a. The temperature was how many degrees above the freezing point of water?

b. The temperature was how many degrees below the boiling point of water?

c. *Discuss* What additional information did we need to know to answer **a** and **b?**

11. (2, 15) For each fraction, write an equivalent fraction that has a denominator of 36.

a. $\frac{5}{12}$ **b.** $\frac{1}{6}$ **c.** $\frac{7}{9}$

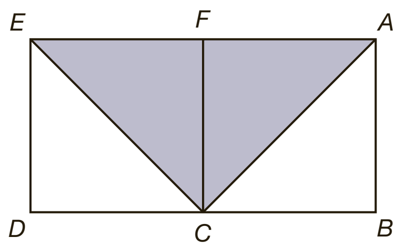
d. *Analyze* What property do we use when we find equivalent fractions?

* **12.** (21) **a.** *Generalize* Write the prime factorization of 576 using exponents.

b. Find $\sqrt{576}$.

* **13.** (26) Write $5\frac{5}{6}$ and $6\frac{6}{7}$ as improper fractions and find their product.

In the figure below, quadrilaterals *ABCF* and *FCDE* are squares. Refer to the figure to answer problems **14–16.**



14. (7) **a.** What kind of angle is $\angle ACD$?

b. Name two segments parallel to $\overline{FC}$.

15. (8) **a.** What fraction of square *CDEF* is shaded?

b. What fraction of square *ABCF* is shaded?

c. What fraction of rectangle *ABDE* is shaded?

16. (19, 20) If *AB* is 3 ft,

a. what is the perimeter of rectangle *ABDE?*

b. what is the area of rectangle *ABDE?*

Solve:

17. (3) $10y = 360°$

18. (3, 20) $p + 2^4 = 12^2$

* **19.** (23) $5\frac{1}{8} - n = 1\frac{3}{8}$

20. (10) $m - 6\frac{2}{3} = 4\frac{1}{3}$

Simplify:

*** 21.** (23) $10 - 1\frac{3}{5}$

*** 22.** (26) $5\frac{1}{3} \cdot 1\frac{1}{2}$

*** 23.** (26) $3\frac{1}{3} \div \frac{5}{6}$

*** 24.** (9, 15) $5\frac{1}{4} \div 3$

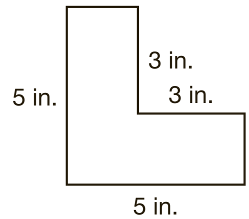
*** 25.** (24) $\frac{5}{4} \cdot \frac{9}{8} \cdot \frac{4}{15}$

26. (9, 15) $\frac{8}{9} - \left(\frac{7}{9} - \frac{5}{9}\right)$

27. (Inv. 2) If the diameter of a circle is half of a yard, then its radius is how many inches?

*** 28.** (27) ***Generalize*** Divide \$12.00 by 16 or find the quotient of an equivalent division problem.

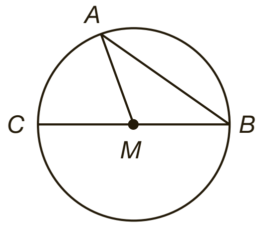
29. (19, 20) A 3-by-3-in. paper square is cut from a 5-by-5-in. paper square as shown.



a. What is the perimeter of the resulting polygon?

b. How many square inches of the 5-by-5-in. square remain?

*** 30.** (Inv. 2) ***Classify*** Refer to this circle with center at point *M* to answer **a–e:**



a. Which chord is a diameter?

b. Which chord is not a diameter?

c. What angle is an acute central angle?

d. Which angles are inscribed angles?

e. Which two sides of triangle *AMB* are equal in length?

LESSON

28

• Two-Step Word Problems
• Average, Part 1

Power Up

Building Power

facts

Power Up F

mental math

a. Number Sense: \$6.23 + \$2.99

b. Decimals: \$1.75 × 100

c. Calculation: \$5.00 − \$1.29

d. Calculation: 8 × 53

e. Calculation: $\frac{5}{8} + \frac{5}{8}$

f. Fractional Parts: $\frac{2}{5}$ of 25

g. Algebra: If $w = 10$, what does $10w$ equal?

h. Calculation: Think of an easier equivalent division for \$56.00 ÷ 14. Then find the quotient.

problem solving

There are two routes that Imani can take to school. There are three routes Samantha can take to school. If Imani is going from her house to school and then on to Samantha's house, how many different routes can Imani take? Draw a diagram that illustrates the problem.

New Concepts

Increasing Knowledge

two-step word problems

Thus far we have considered these six one-step word-problem themes:

1. combining	**4.** elapsed time
2. separating	**5.** equal groups
3. comparing	**6.** parts of a whole

Word problems often require more than one step to solve. In this lesson we will continue practicing problems that require multiple steps to solve. These problems involve two or more of the themes mentioned above.

Example 1

Julie went to the store with \$20. If she bought 8 cans of dog food for 67¢ per can, how much money did she have left?

Solution

This is a two-step problem. First we find out how much Julie spent. This first step is an "equal groups" problem.

Number in group	→	$0.67 each can
Number of groups	→	× 8 cans
Total	→	$5.36

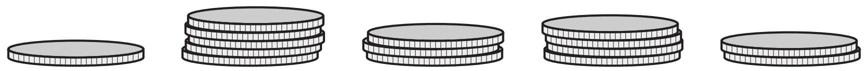
Now we can find out how much money Julie had left. This second step is about separating.

$$\begin{array}{r} \$20.00 \\ -\ \$5.36 \\ \hline \$14.64 \end{array}$$

After spending $5.36 of her $20 on dog food, Julie had **$14.64** left.

average, part 1

Calculating an **average** is often a two-step process. As an example, consider these five stacks of coins:

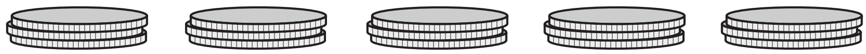


Thinking Skill

Summarize

In your own words, state a rule for finding an average?

There are 15 coins in all. If we made all the stacks the same size, there would be 3 coins in each stack.



Predict If there were 20 coins in all, and we made all the stacks the same size, how many coins would be in each stack?

We say the average number of coins in each stack is 3. Now look at the following problem:

There are 4 squads in the physical education class. Squad A has 7 players, squad B has 9 players, squad C has 6 players, and squad D has 10 players. What is the average number of players per squad?

The average number of players per squad is the number of players that would be on each squad if all of the squads had the same number of players. To find the average of a group of numbers, we combine the numbers by finding their sum.

$$\begin{array}{r} 7 \text{ players} \\ 6 \text{ players} \\ 9 \text{ players} \\ +\ 10 \text{ players} \\ \hline 32 \text{ players} \end{array}$$

Then we form equal groups by dividing the sum of the numbers by the number of numbers. There are 4 squads, so we divide by 4.

$$\frac{\text{sum of numbers}}{\text{number of numbers}} = \frac{32 \text{ players}}{4 \text{ squads}}$$

$$= 8 \text{ players per squad}$$

Finding the average took two steps. First we added the numbers to find the total. Then we divided the total to make equal groups.

Example 2

When people were seated, there were 3 in the first row, 8 in the second row, and 10 in the third row. What was the average number of people in each of the first three rows?

Solution

The average number of people in the first three rows is the number of people that would be in each row if the numbers were equal. First we add to find the total number of people.

$$\begin{array}{r} 3 \text{ people} \\ 8 \text{ people} \\ + \ 10 \text{ people} \\ \hline 21 \text{ people} \end{array}$$

Then we divide by 3 to separate the total into 3 equal groups.

$$\frac{21 \text{ people}}{3 \text{ rows}} = 7 \text{ people per row}$$

The average was **7 people** in each of the first 3 rows. Notice that the average of a set of numbers is *greater than the smallest number* in the set but *less than the largest number* in the set.

Another name for the average is the **mean.** We find the mean of a set of numbers by adding the numbers and then dividing the sum by the number of numbers.

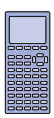
Example 3

In a word game, five students in the class scored 100 points, four scored 95, six scored 90, and five scored 80. What was the mean of the scores?

Solution

First we find the total of the scores.

$$\begin{array}{r} 5 \times 100 = \ 500 \\ 4 \times 95 = \ 380 \\ 6 \times 90 = \ 540 \\ 5 \times 80 = \ \underline{400} \\ 1820 \end{array}$$



Visit www.SaxonPublishers.com/ActivitiesC2 *for a graphing calculator activity.*

Next we divide the total by 20 because there were 20 scores in all.

$$\frac{\text{sum of numbers}}{\text{number of numbers}} = \frac{1820}{20} = 91$$

We find that the mean of the scores was **91.**

Practice Set

Generalize Work each problem as a two-step problem:

a. Jody went to the store with $20 and returned home with $5.36. She bought 3 jars of spaghetti sauce. What was the cost of each jar of sauce?

b. Three-eighths of the 32 wild ducks feeding in the lake were wood ducks, the rest were mallards. How many mallards were in the lake?

c. In Room 1 there were 28 students, in Room 2 there were 29 students, in Room 3 there were 30 students, and in Room 4 there were 25 students. What was the average number of students per room?

d. What is the mean of 46, 37, 34, 31, 29, and 24?

e. What is the average of 40 and 70? What number is halfway between 40 and 70?

f. *Explain* The Central High School basketball team's lowest game score was 80 and highest score was 95. Which of the following could be their average score? Why?

A 80 **B** 84 **C** 95 **D** 96

Written Practice *Strengthening Concepts*

* **1.** (28) Five volunteers collected bottles to be recycled. The number they collected were: 242, 236, 248, 268, and 226. What was the average number of bottles collected by the volunteers?

* **2.** (28) Yori ran a mile in 5 minutes 14 seconds. How many seconds did it take Yori to run a mile?

* **3.** (28) Luisa bought a pair of pants for $24.95 and 3 blouses for $15.99 each. Altogether, how much did she spend?

4. (12) The Italian navigator Christopher Columbus was 41 years old when he reached the Americas in 1492. In what year was he born?

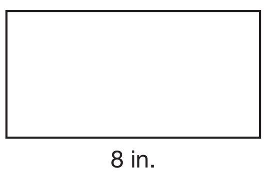
5. (22) In the following statement, change the percent to a reduced fraction. Then diagram the statement and answer the questions.

Salma led for 75% of the 5000-meter race.

a. Salma led the race for how many meters?

b. Salma did not lead the race for how many meters?

6. (19, 20) This rectangle is twice as long as it is wide.



a. What is the perimeter of the rectangle?

b. What is the area of the rectangle?

* **7.** (27) **a.** List the first six multiples of 3.

b. List the first six multiples of 4.

c. *Analyze* What is the LCM of 3 and 4?

d. Use prime factorization to find the least common multiple of 27 and 36.

* **8.** (27) **Predict** On the number line below, 283 is closest to

a. which multiple of 10?

b. which multiple of 100?

* **9.** (24) **Generalize** Write 56 and 240 as products of prime numbers. Then reduce $\frac{56}{240}$.

10. (16) A mile is five thousand, two hundred eighty feet. Three feet equals a yard. So a mile is how many yards?

11. (15) For **a** and **b,** find an equivalent fraction that has a denominator of 24.

a. $\frac{7}{8}$ **b.** $\frac{11}{12}$

c. What property do we use to find equivalent fractions?

12. (21) **a.** Write the prime factorization of 3600 using exponents.

b. Find $\sqrt{3600}$.

* **13.** (28) **Summarize** Describe how to find the mean of 45, 36, 42, 29, 16, and 24.

14. (8, 20) **a.** Draw square *ABCD* so that each side is 1 inch long. What is the area of the square?

b. Draw segments *AC* and *BD*. Label the point at which they intersect point *E*.

c. Shade triangle *CDE*.

d. What percent of the area of the square did you shade?

* **15.** (4, 10) **a.** Arrange these numbers in order from least to greatest:

$$-1, \frac{1}{10}, 1, \frac{11}{10}, 0$$

b. **Classify** Which of these numbers are odd integers?

Solve:

16. (3) $12y = 360°$

17. (3, 20) $10^2 = m + 8^2$

18. (3) $\frac{180}{w} = 60$

Simplify:

19. (9, 15) $4\frac{5}{12} - 1\frac{1}{12}$

20. (10, 15) $8\frac{7}{8} + 3\frac{3}{8}$

* **21.** (23) $12 - 8\frac{1}{8}$

* **22.** (26) $6\frac{2}{3} \cdot 1\frac{1}{5}$

* **23.** (20, 26) $\left(1\frac{1}{2}\right)^2 \div 7\frac{1}{2}$

* **24.** (26) $8 \div 2\frac{2}{3}$

25. (1) $\frac{10{,}000}{80}$

* **26.** (25) $\frac{3}{4} - \left(\frac{1}{2} \div \frac{2}{3}\right)$

27. (1, 20) Evaluate the following expressions for $x = 3$ and $y = 4$:

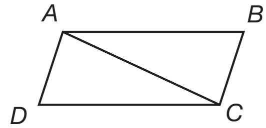
a. x^y

b. $x^2 + y^2$

28. (16) Summarize What is the rule for this function?

Input	Output
1	3
2	5
4	9
5	11
10	21
12	25

29. (18) In the figure below, the two triangles are congruent.



a. Which angle in $\triangle ACD$ corresponds to $\angle CAB$ in $\triangle ABC$?

b. Which segment in $\triangle ABC$ corresponds to $\overline{AD}$ in $\triangle ACD$?

c. If the area of $\triangle ABC$ is $7\frac{1}{2}$ in.^{2}, what is the area of figure $ABCD$?

30. (17) With a ruler draw $\overline{PQ}$ $2\frac{3}{4}$ in. long. Then with a protractor draw $\overrightarrow{QR}$ so that $\angle PQR$ measures 30°. Then, from point P, draw a ray perpendicular to $\overline{PQ}$ that intersects $\overrightarrow{QR}$. (You may need to extend $\overrightarrow{QR}$ to show the intersection.) Label the point where the rays intersect point M. Use a protractor to measure $\angle PMQ$.

Early Finishers

Math and Science

On average, a heart beats about 60 to 80 times a minute when a person is at rest. Silvia wanted to know her average resting heart rate, so she recorded her resting heart rate every morning for one week, as shown below.

80, 75, 77, 66, 61, 73, 65

a. Find Silvia's average resting heart rate.

b. Is her resting heart rate normal? Support your answer.

LESSON 29

• Rounding Whole Numbers
• Rounding Mixed Numbers
• Estimating Answers

Power Up

Building Power

facts Power Up E

mental math

a. Calculation: \$4.32 + \$2.98

b. Decimals: \$12.50 ÷ 10

c. Calculation: \$10.00 − \$8.98

d. Calculation: 9×22

e. Calculation: $\frac{5}{6} + \frac{5}{6}$

f. Fractional Parts: $\frac{3}{5}$ of 20

g. Algebra: If $x = 4$, what does $4x$ equal?

h. Calculation: 6 × 6, ÷ 4, × 3, + 1, ÷ 4, × 8, − 1, ÷ 5, × 2, − 2, ÷ 2

problem solving

The diameter of a penny is $\frac{3}{4}$ inch. How many pennies placed side by side would it take to make a row of pennies 1 foot long?

New Concepts

Increasing Knowledge

rounding whole numbers

The first sentence below uses an exact number to state the size of a crowd. The second sentence uses a rounded number.

There were 3947 fans at the game.

There were about 4000 fans at the game.

Thinking Skill

Analyze

What word in the second sentence of the problem tells you that 4000 is not an exact number?

Rounded numbers are often used instead of exact numbers. One way to round a number is to consider its location on the number line.

Example 1

Use a number line to

a. round 283 to the nearest hundred.

b. round 283 to the nearest ten.

Solution

a. We draw a number line showing multiples of 100 and mark the estimated location of 283.

We see that 283 is between 200 and 300 and is closer to 300. To the nearest hundred, 283 rounds to **300.**

b. We draw a number line showing the tens from 200 to 300 and mark the estimated location of 283.

We see that 283 is between 280 and 290 and is closer to 280. To the nearest ten, 283 rounds to **280.**

Sometimes we are asked to round a number to a certain place value. We can use an underline and a circle to help us do this. We will underline the digit in the place to which we are rounding, and we will circle the next place to the right. Then we will follow these rules:

1. If the circled digit is 5 or more, we add 1 to the underlined digit. If the circled digit is less than 5, we leave the underlined digit unchanged.
2. We replace the circled digit and all digits to the right of the circled digit with zeros.

This rounding strategy is sometimes called the "4-5 split," because if the circled digit is 4 or less we round down, and if it is 5 or more we round up.

Example 2

a. Round 283 to the nearest hundred.

b. Round 283 to the nearest ten.

Solution

a. We underline the 2 since it is in the hundreds place. Then we circle the digit to its right.

2⑧3

Since the circled digit is 5 or more, we add 1 to the underlined digit, changing it from 2 to 3. Then we replace the circled digit and all digits to its right with zeros and get

300

b. Since we are rounding to the nearest ten, we underline the tens digit and circle the digit to its right.

28③

Since the circled digit is less than 5, we leave the 8 unchanged. Then we replace the 3 with a zero and get

280

Discuss Did we find the same answer using the 4–5 split strategy as we did using the number line? How are the two strategies different?

Example 3

Round 5280 so that there is one nonzero digit.

Solution

We round the number so that all but one of the digits are zeros. In this case we round to the nearest thousand, so 5280 rounds to **5000.**

Represent Write the number 5280. Underline the digit in the thousands place, and circle the digit to its right. How do we know to round down?

Example 4

Round 93,167,000 to the nearest million.

Solution

To the nearest million, 93,①67,000 rounds to **93,000,000.**

rounding mixed numbers

When rounding a mixed number to a whole number, we need to determine whether the fraction part of the mixed number is greater than, equal to, or less than $\frac{1}{2}$. If the fraction is greater than or equal to $\frac{1}{2}$, the mixed number rounds up to the next whole number. If the fraction is less than $\frac{1}{2}$, the mixed number rounds down.

A fraction is greater than $\frac{1}{2}$ if the numerator of the fraction is more than half of the denominator. A fraction is less than $\frac{1}{2}$ if the numerator is less than half of the denominator.

Example 5

Round $14\frac{7}{12}$ to the nearest whole number.

Solution

The mixed number $14\frac{7}{12}$ is between the consecutive whole numbers 14 and 15. We study the fraction to decide which is nearer. The fraction $\frac{7}{12}$ is greater than $\frac{1}{2}$ because 7 is more than half of 12. So $14\frac{7}{12}$ rounds to **15.**

estimating answers

Rounding can help us **estimate** the answers to arithmetic problems. Estimating is a quick and easy way to get close to an exact answer. Sometimes a close answer is "good enough," but even when an exact answer is necessary, estimating can help us determine whether our exact answer is reasonable. One way to estimate is to round the numbers before calculating.

Example 6

Barb stopped by the store on the way home to buy two gallons of milk for \$2.79 per gallon, a loaf of bread for \$1.89, and a jar of peanut butter for \$3.15. About how much should she expect to pay for these items?

Solution

By rounding to the nearest dollar, shoppers can mentally keep a running total of the cost of items they are purchasing. Rounding the \$2.79 price per gallon of milk to \$3.00, the \$1.89 price of bread to \$2.00, and the \$3.15 price of peanut butter to \$3.00, we estimate the total to be

$$\$3 + \$3 + \$2 + \$3 = \$11$$

Barb should expect to pay about **\$11.00.**

Example 7

Mentally estimate:

a. $5\frac{7}{10} \times 3\frac{1}{3}$ **b.** 396×312 **c.** $4160 \div 19$

Solution

a. We round each mixed number to the nearest whole number before we multiply.

$$5\frac{7}{10} \times 3\frac{1}{3}$$
$$\downarrow \quad \downarrow$$
$$6 \times 3 = \mathbf{18}$$

b. When mentally estimating we often round the numbers to one nonzero digit so that the calculation is easier to perform. In this case we round to the nearest hundred.

$$400 \times 300 = \mathbf{120{,}000}$$

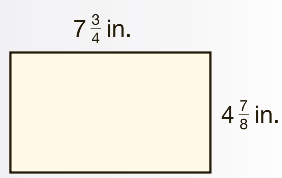
c. We round each number so there is one nonzero digit before we divide.

$$\frac{4160}{19} \longrightarrow \frac{4000}{20} = \mathbf{200}$$

Performing a quick mental estimate helps us determine whether the result of a more complicated calculation is reasonable.

Example 8

Eldon calculated the area of this rectangle to be $25\frac{1}{4}$ sq. in. Is Eldon's calculation reasonable? Why or why not?



Solution

By estimating the area we can decide quickly whether Eldon's answer is reasonable. We round $7\frac{3}{4}$ up to 8 and round $4\frac{7}{8}$ up to 5 and estimate that the area of the rectangle is a little less than 40 sq. in. (8 in. × 5 in.). Based on this estimate, Eldon's calculation seems unreasonably low. Furthermore, by rounding the length and width down to 7 in. and 4 in., we see that the area of the rectangle must be more than 28 sq. in. This confirms that Eldon's calculation is not correct.

Practice Set

a. Round 1760 to the nearest hundred.

b. Round 5489 to the nearest thousand.

c. Round 186,282 to the nearest thousand.

Estimate each answer:

d. $7986 - 3074$

e. 297×31

f. $5860 \div 19$

g. $12\frac{1}{4} \div 3\frac{7}{8}$

h. Calculate the area of this rectangle. After calculating, check the reasonableness of your answer.

Written Practice

Strengthening Concepts

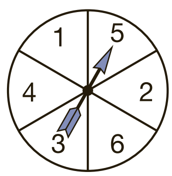
*** 1.** (16, 28) In the 1996 Summer Olympics, Charles Austin won the high jump event by jumping 7 feet 10 inches. How many inches did he jump?

2. (13) **Justify** If 8 pounds of bananas cost $5.52, what does 1 pound of bananas cost? How did you find the cost per pound?

Math Language
The **mean** of a set of numbers is the same as the average of the numbers.

*** 3.** (28) The number of fruit flies in each of Sandra's six samples were: 75, 70, 80, 80, 85, and 90. What was the mean number of fruit flies in her samples?

4. (14, 21) With one spin, what is the probability the arrow will stop on a prime number?



5. (22) **Evaluate** In the following statement, change the percent to a reduced fraction. Then diagram the statement and answer the questions.

Forty percent of the 80 birds were robins.

a. How many of the birds were robins?

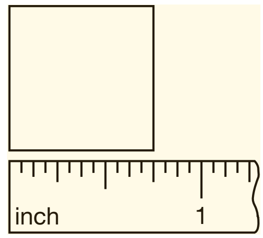
b. How many of the birds were not robins?

* **6.** (27) **a.** What is the least common multiple (LCM) of 4, 6, and 8?

b. *Represent* Use prime factorization to find the LCM of 16 and 36.

7. (19, 20) **a.** What is the perimeter of this square?

b. What is the area of this square?



* **8.** (29) **a.** Round 366 to the nearest hundred.

b. Round 366 to the nearest ten.

* **9.** (29) *Estimate* Mentally estimate the sum of 6143 and 4952 by rounding each number to the nearest thousand before adding.

* **10.** (26, 29) **a.** *Estimate* Mentally estimate the following product by rounding each number to the nearest whole number before multiplying:

$$\frac{3}{4} \cdot 5\frac{1}{3} \cdot 1\frac{1}{8}$$

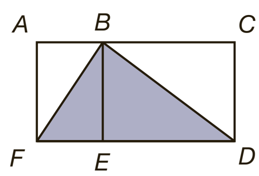
b. *Estimate* Now find the exact product of these fractions and mixed numbers.

11. (15) Complete each equivalent fraction:

a. $\frac{2}{3} = \frac{?}{30}$ **b.** $\frac{?}{6} = \frac{25}{30}$

12. (20, 21) The prime factorization of 1000 is $2^3 \cdot 5^3$. Write the prime factorization of one billion using exponents.

In the figure below, quadrilaterals *ACDF, ABEF,* and *BCDE* are rectangles. Refer to the figure to answer problems **13–15.**



13. (8) **a.** What percent of rectangle *ABEF* is shaded?

b. What percent of rectangle *BCDE* is shaded?

c. What percent of rectangle *ACDF* is shaded?

14. (19, 20) **Infer** The relationships between the lengths of the sides of the rectangles are as follows:

$$AB + FE = BC$$
$$AF + CD = AC$$
$$AB = 2 \text{ in.}$$

a. Find the perimeter of rectangle *ABEF.*

b. Find the area of rectangle *BCDE.*

15. (18) **Infer** Triangle *ABF* is congruent to $\triangle EFB$.

a. Which angle in $\triangle ABF$ corresponds to $\angle EBF$ in $\triangle EFB$?

b. What is the measure of $\angle A$?

Solve:

16. (3, 20) $8^2 = 4m$

* **17.** (23) $x + 4\frac{4}{9} = 15$

18. (10, 15) $3\frac{5}{9} = n - 4\frac{7}{9}$

Simplify:

19. (23) $6\frac{1}{3} - 5\frac{2}{3}$

* **20.** (26) $6\frac{2}{3} \div 5$

* **21.** (26) $1\frac{2}{3} \div 3\frac{1}{2}$

22. (1) $\$7.49 \times 24$

* **23.** (29) **Explain** Describe how to estimate the product of $5\frac{1}{3}$ and $4\frac{7}{8}$.

24. (20) Find the missing exponents.

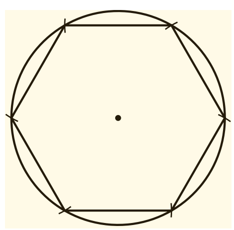
a. $10^3 \cdot 10^3 = 10^m$

b. $\frac{10^6}{10^3} = 10^n$

25. (2) The rule of the following sequence is $k = 2^n + 1$. Find the fifth term of the sequence.

3, 5, 9, 17, ...

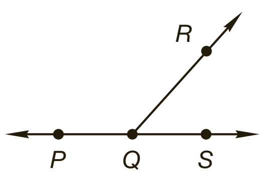
26. (19, Inv. 2) Recall how you inscribed a regular hexagon in a circle in Investigation 2. If the radius of this circle is 1 inch,



a. what is the diameter of the circle?

b. what is the perimeter of the hexagon?

27. (7) Use the figure below to identify the types of angles in **a–c**.



a. $\angle RQS$?

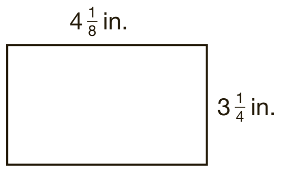
b. $\angle PQR$?

c. $\angle PQS$?

28. (15) Find fractions equivalent to $\frac{2}{3}$ and $\frac{1}{2}$ with denominators of 6. Subtract the smaller fraction you found from the larger fraction.

29. (16) Reggie and Elena each had one cup of water during a break in the soccer game. They took the water from the same 1-quart container. If they took two cups total from the full 1-quart container, how many ounces of water were left?

* **30.** (29) A photograph has the dimensions shown.



a. Estimate the area of the photograph.

b. *Verify* Is the actual area of the photograph more or less than your estimate? How do you know?

Early Finishers
Math and Science

The following list shows the average distance from the Sun to each of the nine planets in kilometers.

a. Round each distance to the nearest million.

b. Is Venus or Mars closer to Earth? Use the rounded distances to support your answers

Planet	Distance (in thousands)
Mercury	57,910
Venus	108,200
Earth	149,600
Mars	227,940
Jupiter	778,330
Saturn	1,426,940
Uranus	2,870,990
Neptune	4,497,070
Pluto	5,913,520

LESSON 30

• Common Denominators
• Adding and Subtracting Fractions with Different Denominators

Power Up

Building Power

facts Power Up F

mental math

a. Number Sense: \$1.99 + \$1.99

b. Decimals: \$0.15 × 1000

c. Equivalent Fractions: $\frac{3}{4} = \frac{?}{12}$

d. Calculation: 5 × 84

e. Calculation: $1\frac{2}{3} + 2\frac{2}{3}$

f. Fractional Parts: $\frac{3}{4}$ of 20

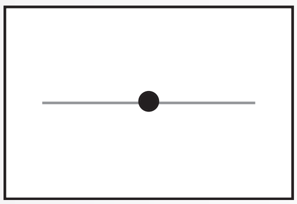
g. Estimation: Estimate the sum of 43 and 23

h. Calculation: Find $\frac{1}{2}$ of 88, + 4, ÷ 8, × 5, − 5, double that number, − 2, ÷ 2, ÷ 2, ÷ 2.

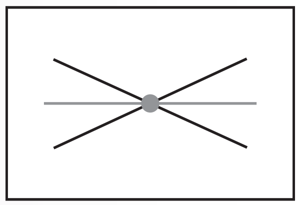
problem solving

Artists since the 14th century have used a geometric illusion in painting and drawing called ***one-point perspective.*** One-point perspective allows the artist to make it appear that objects in the drawing vanish into the distance, even though the drawing is two-dimensional. Follow the five steps provided to create a one-point perspective drawing.

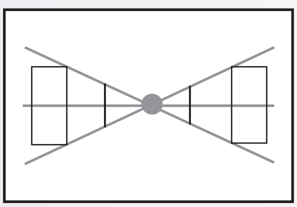
The ***horizon line*** divides the sky from the earth.



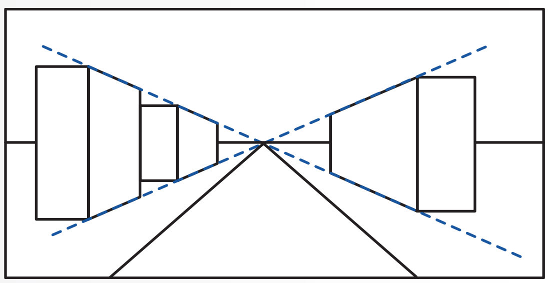
The ***vanishing point*** marks the direction in which you are looking.



The ***construction lines*** show the tops and bottoms of the buildings.



The edges of the buildings' sides will be both perpendicular and parallel to the ***horizon line.***



All ***receding lines*** will merge at the ***vanishing point.*** Erase ***construction lines*** and add details to complete the ***one-point perspective*** drawing.

New Concepts

Increasing Knowledge

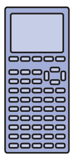
common denominators

If two fractions have the same denominator, we say they have **common denominators.**

$\frac{3}{8}$ $\frac{6}{8}$	$\frac{3}{8}$ $\frac{3}{4}$
These two fractions have common denominators.	These two fractions do not have common denominators.

If two fractions do not have common denominators, then one or both fractions can be renamed so both fractions do have common denominators. We remember that we can rename a fraction by multiplying it by a fraction equal to 1. Thus by multiplying by $\frac{2}{2}$, we can rename $\frac{3}{4}$ so that it has a denominator of 8.

$$\frac{3}{4} \cdot \frac{2}{2} = \frac{6}{8}$$



Visit www.SaxonPublishers.com/ActivitiesC2 *for a graphing calculator activity.*

Thinking Skill

Explain

Describe one way to find the least common multiple of 3 and 4.

Example 1

Rename $\frac{2}{3}$ and $\frac{1}{4}$ so that they have common denominators.

Solution

The denominators are 3 and 4. A common denominator for these two fractions would be any common multiple of 3 and 4. The **least common denominator** would be the least common multiple of 3 and 4, which is 12. We want to rename each fraction so that the denominator is 12.

$$\frac{2}{3} = \frac{}{12} \qquad \frac{1}{4} = \frac{}{12}$$

We multiply $\frac{2}{3}$ by $\frac{4}{4}$ and multiply $\frac{1}{4}$ by $\frac{3}{3}$.

$$\frac{2}{3} \cdot \frac{4}{4} = \frac{8}{12} \qquad \frac{1}{4} \cdot \frac{3}{3} = \frac{3}{12}$$

Thus $\frac{2}{3}$ and $\frac{1}{4}$ can be written with common denominators as

$$\mathbf{\frac{8}{12}} \quad \text{and} \quad \mathbf{\frac{3}{12}}$$

Fractions written with common denominators can be compared by simply comparing the numerators.

Explain In this example, the least common denominator is the product of the two original denominators. Is the product of the denominators always a common denominator? Is the product of the denominators always the least common denominator? Explain.

Example 2

Write these fractions with common denominators and then compare the fractions.

$$\frac{5}{6} \bigcirc \frac{7}{9}$$

Solution

Reading Math

Read the abbreviation LCM as "least common multiple."

The least common denominator for these fractions is the LCM of 6 and 9, which is 18.

$$\frac{5}{6} \cdot \frac{3}{3} = \frac{15}{18} \qquad \frac{7}{9} \cdot \frac{2}{2} = \frac{14}{18}$$

In place of $\frac{5}{6}$ we may write $\frac{15}{18}$, and in place of $\frac{7}{9}$ we may write $\frac{14}{18}$. Then we compare the renamed fractions.

$$\frac{15}{18} \bigcirc \frac{14}{18} \qquad \text{renamed}$$

$$\mathbf{\frac{15}{18} > \frac{14}{18}} \qquad \text{compared}$$

Example 3

Arrange the following fractions in order from least to greatest. (You may write the fractions with common denominators to help you order them.)

$$\frac{2}{3}, \frac{5}{6}, \frac{7}{12}$$

Solution

Since $\frac{2}{3} = \frac{8}{12}$ and $\frac{5}{6} = \frac{10}{12}$, the order is $\mathbf{\frac{7}{12}, \frac{2}{3}, \frac{5}{6}}$.

adding and subtracting fractions with different denominators

To add or subtract two fractions that do not have common denominators, we first rename one or both fractions so they do have common denominators. Then we can add or subtract.

Example 4

Add: $\mathbf{\frac{3}{4} + \frac{3}{8}}$

Solution

First we write the fractions so they have common denominators. The denominators are 4 and 8. The least common multiple of 4 and 8 is 8. We rename $\frac{3}{4}$ so the denominator is 8 by multiplying by $\frac{2}{2}$. We do not need to rename $\frac{3}{8}$. Then we add the fractions and simplify.

$$\begin{aligned} \frac{3}{4} \cdot \frac{2}{2} &= \frac{6}{8} && \text{renamed } \frac{3}{4} \\ + \frac{3}{8} \quad &= \frac{3}{8} \\ \hline &\quad \frac{9}{8} && \text{added} \end{aligned}$$

We finish by simplifying $\frac{9}{8}$.

$$\frac{9}{8} = \mathbf{1\frac{1}{8}}$$

Formulate Write a real world word problem involving the addition of $\frac{3}{4}$ and $\frac{3}{8}$. Then answer your problem.

Example 5

Subtract: $\frac{5}{6} - \frac{3}{4}$

Solution

First we write the fractions so they have common denominators. The LCM of 6 and 4 is 12. We multiply $\frac{5}{6}$ by $\frac{2}{2}$ and multiply $\frac{3}{4}$ by $\frac{3}{3}$ so that both denominators are 12. Then we subtract the renamed fractions.

$$\begin{aligned} \frac{5}{6} \cdot \frac{2}{2} &= \frac{10}{12} && \text{renamed } \frac{5}{6} \\ - \frac{3}{4} \cdot \frac{3}{3} &= \frac{9}{12} && \text{renamed } \frac{3}{4} \\ \hline &\quad \mathbf{\frac{1}{12}} && \text{subtracted} \end{aligned}$$

Example 6

Subtract: $8\frac{2}{3} - 5\frac{1}{6}$

Solution

We first write the fractions so that they have common denominators. The LCM of 3 and 6 is 6. We multiply $\frac{2}{3}$ by $\frac{2}{2}$ so that the denominator is 6. Then we subtract and simplify.

$$\begin{aligned} 8\frac{2}{3} &= 8\frac{4}{6} && \text{renamed } 8\frac{2}{3} \\ - 5\frac{1}{6} &= 5\frac{1}{6} \\ \hline &\quad 3\frac{3}{6} = \mathbf{3\frac{1}{2}} && \text{subtracted and simplified} \end{aligned}$$

Example 7

Add: $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$

Solution

The denominators are 2, 3, and 4. The LCM of 2, 3, and 4 is 12. We rename each fraction so that the denominator is 12. Then we add and simplify.

$$\begin{aligned} &\frac{1}{2} \cdot \frac{6}{6} = \frac{6}{12} && \text{renamed } \tfrac{1}{2} \\ &\frac{2}{3} \cdot \frac{4}{4} = \frac{8}{12} && \text{renamed } \tfrac{2}{3} \\ + &\frac{3}{4} \cdot \frac{3}{3} = \frac{9}{12} && \text{renamed } \tfrac{3}{4} \\ \hline &\qquad\;\; \frac{23}{12} = \mathbf{1\frac{11}{12}} && \text{added and simplified} \end{aligned}$$

Recall from Lesson 27 that prime factorization helps us find the least common multiple. We factor the numbers. Then we find the pool of numbers from which we can form either number. Consider 24 and 32.

$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$

$$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

We can form either number from a pool of factors containing five 2s and one 3. Thus, the LCM of 24 and 32 is

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 96$$

Example 8

Use prime factorization to help you add $\frac{5}{32} + \frac{7}{24}$.

Solution

We write the prime factorization of the denominators for both fractions.

$$\frac{5}{32} = \frac{5}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} \qquad \frac{7}{24} = \frac{7}{2 \cdot 2 \cdot 2 \cdot 3}$$

The least common denominator of the two fractions is the least common multiple of the denominators. So the least common denominator is

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 96$$

To rename the fractions with common denominators, we multiply $\frac{5}{32}$ by $\frac{3}{3}$, and we multiply $\frac{7}{24}$ by $\frac{2 \cdot 2}{2 \cdot 2}$.

$$\begin{aligned} &\frac{5}{32} \cdot \frac{3}{3} \quad\;\; = \frac{15}{96} \\ + &\frac{7}{24} \cdot \frac{2 \cdot 2}{2 \cdot 2} = \frac{28}{96} \\ \hline &\qquad\qquad\quad\;\; \mathbf{\frac{43}{96}} \end{aligned}$$

Practice Set

Write the fractions so that they have common denominators. Then compare the fractions.

a. $\frac{3}{5} \bigcirc \frac{7}{10}$

b. $\frac{5}{12} \bigcirc \frac{7}{15}$

c. Use common denominators to arrange these fractions in order from least to greatest:

$$\frac{1}{2}, \frac{3}{10}, \frac{2}{5}$$

Add or subtract:

d. $\frac{3}{4} + \frac{5}{6} + \frac{3}{8}$

e. $7\frac{5}{6} - 2\frac{1}{2}$

f. $4\frac{3}{4} + 5\frac{5}{8}$

g. $4\frac{1}{6} - 2\frac{5}{9}$

Use prime factorization to help you add or subtract the fractions in problems **h** and **i.**

h. $\frac{25}{36} + \frac{5}{60}$

i. $\frac{3}{25} - \frac{2}{45}$

j. ***Justify*** Choose one of the exercises you answered in this practice set. Explain the steps you took to find the answer.

Written Practice

Strengthening Concepts

*** 1.** *(28)* The 5 starters on the basketball team were tall. Their heights were 76 inches, 77 inches, 77 inches, 78 inches, and 82 inches. What was the average height of the 5 starters?

*** 2.** *(28)* Marie bought 6 pounds of carrots for $0.87 per pound and paid for them with a $10 bill. How much did she get back in change?

*** 3.** *(29)* ***Verify*** While helping her father build a stone fence, Tanisha lifted 17 rocks averaging 8 pounds each. She calculated that she had lifted over 2000 pounds in all. Her father thought Tanisha's calculation was unreasonable. Do you agree or disagree with Tanisha's father? Why?

4. *(14, 24)* One hundred forty of the two hundred sixty students in the auditorium were not seventh graders. What fraction of the students in the auditorium were seventh graders?

5. *(22)* In the following statement, change the percent to a reduced fraction. Then answer the questions.

The Daltons completed 30% of their 2140-mile trip the first day.

a. How many miles did they travel the first day?

b. How many miles of their trip do they still have to travel?

6. *(16, 19)* If the perimeter of a square is 5 feet, how many inches long is each side of the square?

* **7.** (30) *Generalize* Use prime factorization to subtract these fractions:

$$\frac{1}{18} - \frac{1}{30}$$

* **8.** (29) Mt. Whitney in California is 14,494 ft high.

a. What is Mt. Whitney's height to the nearest thousand feet?

b. What is Mt. Whitney's height to the nearest hundred feet?

* **9.** (29) *Estimate* Martin used a calculator to divide 28,910 by 49. The answer displayed was 59. Did Martin enter the problem correctly? (Use estimation to determine whether the displayed answer is reasonable.)

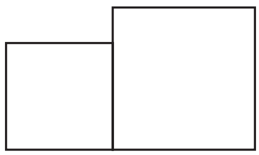
10. (15, 24) **a.** Write 32% as a reduced fraction.

b. Use prime factorization to reduce $\frac{48}{72}$.

11. (30) Write these fractions so that they have common denominators. Then compare the fractions.

$$\frac{5}{6} \bigcirc \frac{7}{8}$$

In the figure below, a 3-by-3-in. square is joined to a 4-by-4-in. square. Refer to the figure to answer problems **12** and **13**.



12. (20) **a.** What is the area of the smaller square?

b. What is the area of the larger square?

c. What is the total area of the figure?

* **13.** (19) **a.** What is the perimeter of the hexagon that is formed by joining the two squares?

b. The perimeter of the hexagon is how many inches less than the combined perimeter of the two squares?

c. *Justify* Explain your answer to **b.**

14. (21) **a.** Write the prime factorization of 5184 using exponents.

b. Use the answer to **a** to find $\sqrt{5184}$.

* **15.** (28) What is the mean of 5, 7, 9, 11, 12, 13, 24, 25, 26, and 28?

16. (28) List the single-digit divisors of 5670.

Solve:

17. (3, 20) $6w = 6^3$

18. (3) $90° + 30° + a = 180°$

19. (3, 13) **Formulate** Write an equal groups word problem for this equation and solve the problem.

$$36x = \$45.00$$

20. (16) To raise funds, the service club washed cars for $6 each. The money earned is a function of the number of cars washed. Make a function table that shows the dollars earned from washing 1, 3, 5, 10, and 20 cars.

Evaluate Simplify:

* **21.** (30) $\frac{1}{2} + \frac{1}{3}$

* **22.** (30) $\frac{3}{4} - \frac{1}{3}$

* **23.** (30) $2\frac{5}{6} - 1\frac{1}{2}$

* **24.** (26) $\frac{4}{5} \cdot 1\frac{2}{3} \cdot 1\frac{1}{8}$

* **25.** (26) $1\frac{3}{4} \div 2\frac{2}{3}$

* **26.** (26) $3 \div 1\frac{7}{8}$

Estimate For exercises **27** and **28,** record an estimated answer and the exact answer.

* **27.** (30) $3\frac{2}{3} + 1\frac{5}{6}$

* **28.** (23, 30) $5\frac{1}{8} - 1\frac{3}{4}$

29. (Inv. 2) **Represent** Draw a circle with a compass, and label the center point *O.* Draw chord *AB* through point *O.* Draw chord *CB* not through point *O.* Draw segment *CO.*

30. (Inv. 2) Refer to the figure drawn in problem **29** to answer **a–c.**

a. Which chord is a diameter?

b. Which segments are radii?

c. Which central angle is an angle of $\triangle OBC$?

Early Finishers
Real-World Application

Half the children at the park are on swings. One eighth of the children are on seesaws. One fourth of the children are on the slides. The other 6 children are playing ball.

Draw a diagram that represents the problem. Then write and solve an equation that shows how many children are in the park. Explain your work.