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Secondary 2 Students' Difficulties in Solving Non-Routine Problems

Abstract

As part of a study on mathematical problem solving of secondary 2 (13- to 14-yearsold) students in Singapore, 56 Secondary two students from ten secondary schools participated in this study. The purpose of this paper is to explore difficulties faced by 56 Secondary 2 students when solving problems. These interviews were analysed using the structure derived from Newman (1983) and Ransley (1979). From the interviews the difficulties experienced by Secondary 2 students who were prevented from obtaining a correct solution were: (a) lack of comprehension of the problem posed, (b) lack of strategy knowledge, (c) inability to translate the problem into mathematical form, and (d) inability to use the correct mathematics.

KEYWORDS: Difficulties, Secondary Two, Interview, Non-Routine Problems

BACKGROUND

Problem solving in mathematics can be explained as "thinking and working mathematically" but the converse is not true. Problem solving in mathematics is an intricate process which calls for a problem solver who is engaged in a mathematical task to organise and deal with domain-specific and domain-general pieces of knowledge. Problem solving has been the main focus of the Singapore mathematics curriculum for the past seventeen years. Singapore's Ministry of Education continued to revise the syllabus for the mathematics curriculum in schools with a primary aim to enable students to develop their abilities in mathematical problem solving (Ministry of Education, 2006). This aim is dependent on five factors; specifically, skills, concepts, processes, metacognition, and attitudes. To successfully solve various types of problems, in particular the non-routine ones, a student has to apply four types of mathematical facilities, namely, specific mathematics concepts, skills, processes, and metacognition to tackle the problem. Mandler (1989) therefore went on to emphasise the importance of analysing the problem task and learner to ensure that surprises, errors and missteps can be handled by some alternatives routes, substitute actions, or a rewording of the task. According to Polya (1962), solving a routine problem did not contribute to the mental development of the student. He believed that to provide an opportunity for students to develop higher-order thinking in the process of understanding, analysis, exploration and application of mathematical concepts, non-routine problems should be employed. However, students generally fear the idea of solving non-routine problems because these problems are usually non-standard, involving unexpected and unfamiliar solutions. Besides, students are also apprehensive, anxious and extremely uncomfortable because they are not able to recall and apply learned procedures in a straightforward way. This was also seemed to reflect in the results of the Fourth National Assessment of Educational Progress (NAEP) of mathematics which showed that students performed poorly on solving word problems (Kouba, Brown, Carpenter, Lindquist, Silver, & Swafford, 1988). Similarly, in a large scale study in Singapore, Kaur and Yap (1996, 1997, 1998) conducted a longitudinal study as part of the KASSEL project on about 2300 Secondary 3 (15 years old) students from different academic streams. They found that the students did not lack any effort in attempting the problems but they appeared to lack the motivation and confidence in attempting unfamiliar problems.

Although it has been widely reported that Singapore emerged top in Mathematics for the Trends in International Mathematics and Science Study or TIMSS, an in-depth analysis of the TIMSS items revealed that Singapore Lower Secondary (grade 7 and grade 8) students did not perform well for items that required them to solve non-routine problems (Kaur & Yap, 1999). A thorough study of the TIMSS test data had confirmed that Singaporean students have indeed performed very well on items that were routine to them and tested what they had been taught at school (Kaur, 2003; Kaur & Yap, 1999; Pereira-Mendoza, Kaur, & Yap, 1999). Moreover, in the 2003 TIMSS study, 44% of the Singapore's students attained the 'Advanced International Benchmark' in Mathematics, compared to 7% internationally (Tan, Chow & Goh, 2008). While this was a remarkable achievement, essential questions was whether which stage of problem solving will influence their abilities in solving non-routine problems and affects their levels of difficulties, are far from clear. The implication is that teachers and mathematics educators should focus on the possible difficulties faced by the students as they interact with the mathematical problem and problem solution. In view of this, the purpose of this paper is to explore difficulties faced by Secondary 2 students when solving problems. The next section aims to review the literature and identify key factors, cognitive and affective factors that contribute to students' difficulties in mathematical problem solving.

LITERATURE REVIEW

Lester and Kehle (2003, p. 510) typify problem solving as an activity that involves the students' engagement in a variety of cognitive actions including accessing and using previous knowledge and experience:

> Successful problem solving involves coordinating previous experiences, knowledge, familiar representations and patterns of inference, and intuition in an effort to generate new representations and related patterns of inference that resolve the tension or ambiguity (i.e., lack of meaningful representations and supportive inferential moves) that prompted the original problem-solving activity.

What does it mean for students to organize previous knowledge and experiences to generate new knowledge? It is clear that if students are to be engaged in problem solving activities they need to develop a way of thinking consistent with mathematical practices, in which problems or tasks are seen as impasses that need to be examined in terms of questions. Thus, students need to problematize their own learning.

Mayer (1982, 1987) explained problem-solving processes as using different forms of knowledge leading to the goal of solving the problem. According to him, the types of knowledge applied in problem solving consisted of:

- linguistic and factual knowledge – about how to encode statements,
- schema knowledge – about relations among problem types,
- algorithmic knowledge – about how to present distinct procedures, and
- strategic knowledge – about how to approach problems.

In fact, according to Newman (1983), difficulty in problem solving may occur at one of the following phases, namely reading, comprehension, strategy know-how, transformation, process skill and solution. Schoenfeld (1985) suggested four aspects that contributed to problem-solving performance. These are the problem solver's: (1) mathematical knowledge, (2) knowledge of heuristics, (3) affective factors which affect the way the problem solver views problem solving, and (4) managerial skills connected with selecting and carrying out appropriate strategies.

In their study of the problem-solving research literature, Kroll and Miller (1993) identified three major cognitive and affective factors; namely, knowledge, control (metacognition) and beliefs and affects that contributed to students' difficulties in problem solving. Further Lester (1994) expressed that difficulties experienced during problem solving could also be caused by the problem solver's characteristics such as:

- traits – such as spatial visualization ability and ability to attend to the structural features of problems,
- dispositions – such as beliefs and attitudes, and
- experiential background – such as instructional history and familiarity with types of problems.

In the early 1970s, research tended to attribute difficulties in solving problems to the various tasks variables such as content and context variables, structure variables, syntax variables, and heuristic behaviour variables (Goldin & McClintoch, 1979). However, Lester (1994) contended that there was a general agreement that problem difficulty is not so much a function of various task variables but rather a function of characteristics of the problem solver. In other words, the knowledge one possesses, one's disposition and one's experiential background often influence problem solving performance. It was also evident in the Singapore's study conducted by Kaur (1995) and Lee (2001). Kaur indicated that Singapore's students experienced problem-solving difficulties such as: (1) lack of comprehension of the problem posed, (2) lack of strategy knowledge, and (3) inability to translate the problem into a mathematical form. Lee who conducted a local study on first year undergraduate students solving routine and non-routine calculus problems, found that the difficulties faced by the students were: (1) lack of experience in defining problems, (2) a tendency to rush toward a solution before the problem has been clearly defined, (3) a tendency to think convergently, and (4) lack of domain-specific knowledge. Similarly, in a recent study which based on their first-person perspective on problem solving and personal experience with the problems, McGinn and Boote (2003) identified four primary factors that affected perceptions of problem difficulty:

- categorisation – ability to recognise that a problem fits into an identifiable category of problems which run a continuum from easily categorisable to uncategorisable,
- goal interpretation – figuring out how a solution would appear which runs a continuum from well defined to undefined,
- resource relevance – referring to how readily resources were recognised as relevant, from highly relevant to peripherally relevant, and
- complexity – performing a number of operations in a solution.

McGinn and Boote suggested that the level of difficulty of the problem depended on problem solvers' perceptions of whether they had suitably categorised the situation, interpreted the intended goal, identified the relevant resources and executed adequate operations to lead toward a solution.

The aforementioned review of the literature on problem solving difficulty suggests that the problem solvers' characteristics are the most important determinants of problem difficulty. The research reported in this paper is part of a larger study investigating the mathematical problems solving of Secondary 2 students when they solve non-routine problems. Therefore, this paper provides findings from interviews conducted with 56 Secondary 2 students related to their difficulties in solving problems. The specific research question to be answered from the analysis of results in this paper is:

- What are the difficulties experienced by Secondary 2 students when solving problems?

RESEARCH METHODOLOGY

It is often tough to analyse the difficulties experienced by students when solving mathematical problems through investigating their written solutions. It may be more productive, when analysing errors, to interview students, noting their verbalisations and thought patterns about the specific problems with which they were faced. It cannot be assumed that when an incorrect answer is given to a mathematical problem that the error occurred because the student lacked the necessary mathematical knowledge or skill (Newman, 1977). In written assignments an interview technique may be used to find out the errors which students have made. A key assumption in this interview technique is that the types of errors students make will be consistent from one session to another. Nevertheless, it seems possible that one-to-one interviews, despite their limitations, do give greater insights into students' thinking and difficulties which would not be possible purely from an analysis of paper and pencil solutions.

Sample

The objective of this study is to gain insights into difficulties experienced by Secondary 2 students when solving problems. As part of a study on mathematical problem solving of secondary 2 (13- to 14-yearsold) students in Singapore, 56 secondary two students from ten secondary schools participated in this study. .

Instruments

An important criterion in choosing suitable mathematical tasks for this study was that they had to be "problems" for a majority of Secondary 2 students. By the definition of a "problem" it had to be reasonably complex but approachable and requiring no specific high level mathematics. The problem had to be relevant to the secondary school mathematics curriculum and at a level which the problem solver did not have a readily accessible procedure that determines the solution. It should also require the problem solver to use heuristic strategies to approach the problem, to understand it and to proceed to a solution. Most importantly, the problem should be capable of stimulating enough interest in an individual to want to attempt a solution. The mathematical problems that were chosen had to have certain general structures that emphasised various components of the problem-solving process so that each could draw out a variety of problem-solving behaviours that were required in the present research. The mathematical problems used in this study can be categorised according to the research literature as "structured problems requiring productive thinking"; that is, tasks where problem-solving heuristics must be used by the problem solver. They are usually referred to as "non-routine or non-standard" process problems in the mathematics classroom context. Such problems are usually not solved by simple recall or the application of familiar algorithms even though a student may possess sufficient mathematical knowledge. A total of three non-routine problems were used in the study. The three problems were assembled from various sources. The Time problem, was adapted from Baroody (1993). The Cat and Rabbit problem was adapted from Stacey and Southwell (1996) and Number problem was adapted from Fong (1994).

The Three Problems

Time Problem

Miss Lee arrived at the concert hall 15 minutes before a concert began. However, due to some technical problems, the concert started 10 minutes later. The whole concert lasted for 2 hours 25 minutes. It was 10.30 pm when Miss Lee left the concert hall. At what time did Miss Lee arrive at the concert hall?

Show all your working and explain it.

Cat and Rabbit Problem

A cat is chasing a rabbit. They are 160 metres apart. For every 9 metres that the cat runs, the rabbit jumps 7 metres. How much further must the cat run in order to overtake the rabbit?

Show all your working and explain it.

Number Problem

The sum of two numbers is 36 and their difference is 12. Find the two numbers.

Show all your working and explain it.

Procedure

The interview structure was developed from the Newman Error Analysis Guidelines (Newman, 1983) and the Ransley's (1979) problem-solving model.

The format used for the interviews consisted of the following oral procedure:

1. **Read the question aloud.**

   (proceed to 2 if the question is read correctly, otherwise show correct words and symbols before proceeding to 2)

2. **What do you need to find? What is the question asking for?**

   (proceed to 3 if student is able to comprehend the problem, otherwise ask questions to ensure understanding and comprehension like "How long was the concert?" before proceeding to 3)

3. **Without doing any working, tell me how are you going to solve this problem? What are you going to have to do in order to solve this problem?**

   (proceed to 4)

4. **What will you have to use to work out (such and such)? How will you work out to solve the problem? Show me how you solve the problem. Explain to me what are you doing as you solve the problem.**

   (proceed to 5)

5. **How can you check to see if your answer is sensible? Study the problem again and decide if your answer is sensible.**

The interviews were conducted one-to-one by the researcher and were audio-taped. The researcher took approximately half an hour to interview each student.

Analysis of Interview Data

The interviews that were conducted with the 56 Secondary 2 students (related to their difficulties in solving problems) were audio-taped. These interviews were analysed using the structure derived from Newman (1983) and Ransley (1979) as follows:

Reading

↓

Comprehension

↓

Strategy Know-How

↓

Transformation

↓

Process Skill

↓

Solution

The following two-point categorisations in behavioral terms for each stage of the above structures were used to analyse the interview protocols.

Reading

1. The student was able to read the problem without any difficulty.
2. The student was not able to read the problem and was helped with some of the words and/or symbols in the question.

As none of the students interviewed had any reading difficulty, there are no examples of difficulties at this stage which may have led to the inability of the students to go beyond this stage during the interview.

Comprehension

1. The student (S) was able to comprehend the problem. This was confirmed by the responses to the questions the researcher (R) asked.
2. The student was not able to comprehend the problem. This was confirmed by the responses to the questions the researcher asked.

The following examples show a series of students' difficulties which hindered their progress in solving the problem at this stage:

Time Problem

R: *Do you understand the problem?*

S: *No.*

R: *Did Miss Lee arrive before or after the concert began?*

S: *Before.*

R: *How long was the concert?*

S: *2 hours and 25 minutes.*

R: *What time did Miss Lee leave the concert hall?*

S: *10.30 pm.*

R: *Do you now understand the problem?*

S: *No, it is very confusing.*

R: *Why is it confusing?*

S: *Too many sentences.*

Cat and Rabbit Problem

R: *Do you understand the problem?*

S: *I really don't understand what the question is wanting me to find.*

R: *How many animals are there?*

S: *Two.*

R: *How far apart were the two animals?*

S: *160 metres.*

R: *For each step, how far did the cat run and how far did the rabbit jump?*

S: *9 metres for the cat and 7 metres for the rabbit.*

R: *Do you understand this problem now?*

S: *I don't know what it means by "for every 9 metres the cat runs, the rabbit jumps 7 metres."*

Numbers Problem

R: *Do you understand the problem?*

S: *Maybe. Quite confusing?*

R: *Why is it confusing?*

S: *Don't know how to say.*

Strategy Know-How

1. The student was able to describe a "method" he or she would use to tackle the problem.
2. The student had no idea of how to approach the solution of the problem at all.

The following examples show students' difficulties for some of the problems at this stage:

Time Problem

R: *Do you understand the problem?*

S: *Yes.*

R: *Did Miss Lee arrive before or after the concert began?*

S: *Before.*

R: *How long was the concert?*

S: *2 hours and 25 minutes.*

R: *What time did Miss Lee leave the concert hall?*

S: *10.30 pm.*

R: Can you explain to me how you will find out what time Miss Lee arrived at the concert hall?

S: I don't know how to explain.

Cat and Rabbit Problem

R Do you understand the problem?

S: Yes.

R: How many animals are there?

S: Two.

R: How far apart were the two animals?

S: 160 metres.

R: For each step, how far did the cat run and how far did the rabbit jump?

S: 9 metres for the cat and 7 metres for the rabbit.

R: Do know how far must the cat run in order to overtake the rabbit?

S: ………….(silence)

R: Do you know how to do it?

S: I don't know.

Number Problem

R: Do you understand the problem?

S: Yes.

R: How many numbers are there?

S: 2.

R: What is the sum of the two numbers?

S: 36.

R: *What is the difference of the two numbers?*

S: *12.*

R: *Can you explain to me what method you used to solve this problem?*

S: *Look easy at first but I don't know what method to use*.

R: *Do you know how to do it?*

S: *No.*

Transformation

1. The student was able to translate the problem into a mathematical form (equation or open sentence),
2. The student was not able to translate the problem into a mathematical form.

The following are examples of students' difficulties for some of the problems at this stage:

Time Problem

R: *Can you explain to me how you would find the solution?*

S: *Take 10.30 am, the duration of the concert, add 10 minutes and subtract 15 minutes.*

R: *How would you do this?*

S: *Don't know.*

Cat and Rabbit Problem

R: *Can you explain to me how you would find the solution?*

S: *Cat run 7 metres and rabbit jump 9 metres. Difference is 2 metres. ...... Not sure.*

R: *How would you do this?*

S: *I don't know. Cannot solve the problem.*

Numbers Problem

R: *Can you explain to me how you will try and solve the problem?*

S: *Factorisation. Let x be the smaller number and y be the bigger number. Don't know how to write the equations.*

R: *How will you find the solution?*

S: *Not sure.*

Process Skill

1. The student was able to do the mathematics and reach a solution.
2. The student was not able to do the mathematics and couldn't continue with the solution.

As none of the students had any difficulty with the Cat and Rabbit problem at this stage, there are only two examples to illustrate the students' difficulties in the Time and Numbers problems.

Time Problem

R: *Can you go over each step of your work and tell me what were you thinking?*

S: First take 2 hours 25 minutes and add 10 minutes. It gives 2 hours 35 minutes. Then 10 hours and 30 minutes minus 2 hours and 35 minutes. 10 hours minus 2 hours equal 8 hours then 35 minutes minus 30 minutes is equal to 5 minutes. Answer is 8 hours 5 minutes. To minus 15 minutes, I take 8 hours 5 minutes minus 15 minutes. Cannot minus therefore I change 8 hours to 7 hours and 5 minutes become 95 minutes.

Numbers Problem

R: *Can you go over each step of your working and tell me what were you thinking?*

S: *Let the 2 numbers be x and y. There x plus y =36 and x – y = 12. I don't think I am confident to solve this. Use trial and error. Don't think can solve.*

Solution

1. The solution obtained was correct.
2. The solution obtained was incorrect.

The following examples illustrate some of the many causes for incorrect solutions.

Time Problem

R: *Show me how you work out your answer? Explain to me what you are doing as you work out your answer.*

S: *10 hours 30 minutes minus 2 hours 25 minutes. 10 hours minus 2 hours equal 8 hours. 30 minutes minus 25 minutes equal 5 minutes. I have 8 hours and 5 minutes which 8.05pm. 8.05 minus 15 minutes equal 7.50 pm.*

Cat and Rabbit Problem

R: *Show me how you work out your answer. Explain to me what you are doing as you work out your answer.*

S: *9 metres minus 7 metres equal to 2 metres. 160 metres divide by 2 metres equal 80 metres. 80 times 9 equal 720 metres. A lot of working and steps. Quite fed up.*

Numbers Problem

R: *Show me how you work out your answer? Explain to me what you are doing as you work your answer.*

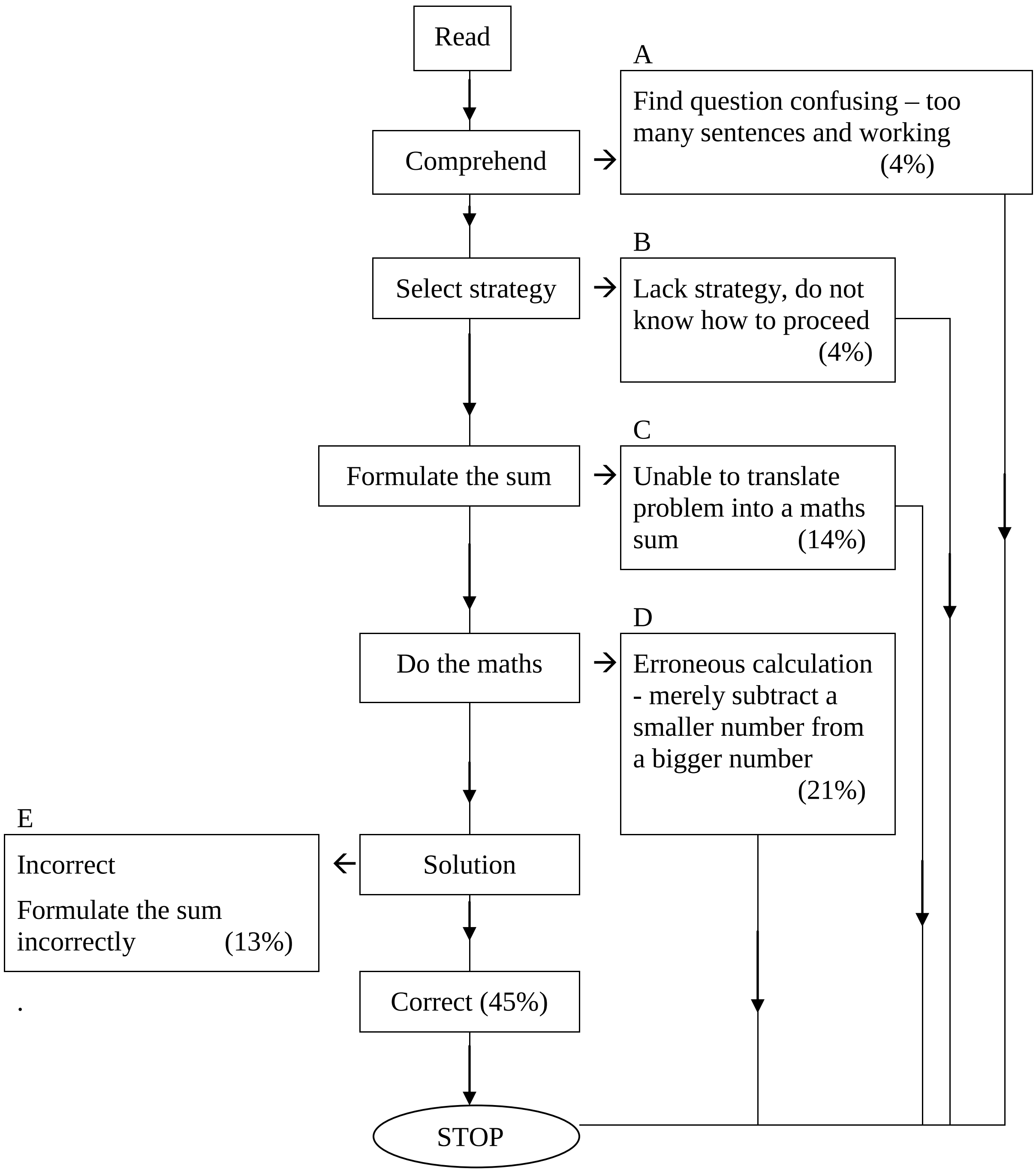
S: *x plus y equal to 36 and name it as equation (1). x minus y equal 12 and equation(2). y equal 36 minus x and call this equation (3). Substitute equation*

(3) into equation (2). We get x minus 36 minus x equal 12 (x - 36 – x = 12). x minus x equal 12 plus 36.

RESULTS AND DISCUSSION

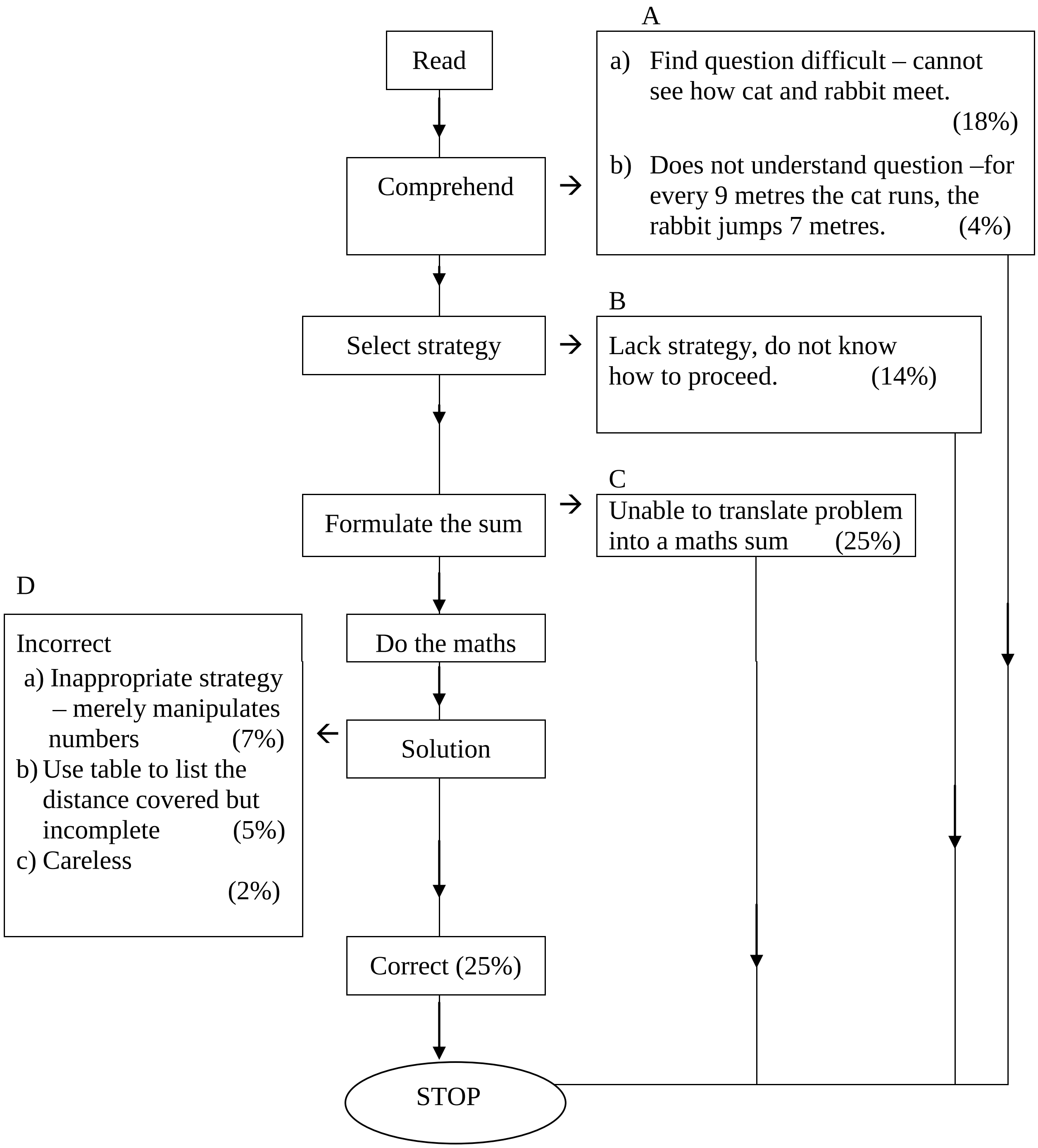
Figures 1, 2, and 3 show the flow charts and accompanying tables for the three problems: Time, Cat and Rabbit, and Numbers respectively. These flow charts provide a summary of the analysis of the interview data. They indicate at which stage the students were unable to proceed with the solution to a problem. The reasons found are also stated and the frequencies given as percentages. Each table gives the breakdown of the occurrences at the various stages for this group of 56 Secondary 2 students.

The table accompanying the flowchart in Figure 1 shows that the students were interviewed about their attempt to solve the Time problem. Of these 56 students, two (i.e., 4%) were classified as “A”. That is, they were unable to proceed beyond the ‘Comprehend’ stage for the reason that they found the question too confusing with “too many sentences and too many workings”. Two (i.e., 4%) were unable to proceed beyond the ‘Select strategy’ stage (‘B’), as they did not know how to proceed with the solution to the problem although they comprehended it. Eight (i.e., 14%) were unable to formulate the sum. The difficulties in using process skills were directly related to the problem rather than a general weakness in using this strategy. Of 12 (i.e., 21%) students, they merely subtracted a smaller number from a bigger number. Thirty-two (i.e., 58%) managed to obtain a solution to the problem. Of these 32 students, seven (i.e., 13%) obtained incorrect solutions and were classified as ‘E’; that is, they “formulated the sum incorrectly”. Similarly, for the other two problems, Figures 2 and 3 show the numbers and percentages of students at the various stages with the reasons for failure shown.



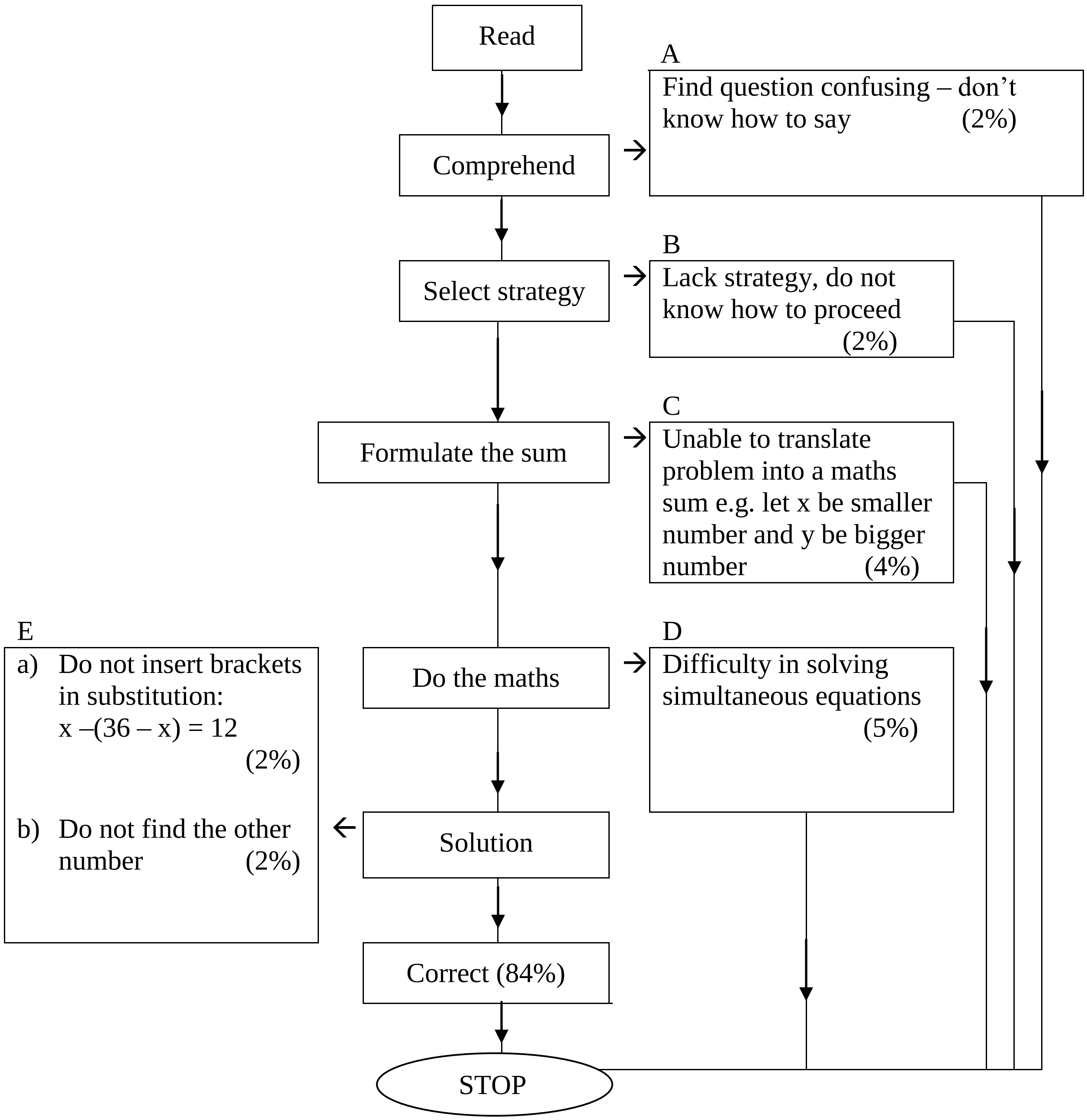
	N (%)					
N (%)	A	B	C	D	E	Correct
56 (100)	2 (4)	2 (4)	8 (14)	12 (21)	7 (13)	25 (45)

Figure 1 Time Problem



	N (%)							
N (%)	Aa	Ab	B	C	Da	Db	Dc	Correct
56 (100)	10 (18)	2 (4)	8 (14)	14 (25)	4 (7)	3 (5)	1 (2)	14 (25)

Figure 2 Cat and Rabbit Problem



	N (%)						
N (%)	A	B	C	D	Ea	Eb	Correct
56 (100)	1 (2)	1 (2)	2 (4)	3 (5)	1 (2)	1 (2)	47 (84)

Figure 3 Numbers Problem

It was observed during the interviews that students were in the habit of trying to solve the current problem using only one strategy. They did not demonstrate any flexibility by trying a strategy and if it did not work, trying another. Students who worked their solutions using an inappropriate strategy were often not aware that the solution was incorrect. Furthermore, students made no attempt to check that their solutions were correct or whether the solutions satisfied the conditions in the problem. The results of the data analysis showed that students were not successful at obtaining solutions for the following reasons:

Lack of comprehension of the problem posed

Some students were impeded in their progress in solving the problem as they did not comprehend the problem; for example, they found the problem confusing as "too many sentences and workings" were involved and did not "know how to say" what was involved in a problem. They were unable to visualise how the cat and rabbit met, and did not comprehend the problem at all.

Lack of strategy knowledge

Some students, who had no difficulty comprehending the problem, were impeded in their progress in solving the problem as they appeared to have no knowledge of ways in which an unfamiliar or non-routine as well routine problem might be approached.

Inability to translate the problem into a mathematical form

Some students, who had a strategy to solve the problem, were impeded in their progress in solving the problem by their inability to translate the problem into a mathematical form (equations or open sentences). For example, for the Time problem, one student knew he had to "work backward" to find the solution but was not able to begin to work using the correct timing. For the Cat and Rabbit problem, a student was unable to use the difference of the distance between the cat and the rabbit for further computation. As for the Numbers problem, a student was unable to form two linear equations.

Inability to use the correct mathematics

Some students, who were able to translate the problem, were impeded in their progress in solving the problem by their inability to use the correct mathematics to solve the problem. Some students identified an appropriate operation or sequence of operations but did not know the procedures necessary to carry out these operations accurately. For example, for the Time problem, a student merely subtracted a smaller number from a bigger number at the hour and minute place value. For the Number problem, one student who had correctly formed two linear equations was unable to solve the two equations simultaneously and did not grasp the procedures involved in solving two linear equations.

The results of the analysis of the data also showed that many times, the solution obtained by the students was incorrect and this could be attributed to the following reasons:

Inappropriate strategy used

The most common inappropriate strategy used to solve the three problems was number manipulation where students merely manipulated the data in the problem by trying to use the four operators (+, -, x, ÷) to arrive at an answer. The use of formulae and tables (incomplete listing) to find the solution to the Cat and Rabbit problem also resulted in a number of incorrect solutions.

Incorrect formulation of the mathematical form

Several students, while working the Time problem, formulated the sums incorrectly where they did not include the 10 minutes due to technical problems.

Computational errors

Several students obtained incorrect solutions owing to careless computations.

Imperfect mathematical knowledge

One student was not successful in obtaining the solution to the Number problem due to an imperfect knowledge of algebraic manipulations.

Misinterpretation of the problem

One student was not able to obtain the solution to the Number problem as he had interpreted that only one number was required to find in the problem.

Therefore, it may be inferred that the difficulties experienced by Secondary 2 students who were prevented from obtaining a correct solution were: (a) lack of comprehension of the problem posed, (b) lack of strategy knowledge, (c) inability to translate the problem into mathematical form, and (d) inability to use the correct mathematics. Students obtained incorrect solutions for the following reasons: (a) an inappropriate strategy used, (b) incorrect formulation of the mathematical form, (c) computational errors, (d) imperfect mathematical knowledge, and (e) misinterpretation of the problem.

Newman (1977) who developed a systematic procedure for analysing errors found that students made a highest proportion of errors on Process Skills, followed by errors on the following order: Comprehension, Carelessness, Reading, Transformation, and Encoding. Figures 1, 2, 3 also show that a high percentage of errors on Process Skills which is similar with the results of Newman's study. Thus, students in both countries seem to have difficulties in using correct processes while solving mathematical problems.

LIMITATIONS

Three major limitations are perceived in this study. Firstly, the difficulties which were discovered in this study may be somewhat limited owing to the problem items which were used in the data collection and hence it cannot be said that the above difficulties are in any way an exhaustive list. Secondly, the interview technique used to document students' difficulties when solving problems has some shortcomings as some students are not likely to be able to construct accounts of their behaviours. The possibility of interviewer bias in making inferences from the interview protocols analysed in this paper cannot be denied which could have been developed in such a study, depending on the position adopted by the investigator on the phenomena under discussion. Thirdly, the qualitative analysis in this paper which involved small samples of Secondary 2 students resulted in most data being used for descriptive purposes only. The aim was to look for behaviours that were both striking and capable of interpretation within the framework adopted for the study. An attempt to consider statistical tests of significance for all the results was unrealistic. For such samples, even if a difference reaches significance statistically, it may not be interpretable or of practical importance.

CONCLUSIONS

It was observed during the interviews that students were in the habit of attempting to solve the current problem using only one heuristic. They did not show any flexibility in seeking to solve the problems using more than one heuristic. This practice has implications for curriculum specialists and teacher educators. If the problem-solving curricula is to be successfully brought into the classrooms in Singapore schools, these practices may have to be carefully examined and mathematics teachers be made aware of how they can successfully implement mathematical problem solving in the classroom. The difficulties experienced by

Secondary 2 students have important implications for classroom teachers. The simple interview format used in this research is easy to implement and could be adapted and used by classroom teachers to analyse their students' difficulties and hence remediate their difficulties.

This study also shows that students must possess relevant knowledge and be able to coordinate their use of appropriate skills to solve problems. Furthermore, knowledge factors (Kroll & Miller, 1993) such as algorithmic knowledge, linguistic knowledge, conceptual knowledge, schematic knowledge and strategic knowledge are vital traits of problem-solving ability. For mathematics teachers to assist their students develop their problem-solving ability, it is essential that they aware of their difficulties first. As this study has shown that diagnostic interviewing (Lankford, 1974) can provided comprehensive knowledge of a student's thinking process. The interview responses are useful in that they can assist the mathematics teachers to focus on their students' difficulties during remediation.

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