Lecture 21: BJTs (Bipolar Junction Transistors)

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Context

In Friday's lecture, we discussed BJTs (Bipolar Junction Transistors)

Today we will find large signal models for the bipolar junction transistor, and start exploring how to use transistors to make amplifiers and other analog devices

Reading

Today's lecture will finish chapter 7, Bipolar Junction Transistors (BJT's)

Then, we will start looking at amplifiers, chapter 8 in the text.

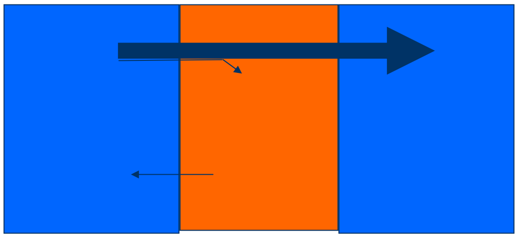
Lecture Outline

- BJT Physics (7.2)
- BJT Ebers-Moll Equations (7.3)
- BJT Large-Signal Models
- BJT Small-Signal Models

Next: Circuits

Currents in the BJT

- A BJT is ordinarily designed so that the minority carrier injection into the base is far larger than the minority carrier injection into the emitter.
- It is also ordinarily designed such that almost all the minority carriers injected into the base make it all the way across to the collector



Current controlled

- So the current is determined by the minority current across the emitter-base junction

$$I_C \approx I_S e^{\frac{qV_{BE}}{kT}}$$

- But since the majority of the minority current goes right through the base to the collector:

$$I_C \approx -I_E$$

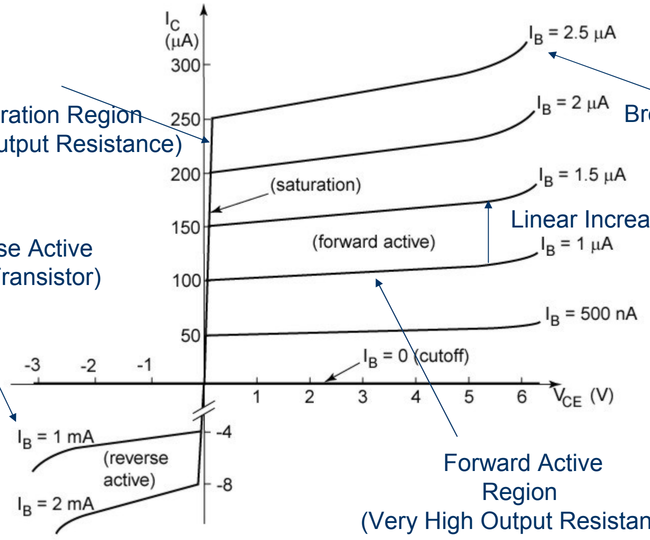
- And so the amount of current that must be supplied by the base is small compared to the current controlled:

$$I_C >> I_B$$

BJT operating modes

- Forward active
  - Emitter-Base forward biased
  - Base-Collector reverse biased
- Saturation
  - Both junctions are forward biased
- Reverse active
  - Emitter-Base reverse biased
  - Base-Collector forward biased
  - Transistor operation is poor in this direction, becauseβ is low: lighter doping of the layer designed to be the collector means that there is a lot of minority carrier injection out of the Base.

Collector Characteristics (I_B)



Saturation Region
(Low Output Resistance)

Breakdown

Linear Increase

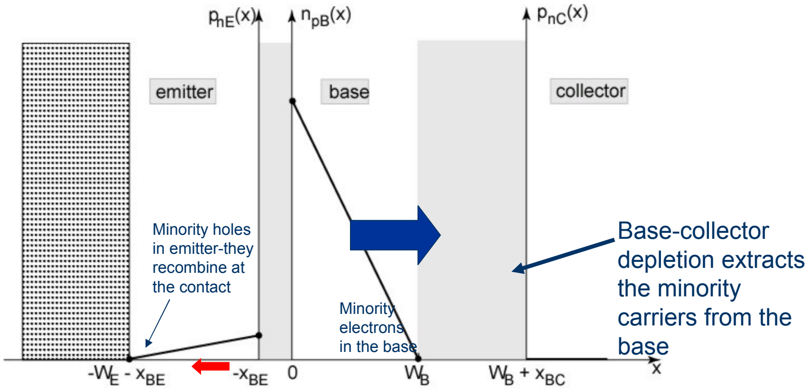
Reverse Active
(poor Transistor)

Forward Active
Region
(Very High Output Resistance)

The origin of current gain in BJT's

- The majority of the minority carriers injected from the emitter go across the base to the collector and are swept out by the electric field in the depletion region of the collector-base junction.
- The base contact doesn't have to supply that current to maintain the voltage of the base—the voltage which is causing the current in the first place.
- The current which does have to be supplied by the base contact comes from two main sources:
  - Recombination in the base (can often neglect in Silicon)
  - Injection of minority carriers into the emitter
- If we find the ratio of the current to the current that must be supplied by the base, that will give us the current gain β.

Diffusion Currents



Minority holes in emitter-they recombine at the contact

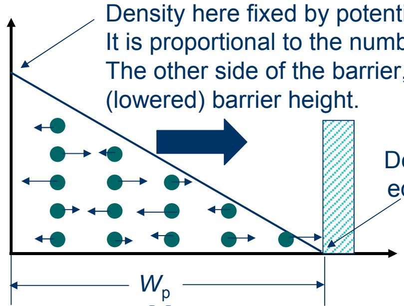
Minority electrons in the base

Base-collector depletion extracts the minority carriers from the base

The minority carriers injected into the base have a concentration gradient, and thus a current. Since emitter doping is higher, this current is much larger than the current due to the minority carriers injected from the base to the emitter. This is the source of BJT current gain.

Diffusion Revisited

- Why is minority current profile a linear function?
- The diffusion current is proportional to the gradient×diffusion constant. Since current is constant→gradient is constant
- Note that diffusion current density is controlled by width of region (base width for BJT):



Density here fixed by potential (injection of carriers)
It is proportional to the number of majority carriers on
The other side of the barrier, and is exponential with the
(lowered) barrier height.

Density at the contact is
equal to the equilibrium
value (strong G/R)

- Decreasing width increases current!

BJT Currents

Collector current is nearly identical to the (magnitude) of the emitter current … define

$$I_C = -\alpha_F I_E \qquad \alpha_F = .999$$

Kirchhoff:

$$-I_E = I_C + I_B$$

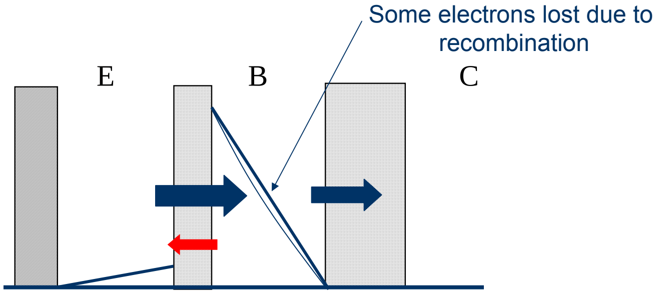
DC Current Gain:

$$I_C = -\alpha_F I_E = \alpha_F (I_B + I_C)$$

$$I_C = \frac{\alpha_F}{1-\alpha_F} I_B = \beta_F I_B \qquad \beta_F = \frac{\alpha_F}{1-\alpha_F} = \frac{.999}{.001} = 999$$

Origin of α_F

Base-emitter junction: some reverse injection of holes into the emitter → base current isn't zero



Typical: $\alpha_F \approx .99$ $\beta_F \approx 100$

Collector Current

Diffusion of electrons across base results in

$$J_n^{diff} = qD_n \frac{dn_p}{dx} = \left(\frac{qD_n n_{pB0}}{W_B} \right) e^{\frac{qV_{BE}}{kT}}$$

$$I_S = \left(\frac{qD_n n_{pB0} A_E}{W_B} \right)$$

$$I_C = I_S e^{\frac{qV_{BE}}{kT}}$$

Base Current

In silicon, recombination of carriers in the base can usually be neglected, so the base current is mostly due to minority injection into the emitter. Diffusion of holes across emitter results in

$$J_p^{diff} = -qD_p \frac{dp_{nE}}{dx} = \left(\frac{qD_p p_{nE0}}{W_E} \right)\left(e^{\frac{qV_{BE}}{kT}} - 1 \right)$$

$$I_B = \left(\frac{qD_p p_{nE0} A_E}{W_E} \right)\left(e^{\frac{qV_{BE}}{kT}} - 1 \right)$$

Current Gain

$$\beta_F = \frac{I_C}{I_B} = \frac{\left(\frac{qD_n n_{pBo} A_E}{W_B} \right)}{\left(\frac{qD_p p_{nEo} A_E}{W_E} \right)} = \left(\frac{D_n}{D_p} \right)\left(\frac{n_{pB0}}{p_{nE0}} \right)\left(\frac{W_E}{W_B} \right)$$

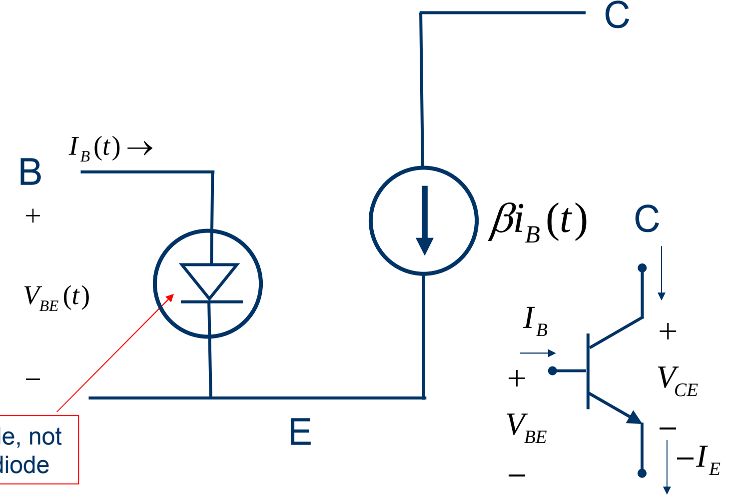
Minimize base width

$$\left(\frac{n_{pB0}}{p_{nE0}} \right) = \frac{\frac{n_i^2}{N_{A,B}}}{\frac{n_i^2}{N_{D,E}}} = \frac{N_{D,E}}{N_{A,B}}$$

Maximize doping in emitter

Simple NPN BJT model

- A simple model for a NPN BJT:



Real diode, not an ideal diode

Ebers-Moll Equations

Exp. 6: measure E-M parameters

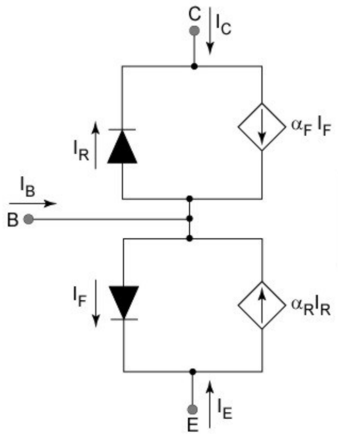
Derivation: Write emitter and collector currents in terms of internal currents at two junctions

$$I_E = -I_{ES}\left(e^{V_{BE}/V_{th}} - 1\right) + \alpha_R I_{CS}\left(e^{V_{BC}/V_{th}} - 1\right)$$

$$I_C = \alpha_F I_{ES}\left(e^{V_{BE}/V_{th}} - 1\right) - I_{CS}\left(e^{V_{BC}/V_{th}} - 1\right)$$

$$\alpha_F I_{ES} = \alpha_R I_{CS}$$

Ebers-Moll Equivalent Circuit



Diode Currents:
$I_F = I_{ES}(e^{V_{BE}/V_{th}}-1)$
$I_R = I_{CS}(e^{V_{BC}/V_{th}}-1)$

Parasitic capacitances

- To model devices adequately at high frequencies, we need to account for the charge that we must move in or out of the devices.
- In the FET, this is clearly a capacitance, but in a BJT the majority of the stored charge is in the form of minority carriers which are diffusing across the device in forward operation, but aren’t there when the transistor is not conducting, so obviously they must be extracted, or allowed to diffuse away.
- This stored charge can be modeled as a capacitance in small signal models.

Diffusion Capacitance

- The total minority carrier charge for a one-sided junction is (area of triangle)

$$Q_n = qA \cdot \frac{1}{2} bh_2 = qA \cdot \frac{1}{2}(W - x_{dep,p})(n_{p0} e^{\frac{qV_D}{kT}} - n_{p0})$$

- For a one-sided junction, the current is dominated by these minority carriers:

$$I_D = \frac{qAD_n}{W_p - x_{dep,p}}(n_{p0} e^{\frac{qV_D}{kT}} - n_{p0})$$

$$\frac{I_D}{Q_n} = \frac{D_n}{\left(W_p - x_{dep,p}\right)^2} \longrightarrow \text{Constant!}$$

Diffusion Capacitance (cont)

- The proportionality constant has units of time

$$\tau_T = \frac{Q_n}{I_D} = \frac{\left(W_p - x_{dep,p}\right)^2}{D_n}$$

Distance across P-type base

Diffusion Coefficient

$$\tau_T = \frac{q}{kT} \frac{\left(W_p - x_{dep,p}\right)^2}{\mu_n}$$

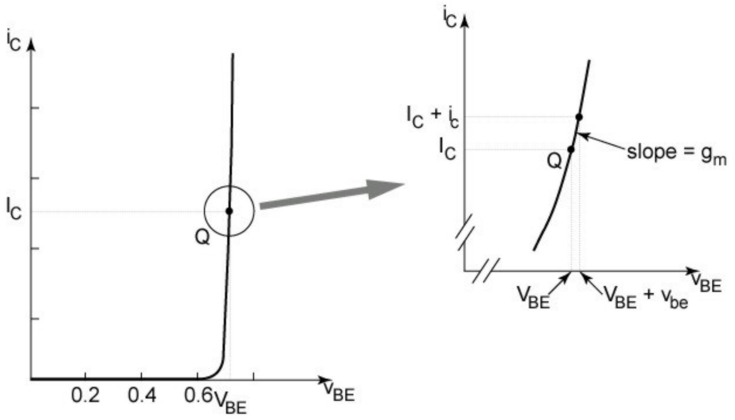
Mobility

Temperature

- The physical interpretation is that this is the *transit* time for the minority carriers to cross the p-type region. Since the capacitance is related to charge:

$$Q_n = \tau_T I_D \longrightarrow C_d = \frac{\partial Q_n}{\partial V} = \tau_T \frac{\partial I}{\partial V} = g_d \tau_T$$

BJT Transconductance g_m



- The transconductance is analogous to diode conductance

Transconductance (cont)

- Forward-active large-signal current:

$$i_C = I_S e^{v_{BE}/V_{th}}(1 + v_{CE}/V_A)$$

Q here means quiescent point not charge

- Differentiating and evaluating at $Q = (V_{BE,}\ V_{CE})$

$$\left.\frac{\partial i_C}{\partial v_{BE}}\right|_Q = \frac{q}{kT} I_S e^{qV_{BE}/kT}(1 + V_{CE}/V_A)$$

$$g_m = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_Q = \frac{qI_C}{kT}$$

Notation Review

$$i_C = f(v_{BE}, v_{CE})$$

Large signal

$$I_C + \Delta i_C = f(V_{BE} + \Delta v_{BE}, V_{CE} + \Delta v_{CE})$$

small signal

DC (bias)

Quiescent Point (bias)

$$I_C + i_c = f(V_{BE} + v_{be}, V_{CE} + v_{ce})$$

small signal (less messy!)

$$Q = (V_{BE}, V_{CE}) \longrightarrow i_c \approx \left.\frac{\partial f}{\partial v_{BE}}\right|_Q v_{be} + \left.\frac{\partial f}{\partial v_{CE}}\right|_Q v_{ce}$$

*trans*conductance

Output conductance

Remember, the point of a small signal model is to produce a set of equations which relate the small variations in currents and voltages to each other linearly → and to create a linear equivalent circuit

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BJT Base Currents

Unlike a MOSFET, there is a DC current into the base terminal of a bipolar transistor:

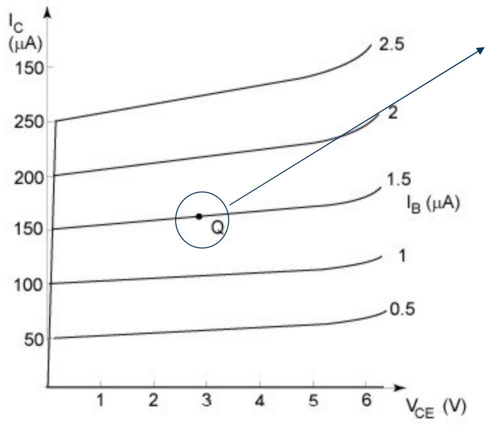
$$I_B = I_C / \beta_F = \left(I_S / \beta_F\right) e^{qV_{BE}/kT} (1 + V_{CE}/V_A)$$

To find the change in base current due to change in base-emitter voltage:

$$i_b = \left.\frac{\partial i_B}{\partial v_{BE}}\right|_Q v_{be} \qquad \left.\frac{\partial i_B}{\partial v_{BE}}\right|_Q = \left.\frac{\partial i_B}{\partial i_C}\right|_Q \left.\frac{\partial i_C}{\partial v_{BE}}\right|_Q = \frac{1}{\beta_F} g_m$$

$$i_b = \frac{g_m}{\beta_F} v_{be}$$

Small Signal Current Gain



$$\beta_0 = \frac{\Delta i_C}{\Delta i_B} = \beta_F$$

$$\Delta i_C = \beta_0 \Delta i_B$$

$$i_c = \beta_0 i_b$$

- Since currents are linearly related, the derivative is a constant (small signal = large signal)

Input Resistance r_π

$$(r_\pi)^{-1} = \left.\frac{\partial i_B}{\partial v_{BE}}\right|_Q = \frac{1}{\beta_F}\left.\frac{\partial i_C}{\partial v_{BE}}\right|_Q = \frac{g_m}{\beta_F}$$

$$r_\pi = \frac{\beta_F}{g_m}$$

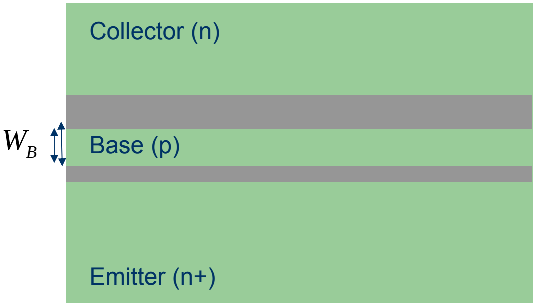
- In practice, the DC current gain β_F and the small-signal current gain β_o are both highly variable (+/- 25%)
- Typical bias point: DC collector current = 100 μA

$$r_\pi = 100\frac{25\,\text{mV}}{.1\,\text{mA}} = 25\,\text{k}\Omega$$

$$R_i = \infty\,\Omega \longleftarrow \text{MOSFET}$$

Output Resistance r_o

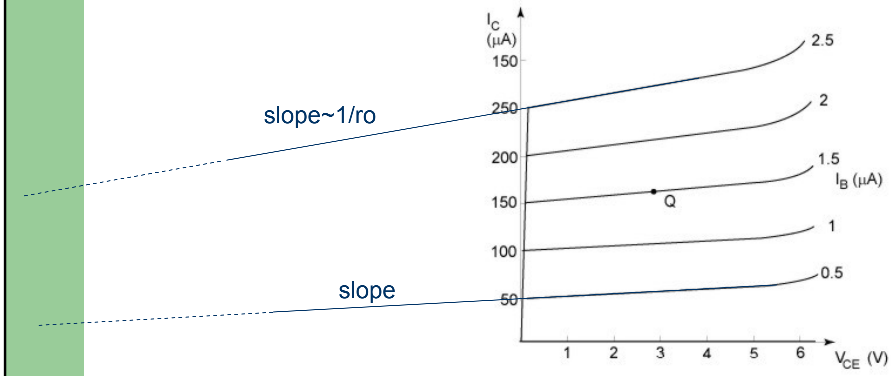
Why does current increase slightly with increasing v_{CE}?

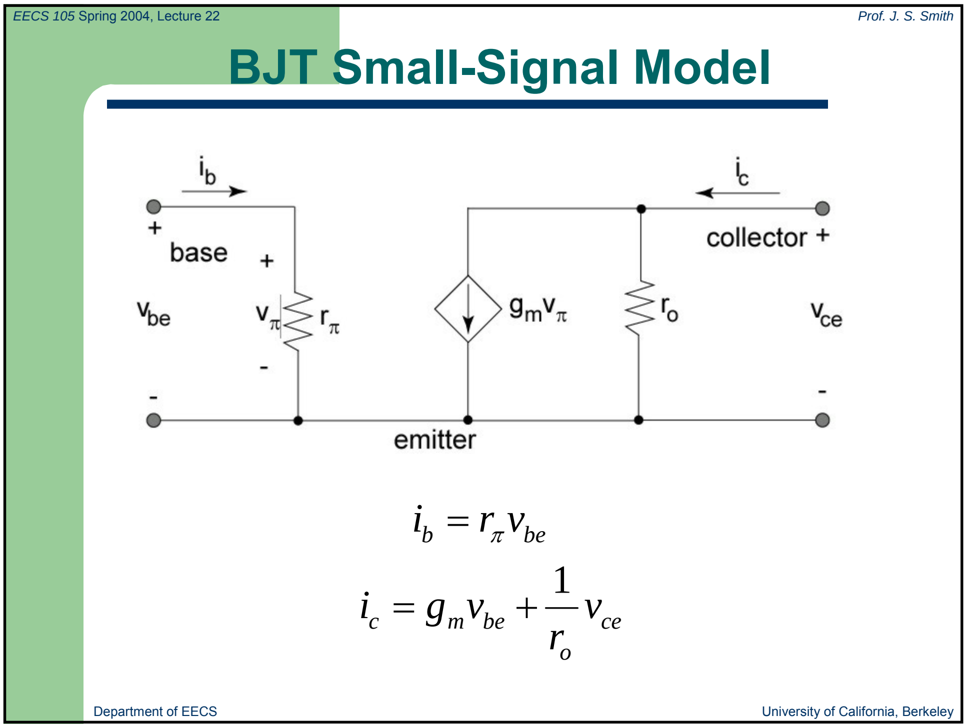


Answer: Base width modulation (similar to CLM for MOS)
Model: Math is a mess, so introduce the Early voltage

$$i_C = I_S e^{v_{BE}/V_{th}}(1+v_{CE}/V_A)$$

Graphical Interpretation of r_o





BJT Small-Signal Model

$$i_b = r_\pi v_{be}$$

$$i_c = g_m v_{be} + \frac{1}{r_o} v_{ce}$$

BJT Parasitic Capacitors

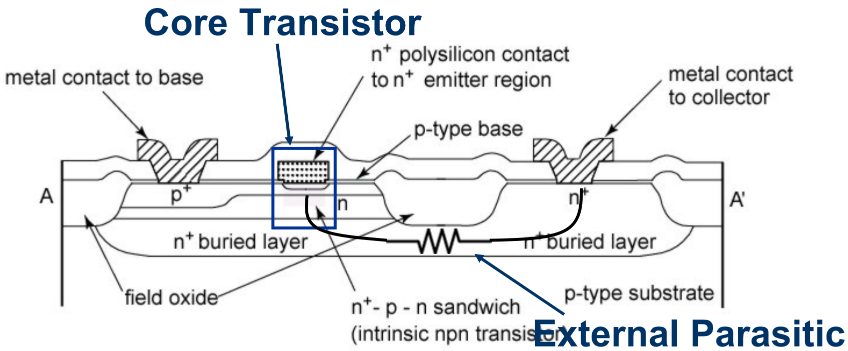
- Emitter-base is a forward biased junction → depletion capacitance:

$$C_{j,BE} \approx 1.4 C_{j,BE0}$$

- Collector-base is a reverse biased junction → depletion capacitance
- Due to minority charge injection into base, we have to account for the diffusion capacitance as well

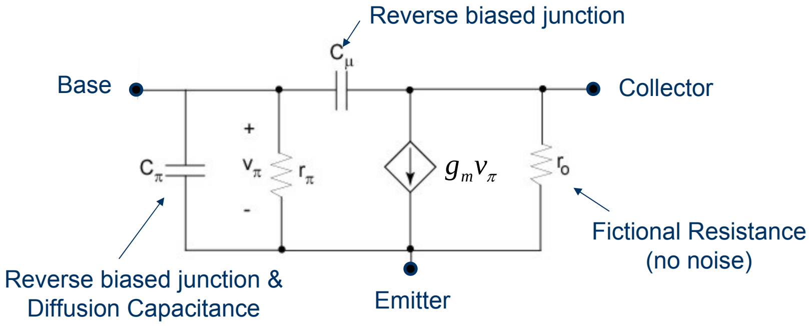
$$C_b = \tau_F g_m$$

BJT Cross Section



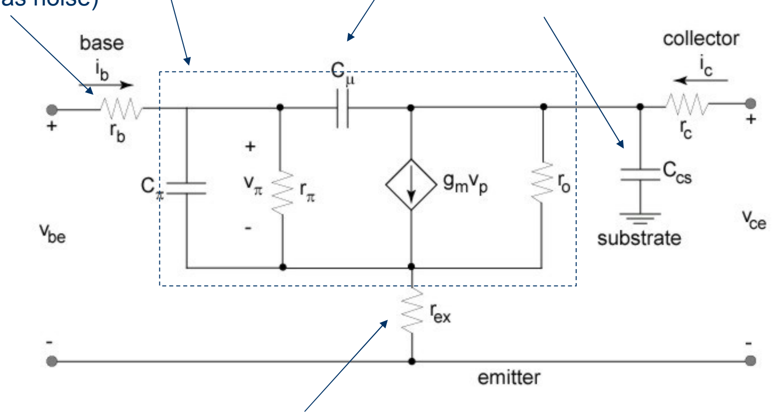
- Core transistor is the vertical region under the emitter contact
- Everything else is "parasitic" or unwanted
- Lateral BJT structure is also possible

Core BJT Model



- Given an ideal BJT structure, we can model most of the action with the above circuit
- For low frequencies, we can forget the capacitors
- Capacitors are *non-linear*! MOS gate & overlap caps are linear

Complete Small-Signal Model



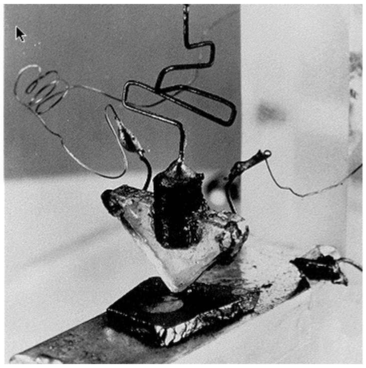
Real Resistance (has noise)

“core” BJT

Reverse biased junctions

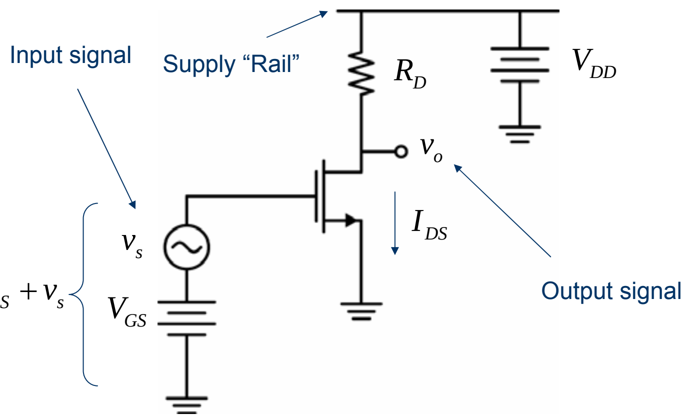
External Parasitics

Circuits!



- When the inventors of the bipolar transistor first got a working device, the first thing they did was to build an audio amplifier to prove that the transistor was actually working!

A Simple Circuit: An MOS Amplifier



$$v_{GS} = V_{GS} + v_s$$