

Lab Manual for PHYSICS 154

General Physics Laboratory II

Heat, Electricity and Magnetism*

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Cover Page

Lab Goal

Historically, in physics it takes a long time and numerous experimental observations before scientists, by inductive thinking/generalization, can come up with a reliable and plausible theory. On the contrary, when one is learning physics she/he moves backwards. One starts with lectures on abstract theoretical concepts and deduces their applicability for description of reality through observations or lab experiments.

“Why do you believe that Earth rotates around Sun and not vice versa?” or “What does make you so sure that the matter around you consists of minute atoms?” or “How is it possible that photons exist, though they have zero mass?” It will be depressing to admit that, for example, the only way for one to answer these or similar questions is to cite words of her/his lecturer (who, probably, cites words heard a decade ago at lectures of another professor).¹

In the lab, the TA will try to help student organize her/his pursuit of a validity check of theoretical physics to be both *logically qualitative* plus *quantitative*, i.e., consistent and clear in physical statements plus exact in numerical results as being supported by error analysis.

Lab Methodology

Physics, just as other sciences, has a goal to discover and to describe new phenomena [1]. In real-life situations, experimentalists may measure something unexpected,² or they may set out to verify something proposed by the theoretician.³ Undergraduate lab is more like the second approach. Consider the lab manual to be a theoretical proposal for your practical verification or disproof. As a consequence, your report is an independent follow up experimental “article”.

Report Style in Short

Writing good reports is a field of rhetorics. And, there is a well explained statement considering rhetorics [2], that it is wrong to present some set of rules to follow but instead one should start, for example, with concept of *Quality* in foundation—though *Quality* is undefinable and is neither a subjective nor objective category—and then all rules of rhetorics will naturally emerge.

One of the main goals of the supplementary “Lab Guideline” is to explain how one can improve *Quality* of her/his research work presentation.

Student-TA Relations

Some students want things to be especially easy—they wish university was like the army, and that they will only need to follow instructions. **NO!** To learn something one must make a personal effort to overcome and outgrow oneself, while the TA has only a consulting role.

The following is the actual passage from the *Instructor’s Manual*: “You will lead your students through some activities. **Lead** is the operative word. Do not tell them exactly what to do – make them figure it out as much as possible, but be helpful, optimistic, encouraging. If you ask the class a question, *wait* for a reply. If they are “stuck”, ask “open” probing questions “Have you thought about this? Have you tried that? How do you think this works? What would happen if you ... ? Does the material in your lab manual say anything that might be helpful?” Avoid questions that can be answered “No. Yes. 3.56.” **Make them think.**”

Quotes from students reports (☺)

- ▶ The aim of this lab is to determine the value of absolute zero based on the fact that the ideal gas law.
- ▶ This experiment was an observational experiment to observe...
- ▶ The goal of the third part of the experiment was to demonstrate the difference between the nature of the graph using a resistor and using a light bulb.
- ▶ In this lab we have learned how to charge and discharge.
- ▶ As we have applied square voltage to oscilloscope we observed how it jumped up and then slowly fell down.
- ▶ In conclusion the process of charging and discharging creates quite a phenomena.
- ▶ Magnetic field could be measured as a distance was moved toward, or away from the energized coil.
- ▶ The slope of this line should go through zero.
- ▶ In this lab, we compared capacitance with resonance frequency.
- ▶ Function generator produced a sin-wave.
- ▶ The line of best fit is a linear line.
- ▶ There is a scaling of magnitude, it is a parabolic shape because it goes back to origin because it is oscillatory. When it is forced, we make it linear, as in part B, but in part B we allow it to be natural and come back.
- ▶ This lab had us test forced oscillations.
- ▶ The second part of the first goal wanted to measure the frequency of the capacitance.

Quotes from Students Evaluation Forms

“Do not use this TA.” — Talk to the professor in charge, Edward Chang. Measures should be taken to improve teaching.

“Lab manual and guideline sucks.” — Bring your dislikes and comments to your TA’s attention. She/he will inform the responsible editor, so the next version of the texts might be improved.

“I like being exposed to the experiment tools for hands-on experience.” — You can do/learn whatever you want in this lab unless you break equipment or harm your classmates. Optional sections may partly satisfy your curiosity.

“Nothing has to be precise. More focus on learning and understanding concepts.” — Purpose of demonstrations during lectures is to illustrate concepts. While purpose of this lab is to give you a flavor of research work [3].

“Talk about applications of the theories before the experiment. So that we can connect the things we learn to our life.” — Again, this is a task of a lecture course to give overview of the concepts. Though, anyway, self-consistent introduction to the theory of studied phenomena is provided for each lab.

“Have the labs correspond to what we are learning in a lecture the same week.” — It is very problematic to synchronize labs and lectures this way.

References

- [1] Contrary to physics, engineering deals with direct application of knowledge. This does not imply that engineering is not creative, see literature about TRIZ, for example, introductory book “And Suddenly the Inventor Appeared” (1996) by G. Altshuller.
- [2] Robert M. Pirsig, *Zen and the Art of Motorcycle Maintenance: An Inquiry into Values*, Bantam, 1984.
- [3] E. Bright Wilson, Jr., *An Introduction to Scientific Research*, McGraw-Hill, New York, 1952.

Leo Kadanoff, *Physics Today*, September (2006)

“... We [physicists] can also serve as examples to show how others might, in their own lives, reach conclusions by careful assessment of evidence. The best scientific work stands in contrast to the self-seeking and evidence-evading characteristic of many people, high and low, in public life. ...

... In the long run, there is something in it for all of us. Education aimed at the evidence-based pursuit of truth can help the community gain tools for a better understanding of the world. Evidence-based argumentation can help scientists, engineers, business people, national leaders—everyone—make better decisions.”

¹ “Doubt is not a threat for ... [physics]. If we have never doubted, how can we say we have really believed? True belief is not about blind submission. It is an open-eyed acceptance, when doubt feeds faith, rather than destroy it.”—not very exact citation of A. Sullivan, *Time*, October 9 (2006)

² For example, Galileo experimentally showed that common belief that more massive objects falls quicker is only due to air friction. If there was no atmosphere, then all objects independent of their mass will fall to the Earth with one and the same acceleration $\approx 9.8 \text{ m/s}^2$.

³ For example, Einstein came up with theory of relativity to be shown later by experiments to be true.

I. LATENT HEAT OF VAPORIZATION OF LIQUID NITROGEN (LN_2)

First of all your TA will explain harmful consequences of careless handling of liquid nitrogen (LN_2).

► To start with illustration—rubber band placed in LN_2 becomes rigid and bends no more losing its elasticity even after it is unfrozen.

► It is not fatal if small droplets of LN_2 will fall on your skin, as they will roll over skin. But if you drink, pour on a cloth, try to keep LN_2 in your palm or immerse a finger in LN_2 , then you can get a **severe** frostbite.

► **You must wear gloves and safety glasses when pouring LN_2 . Plastic aprons are provided to cover your clothing.**

A. Theory

Definition: One single atom can be considered to be a monatomic molecule. So, further only the general term *molecule* will be used here to stand for both terms—atom and molecule.

If one tries to adopt a microscopic approach for the description of gases, liquids, solids, etc., then she/he most probably will start with a mechanical description of motion of each *molecule*, i.e., will figure out forces between *molecules*, will define *molecules*' initial positions and velocities and, finally, will solve equations of motion. But even in 1 cm^3 of air one have an extremely huge amount—about 10^{24} —of *molecules* to handle. Thus such straightforward microscopic approach is an impractical insanity. However, there are alternatives, for example, thermodynamics, statistical physics, kinetic theory, etc. In contrast to classical mechanics, in thermodynamics to describe “behavior” of $\sim 10^{24}$ *molecules* in a given volume one introduces such macroscopic quantities as density, pressure, temperature, etc. For example, temperature of a solid corresponds to average intensity of oscillations of *molecules* around their equilibrium positions.

Consider heating of some system. One increases temperature or in other words intensifies random activity at a microscopic scale. If at some critical temperature there is a qualitative change of the system state, the system is said to undergo a phase transition. For example, a crystal melts losing its rigidity, or a liquid boils transforming into gas. We are going to be interested in the latter process.

In a gas, *molecules* fly independently rarely bumping into one another. In a liquid, *molecules* wander around, but they are kept together as a whole by attraction forces acting among *molecules*. When liquid has reached its boiling temperature its *molecules* are at the highest possible level of random activity. And additional energy one supplies to the liquid does not change its temperature, but is used by some amount of *molecules* to break “bonds” with their neighbors and to evaporate/free out in a gas. Because *molecules* evaporate independently they share the total supplied energy in equal portions. Thus

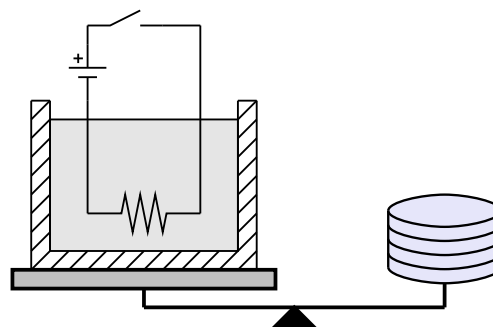


FIG. 1 A double walled styrofoam cup standing on a scale with a resistor suspended within LN_2 .

the number of evaporated *molecules* is proportional to supplied energy, and, finally, at boiling point one gets

$$Energy^{supplied} = L \cdot Mass^{evaporated}, \quad (1)$$

where L is nothing but a coefficient of proportionality. L is called *latent heat of vaporization*. Physically, L is a macroscopic characteristic of boiling. Numerical value of L is specific to a particular liquid. It is sometimes helpful to think of L as the amount of energy required per unit mass of a substance to vaporize the substance at its boiling point.

B. Lab Goal

In this lab you will attempt to measure latent heat of vaporization of liquid nitrogen (LN_2).

C. Experiment Overview/Procedure

Meditating a little on Eq. (1), you will understand that in order to achieve goal of § I.B you need a device where you can control energy input into the liquid and at a same time measure evaporated mass. [Or, a device where you can control rate of energy input into the liquid and at a same time measure rate of evaporated mass.]

Consider apparatus depicted at Fig. 1. You have a double walled styrofoam cup standing on a scale with a resistor “hanging” inside. [A resistor is a device which converts electrical energy into heat. Familiar examples are the filament of a toaster or of a light bulb.] In your thermos bottle you have LN_2 at a boiling temperature of 77 Kelvin (about -321°F). Once you pour LN_2 in a cup, heat from a much warmer atmospheric surrounding environment, namely the room, will be conducted to LN_2 from walls and top of a cup and also along wires. And if you turn on electrical circuit, then in addition the resistor will directly heat the volume of LN_2 . As a result you will

observe LN_2 boiling and its level and mass decreasing as it continuously evaporates into the atmosphere. [Of course, temperature of boiling LN_2 is not changing!]

When heater is on—current through the resistor is switched on—energy balance for some small time Δt is

$$\Delta E_{\text{env}} + V \cdot I \cdot \Delta t = L \cdot \Delta m_{\text{on}}. \quad (2)$$

Energy ΔE_{env} comes from the environment. The battery of constant voltage V creates constant current I in the electrical circuit. As a result the resistor produces $V \cdot I \cdot \Delta t$ amount of energy during time Δt .⁴ During time Δt , all “supplied” energy [left side of Eq. (2)] is used to evaporate mass Δm_{on} of LN_2 [right side of Eq. (2)]. Equation (2) is time dependent; longer you observe the system more energy will be “supplied” and more mass of LN_2 will evaporate. Dividing both sides of Eq. (2) by Δt you can arrive to time independent differential equation:⁵

$$P_{\text{env}} + V \cdot I = L \cdot \frac{dm_{\text{on}}}{dt}, \quad (3)$$

where $P_{\text{env}} = \frac{\Delta E_{\text{env}}}{\Delta t}$ is the rate of energy supply from environment, and $\frac{dm_{\text{on}}}{dt}$ is the rate of mass evaporation with heater on. If [and only if this is actually true in your experiment!] the level of LN_2 in the cup does not vary a lot throughout your measurements, then you can assume constant environmental conditions to be present, i.e., the rate of energy supply from environment, P_{env} , can be assumed to be constant. Heating power of battery, $V \cdot I$, is constant. L is a number. Therefore, according to Eq. (3), the rate of evaporation $\frac{dm_{\text{on}}}{dt}$ of LN_2 should be constant.

Your strategy to evaluate L from Eq. (3) will be: (i) measure constant V and I with voltmeter and ammeter; (ii) determine constant rate of mass evaporation $\frac{dm_{\text{on}}}{dt}$ (see § I.D); (iii) “correct” for environmental heating P_{env} . Note that P_{env} is small but it is not negligible compared with $V \cdot I$.

Step (iii) can be carried out with the heater turned off. The energy balance is

$$\Delta E_{\text{env}} = L \cdot \Delta m_{\text{off}}, \quad (4)$$

where Δm_{off} is evaporated mass of LN_2 during some time Δt when heater is off. Dividing both sides of Eq. (4) by Δt you arrive to equation

$$P_{\text{env}} = L \cdot \frac{dm_{\text{off}}}{dt}. \quad (5)$$

Note the same P_{env} as in Eq. (3). [Formally, you could have derived Eq. (5) by setting V and I to be 0 in Eq. (3).] P_{env} was assumed to be constant. L is a number. Therefore, according to Eq. (5), the rate of evaporation $\frac{dm_{\text{off}}}{dt}$

of LN_2 should be constant. If you measure constant rate of evaporation, $\frac{dm_{\text{off}}}{dt}$, (see § I.D), you will not be able to calculate L from Eq. (5) because you do not know the value of P_{env} . Instead, substitute P_{env} from Eq. (5) into Eq. (3) to obtain the final formula for L

$$L = \frac{V \cdot I}{\frac{dm_{\text{on}}}{dt} - \frac{dm_{\text{off}}}{dt}}. \quad (6)$$

[You can say that the denominator in the equation above is the net rate of evaporation solely corresponding to the heating caused by the resistor.]

To conclude, by analyzing vaporization both with, and without, heating through a resistor, using Eq. (6), you will be able to evaluate latent heat of vaporization L of LN_2 .

D. Details of Experimental Procedure

► Make sure that resistor does not touch walls of cup in the lowest and highest possible positions of a scale, so that it will definitely not stick cup during your manipulations with a scale.

► Making next attempt to measure $\frac{dm}{dt}$ do not forget to refill a cup with LN_2 to the level of previous trial. This will ensure constant environmental conditions (i.e., one and the same P_{env}) in all your trials.

1. How to measure rate of vaporization $\frac{dm}{dt}$

Set timer to cumulative regime and, also, choose seconds for units of a timer scale.

Fill the cup up to the brim with LN_2 . Set the scale to a value a little less than the current total mass of LN_2 and cup. As soon as some LN_2 has evaporated and scale becomes balanced, start timer [this first value of mass you have read from the scale at “zero” time which corresponds to “zero” evaporated mass of LN_2]. Reduce the balance mass, for example, by 2 g and record time t_1 when balance of scale will be achieved again [time t_1 corresponds for evaporation of 2 g of LN_2]. Reduce balance again by 2g and record t_2 [cumulative time t_2 corresponds for evaporation of total 4 g of LN_2]. Having filled table similar to one below

evaporated mass, g	time of evaporation, s
0	0
2	t_1
4	t_2
...	...

plot evaporated mass versus time of evaporation. According to § I.C you would expect a constant rate of vaporization, so you would expect to see that your data is fitted well with a straight line. If this is the case then value of the slope of the best fit line is equivalent to the rate of vaporization $\frac{dm}{dt}$.

Choose 2 g interval, or any other mass interval more suitable from your point of view, so that you will have about 5 readings while level of LN_2 goes from the brim of a cup down to a point when the resistor is still below

⁴ If V is in Volt and I is in Ampere then combination of units for product $V \cdot I$ is Volt · Ampere = Joule/second \equiv Watt.

⁵ Choosing Δt sufficiently small, $\frac{\Delta m}{\Delta t}$ is replaced by the derivative $\frac{dm}{dt}$. Basically, we are deriving differential equations (3) and (5) for time dependant evaporated masses $m_{\text{off}}(t)$ and $m_{\text{on}}(t)$, respectively.

the surface of the liquid. If you choose mass interval to be too small then you will have not enough time to adjust scale; if too big, then you will get bored waiting and, moreover, you will not get enough points to plot.

E. Analysis

Before you start to carry out error propagation for L , Eq. (6), you need to understand first what were the sources of random errors (i.e., what do you estimate to be the accuracy of your measurements of time, mass, voltage and current?).

► Do you observe on your plots the expected constant rate of evaporation $\frac{dm}{dt}$ for both parts of experiment? [In other words, does your linear fit for you data of evaporated mass versus time of evaporation go through error bars of you measurements?] If not, then theory of § I.C fails and as a result you cannot use Eq. (6).

► How does your value of $L = \dots \pm \dots$ for LN_2 compares to the table value? (*Table value of latent heat of vaporization of LN_2 is 199.2 J/g.*)

II. THE IDEAL GAS LAW

A. Theory

Definition: An ideal gas is a non-interacting gas of identical perfectly rigid point-like *particles*.

Real gases are not something trivial. Indeed, at a microscopic level one deals with complex three dimensional *molecules*—usually one has a mixture of different sorts of *molecules* in a gas—that, in course of collisions, start to rotate, vibrate, etc. [Even monoatomic noble gas is not a collection of perfectly rigid billiard balls!] Despite these complications it turns out that the simplest ideal gas approximation is often useful.

For an ideal gas at the equilibrium state it is straightforward to derive two facts. First, for ideal gas its absolute temperature is proportional to average kinetic energy of *particles*.⁶ Second, pressure (P), volume (V) and absolute temperature (T) of an ideal gas can be related by the equation of state, called the Ideal Gas Law:

$$P \cdot V = N \cdot k_B \cdot T, \quad (7)$$

where N is the number of *particles* in the volume and $k_B = 1.38 \cdot 10^{-23}$ J/K is the Boltzmann constant.⁷ Here,

⁶ This fact, along with Eq. (7), is actually used to define absolute/Kelvin (K) temperature scale. 0 K is the smallest possible absolute temperature, as it is related to idealization/practically unachievable situation when gas *molecules* are not moving. And, 1 K temperature change is set to be equal to 1 °C, thus one has a simple conversion from the Kelvin temperature scale to the Celsius scale

$$T_{\text{Celsius}}^{(\circ)} = T_{\text{Kelvin}} - 273.15.$$

⁷ Normalizing N by the Avogadro number $N_A = 6.02 \cdot 10^{23}$ [number

“Optional” Thinking

Can you relate the facts below with the systematic error(s) of your experiment [if you actually had any systematic error(s)]? Maybe there is something else you can come up with!?

► When the cup is not filled to the top, for example is filled to 2/3 or 1/2, there is going to be some space in the cup above liquid for cold vapor to stay. And, it will effectively acts as an insulator preventing direct contact or heating of LN_2 by room air. So, was it important to carry out both parts (heater off/heater on) of the experiment with LN_2 being at about the same level?

► Should you have been careful not to breathe (or to breathe consistently) on the cup throughout your experiment? Does this even matter?

► Do you need to calibrate “zero” of your scale? In other words, think about how scale calibration affects your measurements/data and final results.

the units of P are N/m², of V are m³ and of T are degrees Kelvin (K).

B. Lab Goal

Your tasks are (a) to see if the air in this Lab obeys the Ideal Gas Law,⁸ and (b) to estimate the temperature of absolute zero.⁹

C. Experiment Overview

In order to to check applicability of the Ideal Gas Law, Eq. (7), for approximate description of air one needs to come up with a device where one controls P , V , N and T .

of atoms in 12 g of carbon isotope ¹²C] one can rewrite Eq. (7) in the following equivalent form

$$P \cdot V = \left(\frac{N}{N_A} \right) \cdot (N_A k_B) \cdot T = n \cdot R \cdot T,$$

where $R = N_A k_B = 8.31$ J/(mole·K) is the universal gas constant and $n = N/N_A$ is called number of moles.

⁸ You cannot verify directly, for example, using light microscopy, that air consists of minute *molecules* [about 10⁻¹⁰ m in diameter]. But you can indirectly prove this fact by showing applicability of the Ideal Gas Law for description of air. Note that existence of point-like *particles* is the key assumption in the derivation of the Ideal Gas Law.

⁹ This part of the lab is a good example demonstrating how in some “pathological” cases one can start with relatively precise data and, finally, after some calculations acquire very uncertain outcome.

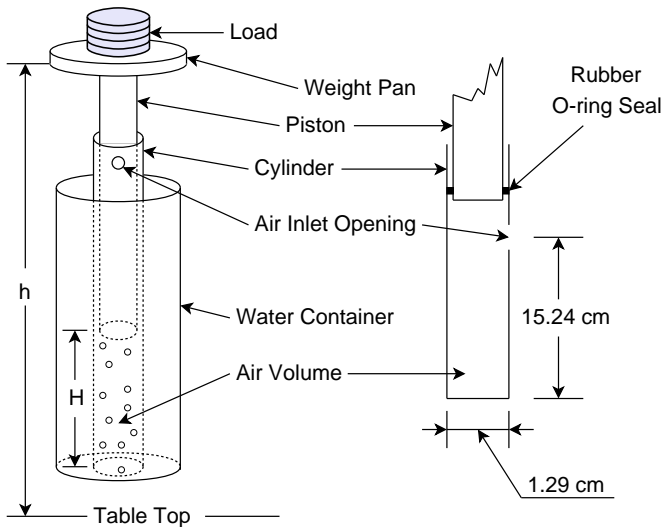


FIG. 2 Sketch of experimental setup with a detailed depiction of “Piston–Cylinder” geometry on the right-hand side.

Consider, for example, apparatus depicted at Fig. 2. It facilitates measurements of pressure and volume as a sample of gas (air) inside a cylinder is compressed isothermally. [A process at a constant temperature is said to be isothermal.]—Once you insert the piston and its weight has pushed the rubber seal below the air inlet hole, you have a fixed amount of air (N molecules) trapped inside the cylinder under the piston. The pressure (P) of air inside is varied by placing different weights on weight pan which is fitted to the top of the piston (see § II.D.2). The gas cylinder is mounted inside another container, which can be filled with water at any desired temperature (T). *The function of the water is to provide a constant temperature environment for the gas.* Volume of the trapped air $V = H \cdot A$, where A is a cross section area of the cylinder, and the height (H) of the air inside the cylinder can be calculated from the height (h) of the pan above the surface of the table (see § II.D.1).

If the air inside the cylinder was an ideal gas then its state should have been described by the Ideal Gas Law, Eq. (7). In the present case, it can be rewritten in the equivalent form

$$H = \frac{N \cdot k_B \cdot T}{A} \cdot \frac{1}{P} = \text{constant} \cdot \frac{1}{P}. \quad (8)$$

Observing how close H of the trapped air follows Eq. (8) as P is varied, by loading the piston with different masses, at two different temperatures (room and ice temperature) will allow you to conclude how accurately air can be approximated to be an ideal gas.

D. Details of Experimental Procedure

- ▶ Before doing anything else, measure the mass of the piston with the weight pan.
- ▶ Learn how to disassemble the weight pan and the cover of the water container without taking the piston out of the cylinder—following this procedure you will be adding ice in part [2].

- ▶ If the piston does not move smoothly inside the cylinder ask the TA to grease it.
- ▶ Look closely at the weights you are going to use; because they have boundary that covers the weight pan, you will need to measure height in a slot cut in them.

1. How to measure the height (H) of the air inside the cylinder

At the very beginning of the experiment, insert the piston in the cylinder and hold it so that you see the bottom edge of the piston in the air inlet opening. In such position, measure reference height h_0 from the table top to the bottom of the weight pan. It will correspond to 15.24 cm of height of the air column inside the cylinder—the numbers 15.24 cm and 1.29 cm [diameter of the cylinder] are the manufacturer’s specification of the experimental setup (see Fig. 2). Knowing h_0 it is obvious how to later recalculate H of the trapped air from measurement of the height (h) for an arbitrary position of the piston in the cylinder.

Friction is important for estimating error of measuring H . At each pressure you can find a range of heights, that is, a maximum and a minimum which depend on whether you last pushed gently the weight platform up or down with your finger.

2. How to determine absolute pressure (P) in the cylinder

Pressure in the cylinder P is given by the atmospheric pressure $P_{\text{atm}} = 1.013 \cdot 10^5 \text{ N/m}^2$ plus the force exerted by the loaded piston per the cross-section area, A :

$$P = P_{\text{atm}} + \frac{M_{\text{tot}} \cdot g}{A}, \quad (9)$$

where M_{tot} is the mass of the piston, the pan and the weight(s) and g is acceleration due to gravity. [Note you can calculate A from the information given on the diagram of Fig. 2.]

You will know values of mass loads with much higher accuracy than heights. Thus in Eq. (9) it is safe to consider P to have no error.

E. Experimental Procedure

Note the temperature each time you begin collecting the data for parts [1] and [2]. [You do not need to keep the thermometer inside the water container during each set of measurements and do not hurry too much as it takes relatively long time for such a big volume of water in the container to change its temperature.]

Part [1]: Start the experiment at the room temperature. Fill the water at approximately room temperature into the water bath. Enclose a sample of room temperature air into the cylinder by lowering the piston carefully into the cylinder. **From this time on, do not remove the piston from the cylinder or raise the rubber seal above the air inlet hole.** The reason is that you want to keep the same number of molecules of gas, N , in the apparatus throughout the experiment.

Measure height H of the gas under the piston for each of 5 to 8 different weights (including “zero” load, when air is compressed only by the weight of the piston and the pan).

Part [2]: Drain some water and add some ice to the water bath (ice should be covered by water!). Wait until temperature stability is established (at a same time you can complete data analysis for part [1]). Then repeat the same measurements as in the part [1] above.

F. Analysis

For both parts [1] and [2] plot H versus $\frac{1}{P}$. If air is well approximated by the Ideal Gas Law at “everyday” temperatures, such as room and ice temperature, then, according to Eq. (8), your data can be fitted by a straight line with y -intercept equal 0. [Formally, according to Eq. (8), H should be equal 0 when P equals ∞ (i.e., when $\frac{1}{P} = 0$). Or, in other words, a totally compressed ideal gas has zero volume, because ideal gas consists of point-like *particles*, see the definition in § II.A.]

► Do your lines of best fit pass through the error bars of all your data points, and do they have zero y -intercept? Can you conclude that the air in the Lab obeys the Ideal Gas Law? [See also Sec. “Optional” Thinking below.]

In addition, calculate the temperature of the absolute zero in degrees Celsius. Let’s pretend that we do not know that the temperature of the absolute zero in degrees Celsius is $T_{\text{abs}}^{(\circ)} = -273.15^\circ\text{C}$, then for conversion of temperature from Kelvin (T) into degrees Celsius ($T^{(\circ)}$) we should write $T^{(\circ)} = T + T_{\text{abs}}^{(\circ)}$. And, if [and only if!] air is well approximated by the ideal gas idealization, then, according to Eq. (8), slopes (S) of the linear fits of the graphs H versus $\frac{1}{P}$ for two temperatures of parts [1] and [2] [note that in Eq. (8) T is in Kelvin] should be

$$\begin{cases} S_1 = \frac{N \cdot k_B \cdot (T_1^{(\circ)} - T_{\text{abs}}^{(\circ)})}{A}, \\ S_2 = \frac{N \cdot k_B \cdot (T_2^{(\circ)} - T_{\text{abs}}^{(\circ)})}{A}. \end{cases} \quad (10)$$

To solve this system of equations for $T_{\text{abs}}^{(\circ)}$ we need to get

rid of the second unknown N . So, we form ratio of slopes and after some math arrive to the final formula:

$$\frac{S_1}{S_2} = \frac{T_1^{(\circ)} - T_{\text{abs}}^{(\circ)}}{T_2^{(\circ)} - T_{\text{abs}}^{(\circ)}} \Rightarrow T_{\text{abs}}^{(\circ)} = \frac{S_1 \cdot T_2^{(\circ)} - S_2 \cdot T_1^{(\circ)}}{S_1 - S_2}. \quad (11)$$

► Calculate your experimental value of $T_{\text{abs}}^{(\circ)} = \dots \pm \dots$ [do error propagation!]. Compare it with the table value $T_{\text{abs}}^{(\circ)} = -273.15^\circ\text{C}$. What are your conclusions?

► When you do the error propagation for $T_{\text{abs}}^{(\circ)}$ note a dramatic increase of error from the denominator of Eq. (11). What are the original physical reason(s) for high uncertainty of the denominator, and, as a result, of your experimental value of $T_{\text{abs}}^{(\circ)}$? How might be improved experiment to determine more precisely value of $T_{\text{abs}}^{(\circ)}$? “Optional” Thinking

► Friction has the most pronounced effect when air is compressed only by relatively small weight of just the piston with the pan (no load).—Piston can get stuck at the “wrong” height.—How have you taken this into consideration during your data analysis?

► There is a possibility that your height measurements will be systematically offset. What could be the reason(s)?

► Estimate what will be the actual H of liquified air in this experiment at very high [“infinite”] pressure. The density of air in the gaseous phase is 1.29 kg/m^3 and the density of liquid air is 1125 kg/m^3 .

► An alternative route to find $T_{\text{abs}}^{(\circ)}$ would be to plot H versus $T^{(\circ)}$ for a given pressure and fixed amount of gas. According to Eq. (8), graph should come out to be a straight line, with x -intercept (when $H = 0$) equal to $T_{\text{abs}}^{(\circ)}$. For a given P (or in other words weight load), you have only two different H measured (one at room and another at ice temperature). Thus you have only two points to do a linear fit of H versus $T^{(\circ)}$ in order to obtain $T_{\text{abs}}^{(\circ)}$, and this it is not “nice”. Why?—Try to follow this alternative route.

III. DC CIRCUITS: VOLTAGE, CURRENT AND RESISTANCE

► For each of the four sections of this experiment (§ III.B, § III.C, § III.D and § III.E) write *only* abstracts (they may turn out to be a little longer than usual ones as they should fully incorporate in themselves “Result, Discussion and Conclusions” section of your regular report). Error analysis is required everywhere.

► Wire each circuit and let your instructor check it out before turning on the power. **There are no hazardous voltages present, but faulty wiring may cause something to overheat.**

A. Theory

Consider a metal at a microscopic scale. Positive ions of metal form a lattice, there are some neutral impurities, and electrons randomly move around. In order to have an organized flow of electrons one creates some favorable conditions for their motion. For example, one can use a battery to set the potential energy of electrons to vary along the wire. As a result, electrons travel along complicated zigzag paths scattering from the ion lattice and impurities (see Fig. 3). The corresponding average electron drifting for common household wiring happens with

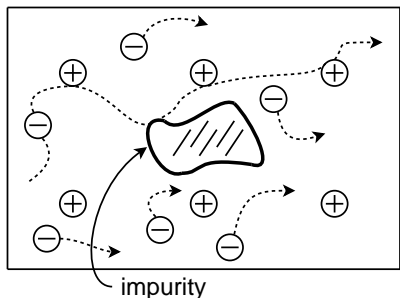


FIG. 3 Current through a piece of a metal conductor (flow of many electrons). Long dotted curve depicts the detailed path of one particular electron.

speeds of about 1 mm/s. There is a negligible kinetic energy of electrons associated with this drifting flow. The dominating effect is the transformation of the potential energy of electrons into vibration (i.e., heat) of metal ions and atoms of impurities during collisions with them.¹⁰

Voltage and current are the main physical quantities that describe the electron flow. Voltage is in one to one correspondence with the electron potential energy. Current is the rate of charge flow (its numerical value equals to the amount of charge that passes through the cross-section of a conductor per unit of time).

It is convenient to discuss how fictitious positive charges—to be called further *charge*—would have flown in a circuit rather than talk about the negative charge flow, i.e., the electron flow, because ...¹¹ [If you are interested in details see your text book, or ask your TA to describe in terms of electrons flow what happens in a circuit you have build.]

For a wide class of objects, the value of current I is directly related to the applied voltage V . Ohm's law is a mathematical representation of this fact

$$V = I \cdot R, \quad (12)$$

where the coefficient of proportionality R is called resistance. The *International System of Units* [abbreviated SI] uses Ohm (Ω) for units of resistance, and also Ohm = Volt/Ampere. The value of resistance is specific for a given conductor. For example, consider the circuit

¹⁰ Situation is somewhat similar to snowfall. Each snowflake aims to Earth surface where its potential energy will be smaller. Due to air friction (i.e., collisions with air *molecules*) snowflake speed sets to some small constant value. The entire huge snowflake potential energy difference corresponding to the cloud-to-Earth height separation is eventually converted into heat during down-fall.

¹¹ Imagine that your friend has given you +20\$. Or, equivalently, you could have said that you have given your friend -20\$, and, correspondingly, your friends wealth has grown by -20\$. Both ways of describing situation are correct, but language of "positive" money is simpler.

Positive charge moves or flows from higher to lower voltage. In contrast, negative charge moves from lower to higher voltage, so one would say that negative charge rises from lower voltage to higher voltage, which is inconvenient. If historically positive charge has been associated with electrons, then wording for metallic systems would have been simpler.

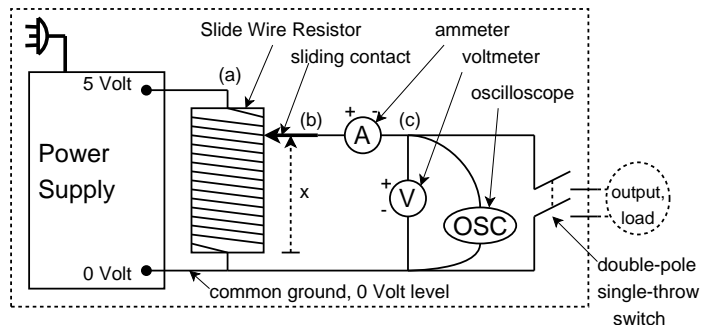


FIG. 4 Voltage source.

in Fig. 5. The *charge* runs through the copper wires more freely, than through a carbon resistor R , since the later has higher concentration of impurities at microscopical scale. Thus $R_{wire} \ll R$, and the *charge* flowing through this circuit losses its potential energy (the voltage drops) solely at the carbon resistor.

The circuits, more complicated compared to the case of a single resistor (see Fig. 5), will include at least two resistors either in parallel or in series (see Fig. 7). For simplicity, all resistors in these circuits are considered to be identical and the applied voltages to be the same. According to Ohm's law, Eq. (12), the current in a single resistor is

$$I_{\text{single}} = \frac{V}{R}. \quad (13)$$

It can be shown that

$$I_{\parallel} = 2I_{\text{single}} \implies I_{\parallel} = \frac{V}{\left(\frac{R}{2}\right)}, \quad (14)$$

$$I_{\text{series}} = \frac{I_{\text{single}}}{2} \implies I_{\text{series}} = \frac{V}{2R}, \quad (15)$$

which is equivalent to the statement that the effective total resistance of the parallel connection is $\frac{R}{2}$ and of the series connection is $2R$.¹²

Nonohmic behavior, for example, is exhibited by a light bulb. Although, the bulb filament is a thin metal wire, its voltage-current characteristic does not follow Ohm's law, Eq. (12). An increase of current corresponds to a dramatic increase of the filament temperature (observed as its brightness variation), and this affects conducting properties of the metal.—Higher temperature manifests itself at the microscopic level in a higher intensity of oscillations of metal ions around their equilibrium positions, which makes the crawling of electrons harder.

¹² In general case of parallel and series connection of two arbitrary resistors R_1 and R_2 , the effective corresponding resistances R^{eff} will be

$$\frac{1}{R_{\parallel}^{\text{eff}}} = \frac{1}{R_1} + \frac{1}{R_2} \implies R_{\parallel}^{\text{eff}} = \frac{R_1 \cdot R_2}{R_1 + R_2},$$

$$R_{\text{series}}^{\text{eff}} = R_1 + R_2.$$

B. Voltage Divider

Consider the circuit in Fig. 4. [When working with analog voltmeters and ammeters, pay attention to the polarity {positive (+) or negative (-)} of the terminals (see Fig. 4). The (-) terminal is often called COM (common). When dealing with light bulbs and resistors, polarity is not important.] The power supply delivers *charge* at constant (DC) voltage of 5 Volt to a point (*a*). *Charge* goes through the slide wire resistor losing its potential energy and eventually arriving to the common ground of 0 Volt. Positioning the sliding contact at some distance x from the bottom of the slide wire resistor, one can pick *charge* at voltages ranging from 0 Volt to 5 Volt at the point (*b*). In other words, this construction serves as a **voltage divider**. On the way to the output, from point (*b*) to (*c*), *charge* goes through the ammeter. Ammeter practically does not disturb the charge flow, but simply counts the total charge q that passes through in time t and calculates the rate of flow, i.e., the current, $I = \frac{dq}{dt}$. At the point (*c*) some tiny fraction of *charge* is taken by the voltmeter and is conducted to the common ground. This allows the voltmeter to determine the potential difference between the two points. Thus the voltmeter measures the output voltage supplied to load.

1. Lab Goal

Build the voltage source depicted in Fig. 4. Test its output characteristics.

2. Experiment Overview/Procedure/Analysis

Put together the voltage source (see Fig. 4). You will use it in later parts of this lab.

Without any load, i.e., the switch is off, measure and plot the output voltage as a function of the sliding contact position (x), measured from the bottom end of the slide wire resistor. Your TA may explain why a linear relationship should be expected.

3. Practice to Use Oscilloscope

The oscilloscope is a fancy voltmeter that shows graphically how the voltage of a signal varies with time. You could have achieved the same results with a simple voltmeter by measuring and plotting voltage versus time.

Connect the oscilloscope channel 1 in parallel to the voltmeter (see Fig. 4). [Connect the ground (-) to the black jack and (+) to red jack.] Set the switch above the input of channel 1 to DC. On the screen, you will see a horizontal line, as the voltage (y -axis on the screen) in your circuit does not change with time (x -axis on the screen). You can offset the voltage by the knob POSITION. Set the switch above the input of channel 1 to GND.—This disconnects the oscilloscope from the circuit and allows you to access the oscilloscope ground.—Center the level of the oscilloscope ground, for example, in the middle of the screen by the POSITION knob (i.e., set the 0 Volt level for the y -axis in the middle of the screen).

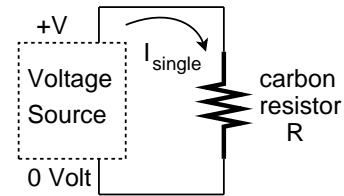


FIG. 5 Single carbon resistor R .

Then switch back to DC. Set appropriate units for the vertical scale by VOLTS/DIV knob [the red calibration knob (CAL) in its center should be clicked fully clockwise].

For different positions of the sliding contact compare the voltage readings of the oscilloscope with the ones of the voltmeter.

C. Ohm's Law

1. Lab Goal

Verify the applicability of the Ohm's law for a carbon resistor and evaluate its resistance.

2. Experiment Overview/Procedure/Analysis

You can burn the fuse of your ammeter if the current that you are trying to measure exceeds the ammeter's range. Therefore *before* turning on the double-pole single-throw switch (see Fig. 4), set the ammeter to the 200 mAmp range.

Load the voltage source that you have build in § III.B with a single small carbon resistor (see Fig. 5). Measure voltage-current characteristic. [Note that 0 Volt / 0 Amp is a perfectly reasonable measurement.] Plot V as a function of I . If the carbon resistor can be described by the Ohm's law, Eq. (12), then your data can be fitted by a straight line that passes through error bars with the y -intercept that equals to 0 Volt. What do *you* observe?

If the resistor is ohmic, determine its resistance R from the slope of the linear voltage-current characteristic. Also figure out R using the color code (see Fig. 6). And, also, measure R using digital multimeter as an ohmmeter. Compare all the values obtained above with each other, and make a statement about their mutual agreement!

D. Resistors in Series and Parallel

1. Lab Goal

Study properties of resistors in parallel and in series.

2. Experiment Overview/Procedure/Analysis

You will be given two identical carbon resistors in a sense that their resistance R will be the same according to their color coding. You can find out R directly from

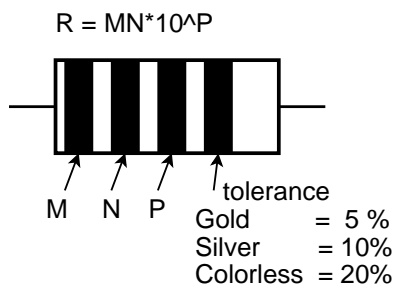


FIG. 6 Color code scheme for labelling resistors.

Colors for M, N, P :	
0 Black	5 Green
1 Brown	6 Blue
2 Red	7 Violet
3 Orange	8 Gray
4 Yellow	9 White

Examples:

a) Brown, Black, Black, Colorless:

$$\hookrightarrow R = 10 \cdot 10^0 \Omega \pm 20\% = 10 \pm 2 \Omega$$

b) Yellow, Violet, Black, Gold:

$$\hookrightarrow R = 47 \cdot 10^0 \Omega \pm 5\% = 47 \Omega \pm 5\%$$

c) Gray, Orange, Brown, Silver:

$$\hookrightarrow R = 83 \cdot 10^1 \Omega \pm 10\% = 830 \Omega \pm 10\%$$

their color codes [or measure resistance of each resistor (R_1 , and R_2) according to the procedures of § III.C]. This allows you to make prediction for the expected value of the effective total resistance of the parallel and series connections to be $\frac{R}{2}$ and $2R$, respectively [see § III.A, and Footnote 12]. Do not forget to carry out error propagation!

Using the procedures of § III.C measure the effective total resistances of the parallel and series connections [also note uncertainty of your measurements]. Are your measurements in the agreement with your expectations?

Optional

For circuits with single resistor (see Fig. 5), two resistors in parallel and two resistors in series (see Fig. 7), measure the total current in these circuits at a fixed voltage, e.g., at 3 Volt. (Check to be sure the voltage has not changed between trials; adjust it if necessary.) How do

IV. CHARGING AND DECAY IN RC CIRCUITS

A. Theory

Charge can be stored on two conductors placed near each other; such a combination is called a capacitor. Usually, but not necessarily, the conductors are in the form of parallel plates. One way of charging a capacitor is to connect the two plates to a battery. In Fig. 8 the battery causes a momentary flow of electrons from the top plate

your findings for the total current in these circuits compare with theory, more specifically with Eqs. (14,15)?

E. Light Bulb

1. Lab Goal

Observe nonohmic voltage-current characteristic of a light bulb.

2. Experiment Overview/Procedure/Analysis

Similar to § III.C measure voltage-current characteristic for a light bulb over as wide range as possible, for example, going from 0 Volt to 3 Volt in 0.5 Volt increments. Plot V as a function of I . Why can (or maybe cannot) you claim that your data represents nonohmic behavior?

Optional

With two light bulbs in parallel and in series you can “visualize” Eqs. (13,14,15). Keep load voltage to be, for example, 3 Volt and observe brightness of a single light bulb, then two bulbs in parallel and then in series. Note that brightness corresponds to the magnitude of current passing through. How do your observations compare with your expectations from Eqs. (13,14,15)?

What happens when you unscrew one bulb in the parallel circuit and in the series circuit?

Relate all your observations above to wiring of light bulbs in a household. Are they wired in parallel or in series? Why?

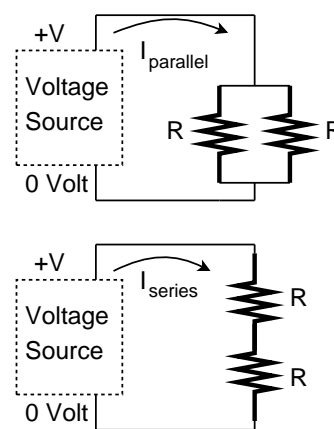


FIG. 7 Circuits with two resistors R in parallel and in series.

through the battery to the bottom plate. One can prove that amount of charge Q on either plate is directly proportional to applied voltage V (potential difference across capacitor created by the battery), thus

$$Q = C \cdot V, \quad (16)$$

where C is the coefficient of proportionality called capacitance. Details of a capacitor construction determine the

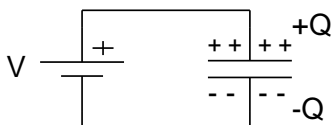


FIG. 8 A parallel plate capacitor connected to a battery.

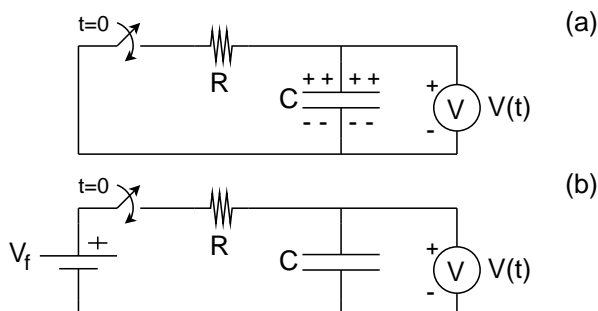


FIG. 9 (a) The upper sketch shows process of a capacitor discharge, and (b) the lower one depicts charging of a capacitor. In both cases the voltmeter on the right side of each diagram is used to monitor the voltage $V(t)$ across the capacitor.

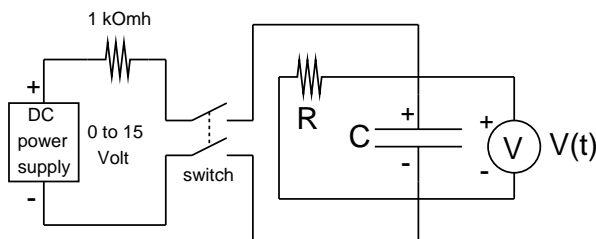


FIG. 10 Circuit to be used to observe slow RC decay.

numerical value of C , for example, for the case of a pair of parallel plates close together $C \propto A/d$, where A is the area of each plate, and d is their separation. The *International System of Units* [abbreviated SI] uses Farad (F) for units of capacitance.

If a charged capacitor is connected across a resistor R as shown in Fig. 9(a), current will begin to flow through the resistor. Solving corresponding differential equation it can be shown that the voltage across the capacitor will decay exponentially from the initial voltage V_i toward zero. The equation governing the time dependence of the voltage decay is

$$V(t) = V_i e^{-t/RC} = V_i e^{-t/\tau}, \quad (17)$$

where $\tau = RC$ is the time constant of the decay. SI units of τ are seconds. [Using Eq. (16) one can rewrite Eq. (17) to describe exponentially decreasing charge $Q(t)$ on the plates of the capacitor.]

This exponential function has some rather remarkable properties. Note that it takes *forever* for the voltage to reach zero. Note also that at the end of each time interval τ , the voltage is less than it was at the beginning of the time interval by a factor of e^{-1} *no matter when the time interval began*. [e is the base of the natural logarithm, and $e^{-1} = 0.3678\dots$ or approximately 37%.] Thus when one decay time has passed ($t = 1\tau$), the voltage is

approximately 37% of its initial value. After two decay times ($t = 2\tau$) the voltage is about 13.5%, after 3τ about 5%, etc. After five decay times ($t = 5\tau$), the voltage is less than one percent of its initial value [$e^{-5} = 0.0067\dots$] and for most practical purposes it is zero.

For charge buildup the same time constant applies. If an uncharged capacitor is connected to a battery of voltage V_f through a resistor R , as shown on Fig. 9(b), the voltage across the capacitor grows exponentially from zero to the battery voltage according to the following equation

$$V(t) = V_f(1 - e^{-t/RC}) = V_f(1 - e^{-t/\tau}). \quad (18)$$

B. RC Decay

1. Lab Goal

Study the decay of charge on the capacitor in the Resistor-Capacitor (RC) circuit by using a digital voltmeter. Obtain the value of capacitance.¹³

2. Experimental Procedure

Slow voltage decay across a capacitor can be monitored directly with a digital voltmeter (DVM).

Wire the circuit shown in Fig. 10. Be sure to observe the polarity for both the capacitor and the DVM. Select the 20 – Volt scale of the DVM. With switch open and R set on the decade resistance box at 100Ω , or less, check the DVM reading. It should be zero, or no more than 1 in the last digit. If it is not, put a short circuit (a wire) across the capacitor terminals, see that the voltage falls to zero, then remove the short.

¹³ You can substitute a physics “study ...” with an engineering “project ...”. For example, for this lab imagine that you are a junior scientist in a hi-tech firm. One day your boss comes and brings a small cylinder-shape electric component. He tells you: “Our new supplier claims that this thing is a good capacitor. Check this claim, and, if it is indeed true, validate specification printed on top of this device. Report is due yesterday.” So, you start to search through your physics text-book, and while reading about RC circuits you get an idea. You build RC circuit with knowingly good resistor and this device, then you measure decay in this RC circuit (see § IV.B.2), and compare your observations with Eq. (17) for a good capacitor, etc. (see § IV.B.3) ... Finally, you write for your boss a logically clear and self-consistent report about your “research”. [Do you think that after this project you will be promoted to a senior scientist position?]

Similarly you can think about all other labs.

► § I: To design cryogenic dewars [thermoses] for liquid nitrogen you need to know/measure latent heat of vaporization of liquid nitrogen.

► § II: Your firm develops air compressors. Is it appropriate to use the Ideal Gas Law for an approximate description of air? [Probably, you will not be interested in $T_{\text{abs}}^{(o)}$.]

► § III: This lab is too simple.

► § V: You want to build and test a magnetic field detector.

► § VI: Playing electric guitar is your hobby. Can you construct a simple analog RLC filter? You will need also to figure out whether electric component that you have soldered out from your grandparents radio is still a good inductor.

Set R to 300 k Ω . With the variable DC power supply on, but turned down, close the switch. Now turn up the voltage to about 10 Volt. Measure the voltage across the discharging capacitor as a function of time by opening the switch and recording $V(t)$. Obtain at least 10 to 15 values. Plan your experiment so that your final measurement of voltage will at least ten times smaller than your initial one. [The actual initial voltage does not matter, but it will be more convenient to make the first measurement at 10 Volt.]

3. Analysis

Check that the measured voltage across the capacitor indeed decays exponentially according to Eq. (17). [Plot your data of V versus t to see how it looks. You cannot yet claim the curve to be exponential, unless you have special software, like Origin, to carry out an exponential fit.] Note that taking a logarithm of both sides of Eq. (17) you arrive at a linear relationship between $\ln V$ and t ,

$$\ln V(t) = \ln V_i - t/\tau. \quad (19)$$

Thus it is convenient to plot voltage versus time using the logarithmic scale for the voltage axis, or, equivalently, you can plot $\ln V$ versus t . If in this representation your data points are close to a straight line, then you have verified that the discharge of the capacitor is described by Eq. (19) and, consequently, by the original Eq. (17). [Moreover, if this is true, then you can also state that you have used in your RC circuit good capacitor and good resistor, because only two basic Eqs. (12,16) were used to derive Eqs. (18,17) describing charging and decay.]

Once the functional form of $V(t)$ is established, find the time constant τ from the slope of the linear fit of $\ln V$ versus t . Finally, calculate the value of C . Do not forget to take into account the uncertainty of your measurements! What is the uncertainty of C after error propagation? Does your result agree with the nominal value of the capacitance?—Look at the capacitor actually used in your circuit. Record the value printed on it; it should be $220 \mu\text{F} \pm 20\%$.

Optional

In anticipation of the next part of the lab, make a “spot” measurement of the time constant by adjusting the initial voltage (V_i) to 10 Volt and timing the fall to 3.7 Volt. Alternatively you could time the drop from an arbitrary V_i to $0.37 \times V_i$. Compare this result for τ with that in the previous part.

“Optional” Thinking

Actually, the effective resistance is not just R that you have set on the decade resistance box, but is instead R in parallel with 10 M Ω input resistance of the DVM. Thus even if you remove R from the circuit, the capacitor will still discharge very slowly through the DVM. To see this,

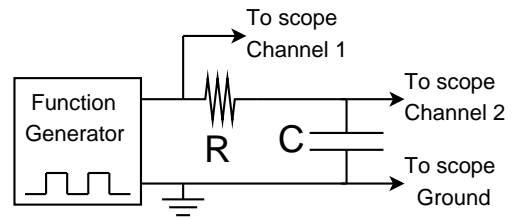


FIG. 11 Circuit to be used to observe fast charging and discharging of capacitor.

repeat the experimental procedure without R in the circuit, except this time measure only a few voltages at intervals of one minute. Find the time constant. Is it what you expect?

C. Capacitors in Series and Parallel

1. Lab Goal

Experimentally establish formulas for the effective capacitance of two capacitors connected in series and parallel.

2. Experimental Procedure

You will be given two capacitors of nominal capacitance $0.5 \mu\text{F} \pm 10\%$, and $1 \mu\text{F} \pm 10\%$, and also a decade capacitance box. Come up with a strategy to invent formulas for effective capacitance of two capacitors connected in series and parallel. For example, carry out some meaningful measurements of the effective capacitance of parallel and series connections (see § IV.C.3), think, put forward a hypothesis, test it, if it does not hold true, think more, come up with some other bright idea...

3. How to measure the capacitance by using a square-wave generator

Learn how to measure the capacitance of a single capacitor!

Wire the circuit shown in Fig. 11, using $R = 2000\Omega$ and $C = 0.5 \mu\text{F}$. The voltage source is a square-wave generator [actually a function generator, set to give square-wave output], and is displayed on channel 1 of the oscilloscope. In a square wave, the voltage jumps from $+V_0$ to $-V_0$ at a preset frequency. This simulates the switch in Fig. 10 being opened and closed repetitively, alternately charging and discharging the capacitor many times each second. This repetition enables you to observe the capacitor voltage decay and buildup on channel 2 as a stationary pattern on the oscilloscope screen. [The instructor will help you with the operation of the oscilloscope as needed.]

Set the generator on the square-wave output at the frequency $\approx 100\text{Hz}$. Adjust the TIME/DIV on the scope to

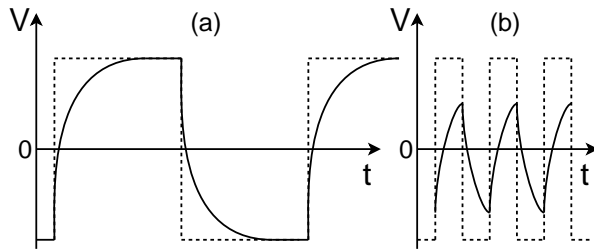


FIG. 12 Both channel 1 and 2 are displayed simultaneously on the screen of the oscilloscope. Solid curves are the time-dependent voltage across the capacitor for different frequencies of the square-wave generator. Dotted lines represent output of the square-wave generator. In the (a) case frequency of the square-wave generator is adjusted correctly, in the (b) case one should decrease frequency.

about 1 ms/DIV. You should see a wave form that looks like Fig. 12(a). If the wave form looks like Fig. 12(b), reduce the frequency of the square-wave generator until the wave looks like Fig. 12(a).

Measure the time constant using either decreasing or increasing capacitor voltage. The way to do this is to find the time it takes the voltage to drop to 37% from its maximum value, or to find the time it takes the voltage to rise to 63% of the way up to the final value. [Note that from Eq. (18) it follows that $V(\tau) = V_f(1 - e^{-1}) \approx 0.63V_f$.] Estimate the uncertainty in your measurement of the time constant. Calculate the capacitance. Does your value agrees with the nominal value?

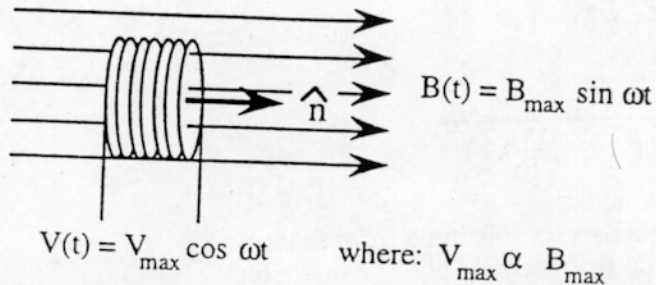
Experiment 5: Magnetic Field Mapping

Purpose

The Purpose of this lab is to determine experimentally how magnetic fields vary in space in the vicinity of various current configurations.

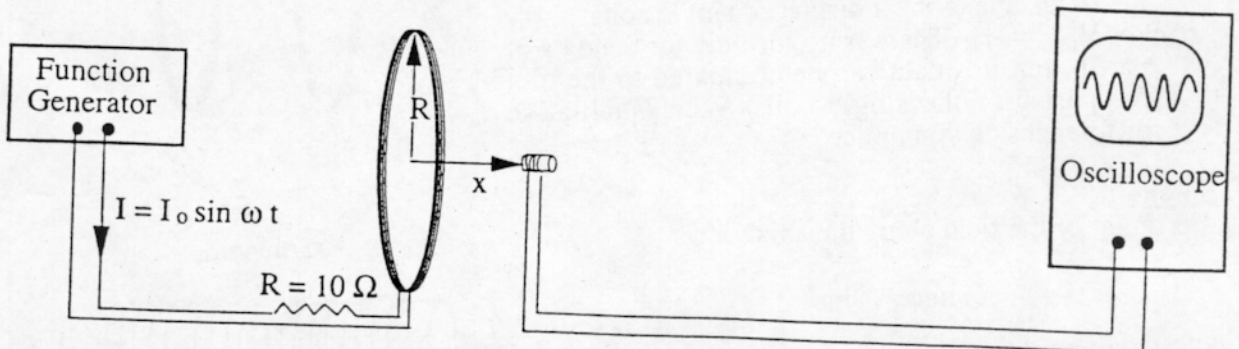
Measurement Technique

The measurement technique utilizes the fact that an oscillating magnetic field can induce an oscillating voltage in a coil of wire, and that the amplitude of the oscillating voltage is directly proportional to the amplitude of the magnetic field oscillation. This is illustrated in the diagram below.



The induced voltage oscillates with the same frequency as the magnetic field, and is due to the changing magnetic flux in the plane of the coil. Notice that the plane of the coil is shown perpendicular to the magnetic field lines (i.e. \hat{n} is parallel to \vec{B}). The size of the voltage "induced" in the coil is also proportional to the number of magnetic flux lines which penetrate the plane of the coil, and this is a maximum in the configuration shown. If the coil is rotated so that its plane is parallel to the magnetic field lines (i.e. \hat{n} is perpendicular to \vec{B}), then the number of field lines which penetrate the plane of the coil becomes zero and no signal will be induced. A small probe coil can thus be used to measure the strength of an oscillating magnetic field, as long as care is taken to orient the coil at whatever angle is necessary to maximize the induced voltage signal.

The diagram below illustrates how this technique can be used to explore the variation of the magnetic field in the vicinity of a current-carrying wire coil of wire:

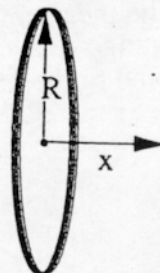


The amplitude of the Voltage induced in the small "probe" coil can be determined using an oscilloscope. As indicated below, the magnetic field near a current-carrying coil should be proportional to $(1 + (x/R)^2)^{-3/2}$. Since the induced voltage is proportional to the magnetic field, a plot of V_{\max} vs $(1 + (x/R)^2)^{-3/2}$ should give a straight line. Remember that, at each position, the plane of the coil should be oriented perpendicular to the magnetic field to get an accurate reading of V_{\max} .

Current Configurations to be Explored

Using this technique, explore as many of the following current configurations as you have time for. For each of these configurations, the formula for the magnetic field is given. Look in your textbook for pictorial diagrams of the expected field around each of the current configurations. In your analysis of the data, see if the data agree with the appropriate formula.

Single Coil



1. Circular Coil of Wire

Formula for the field along the axis:

$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

Hence:

$$\frac{B(x)}{B(0)} = \frac{1}{\left(1 + \left(\frac{x}{R}\right)^2\right)^{3/2}}$$

Suggested Data to be taken:

- V_{\max} along the axis vs x (distance from plane)
- V_{\max} vs distance from center (along a diameter)

Suggested Analysis:

- Plot $V(x)/V(0)$ vs $(1 + (x/R)^2)^{-3/2}$ (Should give a straight line!)
- Plot V vs r (There is no simple formula for this data)

2. Helmholtz Coils (A pair of identical coils separated by a distance equal to their radius)

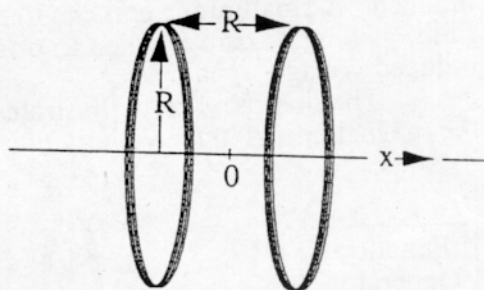
Suggested data (be sure that the current flows in the same direction in the 2 coils):

- V_{\max} along the axis vs distance from center, x
- V_{\max} vs r at $x = 0$

Suggested analysis:

- Show that the field along the axis is just the sum of the fields due to the 2 displaced single coils
- Plot V_{\max} vs r . (There is no formula for the data of part b, but this data can be compared to the plot for part b of the single coil.) What qualitative differences do you notice?

Helmholtz Pair



3. Solenoid

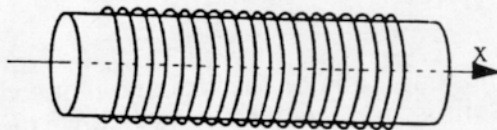
Formula for the field along the axis:

$$B = \frac{\mu_0 NI}{2L} (\sin \phi_2 - \sin \phi_1)$$

Suggested data:

- V_{\max} vs distance along the axis, x (Let $x = 0$ be the center of the coil)
- V_{\max} vs distance along a diameter inside and outside the coil (near center of coil)

Solenoid



Suggested analysis:

- Plot V_{\max} vs x . Show whether or not your data agree with the formula
- Plot V_{\max} vs r . Discuss your results

VI. RLC CIRCUIT

A. Forced Oscillations, Resonance

1. Theory

The term “oscillatory motion” in physics has a wider meaning than just a displacement of an object back and forth. Oscillatory motion is defined to be *any* periodic or almost periodic process, i.e., when one or several values of some physical quantities repeat themselves exactly or almost exactly after equal or almost equal intervals of time.

We are going to discuss oscillatory circuit (see Fig. 13) under both free and forced oscillation conditions. It consists of an inductor L , capacitor C and resistor R connected in series. Oscillations of the charge $Q(t)$ on the plates of the capacitor (and the corresponding current $I(t) = \frac{dQ}{dt}$ in the circuit) are equivalent to a one-dimensional oscillations of the displacement $x(t)$ (and the corresponding velocity $v(t) = \frac{dx}{dt}$) of an object attached to a spring.¹⁴ More specifically, in the oscillatory circuit, L is equivalent to the mass m of the object in the mechanical system, C^{-1} is equivalent to the force constant k of the spring, R is equivalent to the damping coefficient, and the time-varying voltage output $V(t)$ of the generator is equivalent to the external driving force. Such systems are called driven damped harmonic oscillators.

In an oscillatory circuit, the capacitor accumulates energy in the form of electric field produced by the charge $Q(t)$ on its plates. The inductor, e.g., a solenoid, accumulates energy in the form of a magnetic field produced by the current/charge flow $I(t) = \frac{dQ}{dt}$.

In the case of an object attached to a spring, the spring “holds” potential energy when an object is displaced to $x(t)$ from its equilibrium position. The object has a kinetic energy when it is moving with the velocity $v(t) = \frac{dx}{dt}$.

It is very important that the period T of free oscillations (i.e., no external voltage/force) of the harmonic oscillator (i.e., no damping) does not depend on the initial conditions,¹⁵ and, as a consequence, T does not depend on the amplitude of oscillations. For the oscillatory circuit $T = 2\pi\sqrt{LC}$ and for the mechanical system $T = 2\pi\sqrt{\frac{m}{k}}$. In other words, T depends on the parameters of the system that determine the “potential” and the “kinetic” energies. The period T determines the natural frequency f_0 of the free oscillations and the corresponding

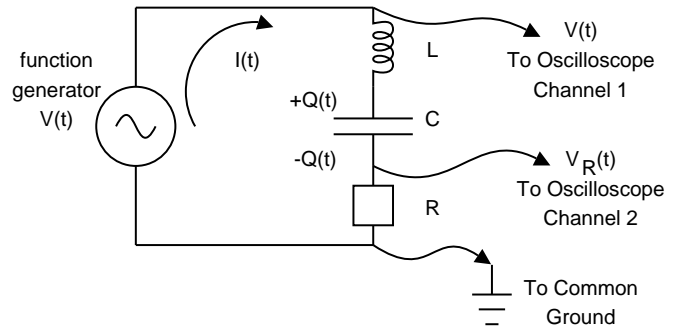


FIG. 13 RLC circuit.

natural angular frequency ω_0 :

$$f_0 \equiv \frac{1}{T} = \frac{1}{2\pi\sqrt{LC}} \quad \text{and} \quad \omega_0 \equiv 2\pi f_0 = \frac{1}{\sqrt{LC}}. \quad (20)$$

Once the alternating with the angular frequency ω driving voltage/force of the amplitude V_0 has been applied,

$$V(t) = V_0 \cdot \sin(\omega t), \quad (21)$$

it can be shown that the current in the circuit, $I(t)$, varies at the same angular frequency ω , but with some phase shift δ :

$$I(t) = \frac{V_0 \cdot \sin(\omega t + \delta)}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}. \quad (22)$$

Measured voltage $V_R(t)$ across the resistor (see Fig. 13) divided by R is the current $I(t)$ flowing in the loop, $I(t) = \frac{V_R(t)}{R}$, since, according to the Ohm's Law, $V_R(t) = I(t) \cdot R$.

The amplitude of oscillations of $I(t)$ in Eq. (22) (see Fig. 14) has its maximum as a function of ω at so-called resonant angular frequency ω_{res} , which minimizes the denominator of Eq. (22),

$$\omega_{\text{res}}L - \frac{1}{\omega_{\text{res}}C} = 0 \quad \Rightarrow \quad \omega_{\text{res}} = \frac{1}{\sqrt{LC}}. \quad (23)$$

Apparently, the resonant angular frequency is equal to the natural angular frequency, $\omega_{\text{res}} = \omega_0$, and $f_{\text{res}} = f_0$. Your TA will explain in detail the physics of this “coincidence” common to most oscillating systems.¹⁶

Note that the resonant frequency f_0 does not depend on the damping R , but R limits the amplitude of the current at resonance.

¹⁴ Both situations are described by one and the same differential equation. Moreover, many other physical systems have equivalent behavior, for example, small oscillations of a pendulum, vibrations of molecules and so on. Sound waves and electromagnetic waves are also closely related phenomena.

¹⁵ Initial conditions of oscillator are, for example, initial capacitor charge/pendulum displacement and initial current in the circuit/pendulum velocity.

¹⁶ To have a child-swing oscillate at maximum amplitude you push it once it comes, for example, to the most right position. This happens about once per period T . So, for the resonance to occur, the frequency of your pushing should be about the natural frequency, $f_0 = \frac{1}{T}$, of the child-swing.

During such forced oscillations, friction limits the amplitude preventing the child-swing from overturning (and, correspondingly, circuit from burning the fuse, spring from breaking, etc.).

$R \rightarrow \text{Ohm}(\Omega)$	$L \rightarrow \text{Henry (H)}$
$C \rightarrow \text{Farad(F)}$	$f, \omega \rightarrow \text{Hertz (Hz)}$

TABLE I International System of Units

2. Lab Goal

Study the resonance in the RLC circuit. Obtain the value of the inductance.

3. Experiment Overview/Procedure/Analysis

To build the RLC circuit you will be given an inductor, the decade capacitance box and the decade resistance box. Suggested range for C is 10–10000nF and 0.5–5k Ω for R . [If $C \lesssim 10\text{nF}$, then stray capacitance will influence your setup affecting the measurements. Choose C to be at least 10 nF!]

[1]: Check the theoretical prediction that the resonant frequency f_0 at the fixed L and C does not depend on the variations of R in the RLC circuit.

► Choose some value of C (for example, 75 nF) and some value of R (for example, 0.5 k Ω). Achieve resonance and measure f_0 (see § VI.A.4). Then, for example, increase R twofold, achieve resonance and measure f_0 . Try some other values of R .

[2]: Once the irrelevance of R to resonant frequency f_0 is established, check that f_0 follows the functional form of Eq. (20) by measuring f_0 at different values of C over as wide range of capacitance as possible.

► In order to have a pronounced response of the RLC circuit to a change of frequency of the driving voltage near the resonant frequency f_0 , choose a “small” value for R , for example, $R \approx 100 \Omega$.

One possible kind of analysis is to plot f_0 versus $C^{-\frac{1}{2}}$. [If you think a little, then you will realize that for such an analysis it would have been nice to choose $C = 10, 16, 28, 59, 204, \dots$ nF.] Do the data lie on a straight line that passes through the origin? Deduce the value for L and its uncertainty from the slope and compare it to the value of inductance written on the top of your inductor (about 130 mH).

A better way to carry out the analysis of presumable power-law-dependant data (see Eq. (20)) is to plot f_0 versus C using logarithmic scales on both axes. This is equivalent to studying the dependance of $\log f_0$ on $\log C$. And, from Eq. (20) it follows that

$$\log f_0 = \log \left(\frac{1}{2\pi\sqrt{LC}} \right) = \log \left(\frac{1}{2\pi\sqrt{L}} \right) - \frac{1}{2} \cdot \log C.$$

Do the data lie on a straight line with the slope $-\frac{1}{2}$? Deduce L and its uncertainty from the y-intercept and compare it to the provided value.

The advantage of the second method is that you do not need to think about the good choice of values of C for measuring f_0 , since for any geometrical progression of values of C , for example, for $C = 10, 100, 1000, 10000$ nF,

the values of $\log C$ are evenly spaced. Do not hesitate to ask your TA about this approach, it will take only 5 minutes. [Note that you cannot actually set 10000 nF on the decade capacitance box, but setting 9999.9 nF will do the trick.]

4. Details of Experimental Procedure

Assemble the RLC circuit shown on Fig. 13. Channel 1 will display the voltage $V(t)$ driving the circuit. Channel 2 will display the response of the circuit $V_R(t)$.

- Set the function generator to produce a **SINE**-wave.
- Connect the frequency counter to the SYNC output of the function generator. This way you will know frequency f of the output signal more accurately than from the reading on the frequency dial of the function generator.
- Set INTERNAL/CHANNEL 1/PEAK-PEAK AUTO for the oscilloscope trigger mode.
- Adjusting the frequency to some specific value, do not forget to set the corresponding frequency range on the function generator. For example, if you have found out that resonant frequency is about 5 kHz, then set function generator to the closest range of 10 kHz.
- Reminder: by the oscilloscope SEC/DIV knob you can adjust the horizontal time scale.

1. How to achieve a resonance (method A): Look at the amplitude V_R^{amp} of the voltage across the resistor $V_R(t)$ (channel 2) as a function of the source frequency (i.e., keep the amplitude of the source voltage constant and vary the source frequency). Having hit the resonant frequency f_0 you will observe the maximum value of V_R^{amp} .

2. How to achieve a resonance (method B): Investigate the relative phase shift of a driving signal (channel 1) and voltage across the resistance (channel 2). The phase shift will vanish at resonance and both signals will almost match each other.¹⁷ The best way to see this is to adjust ground levels of channel 1 and 2 to coincide at the middle of the oscilloscope screen and then switch channel 1 and 2 to AC mode.

Optional

Choose some particular values for R and C . Select different shapes of driving voltage, i.e., **TRIANGLE** or **SQUARE**. Do you see resonance response as you vary the source frequency? Does the resonant frequency differs from the one obtained with **SINE**-wave? Ask your TA for explanation.

¹⁷ It is known that, at resonance, the phase shift δ in Eq. (22) is 0, so $V_R(t) \equiv V(t)$! Thus $I(t) = \frac{V(t)}{R}$ at resonance, meaning that oscillatory and damping parts of the system are effectively decoupled.—The LC circuit experiences oscillations at its natural frequency, while the external driving source perfectly takes care of damping by supplying the necessary power to be dissipated in the resistor.

B. Impedance

1. Theory

For a circuit consisting only of a battery and a resistor, $V = I \cdot R$ according to the Ohm's law. In our case, we have a resistor, an inductor and a capacitor in the circuit, and the circuit is driven by time-varying voltage, Eq. (21). Nevertheless, one can similarly introduce an *effective* resistance (called **impedance**) by

$$V(t) = I(t) \cdot R_{\text{eff}}. \quad (24)$$

For amplitudes V_0 and I^{amp} of $V(t)$ and $I(t)$, respectively, this equation yields

$$V_0 = I^{\text{amp}} \cdot R_{\text{eff}}. \quad (25)$$

Making use of Eq. (22) we obtain

$$\frac{1}{R_{\text{eff}}(\omega)} = \frac{I^{\text{amp}}(\omega)}{V_0} = \frac{1}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}. \quad (26)$$

Here $\frac{1}{R_{\text{eff}}(\omega)}$ is nothing but frequency-dependent effective conductance of the RLC circuit. Curve of $\frac{I^{\text{amp}}}{V_0}$ versus ω (or $f \equiv \frac{\omega}{2\pi}$) for fixed R , L and C is called the resonance curve (see Fig. 14).

At resonance, according to Eq. (26), the effective resistance of the RLC circuit, $R_{\text{eff}}(\omega_0)$, is minimal and its value is equal to R . Thus you can make the following statement: driving RLC circuit by an external voltage of resonant frequency f_0 results in the maximum amplitude of the oscillating current (divergent in the limit $R \rightarrow 0$).

An RLC circuit can be interpreted as a frequency filter. Indeed, relationship between amplitude of the output voltage across resistor and amplitude of the input signal from the function generator

$$V_R^{\text{amp}}(\omega) = I^{\text{amp}}(\omega) \cdot R = V_0 \cdot \frac{R}{R_{\text{eff}}(\omega)}. \quad (27)$$

shows angular frequency dependant scaling of magnitude $\frac{R}{R_{\text{eff}}(\omega)}$. Figure 14 depicts this characteristics of a filter across a wide range of angular frequency ω . Width/broadness/sharpness of the resonance peak determines selectivity of the filter.¹⁸

2. Lab Goal

Measure resonance curve for a given RLC circuit and compare it to the theoretical prediction of Eq. (26).

3. Experiment Overview/Procedure/Analysis

Measure how the amplitude $V_R^{\text{amp}}(\omega)$ changes as you detune f from f_0 for the given RLC circuit. Calculate $\omega = 2\pi f$, and also calculate $\frac{V_R^{\text{amp}}(\omega)}{R \cdot V_0}$.

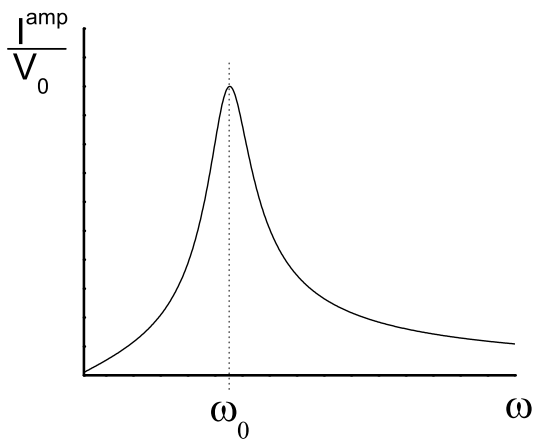


FIG. 14 Resonance curve.—Amplitude of the current in the circuit I^{amp} versus angular frequency ω of the driving voltage.

Plot the theoretical expectation Eq. (26) for resonance curve

$$\frac{1}{R_{\text{eff}}(\omega)} = \frac{I^{\text{amp}}(\omega)}{V_0} = \frac{V_R^{\text{amp}}(\omega)}{R \cdot V_0}$$

versus ω and results of your measurements on the same graph. Do you see an agreement?

4. Details of Experimental Procedure

- ▶ Fix value of C from 10 to 100 nF and R to be in the range 1 – 5 k Ω .¹⁹ [L is fixed in this lab's experimental setup.]
- ▶ Find resonant frequency f_0 . Measure the amplitude of the voltage across the resistor $V_R^{\text{amp}}(\omega_0)$ on the channel 2 at resonance.
- ▶ Measure the amplitude of the driving voltage V_0 on the channel 1. *It should be equal to $V_R^{\text{amp}}(\omega_0)$!*
- ▶ Measure the amplitude of the voltage across the resistor $V_R^{\text{amp}}(\omega)$ on the channel 2 at frequencies ω above and below the resonance, for example, when $V_R^{\text{amp}}(\omega)$ is 75%, 50% and 25% of its resonance value, $V_R^{\text{amp}}(\omega_0)$. *If the amplitude of the driving voltage varies as you take measurement, then inform your TA about this fact!*

Optional

Measure and plot on the same graph *several* resonance curves for *several* values of R at some *fixed* L and C . Observe that the position of resonance peak f_0 does not change, and that the curves are getting broader and shallower with the increase of R .

¹⁸ Note that the circuit that consists of just a single resistor R has $R_{\text{eff}}(\omega) \equiv R$, thus $V_R^{\text{amp}}(\omega) \equiv V_0$ and filter has no selectivity and corresponding resonance curve, Fig. 14, will be a horizontal line.

¹⁹ When $R \gtrsim 1$ k Ω , then R is dominant damping factor in the circuit, since the internal resistance of inductor (≈ 20 Ω) and output resistance on function generator (≈ 50 Ω) both connected in series will be comparably negligible.

VII. HISTORY

This is an “open” document. Suggestions are welcomed. We really do listen, and make text changes based on your personal input. Maybe you have materials to donate? If you know already available good related text, please, send a reference.

Contact responsible editor, *Karén G. Balabanyan*
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◇ Summer 2006

Major revision of the lab manual style was carried out by *K. Balabanyan*.

♣ Spring 2006:

§ IV “Charging and Decay in RC Circuits” was written by *K. Balabanyan* and, then, was revised by *E. Kozik* and *T. Papirio*.

♡ Fall 2005:

§ III “DC Circuits: Voltage, Current and Resistance” was written by *K. Balabanyan* and, then, was revised by *E. Kozik* and *T. Papirio*.

◇ Summer 2005

§ VI “RLC Circuit” was written by *K. Balabanyan*.

All labs produced so far were critically revised by *E. Burovski*, *E. Kozik* and *Tony Papirio*.

Professor *Edward Chang* brought order to § “Optional Cover Page”.

♣ Spring 2005

§ II “The Ideal Gas Law” was written by *K. Balabanyan* and, then, was checked/discussed with *Evgeni Kozik*.

♡ Fall 2004:

“Cover Page” and § I “Latent Heat of Vaporization of Liquid Nitrogen” were written by *Karén G. Balabanyan*. Participation of *Evgeni Burovski* was very fruitful.

✂ Root:

This project grew out from the lab manual “Physics 154 Heat, Electricity and Magnetism” by *prof. Kenneth H. Langley*.