Introduction to Digital Communications

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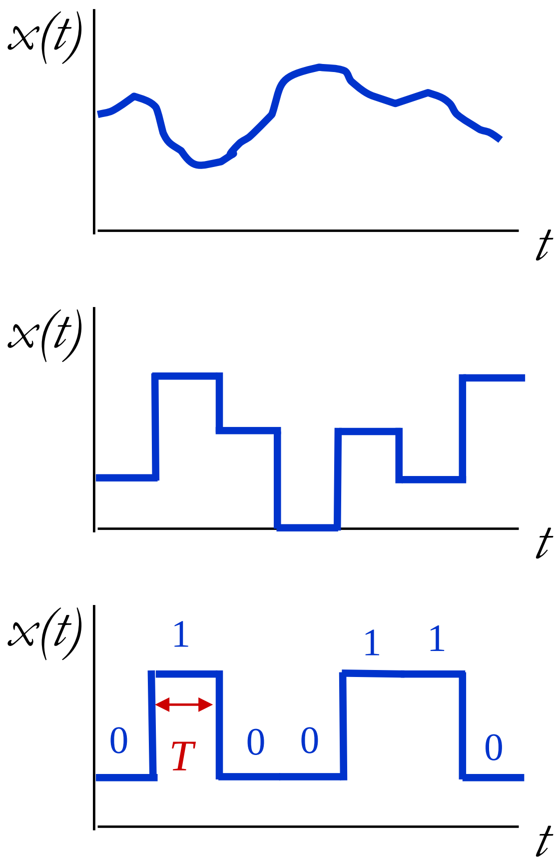
Analog vs. Digital

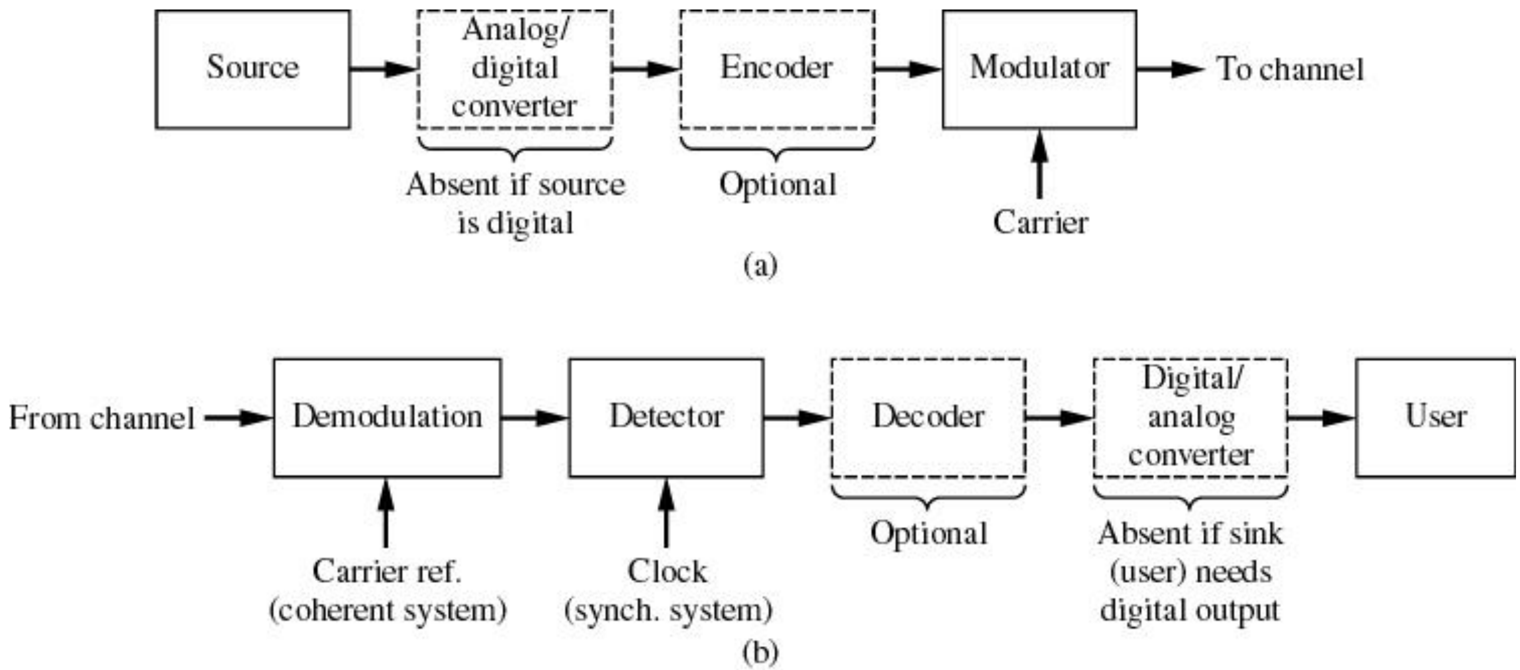
- Analog signals
  - Value varies continuously
- Digital signals
  - Values limited to a finite set
- Binary signals
  - Two valued
  - Time *T* needed to send 1 bit
  - Data rate *R=1/T* bits per second



Information Representation

- Communication systems must convert information into a form suitable for transmission
- Analog systems→Analog signals are directly modulated
  - AM, FM radio
- Digital systems → Generate bits and transmit digital signals
  - Computer communications, Cellular telephones
- Analog signals can be converted into digital signals

Digital Communication System



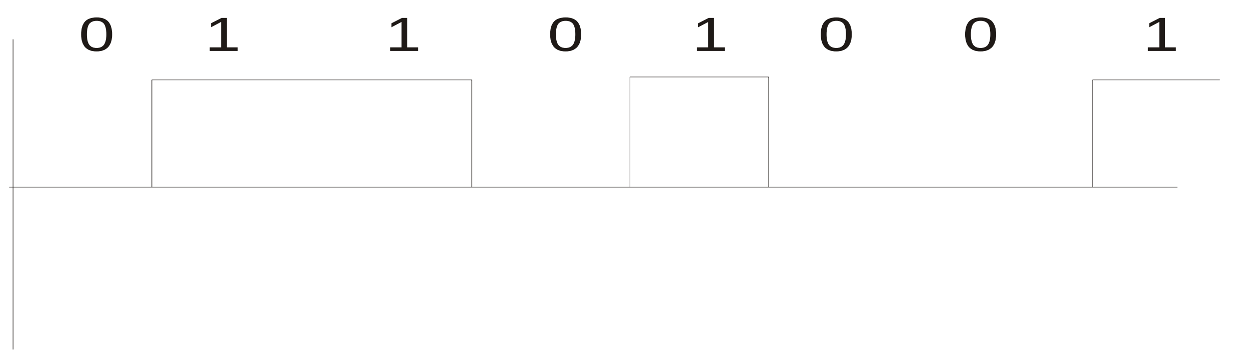
(a) Transmitter. (b) Receiver.

Digital Transmitter

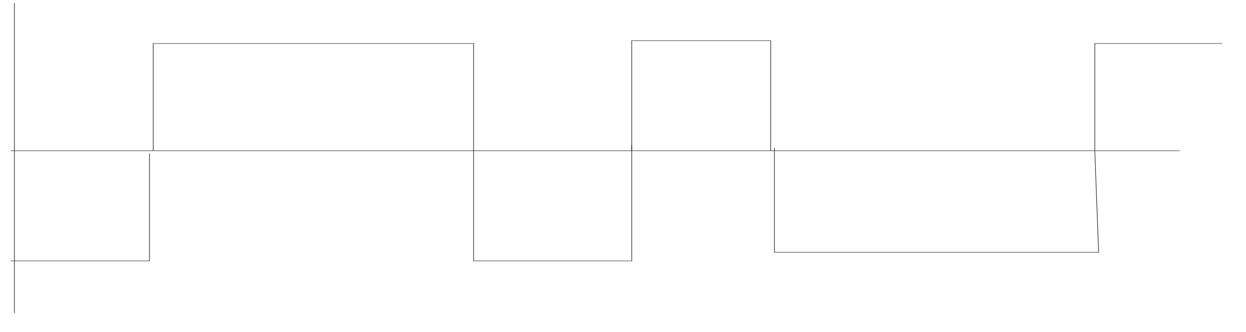
- Matches the message to the channel
- If the message is analog, it must be sampled in time and quantized in amplitude.
  - discrete signal in time and amplitude
- Encoder:
  - adds redundancy for error correction.
- Modulation encodes the message into the amplitude, phase or frequency of the carrier signal (PSK, FSK, QAM, OFDM, PAM, PPM)
- Advantages:
  - Reduces noise and interference
  - Multiplexing
  - Channel assignment
- Examples: television, radio, 802.11, cellphones, bluetooth, GPS, …

Modulation

- Convert the digital information into waveforms suitable for the channel
- OOK



- PAM



Receiver

- Extracts the message from the received signal
- Operations: Filtering, Amplification, Demodulation
- The ideal receiver output is a scaled, delayed version of the message signal
- Decoder:
  - estimates the original message from the received signal.

Channel

- Physical medium that that the signal is transmitted through
- Examples: Air, wires, coaxial cables, fiber optic cables
- Every channel introduces some amount of distortion, noise and interference
- The channel properties determine
  - Data throughput of the system
  - Quality of service (QoS) offered by the system

Noise and Interference

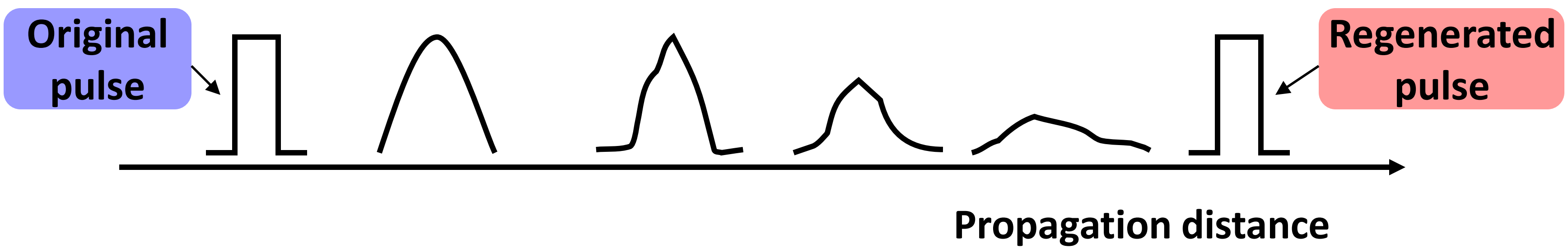
- Internal Noise
  - Generated by components within a communication system (thermal noise)
- External Noise and Interference
  - Atmospheric noise (electrical discharges)
  - Man-made noise (ignition noise)
  - Multipath interference (multiple transmission paths)
  - Multiple access interference (signals from other users)

Advantages

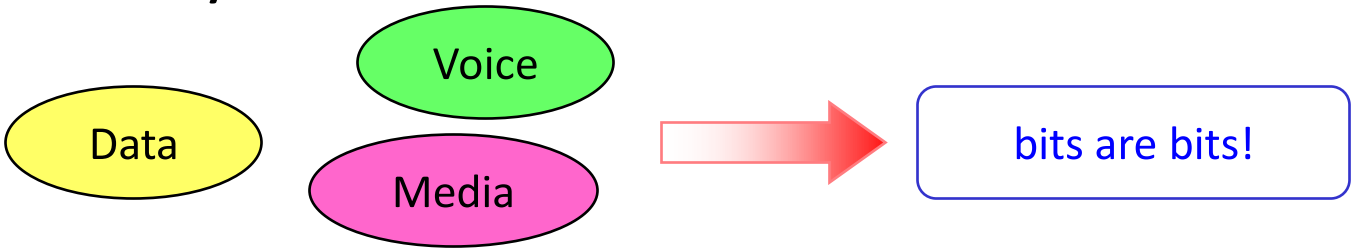
- Many sources are digital in nature
  - Data, images, text, video, music
- Different sources can be treated the same
- Flexibility
  - Encryption
  - Compression (source coding)
  - Error correction/detection
- Reliable reproduction of signals - regeneration
  - Two states vs. infinite variety of shapes
- Greater immunity to noise and interference
- Power efficient and spectral efficient

Digital versus Analog

- Advantages of digital communications
  - Regenerator receiver



  - Different kinds of digital signals can be treated identically.



Digital Advantages

- Source coding compression algorithms can dramatically reduce the bit rate required to represent signals without significant distortion.
- Signal processing and channel coding techniques have significantly increased the bit rate that can be supported by a physical channel.
- Integrated circuits make complex signal processing and coding functions cost effective.

Disadvantages

- Complex signal processing
- Synchronization problems
- Non-graceful degradation in performance as the SNR decreases

Performance Metrics

- In analog communications we want $\hat{m}(t) \cong m(t)$
- In digital communications
  - Data rate (*R* bps) (limited by the Channel Capacity)
  - Resources consumed: bandwidth, power
  - Quality of the communications link : typically measured in terms of the Bit Error Rate (BER) or probability of error, P_E
    - Number of bit errors that occur for a given number of bits transmitted.
    - Optical channels: $P_e = 10^{-9}$
    - Wireless channels: voice $P_e = 10^{-3}$ data $P_e = 10^{-6}$
  - Propagation and processing delay
  - Timing jitter in the bitstream at the receiver

Applications

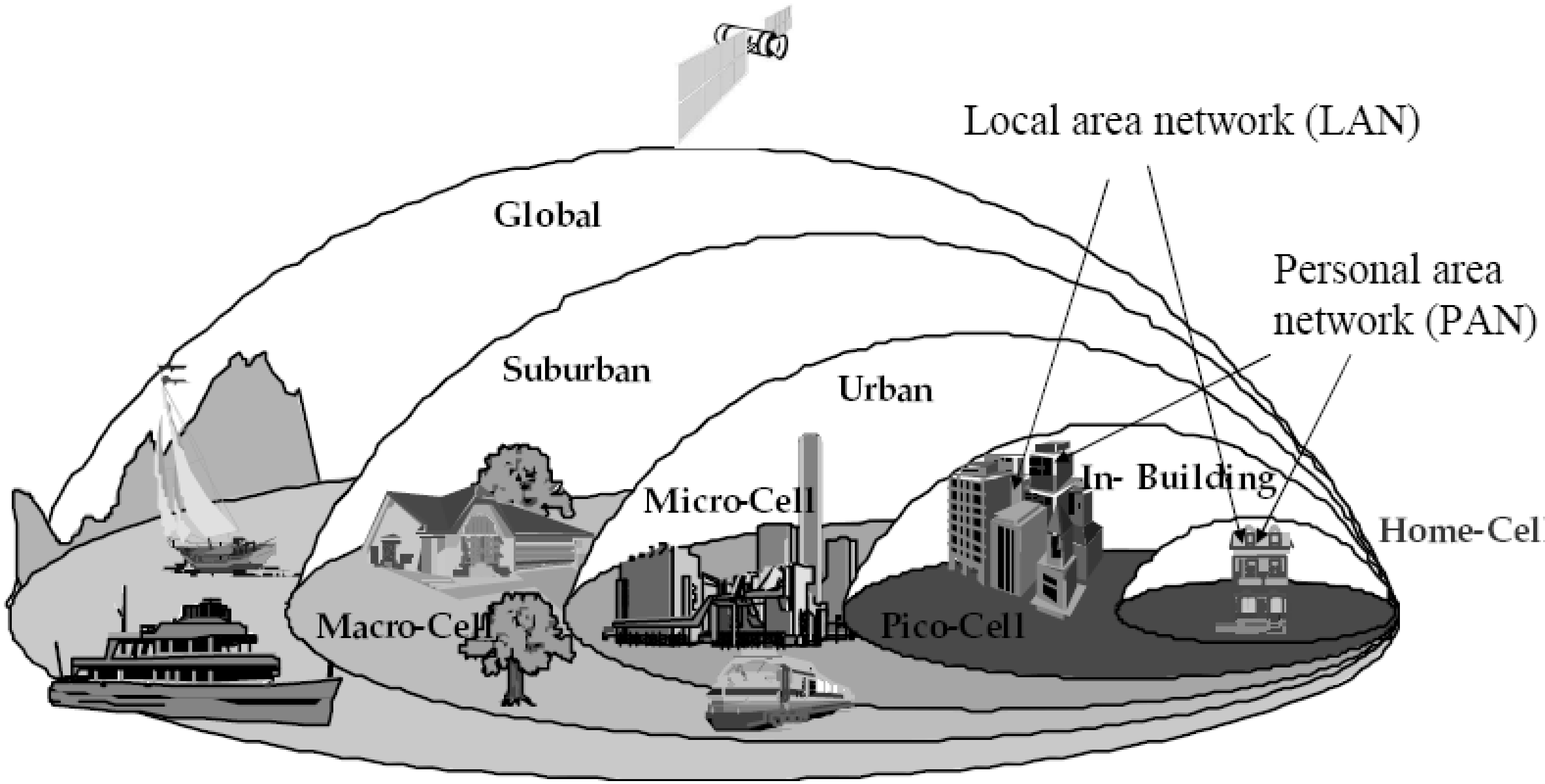
- Internet (last mile, VOIP)
- Local and long distance telephone channels
- Fibre optics (backbone and fibre to the home)
- Satellite communications (HDTV)
- CDs and DVDs
- Digital audio (mp3)

- Wireless Communications
- Cellular Communications
  - GSM, TDMA, FDMA, CDMA, 3G
- Wireless LANs (802.11)
- WiMAX
- Bluetooth (headsets)
- Cordless telephones

Goals of Digital Communications Design

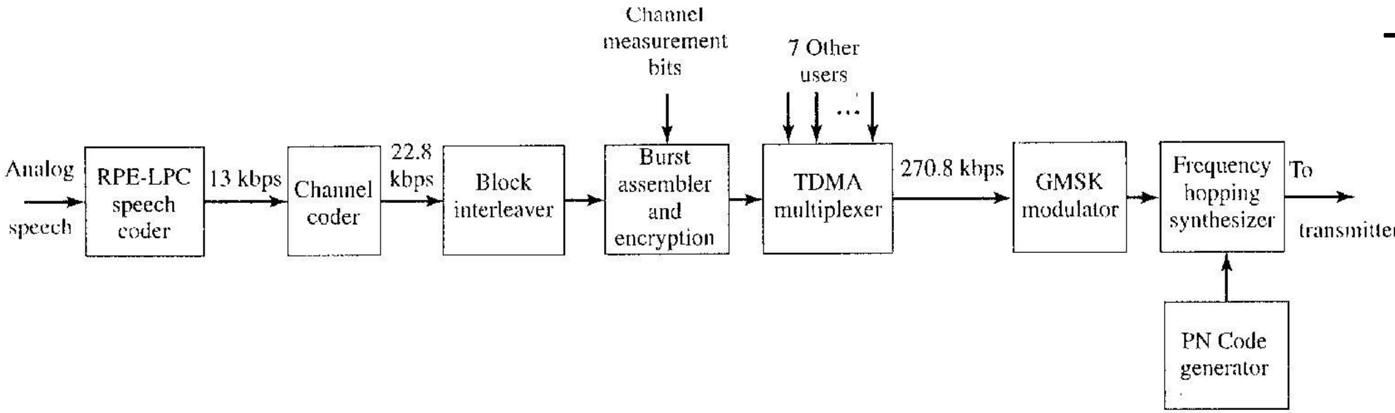
- Maximize bit rate R
- Minimize probability of error P_E
- Minimize required signal-to-noise ratio (SNR)
- Minimize required bandwidth W
- Maximize system utilization Capacity
- Minimize system complexity
- Minimize cost $

Span of Wireless Networks

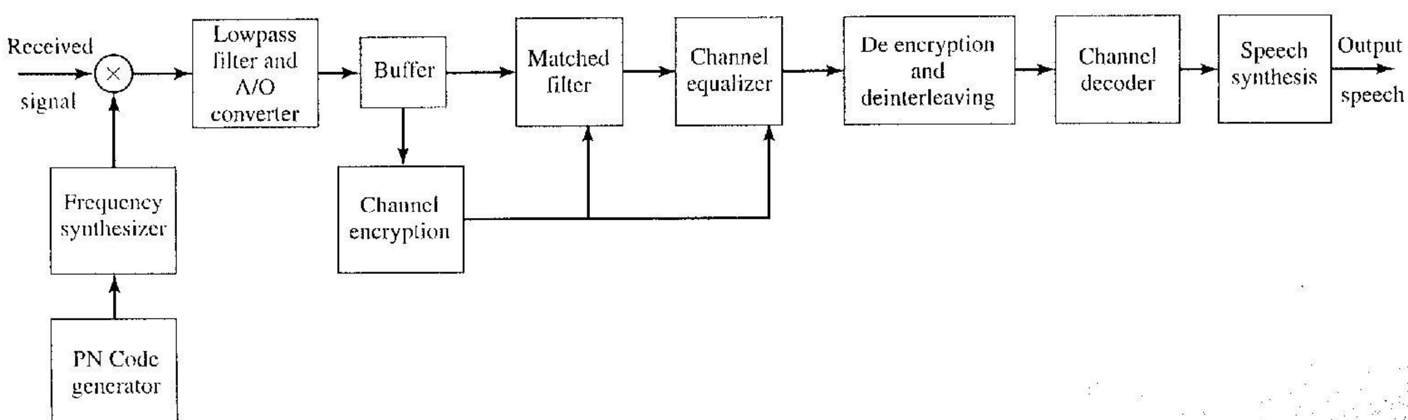


GSM Mobile Phone

Transmitter



(a)

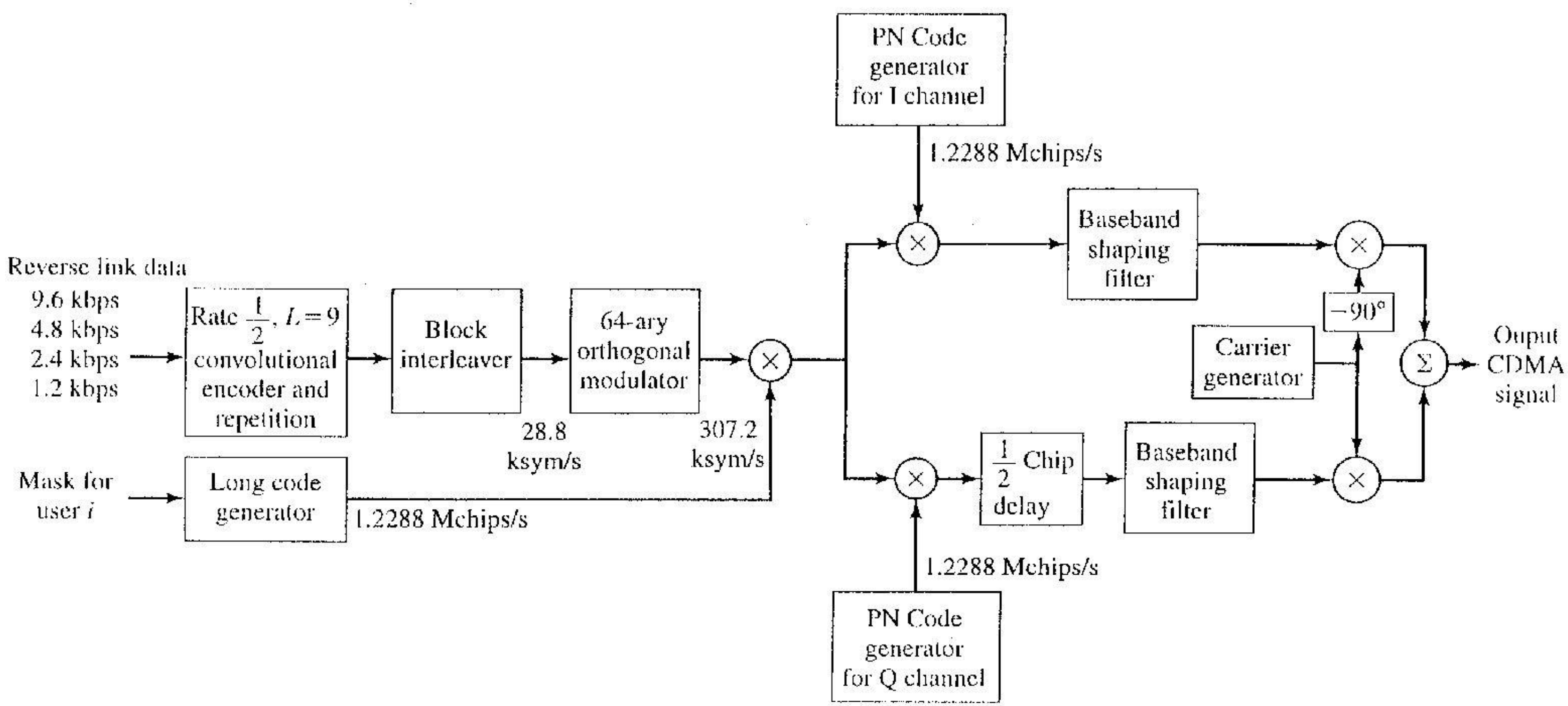


(b)

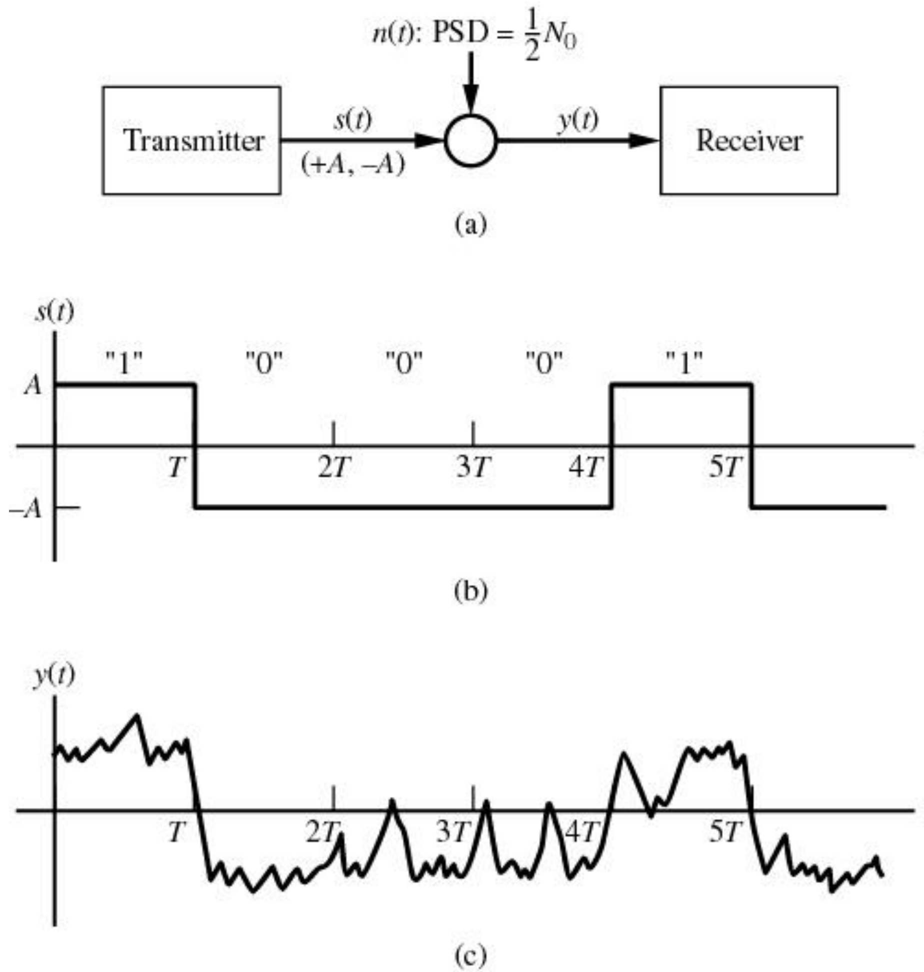
Receiver

CDMA Cell Phone

Mobile phone transmitter



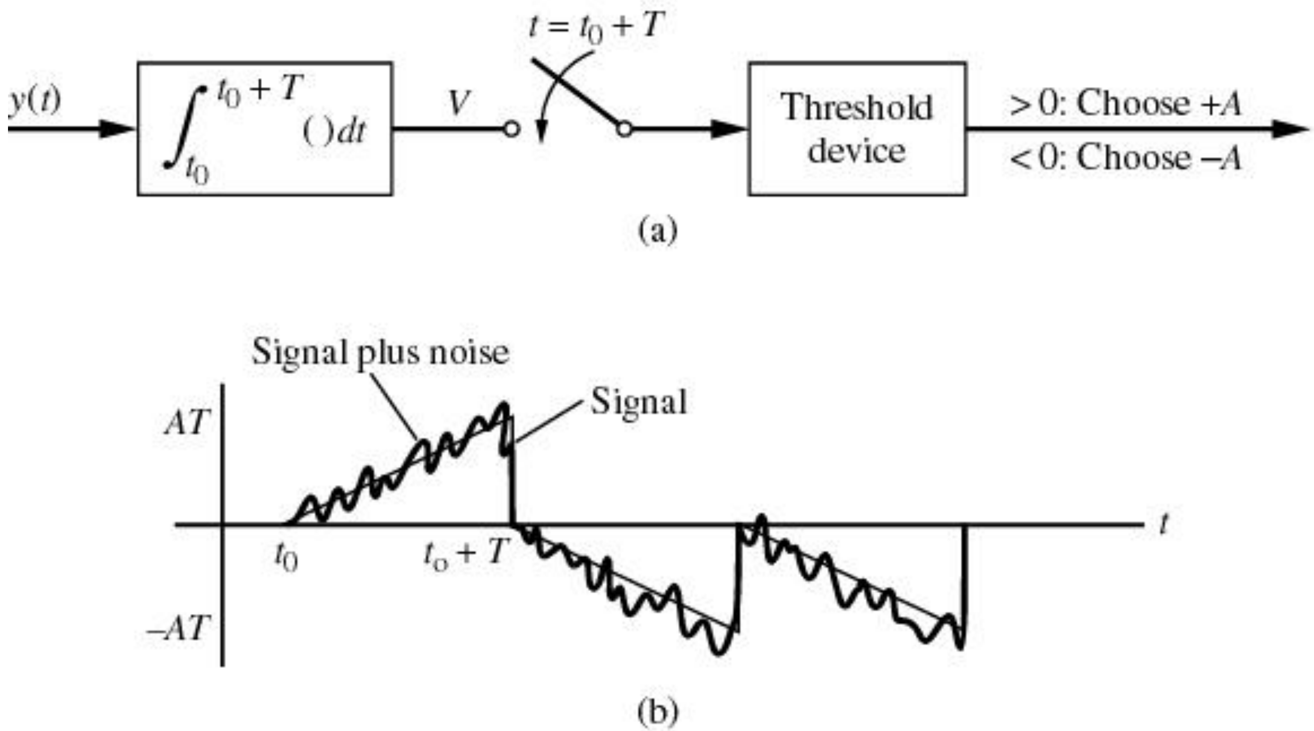
Baseband Data Transmission - PAM



System model and waveforms for synchronous baseband digital data transmission. (a) Baseband digital data communication system. (b) Typical transmitted sequence. (c) Received sequence plus noise.

- Each T second pulse represents a bit of data
- Receiver has to decide whether a 1 or 0 was received (A or –A)
- Integrate-and-dump detector

Receiver Structure



Receiver structure and integrator output. (a) Integrate-and-dump receiver. (b) Output from the integrator.

Receiver Preformance

- The output of the integrator is

$$V = \int_{t_0}^{t_0+T} [s(t) + n(t)]dt$$

$$= \begin{cases} AT + N & A \quad \text{is sent} \\ -AT + N & -A \quad \text{is sent} \end{cases}$$

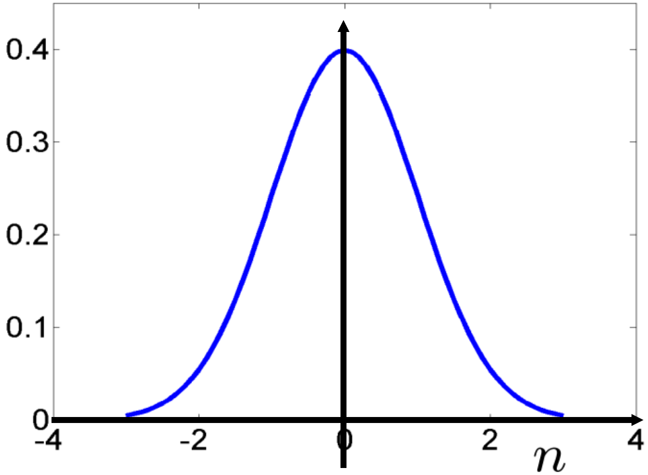
  V is a random variable
- *N* is Gaussian noise

$$N = \int_{t_0}^{t_0+T} n(t)dt$$

Noise in communication systems

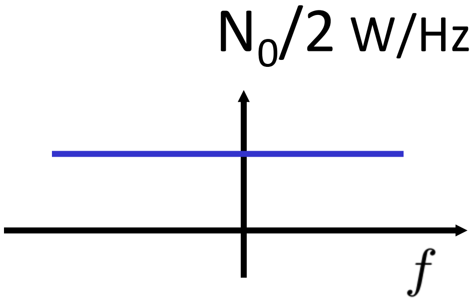
- Thermal noise is described by a zero-mean Gaussian random process, *n*(*t*).
- Its PSD is flat, hence, it is called white noise

$$f_N(n) = \frac{e^{-n^2/(2\sigma^2)}}{\sqrt{2\pi\sigma^2}}$$



Probability density function

Power spectral density



Analysis

$$E[N] = E[\int_{t_0}^{t_0+T} n(t)dt] = \int_{t_0}^{t_0+T} E[n(t)]dt = 0$$

$$Var[N] = E[N^2] - E^2[N]$$

$$= E[N^2] = E\left\{\left[\int_{t_0}^{t_0+T} n(t)dt\right]^2\right\}$$

$$= \int_{t_0}^{t_0+T}\int_{t_0}^{t_0+T} E[n(t)n(s)]dtds$$

$$= \int_{t_0}^{t_0+T}\int_{t_0}^{t_0+T} \frac{N_0}{2}\delta(t-s)dtds = \frac{N_0 T}{2}$$

since AWGN is uncorrelated

Error Analysis

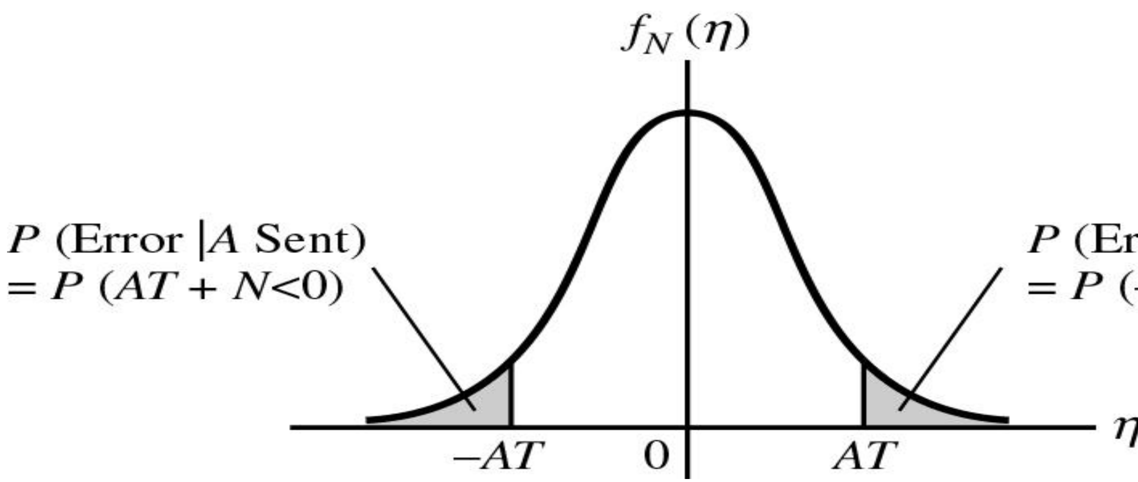
- The pdf of *N* is

$$f_N(n) = \frac{e^{-n^2/(N_0 T)}}{\sqrt{\pi N_0 T}}$$

- In how many different ways can an error occur?

Error Analysis

- Two ways in which errors occur
  - *A* is transmitted, $AT+N<0$ (0 received,1 sent)
  - -*A* is transmitted, $-AT+N>0$ (1 received,0 sent)



Error probabilities for binary signaling.

$$P(Error \mid A) = \int_{-\infty}^{-AT} \frac{e^{-n^2/N_0 T}}{\sqrt{\pi N_0 T}} dn = Q\left(\sqrt{\frac{2A^2T}{N_0}}\right)$$

- Similarly

$$P(Error \mid -A) = \int_{AT}^{\infty} \frac{e^{-n^2/N_0 T}}{\sqrt{\pi N_0 T}} dn = Q\left(\sqrt{\frac{2A^2T}{N_0}}\right)$$

- The average probability of error is

$$P_E = P(E \mid A)P(A) + P(E \mid -A)P(-A)$$

$$= Q\left(\sqrt{\frac{2A^2T}{N_0}}\right)$$

- Energy per bit

$$E_b = \int_{t_0}^{t_0+T} A^2 dt = A^2 T$$

- Therefore, P_E can be written in terms of the energy.
- Define

$$z = \frac{A^2 T}{N_0} = \frac{E_b}{N_0}$$

- Recall: Rectangular pulse of duration *T* seconds has magnitude spectrum

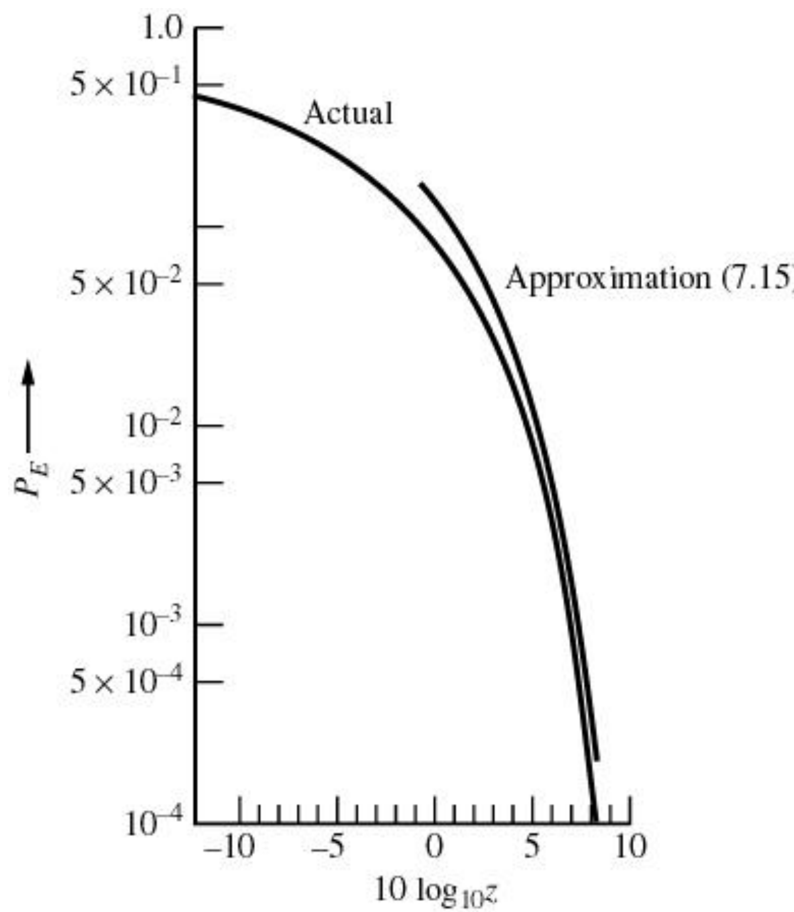
$$AT\mathrm{sinc}(Tf)$$

- Effective Bandwidth $B_p = 1/T$
- Therefore

$$z = \frac{A^2}{N_0 B_p}$$

Probability of Error vs. SNR

P_e for antipodal baseband digital signaling.



Probability of Error Approximation

- Use the approximation

$$Q(u) \cong \frac{e^{-u^2/2}}{u\sqrt{2\pi}}, u >> 1$$

$$P_E = Q\left(\sqrt{\frac{2A^2T}{N_0}}\right) \cong \frac{e^{-z}}{2\sqrt{\pi z}}, z >> 1$$

Example

- Digital data is transmitted through a baseband system with $N_0 = 10^{-7}$ W/Hz, the received pulse amplitude is A = 20mV.

a) If the transmission rate is 1kbps, what is the probability of error?

$$B_p = \frac{1}{T} = \frac{1}{10^{-3}} = 10^3$$

$$SNR = z = \frac{A^2}{N_0 B_p} = \frac{400 \times 10^{-6}}{10^{-7} \times 10^3} = 400 \times 10^{-2} = 4\text{V}$$

$$P_E \cong \frac{e^{-z}}{2\sqrt{\pi z}} = 2.58 \times 10^{-3}$$

b) If 10 kbps are transmitted, what must the value of A be to attain the same probability of error?

$$z = \frac{A^2}{N_0 B_p} = \frac{A^2}{10^{-7} \times 10^4} = 4 \Rightarrow A^2 = 4 \times 10^{-3} \Rightarrow A = 63.2\text{mV}$$

- Conclusion: tradeoff is

Transmission power vs. Bit rate

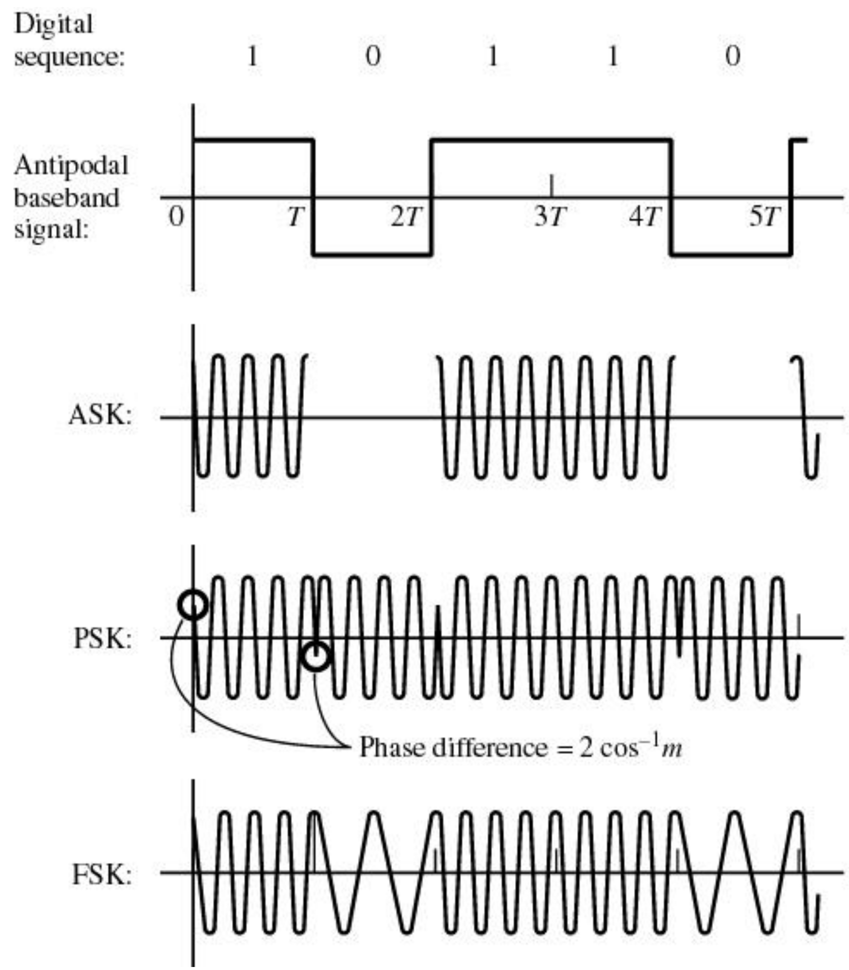
Bandpass Modulation

$$V(t) = A \cos(2\pi f_c t + \Phi)$$

- There are 3 parameters
  - Amplitude A(t) — Amplitude Modulation
  - Frequency f(t) — Frequency Modulation
  - Phase ϕ(t) — Phase Modulation

Binary Signaling Techniques

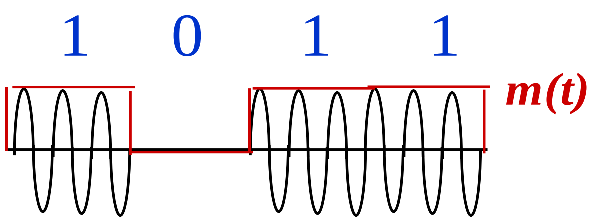
Waveforms for ASK, PSK, and FSK modulation.



ASK, PSK, and FSK

- Amplitude Shift Keying (ASK)

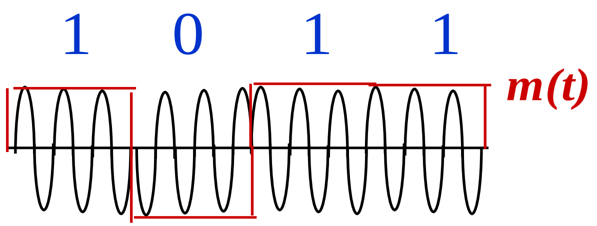
$$s(t) = m(t) A_c \cos(2\pi f_c t) = \begin{cases} A_c \cos(2\pi f_c t) & m(nT_b) = 1 \\ 0 & m(nT_b) = 0 \end{cases}$$



AM Modulation

- Phase Shift Keying (PSK)

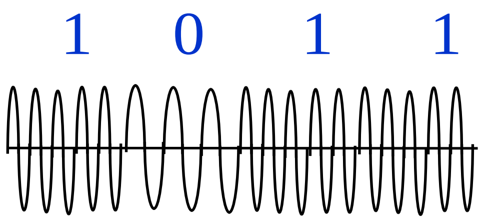
$$s(t) = A_c m(t) \cos(2\pi f_c t) = \begin{cases} A_c \cos(2\pi f_c t) & m(nT_b) = 1 \\ A_c \cos(2\pi f_c t + \pi) & m(nT_b) = -1 \end{cases}$$



PM Modulation

- Frequency Shift Keying

$$s(t) = \begin{cases} A_c \cos(2\pi f_1 t) & m(nT_b) = 1 \\ A_c \cos(2\pi f_2 t) & m(nT_b) = -1 \end{cases}$$



FM Modulation

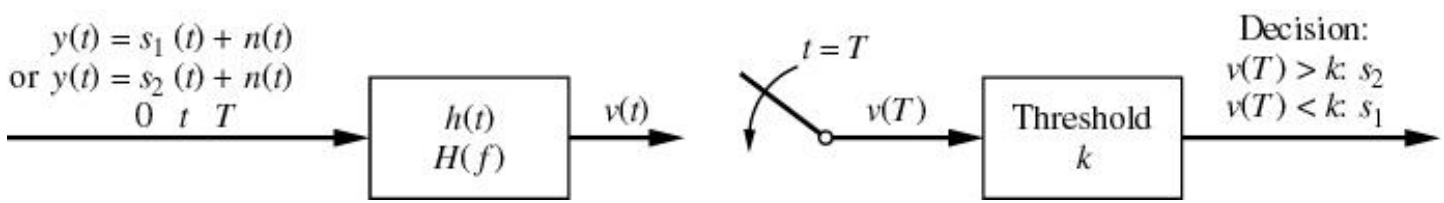
Amplitude Shift Keying (ASK)

$$0 \rightarrow 0$$

$$1 \rightarrow A\cos(2\pi f_c t)$$

What is the structure of the optimum receiver?

Receiver for Binary Signals in Noise



Receiver structure for detecting binary signals in additive white Gaussian noise (AWGN)

Error Analysis

- 0→s1(t), 1→s2(t)
- Received signal:

$$y(t) = s_1(t) + n(t), t_0 \le t \le t_0 + T$$

or

$$y(t) = s_2(t) + n(t), t_0 \le t \le t_0 + T$$

- Noise is white and Gaussian
- Find P_E
  - In how many different ways can an error occur?

Error Analysis (General Case)

- Two types of errors:
  - Receive 1 → Send 0
  - Receive 0 → Send 1
- Decision process:
  - The received signal is filtered
  - Filter output is sampled every *T* seconds
  - Threshold *k*
  - An error occurs when: $v(T) = s_{01}(T) + n_0(T) > k$

or

$$v(T) = s_{02}(T) + n_0(T) < k$$

- s_{01}, s_{02}, n_0 are filtered signal and noise terms.
- Noise term: $n_o(t)$ is filtered white Gaussian noise.
  - therefore it is Gaussian
- The PSD is

$$S_{n_0}(f) = \frac{N_0}{2} \left| H(f) \right|^2$$

  - mean zero
  - variance is equal to the average power of the noise process

$$\sigma^2 = \int_{-\infty}^{\infty} \frac{N_0}{2} \left| H(f) \right|^2 df$$

- The pdf of the noise term is

$$f_N(n) = \frac{e^{-n^2/2\sigma^2{}_0}}{\sqrt{2\pi\sigma^2}}$$

- Note that we still don't know what the filter is.
- Will any filter work? Or is there an optimal one?
- Recall that in the baseband case (no modulation), we used an integrator
  - equivalent to filtering with $H(f) = \frac{1}{j2\pi f}$

- The input to the threshold device is

$$V = v(T) = s_{01}(T) + N$$

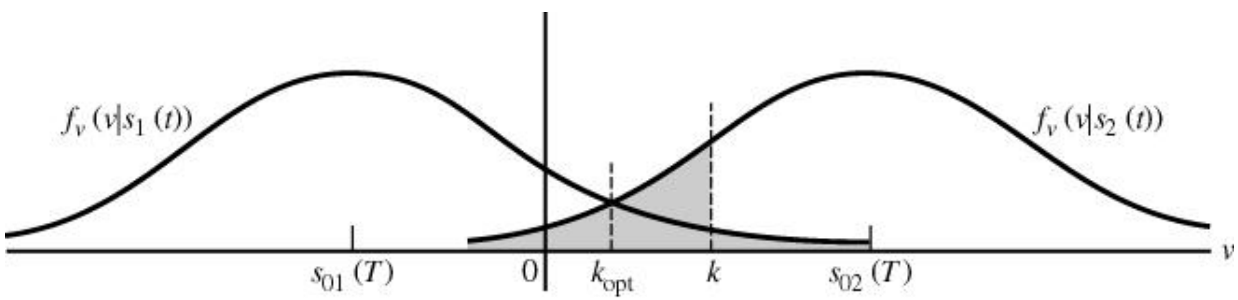
or

$$V = v(T) = s_{02}(T) + N$$

- These are also Gaussian random variables
  - mean: $s_{01}(T)$ or $s_{02}(T)$
  - variance: same as the variance of *N*

Distribution of *V*

- The distribution of *V*, the input to the threshold device is



Conditional probability density functions of the filter output at time $t = T$

Probability of Error

- Two types of errors

$$P(E \mid s_1(t)) = \int_k^{\infty} \frac{e^{-[v-s_{01}(T)]^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}} dv = Q\left(\frac{k - s_{01}(T)}{\sigma}\right)$$

$$P(E \mid s_2(t)) = \int_{-\infty}^{k} \frac{e^{-[v-s_{02}(T)]^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}} dv = 1 - Q\left(\frac{k - s_{02}(T)}{\sigma}\right)$$

- The average probability of error

$$P_E = \frac{1}{2} P[E \mid s_1(t)] + \frac{1}{2} P[E \mid s_2(t)]$$

- Goal: Minimize the average probability of error
  - choose the optimal threshold
- What should the optimal threshold, k_{opt} be?
  - $k_{opt}=0.5[s_{01}(T)+s_{02}(T)]$

$$P_E = Q\left(\frac{s_{02}(T)-s_{01}(T)}{2\sigma}\right)$$

Observations

- P_E is a function of the difference between the two signals.
- Recall: Q-function decreases with increasing argument.
- Therefore, P_E will decrease with increasing distance between the two output signals
- Choose the filter $h(t)$ such that P_E is a minimum
  – maximize the difference between the two signals at the output of the filter

Matched Filter

- Goal: Given $s_1(t), s_2(t)$, choose *H*(f) such that

$$d = \frac{s_{02}(T) - s_{01}(T)}{\sigma}$$

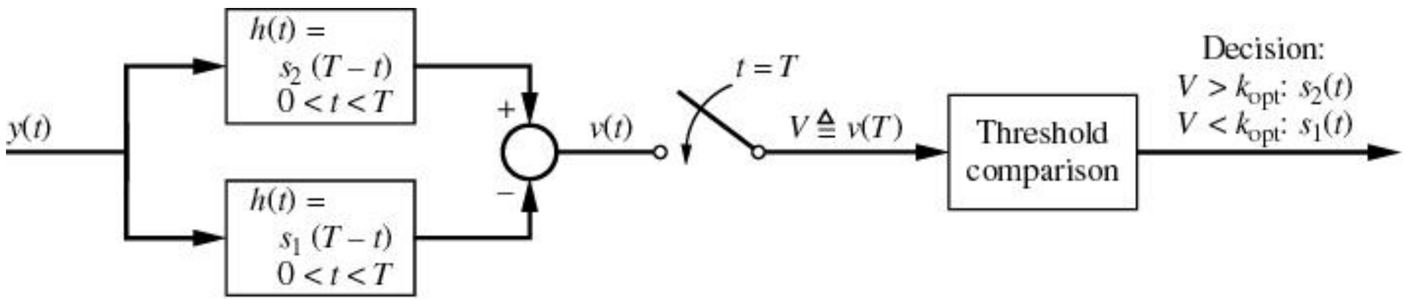
is maximized.

- The solution to this problem is known as the matched filter and is given by

$$h_0(t) = s_2(T-t) - s_1(T-t)$$

- Therefore, the optimum filter depends on the input signals.

Matched Filter Receiver



Matched filter receiver for binary signaling in additive white Gaussian noise (AWGN).

Error Probability for Matched Filter Receiver

- Recall $P_E = Q\left(\frac{d}{2}\right)$

- The maximum value of the distance is

$$d_{\max}{}^2 = \frac{2}{N_0}(E_1 + E_2 - 2\sqrt{E_1 E_2}\,\rho_{12})$$

- E_1 is the energy of the first signal
- E_2 is the energy of the second signal

$$E_1 = \int_{t_0}^{t_0+T} s_1{}^2(t)dt \qquad E_2 = \int_{t_0}^{t_0+T} s_2{}^2(t)dt$$

$$\rho_{12} = \frac{1}{\sqrt{E_1 E_2}} \int_{-\infty}^{\infty} s_1(t)s_2(t)dt$$

- Therefore

$$P_E = Q\left[\left(\frac{E_1 + E_2 - 2\sqrt{E_1 E_2}\,\rho_{12}}{2N_0}\right)^{1/2}\right]$$

- Probability of error depends on the signal energies (just as in the baseband case), noise power, and the similarity between the signals.
- If we make the transmitted signals as dissimilar as possible, then the probability of error will decrease.
- This is achieved with

$$\rho_{12} = -1$$

ASK

$$s_1(t) = 0, s_2(t) = A\cos(2\pi f_c t)$$

- The matched filter: $A\cos(2\pi f_c t)$
- Optimum Threshold: $\frac{1}{4}A^2T$
- Similarity between signals?
- Therefore

$$P_E = Q\left(\sqrt{\frac{A^2T}{4N_0}}\right) = Q\left(\sqrt{z}\right)$$

- 3dB worse than baseband.

PSK

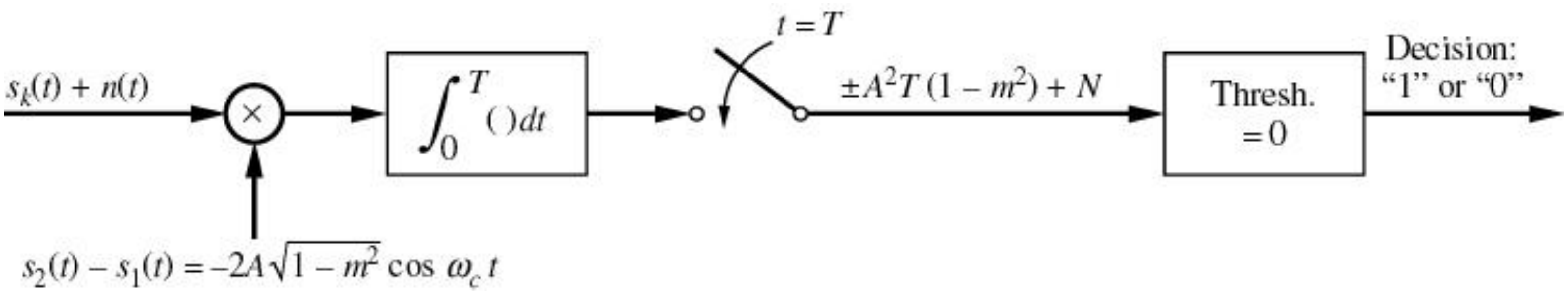
$$s_1(t) = A\sin(2\pi f_c t + \cos^{-1} m)$$
$$s_2(t) = A\sin(2\pi f_c t - \cos^{-1} m)$$

- Modulation index: m (determines the phase shift) $\quad -2A\sqrt{1-m^2}\cos(2\pi f_c t)$
- Matched Filter with threshold 0

$$P_E = Q(\sqrt{2(1-m^2)z})$$

- For *m* = 0, 3dB better than ASK

Matched Filter for PSK



Optimum correlation receiver for PSK.

FSK

$$s_1(t) = A\cos(2\pi f_c t)$$
$$s_2(t) = A\cos(2\pi(f_c + \Delta f)t)$$

- $\Delta f = \dfrac{m}{T}$
- Probability of error: $Q(\sqrt{z})$
- Same as ASK