INSTITUTE OF AERONAUTICAL ENGINEERING
(Autonomous)
Dundigal, Hyderabad – 500 043

MECHANICAL ENGINEERING

PPT ON

KINEMATICS OF MACHINERY

IV SEMESTER

IARE-R16

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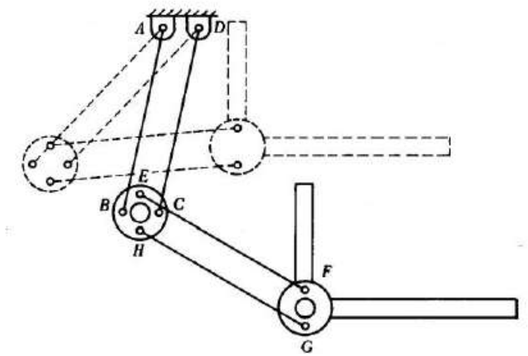
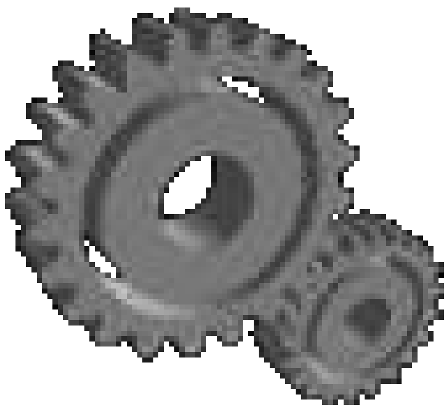
UNIT-I

MACHINE

A machine is a mechanism or collection of mechanisms, which transmit force from the source of power to the resistance to be overcome.

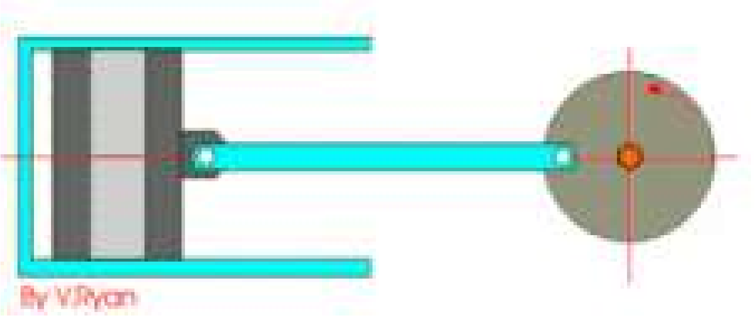
Though all machines are mechanisms, all mechanisms are not machines





DYNAMIC/INERTIA LOAD

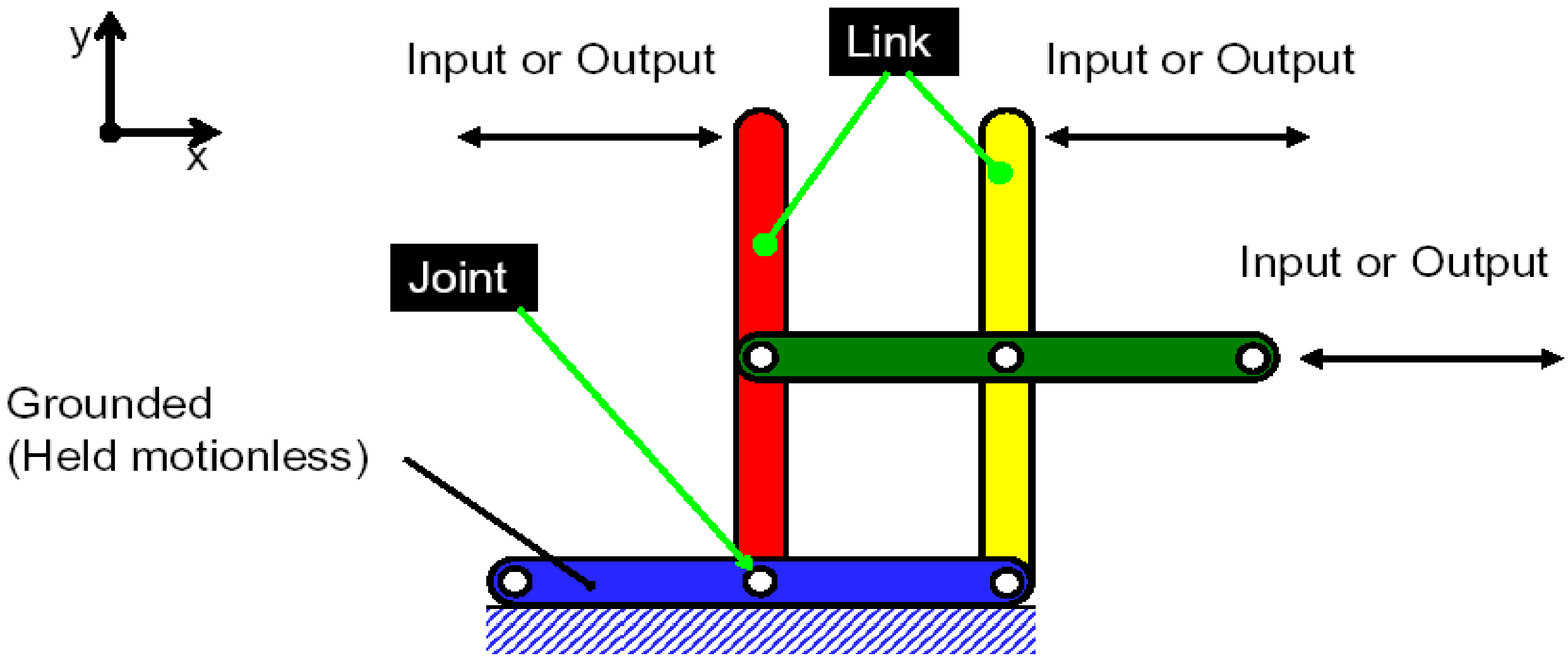
Inertia load require acceleration



What is a linkage?

What is a linkage?

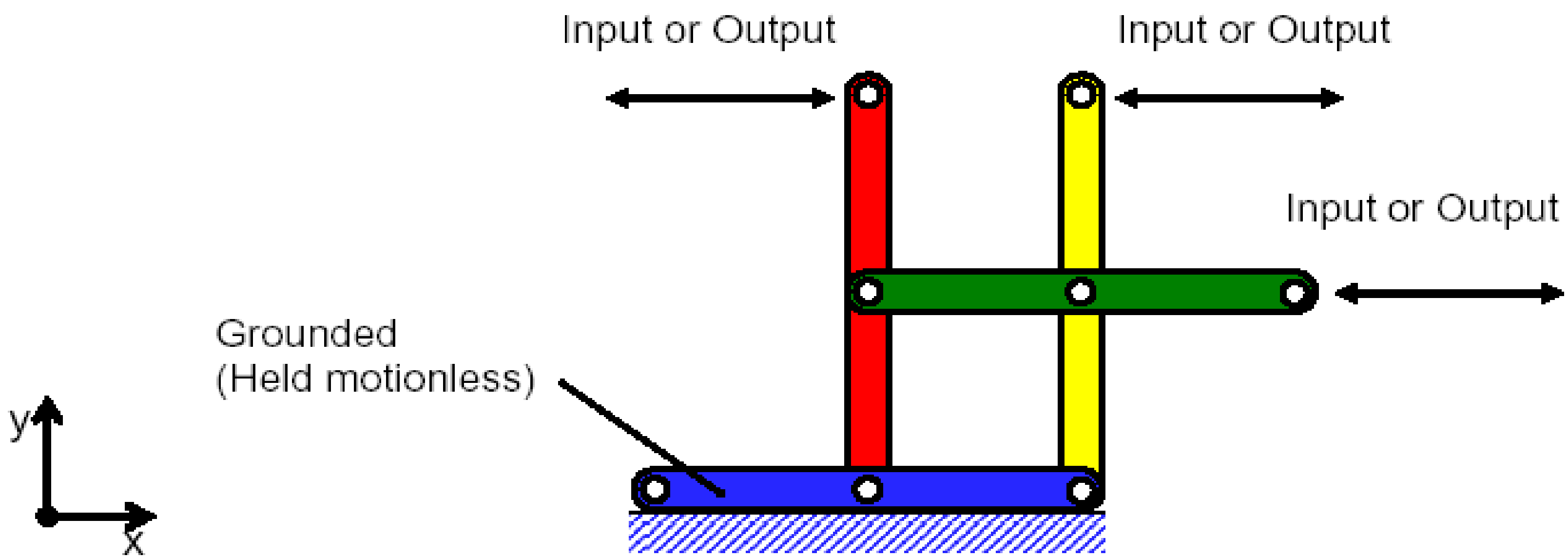
A mechanism used to define motion (kinematics) &/or transfer energy using links & joints



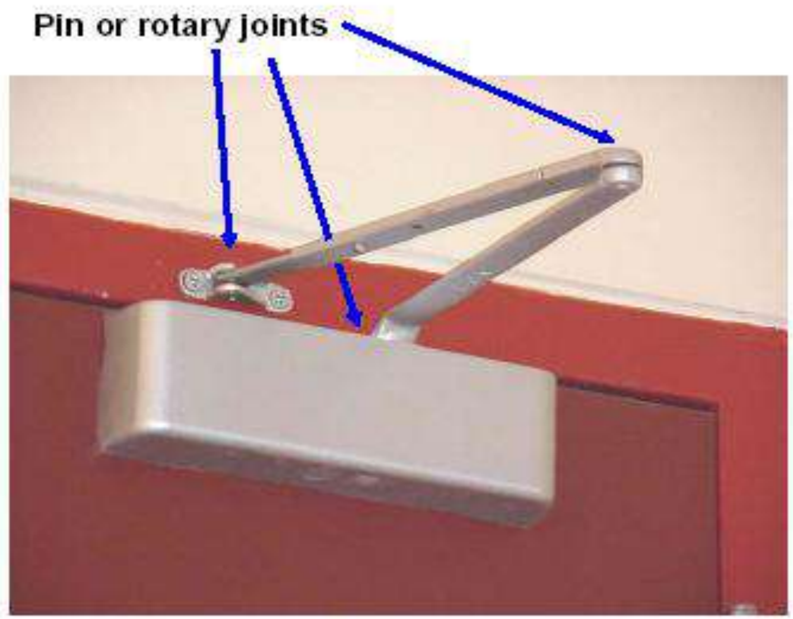
Linkage classification

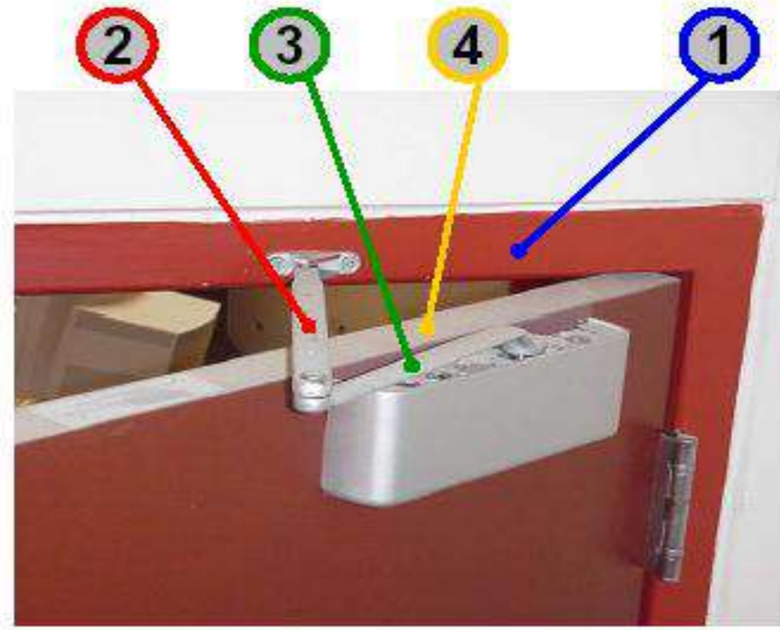
Linkages are classified by:

- The number of links (we will deal with 4-bar linkages)
- Type of links (we will deal with bars and sliders)
- Connection between links (we will deal with pinned, spherical and sliding joints)



Example: 4 bar door damper linkage



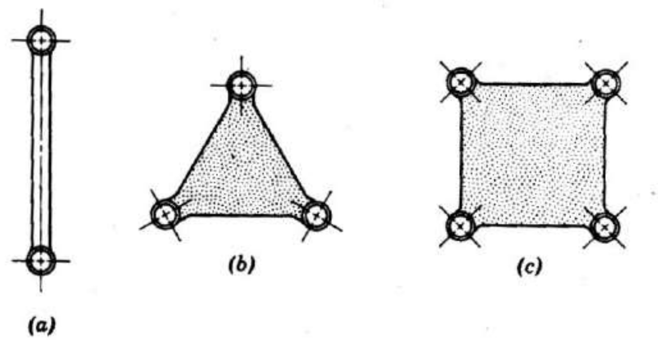


1	= Wall	or	Link 1	This is the grounded (held still)
2	= Bar 2	or	Link 2	
3	= Bar 3	or	Link 3	
4	= Door	or	Link 4	

LINK OR ELEMENT

Any body (normally rigid) which has motion relative to another

- **Binary link**
- **Ternary link**
- **Quaternary link**



KINEMATIC PAIRS

- A mechanism has been defined as a combination so connected that each moves with respect to each other. A clue to the behavior lies in in the nature of connections, known as kinetic pairs. The degree of freedom of a kinetic pair is given by the number independent coordinates required to completely specify the relative movement.

TYPES OF KINEMATIC PAIRS

Based on nature of contact between elements

- (i) **Lower pair** : The joint by which two members are connected has surface contact. A pair is said to be a lower pair when the connection between two elements are through the area of contact. Its 6 types are

Revolute(Or)TurningPair

Prismatic(Or)SlidingPair

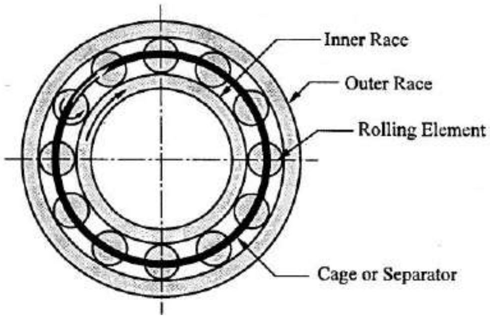
Screw(Or)HelicalPair

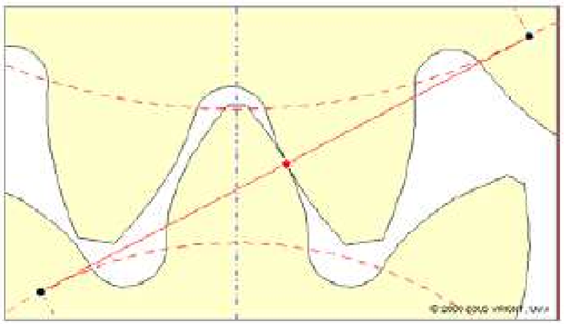
CylindricalPair

Spherical(Or)GlobularPair

Flat(or)PlanarPair

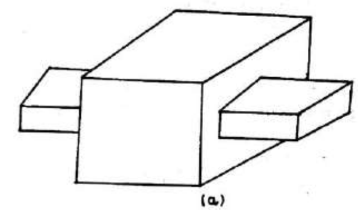
(ii) Higher pair: The contact between the pairing elements takes place at a point or along a line.



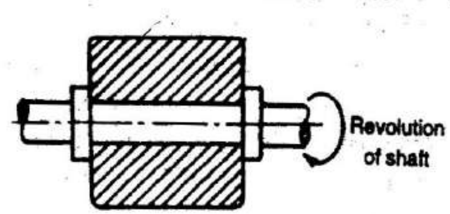


Based on relative motion between pairing elements

(a) Siding pair [DOF = 1]

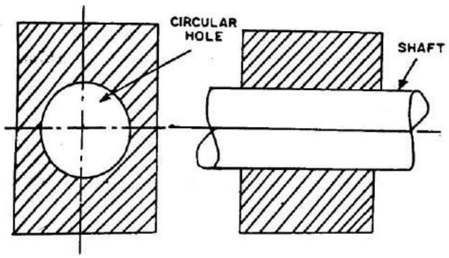


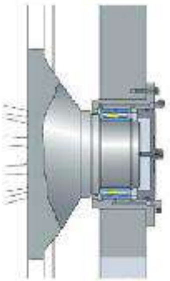
(b) Turning pair (revolute pair) [DOF = 1]



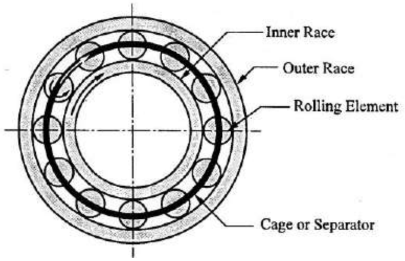
Based on relative motion between pairing elements

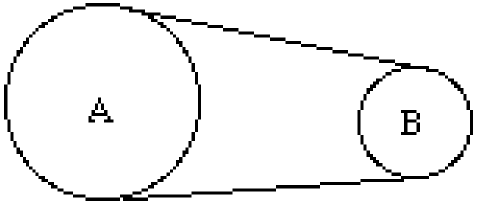
(c) Cylindrical pair [DOF = 2]





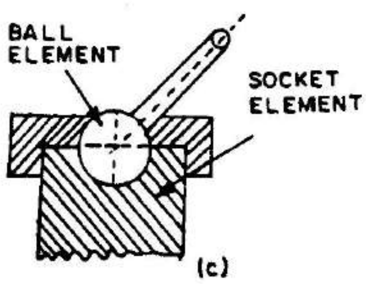
(d) Rolling pair [DOF = 1]

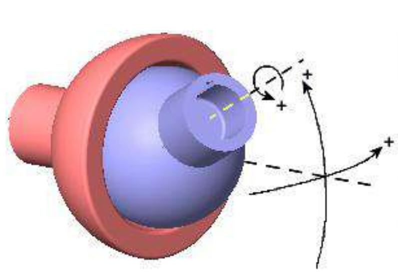


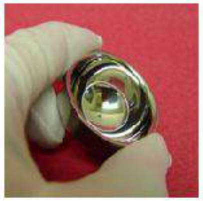


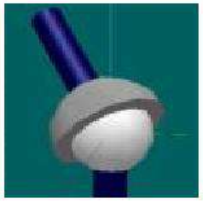
Based on relative motion between pairing elements

(e) Spherical pair [DOF = 3]

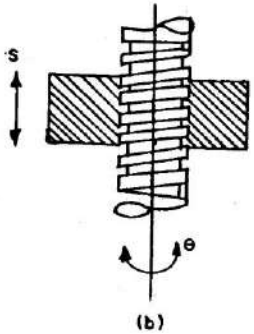


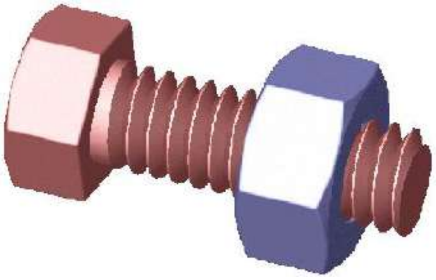






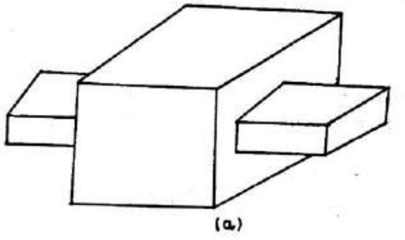
(f) Helical pair or screw pair [DOF = 1]



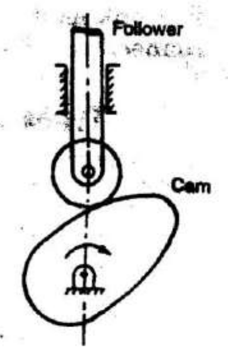


Based on the nature of mechanical constraint

(a) Closed pair



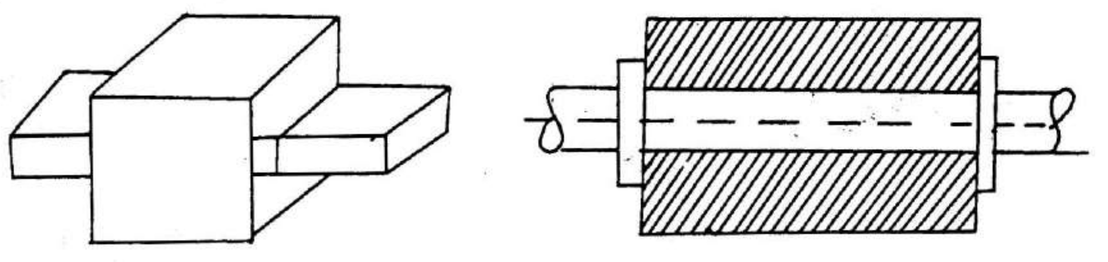
(b) Unclosed or force closed pair



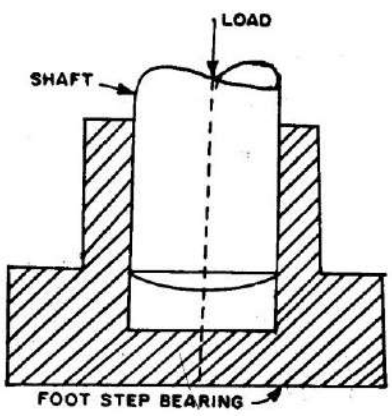
CONSTRAINED MOTION

one element has got only one definite motion relative to the other

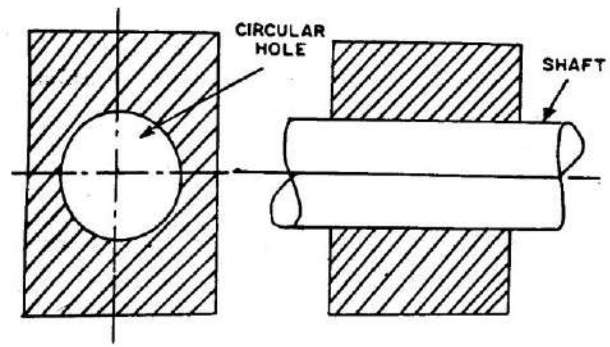
(a) Completely constrained motion



(b) Successfully constrained motion



(c) Incompletely constrained motion



KINEMATIC CHAIN

Group of links either joined together or arranged in a manner that permits them to move relative to one another.

Kinematic Chain

<u>Relation between Links, Pairs and Joints</u>

$L=2P-4$

$J=(3/2)\,L-2$

L => No of Links

P => No of Pairs

J => No of Joints

L.H.S > R.H.S => Locked chain

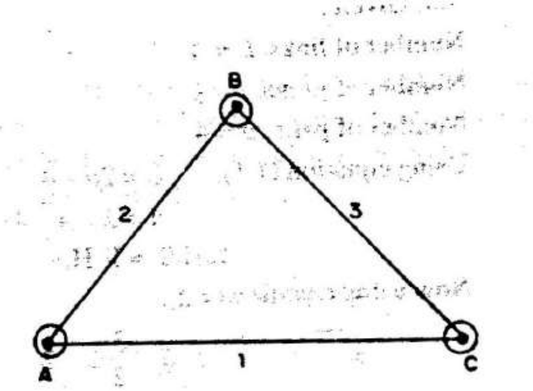
L.H.S = R.H.S => Constrained Kinematic Chain

L.H.S < R.H.S => Unconstrained Kinematic Chain

LOCKED CHAIN (Or) STRUCTURE

Links connected in such a way that no relative motion is possible.

L=3, J=3, P=3 L.H.S>R.H.S

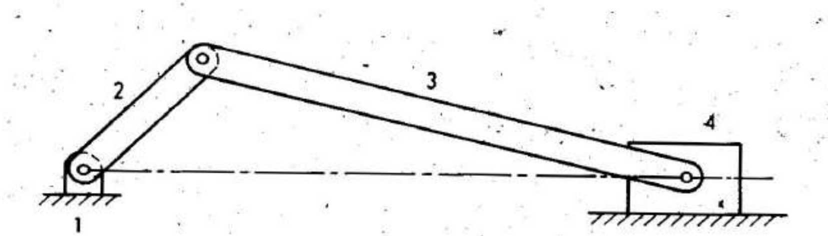


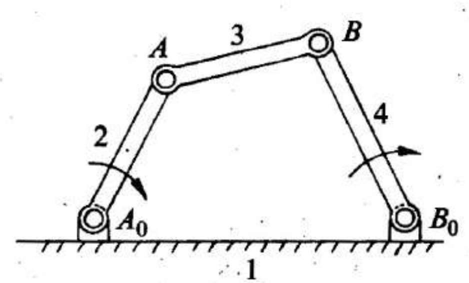
Kinematic Chain Mechanism

Slider crank and four bar mechanisms

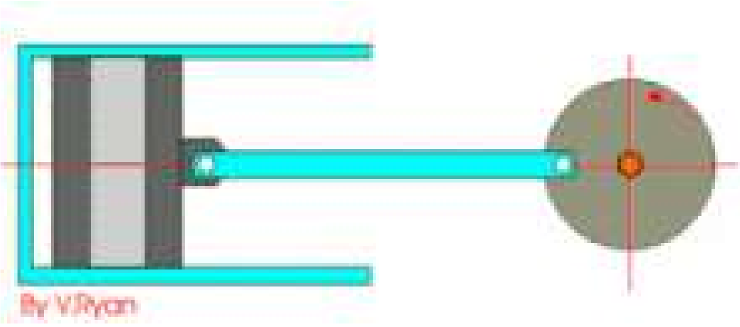
L=4, J=4, P=4

L.H.S=R.H.S





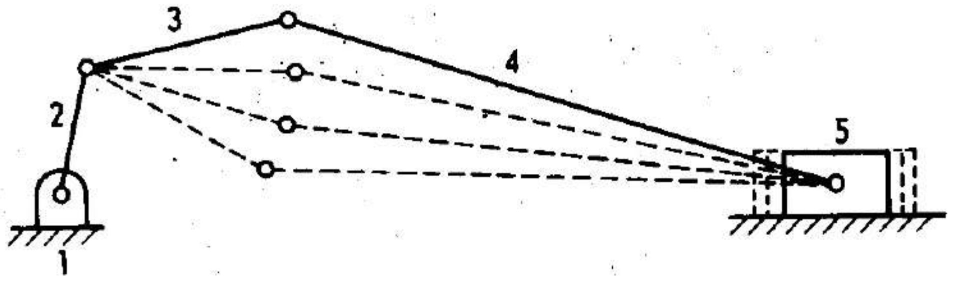
Working of slider crank mechanism



Unconstrained kinematic chain

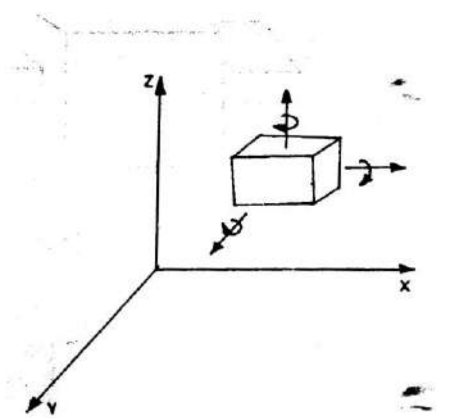
L=5,P=5,J=5

L.H.S < R.H.S



DEGREES OF FREEDOM (DOF):

It is the number of independent coordinates required to describe the position of a body.



Degrees of freedom/mobility of a mechanism

It is the number of inputs (number of independent coordinates) required to describe the configuration or position of all the links of the mechanism, with respect to the fixed link at any given instant.

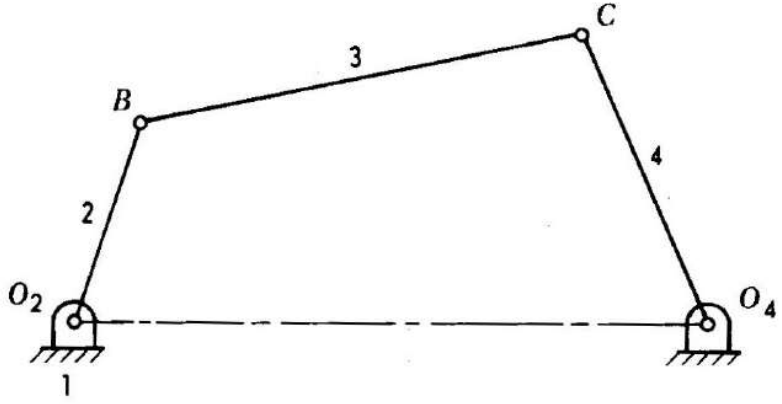
GRUBLER'S CRITERION

Number of degrees of freedom of a mechanism is given by

$$F = 3(n-1)-2l-h.$$ Where,

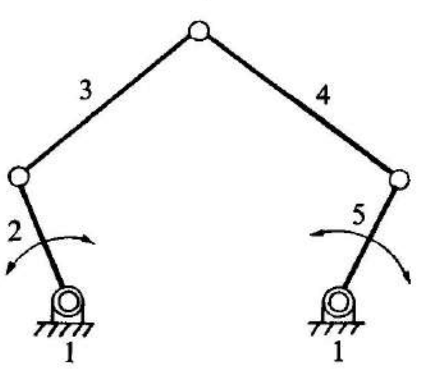
- F = Degrees of freedom
- n = Number of links in the mechanism.
- l = Number of lower pairs, which is obtained by counting the number of joints. If more than two links are joined together at any point, then, one additional lower pair is to be considered for every additional link.
- h = Number of higher pairs

Examples - DOF



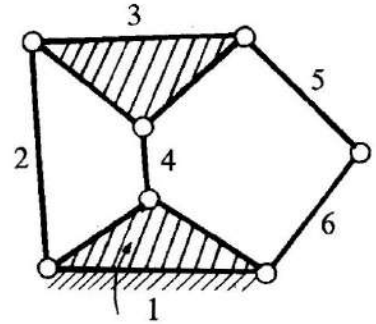
- F = 3(n-1)-2l-h
- Here, n = 4, l = 4 & h = 0.
- F = 3(4-1)-2(4) = 1
- I.e., one input to any one link will result in definite motion of all the links.

Examples - DOF



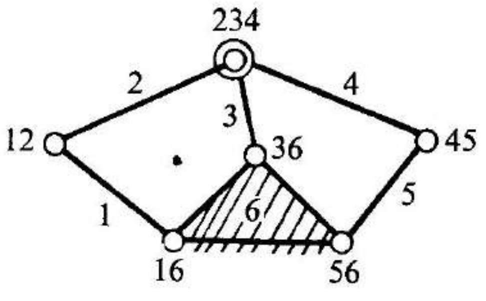
- F = 3(n-1)-2l-h
- Here, n = 5, l = 5 and h = 0.
- F = 3(5-1)-2(5) = 2
- I.e., two inputs to any two links are required to yield definite motions in all the links.

Examples - DOF



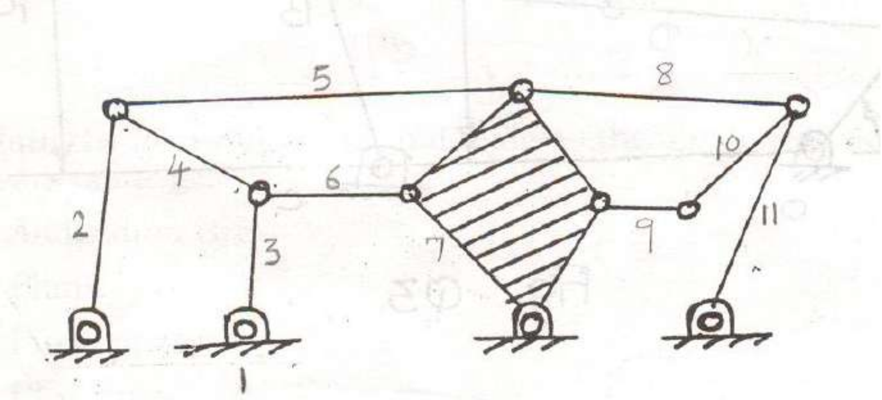
- $F = 3(n-1)-2l-h$
- Here, $n = 6$, $l = 7$ and $h = 0$.
- $F = 3(6-1)-2(7) = 1$
- I.e., one input to any one link will result in definite motion of all the links.

Examples - DOF



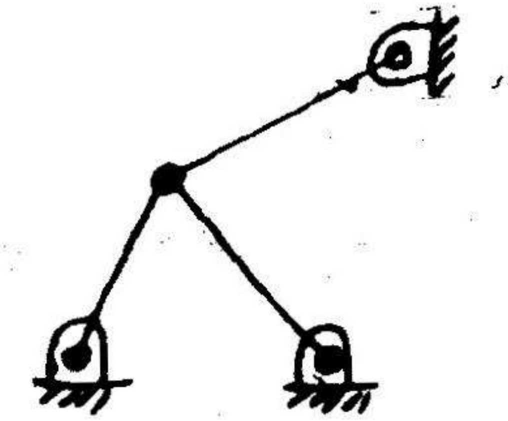
- $F = 3(n-1)-2l-h$
- Here, $n = 6$, $l = 7$ (at the intersection of 2, 3 and 4, two lower pairs are to be considered) and $h = 0$.
- $F = 3(6-1)-2(7) = 1$

Examples - DOF

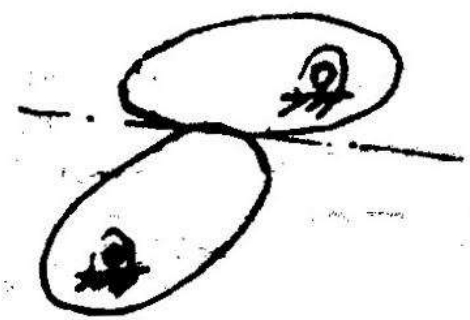


- $F = 3(n-1)-2l-h$
- Here, $n = 11$, $l = 15$ (two lower pairs at the intersection of 3, 4, 6; 2, 4, 5; 5, 7, 8; 8, 10, 11) and $h = 0$.
- $F = 3(11-1)-2(15) = 0$

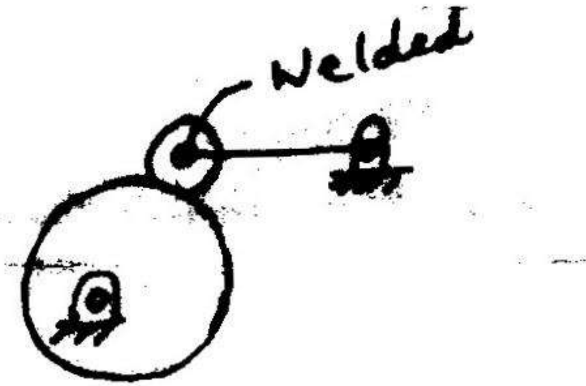
Examples - DOF



(a)
F = 3(n-1)-2l-h
Here, n = 4, l = 5 and h = 0.
F = 3(4-1)-2(5) = -1
I.e., it is a structure



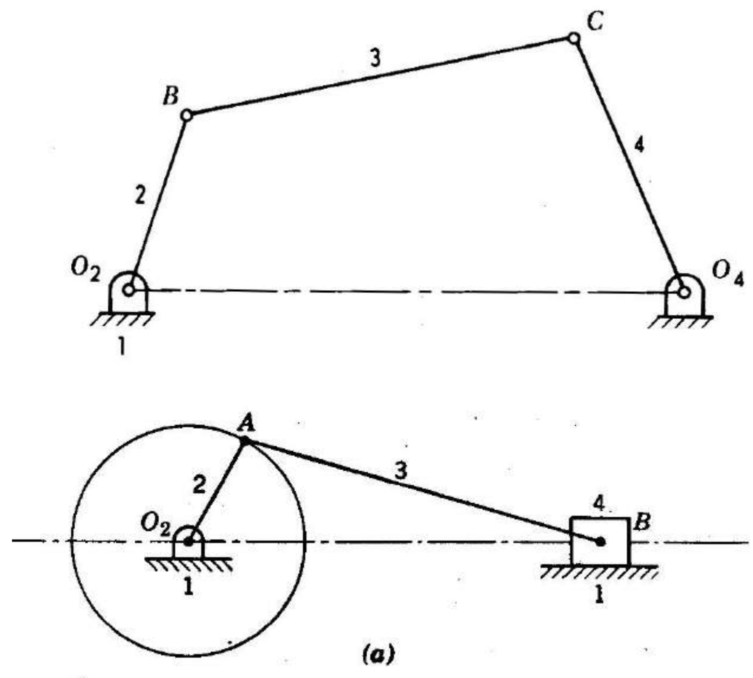
(b)
F = 3(n-1)-2l-h
Here, n = 3, l = 2 and h = 1.
F = 3(3-1)-2(2)-1 = 1



(c)
F = 3(n-1)-2l-h
Here, n = 3, l = 2 and h = 1.
F = 3(3-1)-2(2)-1 = 1

Grashoff Law

- The sum of the shortest and longest link length should not exceed the sum of the other two link lengths.



$s+l < p+q$

(e.x) (1+2) < (3+4)

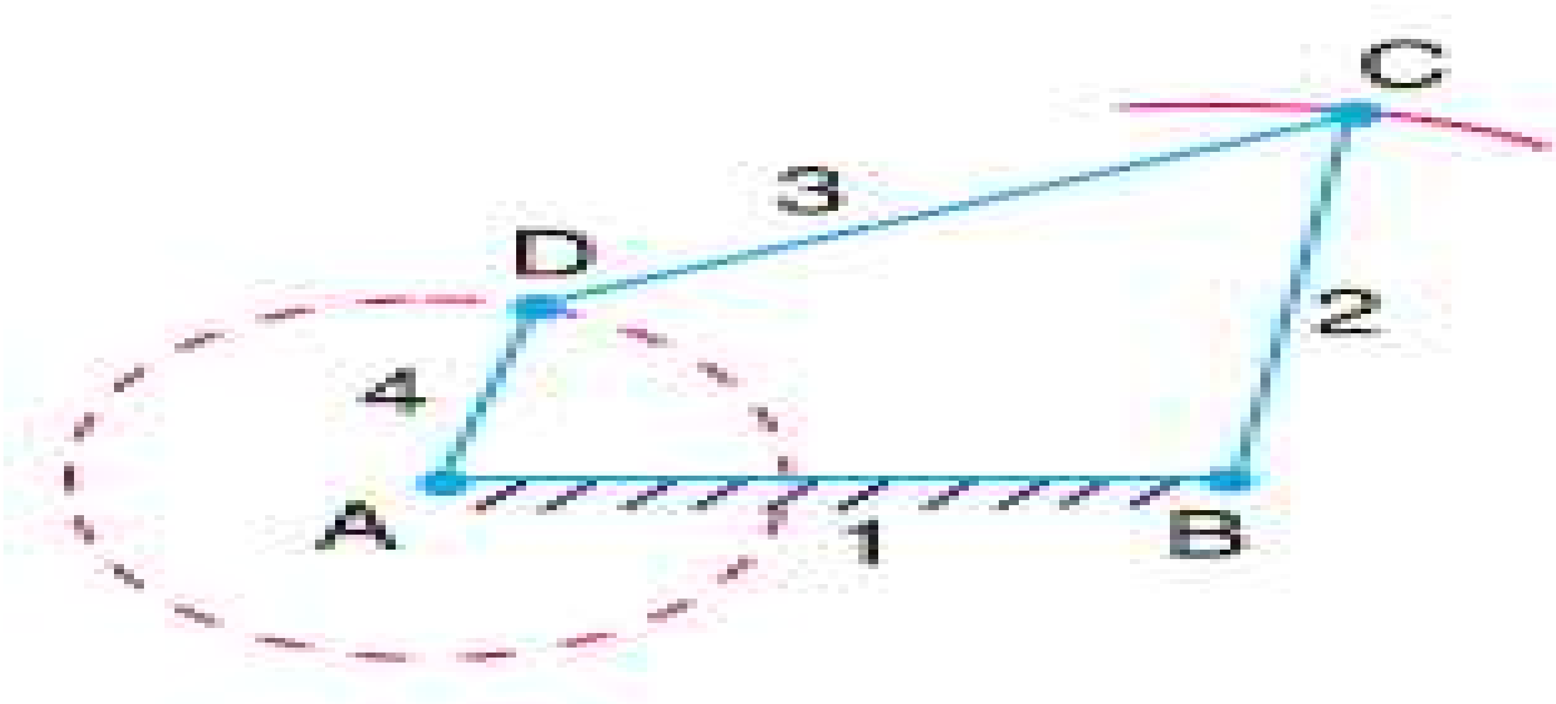
INVERSIONS OF MECHANISM

A mechanism is one in which one of the links of a kinematic chain is fixed. Different mechanisms can be obtained by fixing different links of the same kinematic chain. These are called as inversions of the mechanism.

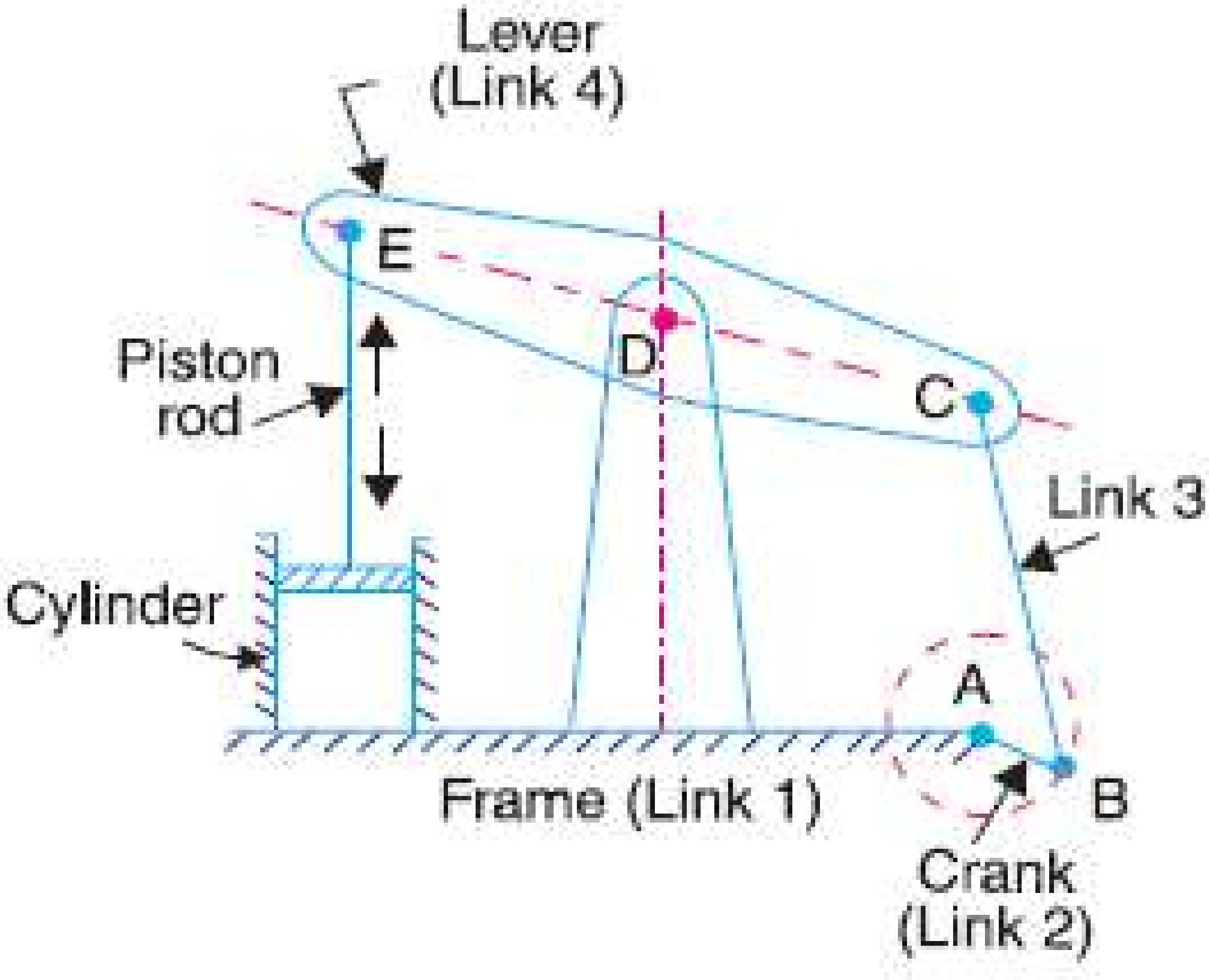
INVERSIONS OF MECHANISM

- 1.Four Bar Chain
- 2.Single Slider Crank
- 3.Double Slider Crank

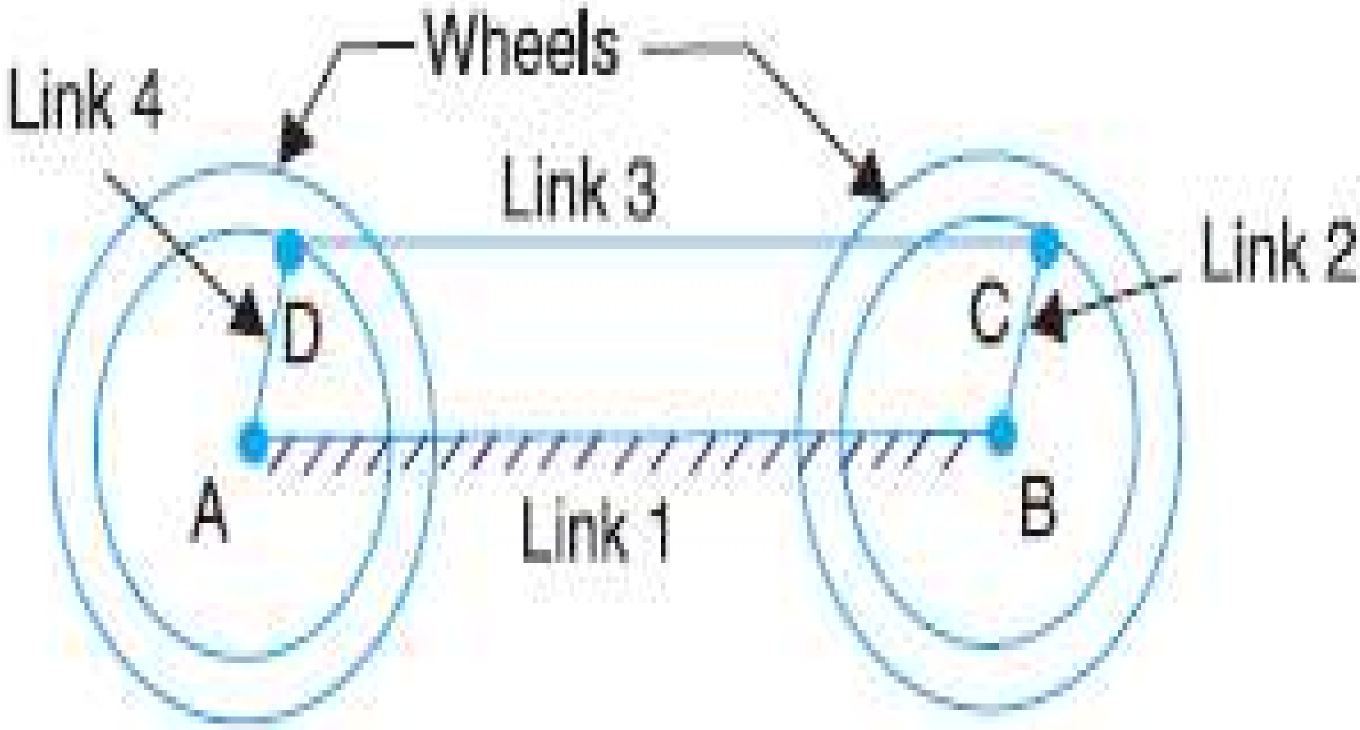
1.Four Bar Chain - Inversions



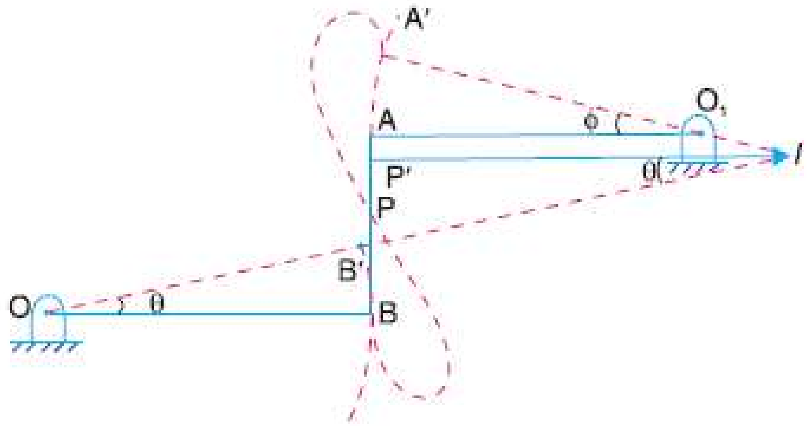
Beam Engine (crank &Lever)



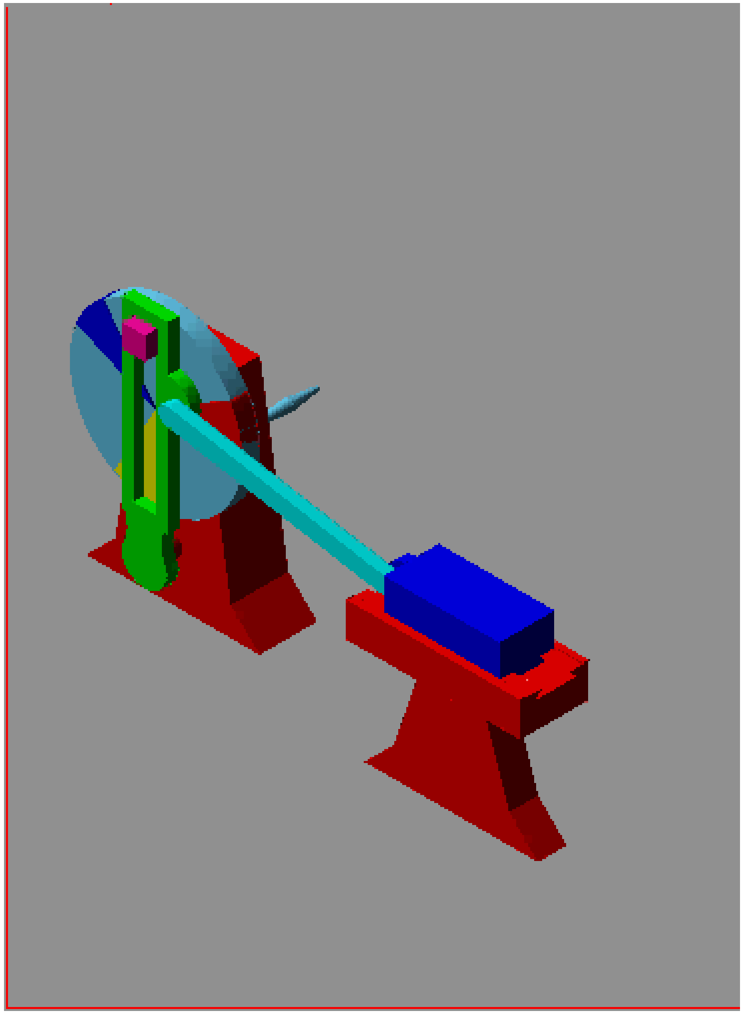
Double Crank Msm



Double Lever



2.Single Slider Crank _ Inversions



Single Slider Crank _ Inversions

1.PENDULUM PUMP

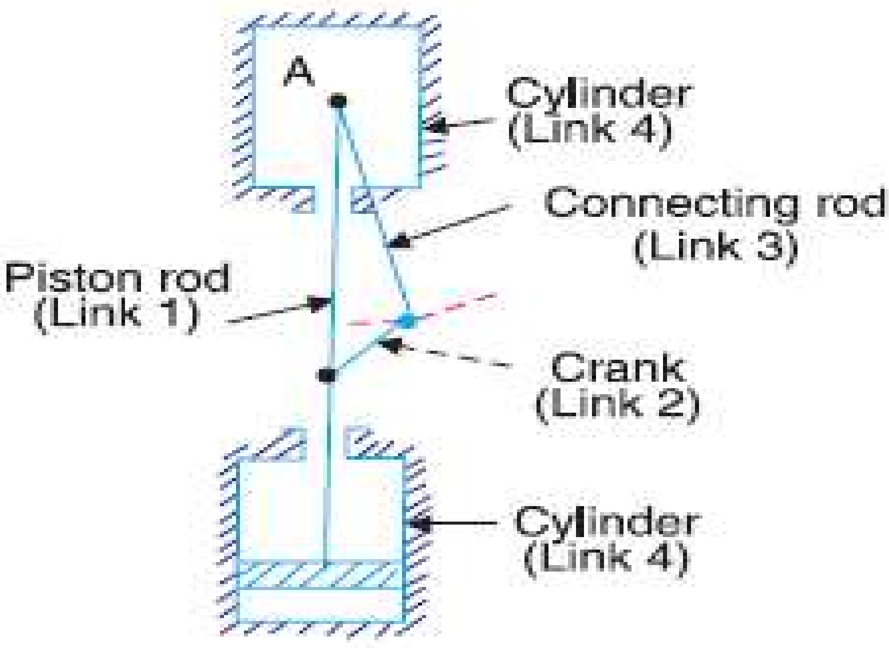
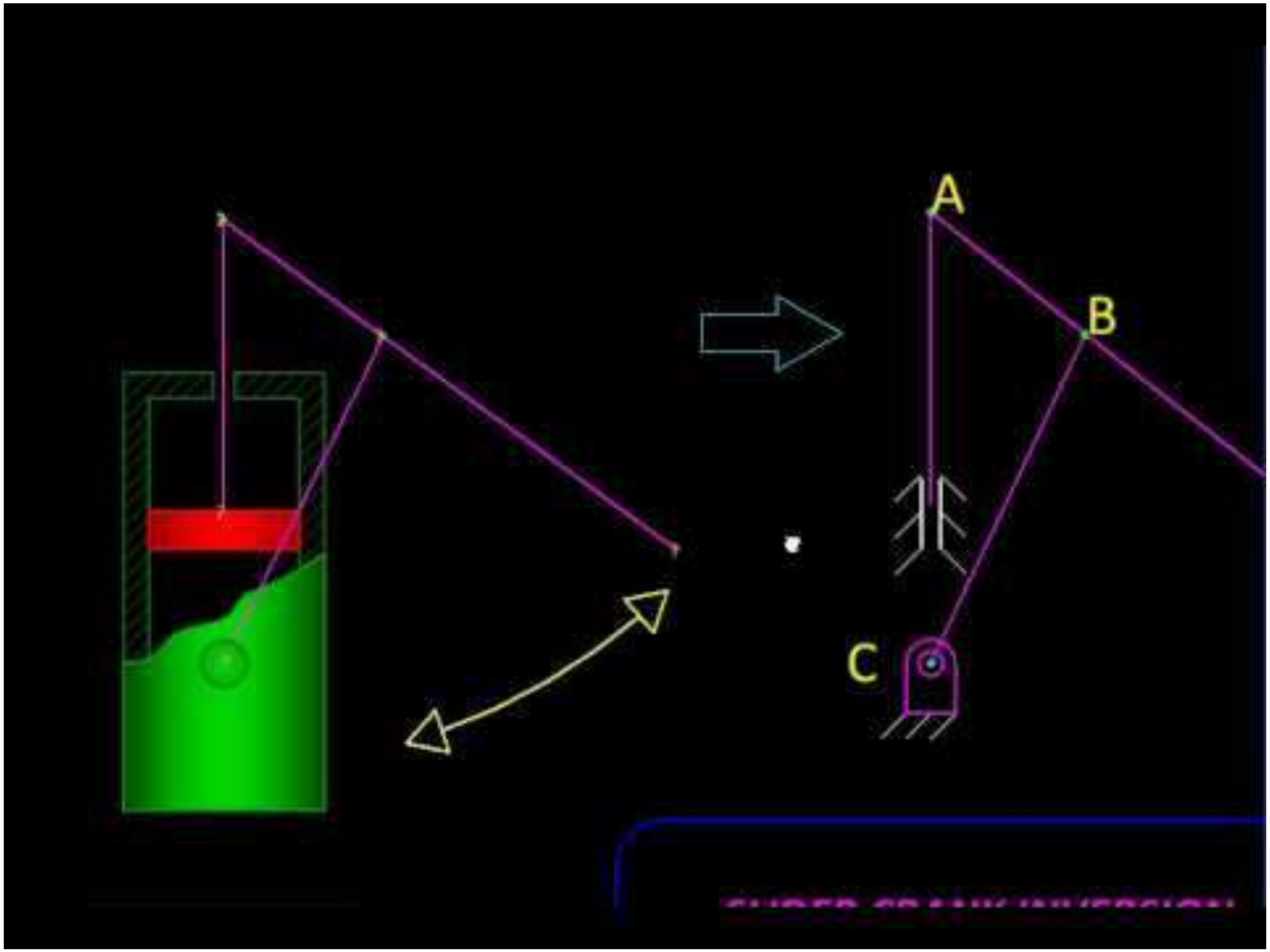
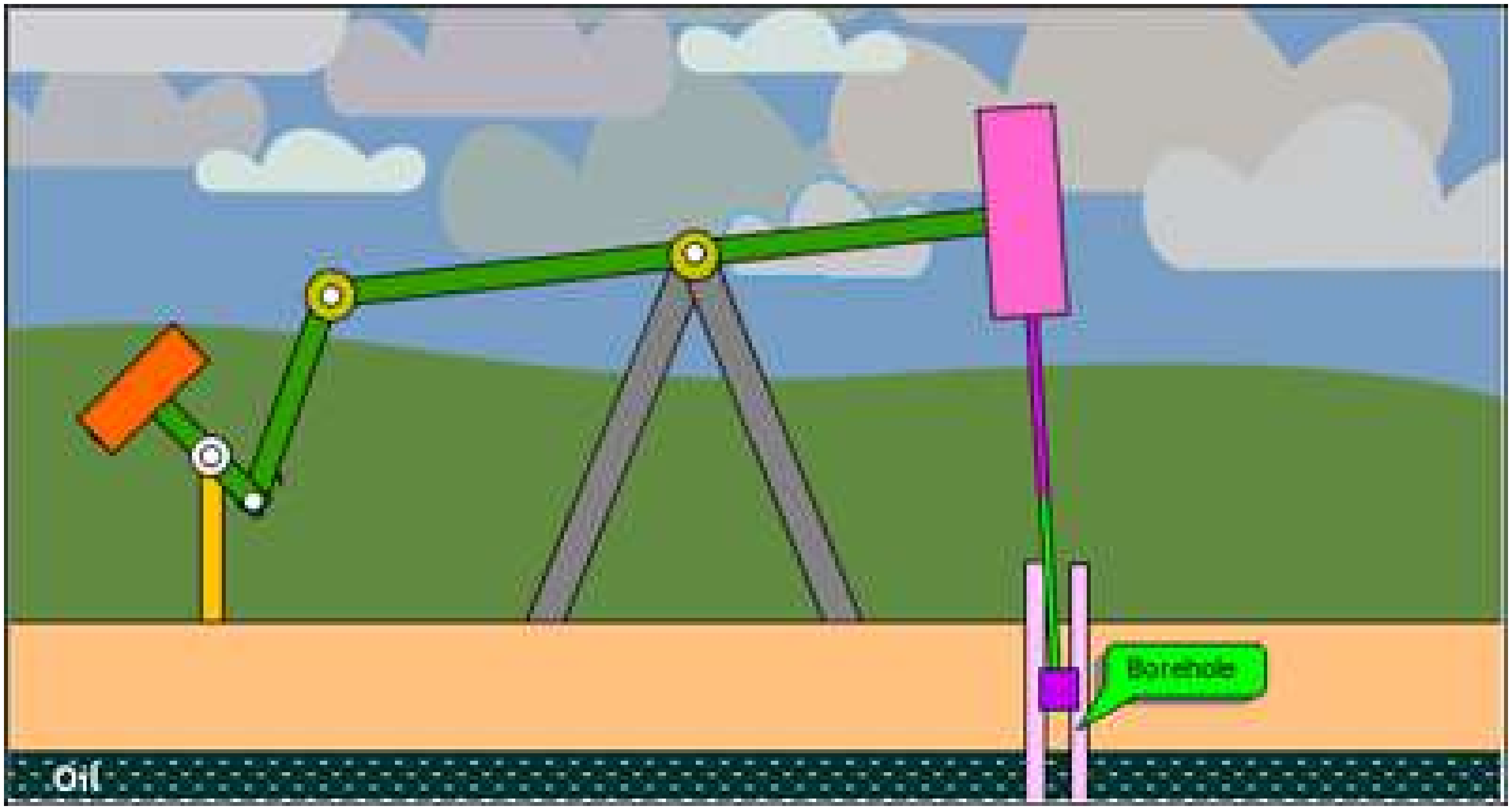


Fig. 5.23. Pendulum pump.


A
B
C


Borehole
Oil

2.OSCILLATING CYLINDER

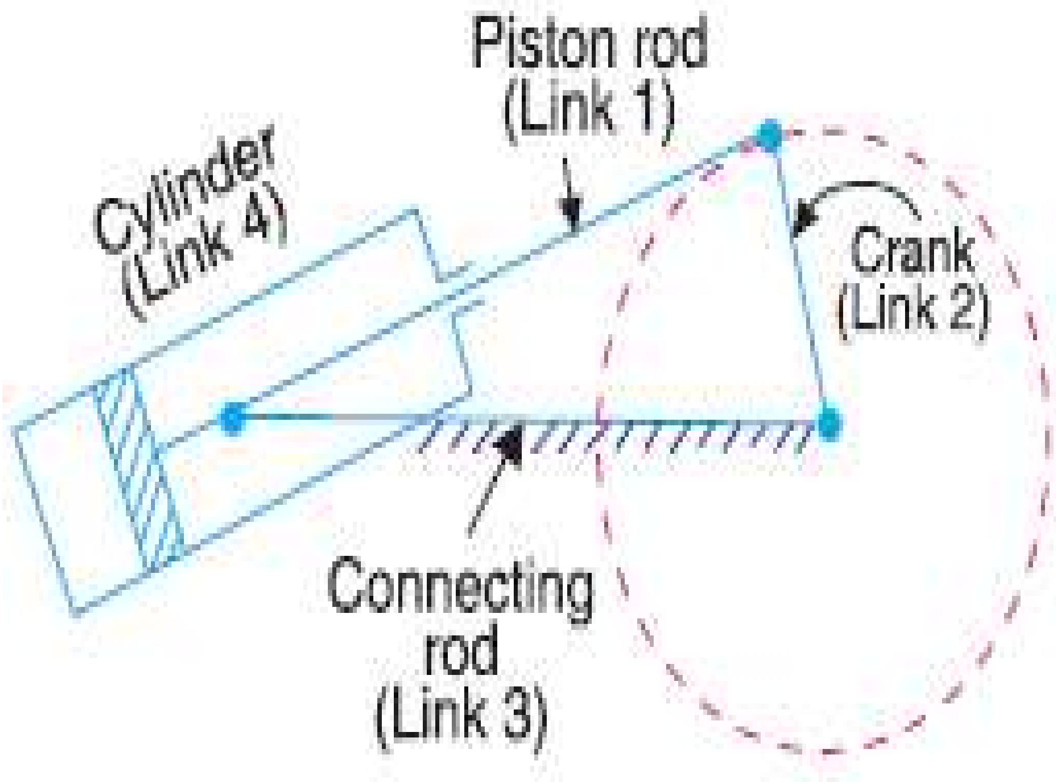
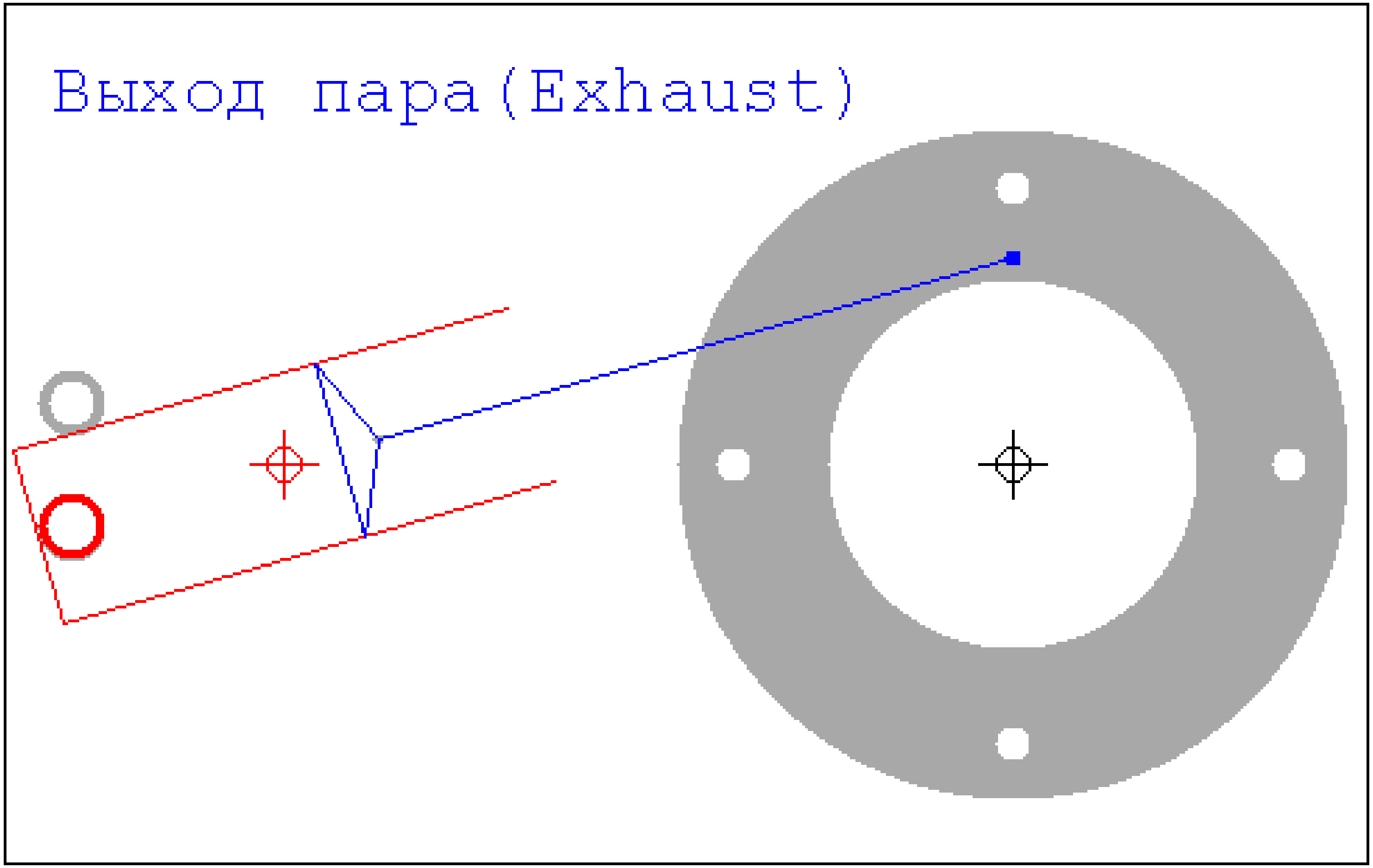
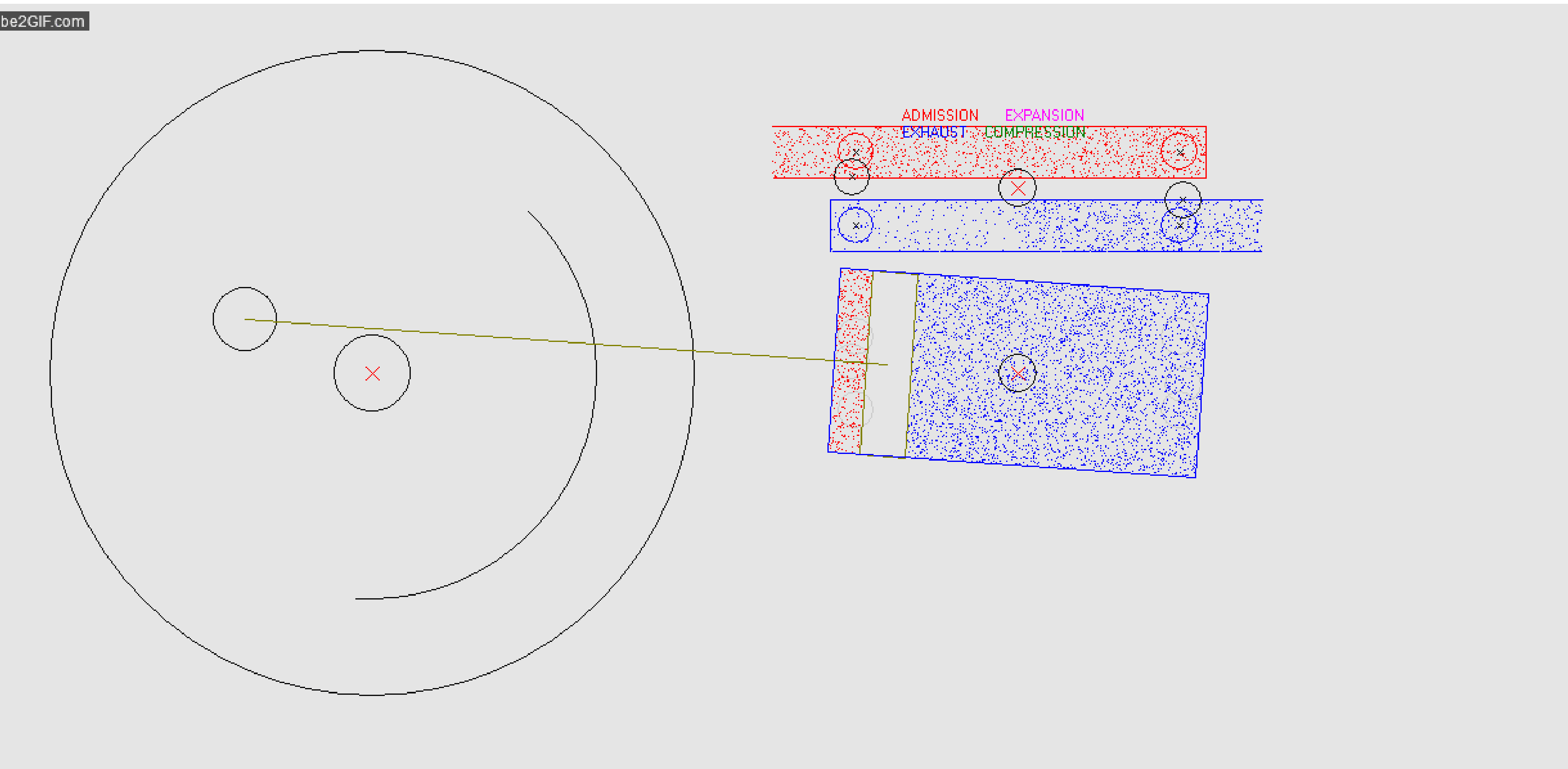


Fig. 5.24. Oscillating cylinder engine.


Выход пара(Exhaust)



be2GIF.com
ADMISSION
EXPANSION
EXHAUST
COMPRESSION

3.ROTARY IC ENGINE

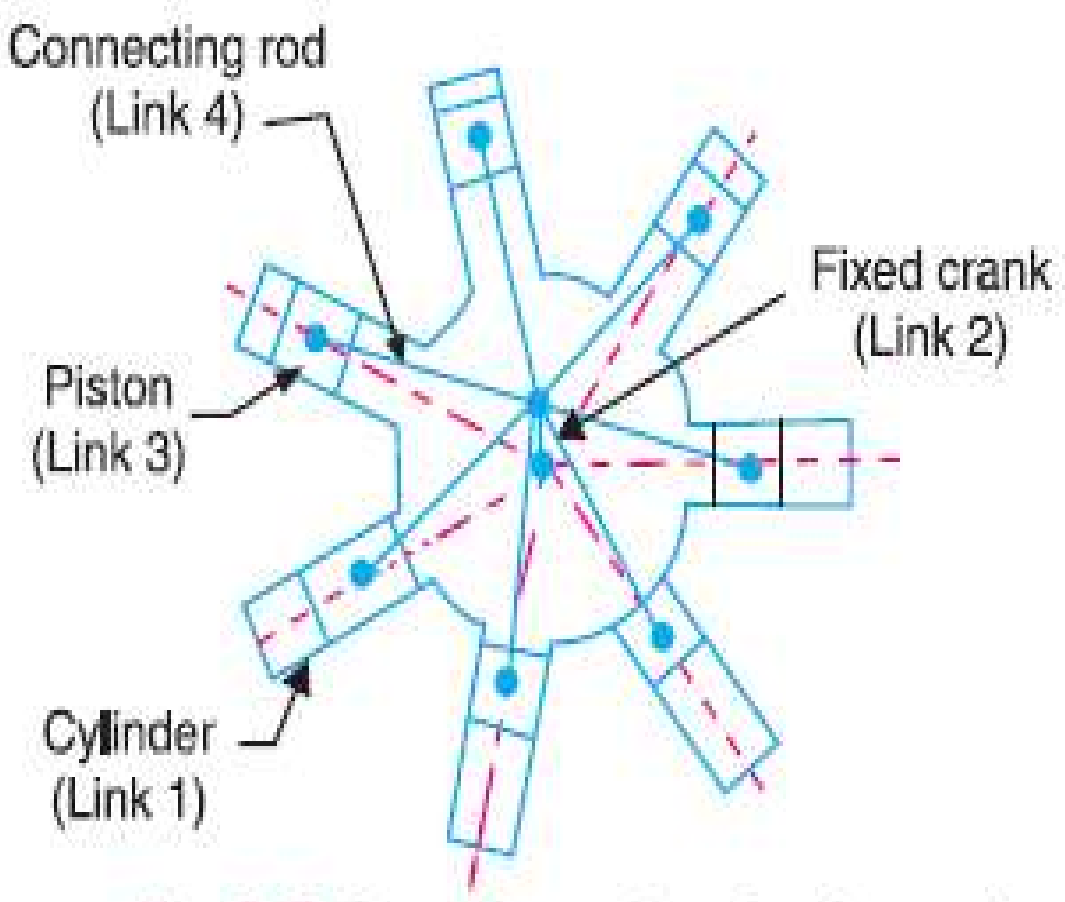
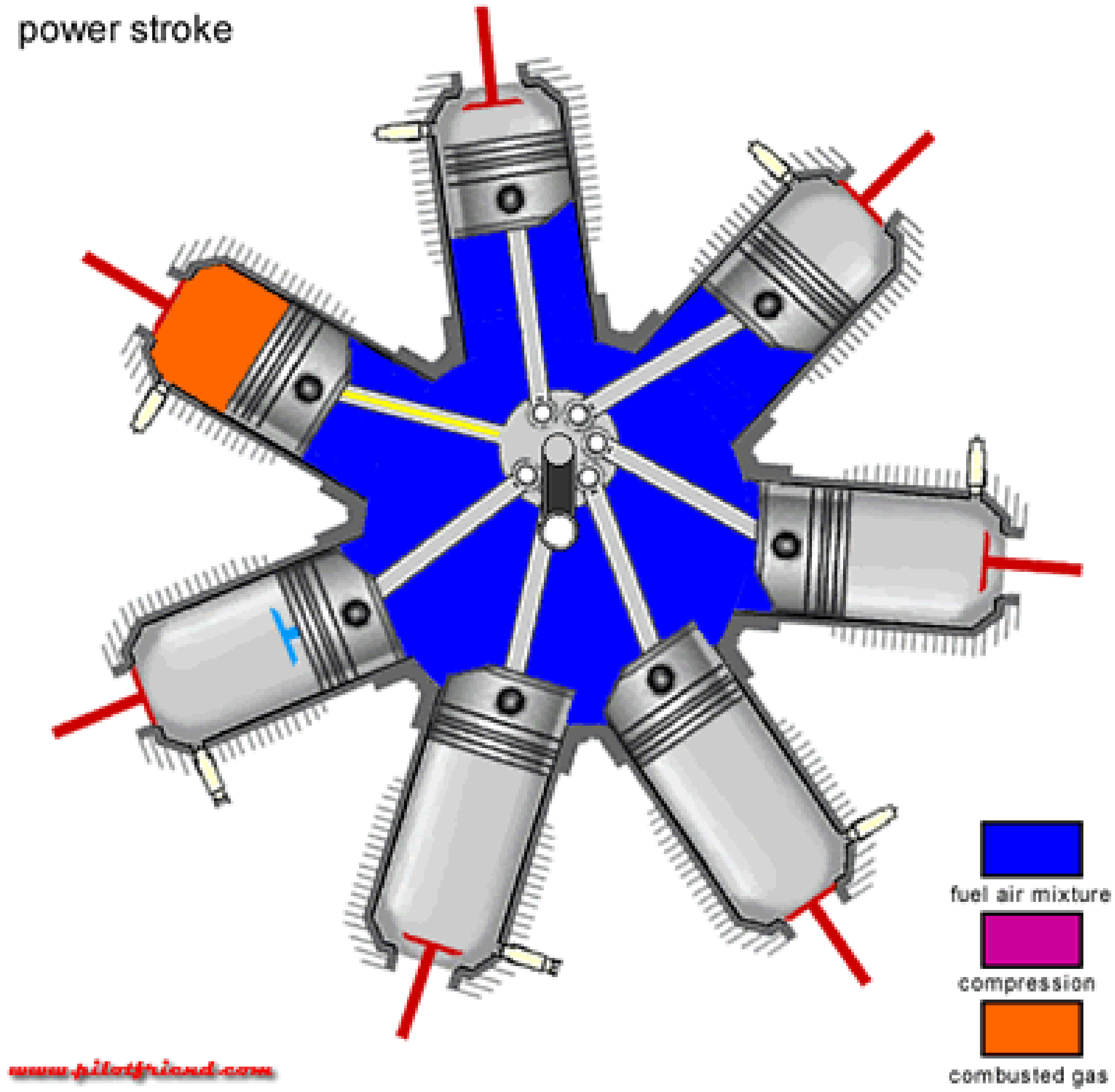
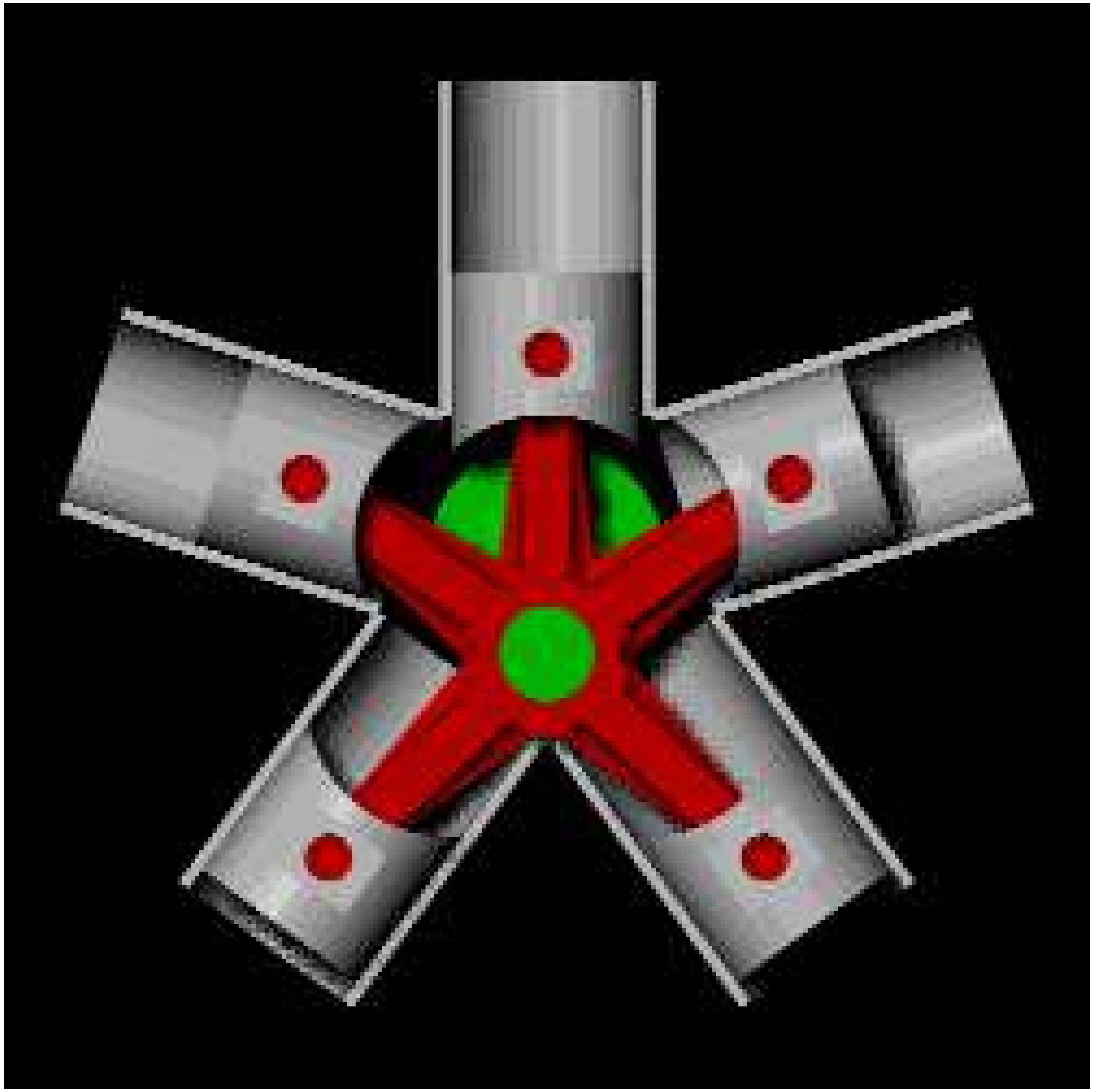
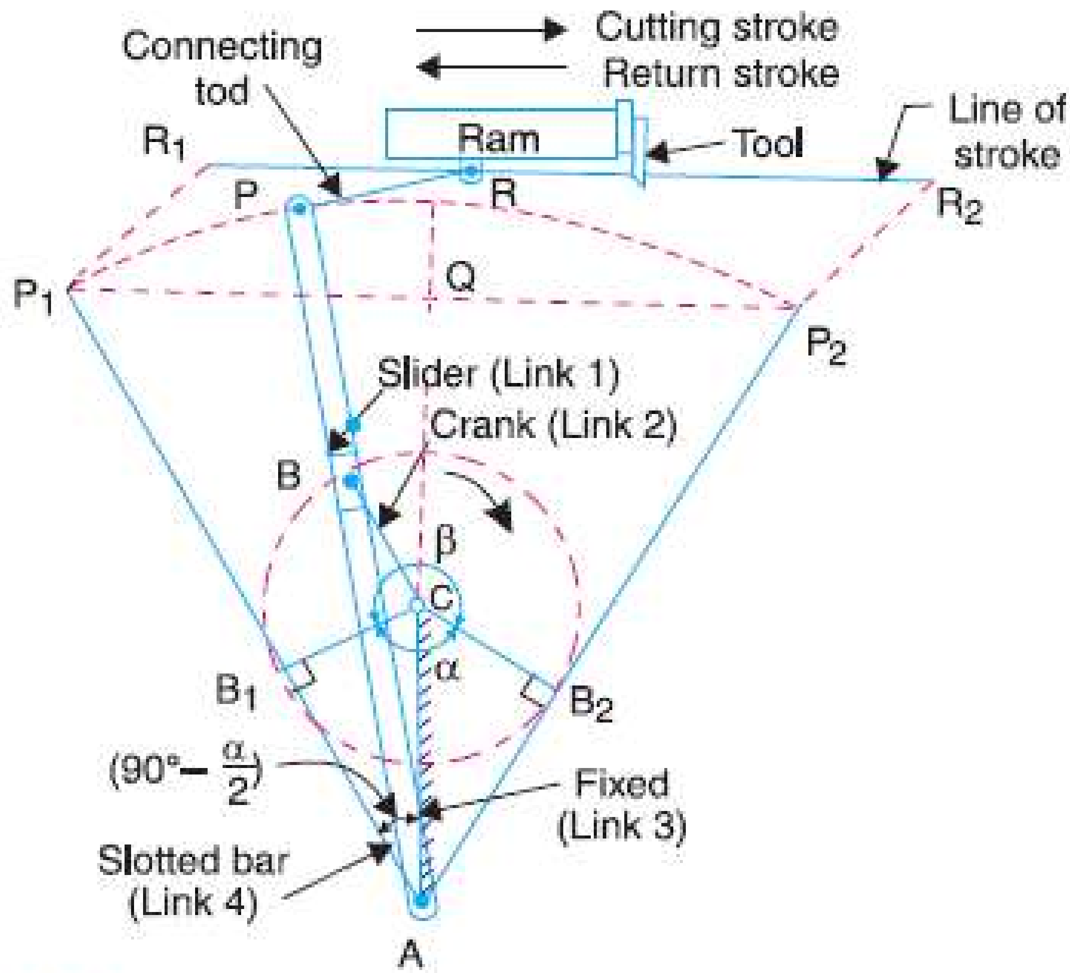


Fig. 5.25. Rotary internal combustion engine.


power stroke
fuel air mixture
compression
combusted gas
www.pilotfriend.com

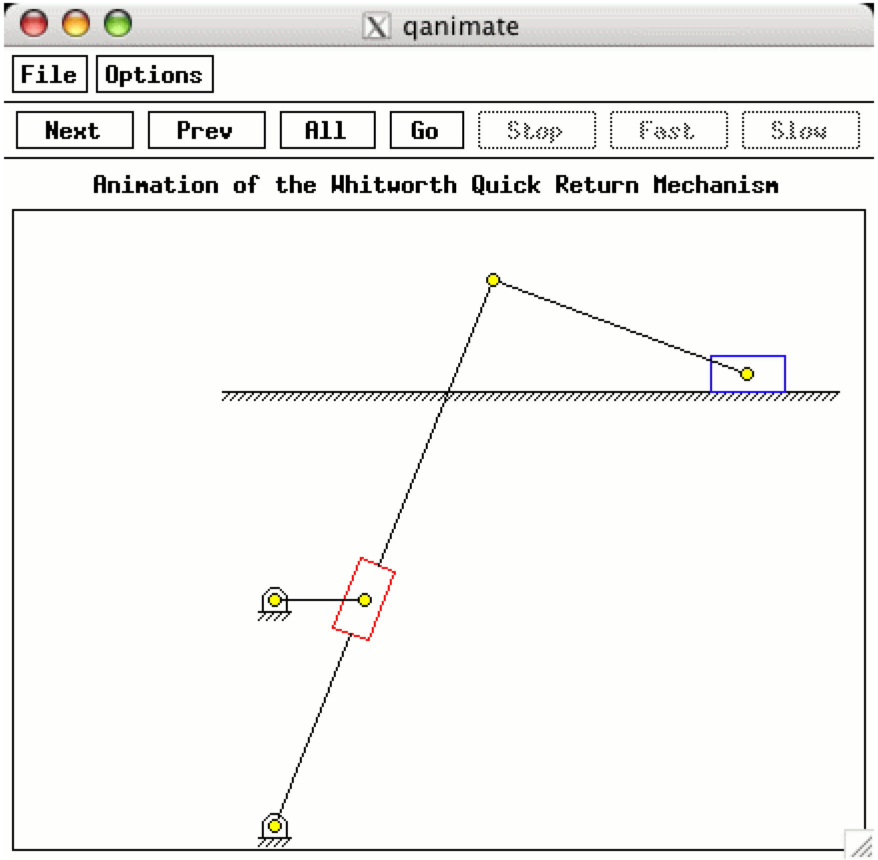


4.Crank and Slotted Lever Mechanism (Quick-Return Motion Mechanism)

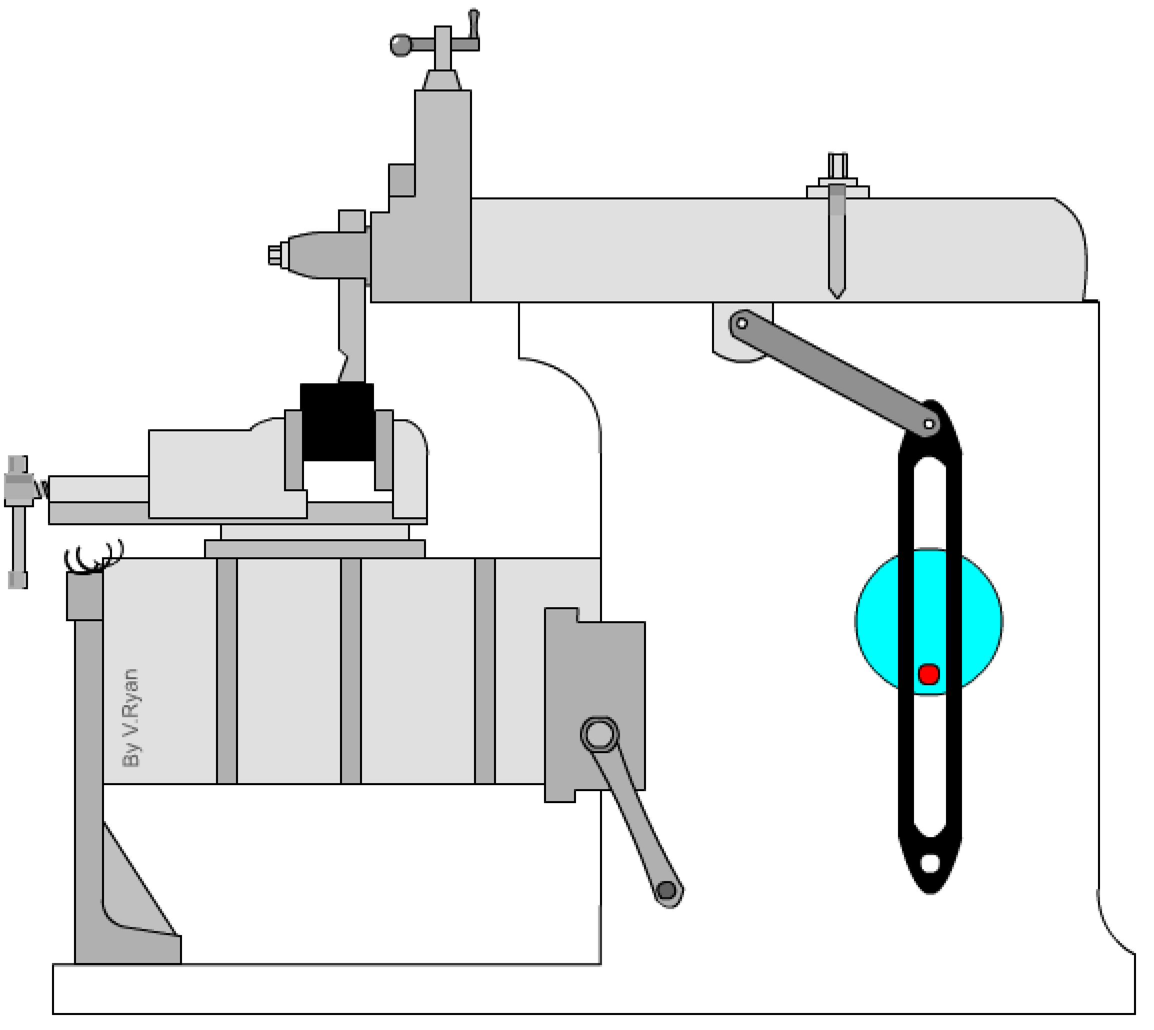


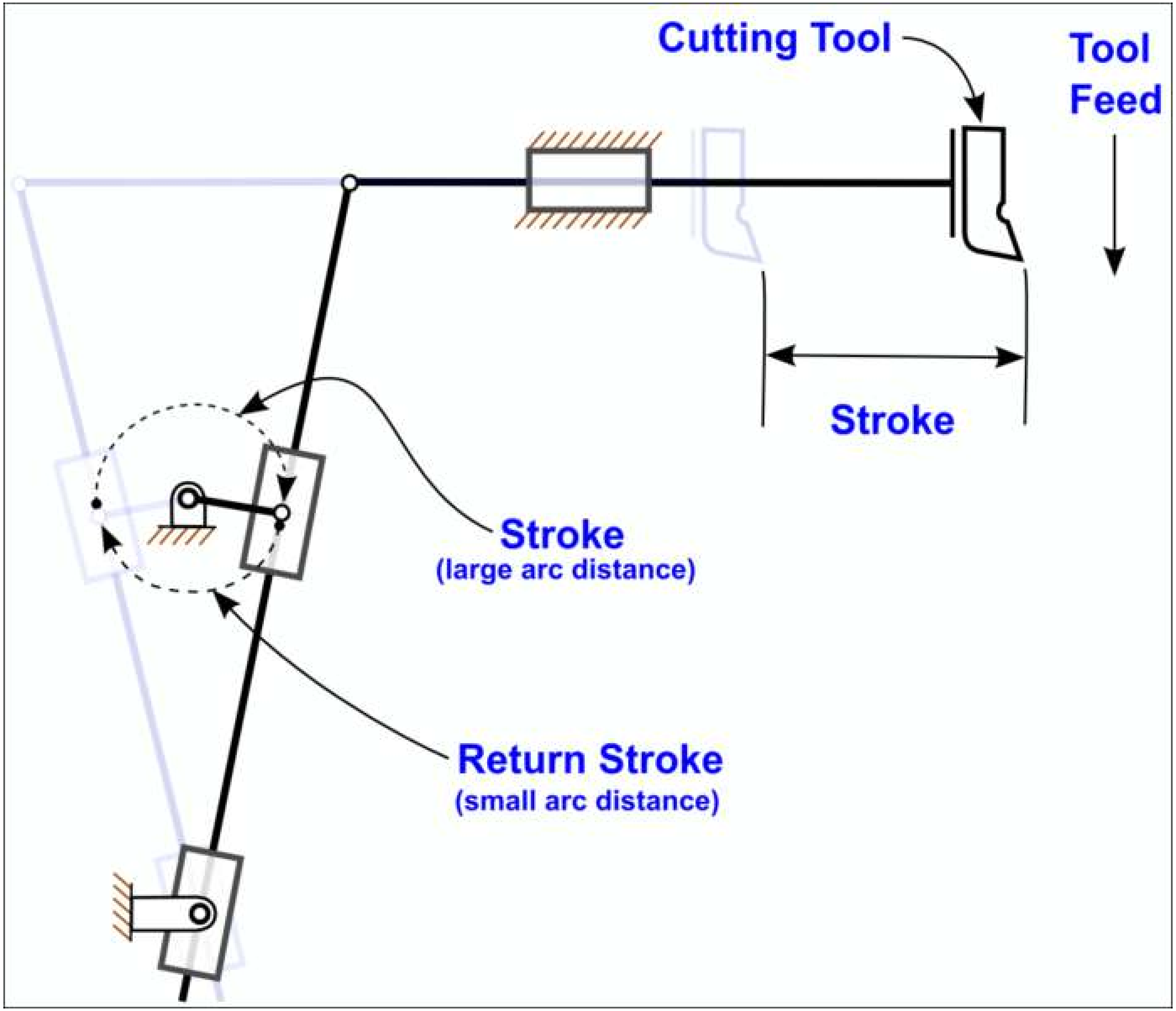
ig. 5.26. Crank and slotted lever quick return motion mechanism.

Basic Quick-Return



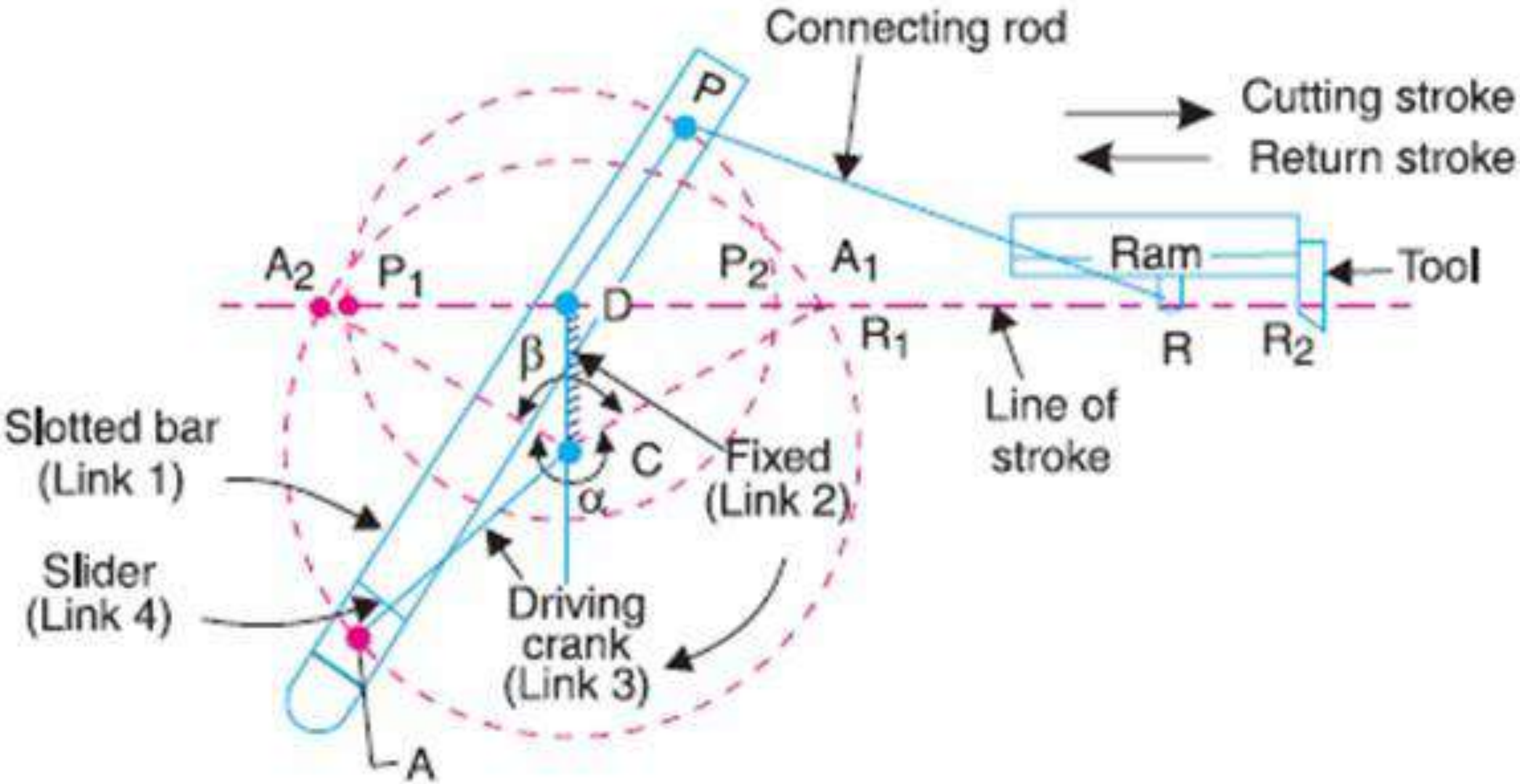
Quick-Return IN SHAPER M/C

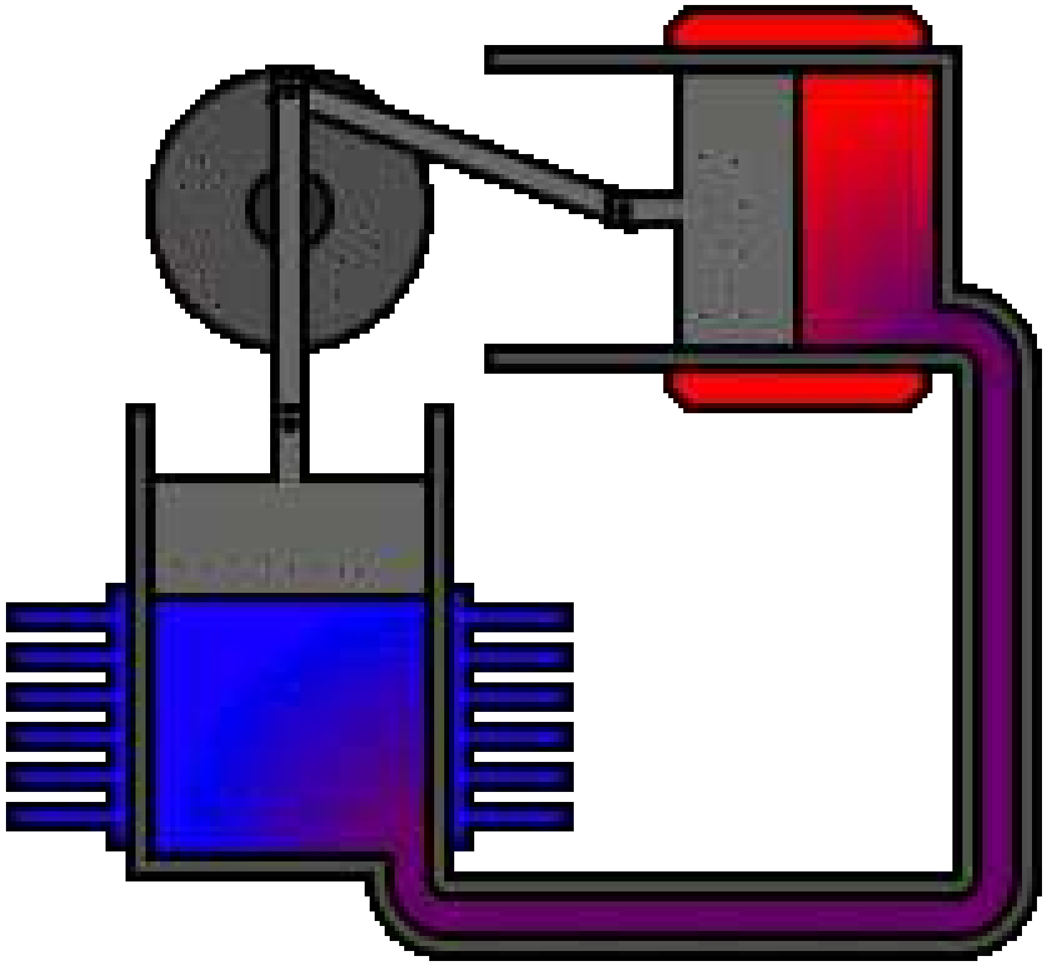




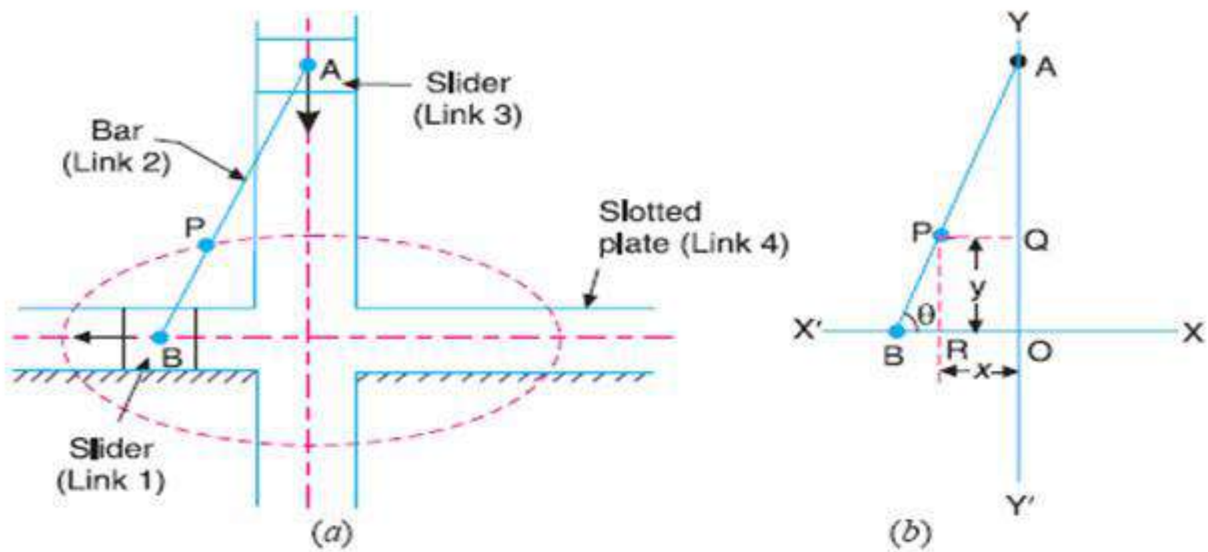
Cutting Tool
Tool
Feed
Stroke
Stroke
(large arc distance)
Return Stroke
(small arc distance)

WHIT WORTH QUICK-RETURN

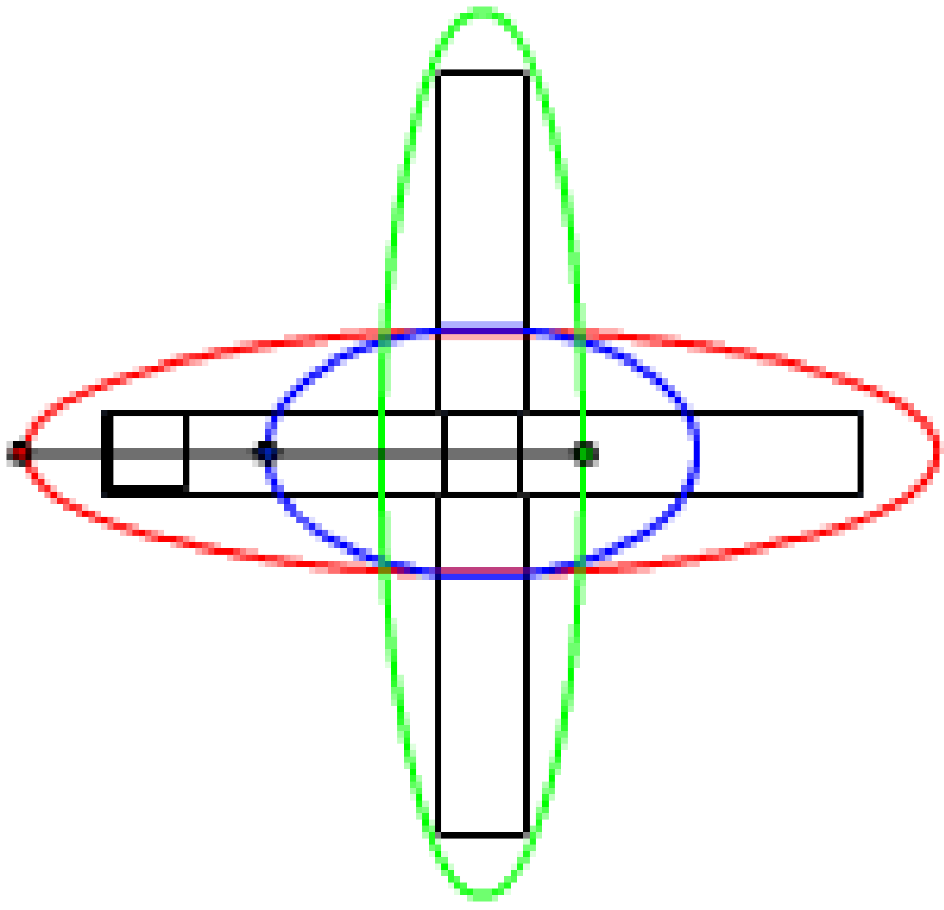


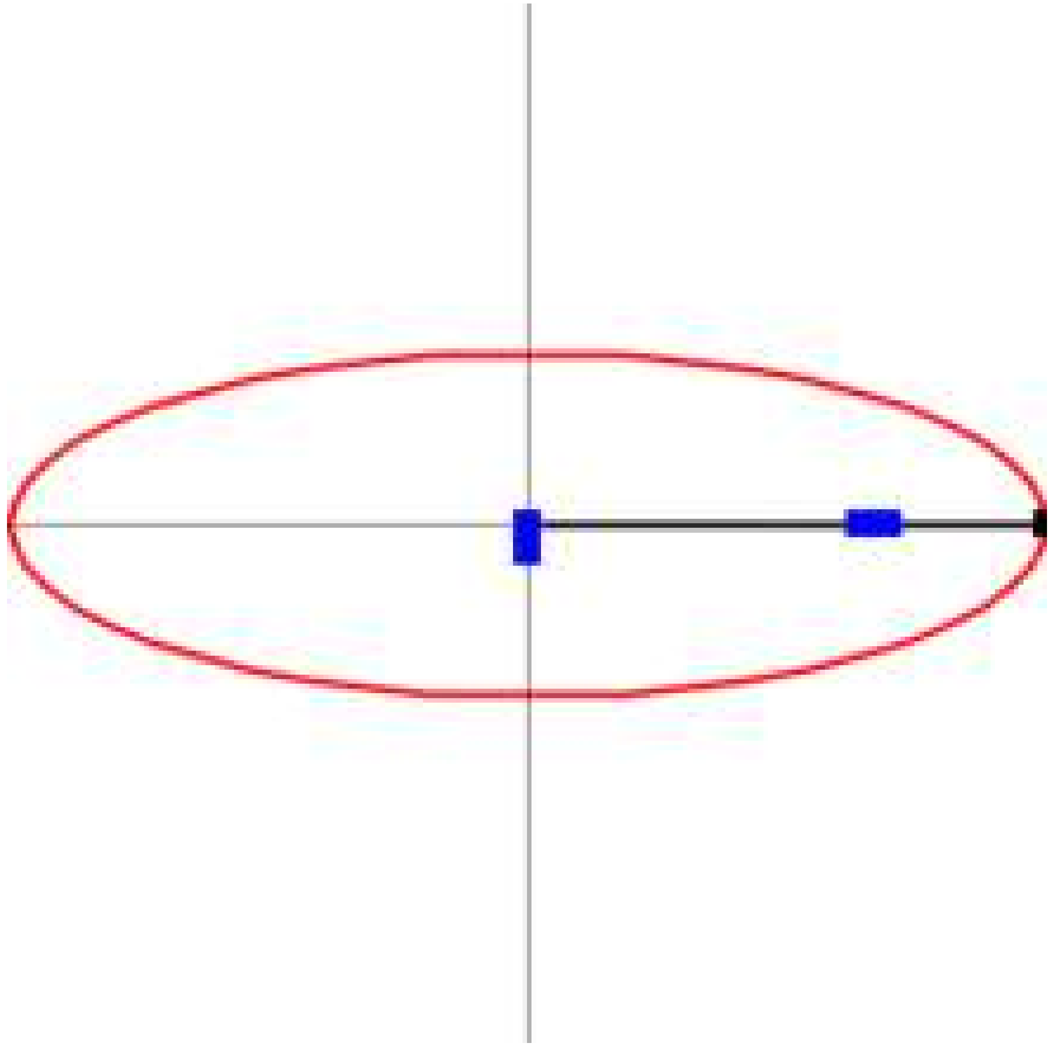


3.Double Slider Crank INVERSIONS

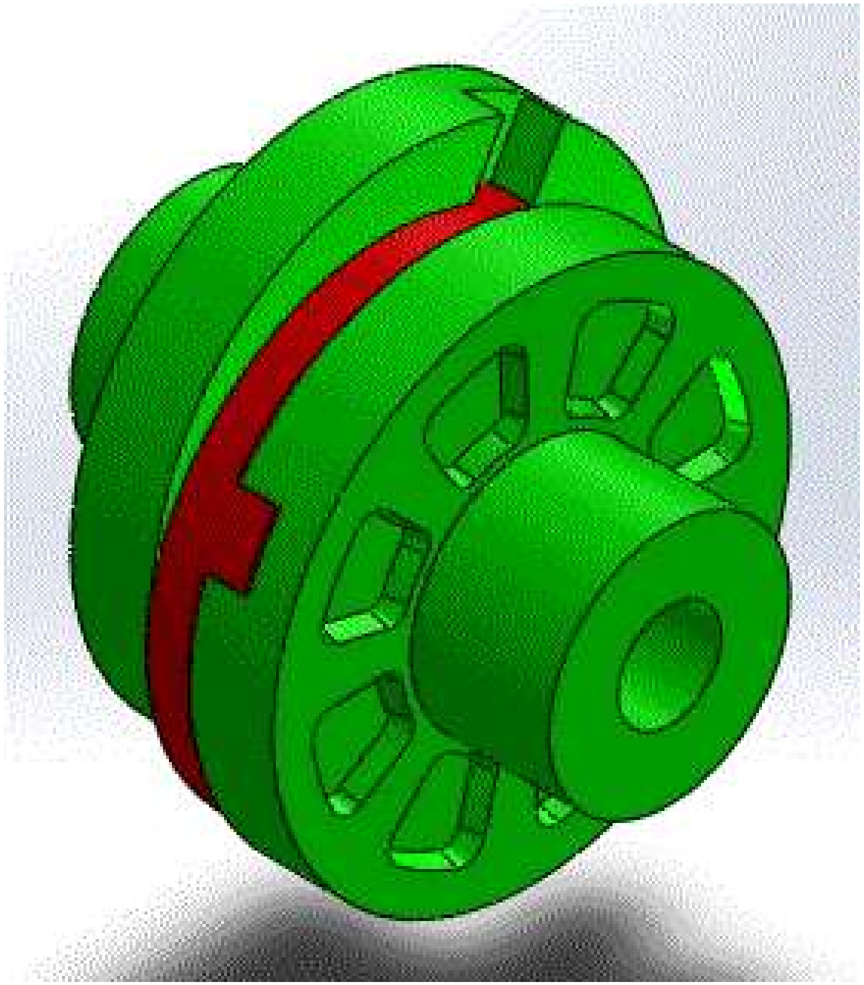


ELLIPTICAL TRAMMEL



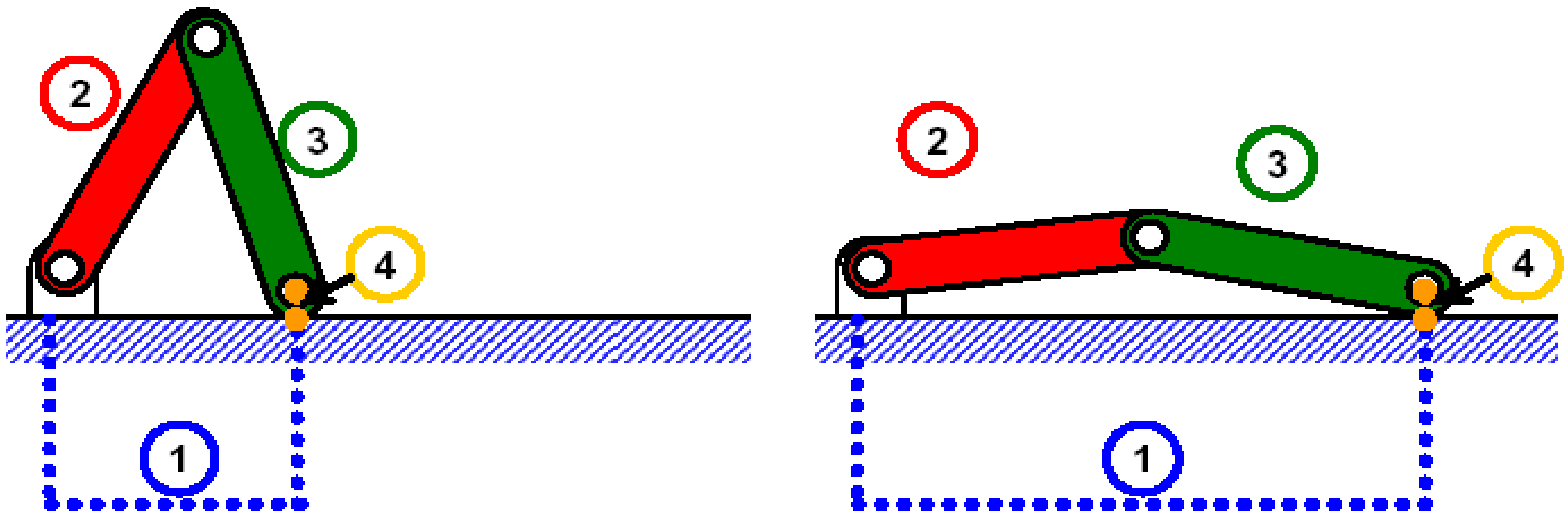


OLD HAMS COUPLING



UNIT - II

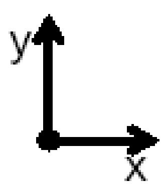
Effective 4-bar linkage



Appears that there are only 3 links

Link 4 is an effective link

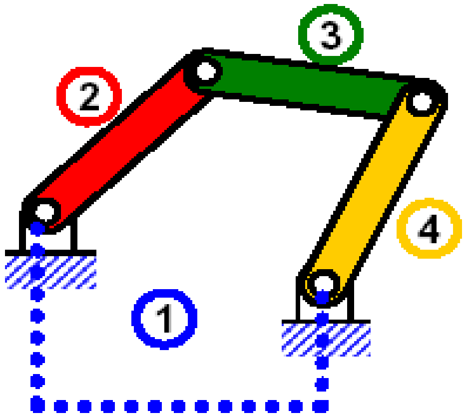
This linkage can be modeled as a 4 bar linkage

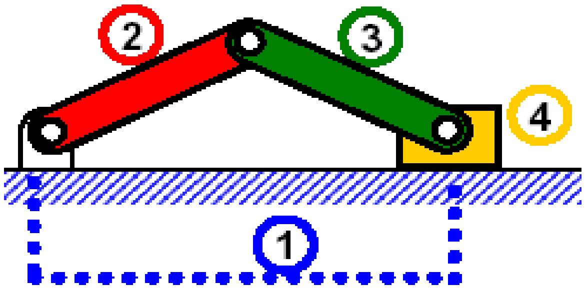


Length of link 1 varies with linkage position

An effective link is characterized by a “rigid” non-changing length that is needed to define the motion of the mechanism

Common 4-bar linkage types





4 BAR COUPLER:

Consists of 4 links:
- 1 = Ground
- 2 = Crank or follower
- 3 = Coupler
- 4 = Crank or follower

Crank is the link driven by force/torque

Links can cross each other

4 BAR SLIDER-CRANK:

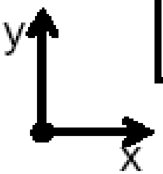
Consists of 3 links and slider:
- 1 = Ground
- 2 = Crank
- 3 = Coupler
- 4 = Slider

Crank is link driven by Force/torque

Link # 4 can be:
- A pin, block, or tube

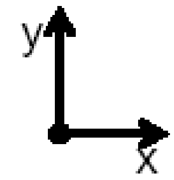
Link # 4 slides in/on one of these:
- Another part, rod or tube

Links can cross each other

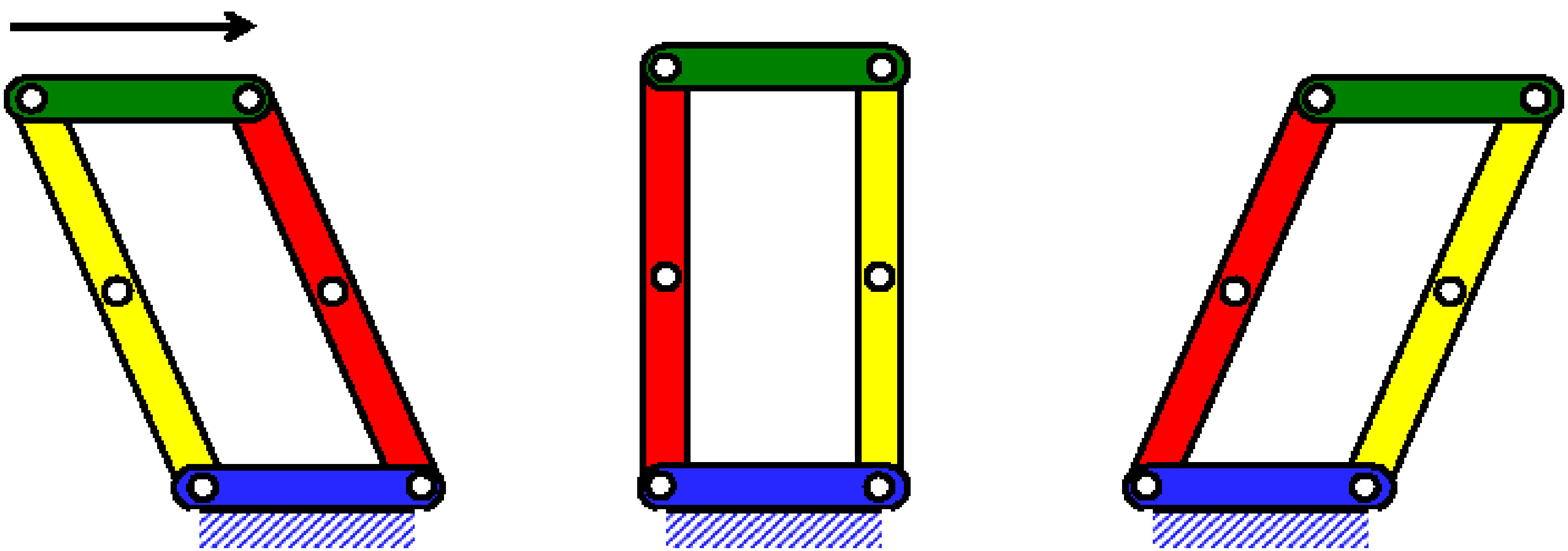


Parallel link 4 bar mechanism

Parallel 4 bar linkage:

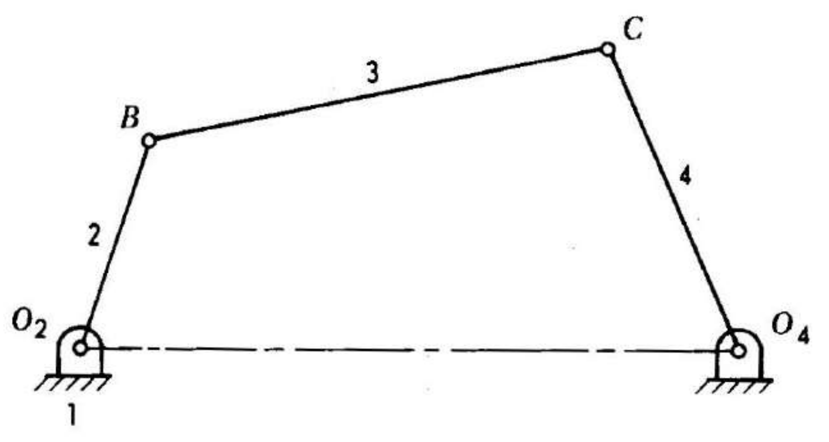


- Opposing links have equal lengths
- $\theta_1 = 180^\circ$ $|R_1| = |R_3|$ $|R_2| = |R_4|$
- Opposing links remain parallel



1. FOUR BAR CHAIN

- (link 1) frame
- (link 2) crank
- (link 3) coupler
- (link 4) rocker

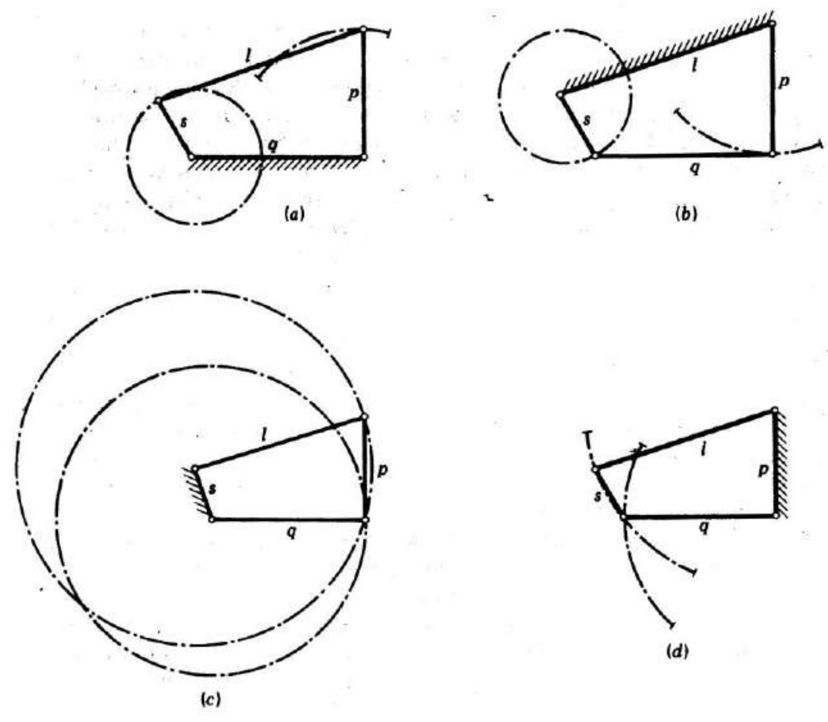


INVERSIONS OF FOUR BAR CHAIN

Fix link 1& 3. Crank-rocker
or Crank-Lever
mechanism

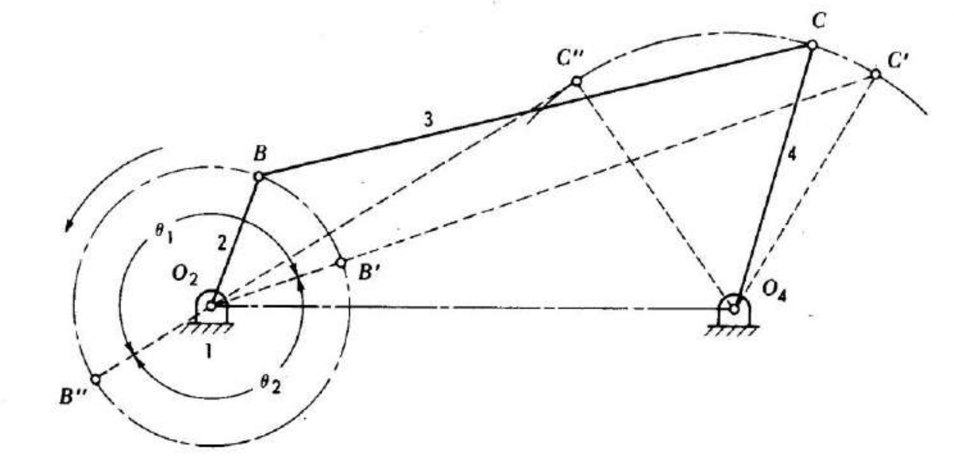
Fix link 2. Drag link
or Double Crank
mechanism

Fix link 4. Double rocker
mechanism
Pantograph

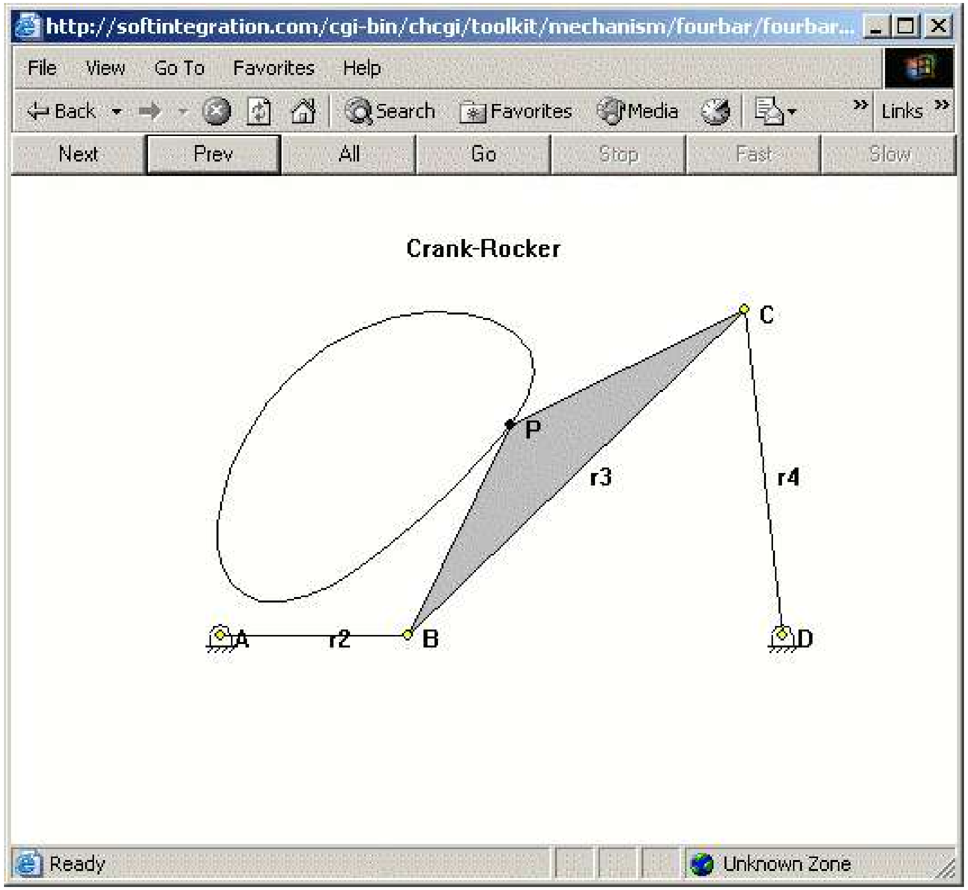


APPLICATION
link-1 fixed-
CRANK-ROCKER MECHANISM

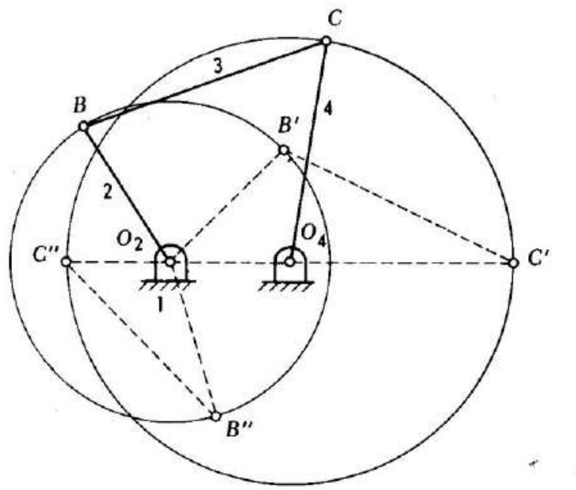
OSCILLATORY MOTION



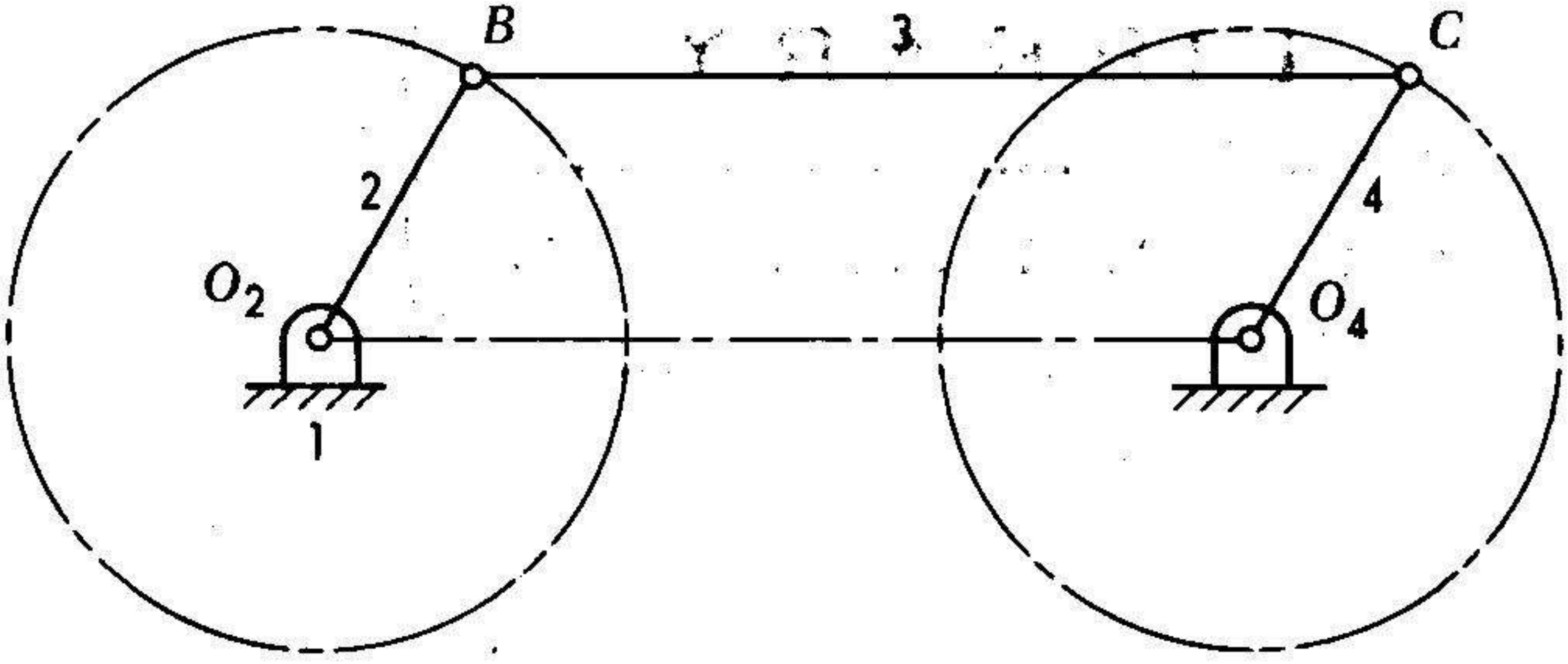
CRANK-ROCKER MECHANISM



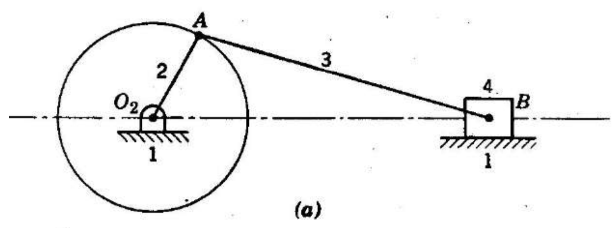
Link 2 Fixed- DRAG LINK MECHANISM



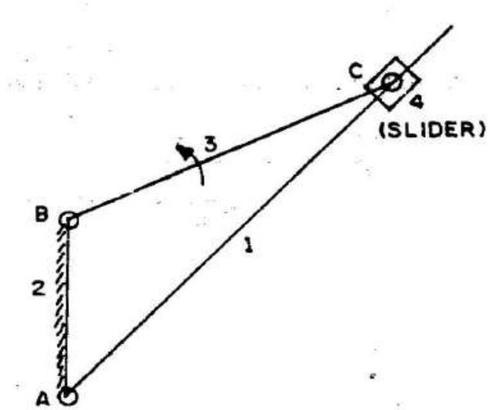
Locomotive Wheel - DOUBLE CRANK MECHANISM



# 2.SLIDER CRANK CHAIN
Link1=Ground
Link2=Crank
Link3=ConnectingRod
Link4=Slider

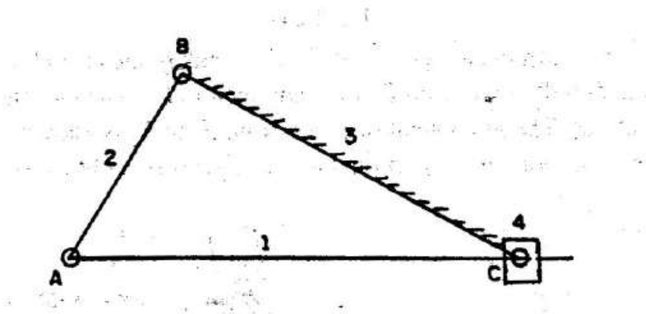


Inversions of slider crank chain



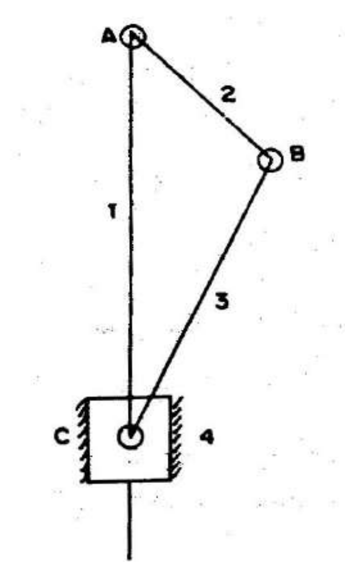
(a) crank fixed

Link 2 fixed



(b) connecting rod fixed

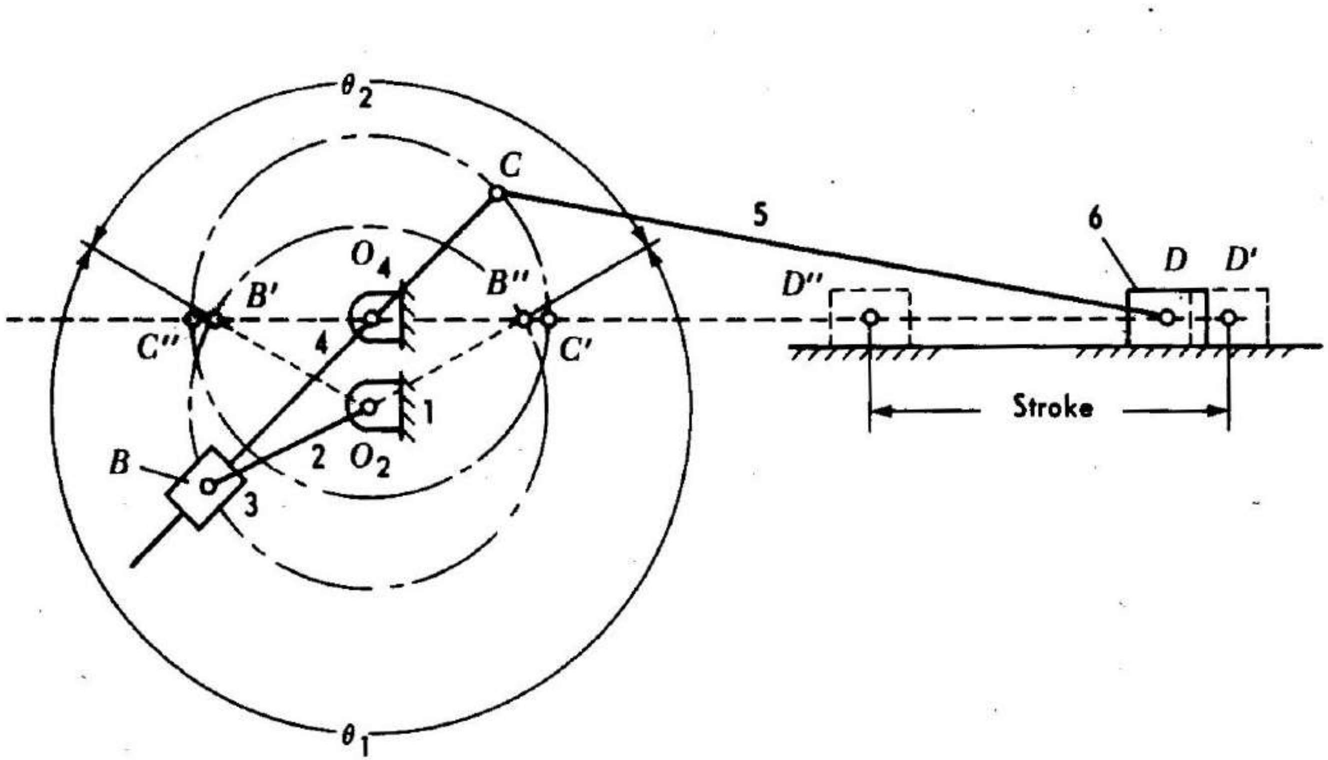
Link 3 fixed



(c) slider fixed

Link 4 fixed

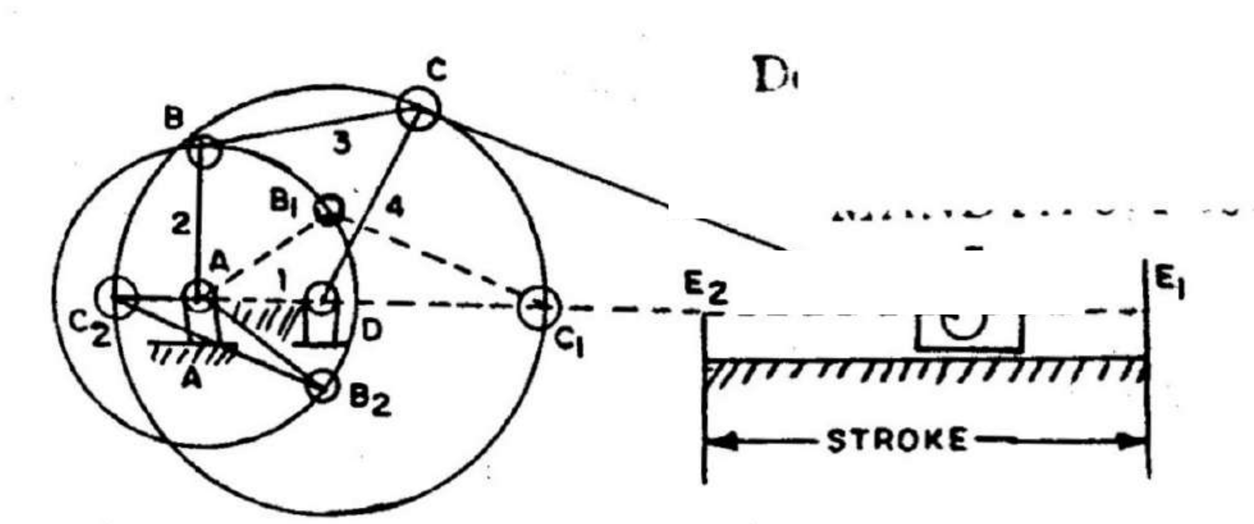
Application
Inversion II – Link 2 Crank fixed
Whitworth quick return motion mechanism



$$\frac{Time for forward stroke}{Time for return stroke} = \frac{B'\hat{o}_2 B''}{B''\hat{o}_2 B'} = \frac{\theta_1}{\theta_2}$$

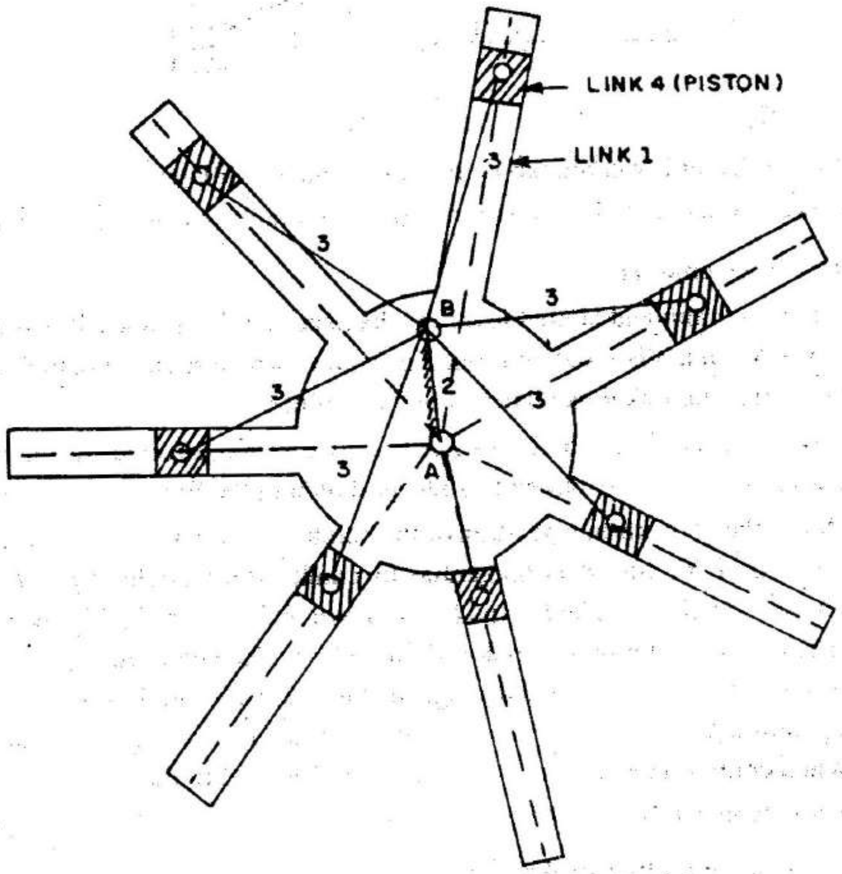
Quick return motion mechanisms

Drag link mechanism

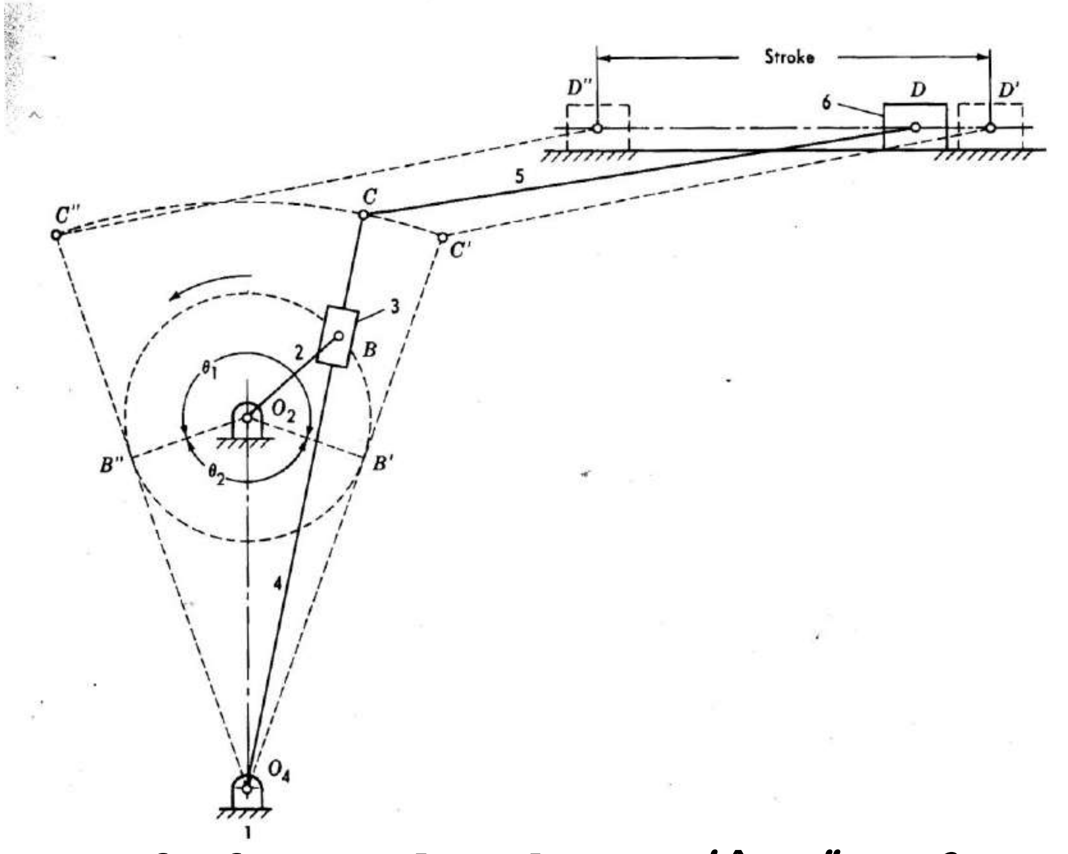


$$\frac{Time for forward stroke}{Time for return stroke} = \frac{B_1 \hat{A} B_2}{B_2 \hat{A} B_1}$$

Rotary engine– II inversion of slider crank mechanism. (crank fixed)

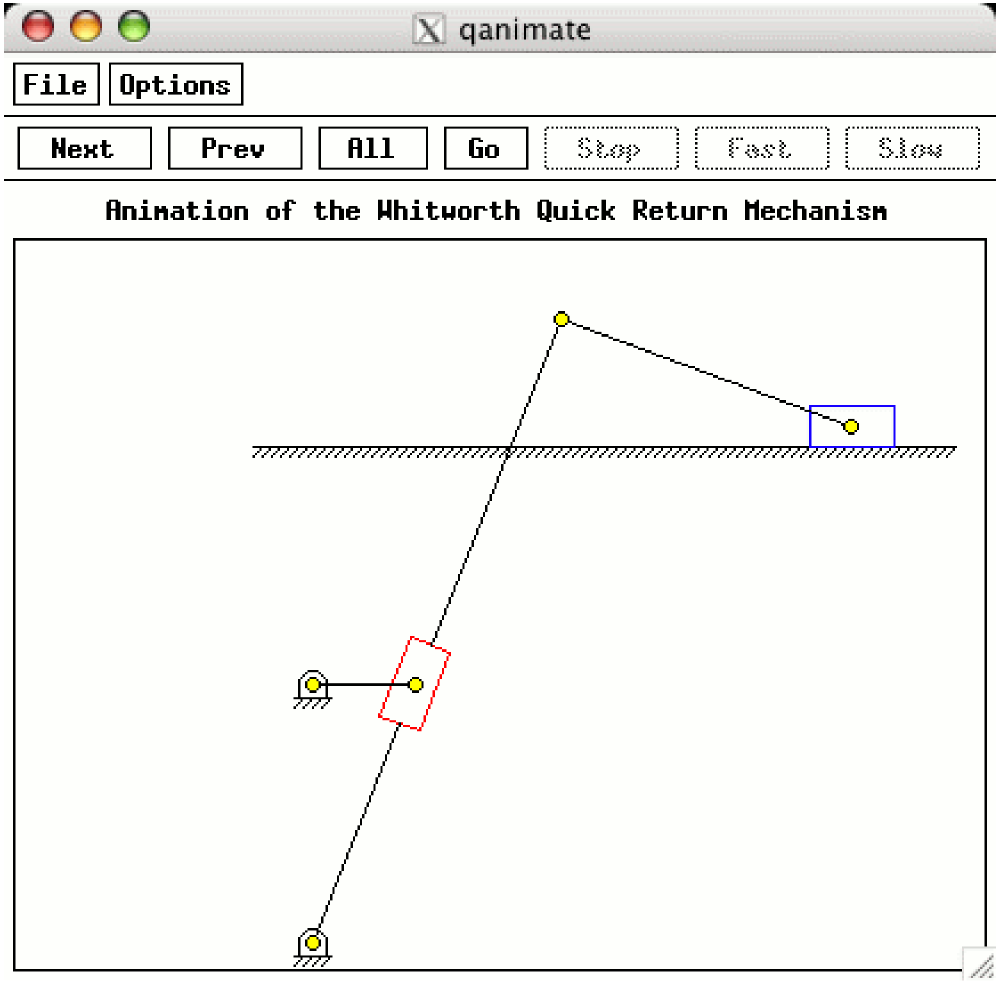


Application
Inversion III -Link 3 Connecting rod fixed
Crank and slotted lever quick return mechanism

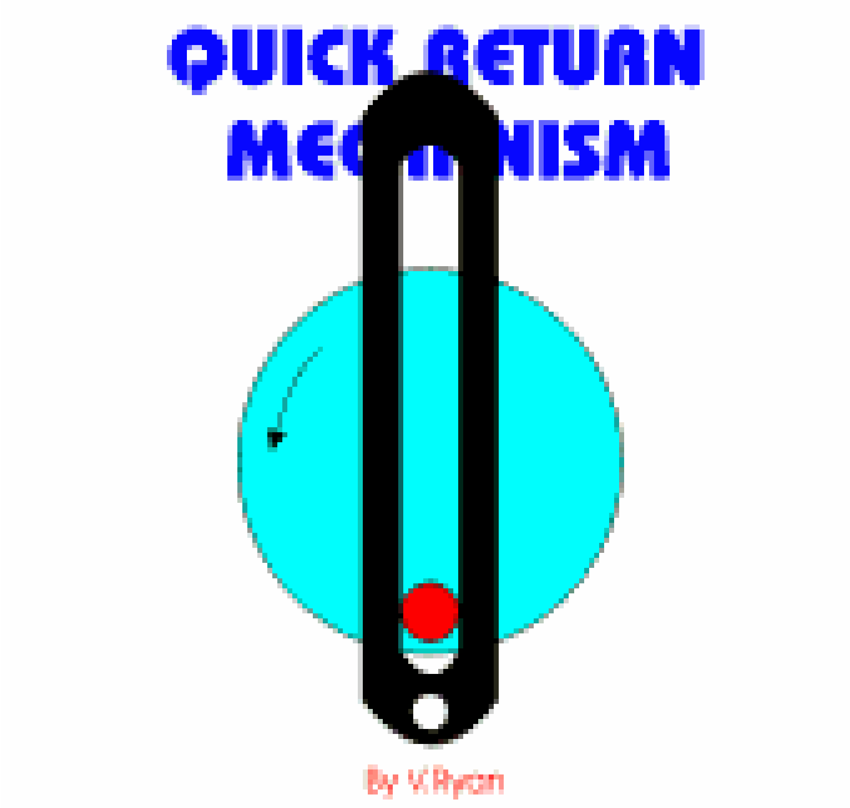


$$\frac{Time for forward stroke}{Time for return stroke} = \frac{B'\hat{o}_2 B''}{B''\hat{o}_2 B'} = \frac{\theta_1}{\theta_2}$$

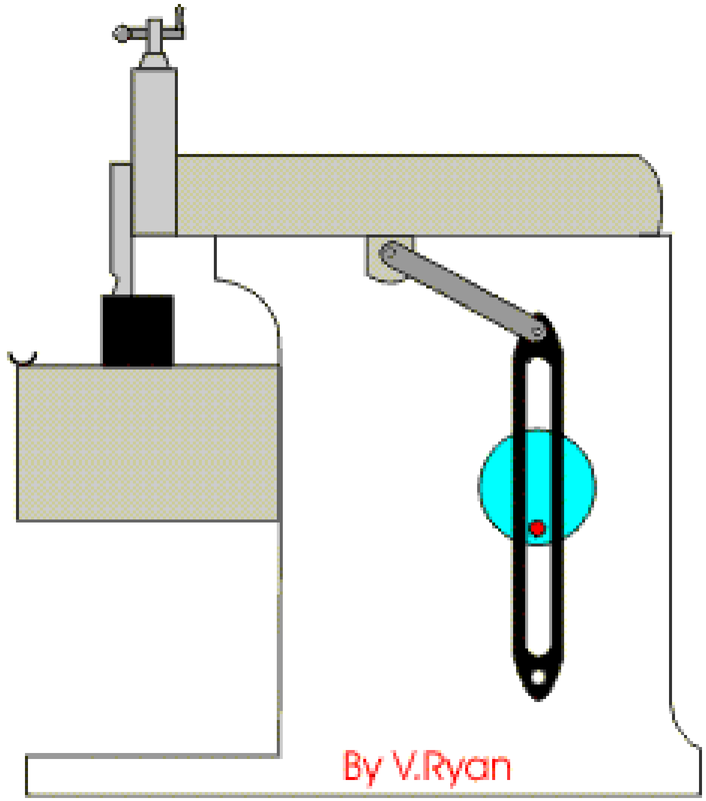
Crank and slotted lever quick return motion mechanism



Crank and slotted lever quick return motion mechanism

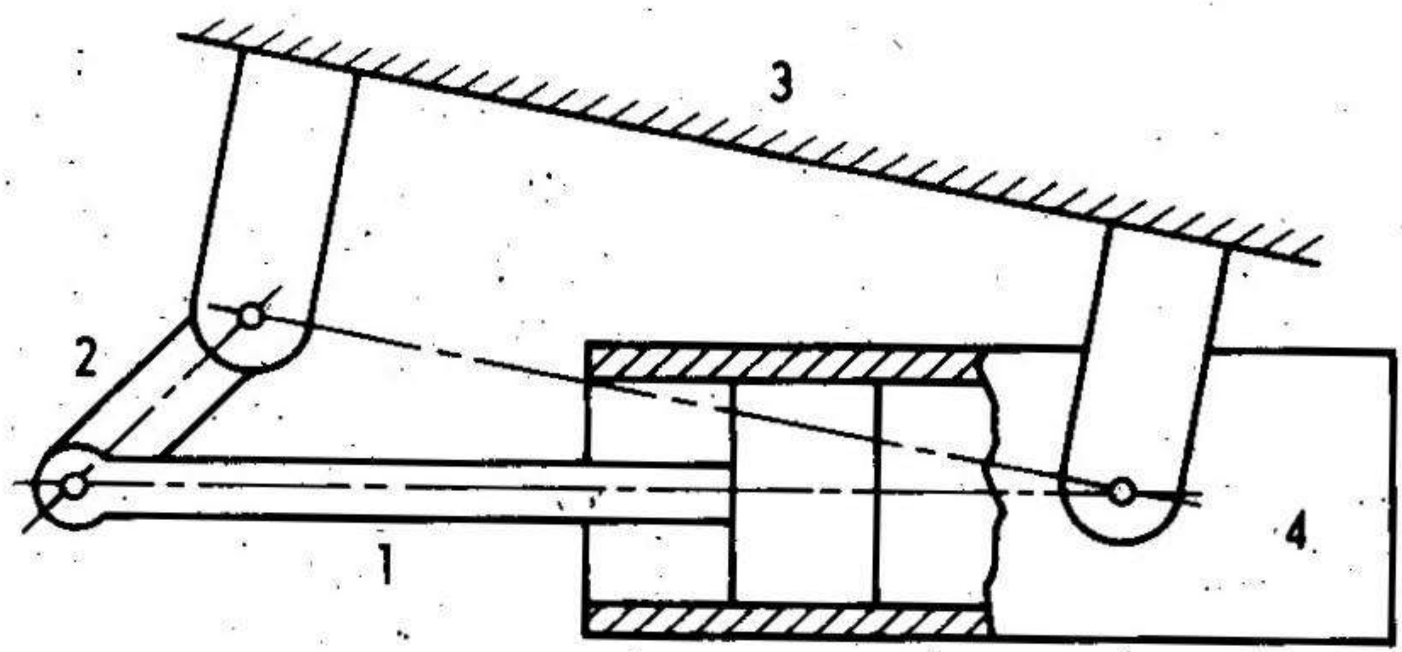


Application of Crank and slotted lever quick return motion mechanism

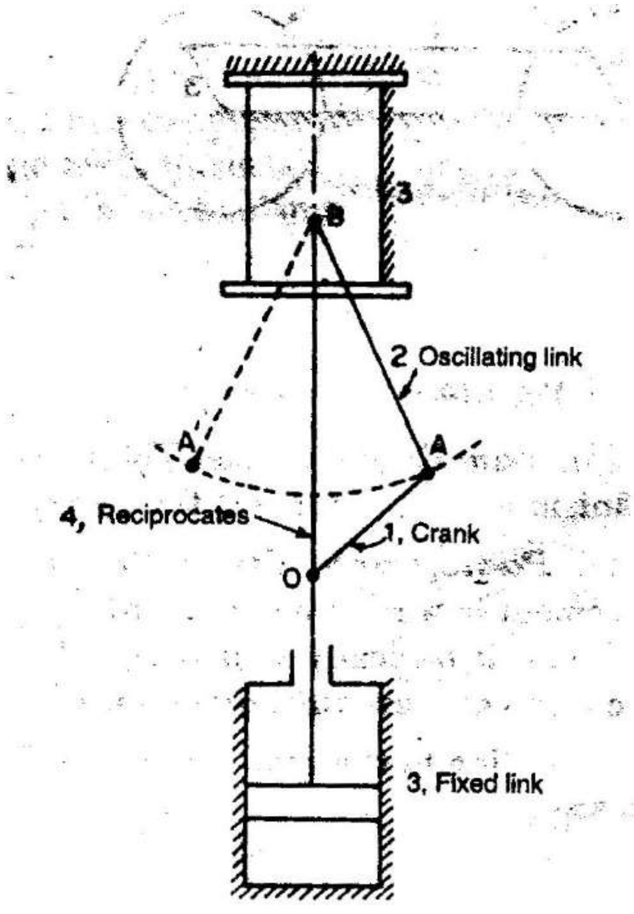


THE SHAPING MACHINE

Oscillating cylinder engine–III inversion of slider crank mechanism (connecting rod fixed)



Application
Inversion IV – Link 4 Slider fixed
Pendulum pump or bull engine



3. DOUBLE SLIDER CRANK CHAIN

It is a kinematic chain consisting of two turning pairs and two sliding pairs.

Link 1 Frame

Link 2 Slider -I

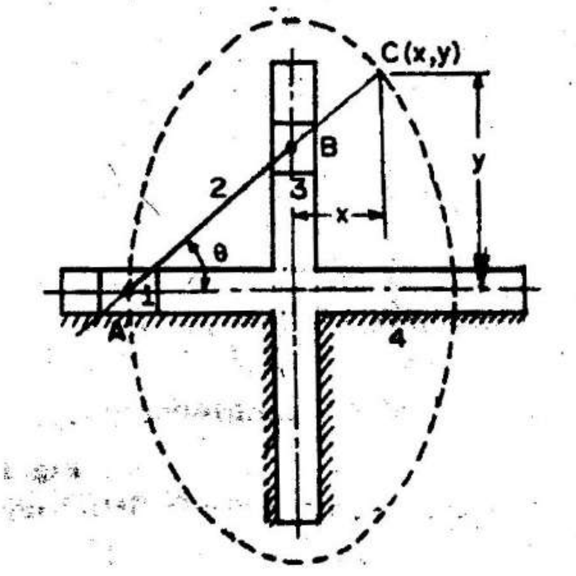
Link 3 Coupler

Link 4 Slider - II

$$\left(\frac{x}{q}\right)^2+\left(\frac{y}{p}\right)^2=\cos^2\theta+\sin^2\theta=1$$

Inversion I – Frame Fixed
Double slider crank mechanism

Elliptical trammel
AC = p and BC = q, then,
x = q.cosθ and y = p.sinθ.
Rearranging,

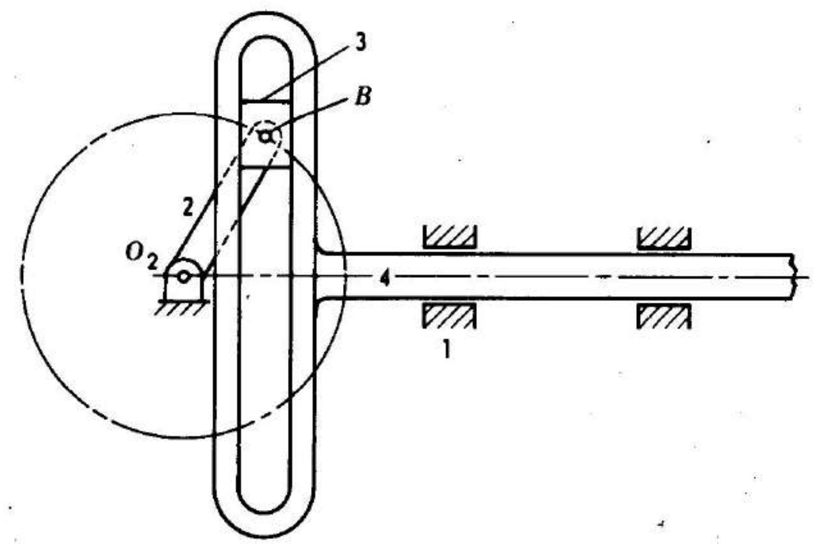


$$\left(\frac{x}{q}\right)^2+\left(\frac{y}{p}\right)^2=\cos^2\theta+\sin^2\theta=1$$

Inversion II – Slider - I Fixed

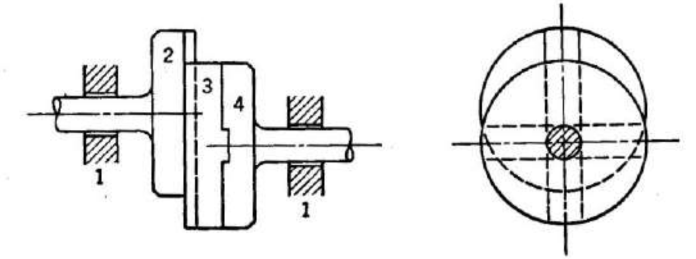
SCOTCH –YOKE MECHANISM

Turning pairs –1&2, 2&3; Sliding pairs – 3&4, 4&1



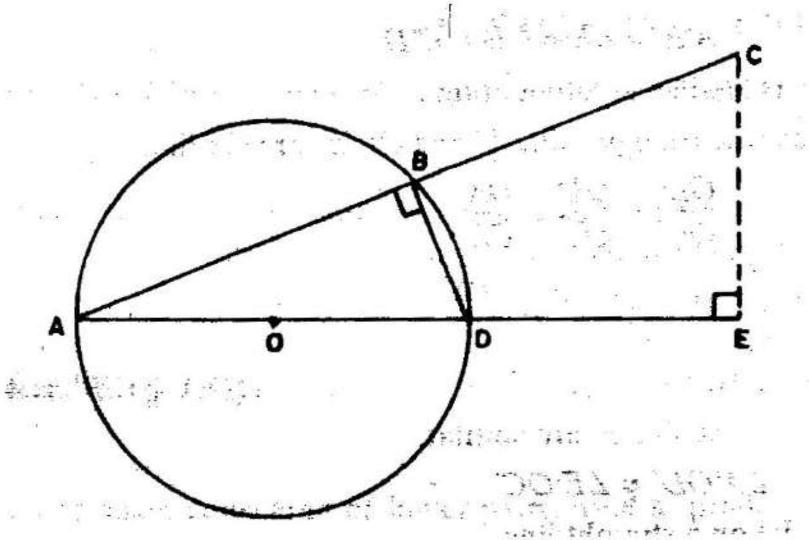
Inversion III – Coupler Fixed

OLDHAM COUPLING



Other Mechanisms
1.Straight line motion mechanisms

Condition for perfect steering



Locus of pt.C will be a straight line, $\perp$ to AE if, $AB \times AC$ is constant.

Proof:

$$\Delta AEC \equiv \Delta ABD$$

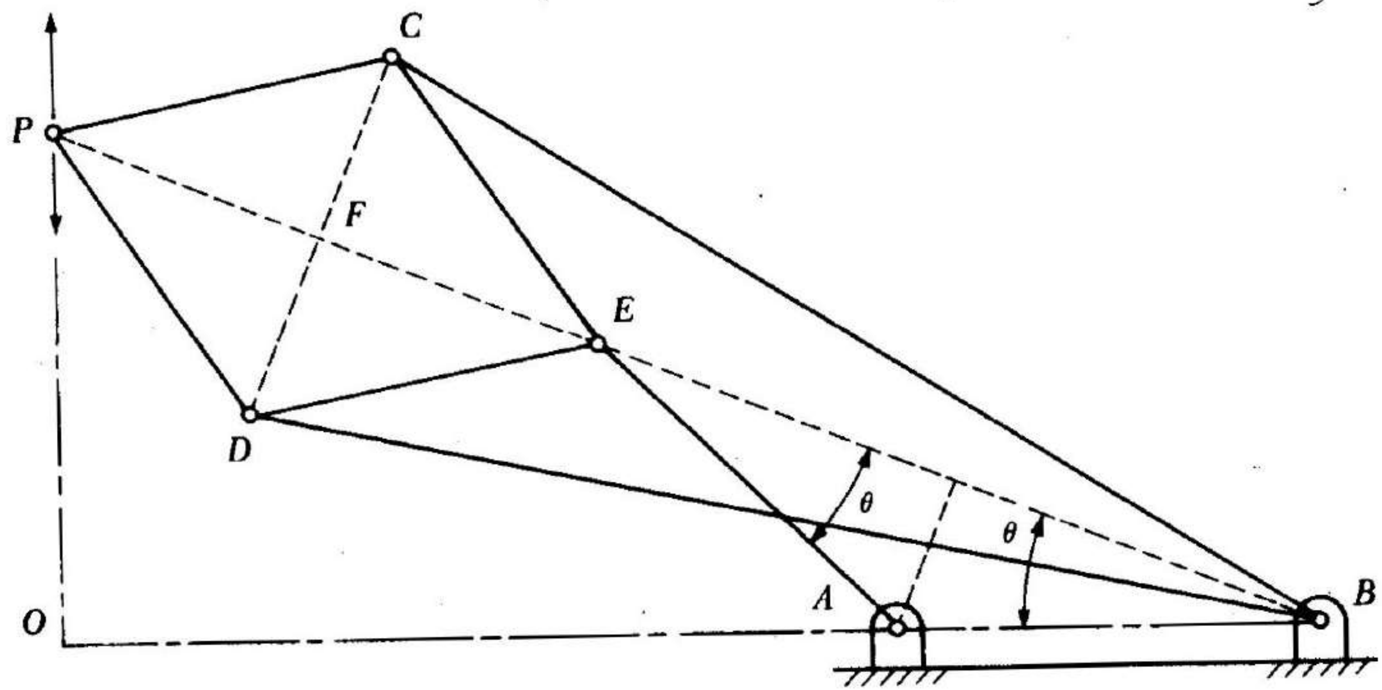
$$\therefore \frac{AD}{AC} = \frac{AB}{AE}$$

$$\therefore AE = \frac{AB \times AC}{AD}$$

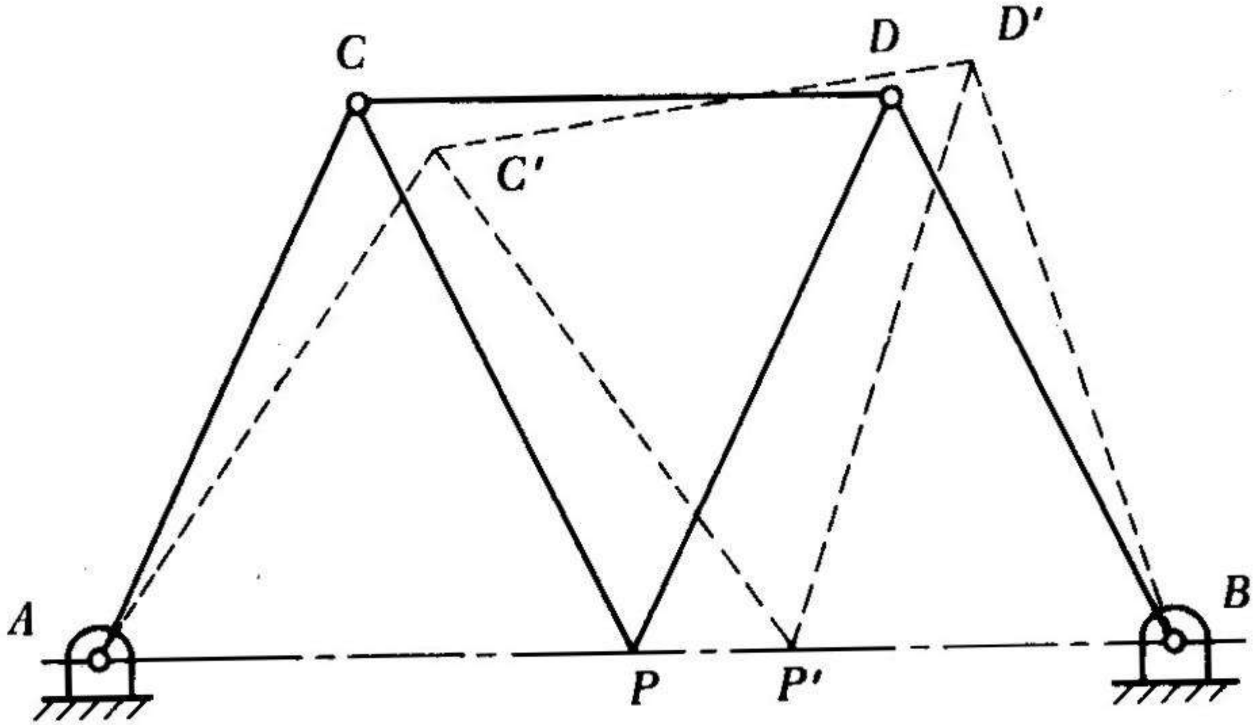
$$but AD = const.$$

$$\therefore AE = const., if AB \times AC = const.$$

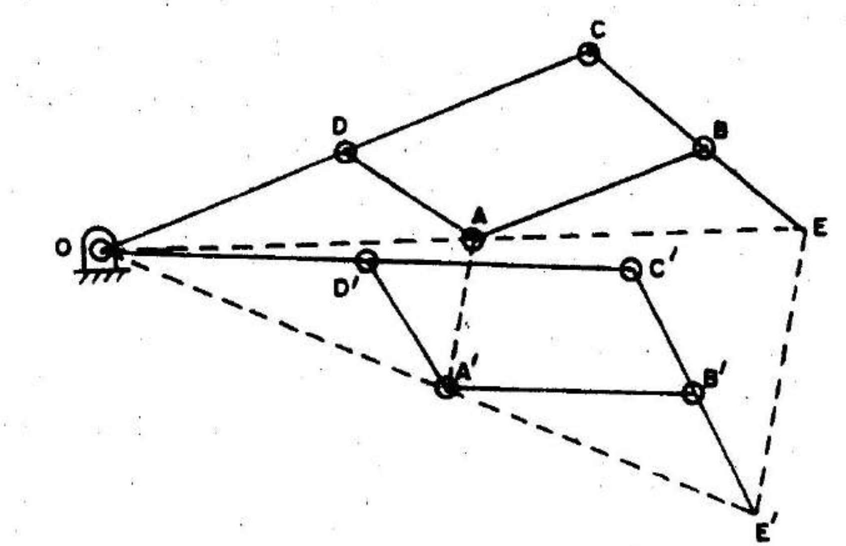
1.a) Peaucellier mechanism

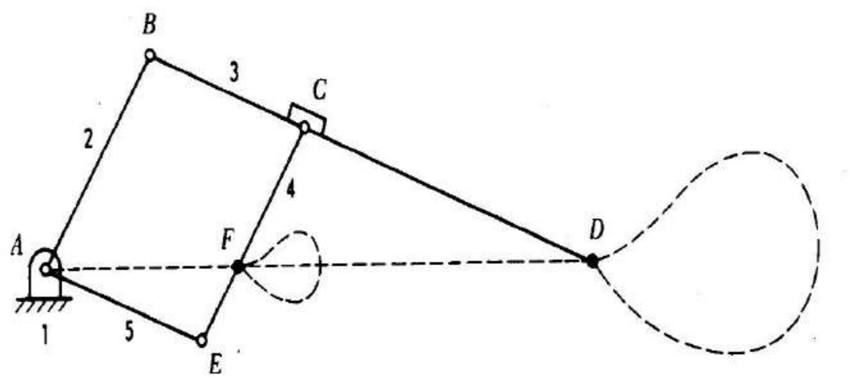


1.b) Robert's mechanism



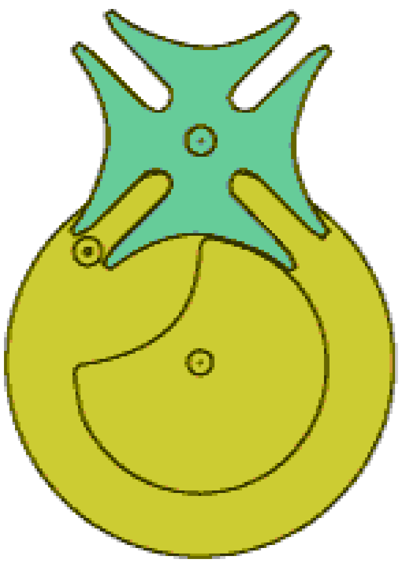
1.c) Pantograph

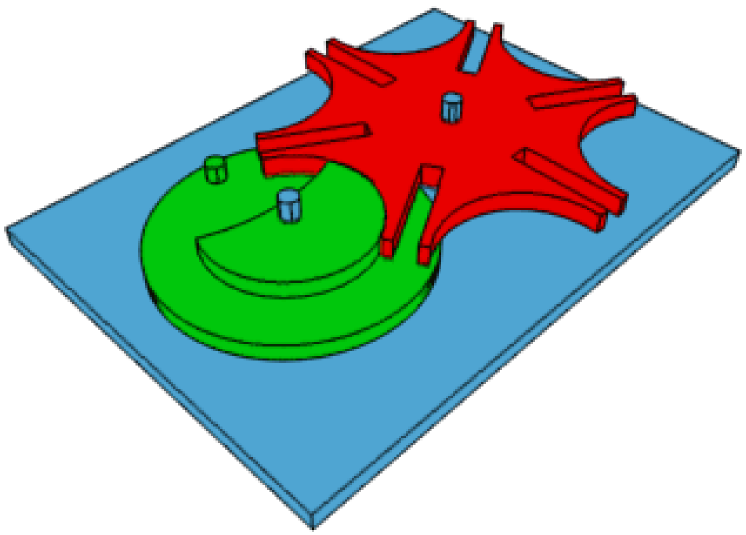


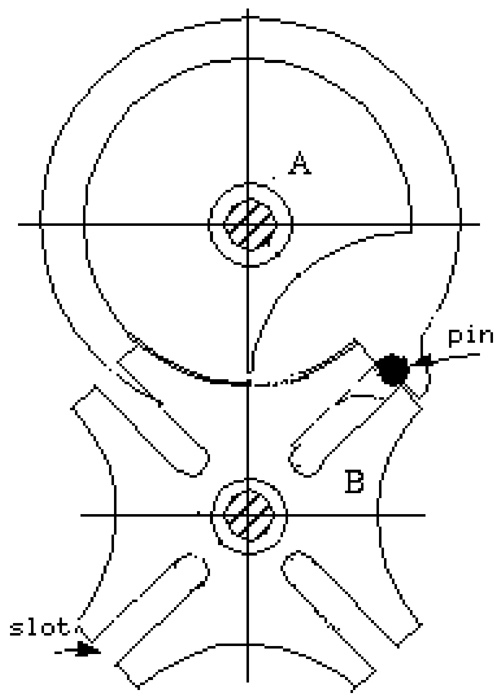


2.Indexing Mechanism

Geneva wheel mechanism

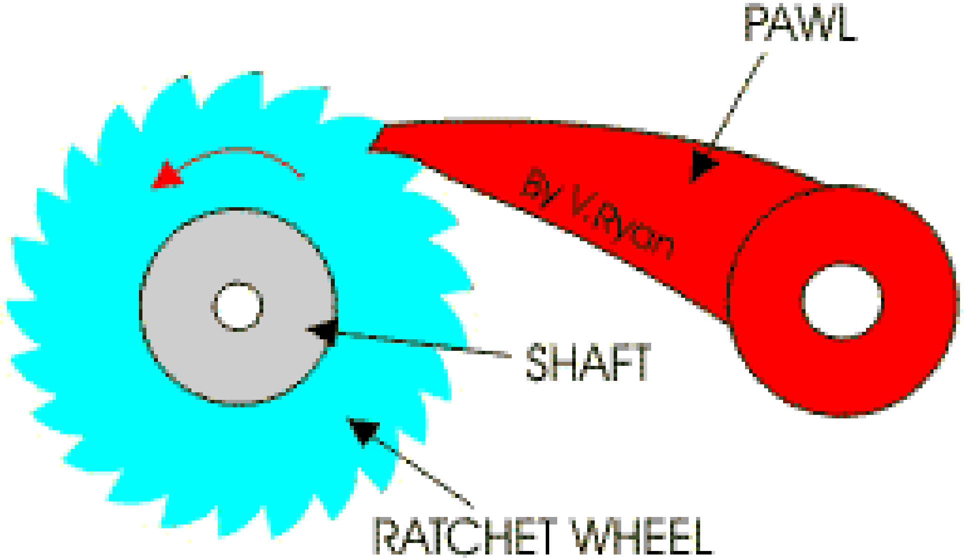


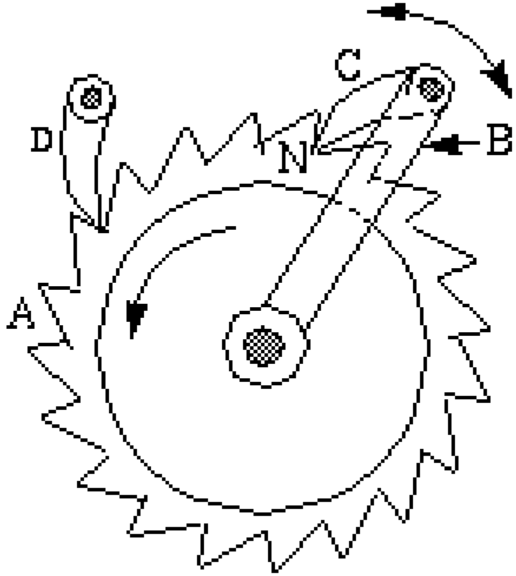




3.Ratchets and Escapements

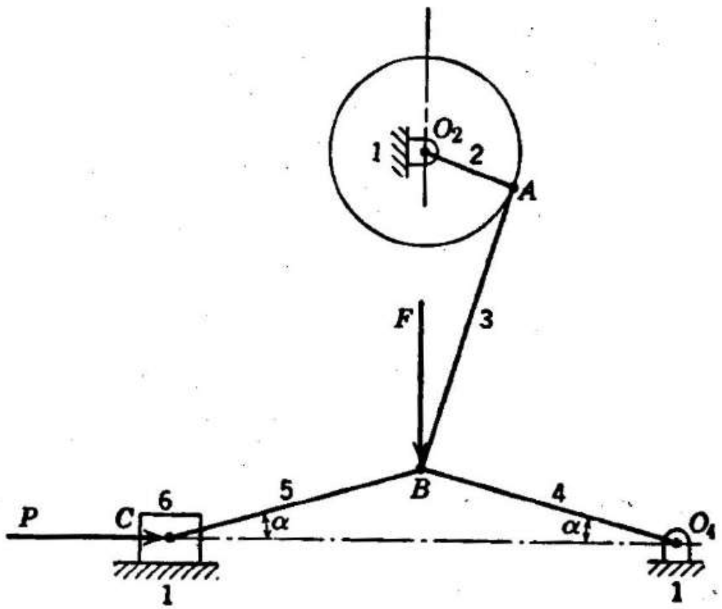
Ratchet and pawl mechanism





Application of Ratchet Pawl mechanism

4. Toggle mechanism



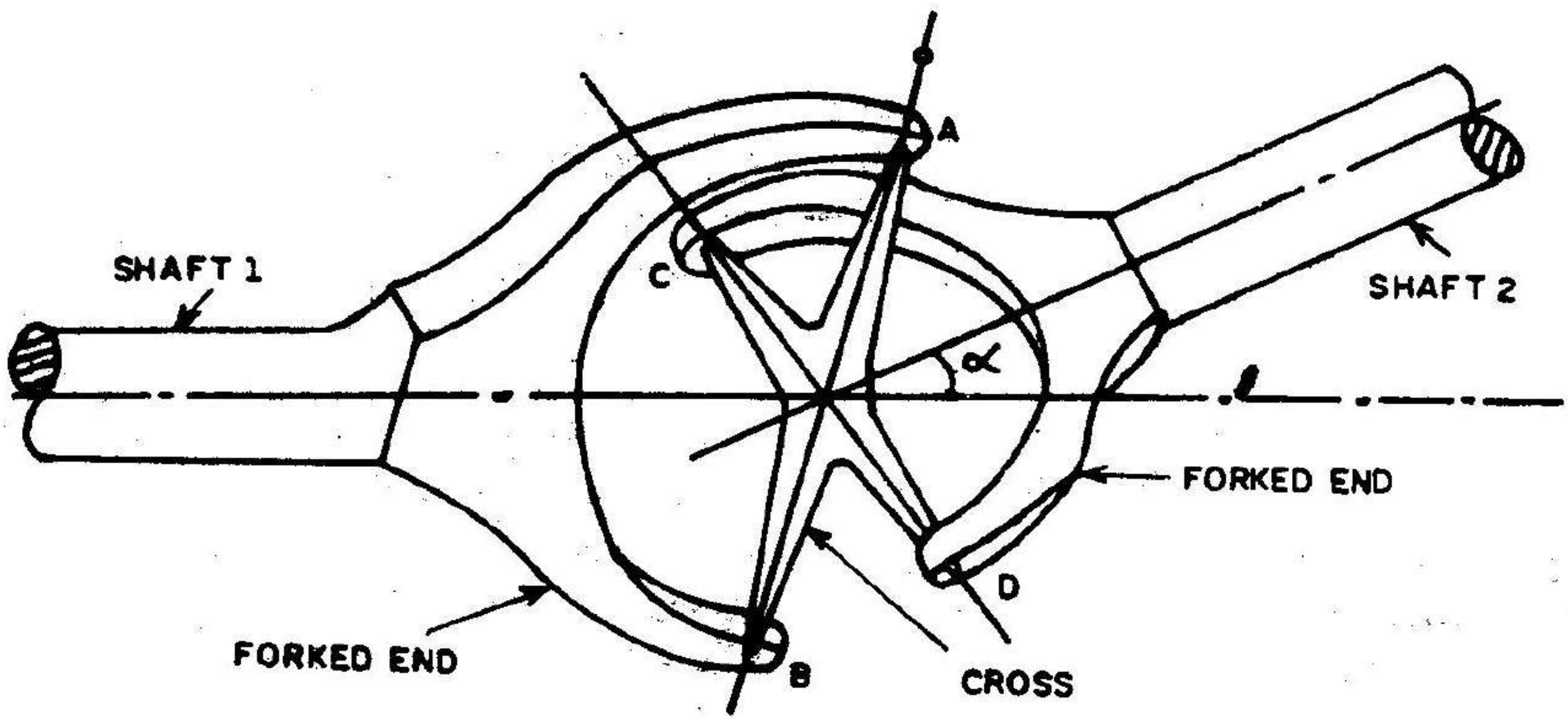
Considering the equilibrium condition of slider 6,

$$\tan\alpha = \frac{F/2}{P}$$

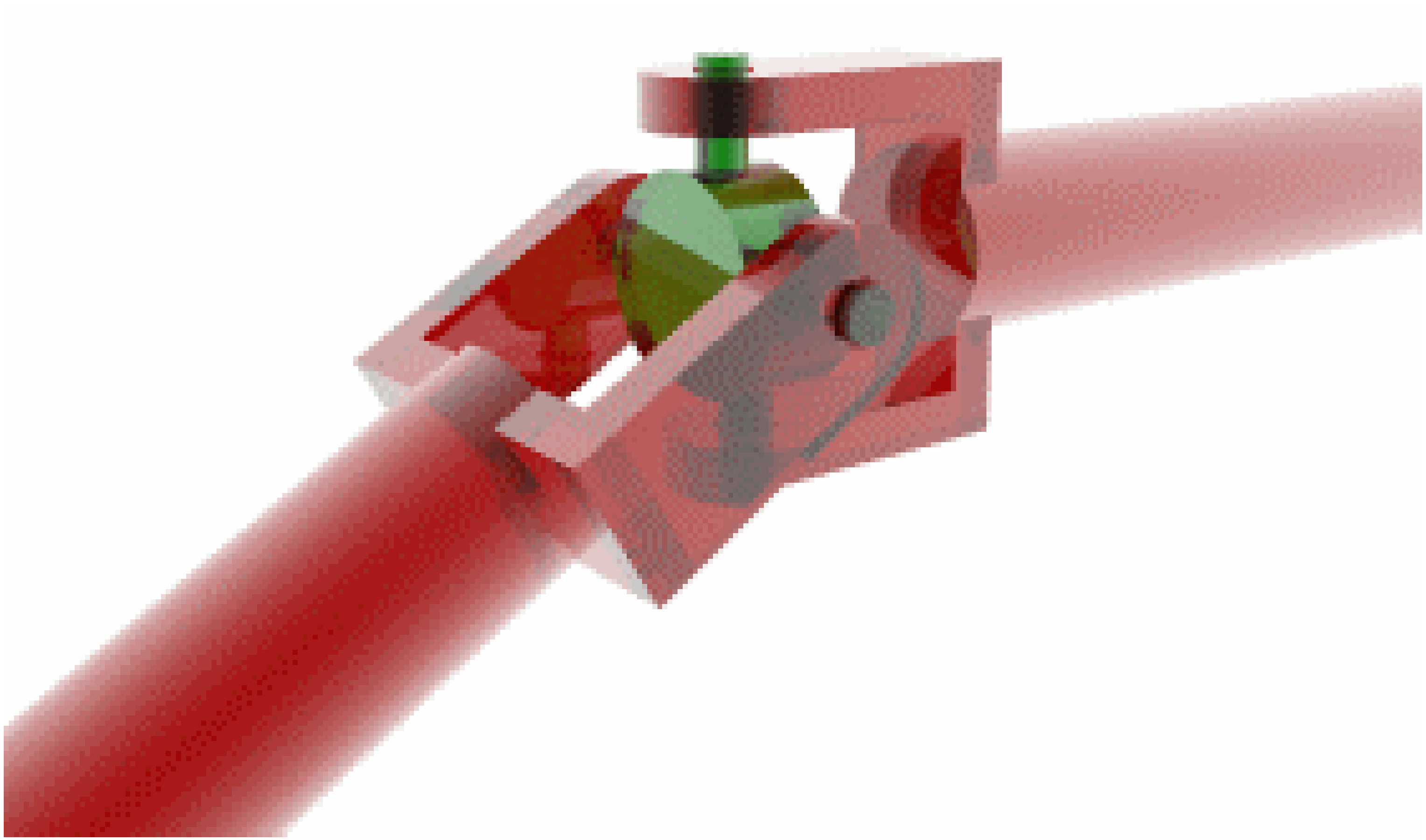
$$\therefore F = 2P\tan\alpha$$

For small angles of α, F is much smaller than P.

5.Hooke’s joint

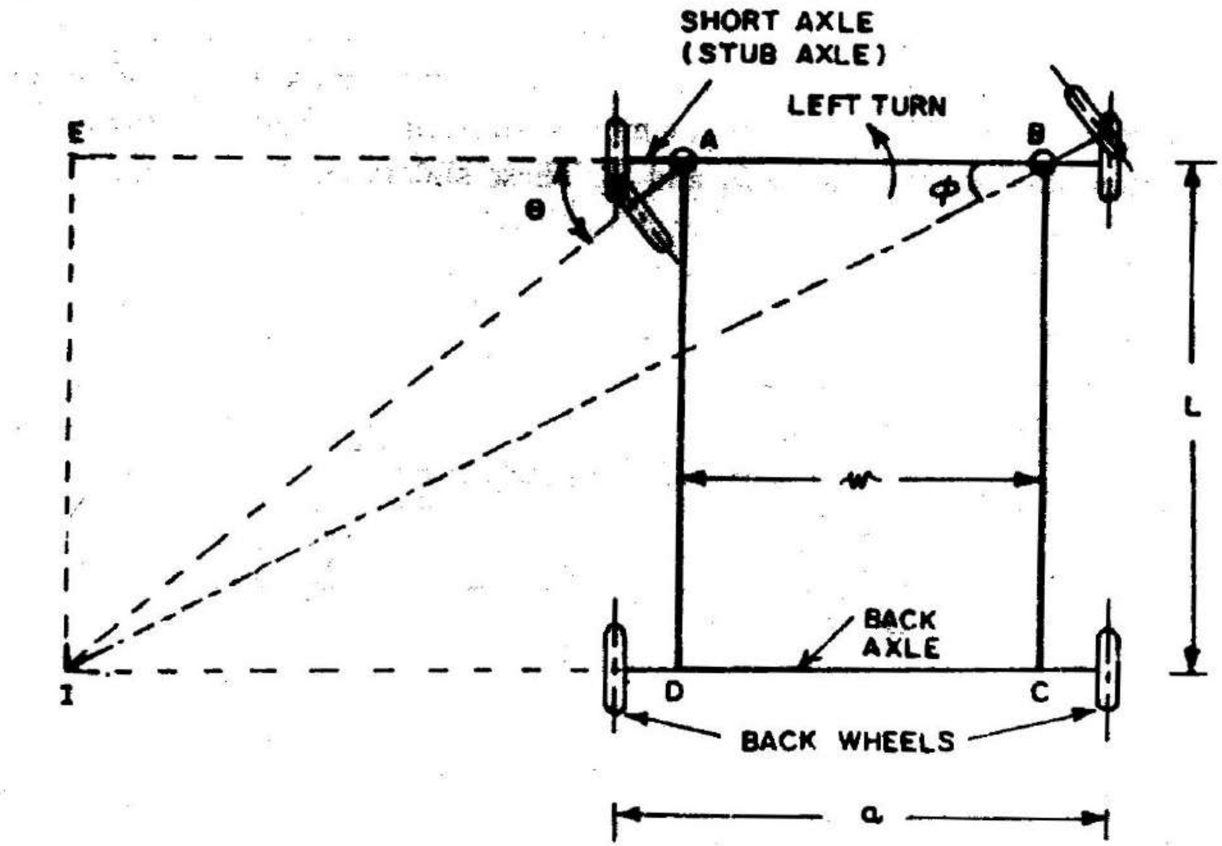


Hooke’s joint

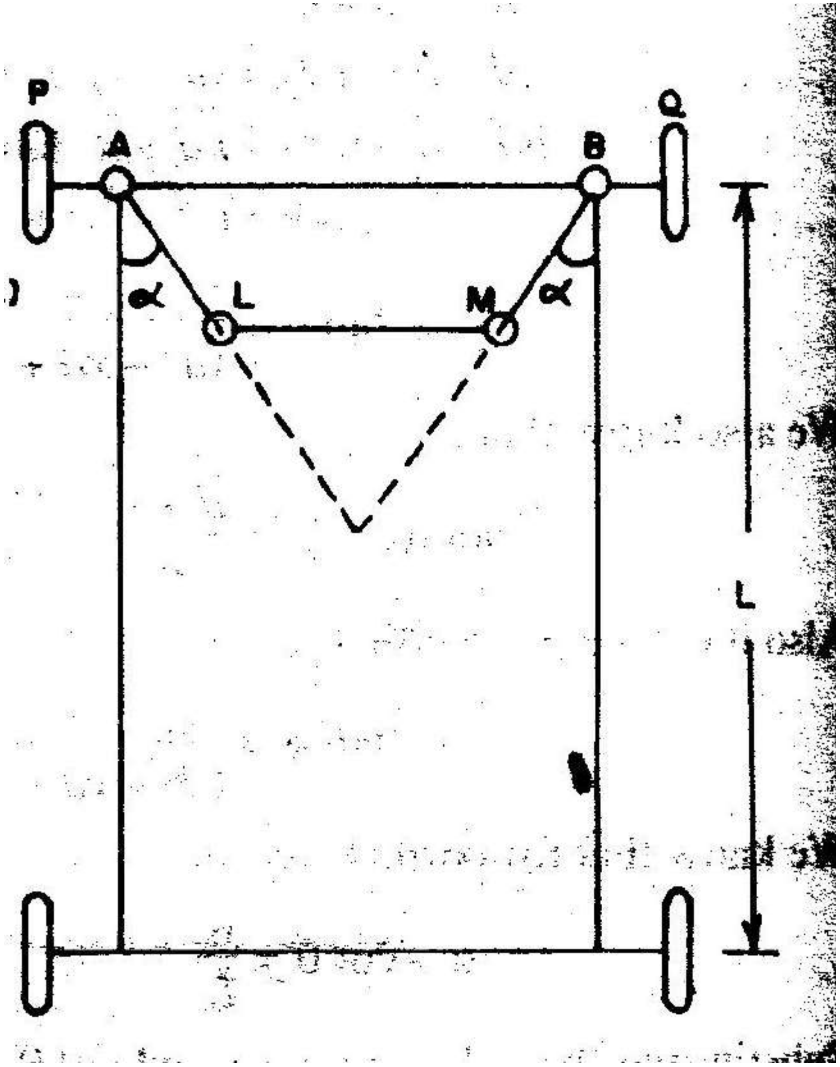


6.Steering gear mechanism

Condition for perfect steering



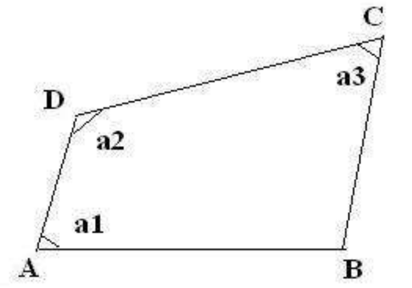
Ackermann steering gear mechanism

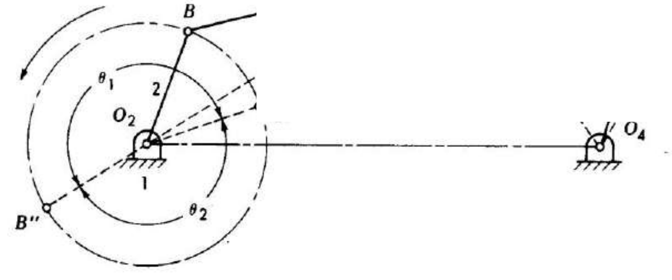


Mechanical Advantage

- Mechanical Advantage of the Mechanism at angle a2 = 0^0 or 180^0

- Extreme position of the linkage is known as toggle positions.

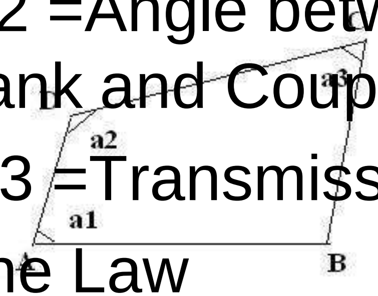




Transmission Angle

θ = a1=Crank Angle

γ = a2 =Angle between crank and Coupler



μ = a3 =Transmission angle

Cosine Law

$$a^2 + d^2 - 2ad \cos\theta = b^2 + c^2 - 2\,bc \cos\mu$$

Where a=AD, b=CD, c=BC, d=AB

Determine μ.

Design of Mechanism

- 1.Slider – Crank Mechanism

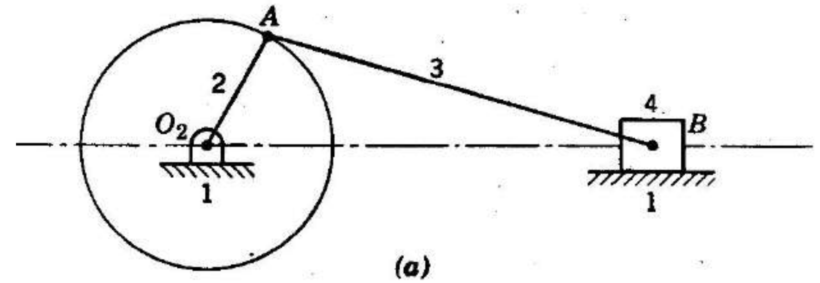
  Link Lengths, Stroke Length, Crank Angle specified.

- 2.Offset Quick Return Mechanism

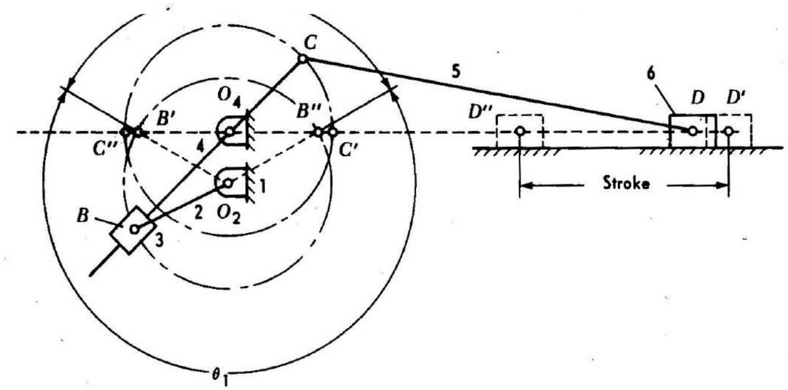
  Link Lengths, Stroke Length, Crank Angle, Time Ratio specified.

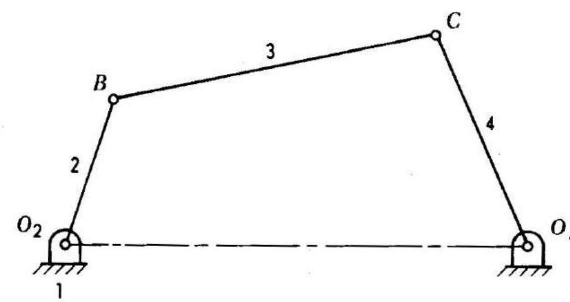
- 3.Four Bar Mechanism – Crank Rocker Mechanism

  Link Lengths and Rocker angle Specified.



(a)





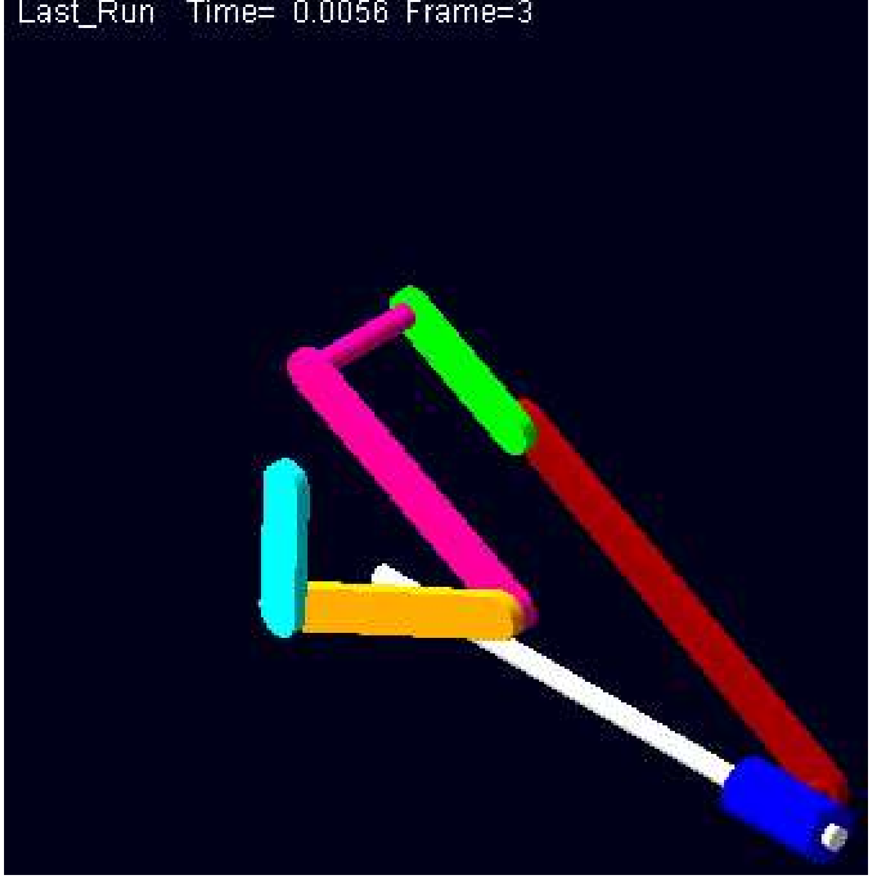
UNIT 2:

Mechanisms:

- **Quick return motion mechanisms:** Drag link mechanism, Whitworth mechanism and Crank and slotted lever Mechanism.
- **Straight line motion mechanisms:** Peaucellier's mechanism and Robert's mechanism.
- **Intermittent Motion mechanisms:** Geneva wheel mechanism and Ratchet and Pawl mechanism.
- Toggle mechanism, Pantograph, Ackerman steering gear mechanism.

Quick return motion mechanisms

- Quick return mechanisms are used in machine tools such as shapers and power driven saws for the purpose of giving the reciprocating cutting tool, a slow cutting stroke and a quick return stroke with a constant angular velocity of the driving crank.
- Some of the common types of quick return motion mechanisms are
  1. **Drag link mechanism**
  2. **Whitworth quick return motion mechanism**
  3. **Crank and slotted lever quick return motion mechanism**
- The ratio of time required for the cutting stroke to the time required for the return stroke is called the time ratio and is greater than unity.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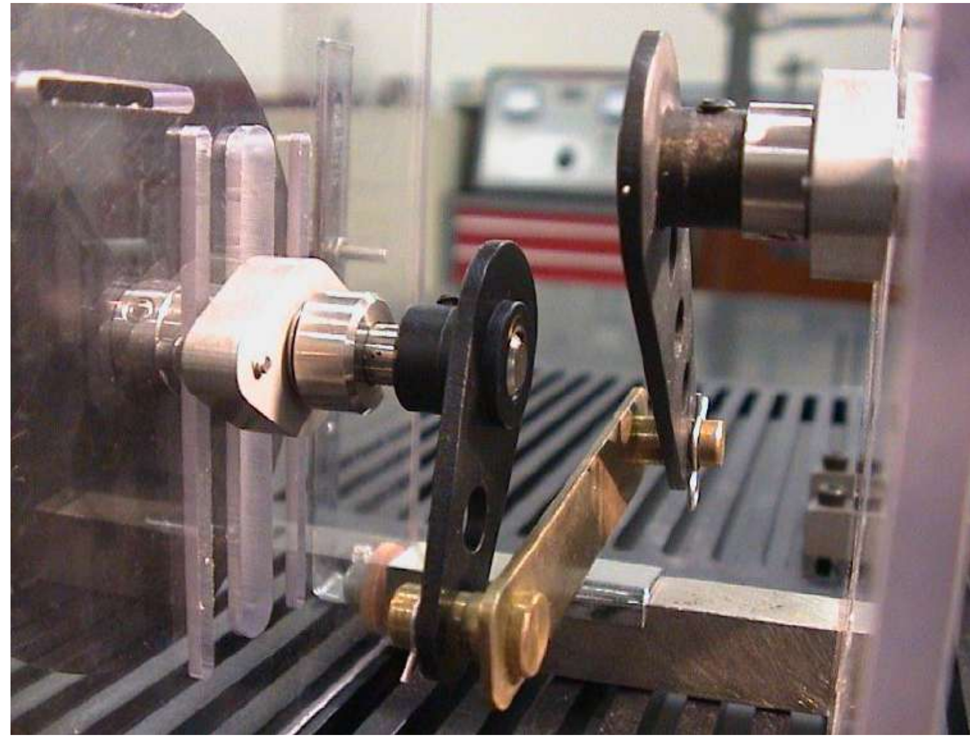
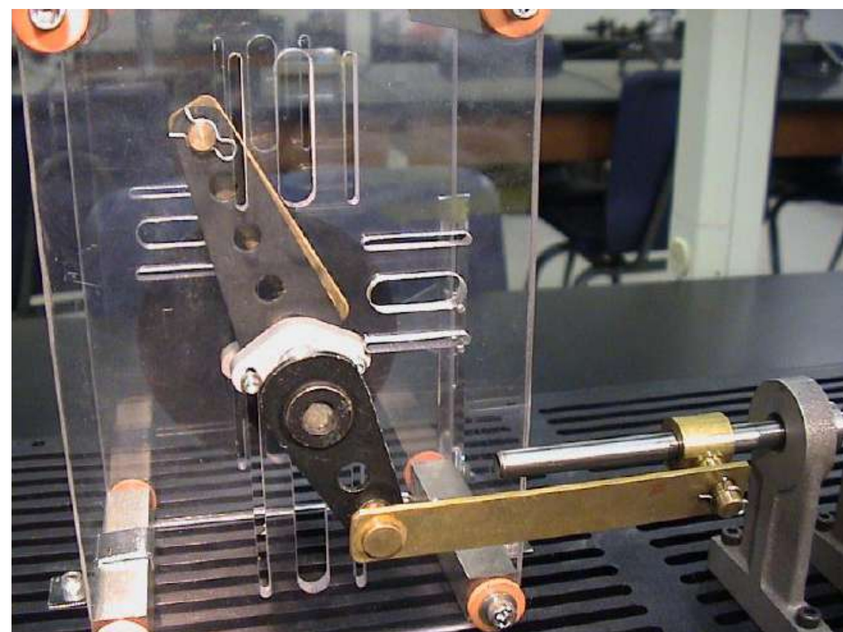
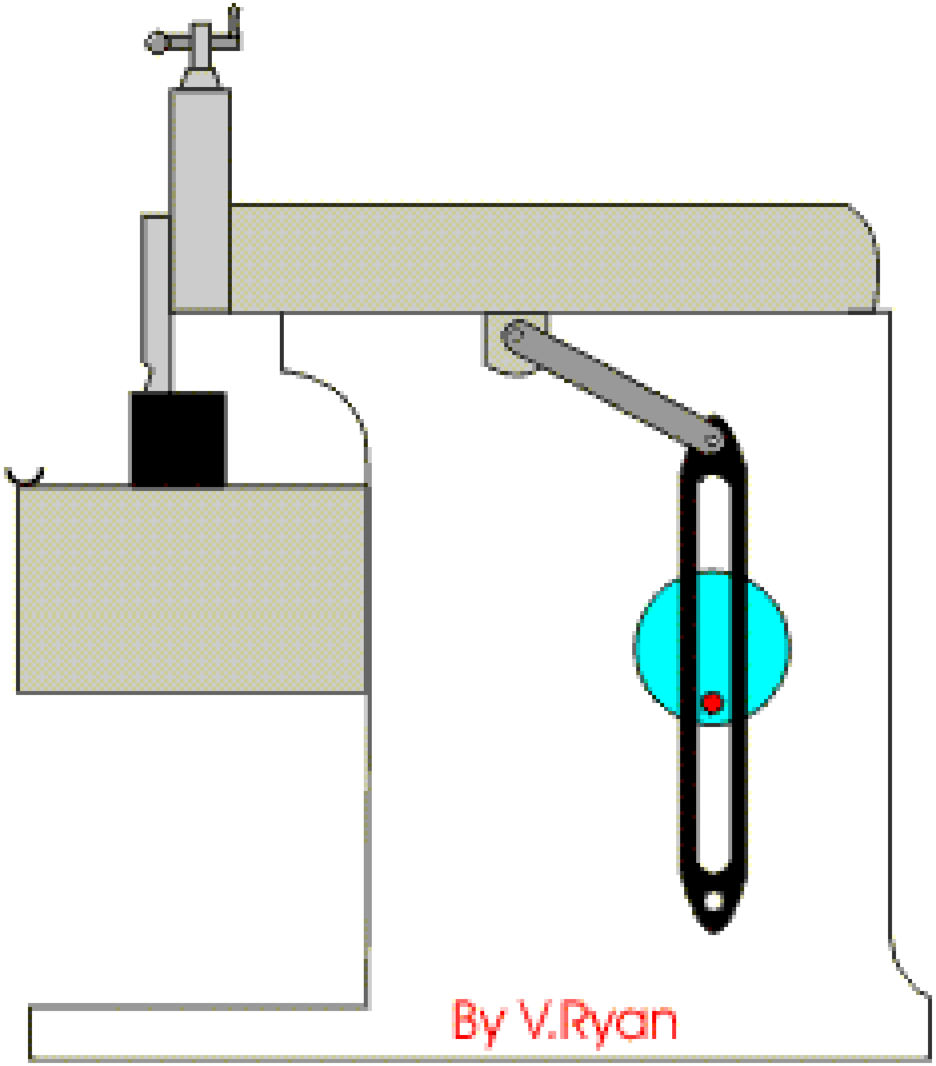
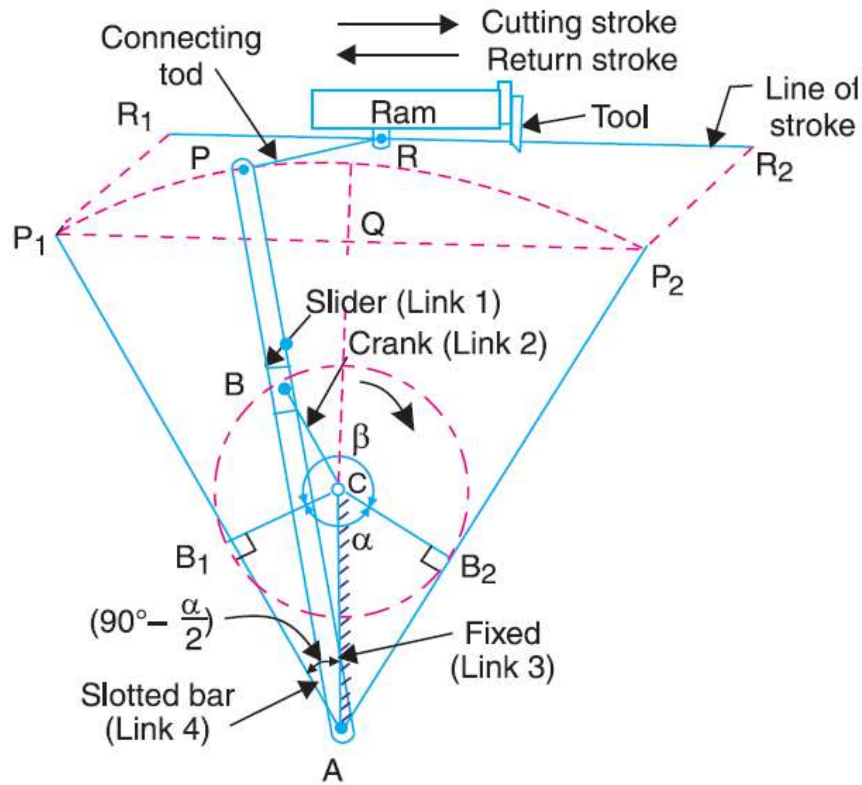




Crank and slotted lever quick return motion mechanism.

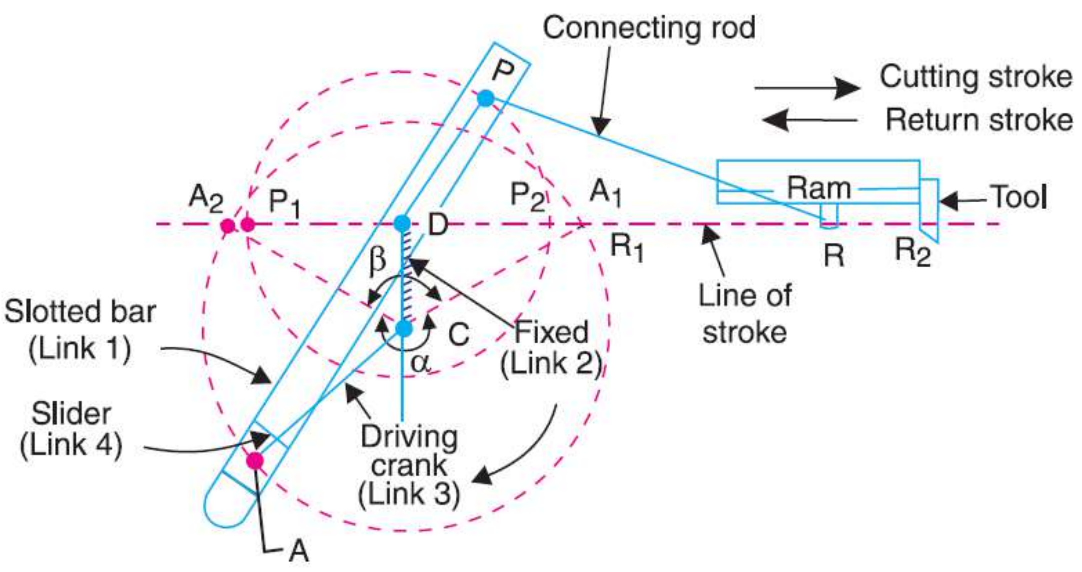


THE SHAPING MACHINE

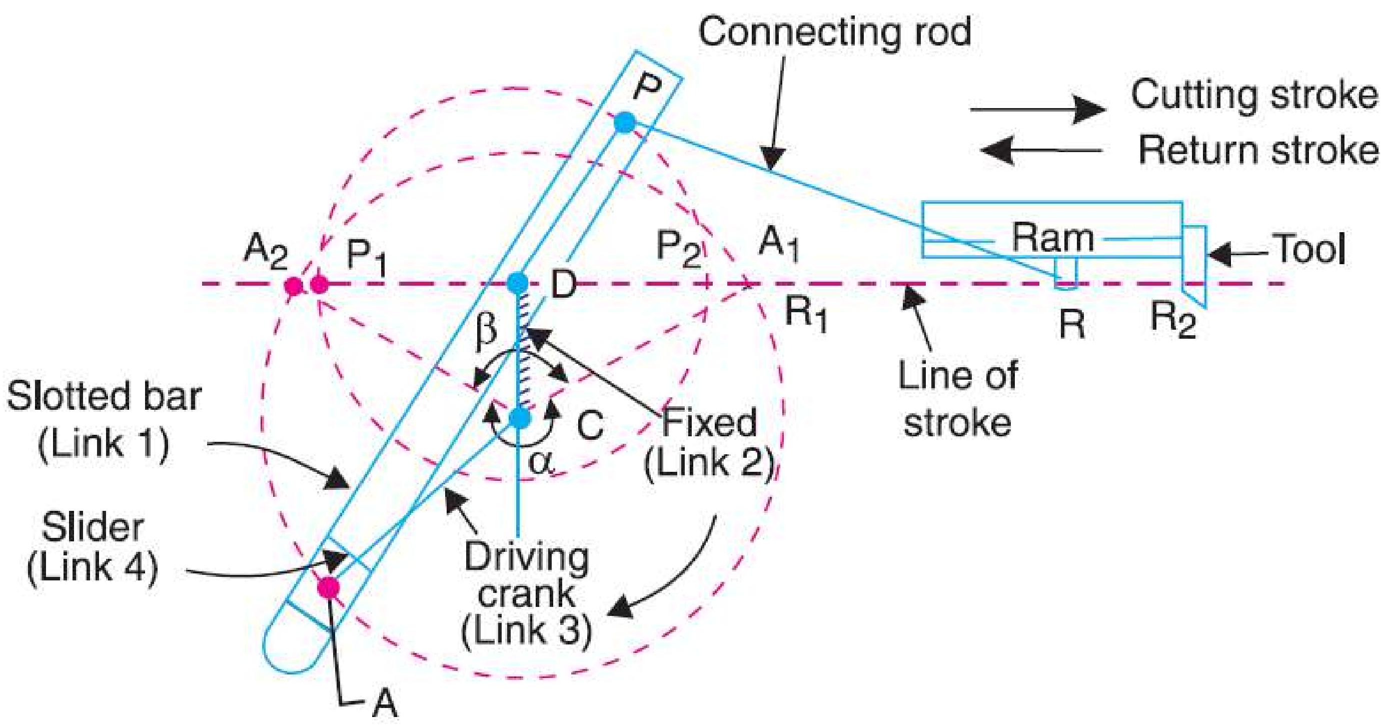


Whitworth quick return motion mechanism

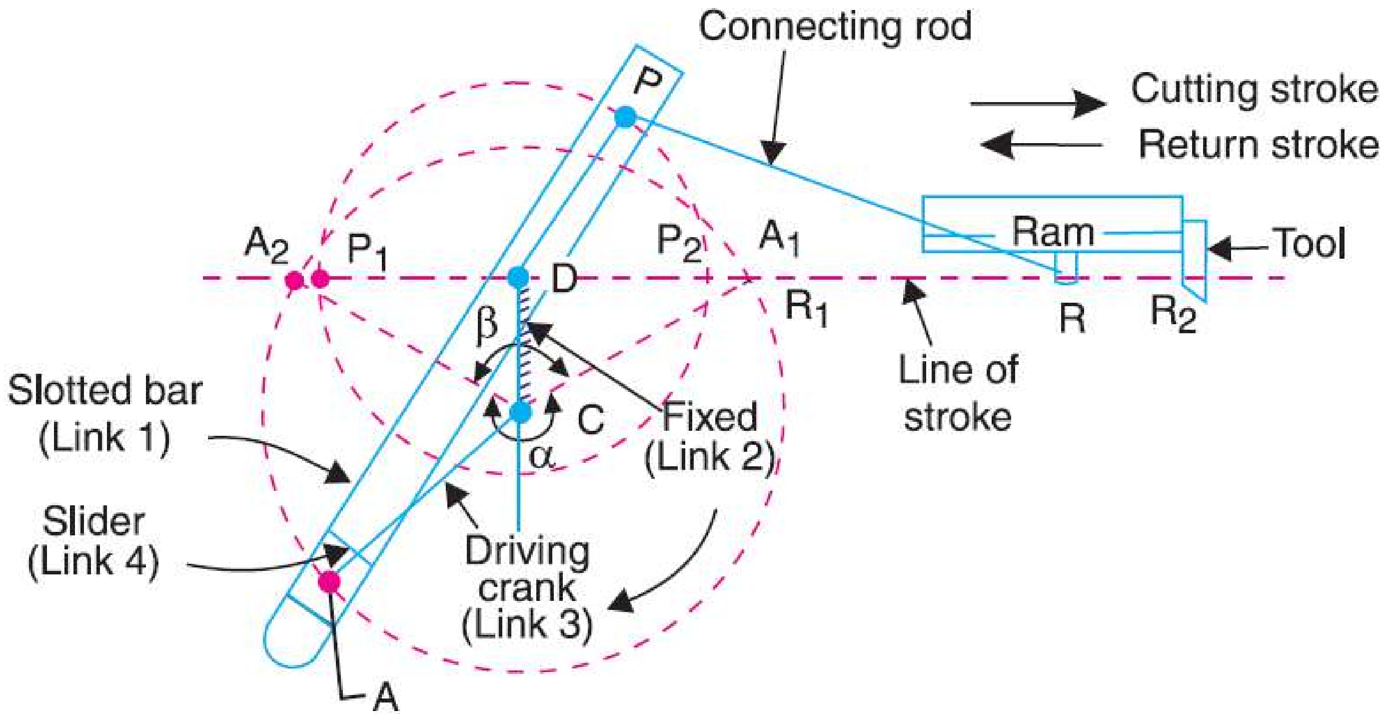
- *This mechanism is mostly used in shaping* and slotting machines.
- In this mechanism, the link *CD (link 2) forming the turning pair is fixed, as*
- shown in Fig.
- The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank *CA (link 3) rotates at a uniform angular speed.*
- *The slider (link 4) attached to the crank pin at A* slides along the slotted bar *PA (link 1) which oscillates at a pivoted point D.*
- *The connecting rod PR* carries the ram at *R to which a cutting tool is fixed.*
- *The motion of the tool is constrained along the* line *RD produced, i.e. along a line passing through D and perpendicular to CD.*



- When the driving crank *CA moves from the position CA*1 to *CA*2 (or the link *DP from the* position *DP*1 to *DP*2) through an angle in the clockwise direction, the tool moves from the left hand end of its stroke to the right hand end through a distance 2*PD.*
- Now when the driving crank moves from the position *CA*2 to *CA*1 (or the link *DP from DP*2 to *DP*1) through an angle in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end.
- A little consideration will show that the time taken during the left to right movement of the ram (*i.e. during forward or cutting stroke) will be equal to the time taken by the driving crank to move* from *CA*1 to *CA*2.



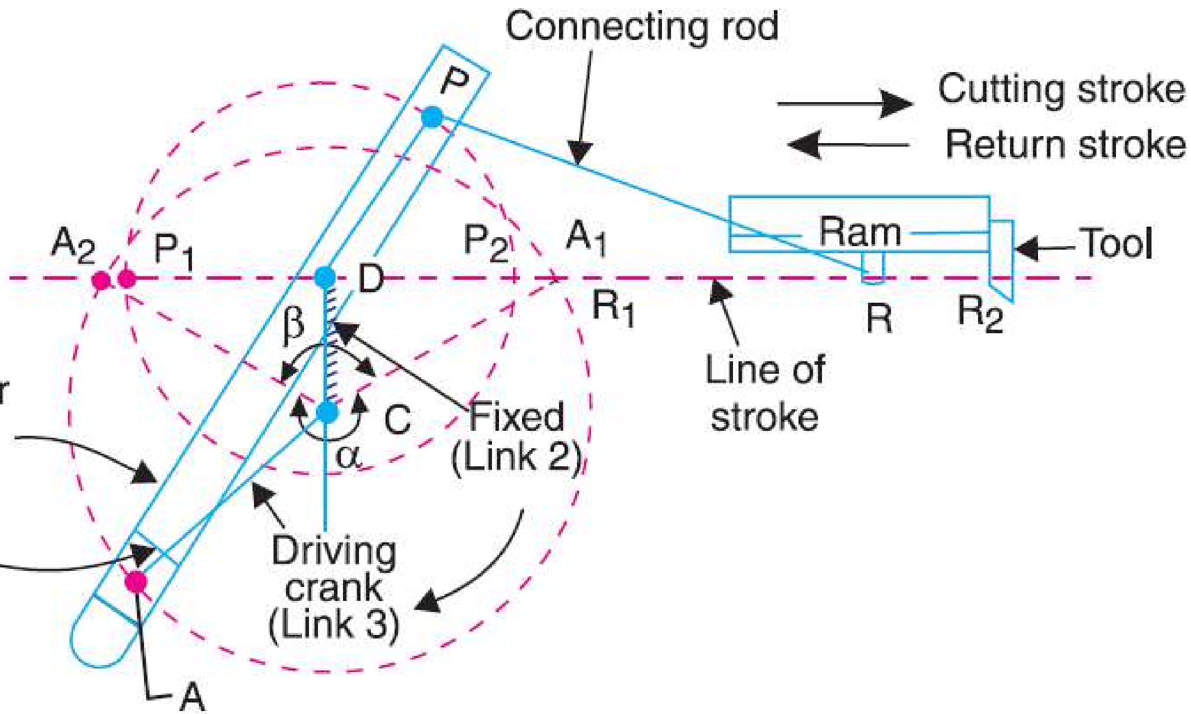
- Similarly, the time taken during the right to left movement of the ram (or during the idle or return stroke) will be equal to the time taken by the driving crank to move from *CA*2 to *CA*1.
- Since the crank link *CA rotates at uniform angular velocity therefore time taken during the* cutting stroke (or forward stroke) is more than the time taken during the return stroke.
- In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke.



- The ratio between the time taken during the cutting and return strokes is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360° - \alpha} \quad \text{or} \quad \frac{360° - \beta}{\beta}$$

Note. In order to find the length of effective stroke $R_1 R_2$, mark $P_1 R_1 = P_2 R_2 = PR$. The length of effective stroke is also equal to 2 PD.



UNIT - III

Straight Line Mechanisms

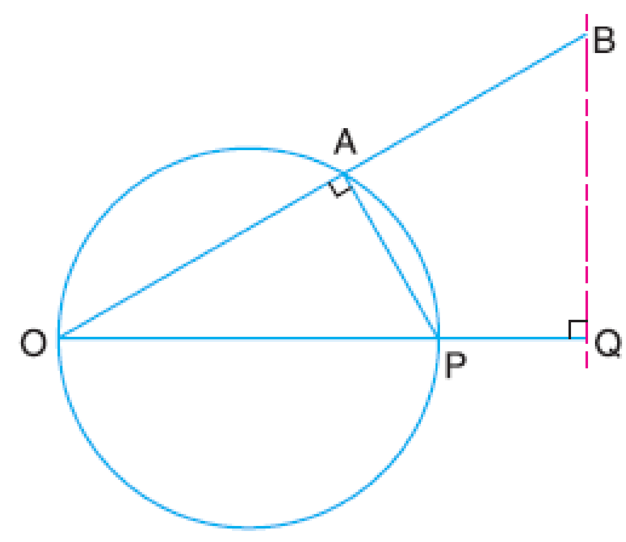
- One of the most common forms of the constraint mechanisms is that it permits only relative motion of an oscillatory nature along a straight line. The mechanisms used for this purpose are called *straight line mechanisms.*
- *These mechanisms are of the following two types:*
    1. in which only turning pairs are used, and
    2. in which one sliding pair is used.
- These two types of mechanisms may produce exact straight line motion or approximate straight line motion, as discussed in the following articles.

Condition for Exact Straight Line Motion Mechanisms

- The principle adopted for a mathematically correct or exact straight line motion is described in Fig.
- Let *O* be a point on the circumference of a circle of diameter *OP. Let OA be any chord and B is a point on OA produced, such* that

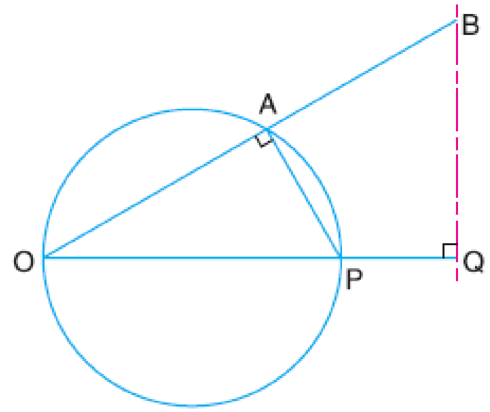
$$OA \times OB = constant$$

- Then the locus of a point *B will be a straight line* perpendicular to the diameter *OP.*



Proof:

- Draw *BQ perpendicular to OP produced. Join AP.* The triangles *OAP and OBQ are similar.*

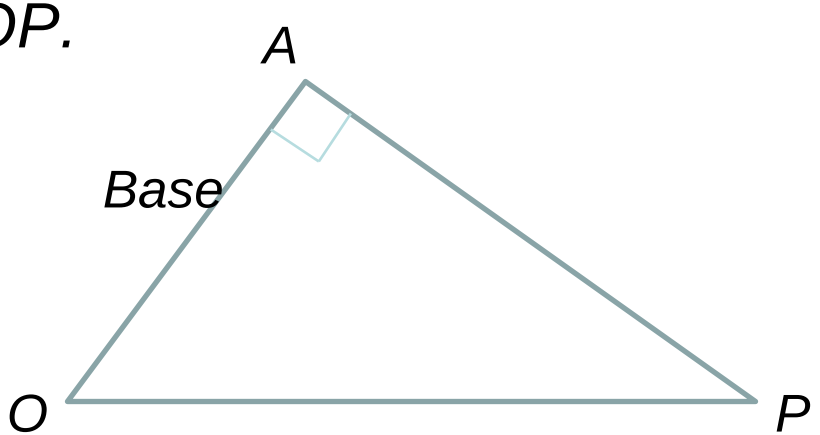


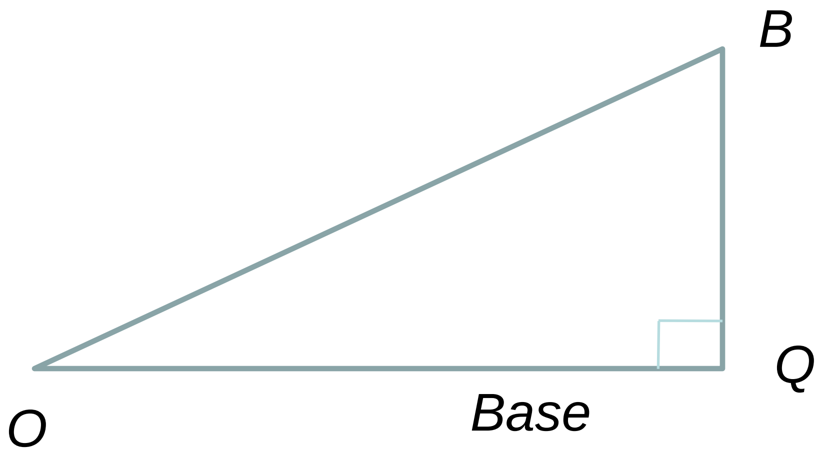
$$\therefore \qquad \frac{OA}{OP} = \frac{OQ}{OB}$$

$$\text{or} \qquad OP \times OQ = OA \times OB$$

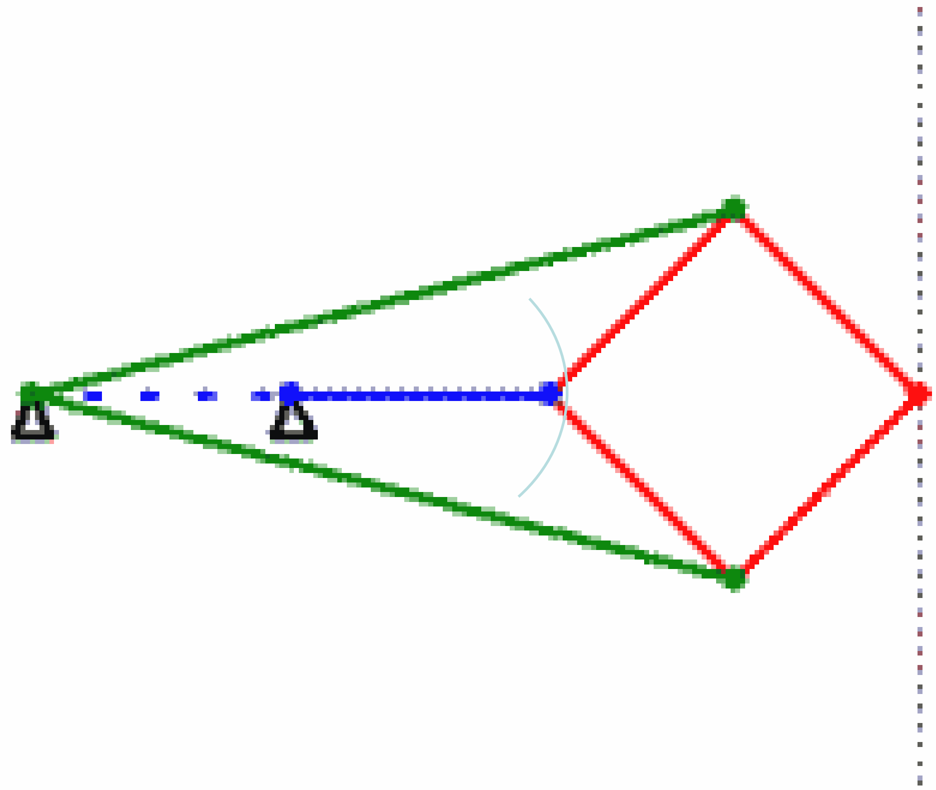
$$\text{or} \qquad OQ = \frac{OA \times OB}{OP}$$

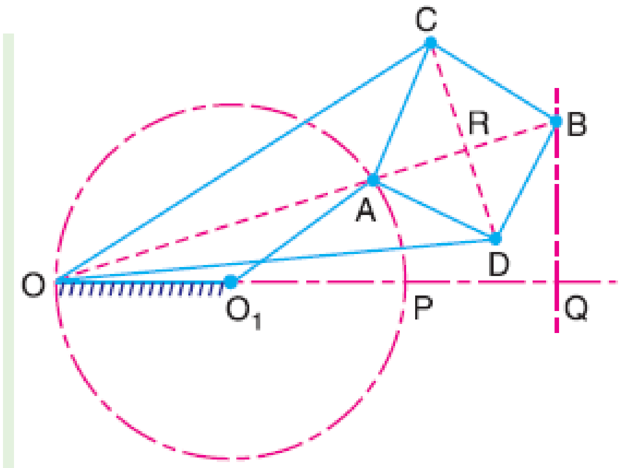
- But *OP is constant as it is the diameter of a circle, therefore,* if *OA × OB is constant, then OQ will be constant. Hence* the point *B moves along the straight path BQ which is perpendicular* to *OP.*





Peaucellier exact straight line motion mechanism:





Peaucellier exact straight line motion mechanism:

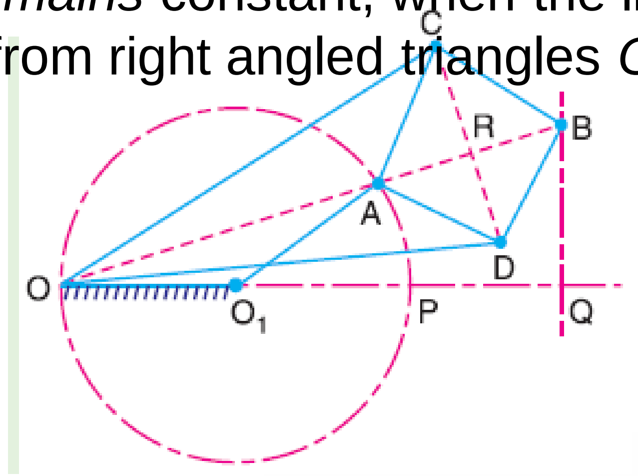
- *It consists of a fixed link* OO_1 and the other straight links O_1A, OC, OD, AD, *DB, BC and CA are connected by turning pairs at their intersections, as shown* in Fig.
- The pin at *A is constrained to move along the circumference* of a circle with the fixed diameter *OP, by means of* the link O_1A.
- *In Fig.,* $AC = CB = BD = DA$; $OC = OD$; and $OO_1 = O_1A$
- It may be proved that the product $OA \times OB$ *remains* constant, when the link O_1A *rotates. Join CD to bisect AB at R.* Now from right angled triangles *ORC and BRC, we have*

$$OC^2 = OR^2 + RC^2$$

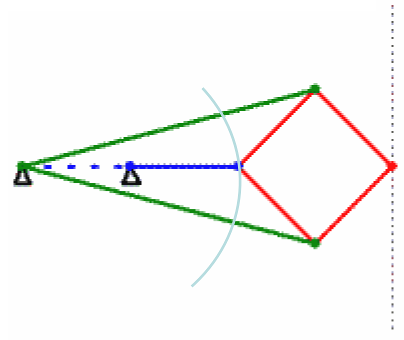
and $$BC^2 = RB^2 + RC^2$$

Subtracting equation (*ii*) from (*i*), we have

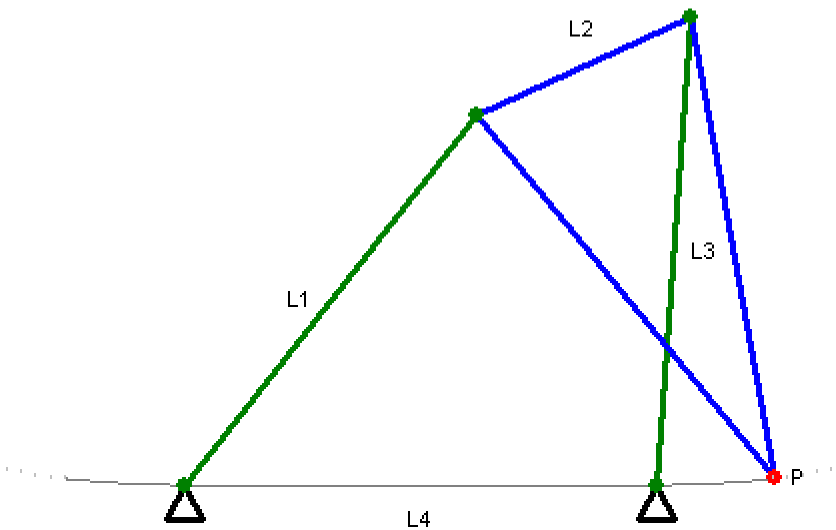
$$OC^2 - BC^2 = OR^2 - RB^2$$
$$= (OR + RB)(OR - RB)$$
$$= OB \times OA$$

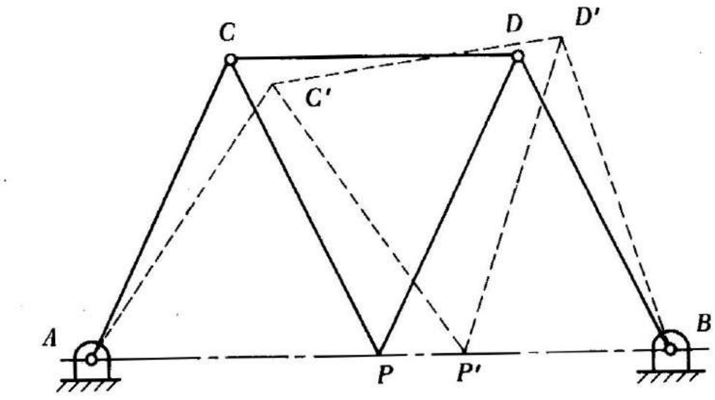


Since *OC and BC are of constant length, therefore* the product $OB \times OA$ *remains constant. Hence the point B* traces a straight path perpendicular to the diameter *OP.*



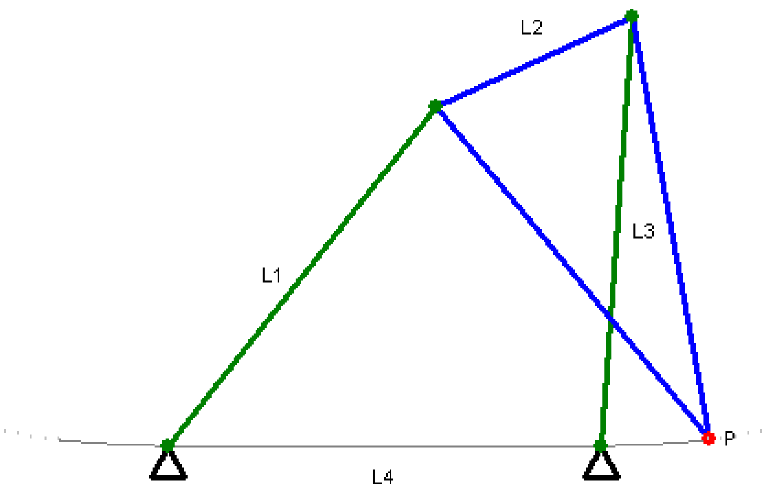
Robert's mechanism

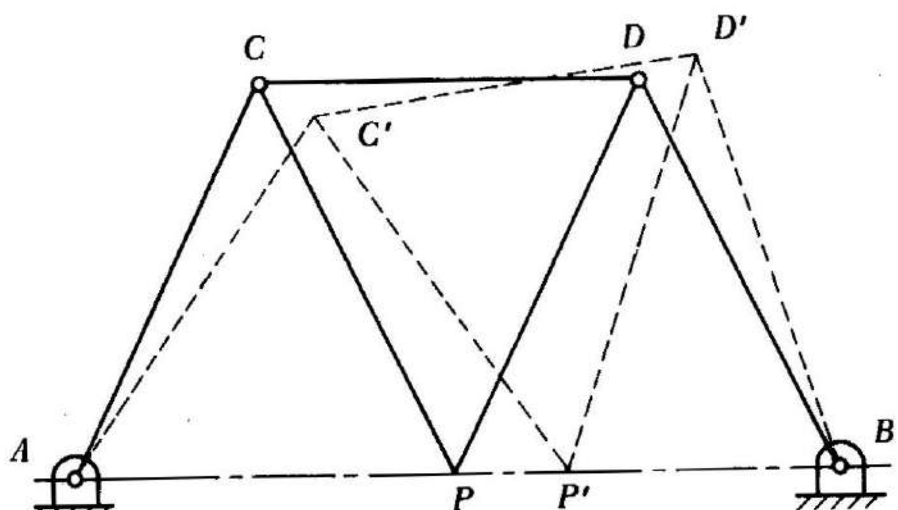




Robert’s mechanism

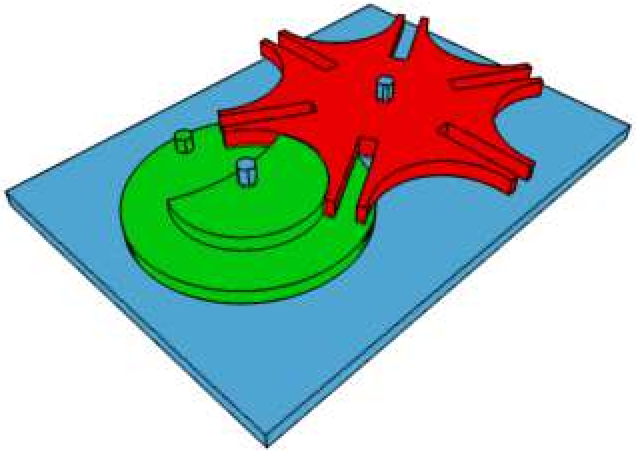
- The Englishman Roberts had proposed an approximate solution, based on a three-rod mechanism and on a CPD blade in the shape of an isosceles triangle.
- This is a four bar mechanism, where, CPD is a single integral link. Also, dimensions AC, BD, CP and PD are all equal and AB = 2 AP
- Point P of the mechanism r

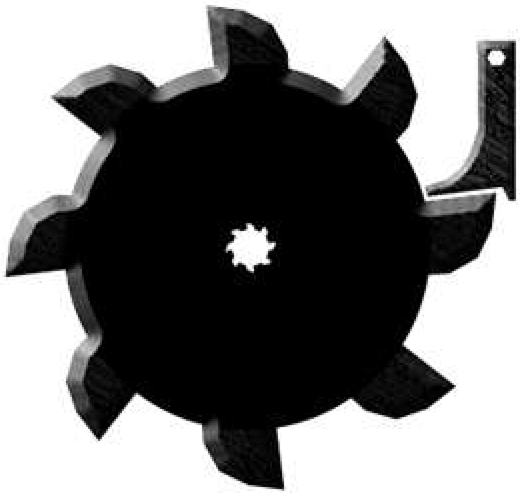


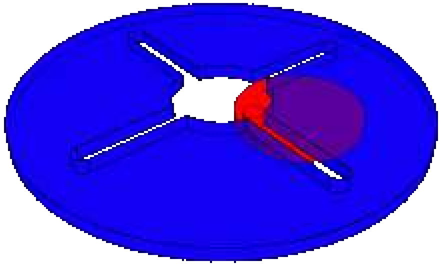


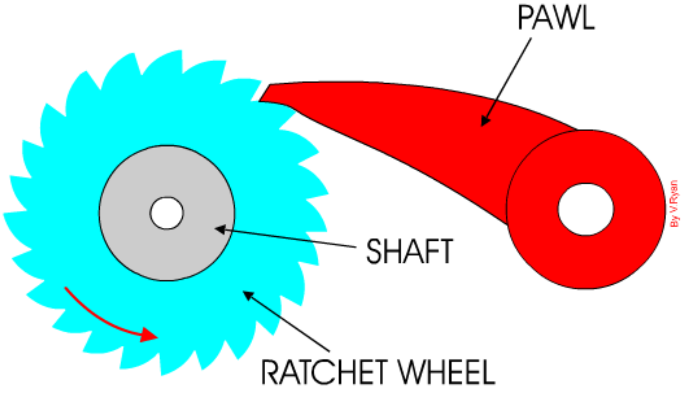
Intermittent Motion Mechanism

- Intermittent motion means that the motion is not continuous, but it ceases after definite intervals.
- Intermittent rotary motion is required generally in machine tools where work table, hexagonal turret, and spindle are to be indexed.



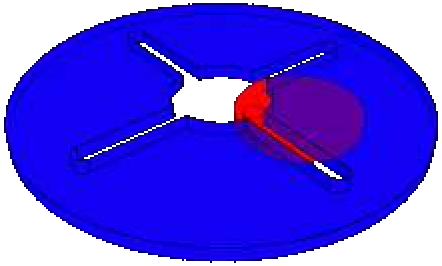


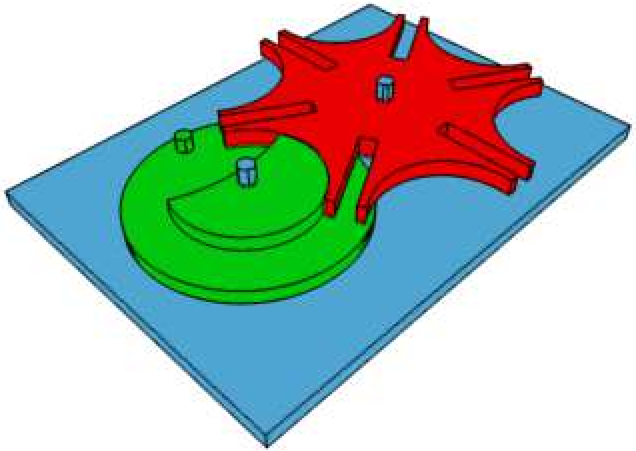


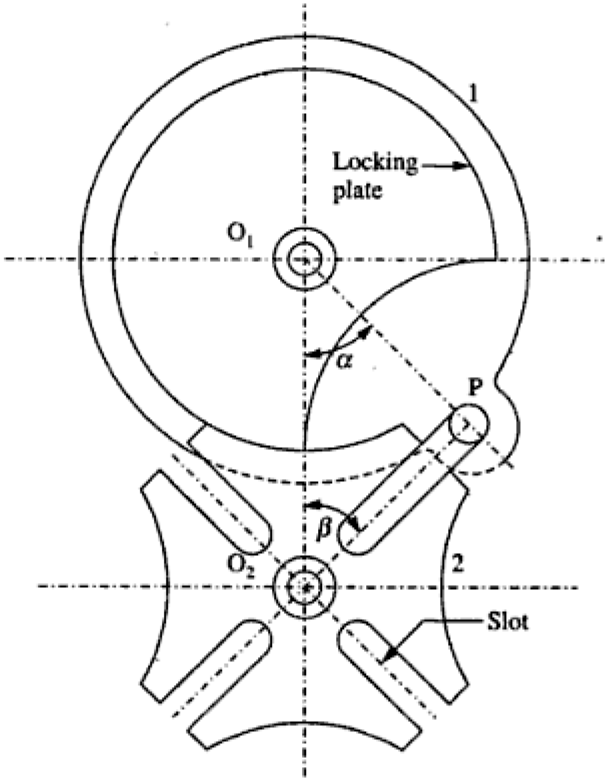


Geneva Mechanism

- Figure shows a Geneva wheel mechanism which consists of a driving wheel 1.
- It rotates continuously, and carries a pin P that engages in a slot in die driven member 2.
- The follower or driven member 2 is turned 1/4th of a revolution for each revolution of plate 1.







Pantograph

- Pantographs are used for reducing or enlarging drawings and maps. They are also used for guiding cutting tools or torches to fabricate complicated shapes.
- In the mechanism shown in fig. path traced by point A will be magnified by point E to scale, as discussed below.
- In the mechanism shown, AB = CD; AD =BC and OAE lie on a straight line.
- When point A moves to A' , E moves to E' and OA'E' also lies on a straight line.

$\Delta ODA \equiv \Delta OCE$ and $\Delta OD'A' \equiv \Delta OC'E'$.

$$\therefore \frac{OD}{OC} = \frac{OA}{OE} = \frac{DA}{CE} \text{ And } \frac{OD'}{OC'} = \frac{OA'}{OE'} = \frac{D'A'}{C'E'}$$

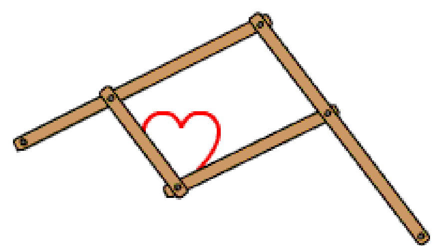
$$\text{But, } \frac{OD}{OC} = \frac{OD'}{OC'}; \therefore \frac{OA}{OE} = \frac{OA'}{OE'}; \therefore \Delta OAA' \equiv \Delta OEE'$$

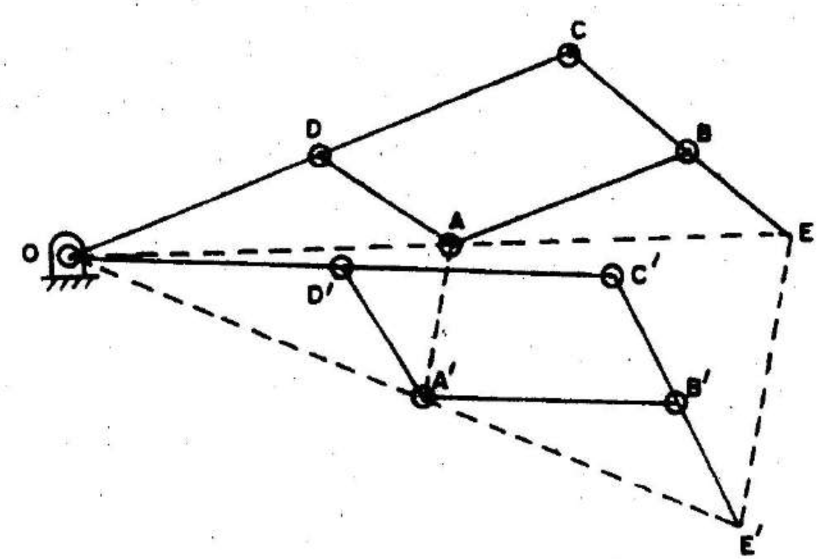
$$\therefore EE' // AA'$$

$$\text{And } \frac{EE'}{AA'} = \frac{OE}{OA} = \frac{OC}{OD}$$

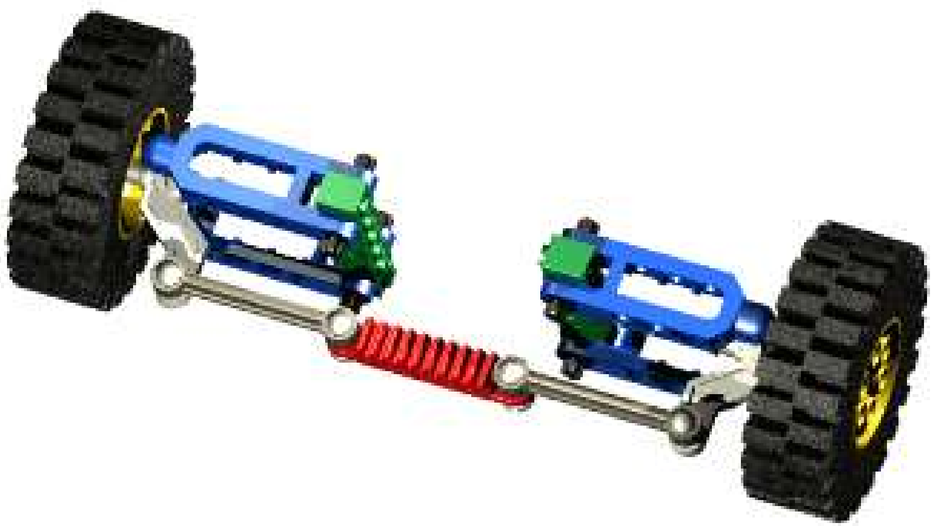
$$\therefore EE' = AA'\left(\frac{OC}{OD}\right)$$

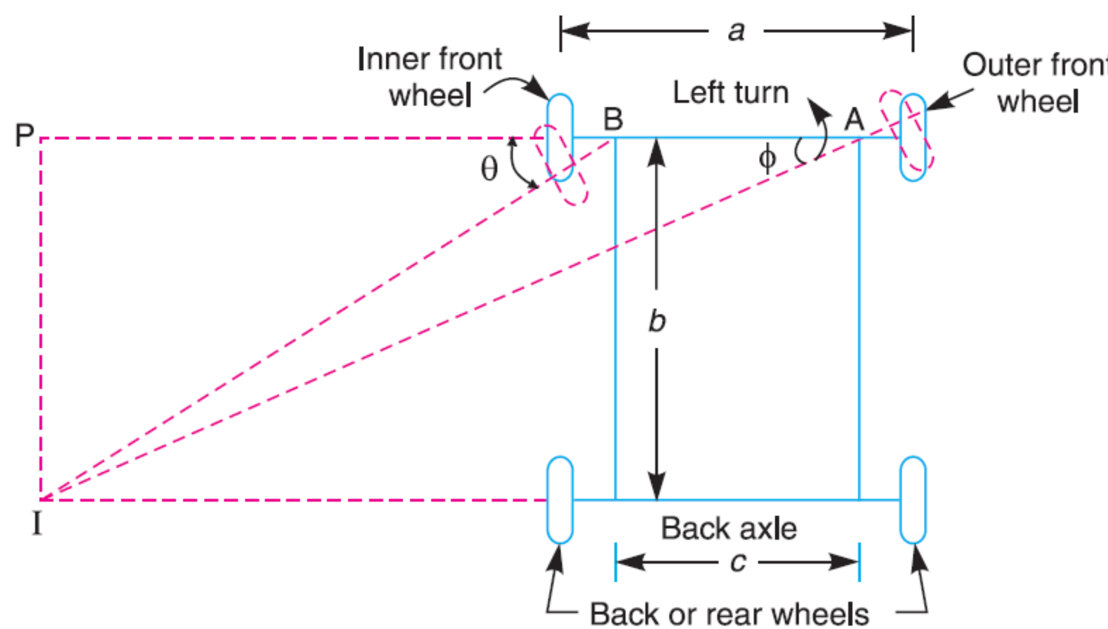
Where $\left(\frac{OC}{OD}\right)$ is the magnification factor





STEERING GEAR MECHANISM

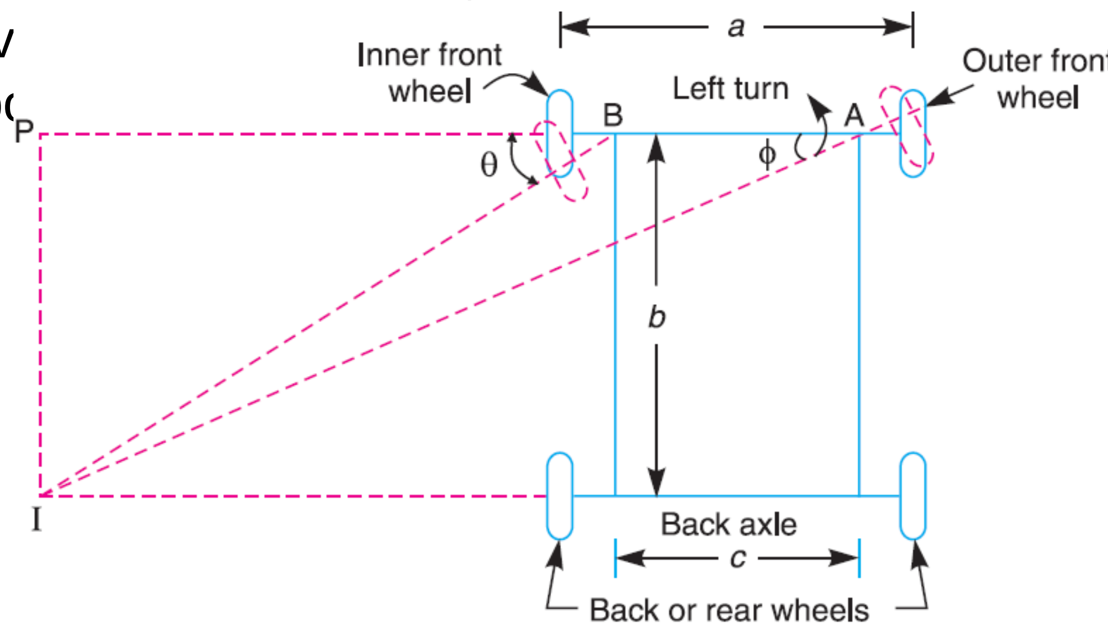




Steering Gear Mechanism

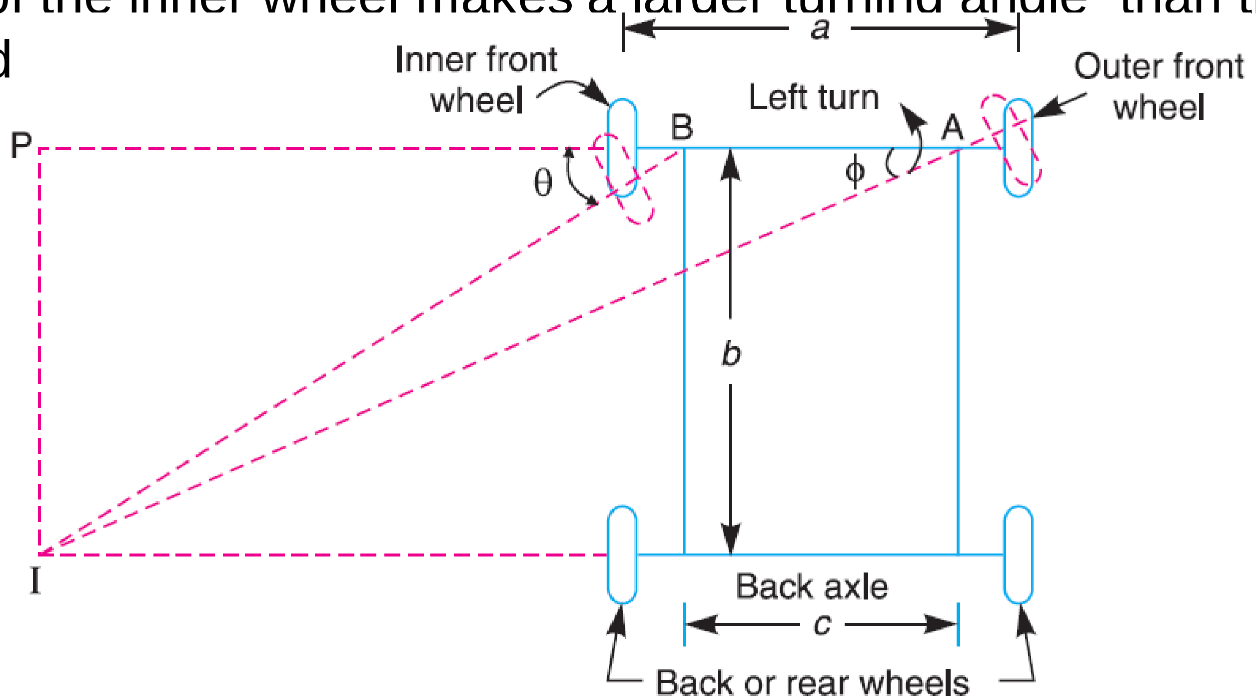
- The steering gear mechanism is used for changing the direction of two or more of the wheel axles with reference to the chassis, so as to move the automobile in any desired path.
- Usually the two back wheels have a common axis, which is fixed in direction with reference to the chassis and the steering is done by means of the front wheels.
- In automobiles, the front wheels are placed over the front axles, which are pivoted at the points *A and B, as shown in Fig. These points are fixed to the chassis.*
- *The back wheels are placed* over the back axle, at the two ends of the differential tube. When the v
the respective axles turn abo

The back wheels remain straight and do not turn. Therefore, the steering is done by means of front wheels only.



Condition for perfect steering

- In order to avoid skidding (*i.e. slipping of the wheels sideways),* the two front wheels must turn about the same instantaneous centre *I which lies on the axis of* the back wheels.
- If the instantaneous centre of the two front wheels do not coincide with the instantaneous centre of the back wheels, the skidding on the front or back wheels will definitely take place, which will cause more wear and tear of the tyres.
- ***Thus, the condition for correct steering is that all the four wheels must turn about the same instantaneous centre.***
- The axis of the inner wheel makes a larger turning angle than the angle subtended



- **Condition for perfect steering**

Let a = Wheel track,

b = Wheel base, and

c = Distance between the pivots A and B of the front axle.

Now from triangle IBP,

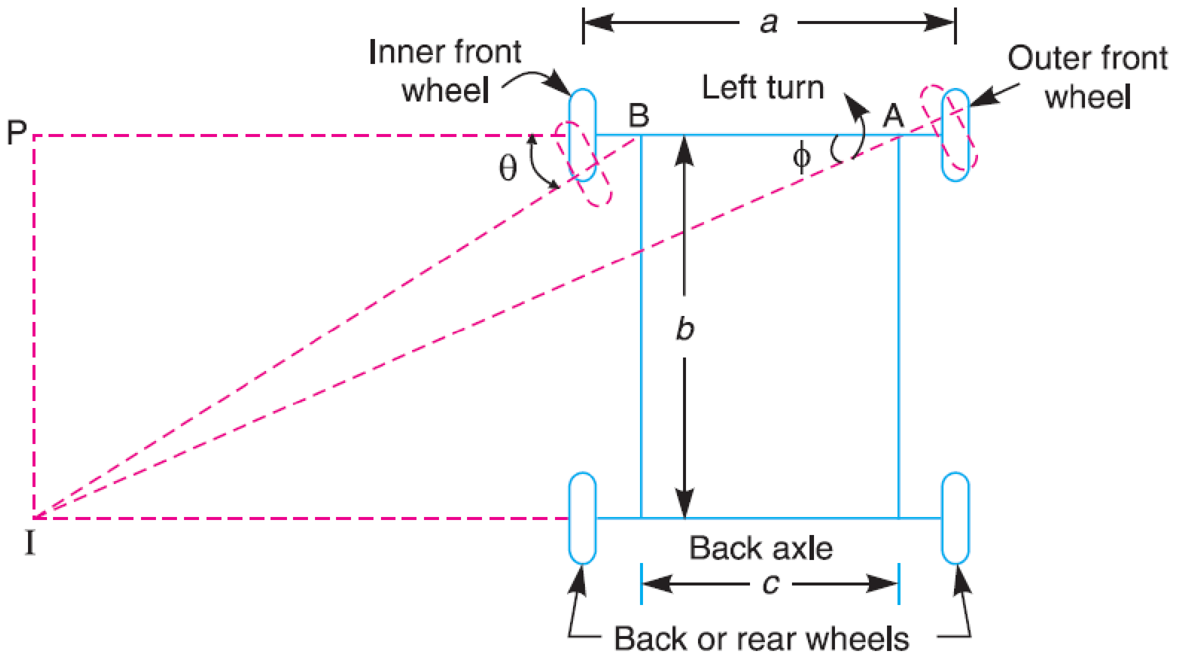
$$\cot\theta = \frac{BP}{IP}$$

and from triangle IAP,

$$\cot\phi = \frac{AP}{IP} = \frac{AB + BP}{IP} = \frac{AB}{IP} + \frac{BP}{IP} = \frac{c}{b} + \cot\theta \qquad ...(\because IP = b)$$

$$\therefore \cot\phi - \cot\theta = c/b$$

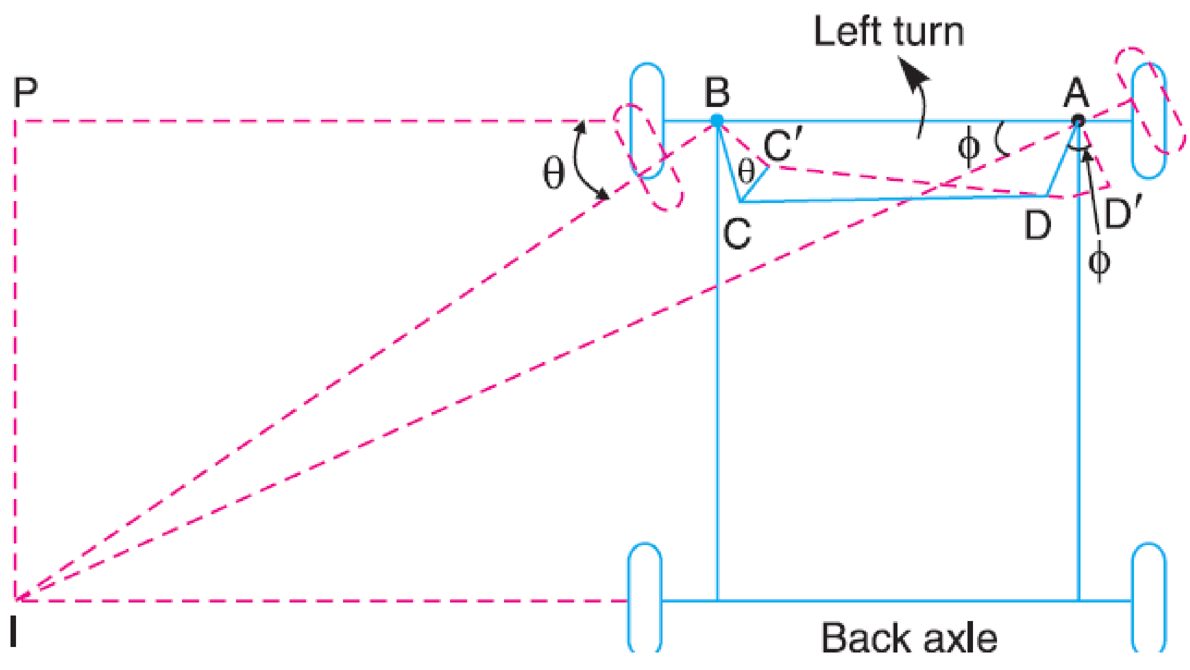
This is the fundamental equation for correct steering. If this condition is satisfied, there will be no skidding of the wheels, when the vehicle takes a turn.



Ackermann steering gear mechanism

- In Ackerman steering gear, the mechanism *ABCD is a four bar crank chain, as shown in Fig*.
- The shorter links *BC and AD are of equal length and are connected by hinge joints with front* wheel axles. The longer links *AB and CD are of unequal length. The following are the only three* positions for correct steering.

1. **When the vehicle moves along a straight path, the longer links *AB and CD are parallel and*** the shorter links *BC and AD are equally inclined to the longitudinal axis of the vehicle, as shown by* firm lines in Fig..
2. **When the vehicle is steering to the left, the position of the gear is shown by dotted lines in** Fig. In this position, the lines of the front wheel axle intersect on the back wheel axle at *I, for* correct steering.
3. **When the** **nay be obtained.**



Velocity and Acceleration Analysis of Mechanisms

(Graphical Methods)

Relative Velocity of Two Bodies Moving in Straight Lines

- Here we shall discuss the application of vectors for the relative velocity of two bodies moving along parallel lines and inclined lines, as shown in Fig.
- Consider two bodies *A and B moving along parallel* lines in the same direction with absolute velocities *v*A and *v*B such that *v*A > *v*B , as shown in Fig. (*a*). *The relative* velocity of *A with respect to B is …….*

$$v_{AB} = \text{Vector difference of } v_A \text{ and } v_B = \overline{v_A} - \overline{v_B}$$

From Fig. 7.1 (*b*), the relative velocity of *A* with respect to *B* (*i.e.* v_{AB}) may be written in the vector form as follows :

$$\overline{ba} = \overline{oa} - \overline{ob}$$

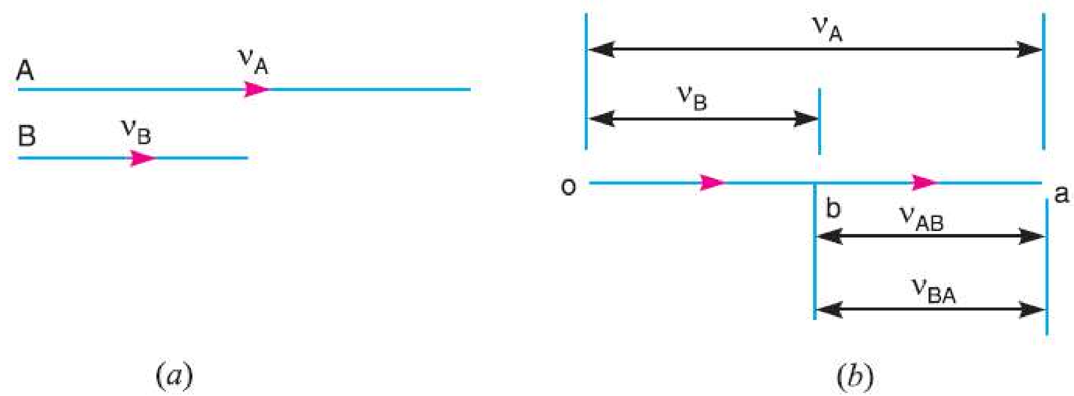


Fig. 7.1. Relative velocity of two bodies moving along parallel lines.

Similarly, the relative velocity of *B* with respect to *A* ,

$$v_{BA} = \text{Vector difference of } v_B \text{ and } v_A = \overline{v_B} - \overline{v_A} \qquad \text{...}(ii)$$

or $$\overline{ab} = \overline{ob} - \overline{oa}$$

- Now consider the body *B moving in an* inclined direction as shown in Fig. 2 (*a*). *The* relative velocity of *A with respect to B may be* obtained by the law of parallelogram of velocities or triangle law of velocities.
- Take any fixed point *o and draw vector oa to represent* v_A in magnitude and direction to some suitable scale.
- Similarly, draw vector *ob to represent* v_B in magnitude and direction to the same scale. Then vector *ba represents the relative velocity of A with* respect to *B as shown in Fig. 2 (b). In the similar* way as discussed above, the relative velocity of *A with respect to B,*

$$v_{AB} = \text{Vector difference of } v_A \text{ and } v_B = \overline{v_A} - \overline{v_B}$$

or $$\overline{ba} = \overline{oa} - \overline{ob}$$

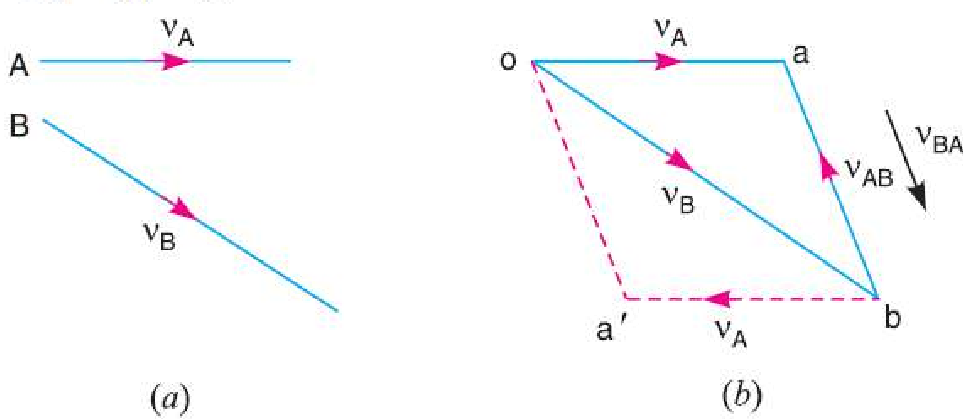


Fig. 7.2. Relative velocity of two bodies moving along inclined lines.

Similarly, the relative velocity of *B* with respect to *A*,

$$v_{BA} = \text{Vector difference of } v_B \text{ and } v_A = \overline{v_B} - \overline{v_A}$$

or $$\overline{ab} = \overline{ob} - \overline{oa}$$

Motion of a link

- Consider two points *A* and *B* on a rigid link *AB* as shown in Fig.
- Let one of the extremities *(B)* of the link move relative to *A,* in a clockwise direction.
- Since the distance from A to *B* remains the same, therefore there can be no relative motion between *A* and *B,* along the line *AB.*

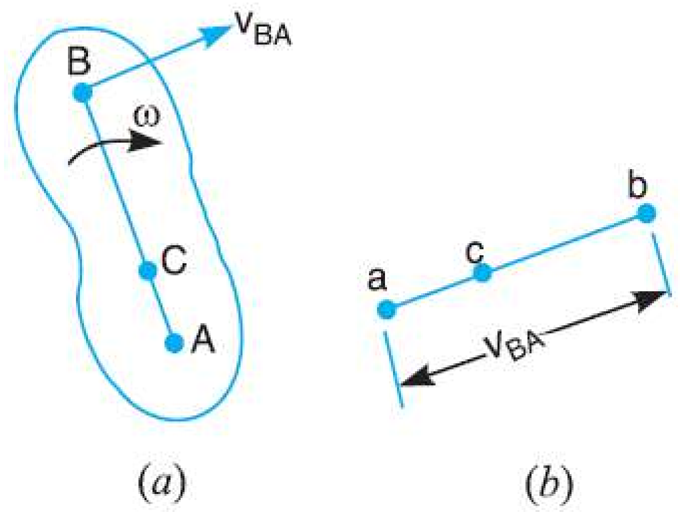


Fig. 7.3. Motion of a Link.

- It is thus obvious, that the relative motion of B with respect to *A* must be perpendicular *to AB.*
- Hence *velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.*

Velocity of a Point on a Link by Relative Velocity Method

- The relative velocity method is based upon the relative velocity of the various points of the link.
- Consider two points *A and B on a link as shown in Fig. 4 (a).*
- *Let the absolute velocity of the* point *A i.e.* v_A is known in magnitude and direction and the absolute velocity of the point *B i.e.* v_B is known in direction only.
- Then the velocity of *B may be determined by drawing the velocity diagram* as shown in Fig. 4 (*b). The velocity diagram is drawn as follows :*

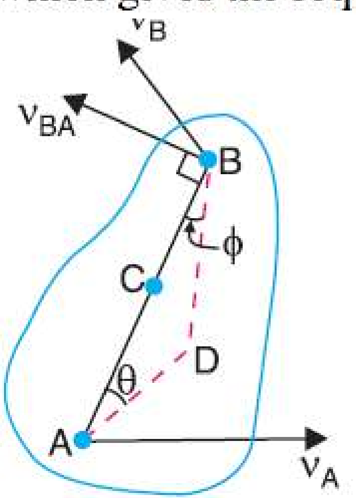
1. Take some convenient point *o*, known as the pole.

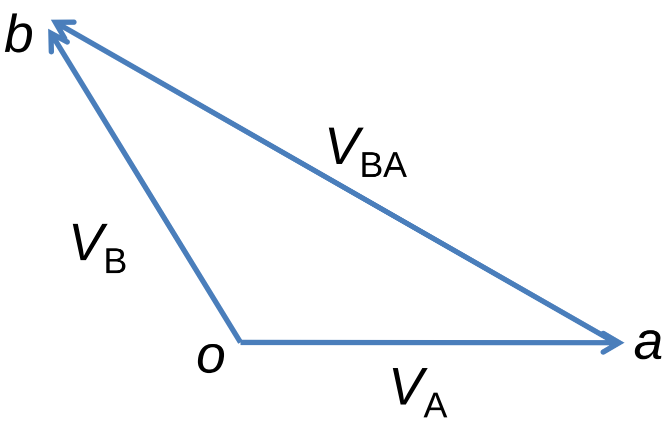
2. Through *o*, draw *oa* parallel and equal to v_A, to some suitable scale.

3. Through *a*, draw a line perpendicular to *A B* of Fig. 7.4 (*a*). This line will represent the velocity of *B* with respect to *A* , *i.e.* v_{BA}.

4. Through *o*, draw a line parallel to v_B intersecting the line of v_{BA} at *b*.

5. Measure *ob*, which gives the required velocity of point *B* (v_B), to the scale.





Notes : **1.** The vector ab which represents the velocity of B with respect to A (v_{BA}) is known as velocity of image of the link AB.

2. The absolute velocity of any point C on AB may be determined by dividing vector ab at c in the same ratio as C divides AB in Fig. 7.4 (a).

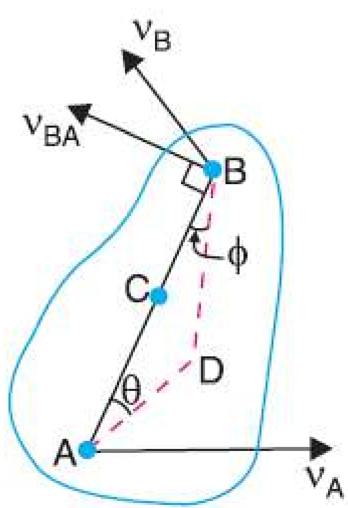
In other words

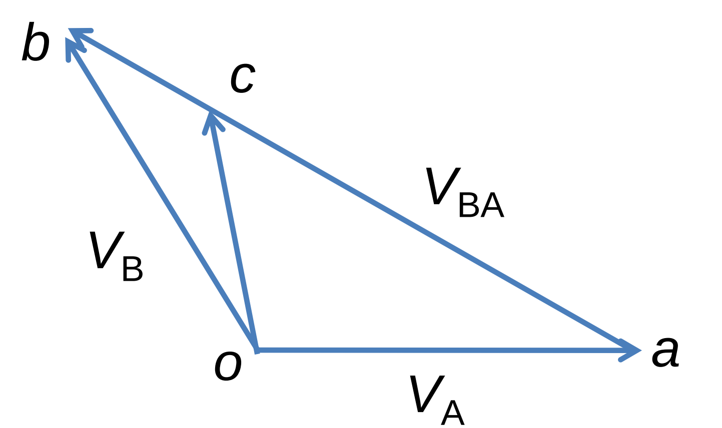
$$\frac{ac}{ab} = \frac{AC}{AB}$$

Join oc. The *vector oc represents the absolute velocity of point C (v_C) and the vector ac represents the velocity of C with respect to A *i.e.* v_{CA}.

4. The angular velocity of the link AB may be found by dividing the relative velocity of B with respect to A (*i.e.* v_{BA}) to the length of the link AB. Mathematically, angular velocity of the link AB,

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB}$$





Rubbing Velocity at pin joint

The links in a mechanism are mostly connected by means of pin joints. The rubbing velocity is defined as **the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.**

Consider two links OA and OB connected by a pin joint at O as shown in Fig. 7.6.

Let ω_1 = Angular velocity of the link OA or the angular velocity of the point A with respect to O.

ω_2 = Angular velocity of the link OB or the angular velocity of the point B with respect to O, and

r = Radius of the pin.

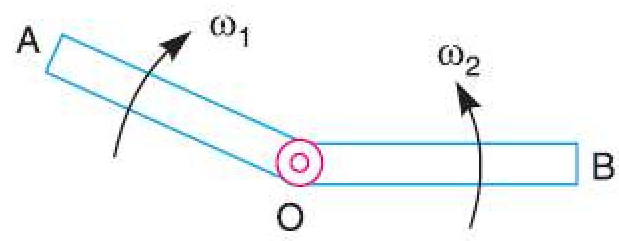


Fig. 7.6. Links connected by pin joints.

According to the definition,

Rubbing velocity at the pin joint O

$= (\omega_1 - \omega_2)\, r$, if the links move in the same direction

$= (\omega_1 + \omega_2)\, r$, if the links move in the opposite direction

Note : When the pin connects one sliding member and the other turning member, the angular velocity of the sliding member is zero. In such cases,

Rubbing velocity at the pin joint $= \omega . r$

where ω = Angular velocity of the turning member, and

r = Radius of the pin.

Acceleration Diagram for a Link

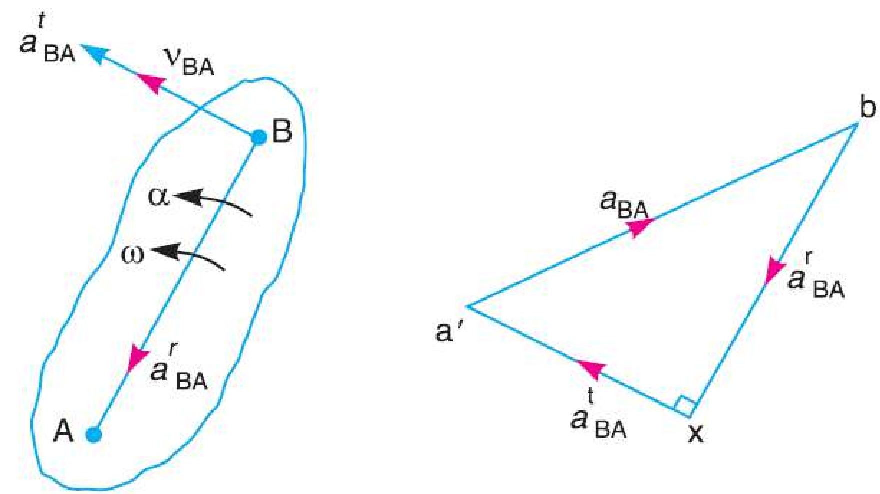
- Consider two points *A and B on a rigid link as shown* in Fig.(*a).*
- *Let the point B moves with respect to A, with* an angular velocity of ω rad/s and let α rad/s^2 be the angular acceleration of the link *AB.*
- Acceleration of a particle whose **velocity changes both in magnitude and direction** at any instant has the following two components :
  1. **The *centripetal or radial component, which is perpendicular to the velocity of the* particle at the given instant.**
  2. **The *tangential component, which is parallel to the velocity of the particle at the given* instant.**
- Thus for a link *AB, the velocity of point B with respect to A (i.e.* v_{BA}) is perpendicular to the link *AB as shown in Fig. 8.1 (a). Since the point B moves with respect to A with an angular velocity of* ω *rad/s, therefore*

mponent of the

$$a^r_{BA} = \omega^2 \times \text{Length of link } AB$$

$$= \omega^2 \times AB = v^2_{BA} / AB \qquad \ldots\left(\because \omega = \frac{v_{BA}}{AB}\right)$$

This radial component of acceleration acts perpendicular to the velocity v_{BA}, In other words, it acts ***parallel to the link AB.***



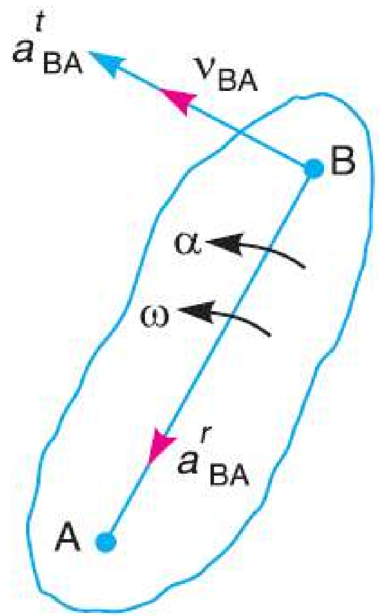
(*a*) Link. (*b*) Acceleration diagram.

We know that tangential component of the acceleration of B with respect to A,

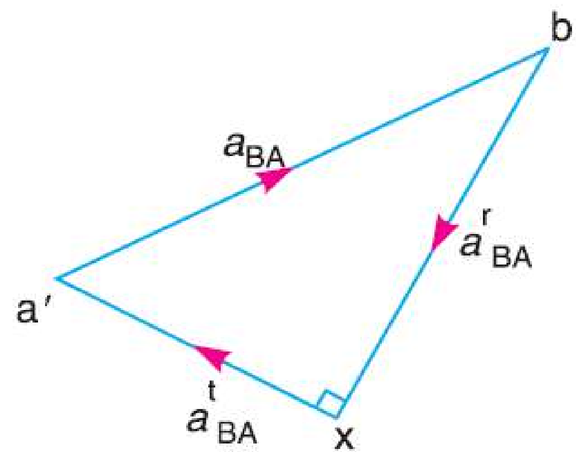
$$a_{BA}^{t} = \alpha \times \text{Length of the link } AB = \alpha \times AB$$

This tangential component of acceleration acts parallel to the velocity v_{BA}. In other words, it acts ***perpendicular*** to the link AB.

In order to draw the acceleration diagram for a link AB, as shown in Fig. 8.1 (*b*), from any point b', draw vector ***b'x parallel to BA*** to represent the radial component of acceleration of B with respect to A *i.e.* a_{BA}^{r} and from point x draw vector xa' perpendicular to BA to represent the tangential component of acceleration of B with respect to A *i.e.* a_{BA}^{t}. ***Join b' a'.*** The vector $b'a'$ (known as ***acceleration image*** of the link AB) represents the total acceleration of B with respect to A (*i.e.* a_{BA}) and it is the vector sum of radial component (a_{BA}^{r}) and tangential component (a_{BA}^{t}) of acceleration.



(*a*) Link.

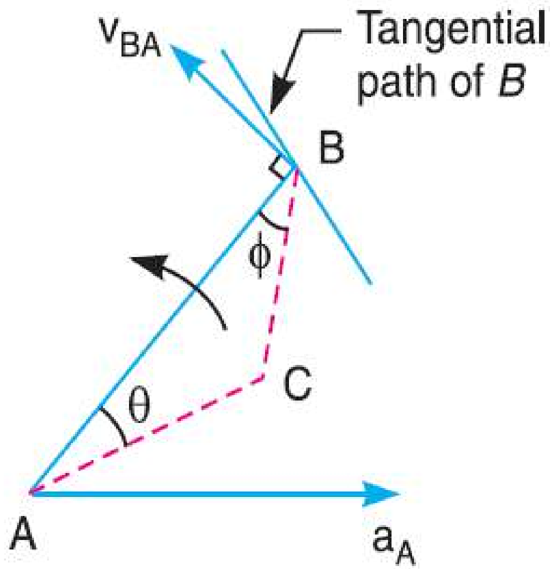


(*b*) Acceleration diagram.

Acceleration of a Point on a Link

- Consider two points *A and B on the rigid link, as shown in Fig. (a). Let the acceleration* of the point *A i.e.* a_A is known in magnitude and direction and the direction of path of *B is given.*
- *The* acceleration of the point *B is determined in magnitude and direction by drawing the acceleration* diagram as discussed below.

1. From any point *o', draw vector o'a' parallel to the direction of absolute acceleration at* point *A i.e.* a_A *, to some suitable scale, as shown in Fig.(b).*



(*a*) Points on a Link.

2. We know that the acceleration of *B with* respect to *A i.e.* a_{BA} has the following two components:

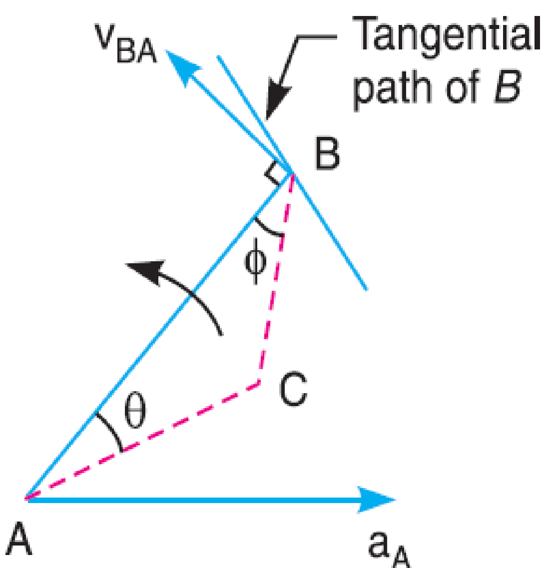
(*i*) *Radial component of the acceleration* of *B with respect to A i.e.* a^r_{BA}, *and*

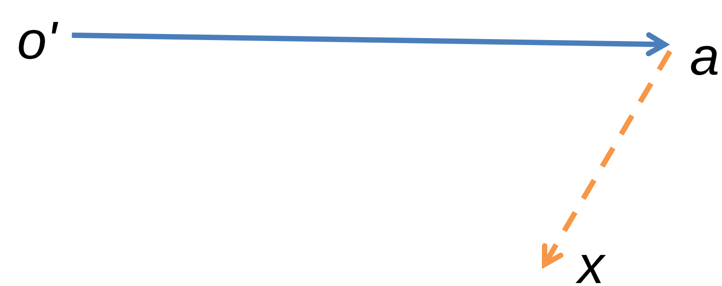
(*ii*) *Tangential component of the* acceleration *B with respect to A i.e.* a^t_{BA}

These two components are mutually perpendicular.

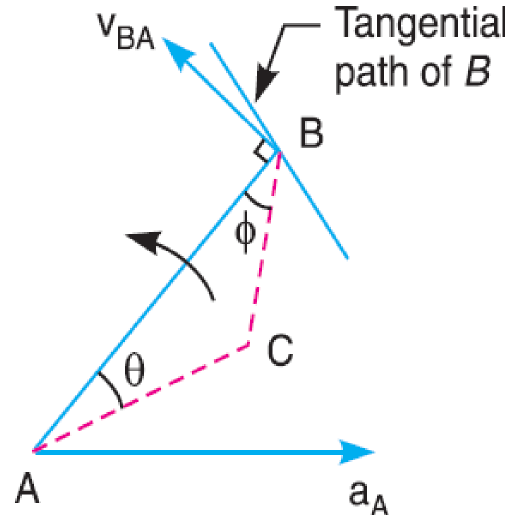
3. Draw ve vector $a'x = a^r_{BA} = v^2_{BA} / AB$ dial component of the accelera *h AB), such that*

where v_{BA} = Velocity of B with respect to A.



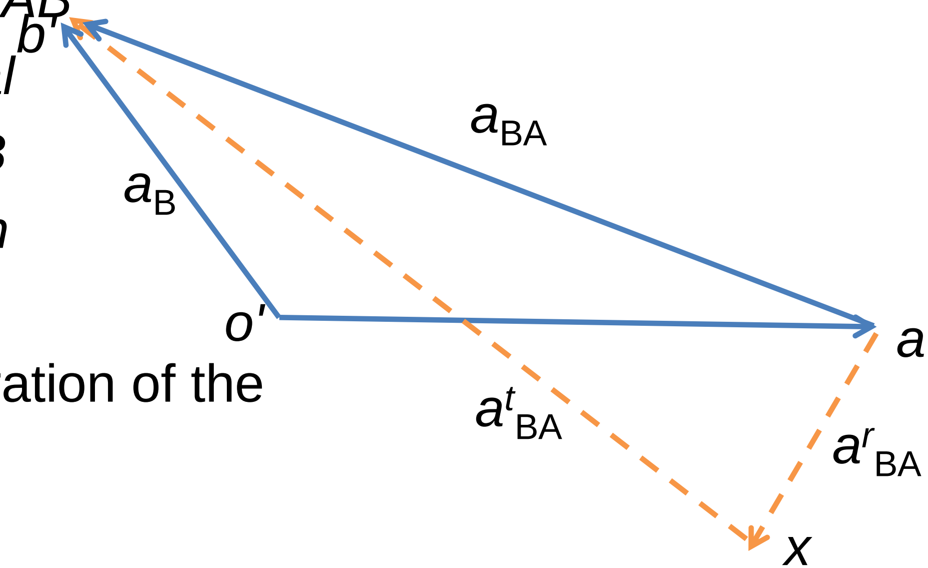


4. From point x, draw vector xb' perpendicular to AB or vector $a'x$ (because tangential component of B with respect to A *i.e.* a^t_{BA}, is perpendicular to radial component a^r_{BA}) and through o' draw a line parallel to the path of B to represent the absolute acceleration of B *i.e.* a_B. The vectors xb' and $o'b'$ intersect at b'. Now the values of a_B and a^t_{BA} may be measured, to the scale.



5. By joining the points *a' and b' we may determine the total acceleration of B with respect* to *A i.e.* a_{BA}. The vector *a' b' is known as acceleration image of the link AB.*

6. The angular acceleration of the link *AB is obtained by dividing the tangential components* of the acceleration of *B with respect to A* (a^t_{BA}) *to the length of the link.*



Mathematically, angular acceleration of the link *AB,*

$$\alpha_{AB} = a^t_{BA} / AB$$

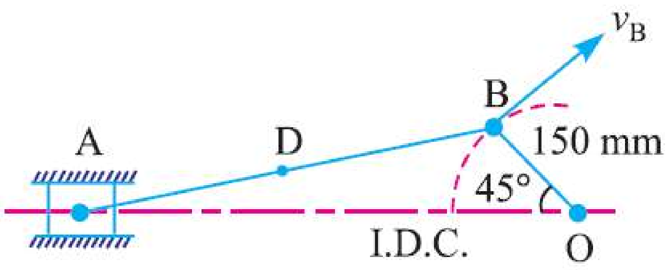
Example 8.1. *The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine :* **1.** *linear velocity and acceleration of the midpoint of the connecting rod, and* **2.** *angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.*

Solution. Given : N_{BO} = 300 r.p.m. or $\omega_{BO} = 2\,\pi \times 300/60 = 31.42$ rad/s; OB = 150 mm = 0.15 m ; BA = 600 mm = 0.6 m

We know that linear velocity of B with respect to O or velocity of B,

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713 \text{ m/s}$$

1. First of all draw the space diagram, to some suitable scale; as shown in Fig. (*a*).



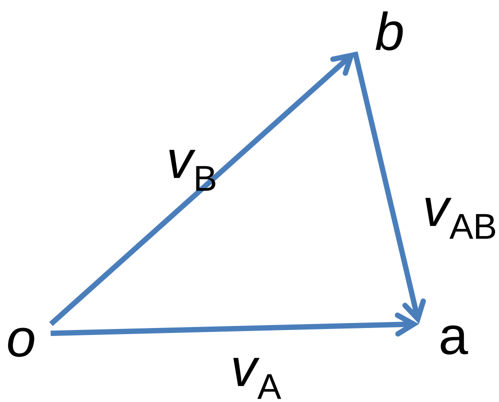
(*a*) Space diagram.

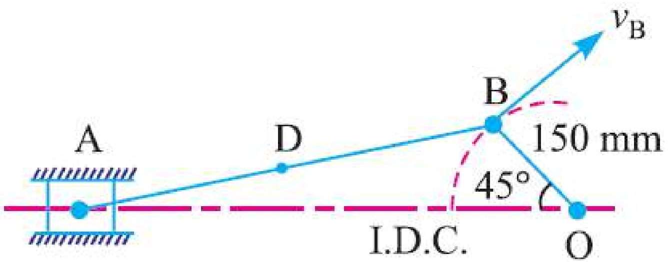
To Draw Velocity Vector polygon

1. Draw vector *ob perpendicular to BO, to some suitable scale, to represent the velocity of B with respect to O or simply velocity of B i.e.* v_{BO} or v_B, such that vector *ob* = $v_{BO} = v_B$ = 4.713 m/s
2. From point *b, draw vector ba perpendicular to BA to represent the velocity of A with* respect to *B i.e.* v_{AB} , and from point *o draw vector oa parallel to the motion of A (which is along AO)* to represent the velocity of *A i.e.* v_A. The vectors *ba and oa intersect at a.*
3. By measurement, we find that velocity of *A with respect to B,*

$$v_{AB} = \text{vector } ba = 3.4 \text{ m/s}$$

$$\text{Velocity of } A,\ v_A = \text{vector } oa = 4 \text{ m/s}$$





(*a*) Space diagram.

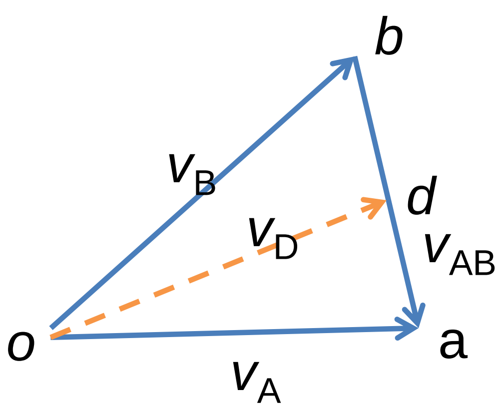
4. In order to find the velocity of the midpoint *D of the connecting rod AB, divide the vector ba at d in the same ratio as D divides AB, in the space diagram.*

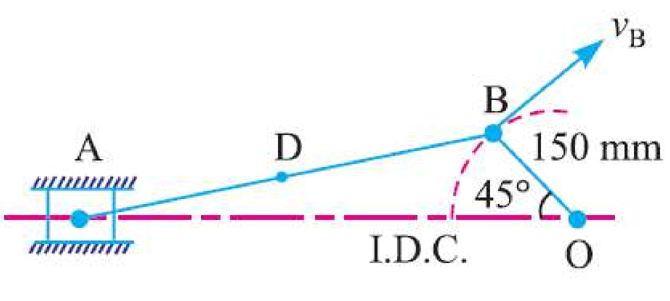
   In other words, bd / ba = BD/BA

   Note: Since *D is the midpoint of AB, therefore d is also midpoint of vector ba.*

5. Join *od. Now the vector od represents the velocity of the midpoint D of the connecting* rod *i.e.* v_D.

   By measurement, we find that v_D = vector *od = 4.1 m/s*





(*a*) Space diagram.

Acceleration of the midpoint of the connecting rod

- We know that the radial component of the acceleration of B with respect to O or the acceleration o

$$a_{BO}^{r} = a_B = \frac{v_{BO}^2}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

and the radial component of the acceleration of *A with respect to B,*

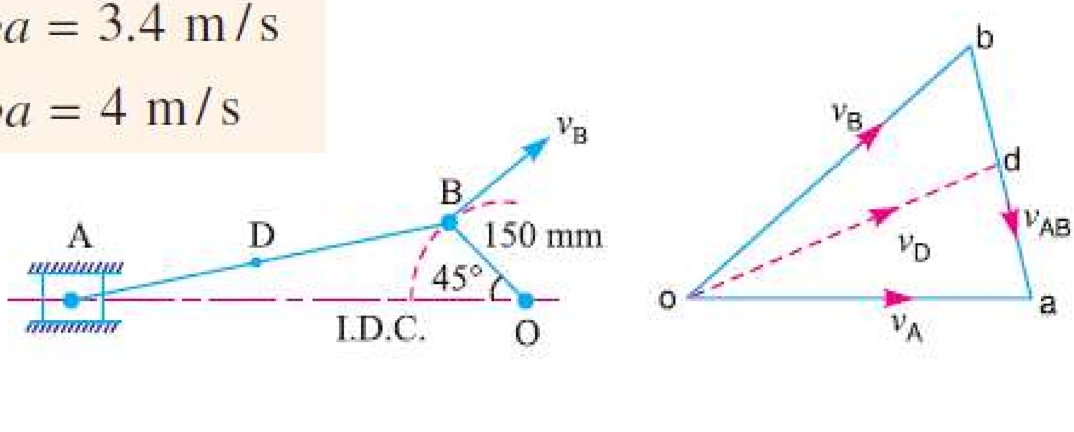
$$a_{AB}^{r} = \frac{v_{AB}^2}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

NOTE:1) A point at the end of a link which moves with constant angular velocity has no tangential component of acceleration.

2) When a point moves along a straight line, it has no centripetal or radial component of the acceleration.

$$v_{AB} = \text{vector } ba = 3.4 \text{ m/s}$$

$$\text{Velocity of } A,\ v_A = \text{vector } oa = 4 \text{ m/s}$$



(*a*) Space diagram. (*b*) Velocity diagram.

Acceleration of the midpoint of the connecting rod

1. Draw vector $o'b'$ parallel to BO, to some suitable scale, to represent the radial component of the acceleration of B with respect to O or simply acceleration of B *i.e.* a_{BO}^r or a_B, such that

$$\text{vector } o'b' = a_{BO}^r = a_B = 148.1 \text{ m/s}^2$$

Note: Since the crank OB rotates at a constant speed, therefore there will be no tangential component of the acceleration of B with respect to O.

2. The acceleration of A with respect to B has the following two components:

(a) The radial component of the acceleration of A with respect to B *i.e.* a_{AB}^r, and

(b) The tangential component of the acceleration of A with respect to B *i.e.* a_{AB}^t. These two components are mutually perpendicular.

Therefore from point b', draw vector $b'x$ parallel to AB to represent $a_{AB}^r = 19.3 \text{ m/s}^2$ and from point x draw vector xa' perpendicular to vector $b'x$ whose magnitude is yet unknown.

3. Now from o', draw vector $o'a'$ parallel to the path of motion of A (which is along AO) to represent the acceleration of A *i.e.* a_A. The vectors xa' and $o'a'$ intersect at a'. Join $a'b'$.

4. In order to find the acceleration of the midpoint D of the connecting rod AB, divide the vector $a'b'$ at d' in the same ratio as D divides AB. In other words

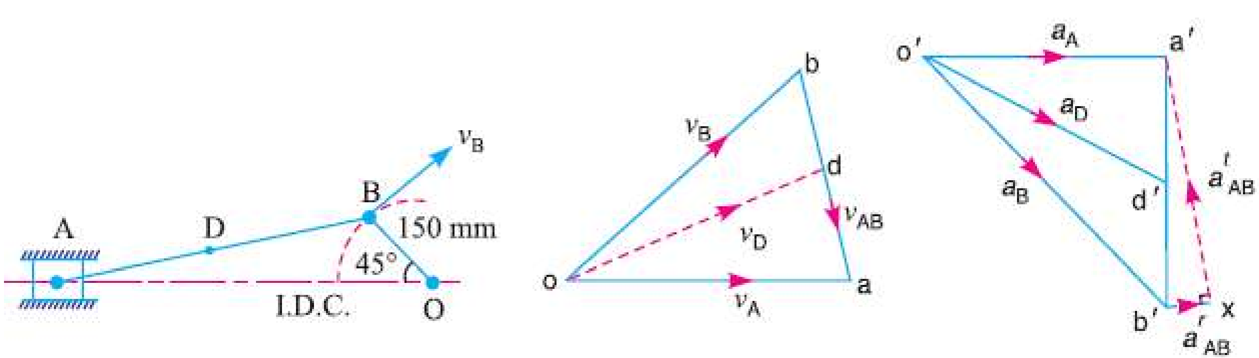
$$b'd' / b'a' = BD / BA$$

Note: Since D is the midpoint of AB, therefore d' is also midpoint of vector $b'a'$.

5. Join $o'd'$. The vector $o'd'$ represents the acceleration of midpoint D of the connecting rod *i.e.* a_D.

By measurement, we find that

$$a_D = \text{vector } o'd' = 117 \text{ m/s}^2 \textbf{ Ans.}$$



2. *Angular velocity of the connecting rod*

We know that angular velocity of the connecting rod AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}$$ **Ans.**

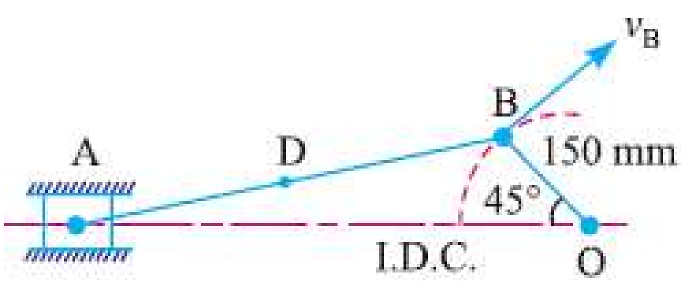
Angular acceleration of the connecting rod

From the acceleration diagram, we find that

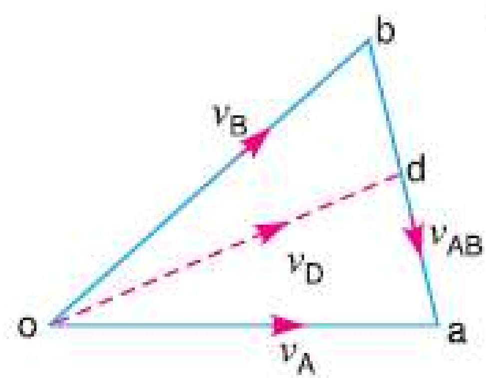
$$a_{AB}^{t} = 103 \text{ m/s}^2$$ ...(By measurement)

We know that angular acceleration of the connecting rod AB,

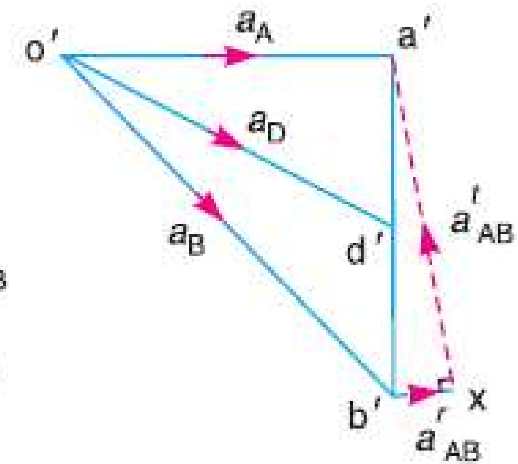
$$\alpha_{AB} = \frac{a_{AB}^{t}}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2$$ **Ans.**



(*a*) Space diagram.



(*b*) Velocity diagram.



(*c*) Acceleration diagram.

Example 8.4. *PQRS is a four bar chain with link PS fixed. The lengths of the links are PQ = 62.5 mm ; QR = 175 mm ; RS = 112.5 mm ; and PS = 200 mm. The crank PQ rotates at 10 rad/s clockwise. Draw the velocity and acceleration diagram when angle QPS = 60° and Q and R lie on the same side of PS. Find the angular velocity and angular acceleration of links QR and RS.*

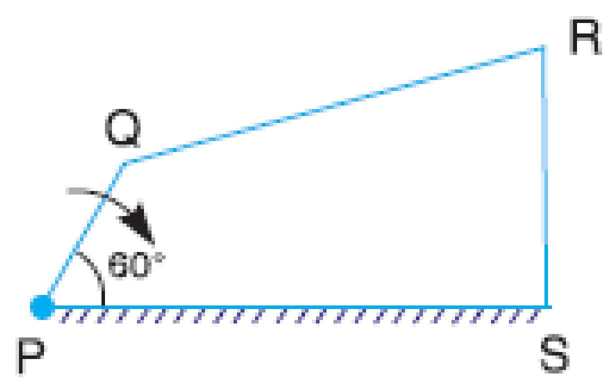
Solution. Given : $\omega_{QP} = 10$ rad/s; $PQ = 62.5$ mm $= 0.0625$ m ; $QR = 175$ mm $= 0.175$ m ; $RS = 112.5$ mm $= 0.1125$ m ; $PS = 200$ mm $= 0.2$ m

We know that velocity of Q with respect to P or velocity of Q,

$$v_{QP} = v_Q = \omega_{QP} \times PQ = 10 \times 0.0625 = 0.625 \text{ m/s}$$

Angular velocity of links QR and RS

First of all, draw the space diagram of a four bar chain, to some suitable scale



(*a*) Space diagram.

1. Since P and S are fixed points, therefore these points lie at one place in velocity diagram. Draw vector pq perpendicular to PQ, to some suitable scale, to represent the velocity of Q with respect to P or velocity of Q *i.e.* v_{QP} or v_Q such that

$$\text{vector } pq = v_{QP} = v_Q = 0.625 \text{ m/s}$$

2. From point q, draw vector qr perpendicular to QR to represent the velocity of R with respect to Q (*i.e.* v_{RQ}) and from point s, draw vector sr perpendicular to SR to represent the velocity of R with respect to S or velocity of R (*i.e.* v_{RS} or v_R). The vectors qr and sr intersect at r. By measurement, we find that

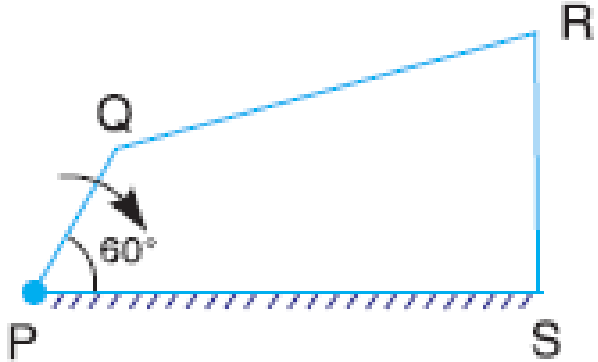
$$v_{RQ} = \text{vector } qr = 0.333 \text{ m/s, and } v_{RS} = v_R = \text{vector } sr = 0.426 \text{ m/s}$$

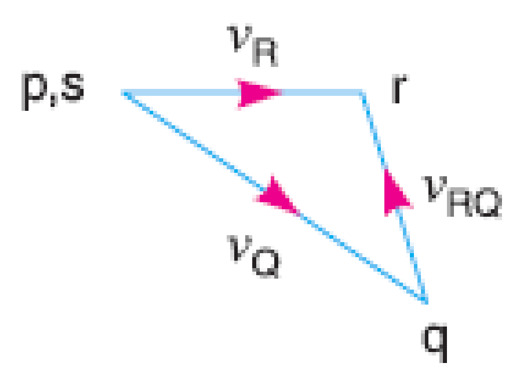
We know that angular velocity of link QR,

$$\omega_{QR} = \frac{v_{RQ}}{RQ} = \frac{0.333}{0.175} = 1.9 \text{ rad/s (Anticlockwise)} \quad \textbf{Ans.}$$

and angular velocity of link RS,

$$\omega_{RS} = \frac{v_{RS}}{SR} = \frac{0.426}{0.1125} = 3.78 \text{ rad/s (Clockwise)} \quad \textbf{Ans.}$$





Angular acceleration of links QR and RS

Since the angular acceleration of the crank PQ is not given, therefore there will be no tangential component of the acceleration of Q with respect to P.

We know that radial component of the acceleration of Q with respect to P (or the acceleration of Q),

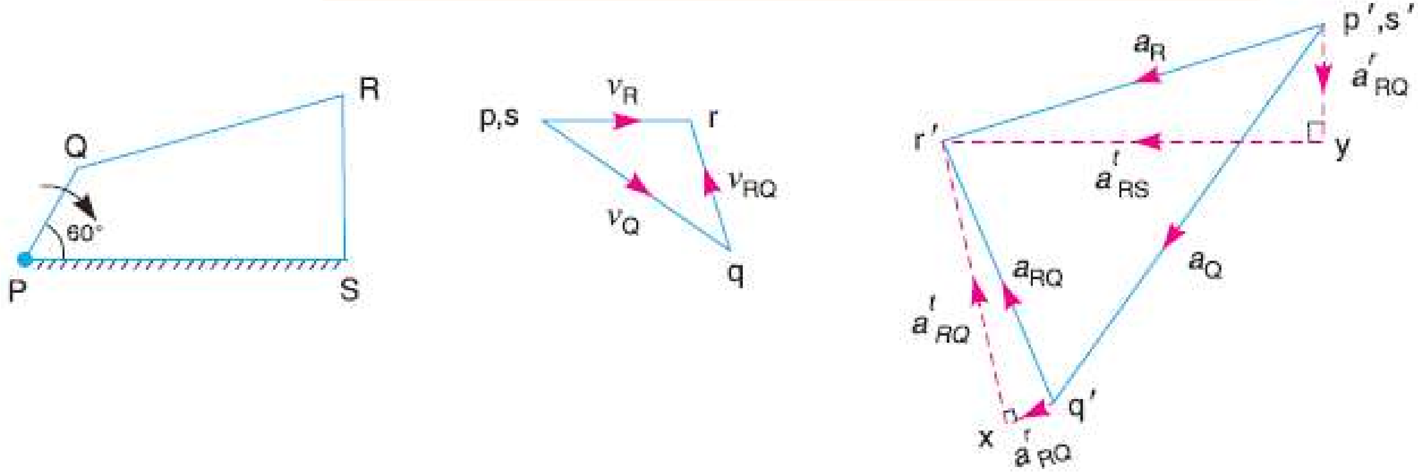
$$a^r_{QP} = a_{QP} = a_Q = \frac{v^2_{QP}}{PQ} = \frac{(0.625)^2}{0.0625} = 6.25 \text{ m/s}^2$$

Radial component of the acceleration of R with respect to Q,

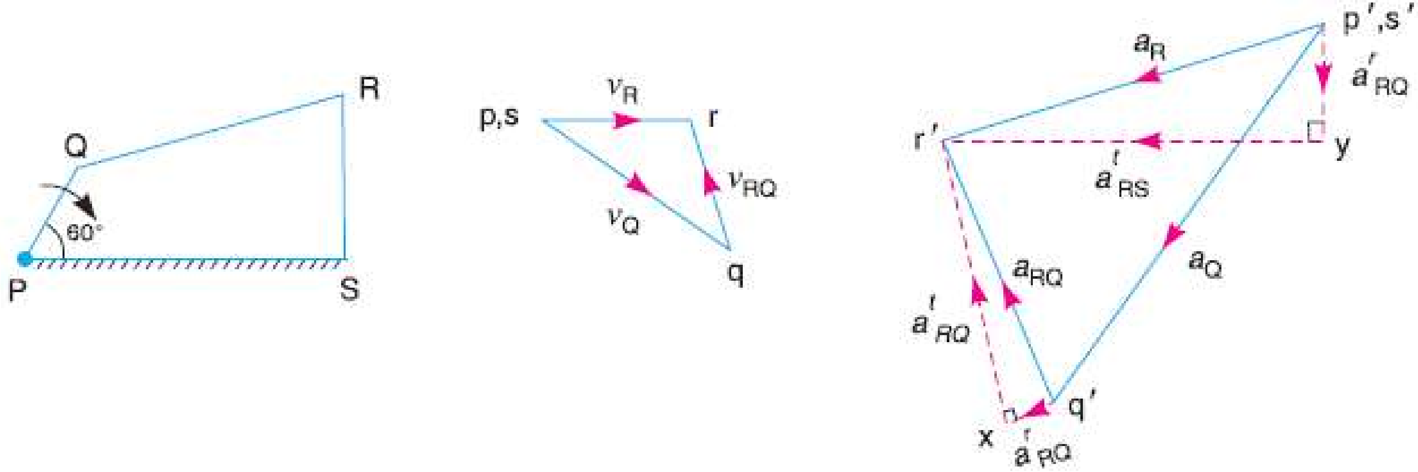
$$a^r_{RQ} = \frac{v^2_{RQ}}{QR} = \frac{(0.333)^2}{0.175} = 0.634 \text{ m/s}^2$$

and radial component of the acceleration of R with respect to S (or the acceleration of R),

$$a^r_{RS} = a_{RS} = a_R = \frac{v^2_{RS}}{SR} = \frac{(0.426)^2}{0.1125} = 1.613 \text{ m/s}^2$$



(*a*) Space diagram. (*b*) Velocity diagram. (c) Acceleration diagram.



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

1. Since *P* and *S* are fixed points, therefore these points lie at one place in the acceleration diagram. Draw vector *p'q'* parallel to *PQ*, to some suitable scale, to represent the radial component of acceleration of *Q* with respect to *P* or acceleration of *Q i.e* a^r_{QP} or a_Q such that

$$\text{vector } p'q' = a^r_{QP} = a_Q = 6.25 \text{ m/s}^2$$

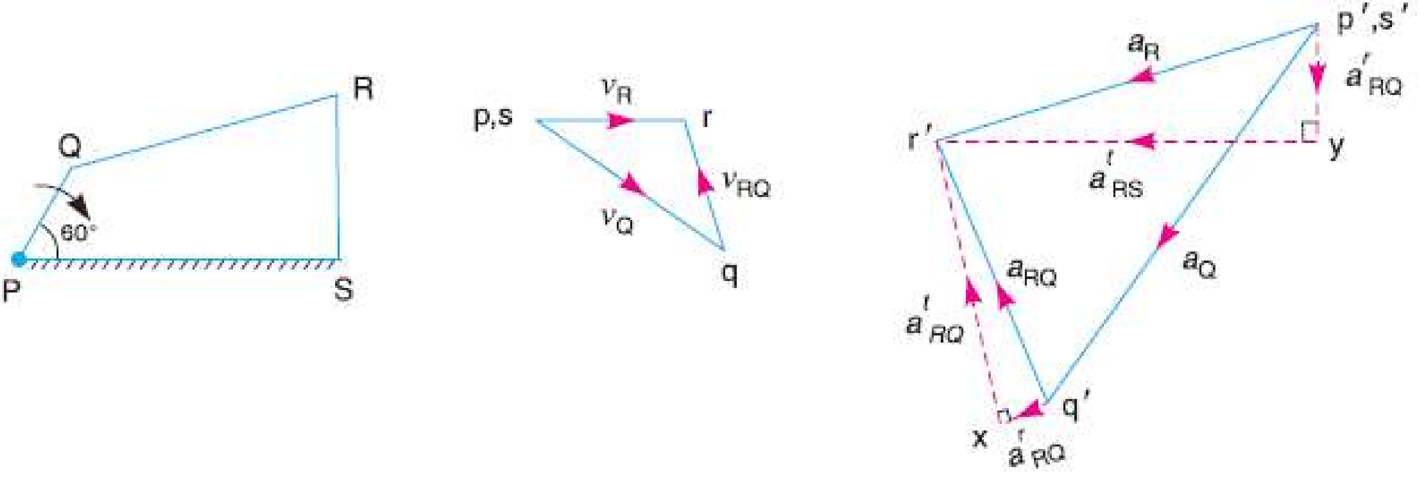
2. From point *q'*, draw vector *q' x* parallel to *QR* to represent the radial component of acceleration of *R* with respect to *Q i.e.* a^r_{RQ} such that

$$\text{vector } q'x = a^r_{RQ} = 0.634 \text{ m/s}^2$$

3. From point *x*, draw vector *xr'* perpendicular to *QR* to represent the tangential component of acceleration of *R* with respect to *Q i.e* a^t_{RQ} whose magnitude is not yet known.

4. Now from point *s'*, draw vector *s'y* parallel to *SR* to represent the radial component of the acceleration of *R* with respect to *S i.e.* a^r_{RS} such that

$$\text{vector } s'y = a^r_{RS} = 1.613 \text{ m/s}^2$$



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

5. From point *y*, draw vector *yr′* perpendicular to *SR* to represent the tangential component of acceleration of *R* with respect to *S* *i.e.* a^t_{RS}.

6. The vectors *xr′* and *yr′* intersect at *r′*. Join *p′r* and *q′ r′*. By measurement, we find that

$$a^t_{RQ} = \text{vector } xr' = 4.1 \text{ m/s}^2 \text{ and } a^t_{RS} = \text{vector } yr' = 5.3 \text{ m/s}^2$$

We know that angular acceleration of link *QR*,

$$\alpha_{QR} = \frac{a^t_{RQ}}{QR} = \frac{4.1}{0.175} = 23.43 \text{ rad/s}^2 \text{ (Anticlockwise)} \textbf{ Ans.}$$

and angular acceleration of link *RS*,

$$\alpha_{RS} = \frac{a^t_{RS}}{SR} = \frac{5.3}{0.1125} = 47.1 \text{ rad/s}^2 \text{ (Anticlockwise)} \textbf{ Ans.}$$

Angular acceleration of links QR and RS

Since the angular acceleration of the crank PQ is not given, therefore there will be no tangential component of the acceleration of Q with respect to P.

We know that radial component of the acceleration of Q with respect to P (or the acceleration of Q),

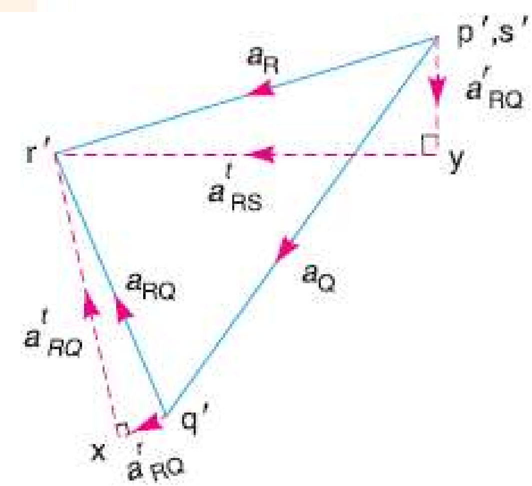
$$a^r_{QP} = a_{QP} = a_Q = \frac{v^2_{QP}}{PQ} = \frac{(0.625)^2}{0.0625} = 6.25 \text{ m/s}^2$$

Radial component of the acceleration of R with respect to Q,

$$a^r_{RQ} = \frac{v^2_{RQ}}{QR} = \frac{(0.333)^2}{0.175} = 0.634 \text{ m/s}^2$$

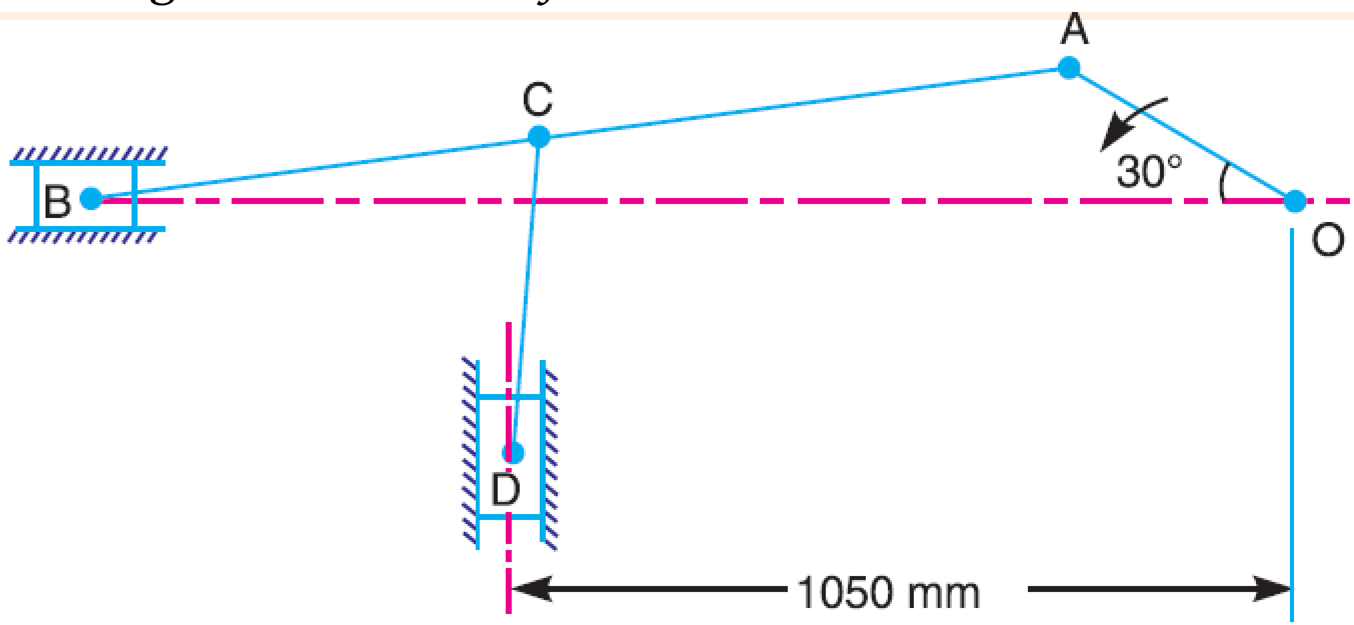
and radial component of the acceleration of R with respect to S (or the acceleration of R),

$$a^r_{RS} = a_{RS} = a_R = \frac{v^2_{RS}}{SR} = \frac{(0.426)^2}{0.1125} = 1.613 \text{ m/s}^2$$



(*c*) Acceleration diagram.

In the mechanism, as shown in Fig., the crank OA rotates at 20 r.p.m. anticlockwise and gives motion to the sliding blocks B and D. The dimensions of the various links are OA = 300 mm; AB = 1200 mm; BC = 450 mm and CD = 450 mm. For the given configuration, determine : 1. velocities of sliding at B and D, 2. angular velocity of CD, 3. linear acceleration of D and 4. angular acceleration of CD.



Solution. Given : $N_{AO} = 20$ r.p.m. or $\omega_{AO} = 2\,\pi \times 20/60 = 2.1$ rad/s ; $OA = 300$ mm $= 0.3$ m ; $AB = 1200$ mm $= 1.2$ m ; $BC = CD = 450$ mm $= 0.45$ m

We know that linear velocity of A with respect to O or velocity of A,

$$v_{AO} = v_A = \omega_{AO} \times OA = 2.1 \times 0.3 = 0.63 \text{ m/s}$$

1. Draw vector oa perpendicular to OA, to some suitable scale, to represent the velocity of A with respect to O (or simply velocity of A), such that

$$\text{vector } oa = v_{AO} = v_A = 0.63 \text{ m/s}$$

2. From point a, draw vector ab perpendicular to AB to represent the velocity of B with respect to A (*i.e.* v_{BA}) and from point o draw vector ob parallel to path of motion B (which is along BO) to represent the velocity of B with respect to O (or simply velocity of B). The vectors ab and ob intersect at b.

3. Divide vector ab at c in the same ratio as C divides AB in the space diagram. In other words,

$$BC/CA = bc/ca$$

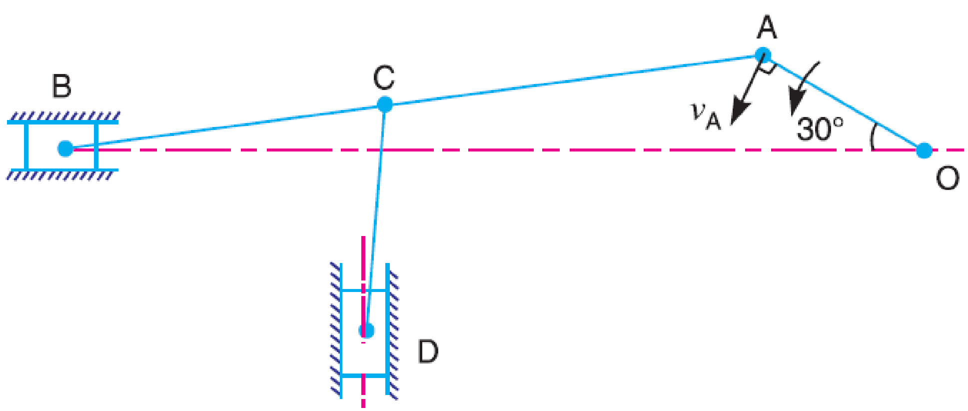
4. Now from point c, draw vector cd perpendicular to CD to represent the velocity of D with respect to C (*i.e.* v_{DC}) and from point o draw vector od parallel to the path of motion of D (which along the vertical direction) to represent the velocity of D.

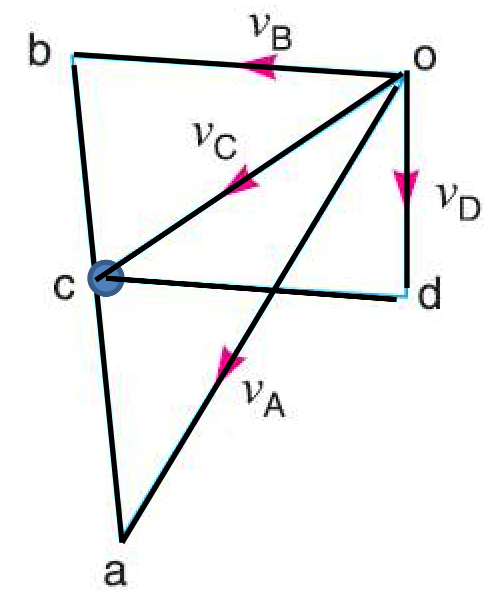
By measurement, we find that velocity of sliding at B,

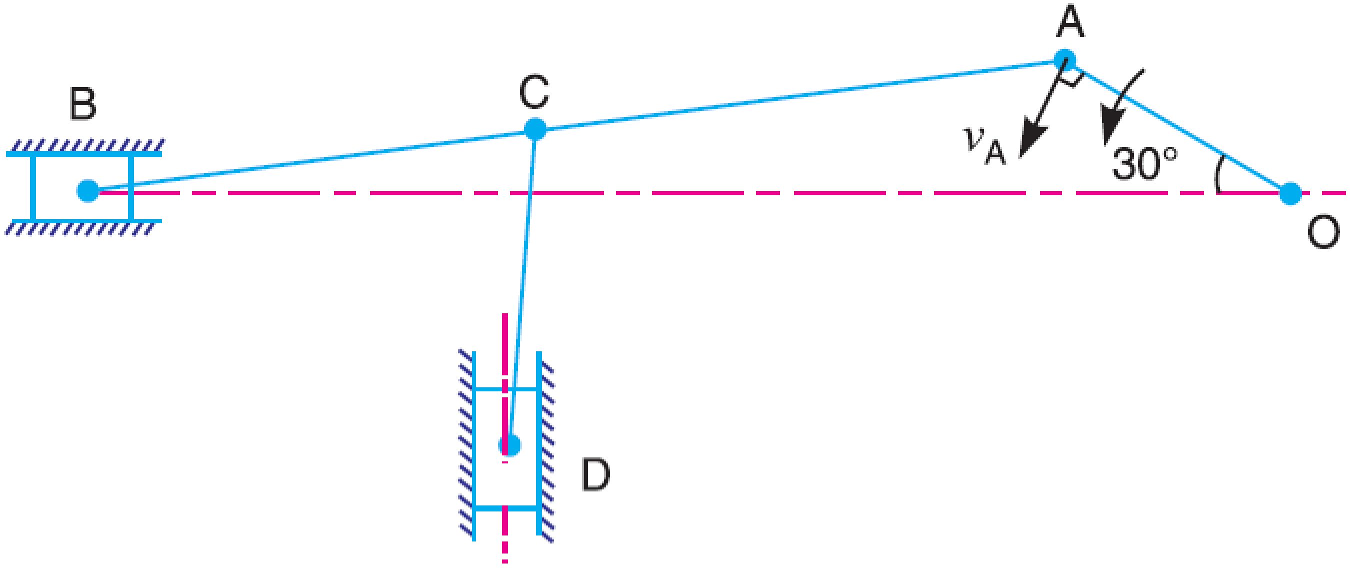
$$v_B = \text{vector } ob = 0.4 \text{ m/s} \quad \textbf{Ans.}$$

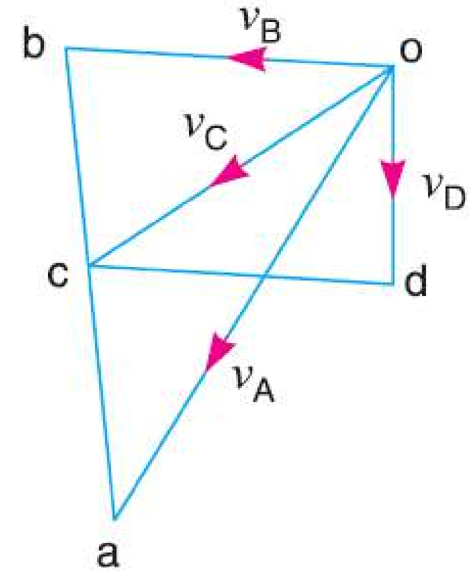
and velocity of sliding at D,

$$v_D = \text{vector } od = 0.24 \text{ m/s} \quad \textbf{Ans.}$$









v_B = vector ob = 0.4 m/s **Ans.**

v_D = vector od = 0.24 m/s **Ans.**

2. *Angular velocity of CD*

By measurement from velocity diagram, we find that velocity of D with respect to C,

$$v_{DC} = \text{vector } cd = 0.37 \text{ m/s}$$

∴ Angular velocity of CD,

$$\omega_{CD} = \frac{v_{DC}}{CD} = \frac{0.37}{0.45} = 0.82 \text{ rad/s (Anticlockwise). } \textbf{Ans.}$$

3. *Linear acceleration of D*

We know that the radial component of the acceleration of A with respect to O or acceleration of A,

$$a_{AO}^r = a_A = \frac{v_{AO}^2}{OA} = \omega_{AO}^2 \times OA = (2.1)^2 \times 0.3 = 1.323 \text{ m/s}^2$$

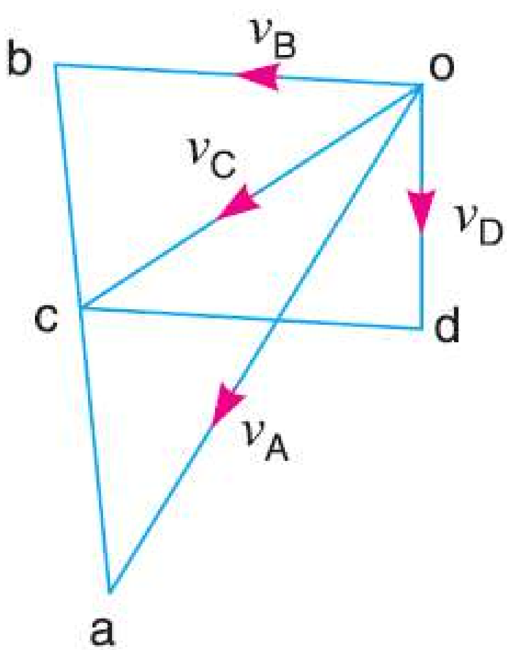
Radial component of the acceleration of B with respect to A,

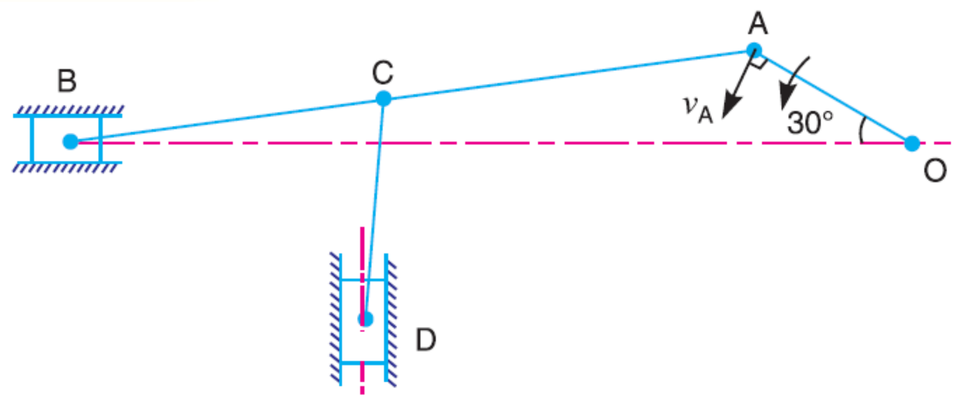
$$a_{BA}^r = \frac{v_{BA}^2}{AB} = \frac{(0.54)^2}{1.2} = 0.243 \text{ m/s}^2$$

...(By measurement, $v_{BA} = 0.54$ m/s)

Radial component of the acceleration of D with respect to C,

$$a_{DC}^r = \frac{v_{DC}^2}{CD} = \frac{(0.37)^2}{0.45} = 0.304 \text{ m/s}^2$$

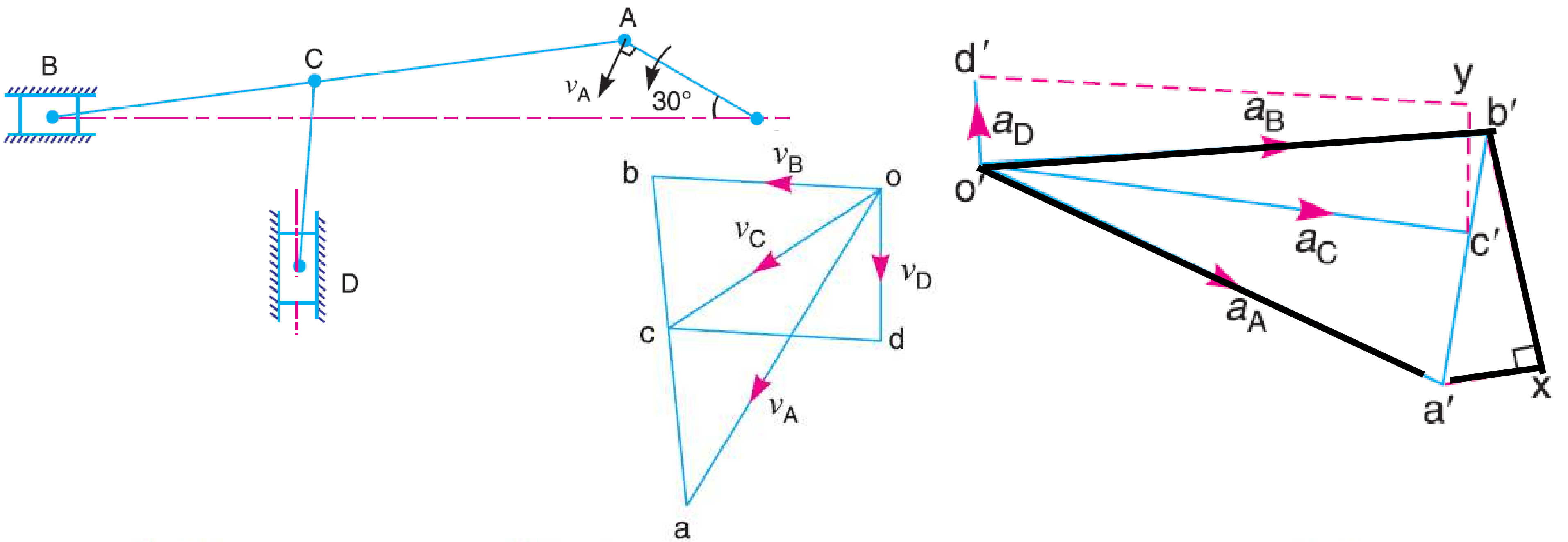




v_B = vector ob = 0.4 m/s **Ans.**

v_D = vector od = 0.24 m/s **Ans.**

$$\omega_{CD} = \frac{v_{DC}}{CD} = \frac{0.37}{0.45} = 0.82 \text{ rad/s}$$



1. Draw vector $o'a'$ parallel to OA, to some suitable scale, to represent the radial component of the acceleration of A with respect to O or simply the acceleration of A, such that

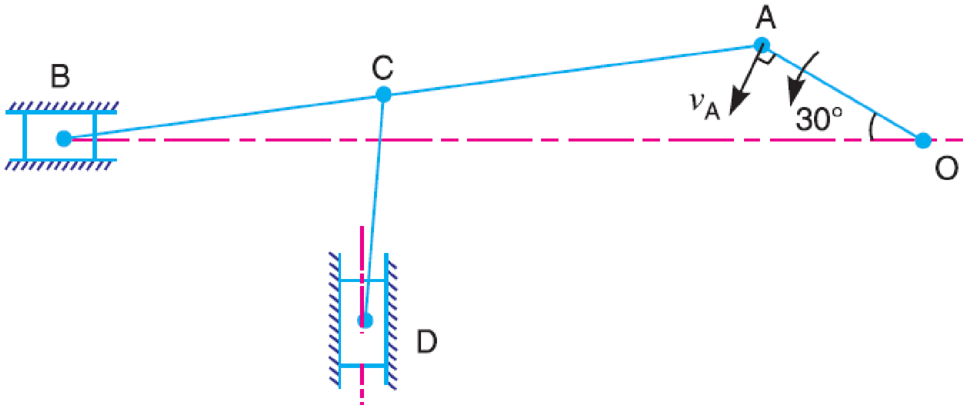
$$\text{vector } o'a' = a^r_{AO} = a_A = 1.323 \text{ m/s}^2$$

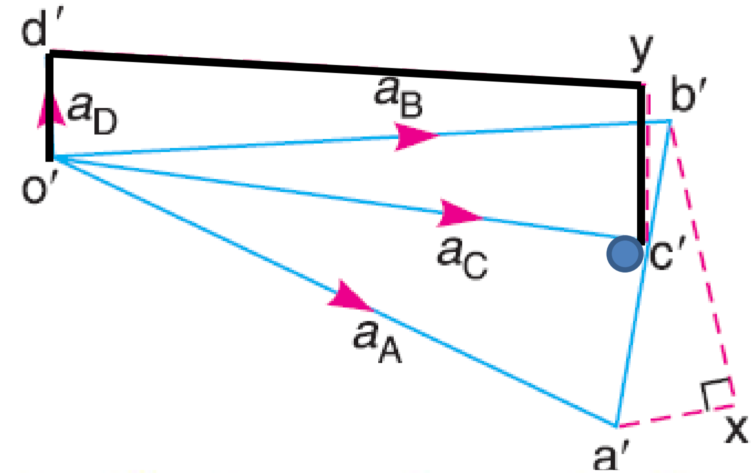
2. From point a', draw vector $a'x$ parallel to AB to represent the radial component of the acceleration of B with respect to A, such that

$$\text{vector } a'x = a^r_{BA} = 0.243 \text{ m/s}^2$$

3. From point x, draw vector xb' perpendicular to AB to represent the tangential component of the acceleration of B with respect to A (*i.e.* a^t_{BA}) whose magnitude is not yet known.

4. From point o', draw vector $o'b'$ parallel to the path of motion of B (which is along BO) to represent the acceleration of B (a_B). The vectors xb' *and* $o'b'$ intersect at b'. Join $a'b'$. The vector $a'b'$ represents the acceleration of B with respect to A.





5. Divide vector $a'b'$ at c' in the same ratio as C divides AB in the space diagram. In other words,

$$BC / BA = b'c'/b'a'$$

6. From point c', draw vector $c'y$ parallel to CD to represent the radial component of the acceleration of D with respect to C, such that

$$\text{vector } c'y = a^r_{DC} = 0.304 \text{ m/s}^2$$

7. From point y, draw yd' perpendicular to CD to represent the tangential component of acceleration of D with respect to C $(i.e.\ a^t_{DC})$ whose magnitude is not yet known.

8. From point o', draw vector $o'd'$ parallel to the path of motion of D (which is along the vertical direction) to represent the acceleration of D (a_D). The vectors yd' and $o'd'$ ***intersect at*** d'.

By measurement, we find that linear acceleration of D,

$$a_D = \text{vector } o'd' = 0.16 \text{ m/s}^2 \qquad \textbf{Ans.}$$

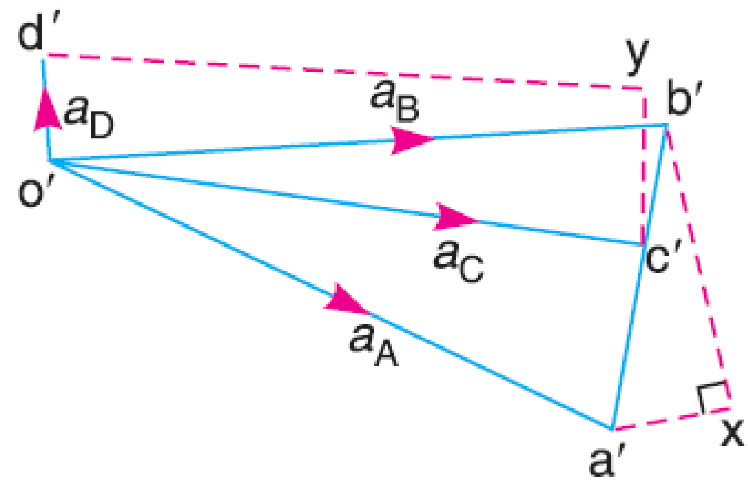
4. *Angular acceleration of CD*

From the acceleration diagram, we find that the tangential component of the acceleration of *D* with respect to *C*,

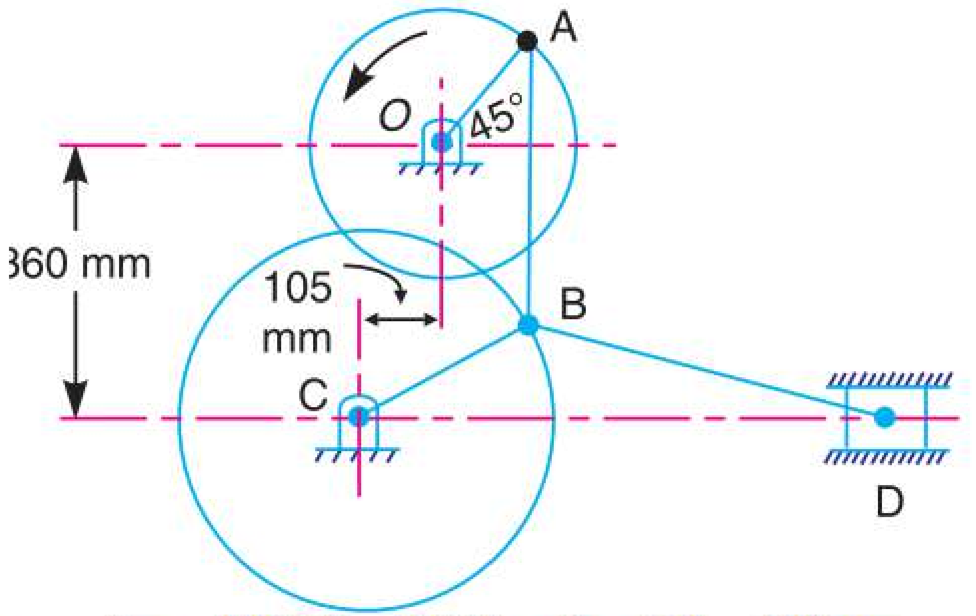
$$a_{DC}^{t} = \text{vector } yd' = 1.28 \text{ m/s}^2 \qquad \text{...(By measurement)}$$

$\therefore$ Angular acceleration of *CD*,

$$\alpha_{CD} = \frac{a_{DC}^{t}}{CD} = \frac{1.28}{0.45} = 2.84 \text{ rad/s}^2$$



In the toggle mechanism shown in Fig., the slider D is constrained to move on a horizontal path. The crank OA is rotating in the counter-clockwise direction at a Speed of 180 r.p.m. increasing at the rate of 50 rad/s^2 . The dimensions of the various links are as follows: OA = 180 mm ; CB = 240 mm ; AB = 360 mm ; and BD = 540 mm. For the given configuration, find 1. Velocity of slider D and angular velocity of BD, and 2. Acceleration of slider D and angular acceleration of BD.



Solution. Given : $N_{AO} = 180$ r.p.m. or $\omega_{AO} = 2\pi \times 180/60 = 18.85$ rad/s ; $OA = 180$ mm $= 0.18$ m ; $CB = 240$ mm $= 0.24$ m ; $AB = 360$ mm $= 0.36$ m ; $BD = 540$ mm $= 0.54$ m

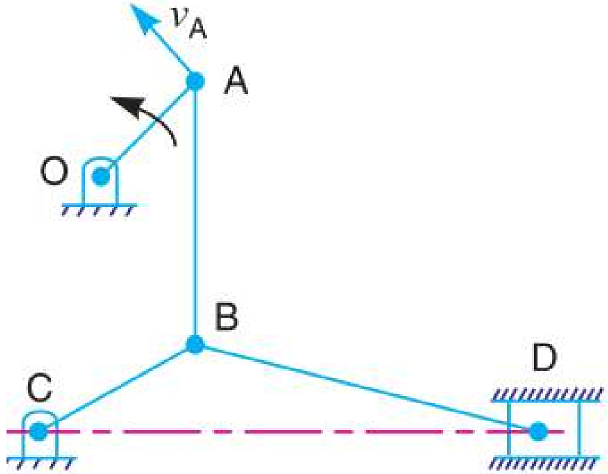
We know that velocity of A with respect to O or velocity of A,

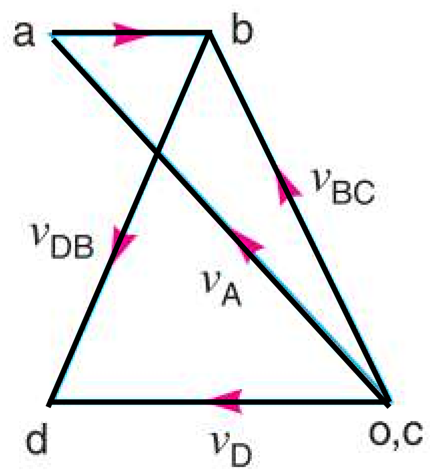
$$v_{AO} = v_A = \omega_{AO} \times OA = 18.85 \times 0.18 = 3.4 \text{ m/s}$$

...(Perpendicular to OA)

1. *Velocity of slider D and angular velocity of BD*

First of all, draw the space diagram to some suitable scale, as shown in Fig.





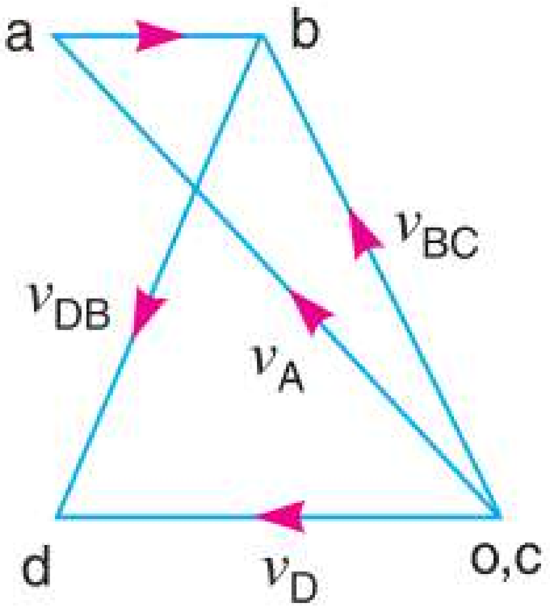
Now the velocity diagram, as shown in Fig.(b), is drawn as discussed

1. Since O and C are fixed points, therefore these points lie at one place in the velocity diagram. Draw vector oa perpendicular to OA, to some suitable scale, to represent the velocity of A with respect to O or velocity of A *i.e.* v_{AO} or v_A, such that

$$\text{vector } oa = v_{AO} = v_A = 3.4 \text{ m/s}$$

2. Since B moves with respect to A and also with respect to C, therefore draw vector ab perpendicular to AB to represent the velocity of B with respect to A *i.e.* v_{BA}, and draw vector cb perpendicular to CB to represent the velocity of B with respect to C *ie.* v_{BC}. The vectors ab and cb intersect at b.

3. From point b, draw vector bd perpendicular to BD to represent the velocity of D with respect to B *i.e.* v_{DB}, and from point c draw vector cd parallel to CD (*i.e.*, in the direction of motion of the slider D) to represent the velocity of D *i.e.* v_D.



By measurement, we find that velocity of B with respect to A,

$$v_{BA} = \text{vector } ab = 0.9 \text{ m/s}$$

Velocity of B with respect to C,

$$v_{BC} = \text{vector } cb = 2.8 \text{ m/s}$$

Velocity of D with respect to B,

$$v_{DB} = \text{vector } bd = 2.4 \text{ m/s}$$

and velocity of slider D, $\quad v_D = \text{vector } cd = 2.05 \text{ m/s}$ **Ans.**

Angular velocity of BD

We know that the angular velocity of BD,

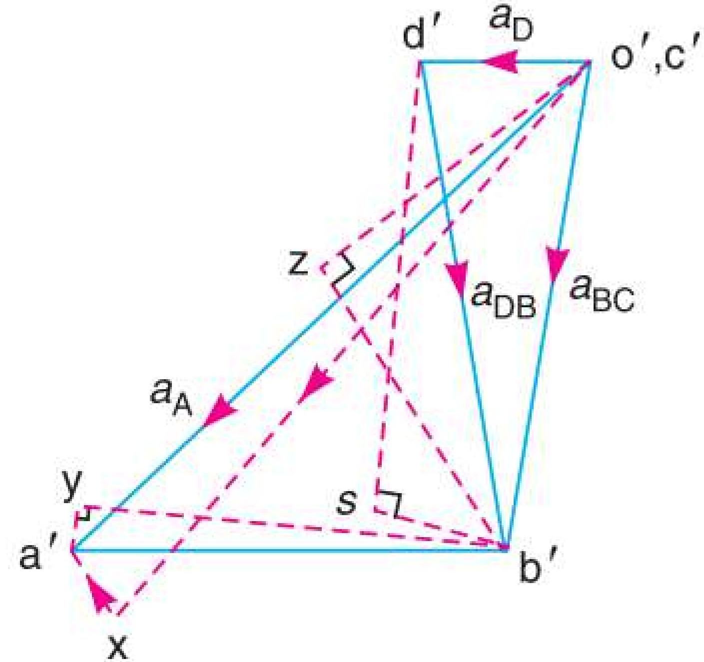
$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{2.4}{0.54} = 4.5 \text{ rad/s} \quad \textbf{Ans.}$$

2. Acceleration of slider D and angular acceleration of BD

Since the angular acceleration of OA increases at the rate of 50 rad/s^2, *i.e.* $\alpha_{AO} = 50$ rad/s^2, therefore

Tangential component of the acceleration of A with respect to O,

$$a_{AO}^t = \alpha_{AO} \times OA = 50 \times 0.18 = 9 \text{ m/s}^2$$



Radial component of the acceleration of A with respect to O,

$$a_{AO}^r = \frac{v_{AO}^2}{OA} = \frac{(3.4)^2}{0.18} = 63.9 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to A,

$$a_{BA}^r = \frac{v_{BA}^2}{AB} = \frac{(0.9)^2}{0.36} = 2.25 \text{ m/s}^2$$

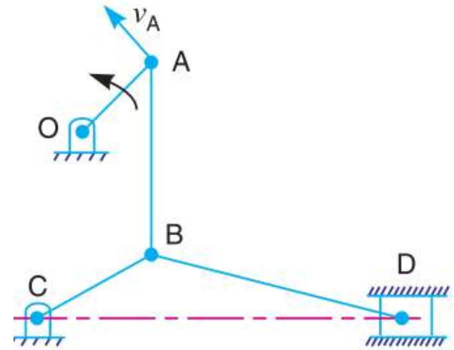
Radial component of the acceleration of B with respect to C,

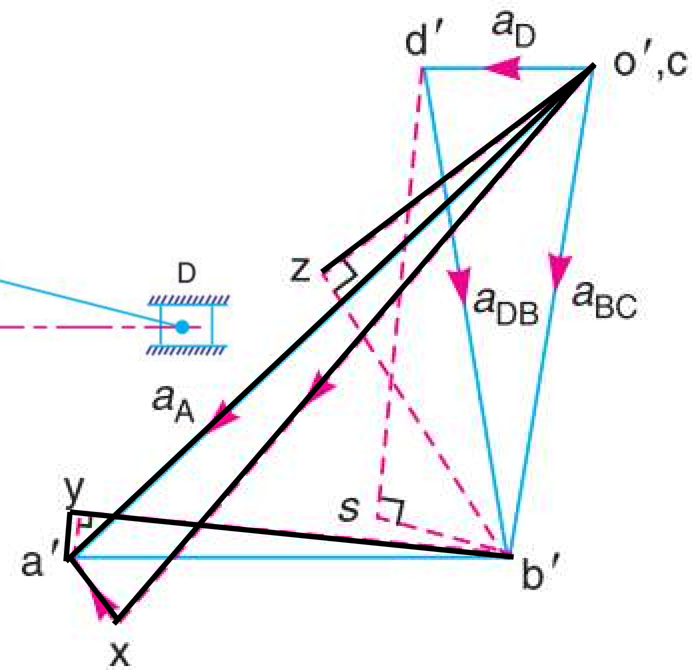
$$a_{BC}^r = \frac{v_{BC}^2}{CB} = \frac{(2.8)^2}{0.24} = 32.5 \text{ m/s}^2$$

radial component of the acceleration of D with respect to B,

$$a_{DB}^r = \frac{v_{DB}^2}{BD} = \frac{(2.4)^2}{0.54} = 10.8 \text{ m/s}^2$$

1. Since O and C are fixed points, therefore these points lie at one place in the acceleration diagram. Draw vector $o'x$ parallel to OA, to some suitable scale, to represent the radial component of the acceleration of A with respect to O *i.e.* a^r_{AO}, such that vector $o'x = a^r_{AO} = 63.9 \text{ m/s}^2$





2. From point x, draw vector xa' perpendicular to vector $o'x$ or OA to represent the tangential component of the acceleration of A with respect to O *i.e.* a^t_{AO}, such tha vector $xa' = a^t_{AO} = 9 \text{ m/s}^2$

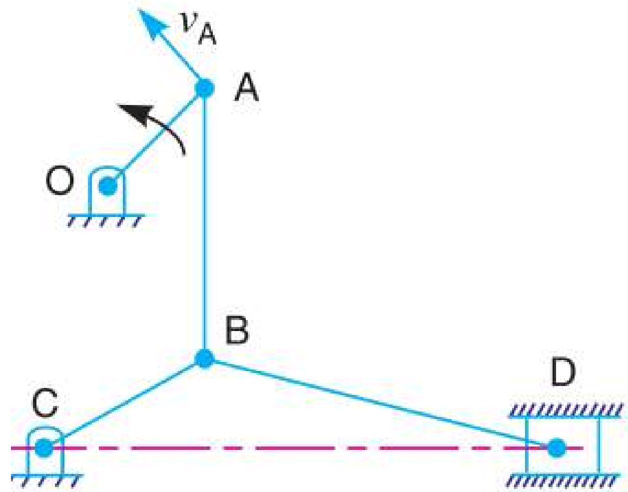
3. Join $o'a'$. The vector $o'a'$ represents the total acceleration of A with respect to O or acceleration of A *i.e.* a_{AO} or a_A.

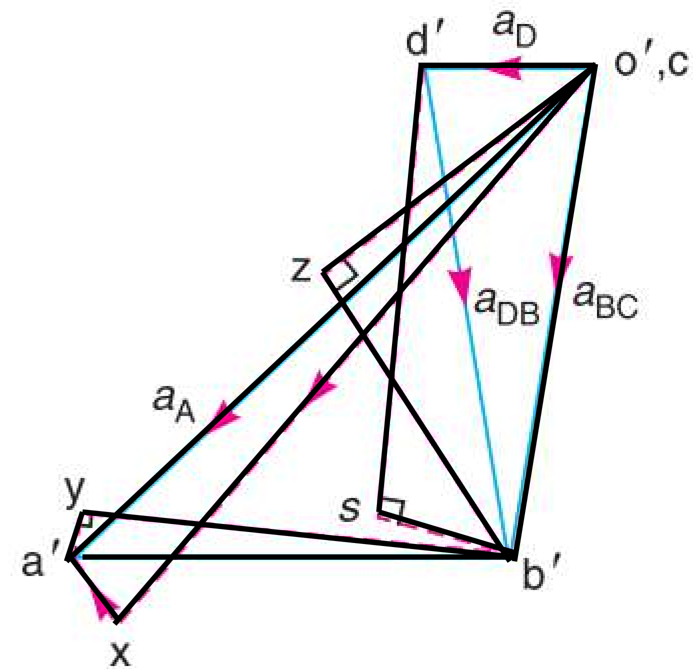
4. Now from point a', draw vector $a'y$ parallel to AB to represent the radial component of the acceleration of B with respect to A *i.e.* a^r_{BA}, such that

$$\text{vector } a'y = a^r_{BA} = 2.25 \text{ m/s}^2$$

5. From point y, draw vector yb' perpendicular to vector $a'y$ or AB to represent the tangential component of the acceleration of B with respect to A *i.e.* a^t_{BA} whose magnitude is yet unknown.

6. Now from point c', draw vector $c'z$ parallel to CB to represent the radial component of the acceleration of B with respect to C *i.e.* a^r_{BC}, such that vector $c'z = a^r_{BC} = 32.5 \text{ m/s}^2$





7. From point z, draw vector zb' perpendicular to vector $c'z$ or CB to represent the tangential component of the acceleration of B with respect to C *i.e.* a^t_{BC}. The vectors yb' and zb' intersect at b'. Join $c'b'$. The vector $c'b'$ represents the acceleration of B with respect to C *i.e.* a_{BC}.

8. Now from point b', draw vector $b's$ parallel to BD to represent the radial component of the acceleration of D with respect to B *i.e.* a^r_{DB}, such that

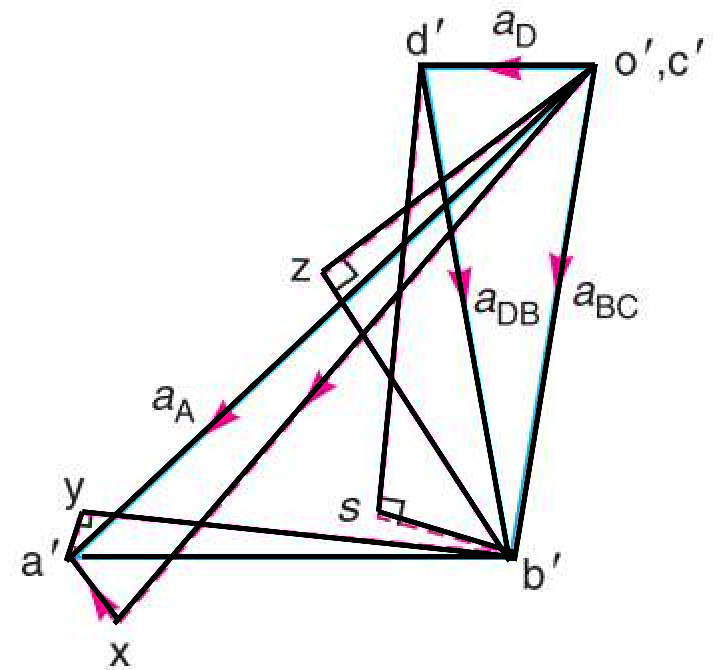
$$\text{vector } b's = a^r_{DB} = 10.8 \text{ m/s}^2$$

9. From point s, draw vector sd' perpendicular to vector $b's$ or BD to represent the tangential component of the acceleration of D with respect to B *i.e.* a^t_{DB} whose magnitude is yet unknown.

10. From point c', draw vector $c'd'$ parallel to the path of motion of D (which is along CD) to represent the acceleration of D *i.e.* a_D. The vectors sd' and $c'd'$ intersect at d'.

By measurement, we find that acceleration of slider D,

$$a_D = \text{vector } c'd' = 13.3 \text{ m/s}^2 \quad \textbf{Ans.}$$



Angular acceleration of BD

By measurement, we find that tangential component of the acceleration of D with respect to B,

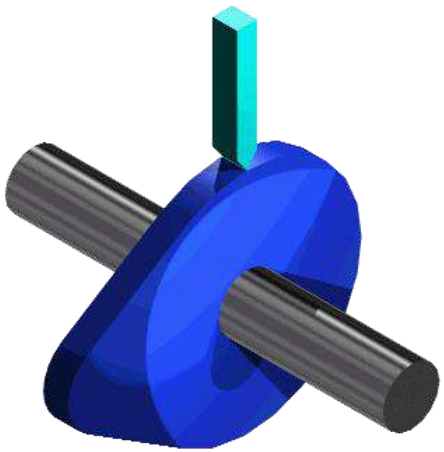
$$a^t_{DB} = \text{vector } sd' = 38.5 \text{ m/s}^2$$

We know that angular acceleration of BD,

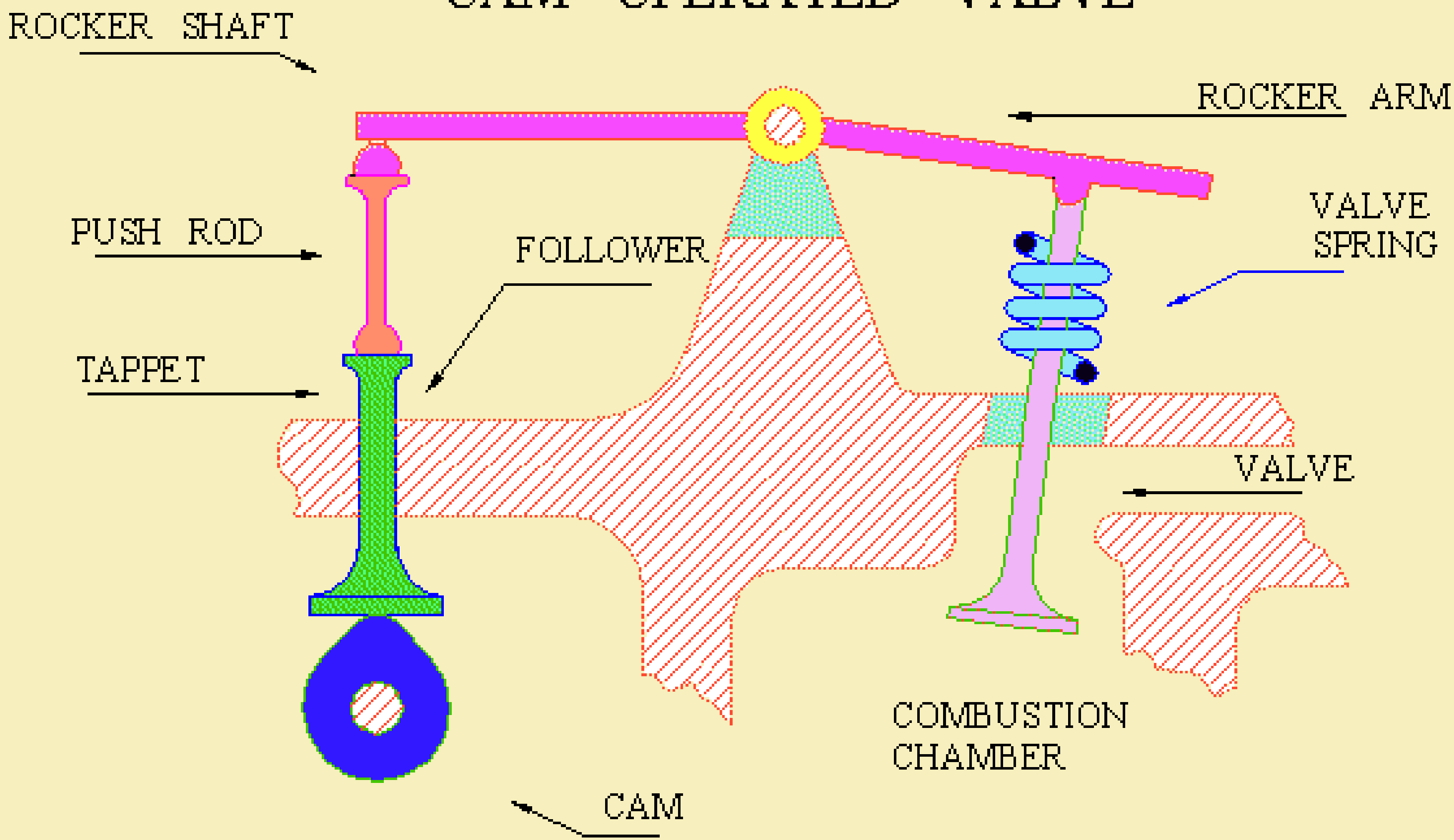
$$\alpha_{BD} = \frac{a^t_{DB}}{BD} = \frac{38.5}{0.54} = 71.3 \text{ rad/s}^2 \text{ (Clockwise)} \quad \textbf{Ans.}$$

UNIT - IV

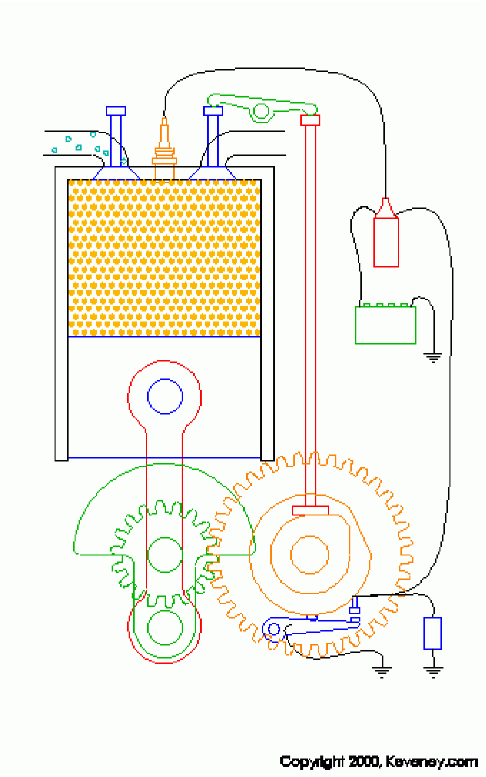
CAMS

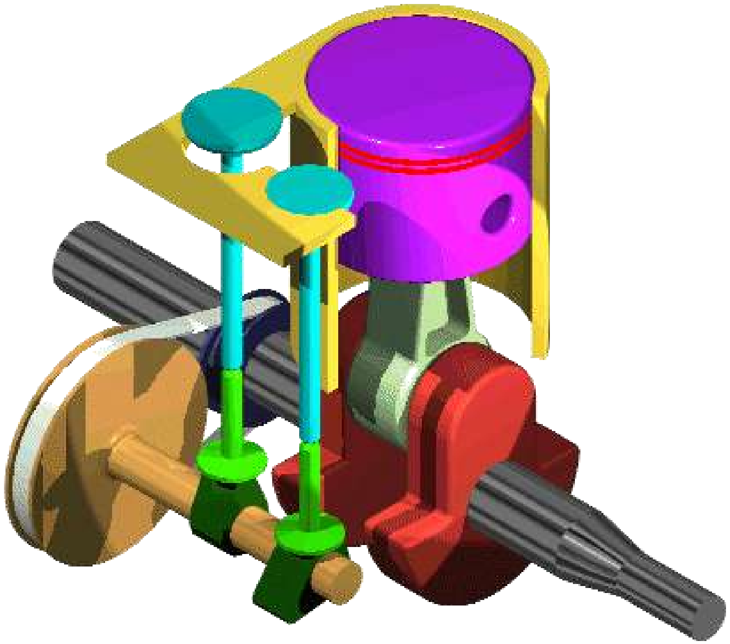


CAM OPERATED VALVE



Examples for cam

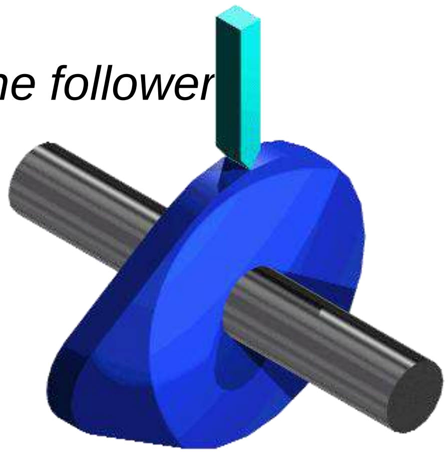




- In IC engines to operate the inlet and exhaust valves

Introduction

- A cam is a mechanical member used to impart desired motion to a follower by direct contact.
- The cam may be rotating or reciprocating whereas the follower may be rotating, reciprocating or oscillating.
- Complicated output motions which are otherwise difficult to achieve can easily be produced with the help of cams.
- Cams are widely used in automatic machines, internal combustion engines, machine tools, printing control mechanisms, and so on.
- They are manufactured usually by die-casting, milling or by punch-presses.
- A cam and the follower combination belong to the category of higher pairs.
- Necessary elements of a cam mechanism are
  - A driver member known as the *cam*
  - A driven member called *the follower*
  - *A frame which supports the cam and guides the follower*



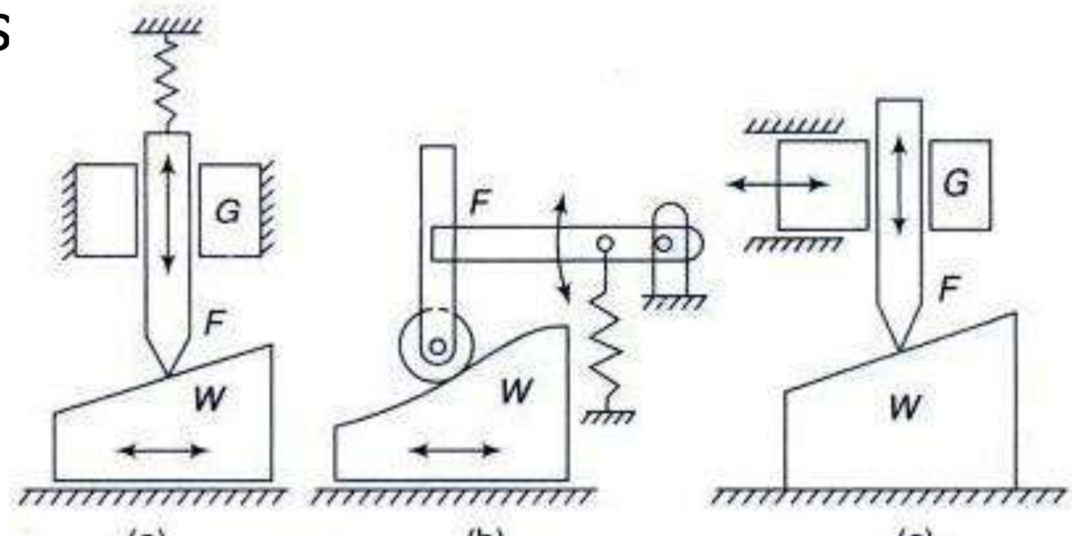
TYPES OF CAMS

- Cams are classified according to
    1. shape,
    2. follower movement, and
    3. manner of constraint of the follower.

I. According to Shape

1) Wedge and Flat Cams

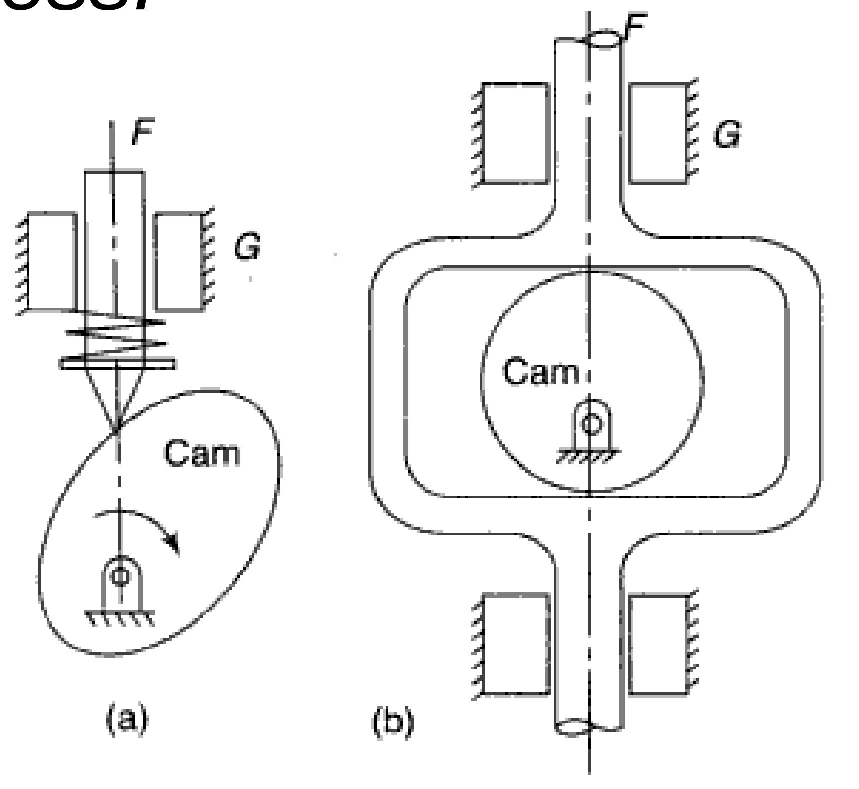
- *A wedge cam has a wedge W which, in general, has a translational motion.*
- *The follower F can either translate [Fig.(a)] or oscillate [Fig.(b)].*
- *A spring is, usually, used to maintain the contact between the cam and t*he follower.
- In Fig.(c), the cam is stationary and the follower constraint or guide *G caus* ... *ıd the follower.*



(a) (b) (c)

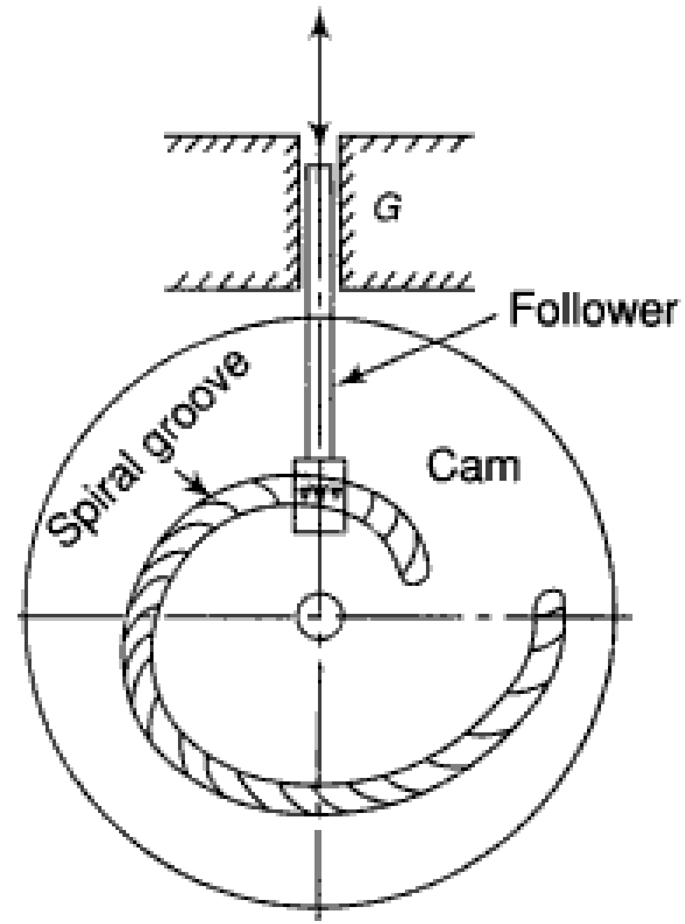
2. Radial or Disc Cams

- *A cam in which the follower moves radially from the centre of rotation of the cam is known as a radial or a disc cam (Fig. (a) and (b)].*
- *Radial cams are very popular due to their simplicity and compactness.*



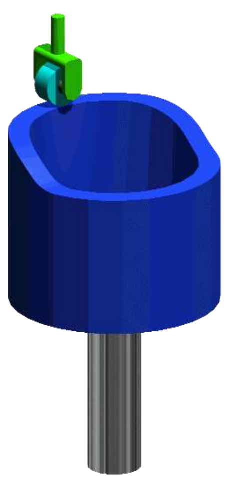
3. Spiral Cams

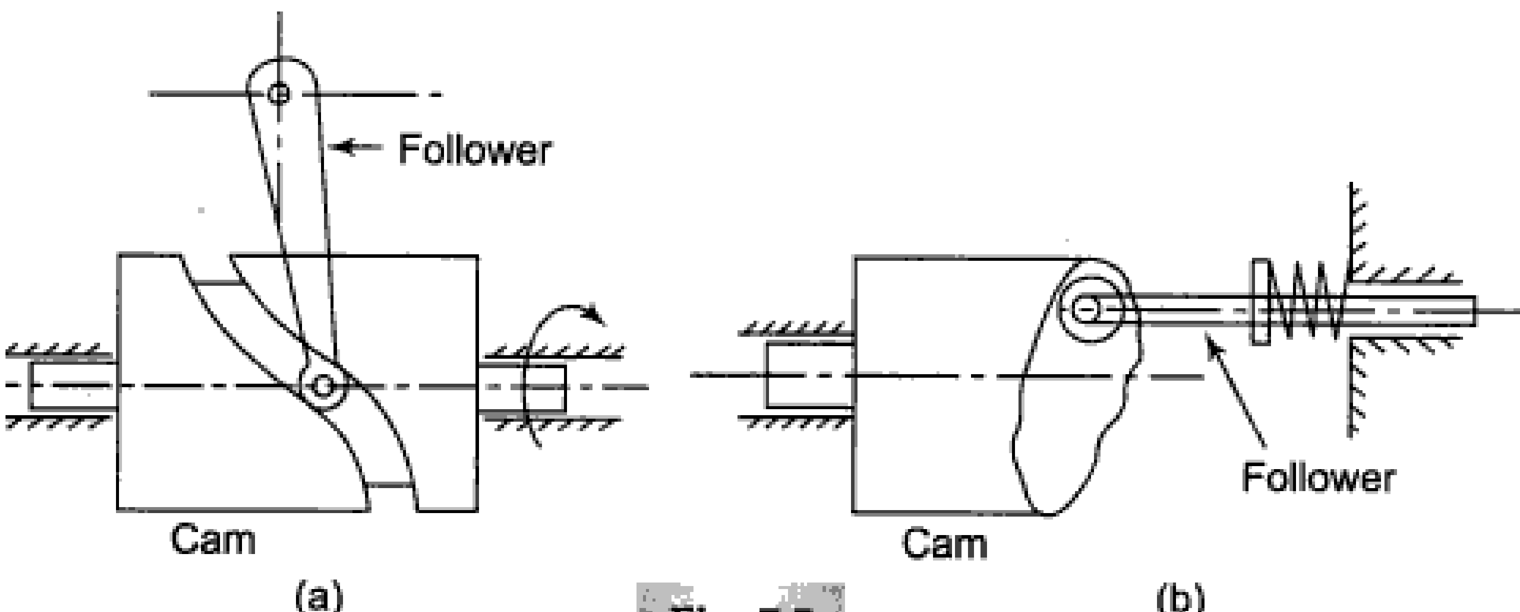
- *A spiral cam is a face cam in which a groove is cut in the form of a spiral as shown in Fig.*
- *The spiral groove consists of teeth which mesh with a pin gear follower.*
- *The velocity of the follower is proportional to the radial distance of the groove from the axis of the cam.*
- The use of such a cam is limited as the cam has to reverse the direction to reset the position of the follower. It finds its use in computers.



4. *Cylindrical Cams*

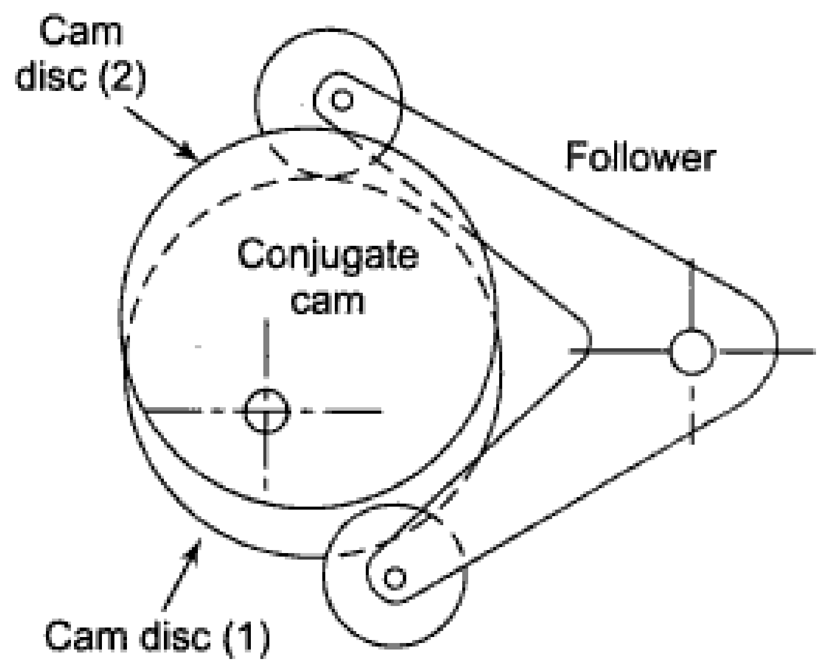
- *In a cylindrical cam, a cylinder which has a circumferential contour cut in the surface, rotates about its axis.*
- *The follower motion can be of two types as follows:* In the first type, a groove is cut on the surface of the cam and a roller follower has a constrained (or positive) oscillating motion [Fig.(a)].
- Another type is an end cam in which the end of the cylinder is the working surface (b).
- A spring-loaded follower translates along or parallel to the axis of the rotating cylinder.





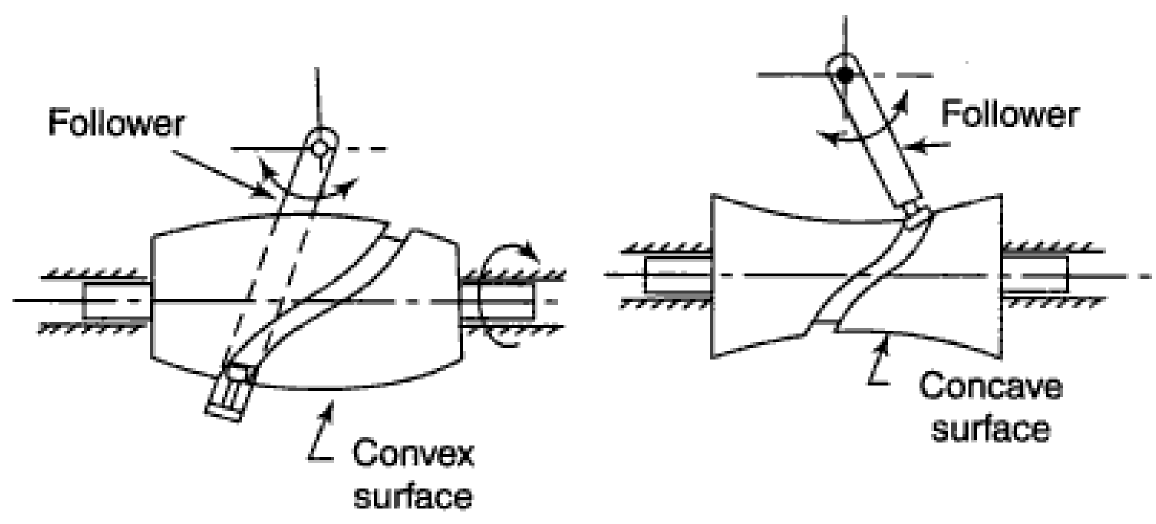
5. *Conjugate Cams*

- *A conjugate cam is a double-disc cam, the two discs being keyed together and are in constant touch with the two rollers of a follower (shown in Fig.).*
- *Thus, the follower has a positive constraint.*
- *Such a type of cam is preferred when the requirements are low wear, low noise, better control of the follower, high speed, high dynamic loads, etc.*



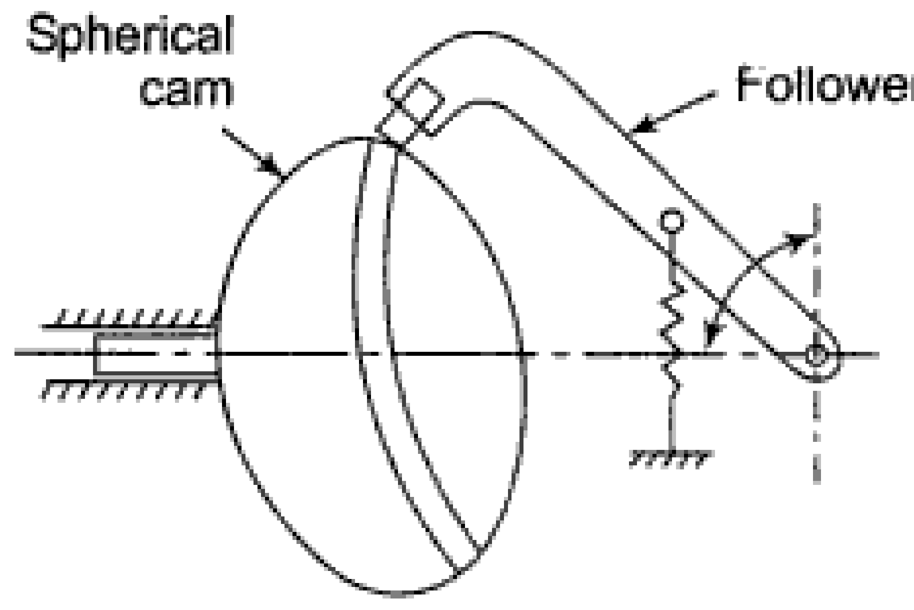
6. *Globoidal Cams*

- *A globoidal cam can have two types of surfaces, convex or concave.*
- *A circumferential contour is cut on the surface of rotation of the cam to impart motion to the follower which has an oscillatory motion (Fig.).*
- *The application of such cams is limited to moderate speeds and where the angle of oscillation of the follower is large.*



7. Spherical Cams

- *In a spherical* cam, the follower oscillates about an axis perpendicular to the axis surface of rotation of the cam.
- Note that in a disc cam, the follower oscillates about an axis parallel to the axis of rotation of the cam.
- A spherical cam is in the form of a spherical surface which transmits motion to the follower (Fig.).

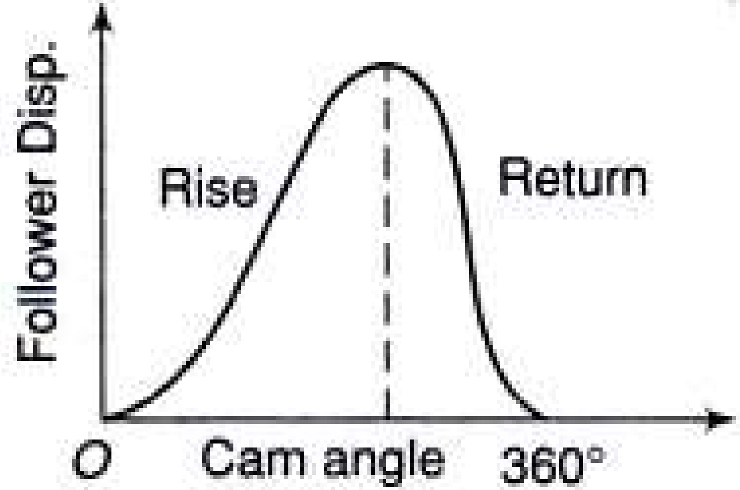


II. According to Follower Movement

- The motions of the followers are distinguished from each other by the dwells they have.
- A dwell is the zero displacement or the absence of motion of the follower during the motion of the cam.
- Cams are classified according to the motions of the followers in the following ways:

1. Rise-Return-Rise (R-R-R)

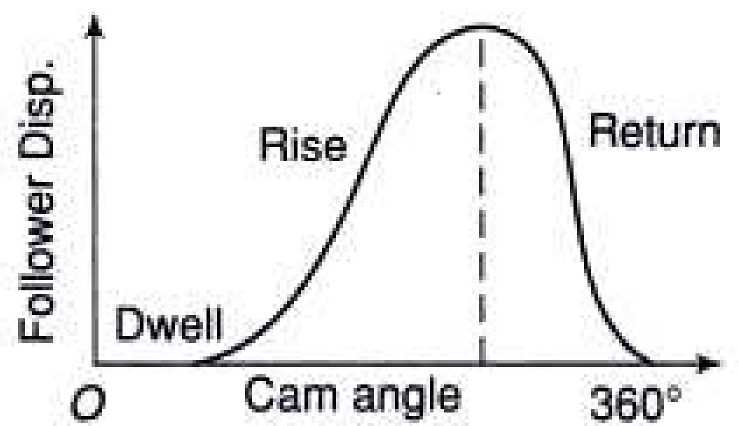
- *In* this, there is alternate rise and return of the follower with no periods of dwells (Fig. a).
- Its use is very limited in the industry.
- The follower has a linear or an angular displacement.



(a)

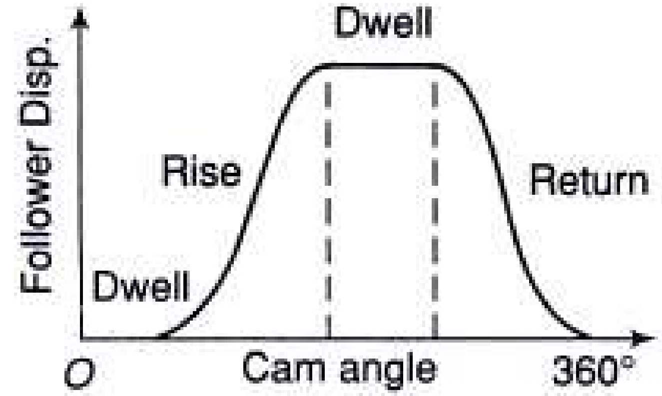
2. Dwell-Rise-Return-Dwell (D-R-R-D)

- *In such a type of cam, there is rise and return of the follower after a dwell Fig.(b).*
- *his type is used more frequently than the R-R-R type of cam.*



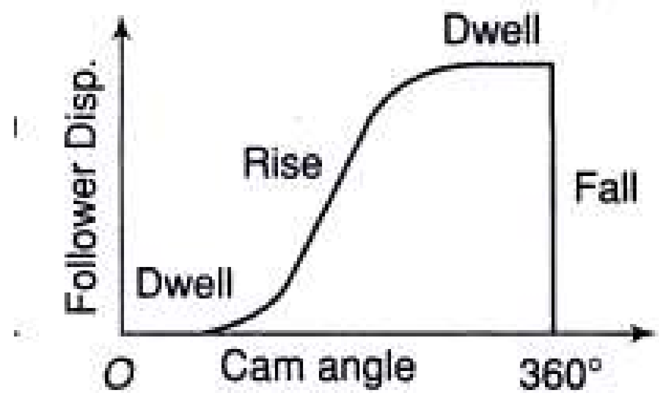
(b)

3. *Dwell-Rise-Dwell-Return-Dwell (D-R-D-R-D)*



(c)

- *It is the most widely used type of cam.*
- *The dwelling of the cam is followed by* rise and dwell and subsequently by return and dwell as shown in rig. (c).
- In case the return of the follower is by a fall [Fig.(d)], the motion may be known as Dwell-Rise-Dwell (D-R-D).



(d)

III. According to Manner of Constraint of the Follower

- To reproduce exactly the motion transmitted by the cam to the follower, it is necessary that the two remain in touch at all speeds and at all times.
- The cams can be classified according to the manner in which this is achieved.

1. ***Pre-loaded Spring Cam***

   A pre-loaded compression spring is used for the purpose of keeping the contact between the cam and the follower.

2. ***Positive-drive Cam***

   In this type, constant touch between the cam and the follower is maintained by a roller follower operating in the groove of a cam.

   The follower cannot go out of this groove under the normal working operations.

   A constrained or positive drive is also obtained by the use of a conjugate cam

3. ***Gravity Cam***

   If the rise of the cam is achieved by the rising surface of the cam and the return by the force of gravity or due to the weight of the cam, the cam is known as a gravity cam.

   However, these cams are not preferred due to their uncertain behavior.

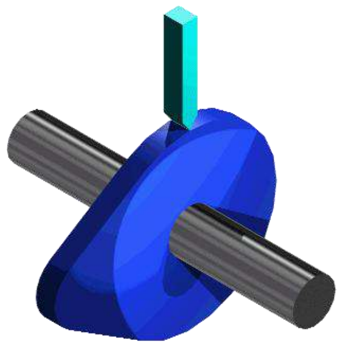
Classification of Followers

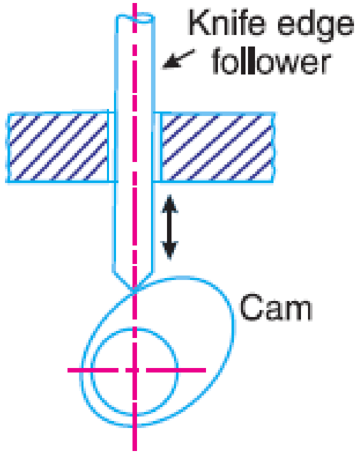
1. According to the surface in contact.

a) Knife edge follower.

When the contacting end of the follower has a sharp knife edge, it is called a knife edge follower, as shown in Fig.(*a).*

- *The* sliding motion takes place between the contacting surfaces (*i.e. the knife edge and the cam surface).*
- *It* is seldom used in practice because the small area of contacting surface results in excessive wear.
- In knife edge followers, a considerable side thrust exists between the follower and the guide.

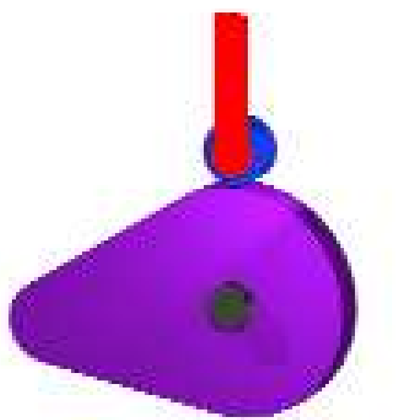


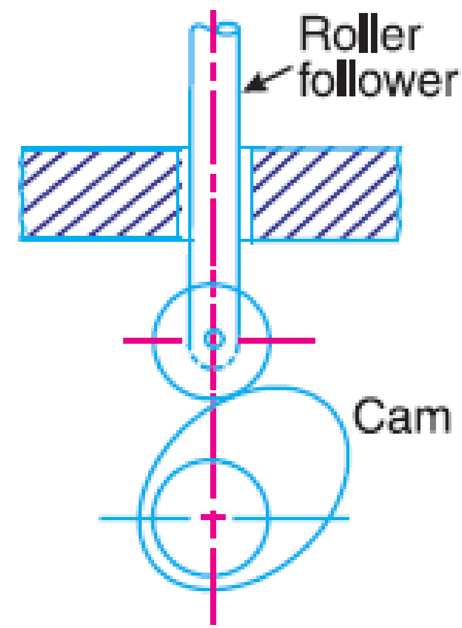


(*a*) Cam with knife edge follower.

(*b*) Roller follower.

- ***When the contacting end of the follower is a roller, it is called a roller*** follower, as shown in Fig. (*b*).
- *Since the rolling motion takes place between the* contacting surfaces (*i.e. the roller and the cam*), *therefore the rate of wear is greatly reduced.*
- In roller followers also the side thrust exists between the follower and the guide.
- The roller followers are extensively used where more space is available
r





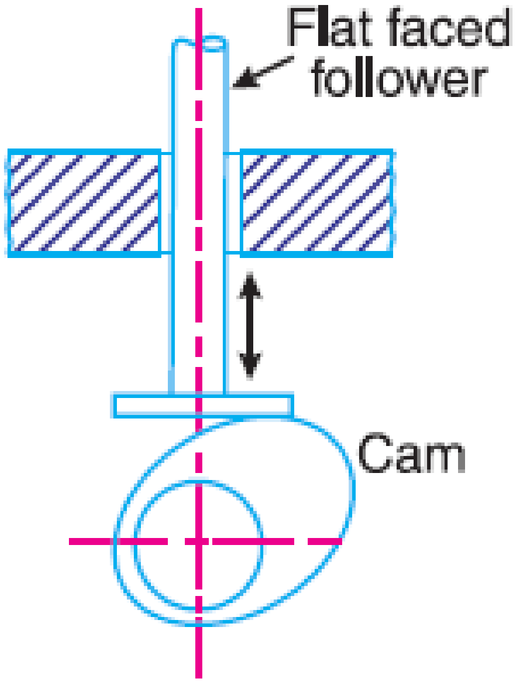
(*b*) Cam with roller follower.

(c) Flat faced or mushroom follower.

- *When the contacting end of the follower is a perfectly* flat face, it is called a flat-faced follower, as shown in Fig. 20.1 (*c).*
- *It may be noted that* the side thrust between the follower and the guide is much reduced in case of flat faced followers.
- The only side thrust is due to friction between the contact surfaces of the follower and the cam.

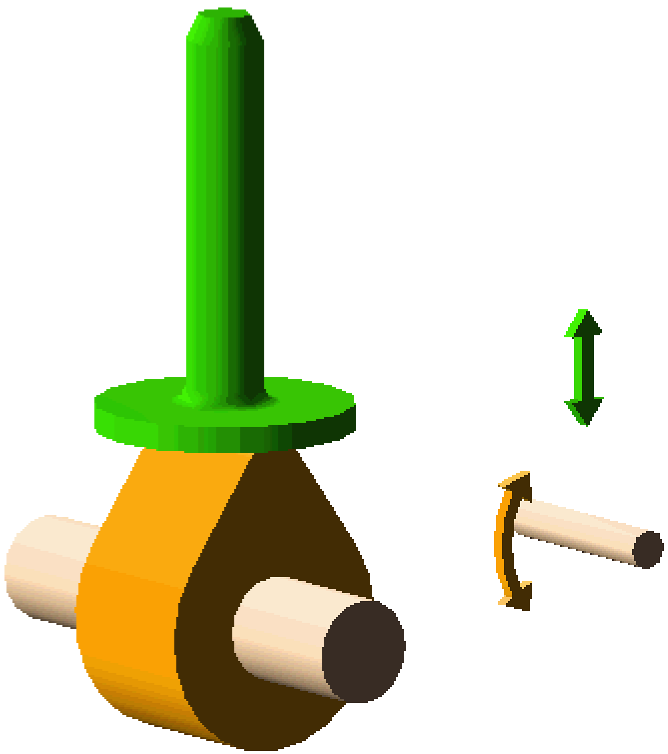
•The flat faced followers are generally used where space is limited such as in cams which operate the valves of automobile engines.

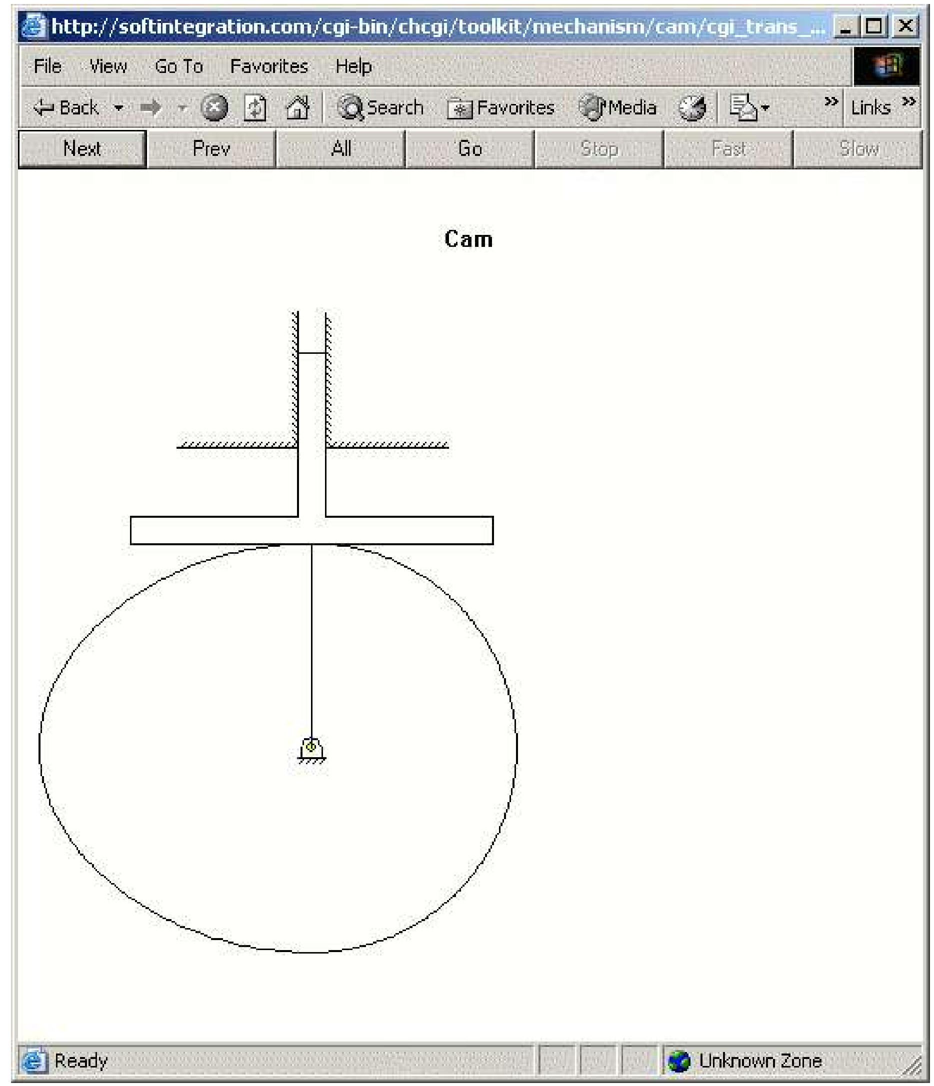
•Note : When the flat faced follower is circular, it is then called a mushroom follower.



(*c*) Cam with flat faced follower.

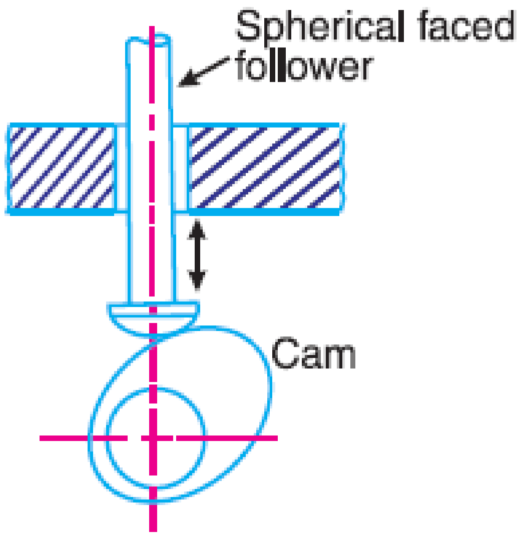
C) Flat faced or mushroom follower.



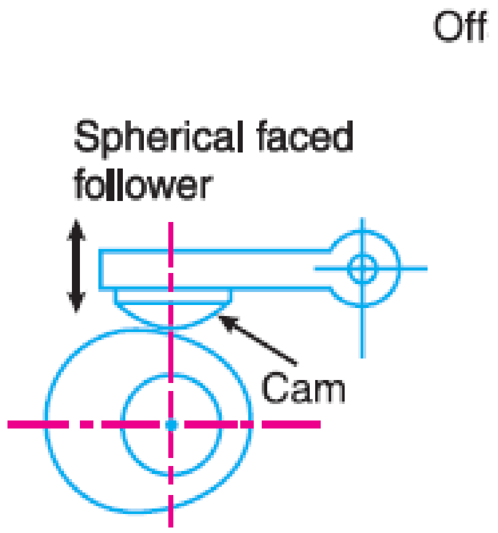


(d) Spherical faced follower.

- *When the contacting end of the follower is of spherical shape,* it is called a spherical faced follower, as shown in Fig. (*d).*
- *It may be noted that when* a flat-faced follower is used in automobile engines, high surface stresses are produced.
- In order to minimise these stresses, the flat end of the follower is machined to a spherical shape.



(*d*) Cam with spherical faced follower.

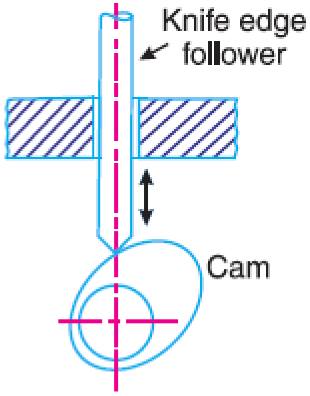


(*e*) Cam with spherical faced follower.

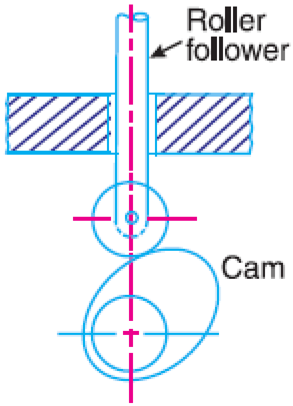
2. *According to the motion of the follower*

(a) Reciprocating or translating follower.

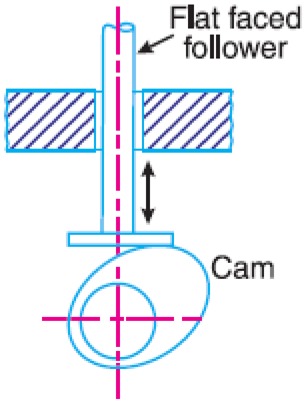
- *When the follower reciprocates in guides as the* cam rotates uniformly, it is known as reciprocating or translating follower.
- The followers as shown in Fig. (*a) to (d) are all reciprocating or translating followers.*



(*a*) Cam with knife edge follower.



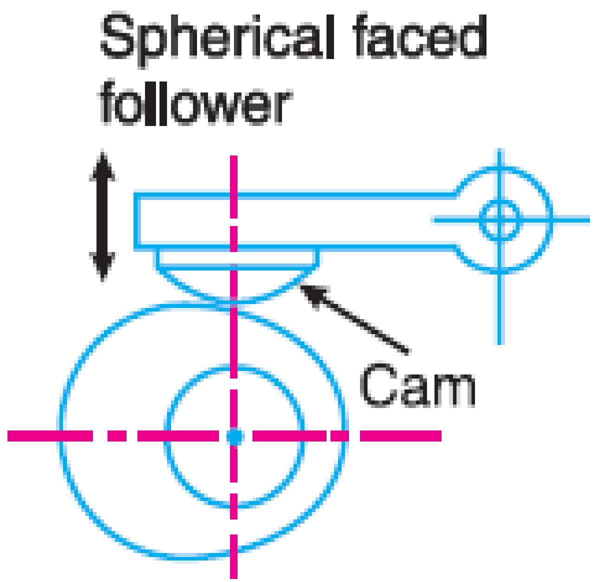
(*b*) Cam with roller follower.



(*c*) Cam with flat faced follower.

(b) Oscillating or rotating follower.

- *When the uniform rotary motion of the cam is converted* into predetermined oscillatory motion of the follower, it is called oscillating or rotating follower.
- The follower, as shown in (*e*)*, is an oscillating or rotating follower.*

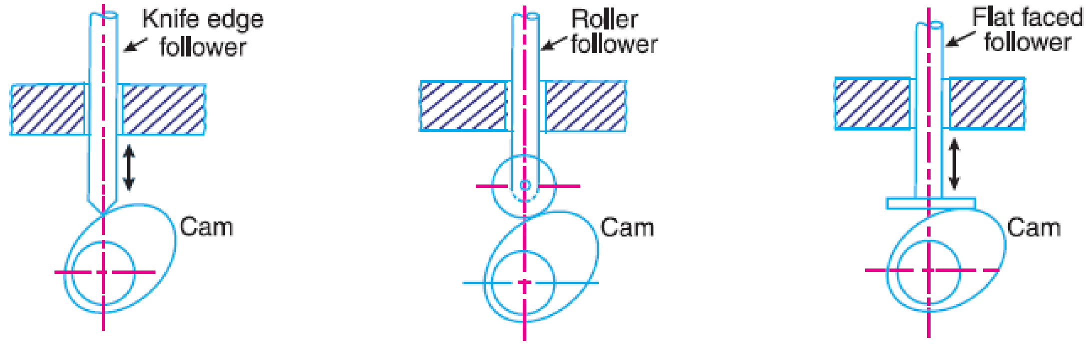


(*e*) Cam with spherical faced follower.

3. *According to the path of motion of the follower.*

(*a*) *Radial follower. When the motion of the follower is along an axis passing through the* centre of the cam, it is known as radial follower.

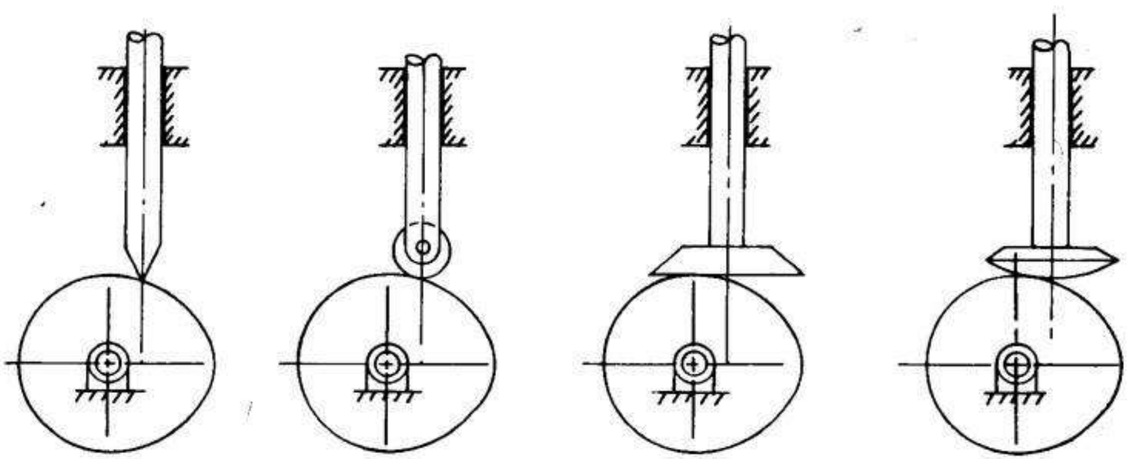
The followers, as shown in Fig. (to (*c*), *are all radial followers.*



(*a*) Cam with knife edge follower. (*b*) Cam with roller follower. (*c*) Cam with flat faced follower.

(b) Off-set follower.

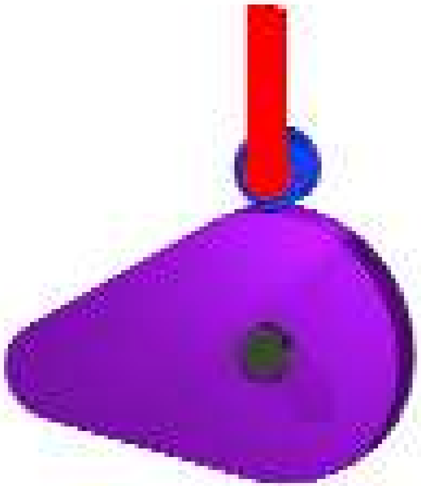
- *When the motion of the follower is along an axis away from the axis of* the cam centre, it is called off-set follower. The follower, as shown in Fig. (*f*), *is an* off-set



(a) (b) (c) (d)

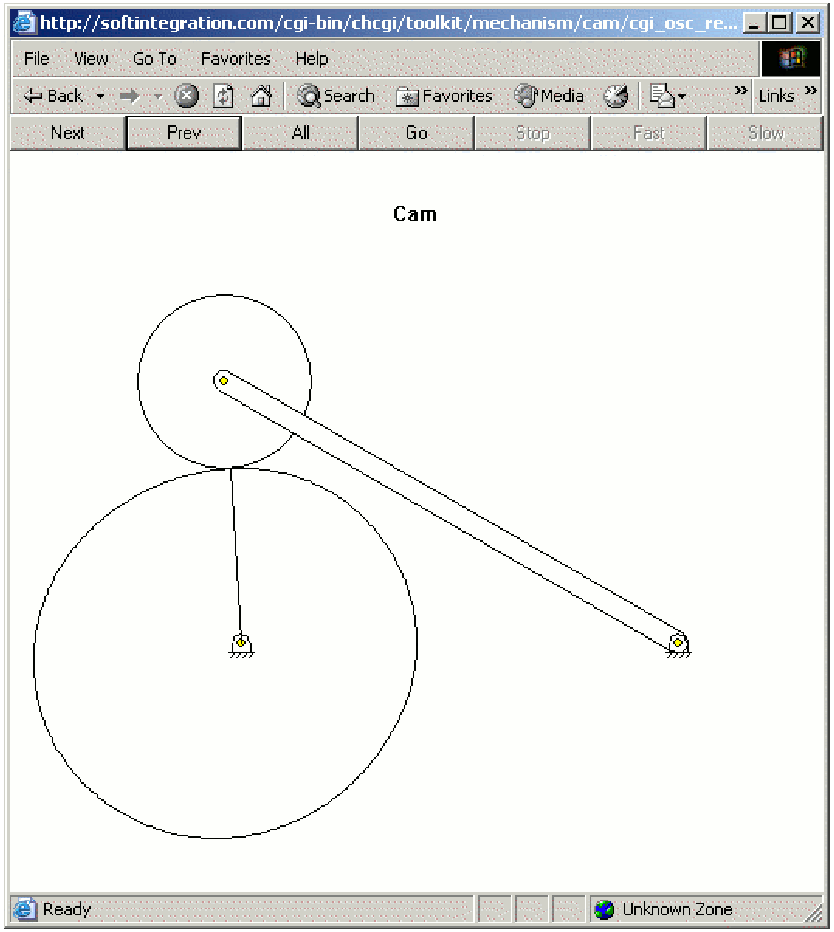
Based on modes of Input / Output motion

Rotating cam – Translating follower



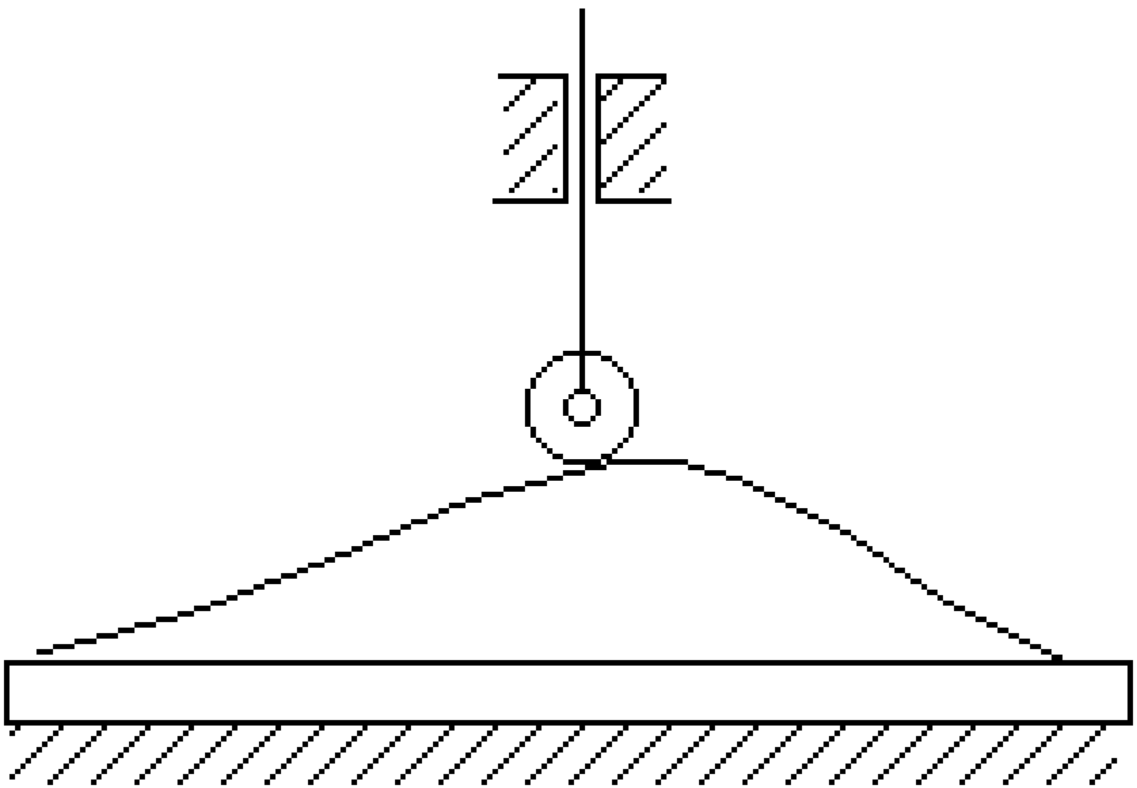
Based on modes of Input / Output motion

Rotating cam – Oscillating follower



Based on modes of Input / Output motion

Translating cam – Translating follower

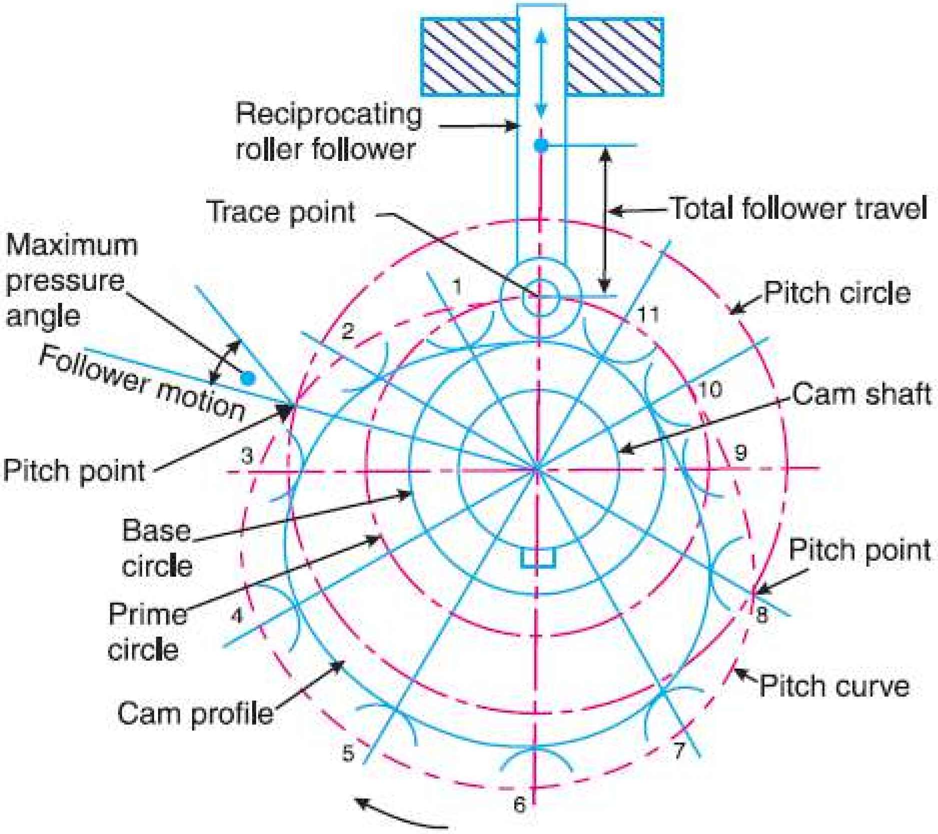


Terms Used in Radial Cams

- Fig. shows a radial cam with reciprocating roller follower. The following terms are important in order to draw the cam profile.

1. *Base circle.* *It is the smallest circle that can be drawn to the cam profile.*

2. *Trace point.* *It is a reference point on the follower and is used to generate the pitch curve.* In case of knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In a roller follower, the centre of the roller represents the trace point.



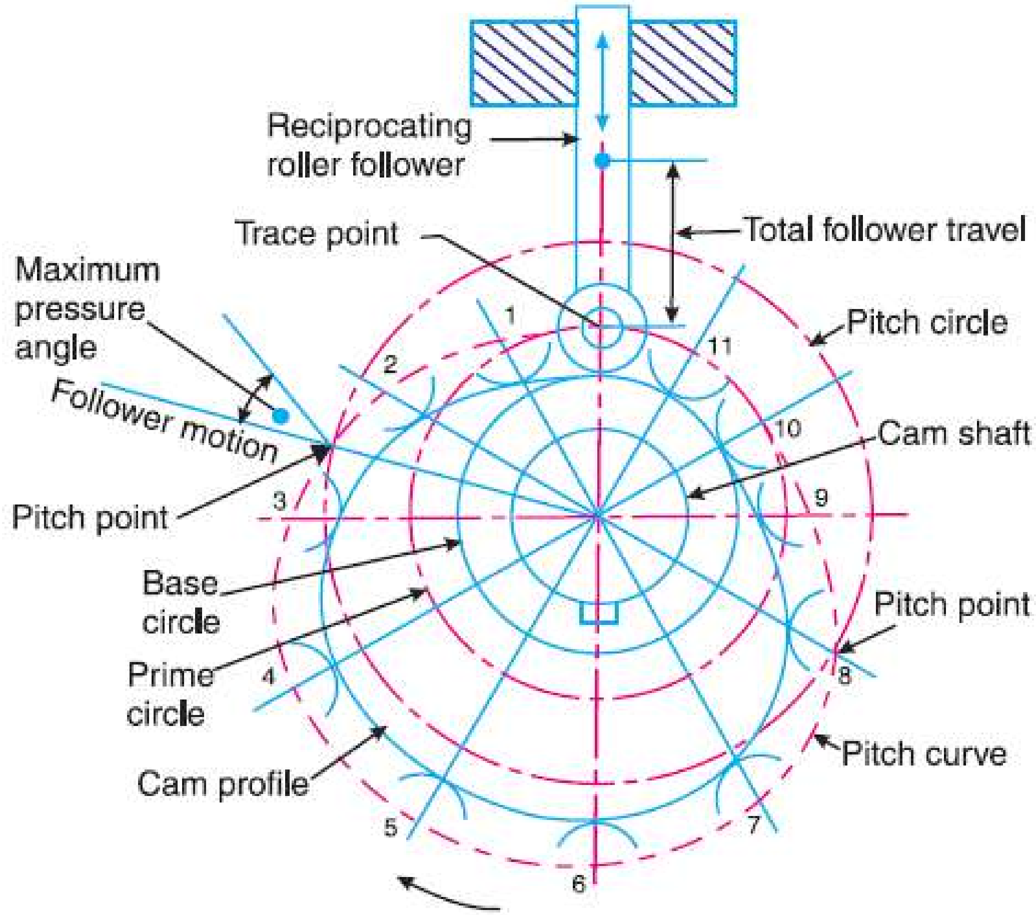
Terms Used in Radial Cams

3. *Pressure angle.* *It is the angle between the direction of the follower motion and a normal* to the pitch curve. This angle is very important in designing a cam profile. If the pressure angle is too large, a reciprocating follower will jam in its bearings.

4. *Pitch point.* *It is a point on the pitch curve having the maximum pressure angle.*

5. *Pitch circle.* *It is a circle drawn from the centre of the cam through the pitch points***.**

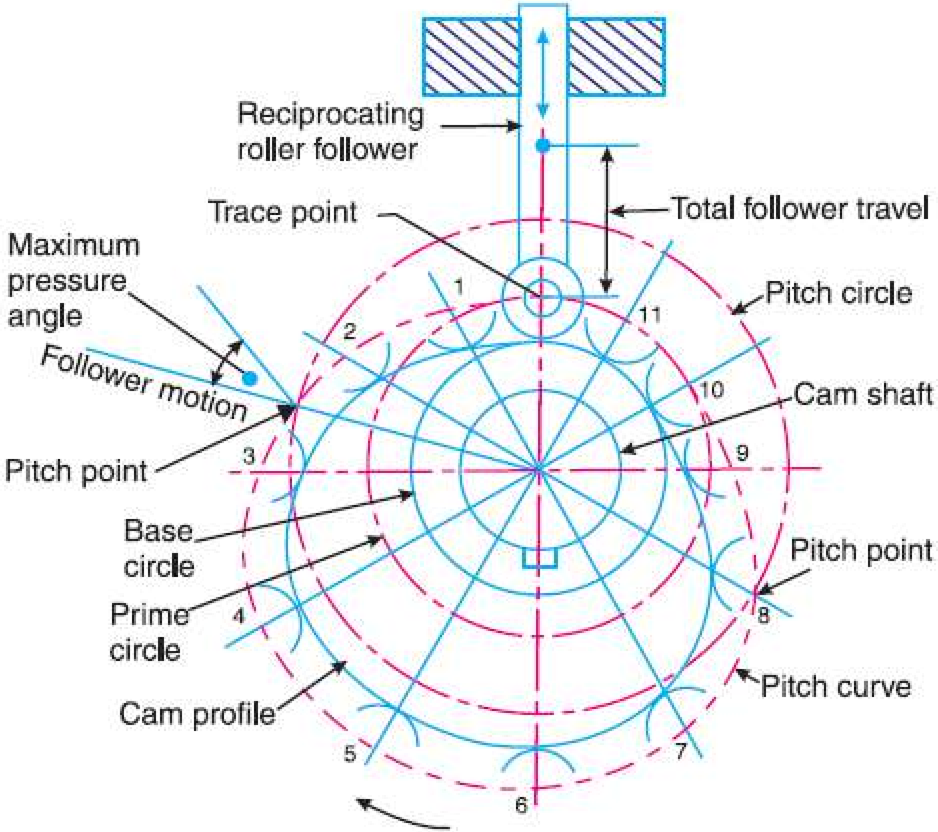
6. *Pitch curve.* *It is the curve generated by the trace point as the follower moves relative to* the cam. For a knife edge follower, the pitch curve and the cam profile are same whereas for a roller follower, they are separated by the radius of the roller.



Terms Used in Radial Cams

7. ***Prime circle.*** *It is the smallest circle that can be drawn from the centre of the cam and* tangent to the pitch curve. For a knife edge and a flat face follower, the prime circle and the base circle are identical. For a roller follower, the prime circle is larger than the base circle by the radius of the roller.

8. ***Lift or stroke.*** *It is the maximum travel of the follower from its lowest position to the* topmost position.

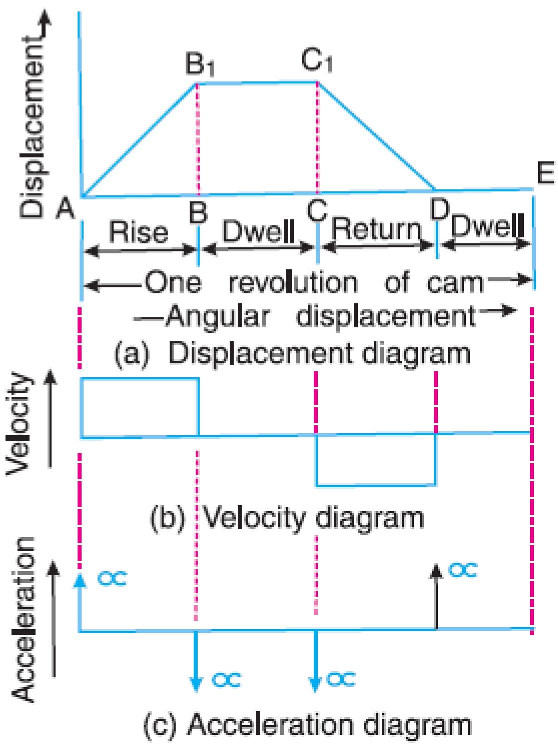


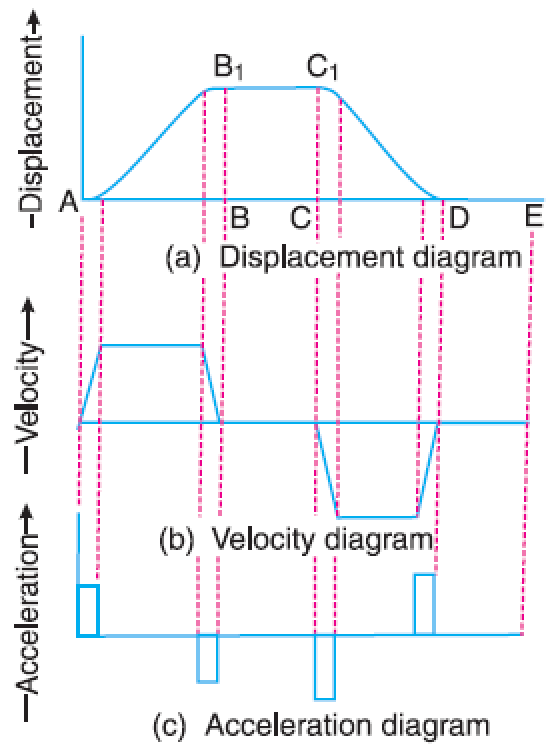
Motion of the Follower

- The follower, during its travel, may have one of the following motions.
  - **Uniform velocity**
  - **Simple harmonic motion**
  - **Uniform acceleration and retardation**
  - **Cycloidal motion**

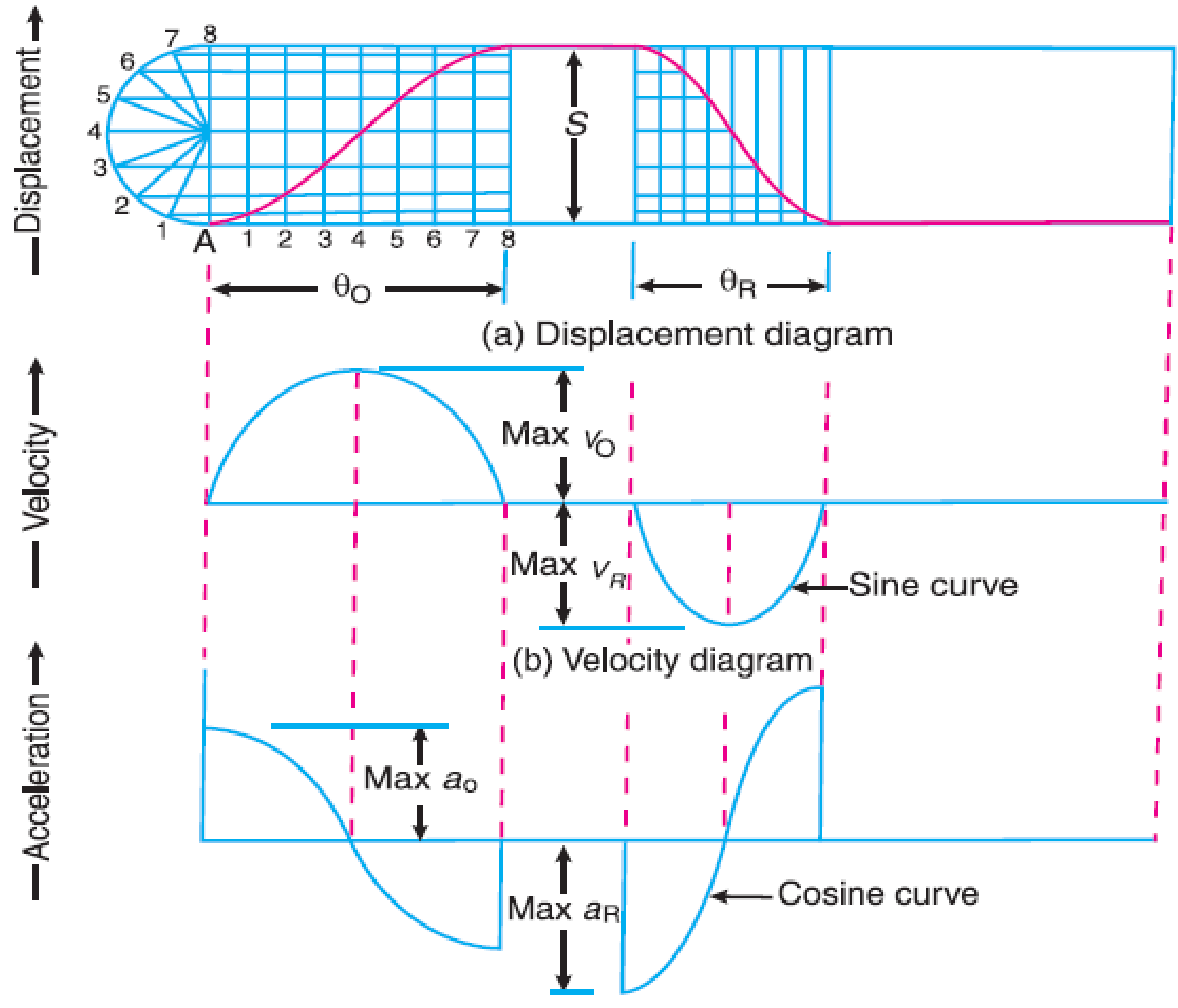
Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Velocity

- The displacement, velocity and acceleration diagrams when a knife-edged follower moves with uniform velocity are shown in Fig. (*a), (b) and (c) respectively.*
- The abscissa (base) represents the time (*i.e. the number of seconds required for the cam to complete one revolution) or* it may represent the angular displacement of the cam in degrees. The ordinate represents the displacement or velocity or acceleration of the follower





Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Simple Harmonic Motion



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Simple Harmonic Motion

Let S = Stroke of the follower,

θ_O and θ_R = Angular displacement of the cam during out stroke and return stroke of the follower respectively, in radians, and

ω = Angular velocity of the cam in rad/s.

maximum velocity of the follower on the outstroke, $\dfrac{\pi \omega S}{2\theta_O}$

$\therefore$ Maximum acceleration of the follower on the outstroke,

$$a_O = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2}$$

Similarly, maximum velocity of the follower on the return stroke,

$$v_R = \frac{\pi \omega . S}{2\theta_R}$$

and maximum acceleration of the follower on the return stroke,

$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2}$$

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Acceleration and Retardation

velocity of the follower during outstroke,

$$v_O = \frac{2\omega.S}{\theta_O}$$

maximum velocity of the follower during return

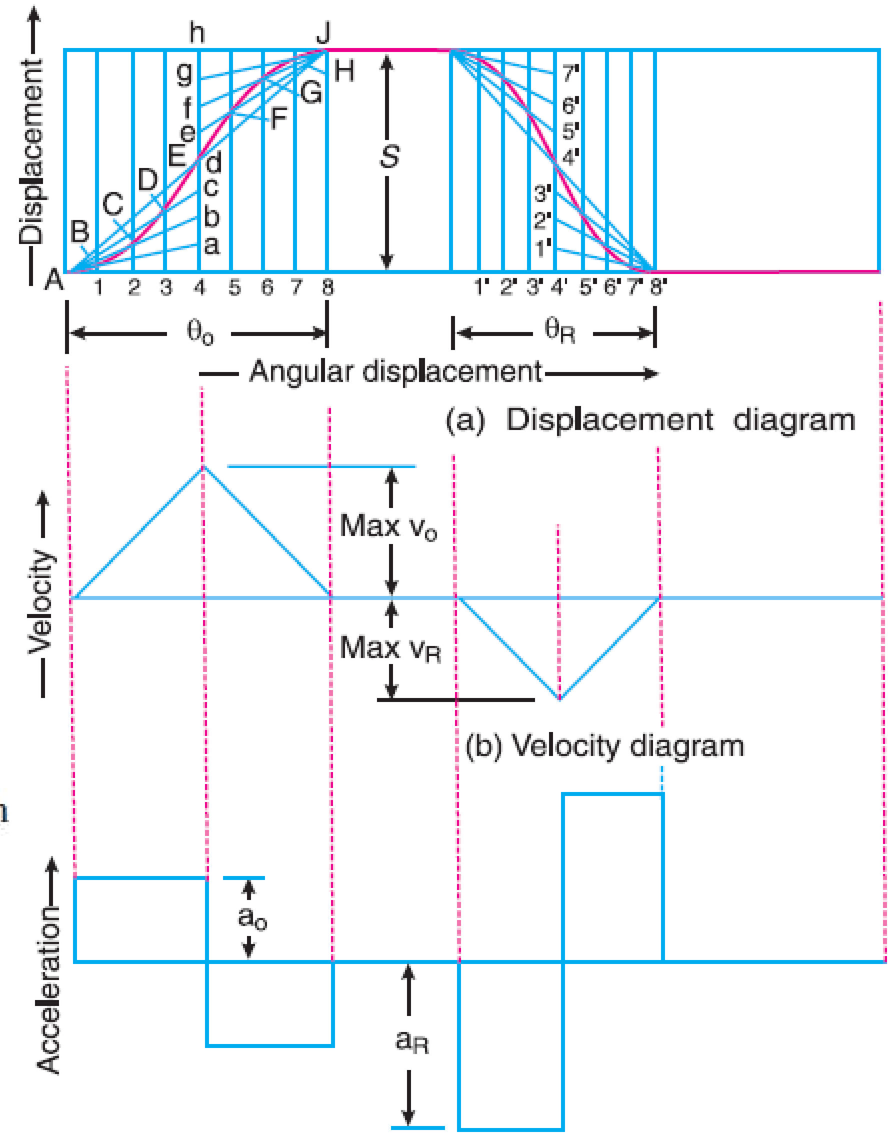
$$v_R = \frac{2\omega.S}{\theta_R}$$

Maximum acceleration of the follower during outstroke,

$$a_O = \frac{4\omega^2.S}{(\theta_O)^2}$$

Similarly, maximum acceleration of the follower during return

$$a_R = \frac{4\omega^2.S}{(\theta_R)^2}$$



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

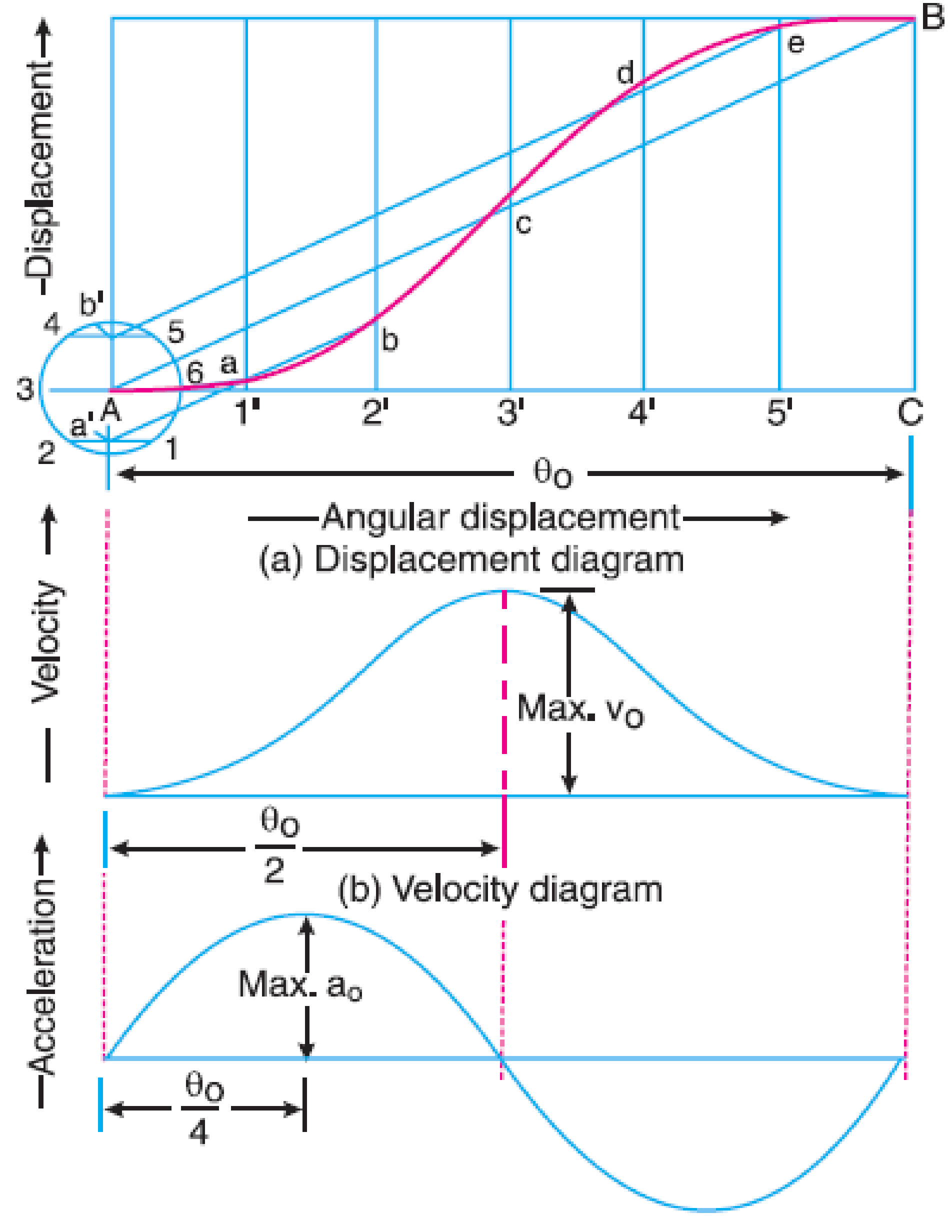
Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Cycloidal Motion

$$v_O = \frac{2\omega S}{\theta_O}$$

$$a_O = \frac{2\pi\omega^2 . S}{(\theta_O)^2}$$

$$v_R = \frac{2\omega . S}{\theta_R}$$

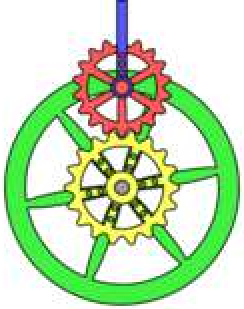
$$a_R = \frac{2\pi\omega^2 . S}{(\theta_R)^2}$$



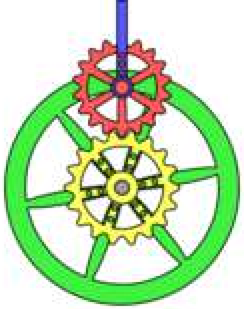
(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram



GEAR TRAINS



intro. about gear train of wristwatch - YouTube.flv

Transmission Animation - YouTube.flv

Manual Transmission Animation - DnaTube.com - Scientific Video and Animation Site.flv

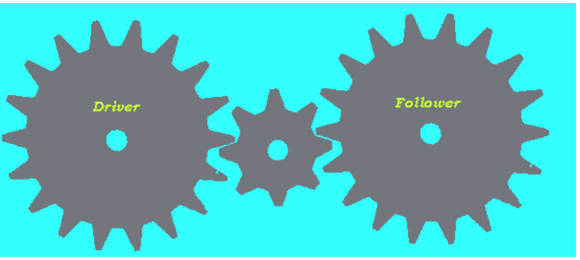
Watch movement animation - YouTube.flv

Introduction

- Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called ***gear train or train of toothed wheels.***
- The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts.
- A gear train may consist of spur, bevel or spiral gears.

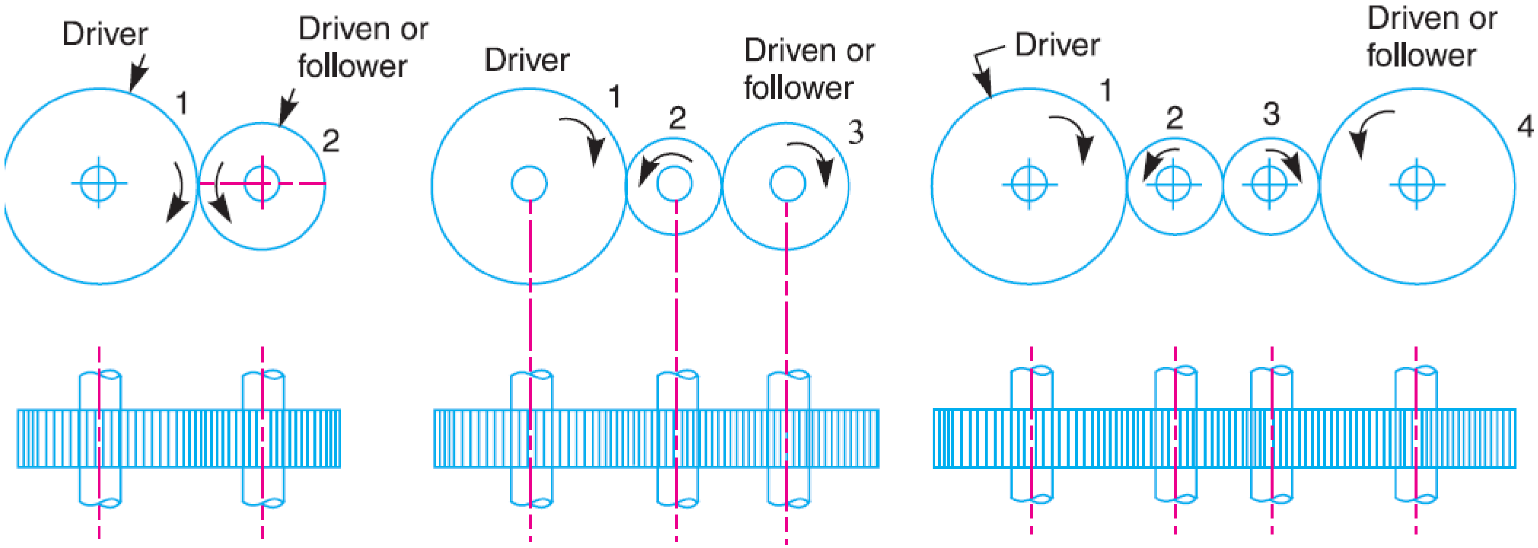
Types of Gear Trains

- Following are the different types of gear trains, depending upon the arrangement of wheels :
    1. Simple gear train
    2. Compound gear train
    3. Reverted gear train
    4. Epicyclic gear train
- In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other.
- But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

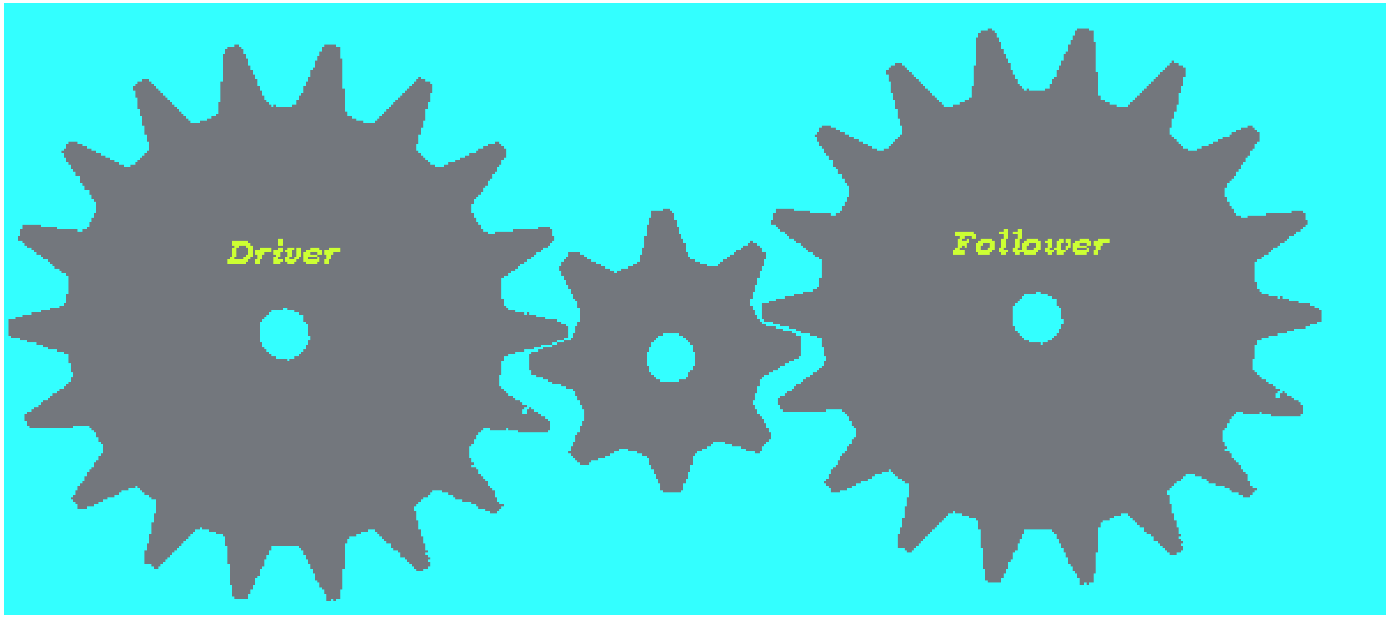


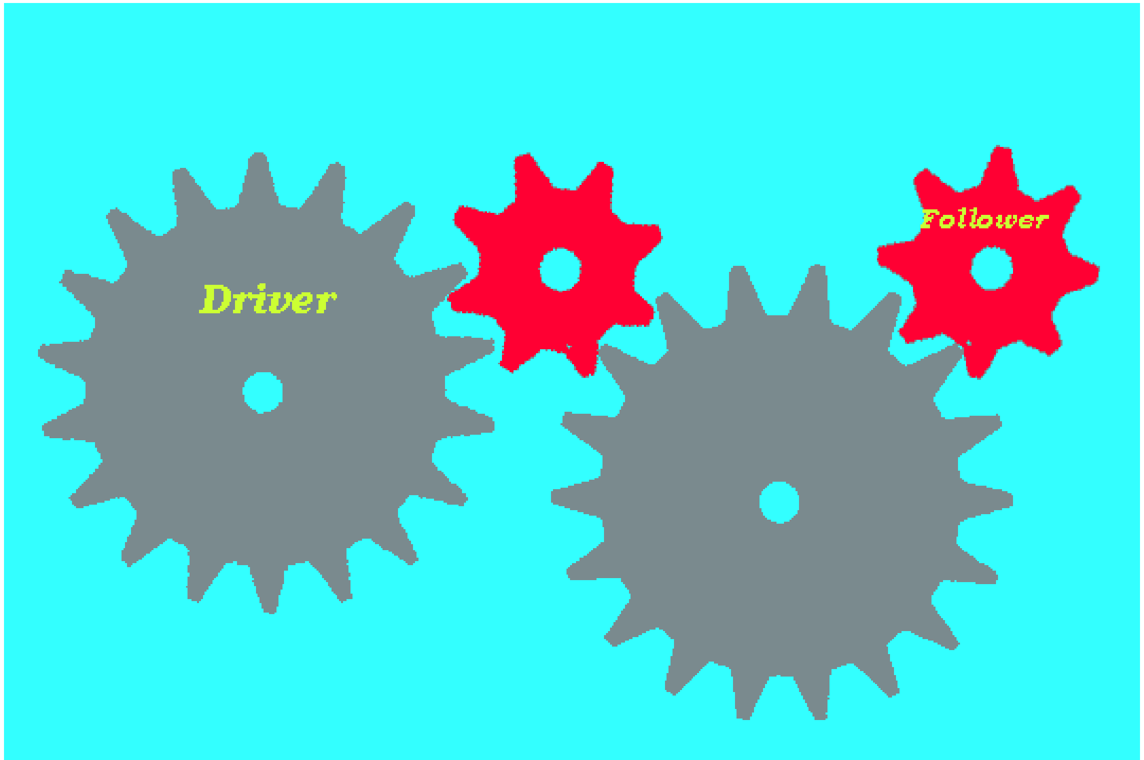
Simple Gear Train

- When there is only one gear on each shaft, as shown in Fig., it is known as *simple gear train.*
- *The gears are* represented by their pitch circles.
- When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig. (*a).*
- *Since the gear 1 drives the gear* 2, therefore gear 1 is called the *driver and the gear 2 is called the driven or follower.*
- *It may be noted* that the motion of the driven gear is opposite to the motion of driving gear.



(*a*) (*b*) (*c*)


Driver
Follower


Driver
Follower

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as ***train value*** of the gear train. Mathematically,

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

From above, we see that the train value is the reciprocal of speed ratio.

Sometimes, the distance between the two gears is large. The motion from one gear to another, in such a case, may be transmitted by either of the following two methods :

1. By providing the large sized gear, or **2.** By providing one or more intermediate gears.

A little consideration will show that the former method (*i.e.* providing large sized gears) is very inconvenient and uneconomical method ; whereas the latter method (*i.e.* providing one or more intermediate gear) is very convenient and economical.

It may be noted that when the number of intermediate gears are ***odd,*** the motion of both the gears (*i.e.* driver and driven or follower) is ***like*** as shown in Fig. 13.1 (*b*).

But if the number of intermediate gears are ***even,*** the motion of the driven or follower will be in the opposite direction of the driver as shown in Fig. 13.1 (*c*).

Now consider a simple train of gears with one intermediate gear as shown in Fig. 13.1 (*b*).

Let N_1 = Speed of driver in r.p.m.,

N_2 = Speed of intermediate gear in r.p.m.,

N_3 = Speed of driven or follower in r.p.m.,

T_1 = Number of teeth on driver,

T_2 = Number of teeth on intermediate gear, and

T_3 = Number of teeth on driven or follower.

Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \qquad \ldots(i)$$

Similarly, as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is

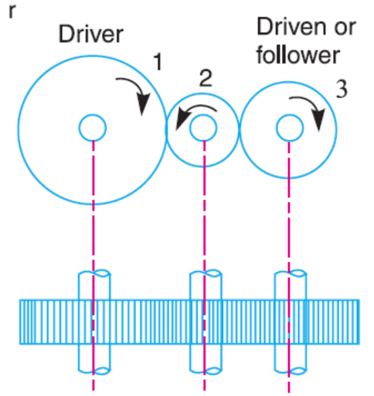
$$\frac{N_2}{N_3} = \frac{T_3}{T_2} \qquad \ldots(ii)$$

The speed ratio of the gear train as shown in Fig. 13.1 (*b*) is obtained by multiplying the equations (*i*) and (*ii*).

$$\therefore \qquad \frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2} \qquad \text{or} \qquad \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

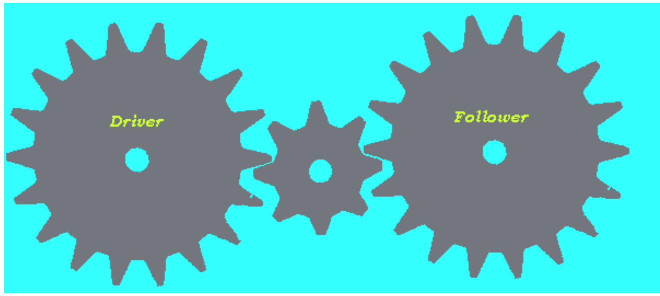
i.e.
$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

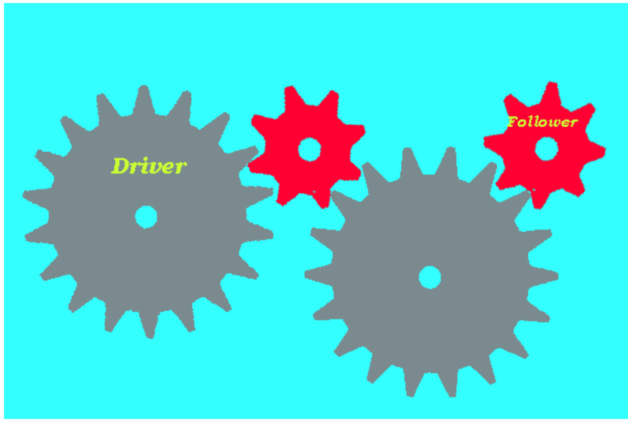
and
$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$



(*b*)

Similarly, it can be proved that the above equation holds good even if there are any number of intermediate gears. From above, we see that the speed ratio and the train value, in a simple train of gears, is independent of the size and number of intermediate gears. These intermediate gears are called ***idle gears***, as they do not effect the speed ratio or train value of the system. The idle gears are used for the following two purposes :





1. To connect gears where a large centre distance is required, and

2. To obtain the desired direction of motion of the driven gear (*i.e.* clockwise or anticlockwise).

Compound Gear Train

- When there are more than one gear on a shaft, as shown in Fig. , it is called a *compound train of gear.*
- We have seen that the idle gears, in a simple train of gears do not effect the speed ratio of the system.
- But these gears are useful in bridging over the space between the driver and the driven.

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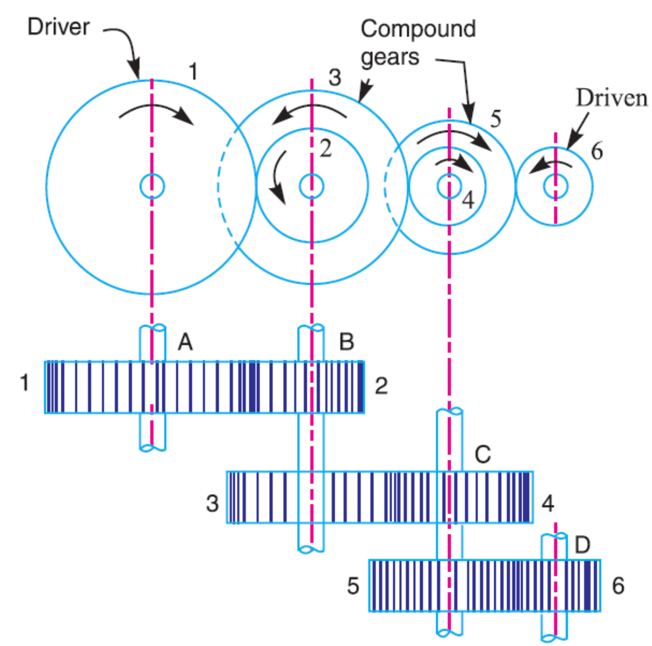
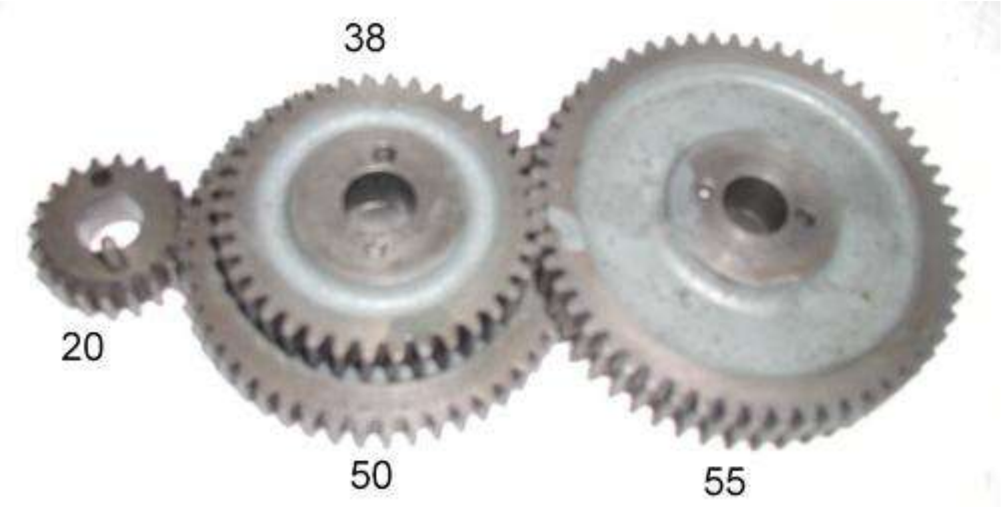


Compound Gear Train (Continued)

- But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is increased by providing compound gears on intermediate shafts.
- In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed.
- One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in Fig.



In a compound train of gears, as shown in Fig. 13.2, the gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let N_1 = Speed of driving gear 1,

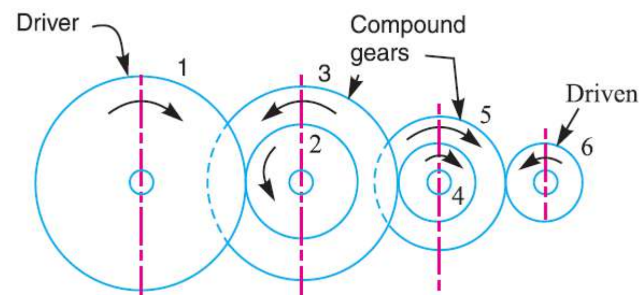
T_1 = Number of teeth on driving gear 1,

$N_2, N_3 ..., N_6$ = Speed of respective gears in r.p.m., and

$T_2, T_3 ..., T_6$ = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad ...(i)$$



Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad ...(ii)$$

and for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad ...(iii)$$

The speed ratio of compound gear train is obtained by multiplying the equations (*i*), (*ii*) and (*iii*),

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \quad \text{or} \quad {}^*\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

* Since gears 2 and 3 are mounted on one shaft B, therefore $N_2 = N_3$. Similarly gears 4 and 5 are mounted on shaft C, therefore $N_4 = N_5$.

i.e.

$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}}$$

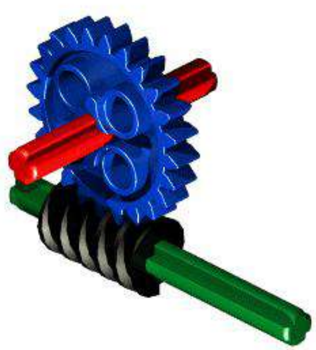
$$= \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

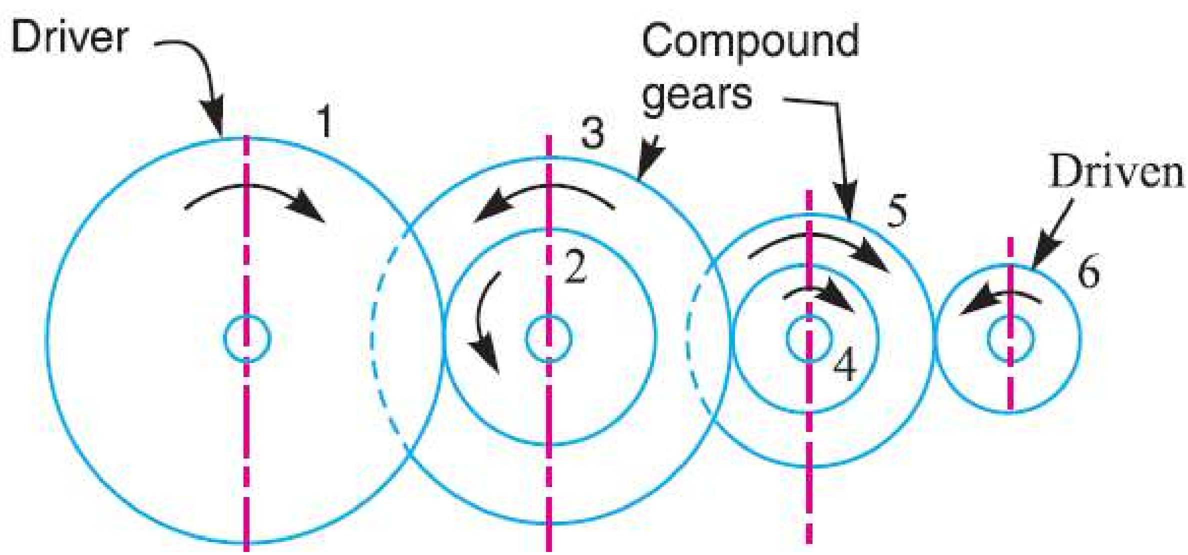
and

$$\text{Train value} = \frac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}}$$

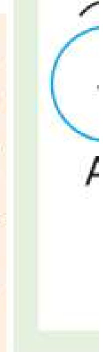
$$= \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}}$$

The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears. If a simple gear train is used to give a large speed reduction, the last gear has to be very large. Usually for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed.

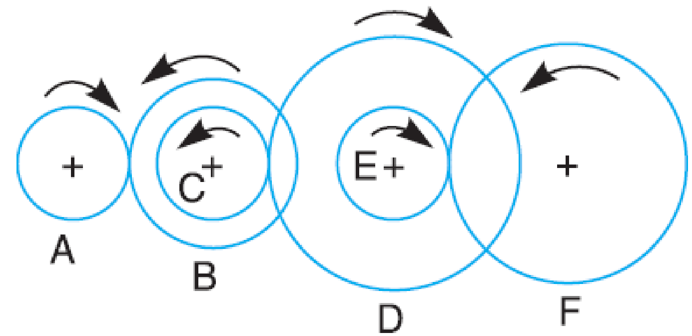




Example 13.1. *The gearing of a machine tool is shown in Fig. 13.3. The motor shaft is connected to gear A and rotates at 975 r.p.m. The gear wheels B, C, D and E are fixed to parallel shafts rotating together. The final gear F is fixed on the output shaft. What is the speed of gear F ? The number of teeth on each gear are as given below :*



Gear	*A*	*B*	*C*	*D*	*E*	*F*
No. of teeth	*20*	*50*	*25*	*75*	*26*	*65*



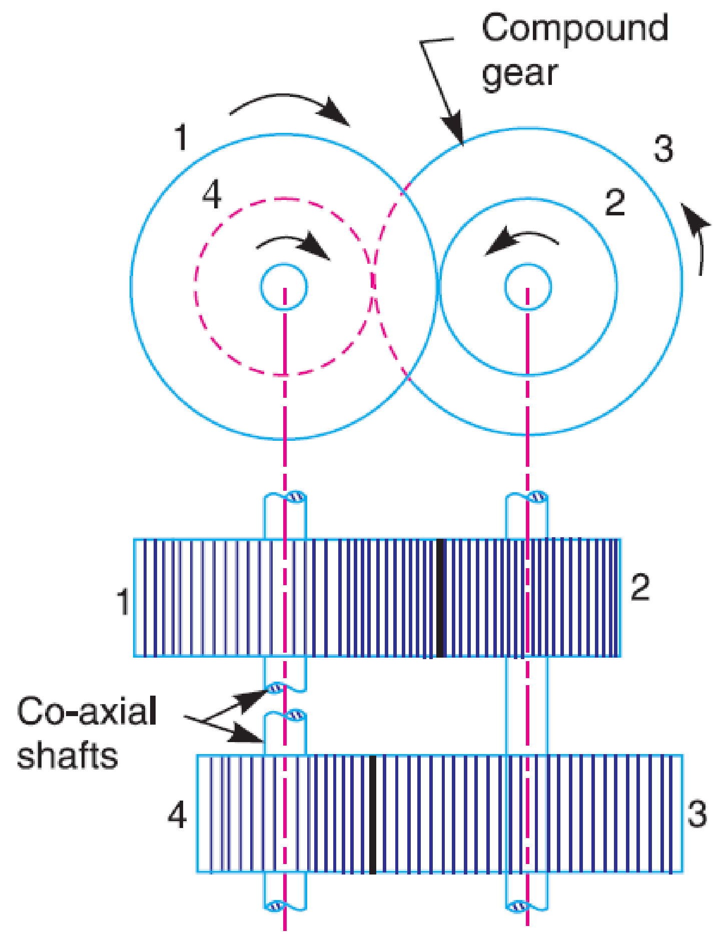
$$\frac{\text{Speed of the first driver}}{\text{Speed of the last driven}} = \frac{\text{Product of no. of teeth on drivens}}{\text{Product of no. of teeth on drivers}}$$

$$\frac{N_A}{N_F} = \frac{T_B \times T_D \times T_F}{T_A \times T_C \times T_E} = \frac{50 \times 75 \times 65}{20 \times 25 \times 26} = 18.75$$

$$N_F = \frac{N_A}{18.75} = \frac{975}{18.75} = 52 \text{ r. p. m.}$$ **Ans.**

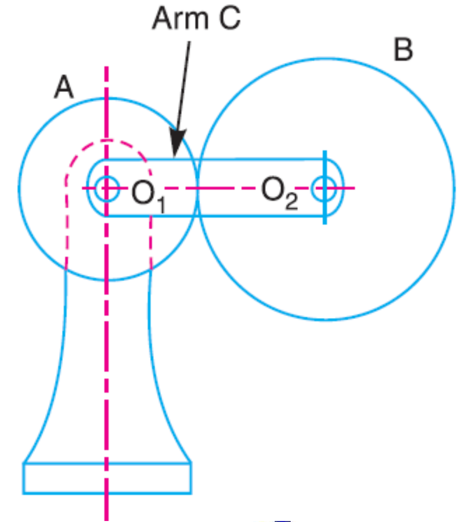
Reverted Gear Train

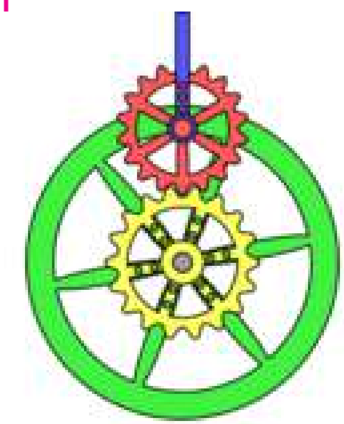
- When the axes of the first gear (*i.e. first driver)* and the last gear (*i.e. last driven or follower) are co-axial,* then the gear train is known as ***reverted gear train as*** shown in Fig.
- We see that gear 1 (*i.e. first driver) drives the* gear 2 (*i.e. first driven or follower) in the opposite direction.*
- Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2.
- The gear 3 (which is now the second driver) drives the gear 4 (*i.e. the last driven or follower) in the same direction as* that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is ***like.***



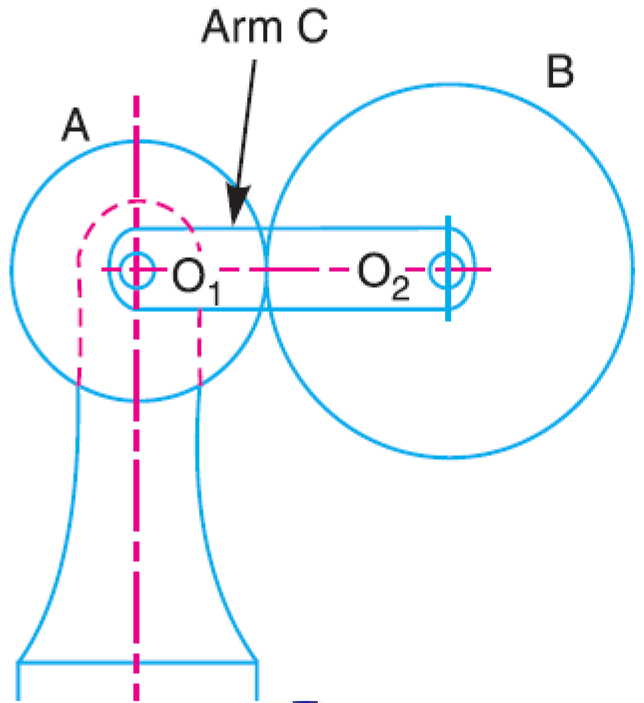
Epicyclic Gear Train

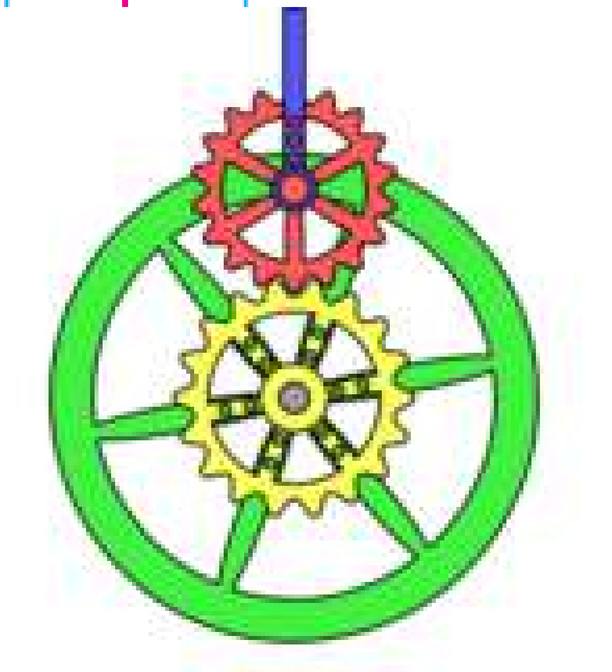
- In an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis.
- A simple epicyclic gear train is shown in Fig., where a gear *A and the arm C have a common axis at O*1 about which they can rotate.
- The gear *B meshes with gear A and has its axis on the arm at O*2, about which the gear *B can rotate.*





- *If the* arm is fixed, the gear train is simple and gear *A can drive gear B* or *vice-versa, but if gear A is fixed and the arm is rotated about* the axis of gear *A (i.e. O*1), then the gear *B is forced to rotate upon and around gear A.*
- *Such a motion is called epicyclic and* the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as *epicyclic gear trains (epi. means upon and cyclic* means around).
- The epicyclic gear trains may be *simple or compound.*
- The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space.
- The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.





Algebraic method

Let the arm C be fixed in an epicyclic gear train as shown in Fig. 13.6. Therefore speed of the gear A relative to the arm C

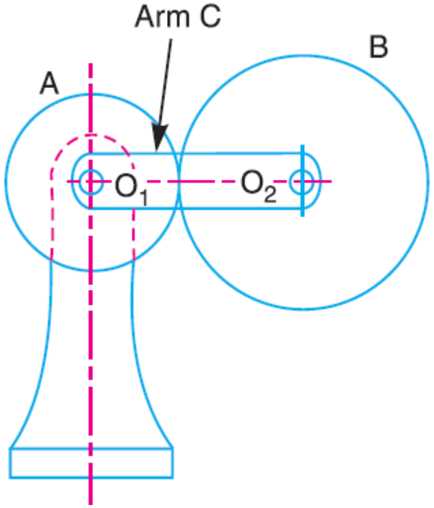
$$= N_A - N_C$$

and speed of the gear B relative to the arm C,

$$= N_B - N_C$$

Since the gears A and B are meshing directly, therefore they will revolve in *opposite* directions.

$$\therefore \qquad \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$



Since the arm C is fixed, therefore its speed, $N_C = 0$.

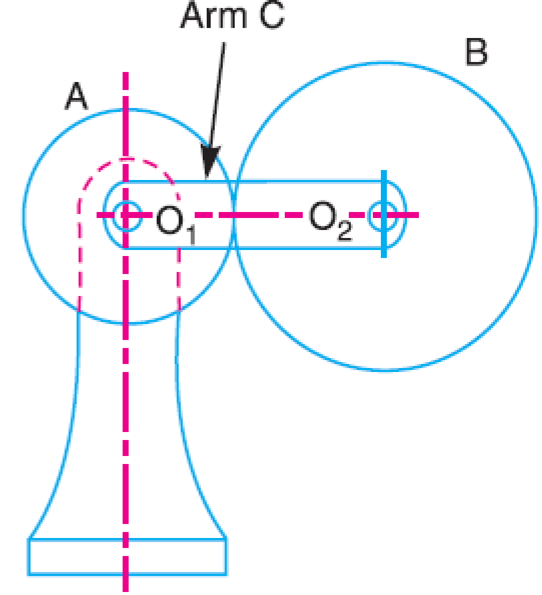
$$\therefore \qquad \frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

If the gear A is fixed, then $N_A = 0$.

$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B} \qquad \text{or} \qquad \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$

Tabular method.

Consider an epicyclic gear train as shown in Fig.



- Let *T*A = Number of teeth on gear *A, and T*B = Number of teeth on gear *B.*
- First of all, let us suppose that the arm is fixed. Therefore the axes of both the gears are also fixed relative to each other.
- When the gear *A makes one* revolution anticlockwise, the gear *B will* make ***T*** A / *T*B revolutions, clockwise.

We know that $N_B / N_A = T_A / T_B$. Since $N_A = 1$ revolution, therefore $N_B = T_A / T_B$.

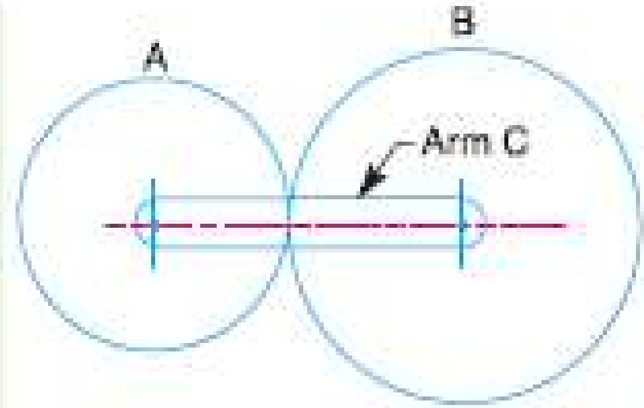
Step No.	*Conditions of motion*	*Revolutions of elements*		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear *A* rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\dfrac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	+ *x*	$-x \times \dfrac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \dfrac{T_A}{T_B}$

Tabular method.

- when gear *A makes + 1* revolution, then the gear *B will make* (– *T*A / *T*B) revolutions. This statement of relative motion is entered in the first row of the table.
- Secondly, if the gear *A makes + x revolutions, then the gear B will* make – *x* × *T*A / *T*B revolutions. This statement is entered in the second row of the table. In other words, multiply the each motion (entered in the first row) by *x.*
- Thirdly, each element of an epicyclic train is given + *y revolutions and entered in the third* row.
- Finally, the motion of each element of the gear train is added up and entered in the fourth row.

Step No.	*Conditions of motion*	*Revolutions of elements*		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear *A* rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	+ *x*	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_A}{T_B}$

Example 13.4. *In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B ?*



Solution. Given : $T_A = 36$; $T_B = 45$; $N_C = 150$ r.p.m. (anticlockwise)

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear *A* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	+ *x*	$-x \times \frac{T_A}{T_B}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y
4.	Total motion	+ y	*x* + y	$y - x \times \frac{T_A}{T_B}$

Speed of gear B when gear A is fixed

Since the speed of arm is 150 r.p.m. anticlockwise, therefore from the fourth row of the table,

$$y = +150 \text{ r.p.m.}$$

Also the gear A is fixed, therefore

$$x + y = 0 \qquad \text{or} \qquad x = -y = -150 \text{ r.p.m.}$$

$\therefore$ Speed of gear B, $\quad N_B = y - x \times \frac{T_A}{T_B} = 150 + 150 \times \frac{36}{45} = +270 \text{ r.p.m.}$

$$= 270 \text{ r.p.m. (anticlockwise)} \quad \textbf{Ans.}$$

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear A makes 300 r.p.m.clockwise, therefore from the fourth row of the table,

$$x + y = -300 \qquad \text{or} \qquad x = -300 - y = -300 - 150 = -450 \text{ r.p.m.}$$

$\therefore$ Speed of gear B,

$$N_B = y - x \times \frac{T_A}{T_B} = 150 + 450 \times \frac{36}{45} = +510 \text{ r.p.m}$$

$$= 510 \text{ r.p.m. (anticlockwise)} \qquad \textbf{Ans.}$$

Example 13.7. *An epicyclic train of gears is arranged as shown in Fig. 13.11. How many revolutions does the arm, to which the pinions B and C are attached, make :*

1. when A makes one revolution clockwise and D makes half a revolution anticlockwise, and

2. when A makes one revolution clockwise and D is stationary ?

The number of teeth on the gears A and D are 40 and 90 respectively.

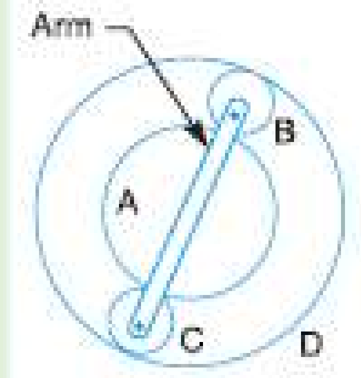


Fig. 13.11

Solution. Given : $T_A = 40$; $T_D = 90$

$$d_A + d_B + d_C = d_D \quad \text{or} \quad d_A + 2\,d_B = d_D \qquad ...(\because d_B = d_C)$$

Since the number of teeth are proportional to their pitch circle diameters, therefore,

$$T_A + 2\,T_B = T_D \quad \text{or} \quad 40 + 2\,T_B = 90$$

$$\therefore \qquad T_B = 25, \quad \text{and} \quad T_C = 25 \qquad ...(\because T_B = T_C)$$

Step No.	*Conditions of motion*	*Revolutions of elements*			
		Arm	*Gear A*	*Compound gear B-C*	*Gear D*
1.	Arm fixed, gear *A* rotates through – 1 revolution (*i.e.* 1 rev. clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B} \times \frac{T_B}{T_D} = +\frac{T_A}{T_D}$
2.	Arm fixed, gear *A* rotates through – *x* revolutions	0	$-x$	$+x \times \frac{T_A}{T_B}$	$+x \times \frac{T_A}{T_D}$
3.	Add – *y* revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x \times \frac{T_A}{T_B} - y$	$x \times \frac{T_A}{T_D} - y$

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound gear B-C	Gear D
1.	Arm fixed, gear A rotates through $-$ 1 revolution (*i.e.* 1 rev. clockwise)	0	-1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_D}=+\frac{T_A}{T_D}$
2.	Arm fixed, gear A rotates through $-x$ revolutions	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_D}$
3.	Add $-y$ revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x\times\frac{T_A}{T_B}-y$	$x\times\frac{T_A}{T_D}-y$

1. *Speed of arm when A makes 1 revolution clockwise and D makes half revolution anticlockwise*

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x-y=-1 \quad \text{or} \quad x+y=1 \qquad \ldots(i)$$

Also, the gear D makes half revolution anticlockwise, therefore

$$x\times\frac{T_A}{T_D}-y=\frac{1}{2} \quad \text{or} \quad x\times\frac{40}{90}-y=\frac{1}{2}$$

$$\therefore \quad 40\,x-90\,y=45 \quad \text{or} \quad x-2.25\,y=1.125 \qquad \ldots(ii)$$

From equations *(i)* and *(ii)*, $x=1.04$ and $y=-0.04$

$$\therefore \quad \text{Speed of arm} = -y = -(-0.04) = +0.04$$

$$= 0.04 \text{ revolution anticlockwise} \quad \textbf{Ans.}$$

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound gear B-C	Gear D
1.	Arm fixed, gear A rotates through – 1 revolution (*i.e.* 1 rev. clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_D}=+\frac{T_A}{T_D}$
2.	Arm fixed, gear A rotates through $-x$ revolutions	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_D}$
3.	Add $-y$ revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x\times\frac{T_A}{T_B}-y$	$x\times\frac{T_A}{T_D}-y$

2. *Speed of arm when A makes 1 revolution clockwise and D is stationary*

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x-y=-1 \quad \text{or} \quad x+y=1 \quad \text{...}(iii)$$

Also the gear D is stationary, therefore

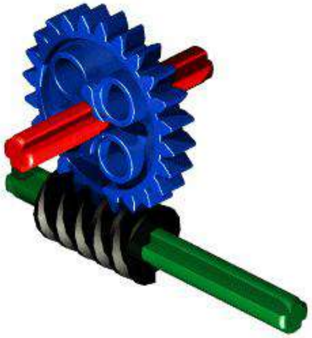
$$x\times\frac{T_A}{T_D}-y=0 \quad \text{or} \quad x\times\frac{40}{90}-y=0$$

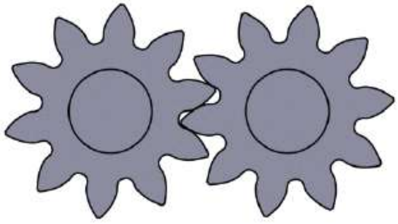
$$\therefore \quad 40\,x-90\,y=0 \quad \text{or} \quad x-2.25\,y=0 \quad \text{...}(iv)$$

From equations ***(iii)*** and ***(iv)***,

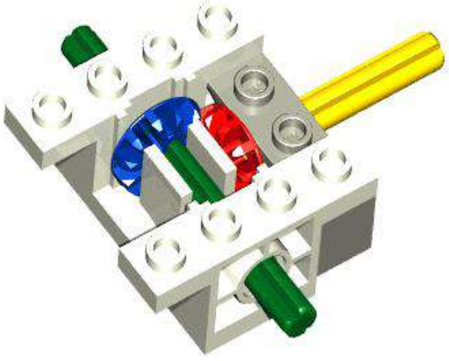
$$x=0.692 \quad \text{and} \quad y=0.308$$

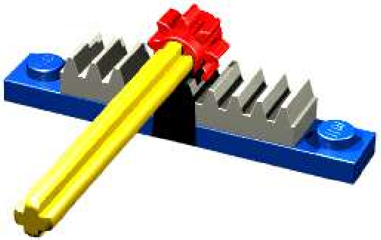
$\therefore$ Speed of arm $=-y=-0.308=0.308$ revolution clockwise **Ans.**



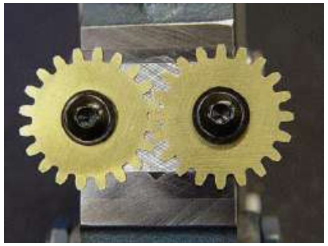


UNIT - V



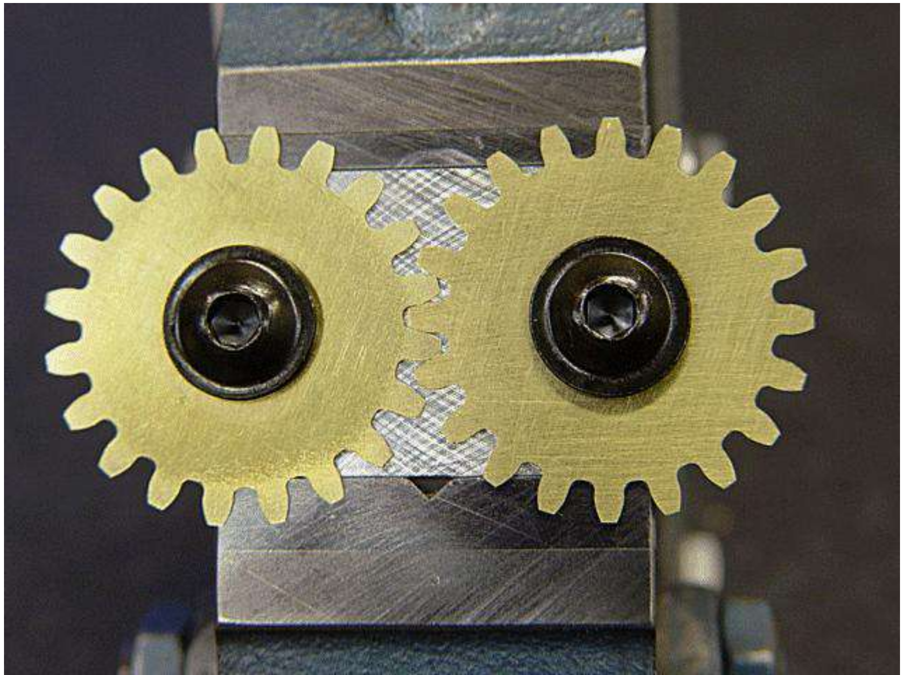


TOOTHED GEARING



SPUR GEARS

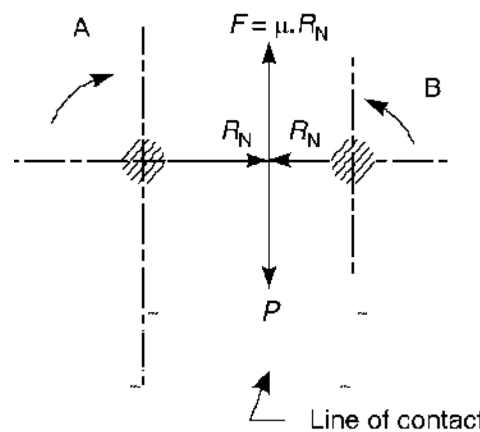
- Gear terminology, law of gearing, Characteristics of involute action, Path of contact, Arc of contact, Contact ratio of spur, helical, bevel and worm gears.
- Interference in involute gears.
- Methods of avoiding interference and Back lash.
- Comparison of involute and cycloidal teeth, Profile modification.

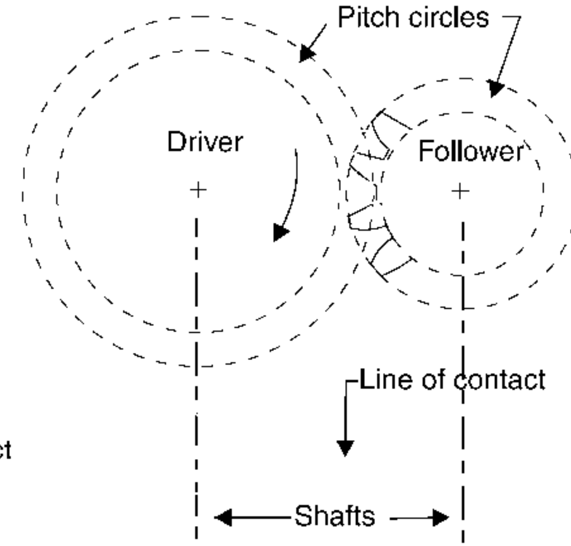


Introduction

- Let the wheel *A be keyed to the rotating shaft and the wheel B to the shaft, to be rotated.*
- *A* little consideration will show, that when the wheel *A is rotated by a rotating shaft, it will rotate the* wheel *B in the opposite direction as shown in Fig. (a).*
- The wheel *B will be rotated (by the wheel A) so long as the tangential force exerted by the* wheel *A does not exceed the maximum frictional resistance between the two wheels.*
- *But when the* tangential force (*P) exceeds the* ***frictional resistance (F), slipping will take place between the two*** wheels. Thus the friction drive is not a positive drive.

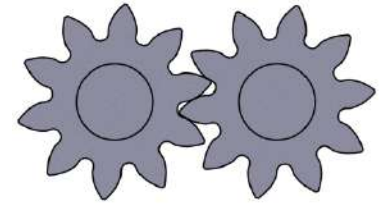
In order to avoid the slipping, a number of projections (called teeth) as shown in Fig. (*b), are provided on the periphery of the wheel A, which will fit into the corresponding* recesses on the periphery of the wheel *B. A friction wheel with the teeth cut on it is known as* ***toothed wheel or gear.***





(*b*) Toothed wheels.

TYPES OF GEARS



1. According to the position of axes of the shafts.

a. Parallel

   1.Spur Gear

   2.Helical Gear

   3.Rack and Pinion

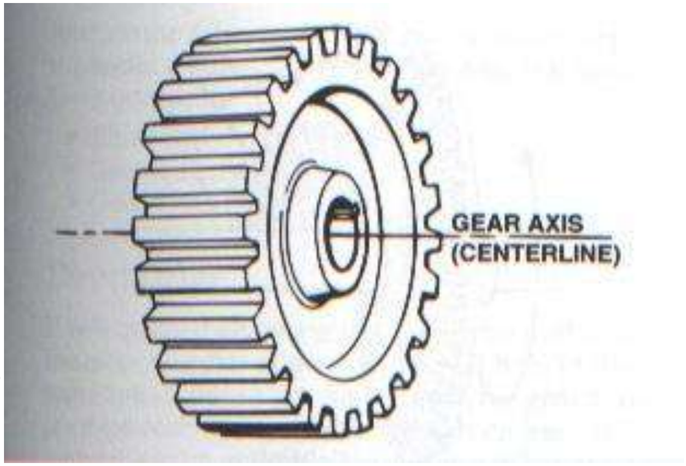
b. Intersecting

   Bevel Gear

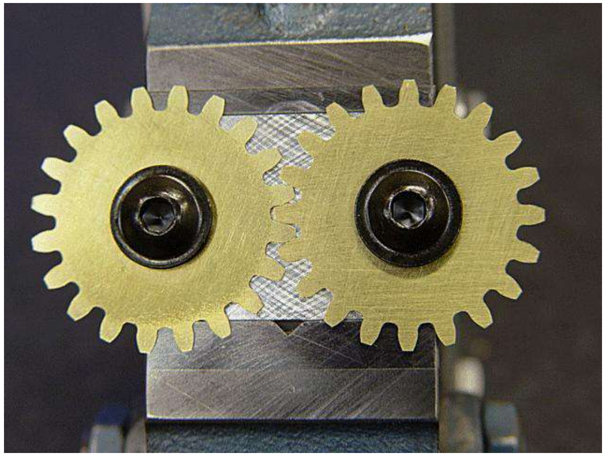
c. Non-intersecting and Non-parallel

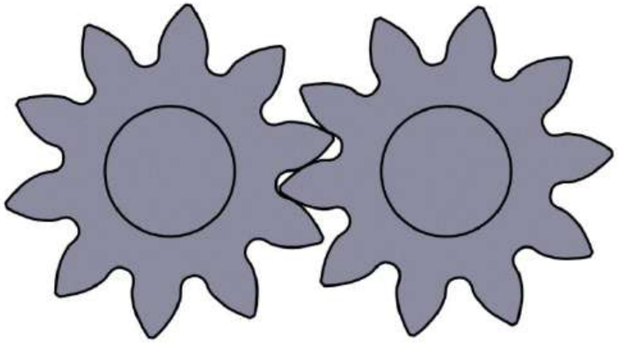
   worm and worm gears

SPUR GEAR

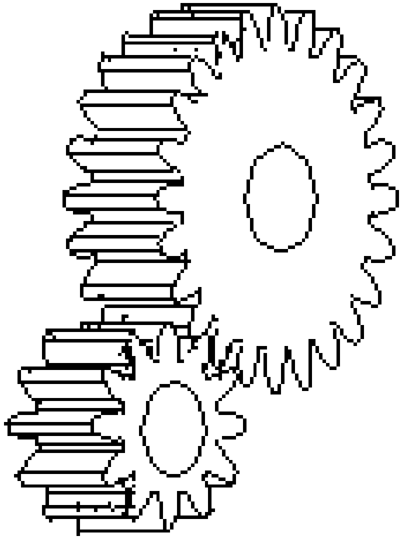


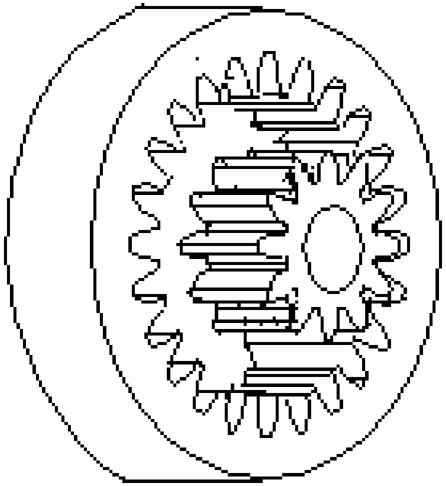
- Teeth is parallel to axis of rotation
- Transmit power from one shaft to another parallel shaft
- Used in Electric screwdriver, oscillating sprinkler, windup alarm clock, washing machine and clothes dryer





External and Internal spur Gear…

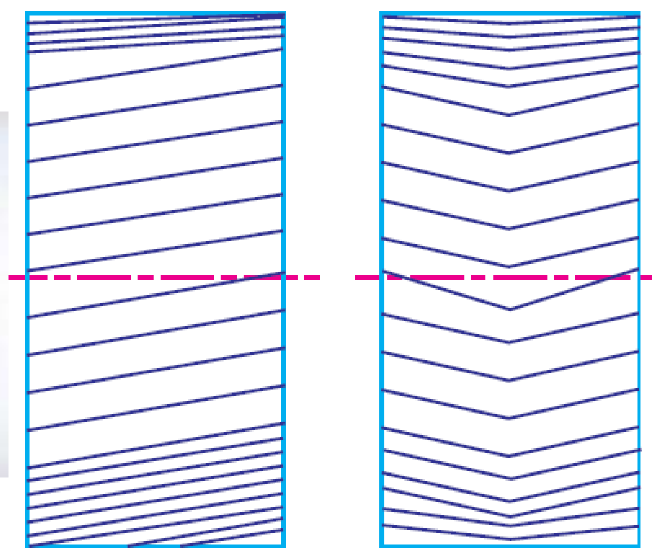




Internal gear animation with solidworks.mp4

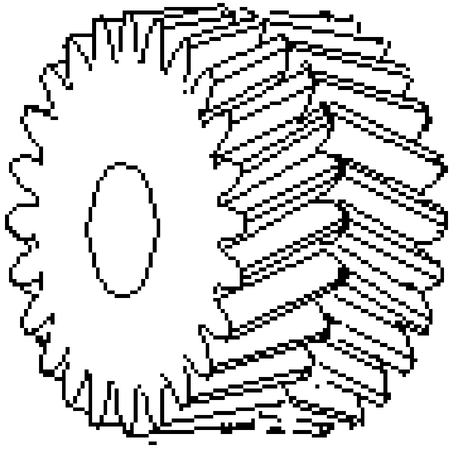
Helical Gear

- The teeth on helical gears are cut at an angle to the face of the gear
- This gradual engagement makes helical gears operate much more smoothly and quietly than spur gears
- One interesting thing about helical gears is that if the angles of the gear teeth are correct, they can be mounted on perpendicular shafts, adjusting the rotation angle by 90 degrees



Herringbone gears

- To avoid axial thrust, two helical gears of opposite hand can be mounted side by side, to cancel resulting thrust forces

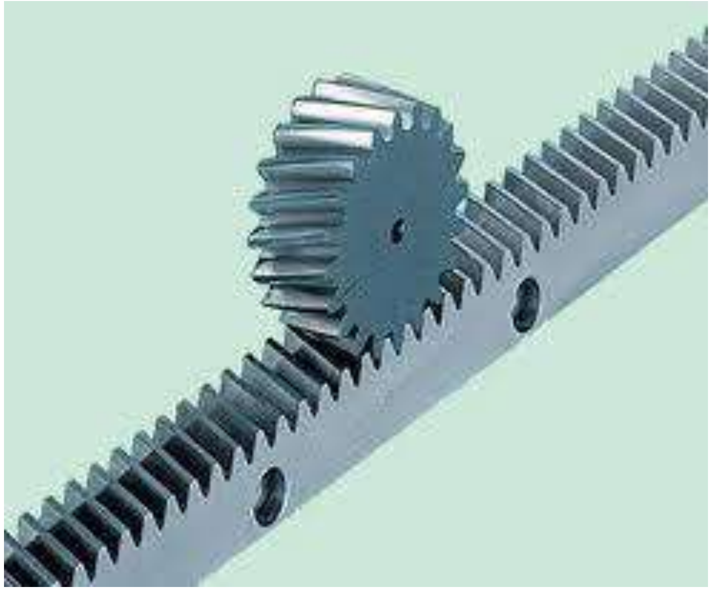


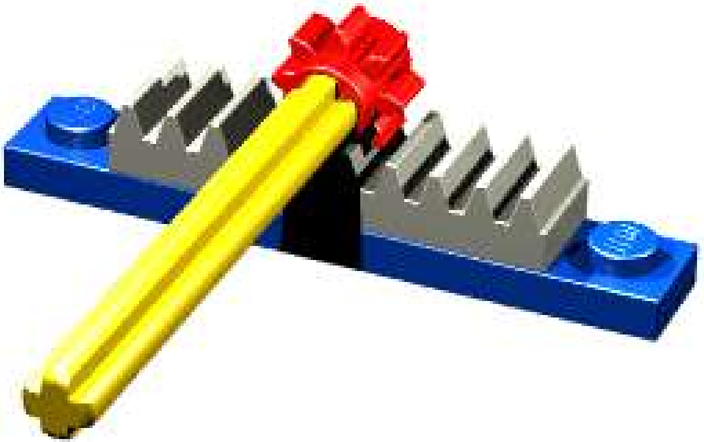
- Herringbone gears are mostly used on heavy machinery.

Rack and pinion

- **Rack and pinion gears** are used to convert rotation (From the pinion) into linear motion (of the rack)

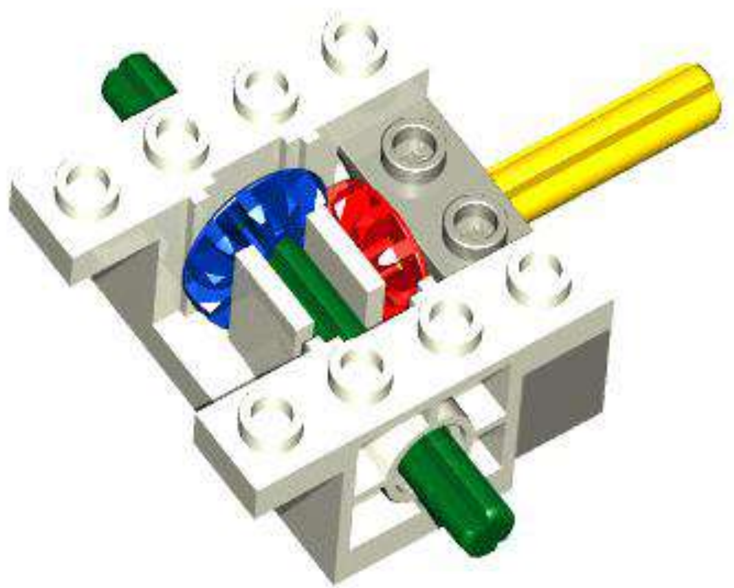
- A perfect example of this is the steering system on many cars

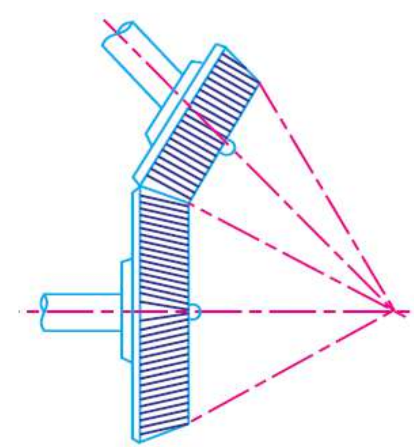




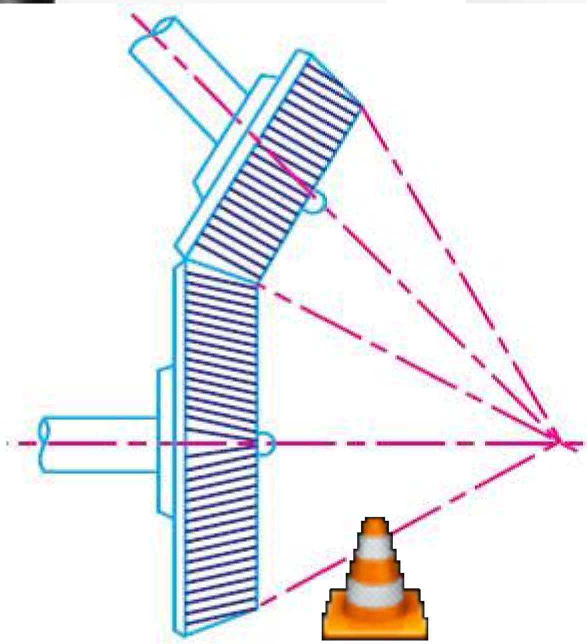
Bevel gears

- **Bevel gears** are useful when the direction of a shaft's rotation needs to be changed
- They are usually mounted on shafts that are 90 degrees apart, but can be designed to work at other angles as well
- The teeth on bevel gears can be **straight**, **spiral** or **hypoid**
- locomotives, marine applications, automobiles, printing presses, cooling towers, power plants, steel plants, railway track inspection machines, etc.





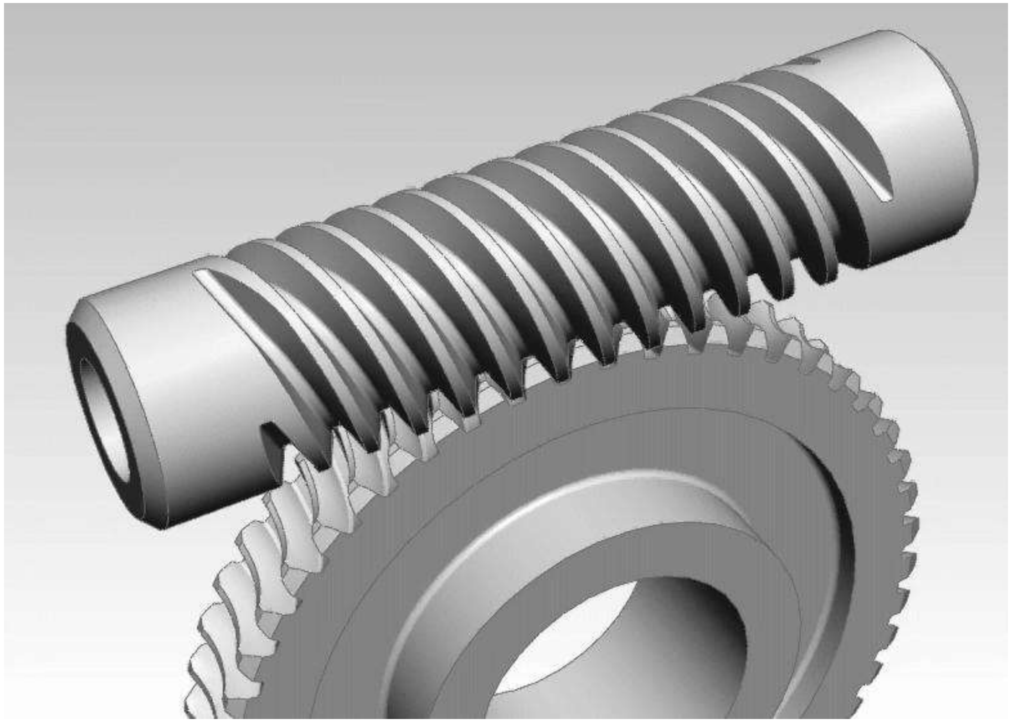
Straight and Spiral Bevel Gears

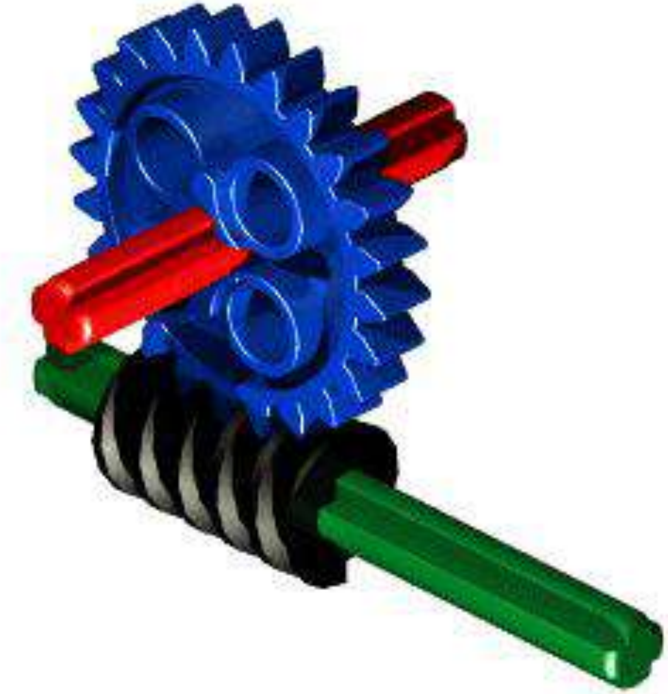


Bevel Gear Differential -- Simple Addition Demo 1.mp4

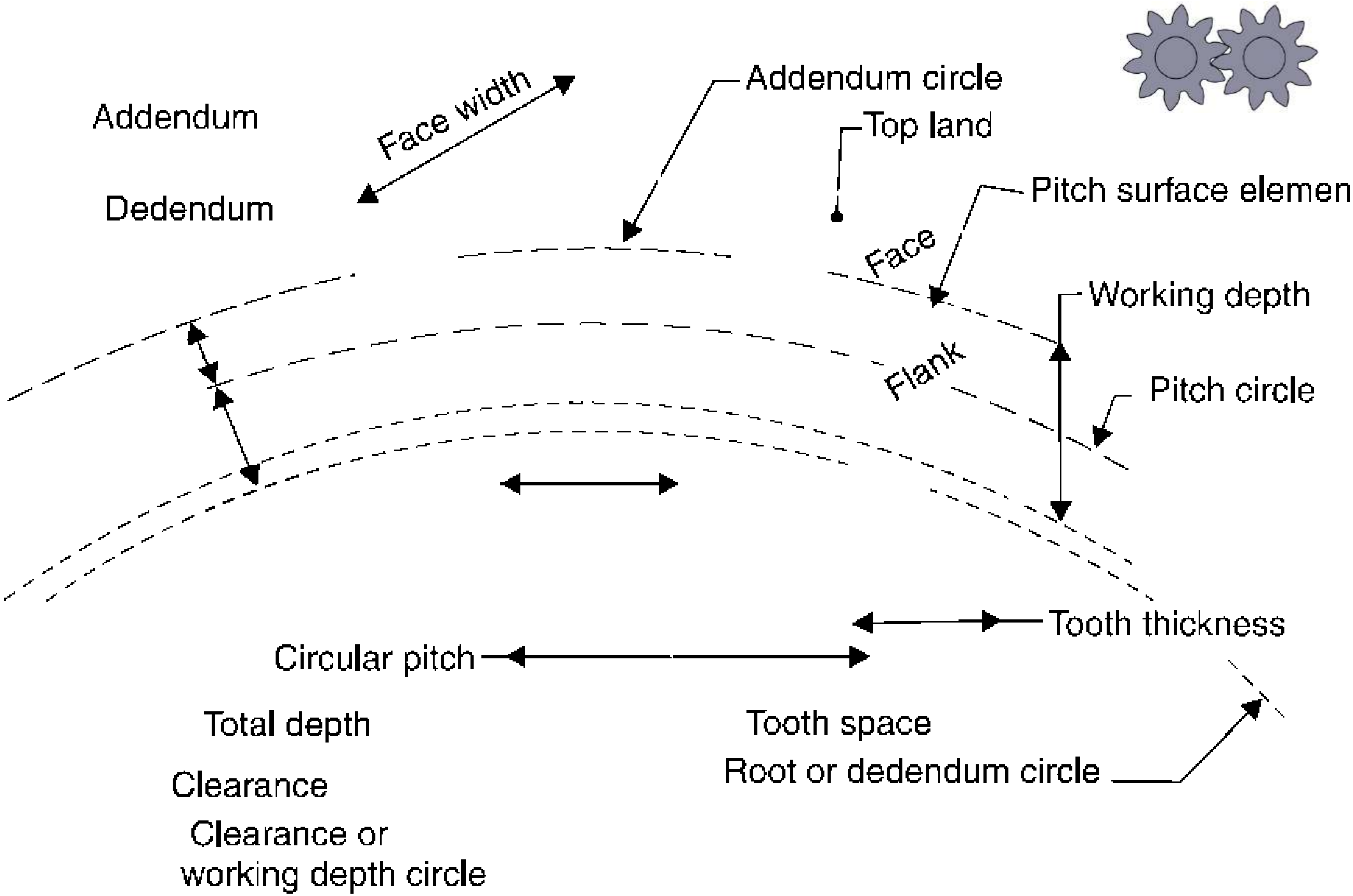
WORM AND WORM GEAR

- **Worm gears** are used when large gear reductions are needed. It is common for worm gears to have reductions of 20:1, and even up to 300:1 or greater
- Many worm gears have an interesting property that no other gear set has: the worm can easily turn the gear, but the gear cannot turn the worm
- Worm gears are used widely in material handling and transportation machinery, machine tools, automobiles etc

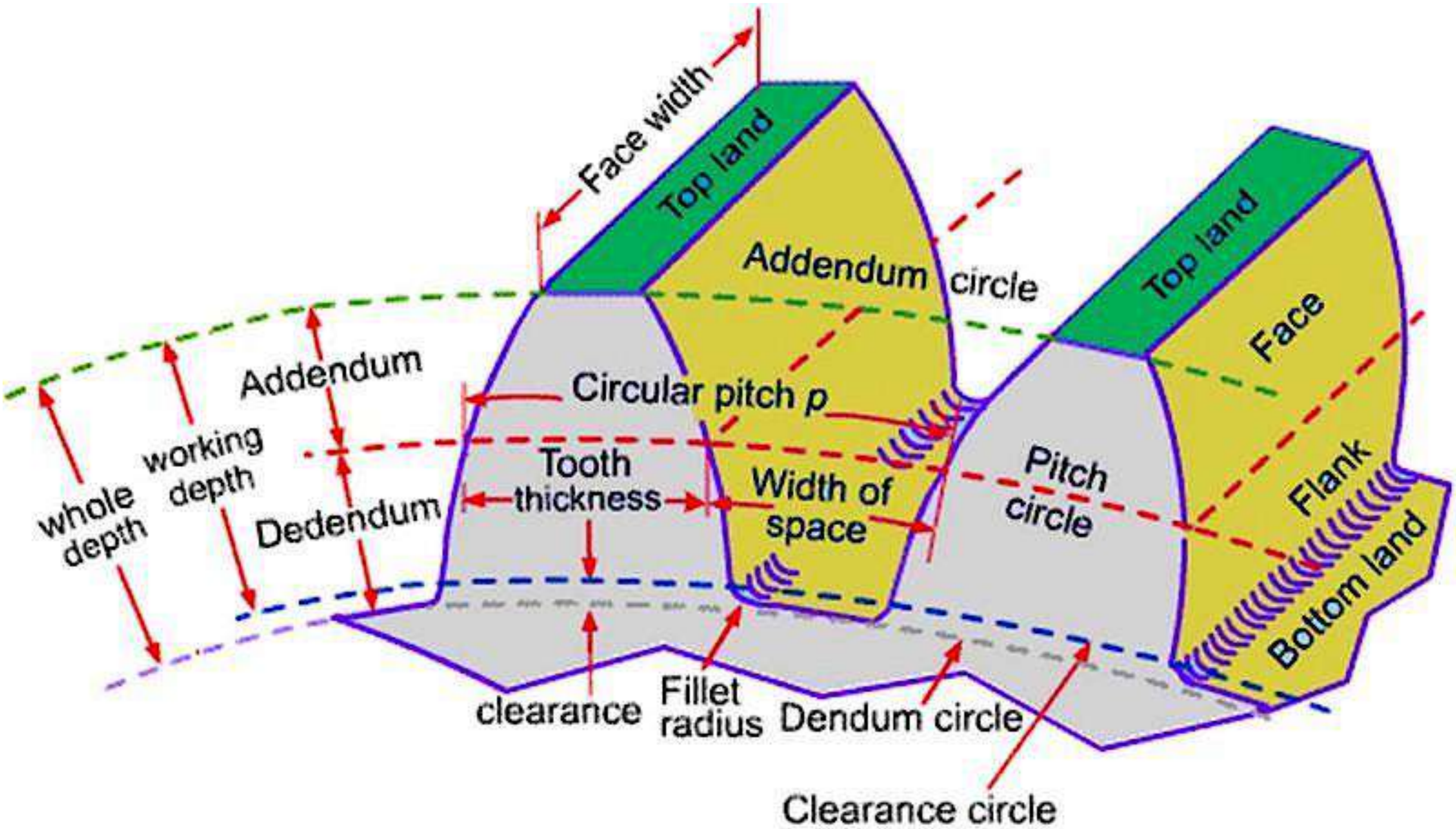




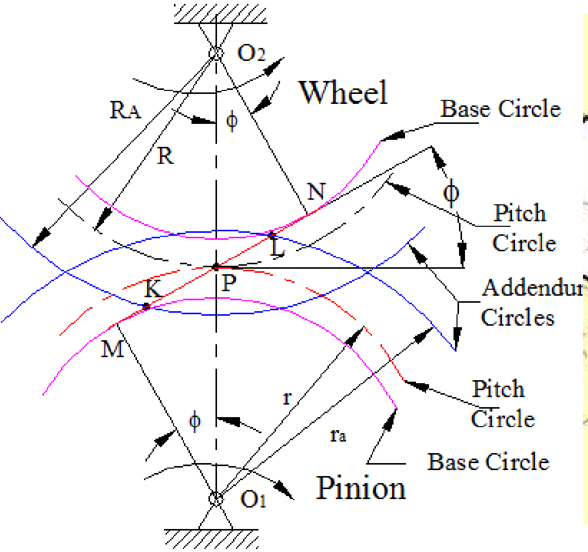
NOMENCLATURE OF SPUR GEARS

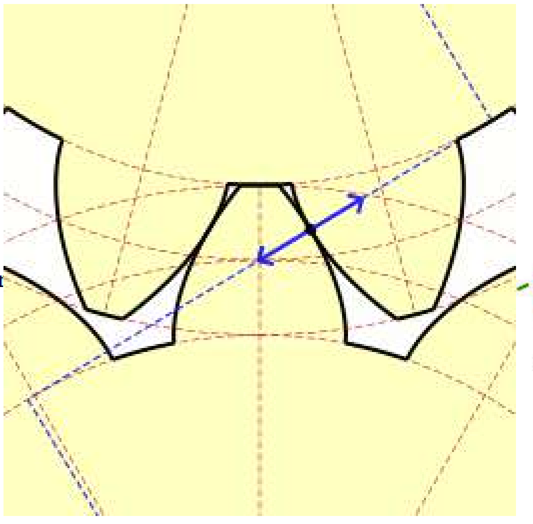


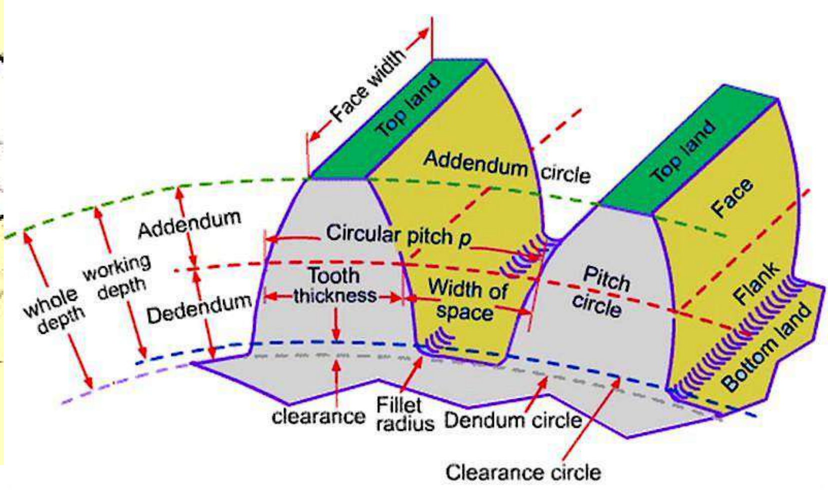
NOMENCLATURE OF SPUR GEARS



- ***Pitch circle:*** *It is an imaginary circle which by pure rolling action would give the same* motion as the actual gear.
- ***Pitch circle diameter:*** *It is the diameter of the pitch circle. The size of the gear is usually* specified by the pitch circle diameter. It is also known as *pitch diameter.*
- ***Pitch point:*** *It is a common point of contact between two pitch circles.*
- ***Pressure angle or angle of obliquity:*** *It is the angle between the common normal to two* gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by φ. The standard pressure angles are 14 1/2 ° and 20°.

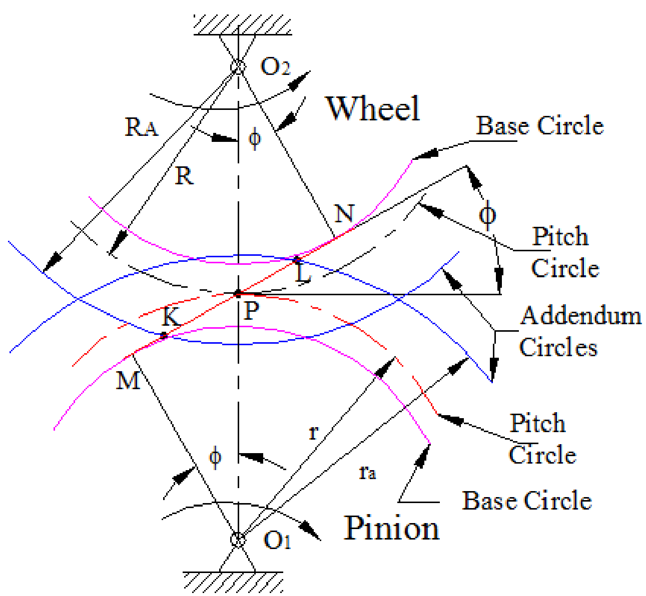


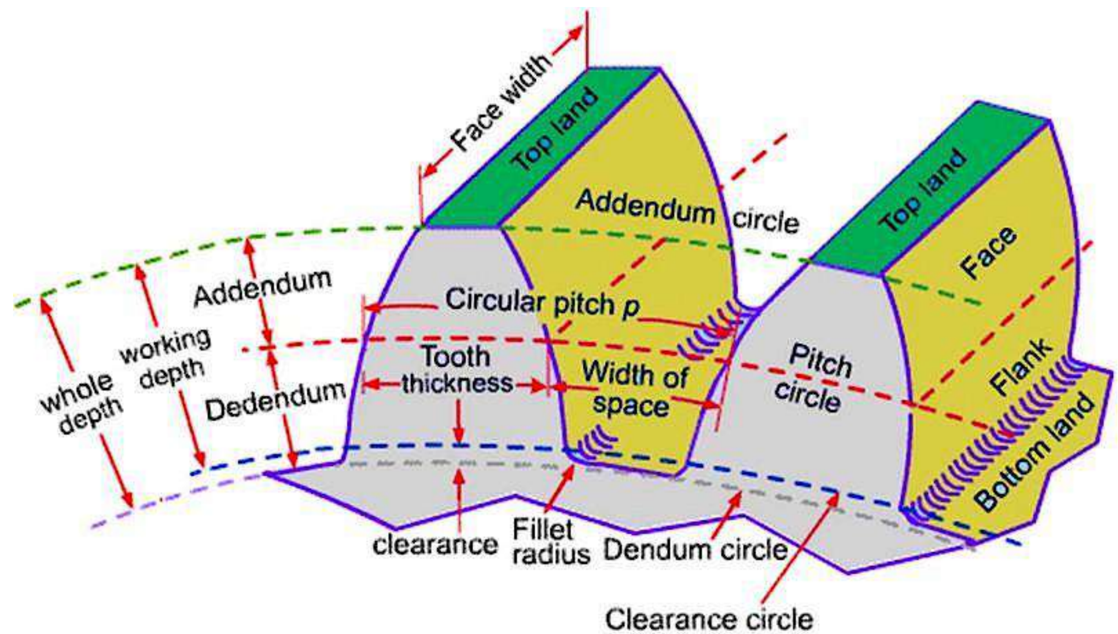




- ***Addendum:*** *It is the radial distance of a tooth from the pitch circle to the top of the tooth.*
- ***Dedendum:*** *It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.*
- ***Addendum circle:*** *It is the circle drawn through the top of the teeth and is concentric with* the pitch circle.
- ***Dedendum circle:*** *It is the circle drawn through the bottom of the teeth. It is also called* root circle.

Note : Root circle diameter = Pitch circle diameter × cos φ
where φ is the pressure angle.





Circular pitch: *It is the distance measured on the circumference of the pitch circle from* a point of one tooth to the corresponding point on the next tooth. It is usually denoted by

$$\text{Circular pitch,} \quad p_c = \pi D/T$$

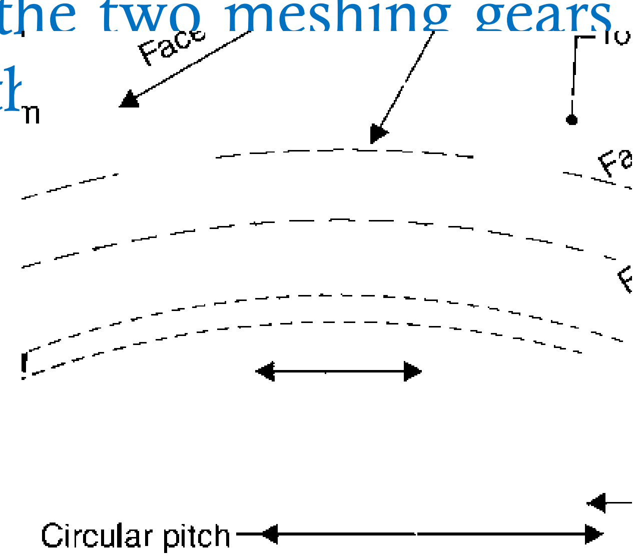
where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

A little consideration will show that the **two gears will mesh together correctly, if the two wheels have the same circular pitch.**

Note : If D … the two meshing gears, having the … th… correctly,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$



Diametral pitch: *It is the ratio of number of teeth to the pitch circle diameter in millimetres.* It is denoted by p_d. Mathematically,

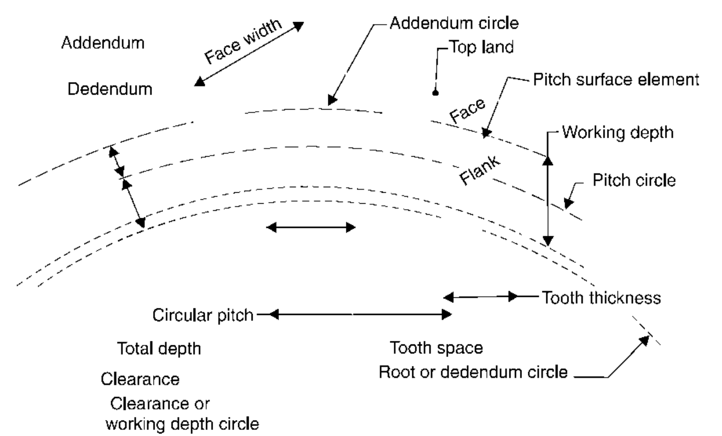
$$p_d = \frac{T}{D} = \frac{\pi}{p_c} \qquad \ldots \left(\because\ p_c = \frac{\pi D}{T} \right)$$

Module: *It is the ratio of the pitch circle diameter in millimeters to the number of teeth.* It is usually denoted by *m*.

Mathematically, Module, $m = D/T$

Clearance: *It is the radial distance from the top of the tooth to the bottom of the tooth, in* a meshing gear. A circle passing through the top of the meshing gear is known as ***clearance circle.***

Total depth: *It is the radial distance between the addendum and the dedendum circles of* a gear. It is equal to the sum of the addendum and dedendum.

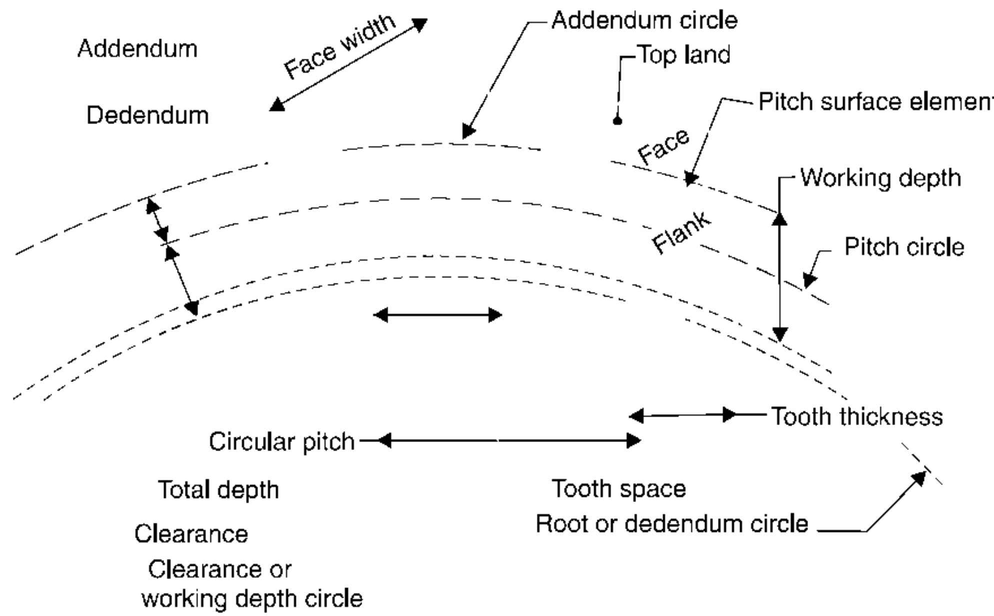


Working depth: *It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.*

Tooth thickness: *It is the width of the tooth measured along the pitch circle.*

Tooth space: *It is the width of space between the two adjacent teeth measured along the pitch circle.*

Backlash: *It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.*



Face of tooth: *It is the surface of the gear tooth above the pitch surface.*

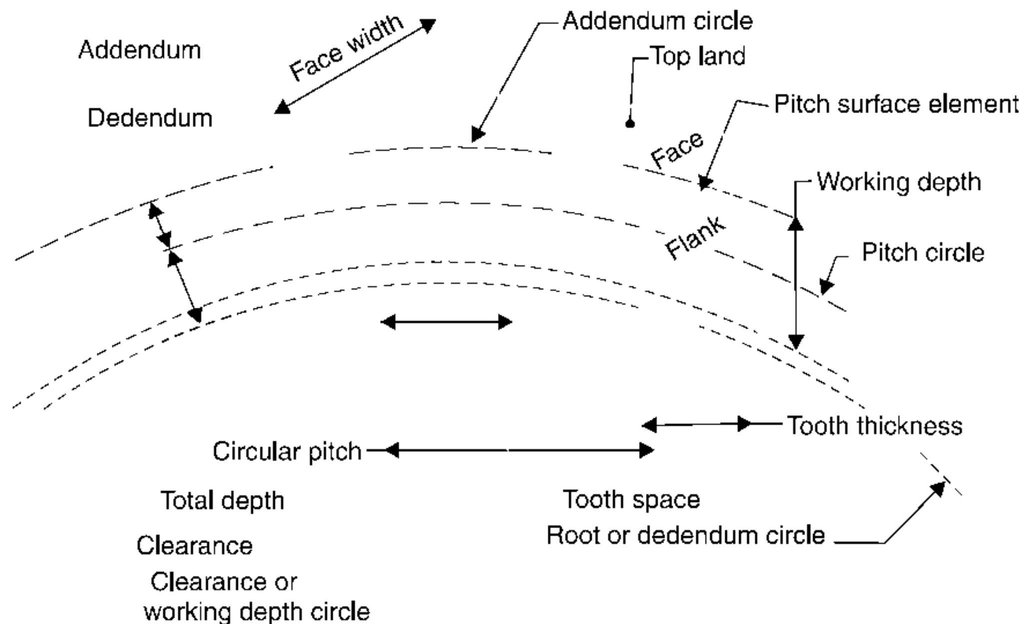
Flank of tooth: *It is the surface of the gear tooth below the pitch surface.*

Top land: *It is the surface of the top of the tooth.*

Face width: *It is the width of the gear tooth measured parallel to its axis.*

Profile: *It is the curve formed by the face and flank of the tooth.*

Fillet radius: *It is the radius that connects the root circle to the profile of the tooth.*



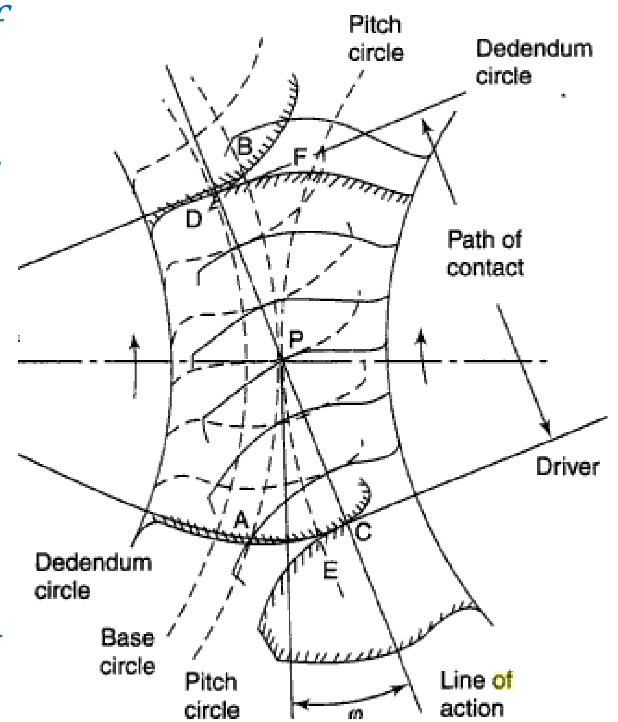
Path of contact: *It is the path traced by the point of contact of two teeth from the* beginning to the end of engagement.

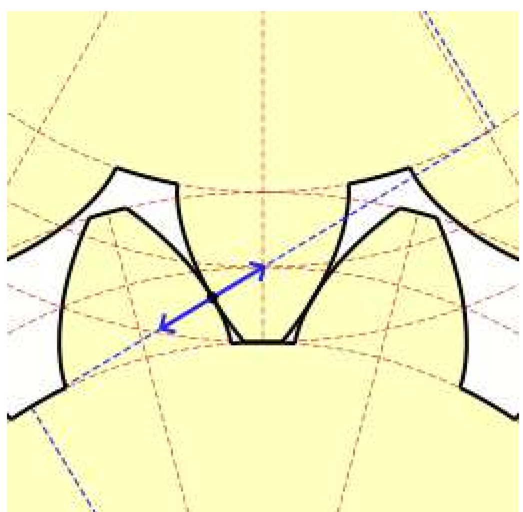
Length of the path of contact: *It is the length of the common normal cut-off by the* addendum circles of the wheel and pinion.

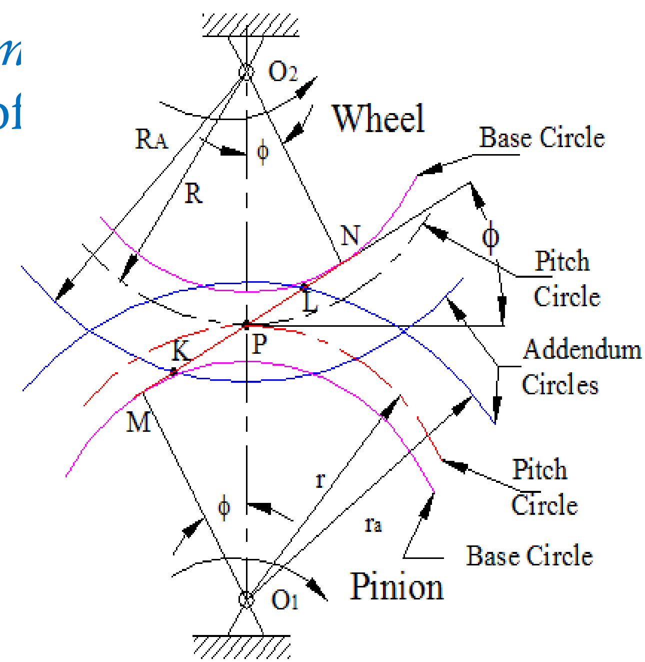
Arc of contact: *It is the path traced by a point on the pitch circle from the beginning* to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, *i.e.*

(a) Arc of approach. *It is the portion of the arc of contact from the beginning of the* engagement to the pitch point.

(b) Arc of recess: *It is the portion of the arc of contact from the pitch point to the end of the* engagement of a pair of teeth.







Forms of Teeth

- In actual practice following are the two types of teeth commonly used

  1. *Cycloidal teeth ; and 2. Involute teeth.*

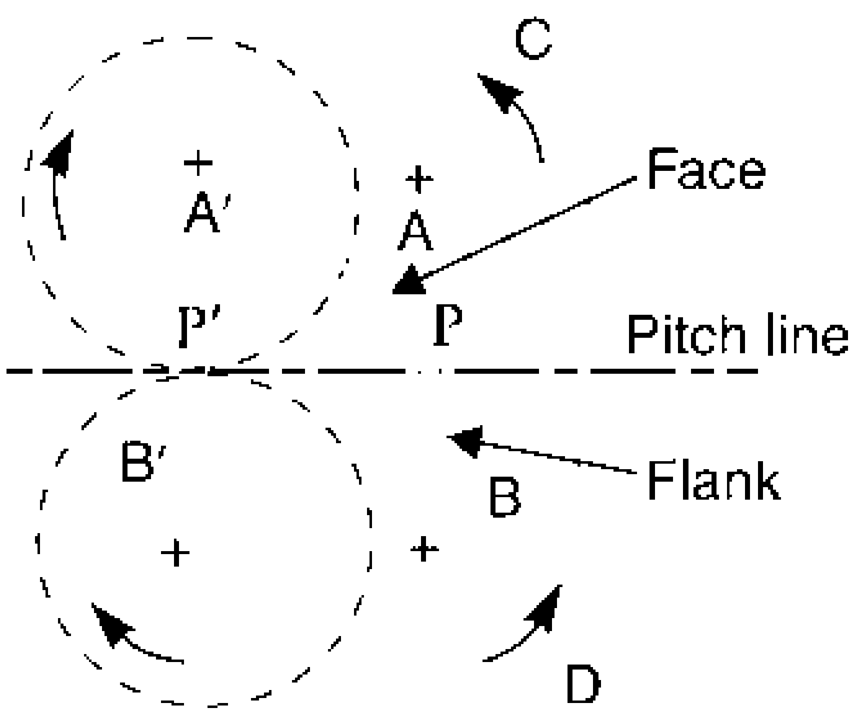
Cycloidal Teeth

- A *cycloid is the curve traced by a point on the circumference of a circle which rolls without* slipping on a fixed straight line.
- When a circle rolls without slipping on the outside of a fixed circle, the curve traced by a point on the circumference of a circle is known as ***epi-cycloid***.
- *On the other* hand, if a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a point on the circumference of a circle is called ***hypo-cycloid*****.**

Construction of cycloidal teeth for Rack

- *In Fig. (a), the fixed line or pitch line of a rack is shown. When the circle C rolls without slipping above the pitch line in the direction as indicated in Fig, then the point P on the circle traces epi-cycloid PA. This represents the face of the cycloidal tooth profile.*
- *When the circle D rolls without slipping below the pitch line, then the point P on the circle D traces hypo-cycloid PB, which represents the flank of the cycloidal tooth. The profile BPA is one side of the cycloidal rack tooth.*

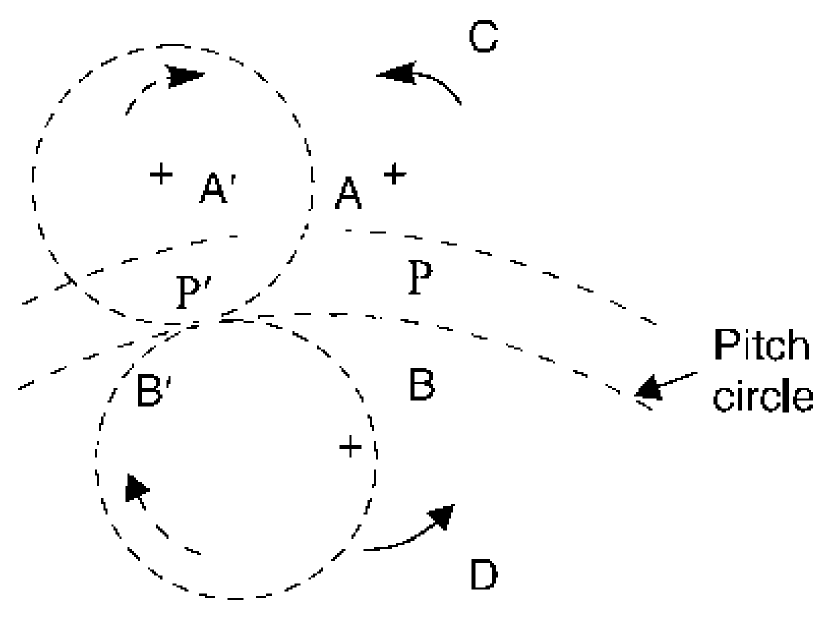
Similarly, the two curves P' A' and P'B' forming the opposite side of the tooth profile are traced by the point *P' when the circles C and D roll in the opposite directions.*



Construction of cycloidal teeth for gear

- The cycloidal teeth of a gear may be constructed as shown in Fig.
- The circle *C is rolled without slipping on the outside of the pitch circle and the point P on the circle C traces epi-cycloid PA, which represents the face of the cycloidal tooth.*
- *The circle D is rolled on the* inside of pitch circle and the point *P on the circle D traces hypo-cycloid PB, which represents the* flank of the tooth profile.

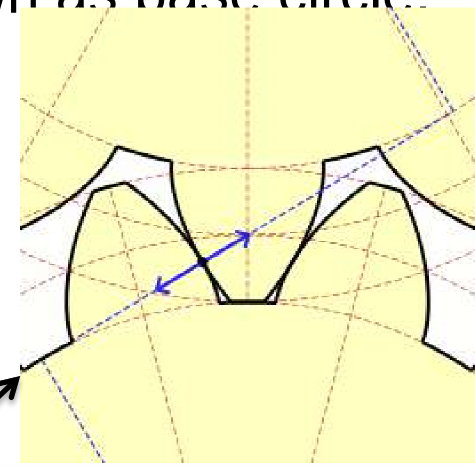
The profile *BPA is one side of the cycloidal tooth. The opposite side of the* tooth is traced as explained above.



(b)

Involute Teeth

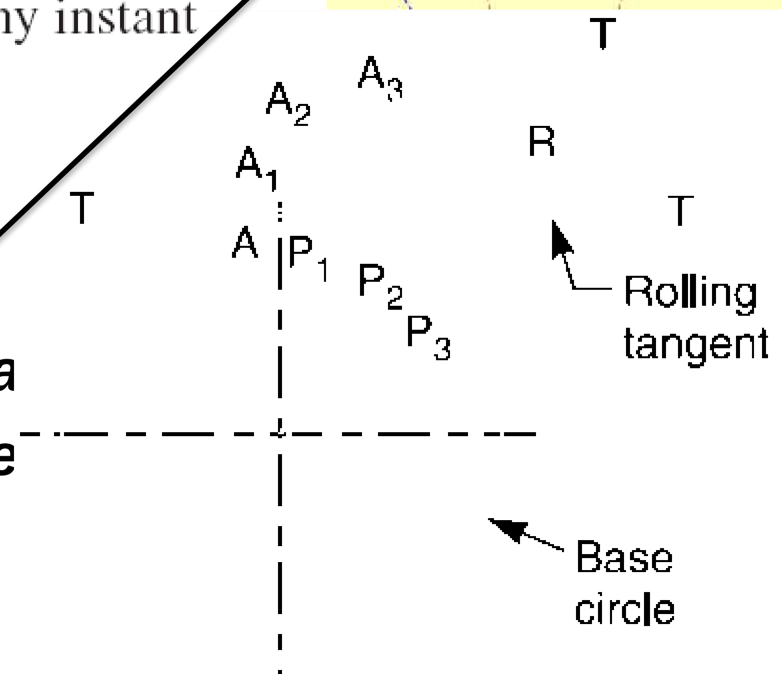
- An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping as shown in Fig.
- In connection with toothed wheels, the circle is known as base circle. The involute is traced as follows :



Let A be the starting point of the involute. The base circle is divided into equal number of parts *e.g.* AP_1, P_1P_2, P_2P_3 etc. The tangents at P_1, P_2, P_3 etc. are drawn and the length P_1A_1, P_2A_2, P_3A_3 equal to the arcs AP_1, AP_2 and AP_3 are set off. Joining the points A, A_1, A_2, A_3 etc. we obtain the involute curve AR. A little consideration will show that at any instant

A_3, the tangent A_3T *to the involute is perpendicular to* P_3A_3 and P_3A_3 is the normal to the involute.

In other words, ***normal at any point of a involute is a tangent to the base circle***



Comparison Between Involute and Cycloidal Gears

- In actual practice, the involute gears are more commonly used as compared to cycloidal gears, due to the following advantages :

Advantages of involute gears

- The most important advantage of the involute gears is that **the centre distance for a pair of involute gears can be varied within limits** without changing the velocity ratio. This is not true for cycloidal gears which requires exact centre distance to be maintained.
- **In involute gears, the pressure angle, from the start of the engagement of teeth to the end of the engagement, remains constant**. It is necessary for smooth running and less wear of gears. But in cycloidal gears, the pressure angle is maximum at the beginning of engagement, reduces to zero at pitch point, starts decreasing and again becomes maximum at the end of engagement. This results in less smooth running of gears.
- The face and flank of involute teeth are generated by a single curve where as in cycloidal gears, double curves (*i.e. epi-cycloid and hypo-cycloid) are required for the face and flank respectively.* Thus the **involute teeth are easy to manufacture than cycloidal teeth**. In involute system, the basic rack has straight teeth and the same can be cut with simple tools.
- *Note :* ***The only disadvantage of the involute teeth is that the interference occurs with pinions having smaller number of teeth****. This may be avoided by altering the heights of addendum and dedendum of the mating teeth or the angle of obliquity of the teeth.*

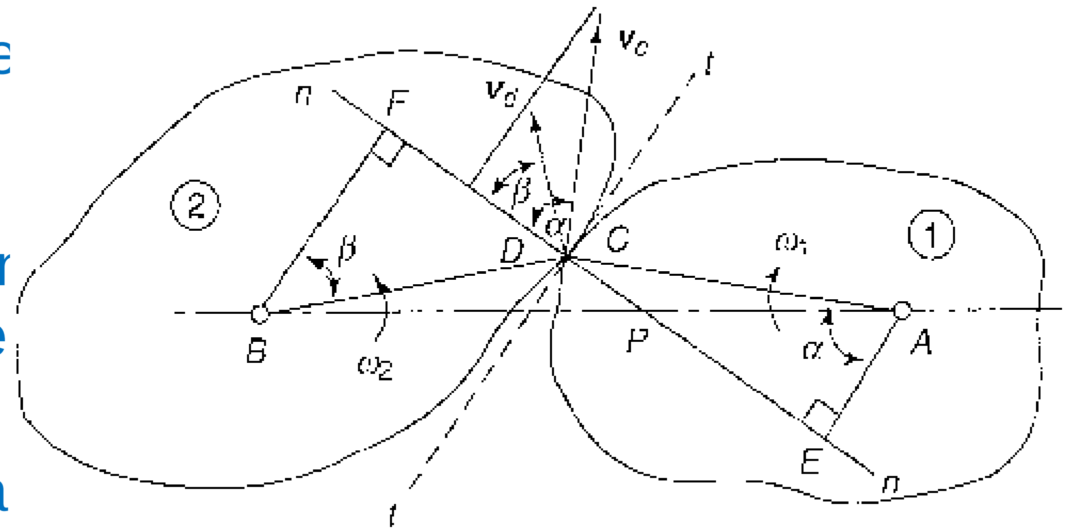
Advantages of cycloidal gears

Following are the advantages of cycloidal gears :

- Since the cycloidal teeth have wider flanks, therefore the **cycloidal gears are stronger than the involute gears**, for the same pitch.
- In cycloidal gears, the contact takes place between a convex flank and concave surface, whereas in involute gears, the convex surfaces are in contact. This condition results in **less wear in cycloidal gears as compared to involute gears**. However the difference in wear is negligible.
- **In cycloidal gears, the interference does not occur at all**. Though there are advantages of cycloidal gears but they are outweighed by the greater simplicity and flexibility of the involute gears.

Condition for Constant Velocity Ratio of Toothed Wheels–Law of Gearing

- The law of gearing states the condition which must be fulfilled by the gear tooth profiles to maintain a constar angular velocity ratio betwee two gears.
- Figure shows two bodies 1 a 2 representing a portion of the two gears in mesh.



A point C on the tooth profile of the gear 1 is in contact with a point *D* on the tooth profile of the gear 2. The two curves in contact at points *C* or *D* must have a common normal at the point. Let it be *n - n.*

Let,

ω_1= instantaneous angular velocity of the gear 1 (CW)

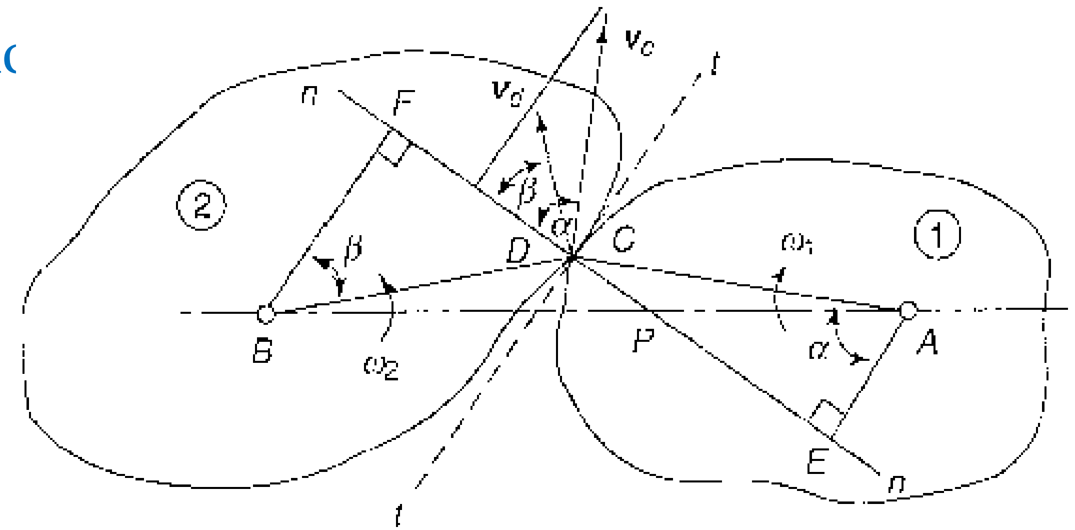
ω_2= instantaneous angular velocity of the gear 2 (CCW)

v_c = linear velocity of C

v_d = linear velocity of *D*

Condition for Constant Velocity Ratio of Toothed Wheels–Law of Gearing

- Then $v_c = \omega_1$ AC ; in a directic perpendicular to *AC* or at an angle α to *n - n*,
- $v_d = \omega_2$ BD ; in a direction perpendicular to *BD* or at an angle β to *n - n*.



Now, if the curved surfaces of the teeth of two gears are to remain in contact, one surface may slide relative to the other along the common tangent t-t.

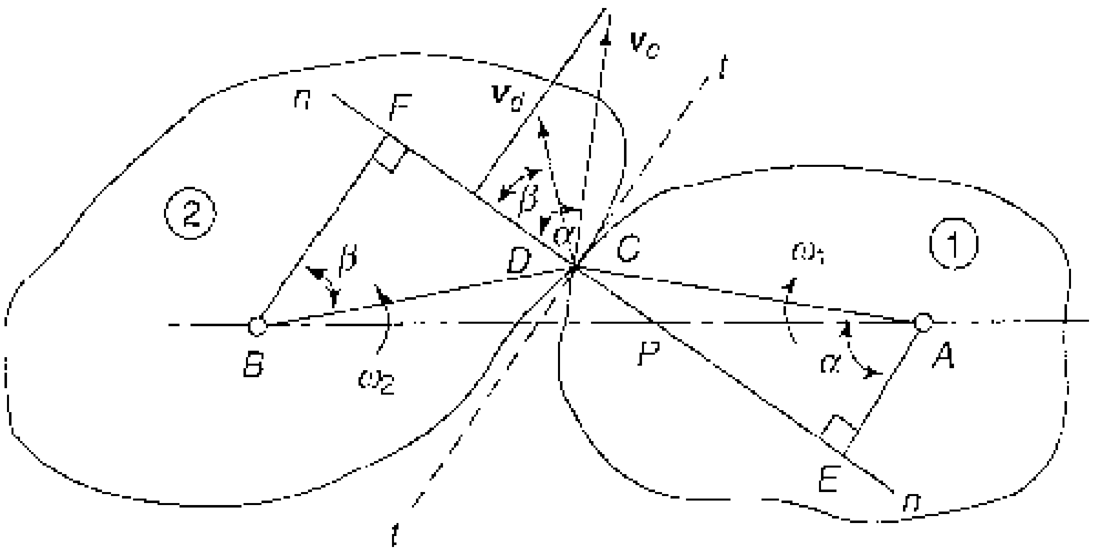
The relative motion between the surfaces along the common normal *n - n* must be zero to avoid the separation, or the penetration of the two teeth into each other.

Component of v_c along *n - n* = $v_c \cos \alpha$

Component of v_d along *n - n* = $v_d \cos \beta$

Relative motion along *n - n* = $v_c \cos \alpha - v_d \cos \beta$

Condition for Constant Velocity Ratio of Toothed Wheels–Law of Gearing



- Draw perpendiculars *AE* and *on n-n* from points *A* and *B* respectively.

  Then $\angle$ *CAE* $=\alpha$ and $\angle$ *DBF* $=\beta$

For proper contact,

$v_c \cos\alpha - v_d \cos\beta = 0$

$\omega_1 AC \cos\alpha - \omega_2 BD \cos\beta = 0$

$v_c = \omega_1 AC \qquad v_d = \omega_2 BD$

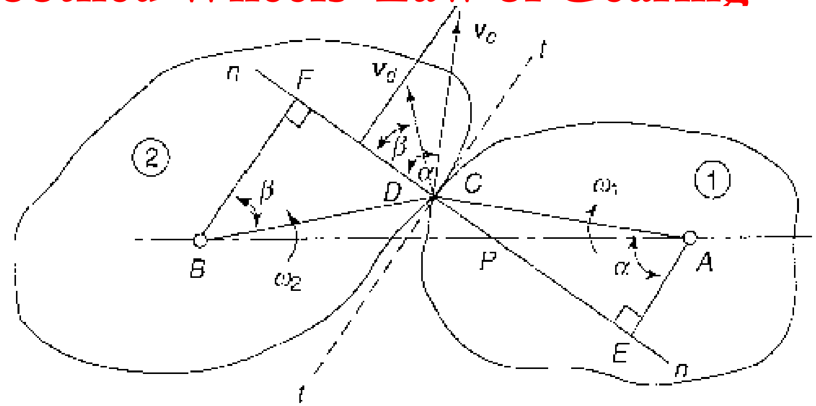
$$\omega_1 AC \frac{AE}{AC} - \omega_2 BD \frac{BF}{BD} = 0$$

$$\omega_1 AE - \omega_2 BF = 0$$

$$\frac{\omega_1}{\omega_2} = \frac{BF}{AE} = \frac{BP}{AP}$$

Condition for Constant Velocity Ratio of Toothed Wheels–Law of Gearing

Also, as the ΔAEP and $\Delta\ BFP$ are similar.



$$\frac{BP}{AP} = \frac{FP}{EP}$$

$$\frac{\omega_1}{\omega_2} = \frac{BF}{AE} = \frac{BP}{AP} = \frac{FP}{EP}$$

- Thus, it is seen that the **centre line AB is divided at P by the common normal in the inverse ratio of the angular velocities of the two gears.**
- If it is desired that the angular velocities of two gears remain constant, **the common normal at the point of contact of the two teeth should always pass through a fixed point P** which divides the line of centres in the inverse ratio of angular velocities of two gears.
- As seen earlier, **P is also the point of contact of two pitch circles** which divides the line of centres in the inverse ratio of the angular velocities of the two circles and is the pitch point.
- Thus, for constant angular velocity ratio of the two gears, **the common normal at the point of contact of the two mating teeth must pass through the pitch point.**

VELOCITY OF SLIDING

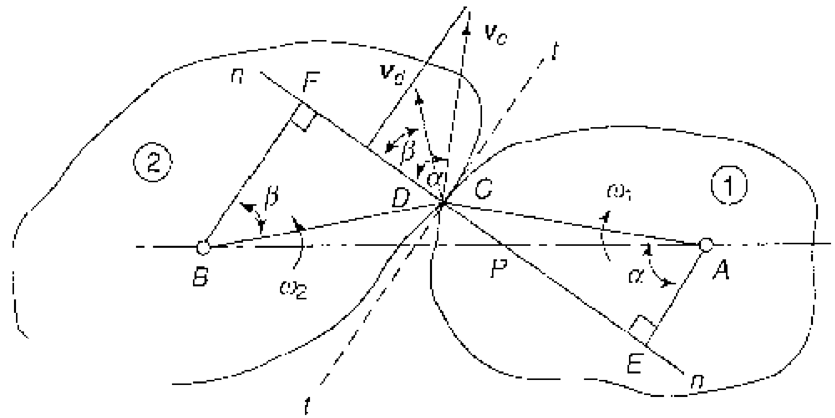
If the curved surfaces of the two teeth of the gears 1 and 2 are to remain in contact, one can have a sliding motion relative to the other along the common tangent t-t at C or *D.*

Component of v_c along t - t = $v_c \sin \alpha$

Component of v_d along t - t= $v_d \sin \beta$

Relative motion along $n - n$

$$= v_c \sin \alpha - v_d \sin \beta$$



AC and $v_d = \omega_2 BD$

$$= \omega_1 AC \frac{EC}{AC} - \omega_2 BD \frac{FD}{BD}$$

$$= \omega_1 EC - \omega_2 FD$$

$$= \omega_1 (EP + PC) - \omega_2 (FP - PD)$$

$$= \omega_1 EP + \omega_1 PC - \omega_2 FP + \omega_2 PC$$

(C and D are the coinciding points)

$$= (\omega_1 + \omega_2) PC + \omega_1 EP - \omega_2 FP$$

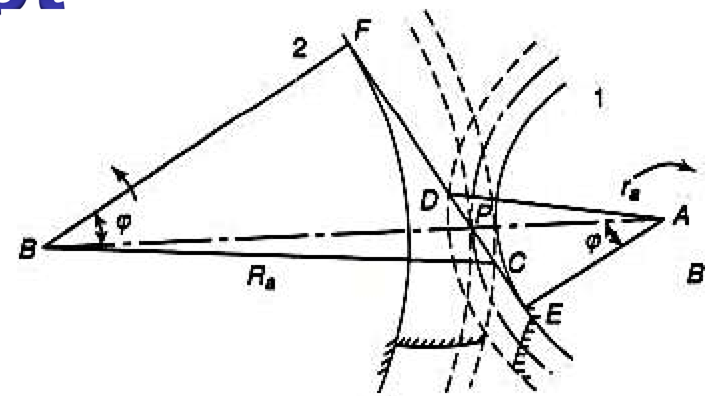
$$= (\omega_1 + \omega_2) PC \qquad [\omega_1 EP = \omega_2 FP,$$

= sum of angular velocities × distance between the pitch point and the point of contact

$$\frac{BP}{AP} = \frac{FP}{EP}$$

$$\frac{\omega_1}{\omega_2} = \frac{BF}{AE} = \frac{BP}{AP} = \frac{FP}{EP}$$

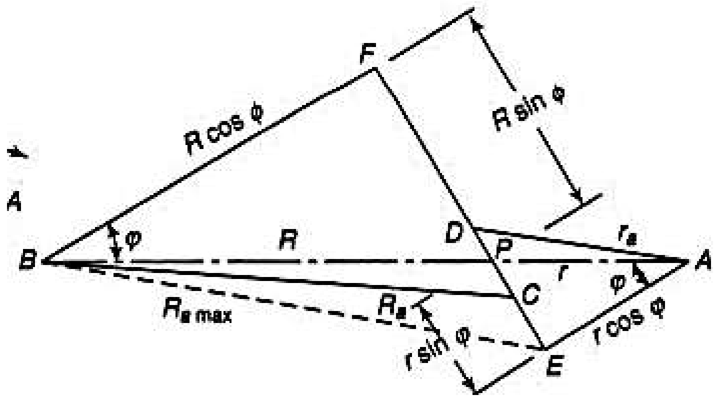
Path of contact



The pinion 1 is the driver and is rotating clockwise. The wheel 2 is driven in the counter-clockwise direction. *EF* is their common tangent to the base circles.

Contact of the two teeth is made where the addendum circle of the wheel meets the line of action EF, i.e., at *C* and is broken where the addendum circle of the pinion meets the line of action, i.e., at *D.*

CD is then the path of contact.



Let r = pitch circle radius of pinion

R = pitch circle radius of wheel

r_a = addendum circle radius of pinion

R_a = addendum circle radius of wheel.

Path of contact = path of approach + path of recess

CD = CP + PD

= (CF − PF) + (DE − PE)

$$= \left(\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right) + \left(\sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi\right)$$

$$= \left(\sqrt{R_a^2 - R^2\cos^2\phi}\right) + \left(\sqrt{r_a^2 - r^2\cos^2\phi} + (R+r)\sin\phi\right)$$

Observe that the path of approach can be found if the dimensions of the driven wheel are known. Similarly, the path of recess is known from the dimensions of the driving wheel.

F

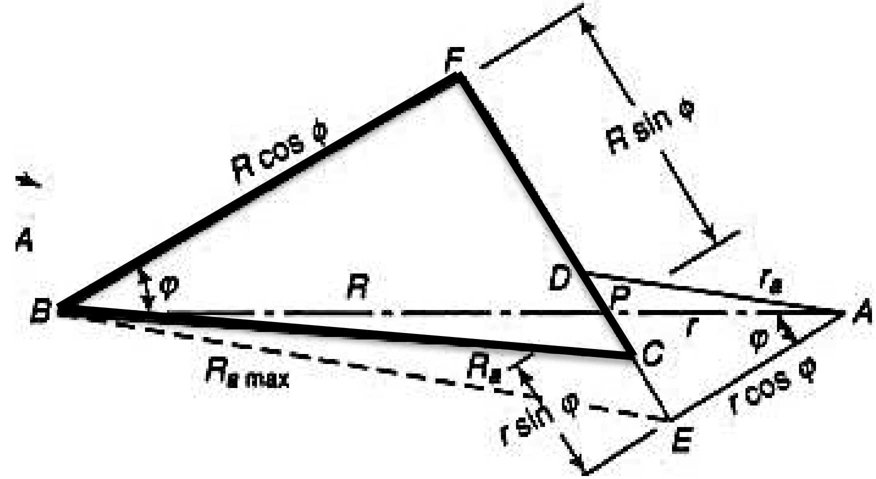
$R\cos\varphi$

$$\sqrt{R_a^2 - R^2\cos^2\phi}$$

B

R_a

C



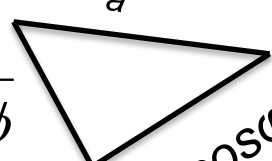
$$CD = CP + PD$$

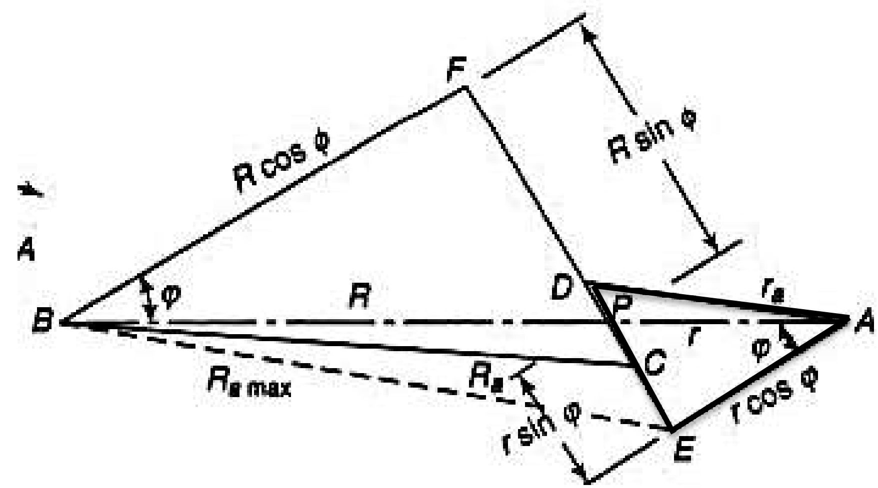
$$= (CF\text{-}PF) + (DE\text{-}PE)$$

$$= \left(\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right) + \left(\sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi\right)$$

$$= \left(\sqrt{R_a^2 - R^2\cos^2\phi}\right) + \left(\sqrt{r_a^2 - r^2\cos^2\phi} + (R+r)\sin\phi\right)$$

$$\sqrt{r_a^2 - r^2\cos^2\phi}$$





ARC OF CONTACT

- The arc of contact is the distance travelled by a point on either pitch circle of the two wheels during the period of contact of a pair of teeth.
- In Fig., at the beginning of engagement, the driving involute is shown as *GH;* when the point of contact is at *P* it is shown as *JK* and when at the end of engagement, it is *DL.*
- The arc of contact is *P'P"* and it consists of the arc of approach *P'P* and the arc of recess *PP".*

Let the time to traverse the arc of approach is t_a.

Then,

Arc of approach = *P'P* = Tangential velocity of *P'* x Time of approach

$$= \omega_a r \times t_a$$

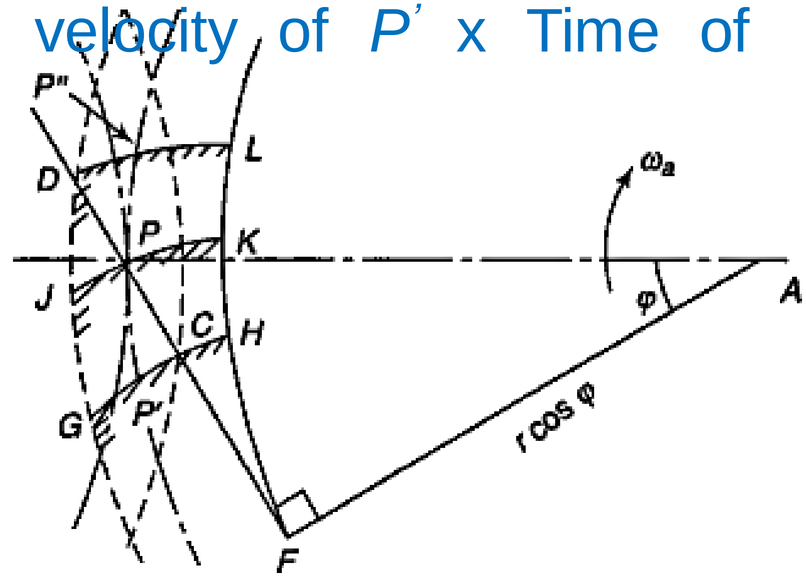
$$= \omega_a (r\cos\varphi)\frac{1}{\cos\varphi}t_a$$

$$= (\text{Tang. vel. of } H)\, t_a \frac{1}{\cos\varphi}$$

$$= \frac{\text{Arc } HK}{\cos\varphi}$$

$$= \frac{\text{Arc } FK - \text{Arc } FH}{\cos\varphi}$$

$$= \frac{FP - FC}{\cos\varphi} = \frac{CP}{\cos\varphi}$$



ARC OF CONTACT

Arc *FK* is equal to the path *FP* as the point *P* is on the generator *FP* that rolls on the base circle *FHK* to generate the involute *PK*.
Similarly, arc *FH* = Path *FC*.
Arc of recess = *PP''* = Tang. vel. of *P* x Time of recess

$$= \omega_a r \times t_r$$

$$= \omega_a (r\cos\varphi)\frac{1}{\cos\varphi}t_r$$

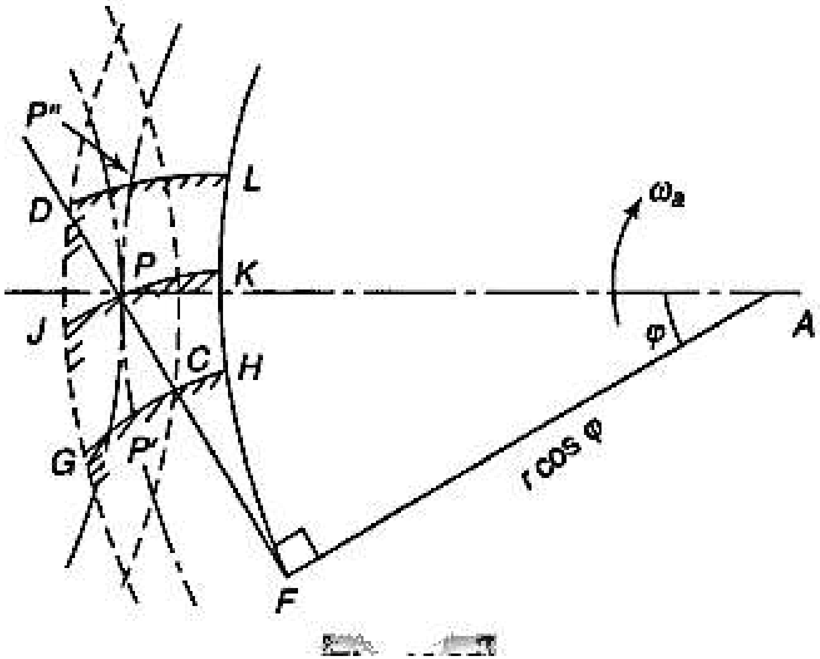
$$= (\text{Tang. vel. of } K)\, t_r \frac{1}{\cos\varphi}$$

$$= \frac{\text{Arc } KL}{\cos\varphi} = \frac{\text{Arc } FL - \text{Arc } FK}{\cos\varphi}$$

$$PP'' = \frac{FD - FP}{\cos\varphi} = \frac{PD}{\cos\varphi}$$

$$\text{Arc of contact} = \frac{CP}{\cos\varphi} + \frac{PD}{\cos\varphi} = \frac{CP + PD}{\cos\varphi} = \frac{CD}{\cos\varphi}$$

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\varphi}$$



NUMBER OF PAIRS OF TEETH IN CONTACT (CONTACT RATIO)

- The arc of contact is the length of the pitch circle traversed by a point on it during the mating of a pair of teeth.
- Thus, all the teeth lying in between the arc of contact will be meshing with the teeth on the other wheel.

Therefore, the number of teeth in contact =

$$\frac{\text{Arc of contact}}{\text{Circular pitch}} = \frac{\text{path of contact}}{\cos\phi} \times \frac{1}{p_c} \quad \text{Where } P_c = \pi D/T$$

- As the ratio of the arc of contact to the circular pitch is also the *contact ratio,* the number of teeth is also expressed in terms of contact ratio.
- For continuous transmission of motion, at least one tooth of one wheel must be in contact with another tooth of the second wheel.
- Therefore, *n* must be greater than unity.

Each of two gears in a mesh has 48 teeth and a module of 8 mm. The teeth are of 20° involute profile. The arc of contact is 2.25 times the circular pitch. Determine the addendum.

Solution $\varphi = 20°;\ t = T = 48;\ m = 8$ mm;

$$R = r = \frac{mT}{2} = \frac{8 \times 48}{2} = 192 \text{ mm};\ R_a = r_a$$

Arc of contact = 2.25 × Circular pitch = 2.25π m

$= 2.25\pi \times 8 = 56.55$ mm

$$p_c = \pi D/T$$

Path of contact = 56.55 × cos 20° = 53.14 mm

or $$\left(\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi\right)$$

$$+\left(\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi\right) = 53.14$$

or $$2\left(\sqrt{R_a^2 - 192^2\cos^2 20°} - 192\sin 20°\right)$$

$$= 53.14 \quad \text{or} \quad R_a = 202.6 \text{ mm}$$

Addendum = $R_a - R = 202.6 - 192 = 10.6$ mm

A pinion having 30 teeth drives a gear having 80 teeth. The profile of the gears is involute with 20° pressure angle, 12 mm module and 10 mm addendum. Find the length of path of contact, arc of contact and the contact ratio.

Solution. Given : $t = 30$; $T = 80$; $\phi = 20°$; $m = 12$ mm ; Addendum = 10 mm

Length of path of contact

We know that pitch circle radius of pinion,

$$r = m.t / 2 = 12 \times 30 / 2 = 180 \text{ mm}$$

and pitch circle radius of gear,

$$R = m.T / 2 = 12 \times 80 / 2 = 480 \text{ mm}$$

∴ Radius of addendum circle of pinion,

$$r_A = r + \text{ Addendum} = 180 + 10 = 190 \text{ mm}$$

and radius of addendum circle of gear,

$$R_A = R + \text{Addendum} = 480 + 10 = 490 \text{ mm}$$

We know that length of the path of approach,

$$KP = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi$$

$$= \sqrt{(490)^2 - (480)^2 \cos^2 20°} - 480 \sin 20° = 191.5 - 164.2 = 27.3 \text{ mm}$$

A pinion having 30 teeth drives a gear having 80 teeth. The profile of the gears is involute with 20° pressure angle, 12 mm module and 10 mm addendum. Find the length of path of contact, arc of contact and the contact ratio.

and length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(190)^2 - (180)^2\cos^2 20°} - 180\sin 20° = 86.6 - 61.6 = 25 \text{ mm}$$

We know that length of path of contact,

$$KL = KP + PL = 27.3 + 25 = 52.3 \text{ mm} \quad \textbf{Ans.}$$

Length of arc of contact

We know that length of arc of contact

$$= \frac{\text{Length of path of contact}}{\cos\phi} = \frac{52.3}{\cos 20°} = 55.66 \text{ mm} \quad \textbf{Ans.}$$

Contact ratio

We know that circular pitch,

$$p_c = \pi.m = \pi \times 12 = 37.7 \text{ mm}$$

$$\therefore \quad \text{Contact ratio} = \frac{\text{Length of arc of contact}}{p_c} = \frac{55.66}{37.7} = 1.5$$

Two involute gears in mesh have 20º pressure angle. The gear ratio is 3 and the number of teeth on the pinion is 24. The teeth have a module of 6 mm. The pitch line velocity is 1.5 m/s and the addendum equal to one module. Determine the angle of action of the pinion (the angle turned by the pinion when one pair of teeth is in the mesh) and the maximum velocity of sliding.

Solution $\varphi = 20°;\ t = 24;\ m = 6$ mm;

$T = 24 \times 3 = 72;$

$$r = \frac{mt}{2} = \frac{6 \times 24}{2} = 72 \text{ mm};$$

$R = 72 \times 3 = 216$ mm; $r_a = 72 + 6 = 78$ mm ;
$R_a = 216 + 6 = 222$ mm

$$\text{Path of contact} = \left(\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi\right) + \left(\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi\right)$$

$$= \left(\sqrt{222^2 - 216^2\cos^2 20°} - 216\sin 20°\right) + \left(\sqrt{78^2 - 72^2\cos^2 20°} - 72\sin 20°\right)$$

$$= 16.04 + 14.18 = 30.22 \text{ mm}$$

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\varphi} = \frac{30.22}{\cos 20°}$$

$= 32.16$ mm

$$\text{Angle of action} = \frac{\text{Arc of contact}}{r} = \frac{32.16}{72}$$

$= 0.4467 \text{ rad} = 0.4467 \times 180/\pi = 25.59°$

Velocity of sliding $= (\omega_p + \omega_g) \times$ Path of approach

$$= \left(\frac{v}{r} + \frac{v}{R}\right) \times \text{Path of approach}$$

$$= \left(\frac{1500}{72} + \frac{1500}{216}\right) \times 16.04 = 445.6 \text{ mm/s}$$

Two involute gears in a mesh have a module of 8 mm and a pressure angle of 20°. The larger gear has 57 while the pinion has 23 teeth. If the addenda on pinion and gear wheels are equal to one module, find the (i) contact ratio (ii) angle of action of the pinion and the gear wheel (iii) ratio of the sliding to rolling velocity at the (a) beginning of contact (b) pitch point (c) end of contact

Solution $\varphi = 20°;\ T = 57;\ t = 23;\ m = 8$ mm;

addendum $= m = 8$ mm

$$R = \frac{mT}{2} = \frac{8 \times 57}{2} = 228 \text{ mm};$$

$$R_a = R + m = 228 + 8 = 236 \text{ mm}$$

$$r = \frac{mt}{2} = \frac{8 \times 23}{2} = 92 \text{ mm};$$

$$r_a = r + m = 92 + 8 = 100 \text{ mm}$$

(i) $$n = \frac{\text{Arc of contact}}{\text{Circular pitch}} = \left(\frac{\text{Path of contact}}{\cos\varphi}\right)$$

$$\times \frac{1}{\pi m} = \frac{\text{Path of approach} + \text{Path of recess}}{\cos\varphi \times \pi m}$$

$$= \frac{\left[\begin{array}{l}\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi \\ +\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi\end{array}\right]}{\cos\varphi \times \pi m}$$

$$\frac{\left[\begin{array}{l}\sqrt{(236)^2 - (228^2\cos^2 20^\circ} - 228\sin 20^\circ \\ +\sqrt{(100)^2 - (92)^2\cos^2 20^\circ} - 92\sin 20^\circ\end{array}\right]}{\cos 20^\circ \pi \times 8}$$

$$= \frac{20.97 + 18.79}{\cos 20° \times \pi \times 8} = 42.31 \times \frac{1}{\pi \times 8} = \underline{1.68}$$

Two involute gears in a mesh have a module of 8 mm and a pressure angle of 20°. The larger gear has 57 while the pinion has 23 teeth. If the addenda on pinion and gear wheels are equal to one module, find the (i) contact ratio (ii) angle of action of the pinion and the gear wheel (iii) ratio of the sliding to rolling velocity at the (a) beginning of contact (b) pitch point (c) end of contact

(ii) Angle of action, $\delta_p = \dfrac{\text{Arc of contact}}{r} = \dfrac{42.31}{92}$

$= 0.46$ rad or $0.46 \times 180/\pi = 26.3°$

$$\delta_g = \frac{\text{Arc of contact}}{R} = \frac{42.31}{228} = 0.1856 \text{ rad}$$

or $0.1856 \times 180/\pi = 10.63°$

(iii) (a) $\dfrac{\text{Sliding velocity}}{\text{Rolling velocity}}$

$$= \frac{(\omega_p + \omega_g) \times \text{Path of approach}}{\text{Pitch line velocity } (= \omega_p \times r)}$$

$$= \frac{\left(\omega_p + \frac{23}{57}\omega_p\right) \times 20.97}{\omega_p \times 92} = \underline{0.32}$$

(b) $$\frac{\text{Sliding velocity}}{\text{Rolling velocity}} = \frac{(\omega_p + \omega_g) \times 0}{\text{Pitch line velocity}} = 0$$

(c) $\dfrac{\text{Sliding velocity}}{\text{Rolling velocity}}$

$$= \frac{\left(\omega_p + \frac{23}{57}\omega_p\right) \times \text{Path of recess}}{\omega_p \times r}$$

$$= \frac{\left(1 + \frac{23}{57}\right) \times 18.79}{92} = \underline{0.287}$$

Two 20° gears have a module pitch of 4 mm. The number of teeth on gears 1 and 2 are 40 and 24 respectively. If the gear 2 rotates at 600 rpm, determine the velocity of sliding when the contact is at the lip of the tooth of gear 2. Take addendum equal to one module. Also, find the maximum velocity of sliding.

Solution 1 is the gear wheel and 2 is the pinion.

$\varphi = 20°$; $T = 40$; $N_p = 600$ mm; $t = 24$; $m = 4$ mm

Addendum = 1 module = 4 mm

$$R = \frac{mT}{2} = \frac{4 \times 40}{2} = 80 \text{ mm}; \ R_a = 80 + 4 = 84 \text{ mm}$$

$$r = \frac{mt}{2} = \frac{4 \times 24}{2} = 48 \text{ mm}; \ r_a = 48 + 4 = 52 \text{ mm}$$

$$N_g = N_p \times \frac{t}{T} = 600 \times \frac{24}{40} = 360 \text{ rpm}$$

Let pinion (gear 2) be the driver. The tip of the driving wheel is in contact with a tooth of the driven wheel at the end of engagement. Thus, it is required to find the path of recess which is obtained from the dimensions of the driving wheel.

$$\text{Path of recess} = \sqrt{r_a^2 - (r\cos\varphi)^2} - r\sin\varphi$$

$$= \sqrt{(52)^2 - (48\cos 20°)^2} - 48\sin 20°$$

$= 9.458$ mm

Velocity of sliding $= (\omega_p + \omega_g) \times$ Path of recess

$= 2\pi\,(N_p + N_g) \times 9.458$

$= 2\pi\,(600 + 360) \times 9.458$

$= 57\,049$ mm/min

$= \underline{950.8 \text{ mm/s}}$

(ii) In case the gear wheel is the driver, the tip of the pinion will be in contact with the flank of a tooth of the gear wheel at the beginning of contact. Thus, it is required to find the distance of the point of contact from the pitch point, i.e.. path of approach. The path of approach is found from the dimensions of the driven wheel which is again pinion.

Thus, path of approacl $= \sqrt{r_a^2 - (r\cos\varphi)^2} - r\sin\varphi$

=9.458mm, same as before and velocity of sliding = 950.8 mm/s

Thus, it is immaterial whether the driver is the gear wheel or the pinion, the velocity of

sliding is the same when the contact is at the tip of the pinion. The maximum velocity of sliding will depend upon the larger path considering any of the wheels to be the driver. Consider pinion to be the driver. Path of recess = 9.458 mm

Path of approac
$$= \sqrt{R_a^2 - (R\cos\varphi)^2} - R\sin\varphi$$
$$= \sqrt{(84)^2 - (80\cos 20°)} - 80\sin 20°$$
$$= 10.117 \text{ mm}$$

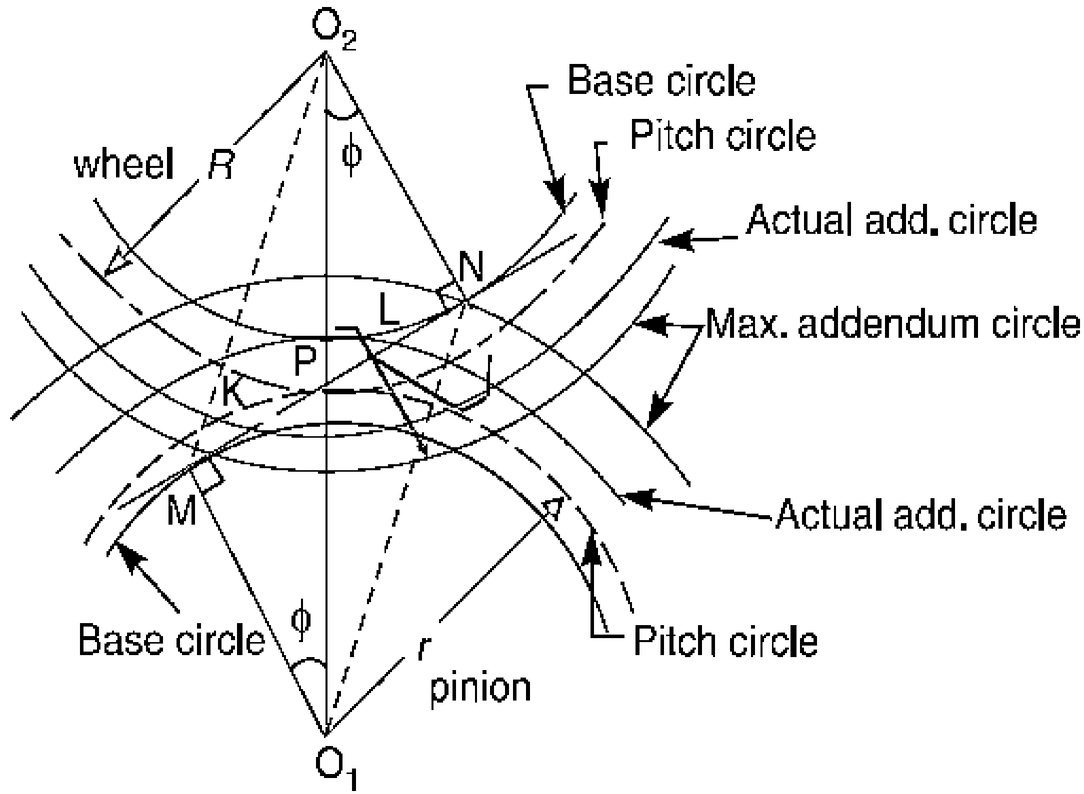
This is also the path of recess if the wheel becomes the driver.

Maximum velocity of sliding

$$= (\omega_p + \omega_g) \times \text{Maximum path}$$
$$= 2\pi(600 + 360) \times 10.117$$
$$= 61024 \text{ mm/min}$$
$$= \underline{1017.1 \text{ mm/s}}$$

Interference in Involute Gears

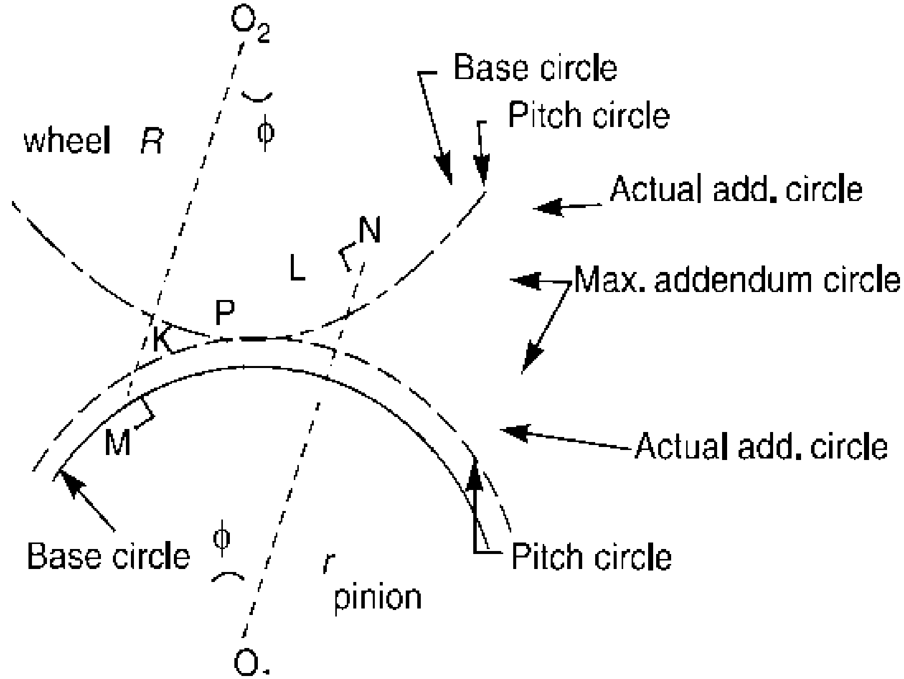
- Fig. shows a pinion with centre *O*1, in mesh with wheel or gear with centre *O*2.
- *MN is* the common tangent to the base circles and *KL is the path of contact between the two mating teeth.*
  - A little consideration will show, that if the radius of the addendum circle of pinion is increased to O_1 *N, the point of contact L will move from L to N.*
  - *When this radius is further increased,* the point of contact *L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel.*



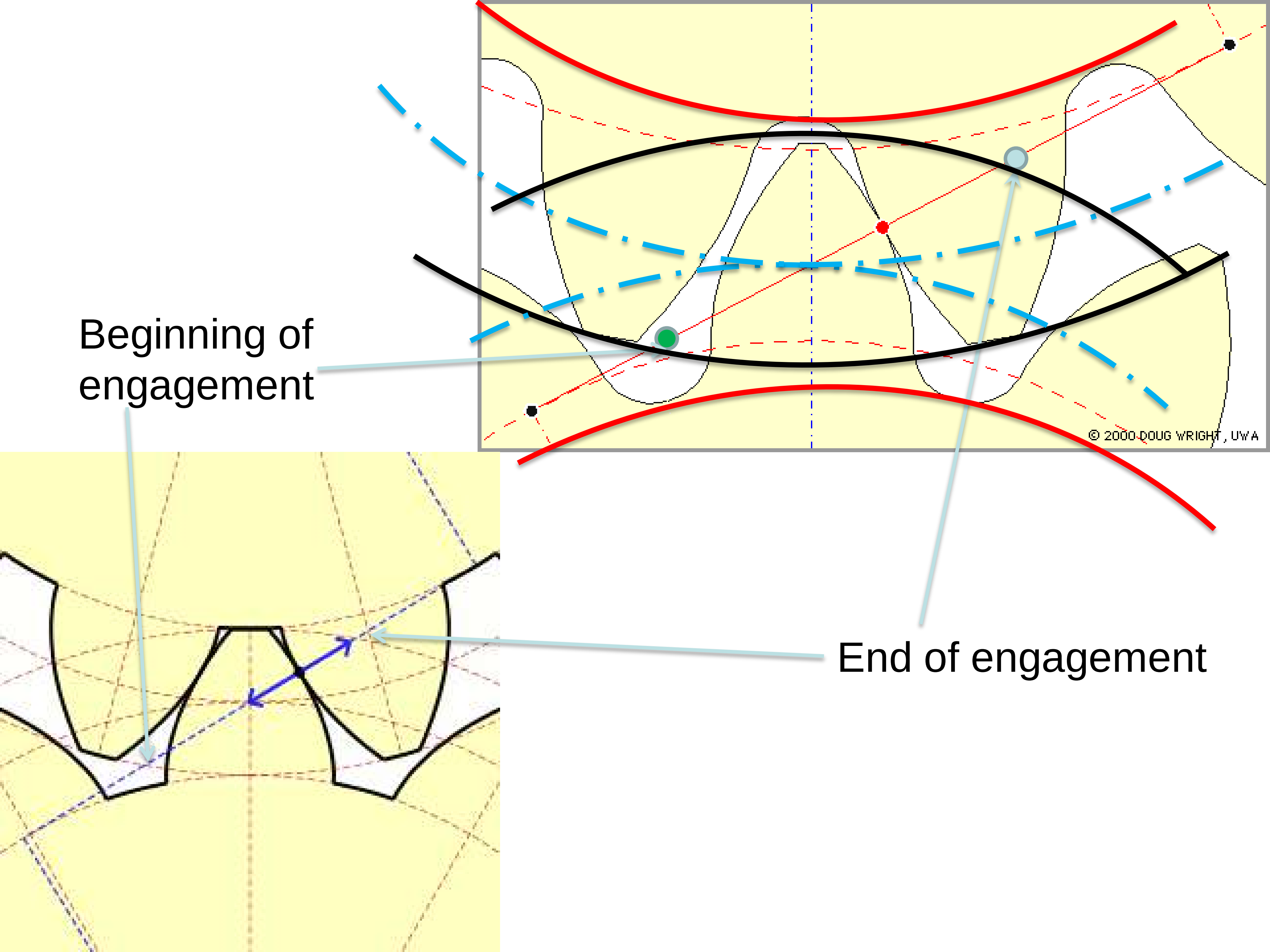
- The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as *interference, and* occurs when the teeth are being cut.
- In brief, *the phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.*

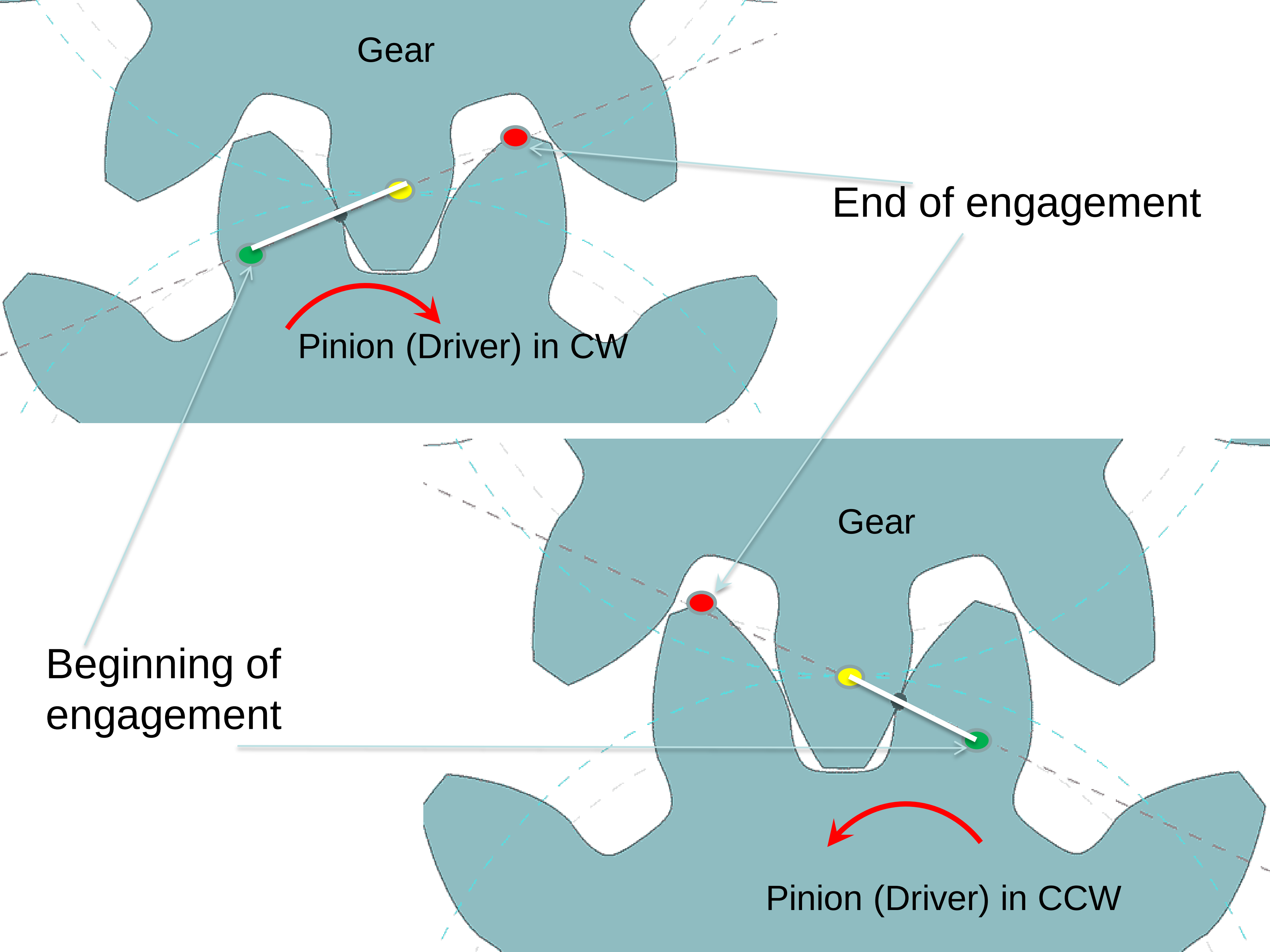
- Similarly, if the radius of the addendum circle of the wheel increases beyond *O2M, then the* tip of tooth on wheel will cause interference with the tooth on pinion. The points *M and N are called interference points.*

Obviously, interference may be avoided if the path of contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is *O*1*N and of the wheel is O2M.*



- From the above discussion, we conclude that the interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both the teeth. In other words, *interference may only be prevented, if the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency.*
- When interference is just avoided, the maximum length of path of contact is *MN when the* maximum addendum circles for pinion and wheel pass through the points of tangency *N and M respectively* as shown in Fig.


Beginning of engagement
End of engagement
© 2000 DOUG WRIGHT, UWA


Gear
End of engagement
Pinion (Driver) in CW
Gear
Beginning of engagement
Pinion (Driver) in CCW

Methods of elimination of Gear tooth Interference

In certain spur designs if interference exists, it can be overcome by:

1. Removing the cross hatched tooth tips i.e., using stub teeth.
2. Increasing the number of teeth on the mating pinion.
3. Increasing the pressure angle
4. Tooth profile modification or profile shifting
5. Increasing the centre distance.

Minimum Number of Teeth on the wheel in Order to Avoid Interference

- We have already discussed that in order to avoid interference, the addendum circles for the two mating gears must cut the common tangent to the base circles between the points of tangency. The limiting condition es of pinion and wheel pass through

t = Number of teeth on the pinion,,

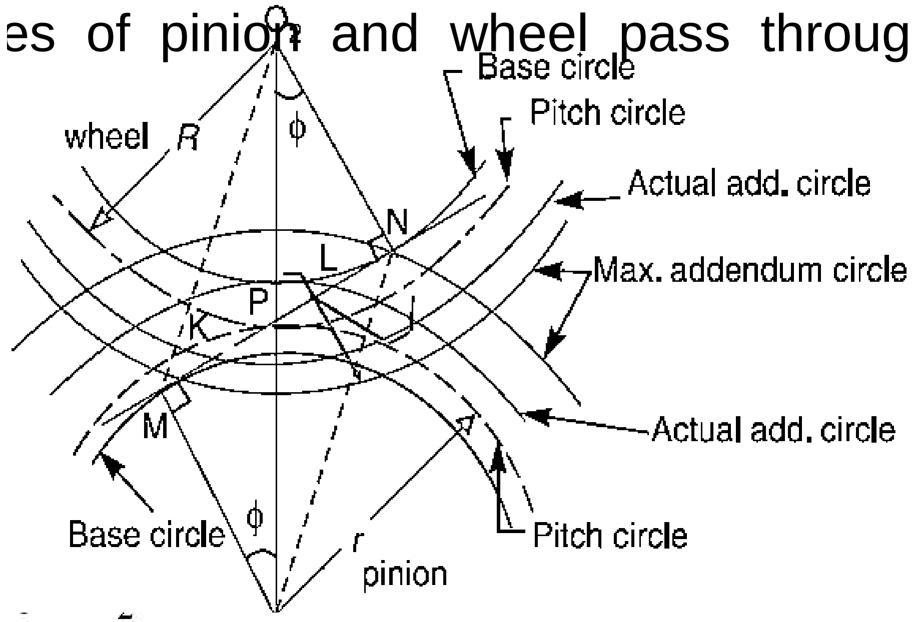
T = Number of teeth on the wheel,

m = Module of the teeth,

r = Pitch circle radius of pinion = $m.t / 2$

G = Gear ratio = $T / t = R / r$

ϕ = Pressure angle or angle of obliquity.



we have from triangle O_2MP

$$(O_2M)^2 = (O_2P)^2 + (PM)^2 - 2 \times O_2P \times PM \cos O_2PM$$

$$= R^2 + r^2 \sin^2\phi - 2\, R.r \sin\phi \cos(90° + \phi)$$

$$\ldots(\because PM = O_1P \sin\phi = r)$$

$$= R^2 + r^2 \sin^2\phi + 2R.r \sin^2\phi$$

$$= R^2\left[1 + \frac{r^2 \sin^2\phi}{R^2} + \frac{2r\sin^2\phi}{R}\right] = R^2\left[1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi\right]$$

$\therefore$ Limiting radius of wheel addendum circle,

$$O_2M = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi} = \frac{m.T}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi}$$

$A_W m$ = Addendum of the wheel, where A_W is a fraction by which the standard addendum for the wheel should be multiplied.

We know that the addendum of the wheel

$$= O_2M - O_2P$$

$$\therefore \quad A_W m = \frac{m.T}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - \frac{m.T}{2} \quad \ldots(\because O_2P = R = m.T/2)$$

$$= \frac{m.T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$A_W = \frac{T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

Note: Use this formula while calculating minimum number of teeth of wheel and then use gear ration to calculate the teeth of pinion

$$T = \frac{2A_W}{\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1} = \frac{2A_W}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

Minimum Number of Teeth on the Pinion in Order to Avoid Interference

- We have already discussed that in order to avoid interference, the addendum circles for the two mating gears must cut the common tangent to the base circles between the points of tangency. The limiting condition reaches, when the addendum circles of pinion and wheel pass through

t = Number of teeth on the pinion,,

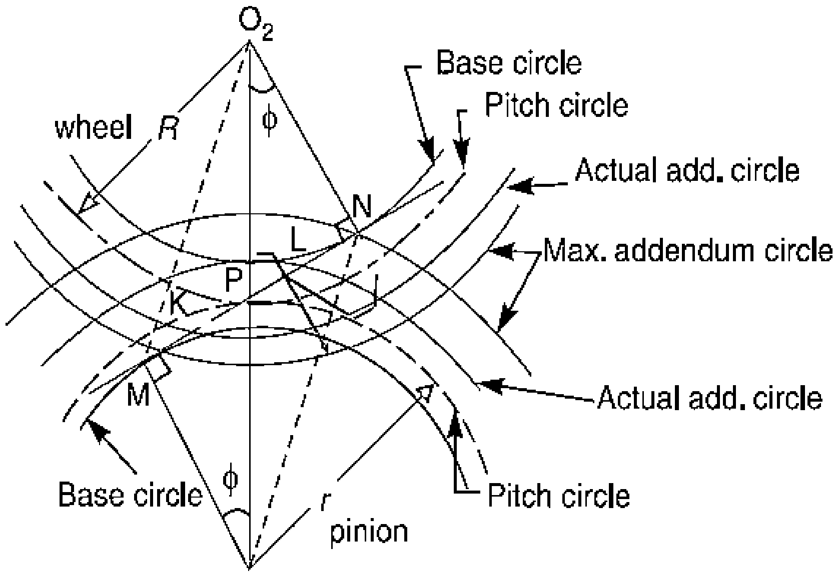
T = Number of teeth on the wheel,

m = Module of the teeth,

r = Pitch circle radius of pinion = $m.t / 2$

G = Gear ratio = $T / t = R / r$

ϕ = Pressure angle or angle of obliquity.



From triangle O_1NP,

$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \cos O_1PN$$

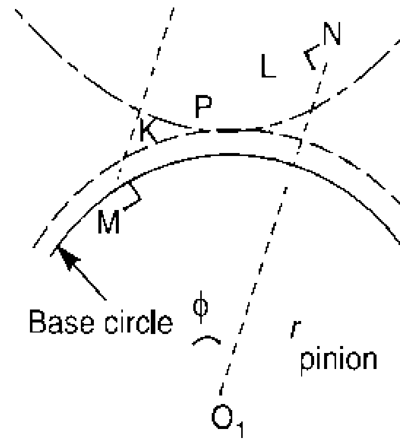
$$= r^2 + R^2 \sin^2 \phi - 2r.R \sin\phi \cos(90° + \phi) \qquad ...(\because PN = O_2P \sin\phi = R\sin\phi)$$

$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \cos O_1PN$$

$$= r^2 + R^2 \sin^2\phi - 2r.R \sin\phi \cos(90° + \phi)$$

$$= r^2 + R^2 \sin^2\phi + 2r.R \sin^2\phi$$

$$= r^2\left[1 + \frac{R^2 \sin^2\phi}{r^2} + \frac{2R\sin^2\phi}{r}\right] = r^2\left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$



∴ Limiting radius of the pinion addendum circle,

$$O_1N = r\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi} = \frac{m.t}{2}\sqrt{1 + \frac{T}{t}\left[\frac{T}{t} + 2\right]\sin^2\phi}$$

Let $A_P.m$ = Addendum of the pinion, where A_P is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

We know that the addendum of the pinion

$$= O_1N - O_1P$$

$$A_P.m = \frac{m.t}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - \frac{m.t}{2}$$

$$= \frac{m.t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$A_P = \frac{t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$t = \frac{2A_P}{\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1}$$

$$= \frac{2A_P}{\sqrt{1 + G(G + 2)\sin^2\phi} - 1}$$

Two 20° involute spur gears mesh externally and give a velocity ratio of 3. Module is 3 mm and the addendum is equal to 1.1 module. If the pinion rotates at 120 rpm, determine (i) the minimum number of teeth on each wheel to avoid interference (ii) the number of pairs of teeth in cor

Solution $\varphi = 20°$ $N_p = 120$ rpm

$VR = 3$ Addendum = 1.1 m

$m = 3$ mm $a_w = 1.1$

(i) $$T = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi}-1}$$

$$= \frac{2\times 1.1}{\sqrt{1+\frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20°}-1} = 49.44$$

Taking the higher whole number divisible by the velocity ratio,

i.e., $T = 51$ and $t = \frac{51}{3} = \underline{17}$

(ii) Contact ratio or number of pairs of teeth in contact,

$$n = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

$$= \left(\frac{\text{Path of contact}}{\cos\varphi}\right)\times\frac{1}{\pi m}$$

or

$$n = \frac{\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi + \sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi}{\cos\varphi\times\pi m}$$

We have, $R = \frac{mT}{2} = \frac{3\times 51}{2} = 76.5$ mm

$R_a = R + 1.1\,m = 76.5 + 1.1\times 3 = 79.8$ mm

$r = \frac{mt}{2} = \frac{3\times 17}{2} = 25.5$ mm

$r_a = 25.5 + 1.1\times 3 = 28.8$ mm

Two 20° involute spur gears mesh externally and give a velocity ratio of 3. Module is 3 mm and the addendum is equal to 1.1 module. If the pinion rotates at 120 rpm, determine (i) the minimum number of teeth on each wheel to avoid interference (ii) the number of pairs of teeth in contact.

$$n = \frac{\left[\begin{array}{l}\sqrt{(79.8)^2 - (76.5\cos 20^\circ)^2} - 76.5\sin 20^\circ \\ +\sqrt{(28.8)^2 - (25.5\cos 20^\circ)^2} - 25.5\sin 20^\circ\end{array}\right]}{\cos 20^\circ \times \pi \times 3}$$

$$= \frac{34.646 - 26.165 + 15.977 - 8.720}{\cos 20^\circ \times \pi \times 3}$$

$$= 1.78$$

Thus, 1 pair of teeth will always remain in contact whereas for 78% of the time, 2 pairs of teeth will be in contact.

Two 20° involute spur gears have a module of 10 mm. The addendum is equal to one module. The larger gear has 40 teeth while the pinion has 20 teeth. Will the gear interfere with the pinion?

Solution

$\varphi = 20°$, $T = 40$, $m = 10$ mm, $t = 20$

Addendum = 1 m = 10 mm

$$R = \frac{mT}{2} = \frac{10 \times 40}{20} = 200 \text{ mm}$$

$$R_a = 200 + 10 = 210 \text{ mm}$$

$$r = \frac{mt}{2} = \frac{10 \times 20}{2} = 100 \text{ mm}$$

$$r_a = 100 + 10 = 110 \text{ mm}$$

Let pinion be the driver (Refer Fig.10.24).

$$\text{Path of approach, } PC = \sqrt{R_a^2 - (R\cos\varphi)^2} - R\sin\varphi$$

$$= \sqrt{(210)^2 - (200 \times \cos 20°)^2} - 200\sin 20°$$

$$= 25.3 \text{ mm}$$

To avoid interference, maximum length of the path of approach can be PE.

$$PE = r\sin\varphi = 100\sin 20° = 34.2 \text{ mm}$$

Since the actual path of approach is within the maximum limit, no interference occurs.

A pair of spur gears with involute teeth is to give a gear ratio of 4: 1. The arc of approach is not to be less than the circular pitch and smaller wheel is the driver. The angle of pressure is 14.5°. Find :
1. the least number of teeth that can be used on each wheel, and
2. the addendum of the wheel in terms of the circular pitch.

Solution. *Given* : $G = T/t = R/r = 4$; $\phi = 14.5°$

1. ***Least number of teeth on each wheel***

We know that the maximum length of the arc of approach

$$= \frac{\text{Maximum length of the path of approach}}{\cos\phi} = \frac{r\sin\phi}{\cos\phi} = r\tan\phi$$

circular pitch, $$p_c = \pi m = \frac{2\pi r}{t}$$

Since the arc of approach is not to be less than the circular pitch, therefore

$$r\tan\phi = \frac{2\pi r}{t} \quad \text{or} \quad t = \frac{2\pi}{\tan\phi} = \frac{2\pi}{\tan 14.5°} = 24.3 \text{ say } 25$$

$$T = G.t = 4 \times 25 = 100 \textbf{ Ans.}$$

A pair of spur gears with involute teeth is to give a gear ratio of 4: 1. The arc of approach is not to be less than the circular pitch and smaller wheel is the driver. The angle of pressure is 14.5°. Find : 1. the least number of teeth that can be used on each wheel, and 2. the addendum of the wheel in terms of the circular pitch.

2. *Addendum of the wheel*

We know that addendum of the wheel

$$= \frac{m.T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{m \times 100}{2}\left[\sqrt{1 + \frac{25}{100}\left(\frac{25}{100} + 2\right)\sin^2 14.5°} - 1\right]$$

$$= 50\,m \times 0.017 = 0.85\ m = 0.85 \times p_c / \pi = 0.27\ p_c$$

Two 20° involute spur gears have a module of 10 mm. The addendum is one module. The larger gear has 50 teeth and the pinion has 13 teeth. Does interference occur? If it occurs, to what value should the pressure angle be changed to eliminate interference?

Solution $\varphi = 20°$; $T = 50$; $m = 10$ mm; $t = 13$;

Addendum = 1 m = 10 mm

$$R = \frac{mT}{2} = \frac{10 \times 50}{2} = 250 \text{ mm};$$

$$R_a = 250 + 10 = 260 \text{ mm}$$

$$r = \frac{mt}{2} = \frac{10 \times 13}{2} = 65 \text{ mm}$$

Maximum addendum radius can also be found using the relation

$$R_{a\,\max} = R\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\varphi}$$

$$= 250\sqrt{1 + \frac{13}{50}\left(\frac{13}{50} + 2\right)\sin^2\varphi} = 258.45 \text{ mm}$$

The actual addendum radius R_a is more than the maximum value $R_{a\,\max}$, and therefore, interference occurs.

The new value of φ can be found by taking $R_{a\,\max}$ equal to R_a.

i.e., $$260 = \sqrt{(250\cos\varphi)^2 + (315\sin\varphi)^2}$$

or $$(260)^2 = (250)^2\cos^2\varphi + (315)^2(1 - \cos^2\varphi)$$
$$= (250)^2\cos^2\varphi + (315)^2 - (315)^2\cos^2\varphi$$

or $$\cos^2\varphi = \frac{(315)^2 - (260)^2}{(315)^2 - (250)^2} = 0.861$$

$$\cos\varphi = 0.928 \text{ or } \varphi = 21.88° \text{ or } 21°52'$$

Thus, if the pressure angle is increased to 21°52′, the interference is avoided.

The following data relate to two meshing involute gears: Number of teeth on the gear wheel = 60 ; pressure angle = 20º ; Gear ratio = 1.5; Speed of the gear wheel = 100 rpm; Module = 8 mm; The addendum on each wheel is such that the path of approach and the path of recess on each side are 40% of the maximum possible length each. Determine the addendum for the pinion and the gear and the length of the arc of contact.

Solution $R = \dfrac{mT}{2} = \dfrac{8 \times 60}{2} = 240$ mm;

$$r = \frac{mT}{2} = \frac{8 \times (60/1.5)}{2} = 160 \text{ mm}$$

Refer Fig. 10.24 and let the pinion be the driver.

Maximum possible length of path of approach = $r \sin \varphi$

Actual length of path of approach = $0.4 \times r \sin \varphi$

Similarly, actual length of path of recess = $0.4 R \sin \varphi$

Thus, we have

$$0.4 r \sin \varphi = \sqrt{R_a^2 - (R \cos \varphi)^2} - R \sin \varphi$$

$$0.4 \times 160 \sin 20° = \sqrt{R_a^2 - (240 \cos 20°)^2} - 240 \sin 20°$$

$$R_a^2 - 50\,862 = 10\,809.8$$

$$R_a^2 = 61671.8$$

$$R_a = 248.3 \text{ mm}$$

Addendum of the wheel = 248.3 – 240 = 8.3 mm

Also, $0.4 R \sin \varphi = \sqrt{r_a^2 - (r \cos \varphi)^2} - r \sin \varphi$

$$0.4 \times 240 \sin 20° = \sqrt{r_a^2 - (160 \cos 20°)^2} - 160 \sin 20°$$

or $\quad r_a^2 - 22\,605 = 7666$

or $\quad r_a^2 = 30\,271$

or $\quad r_a = 174$ mm

Addendum of the pinion = 174 – 160 = 14 mm

A pinion of 20º involute teeth rotating at 275 rpm meshes with a gear and provides a gear ratio of 1.8. The number of teeth on the pinion is 20 and the module is 8 mm. If the interference is just avoided, determine (i) the addenda on the wheel and the pinion (ii) the path of contact, and (iii) the maximum velocity of sliding on both sides of the pitch point.

Solution $\varphi = 20°$; $VR = 1.8$; $m = 8$ mm; $t = 20$; $G = 1.8$; $T = 20 \times 1.8 = 36$; $N = 275$ rpm

$$R = \frac{mT}{2} = \frac{8 \times 36}{2} = 144 \text{ mm}; \; r = \frac{144}{1.8} = 80 \text{ mm}$$

Maximum addendum of the wheel,

$$a_{w\max} = R\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2 \varphi} - 1\right]$$

$$= 144\left[\sqrt{1 + \frac{1}{1.8}\left(\frac{1}{1.8} + 2\right)\sin^2 20^\circ} - 1\right]$$

$$= 144\,(1.08 - 1) = 11.5 \text{ mm}$$

Maximum addendum of the pinion,

$$a_{p\max} = r\left[\sqrt{1 + G\,(G + 2)\sin^2 \varphi} - 1\right]$$

$$= 80\left[\sqrt{1 + 1.8\,(1.8 + 2)\sin^2 20^\circ} - 1\right] = 27.34 \text{ mm}$$

Path of contact when the interference is just avoided

= maximum length of path of approach + maximum length of path of recess

$= r \sin \varphi + R \sin \varphi = 80 \sin 20° + 144 \sin 20°$

$= 27.36 + 49.24 = 76.6$ mm

$$\omega_p = \frac{2\pi \times 275}{60} = 28.8 \text{ rad/s}; \; \omega_g = \frac{28.8}{1.8} = 16 \text{ rad/s}$$

Velocity of sliding on one side $= (\omega_p + \omega_g) \times$ Path of approach

$= (28.8 + 16) \times 27.36 = 1226$ mm/s or 1.226 m/s

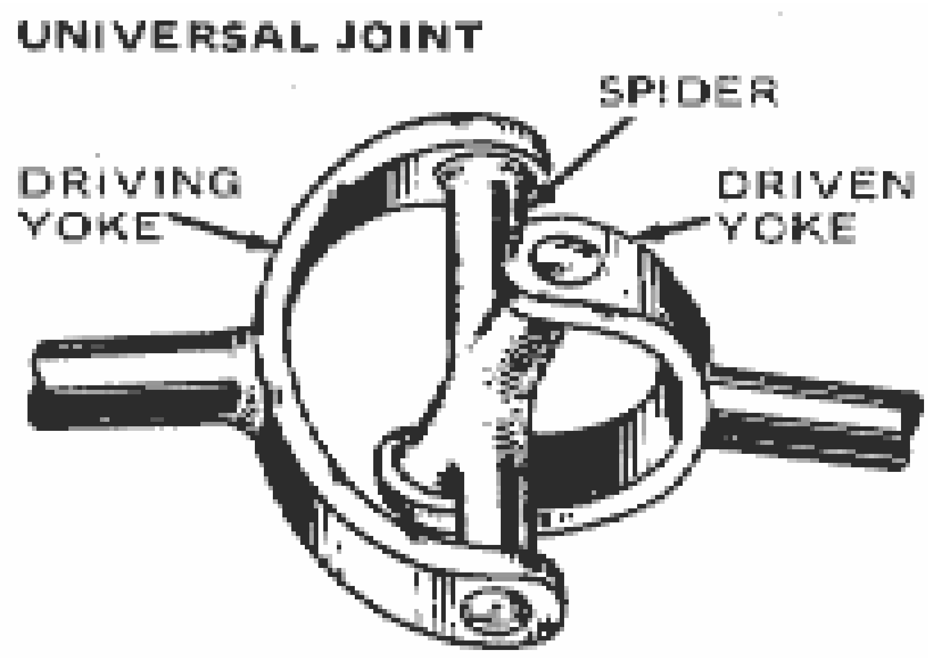
Velocity of sliding on other side $= (\omega_p + \omega_g) \times$ Path of recess

$= (28.8 + 16) \times 49.24 = 2206$ mm/s or 2.206 m/s

UNIVERSAL JOINT

- A **universal joint**, (**universal coupling**, **U-joint** or **Hooke's joint**) is a joint or coupling in a rigid rod that allows the rod to 'bend' in any direction, and is commonly used in shafts that transmit rotary motion.

- To give drive at varying angles (up to 20 degree)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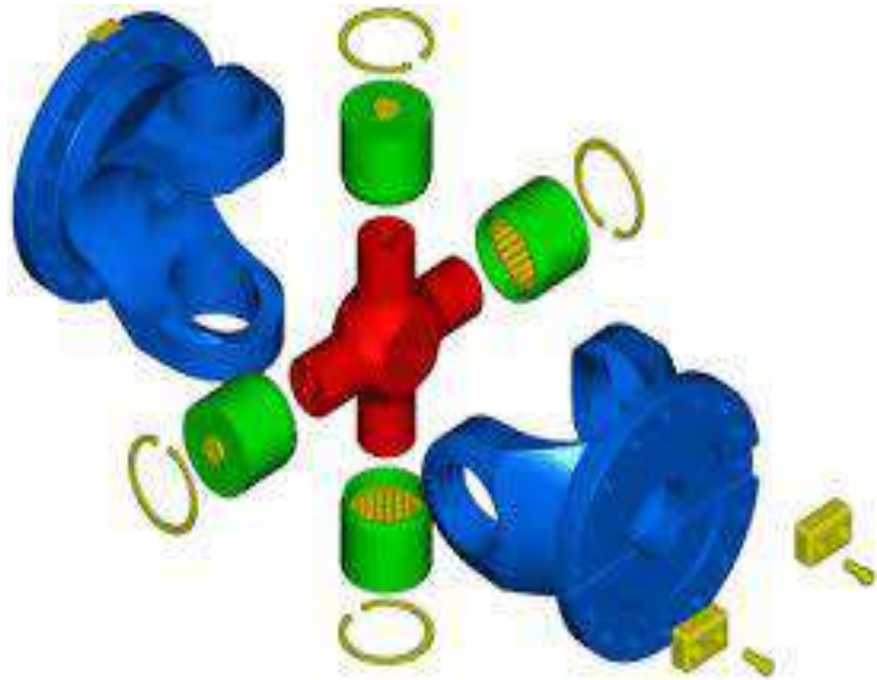
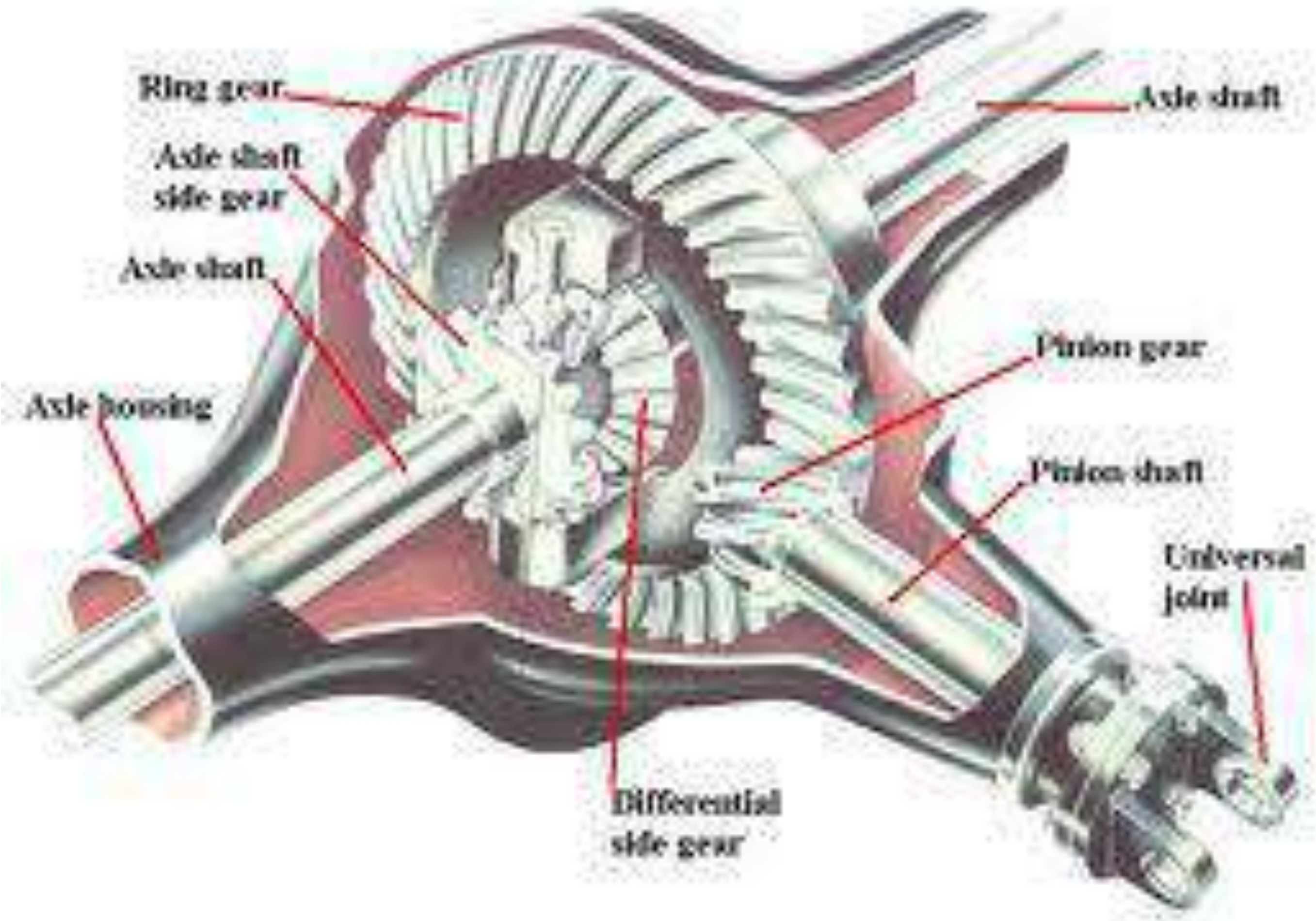


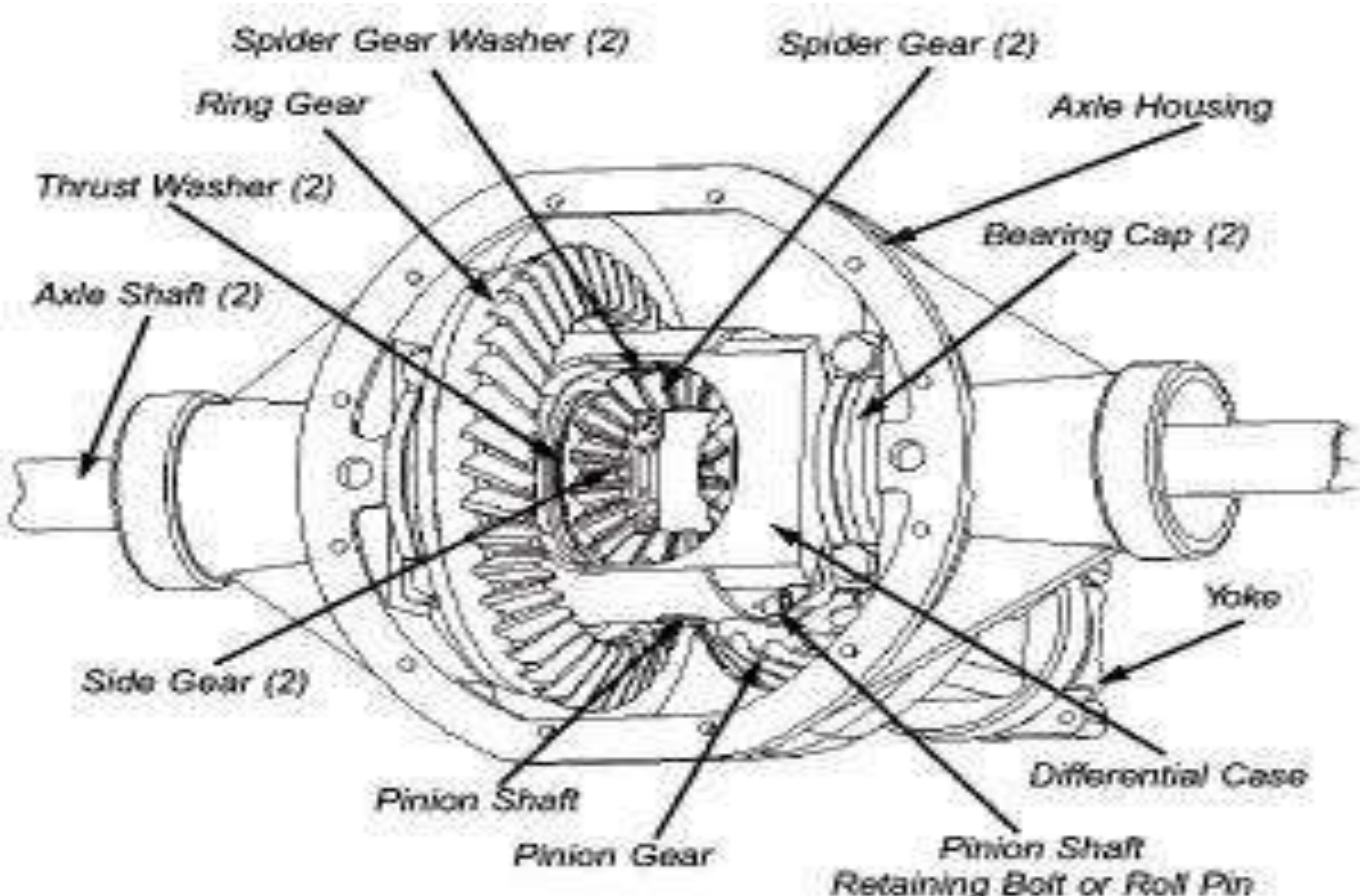
DIFFERENTIAL



Differential

- Differential is a very important part in a vehicle, as a component transfer the engine power is transmitted to the wheels. Engine power is transferred by a rear propeller shaft to wheel first changed direction by differential rotation are then referred to rear axle shafts after that to the rear wheels.

How It Works differential

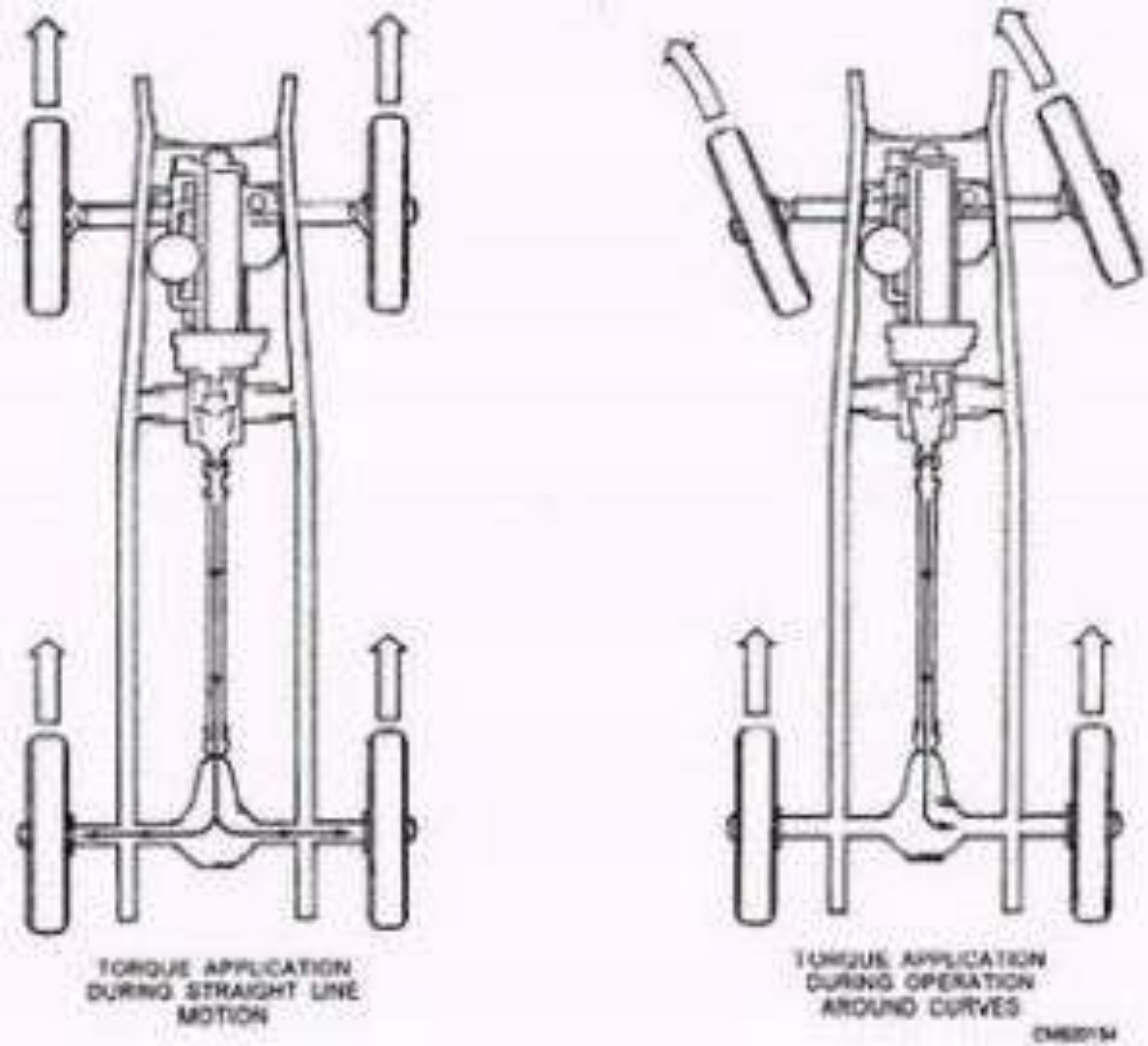


Main Parts of Differential

1. Bevel Pinion
2. Crown Wheel / Ring Gear
3. Half Axle
4. Sun Gear
5. Star Gear
6. Cage
7. Bearing

How It Works differential

- At the time of straight road.
  During the vehicle runs straight, the wheels of the rear axle will be screened by the drive pinion through the ring gear differential case, wheel-wheel differential gear pinion shaft, wheel-pinion differential gears, side gear teeth is not spinning, remain to be drawn into the ring gear rotation. Thus the spin on the wheel left and right alike.



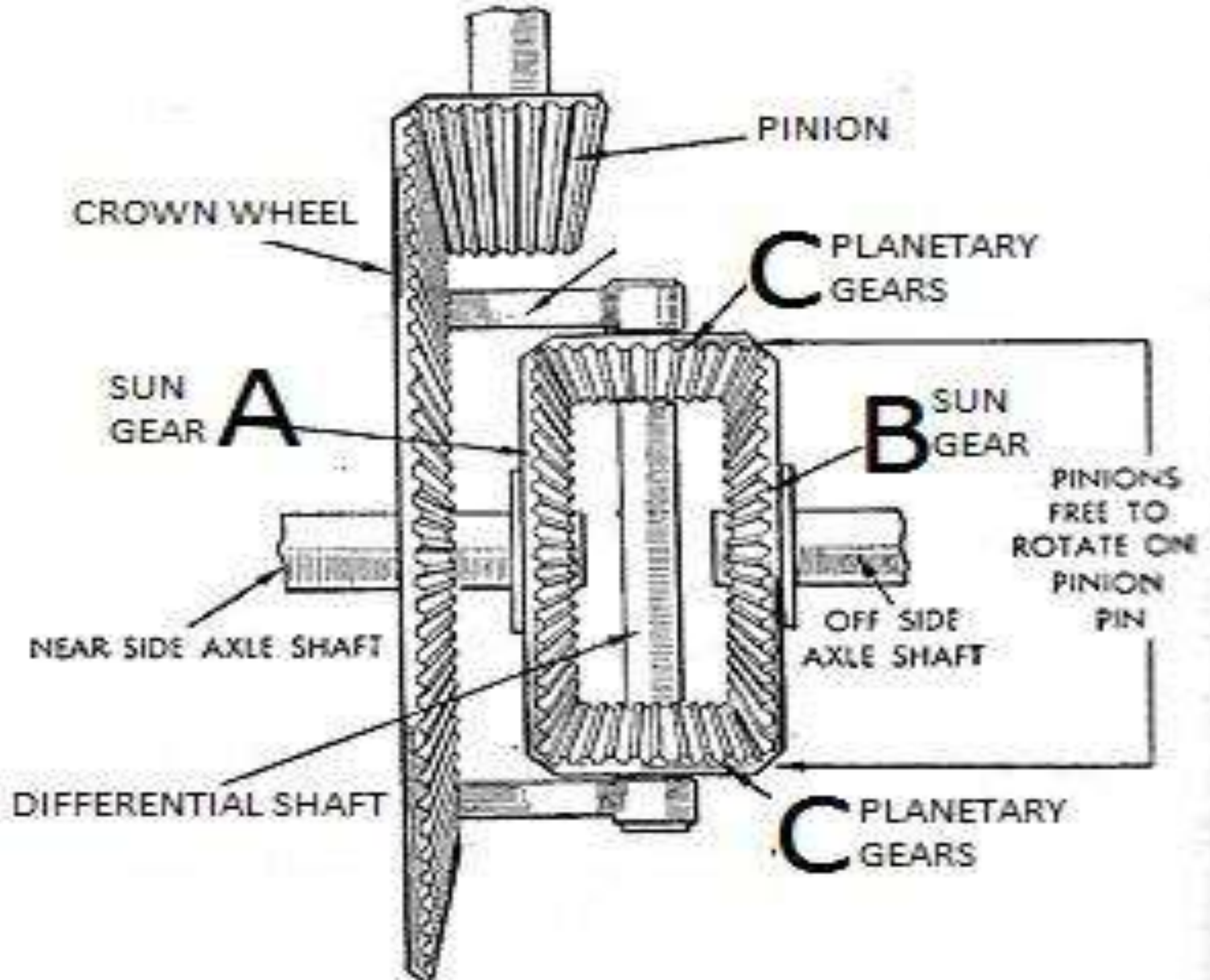
TORQUE APPLICATION DURING STRAIGHT LINE MOTION
TORQUE APPLICATION DURING OPERATION AROUND CURVES

Continue

- At the time of turning.
  At the time of vehicle turning left prisoners left wheel is bigger than the right wheel. If the differential case with the ring gear rotates the pinion will rotate on its axis and also the movement around the left side gear, so round the right hand side gear increases, the side where the number of revolutions of the gear which is 2 times round the ring gear. It can be said that the average second round gear is comparable with the rotary ring gear. as it should.

Working principle of differential

1. The basic principle of the differential gear unit can be understood by using equipment that consists of two gears pinion and rack.
2. Both rack can be moved in the vertical direction as far as the weight rack and slip resistance will be lifted simultaneously. Placed between the tooth pinion rack and pinion gear connected to the braces and can be moved by these braces.



PINION
CROWN WHEEL
C PLANETARY GEARS
SUN GEAR A
B SUN GEAR
PINIONS FREE TO ROTATE ON PINION PIN
NEAR SIDE AXLE SHAFT
OFF SIDE AXLE SHAFT
DIFFERENTIAL SHAFT
C PLANETARY GEARS

Continue……

1. When the same load “W” placed on each rack then braces (Shackle) is pulled up the second rack would be lifted at the same distance, this will prevent the pinion gear does not rotate.

2. But if a greater burden placed on the left rack and pinion buffer will then be drawn up along the gear rack rotates the load gets heavier, which is attributed to differences in prisoners who are given the pinion gear, so the smaller the burden will be lifted.

1. The raised rack spacing is proportional to the number of turns pinion gear. In other words that rack gets custody larger still and while prisoners who received a smaller load will move. This principle is used in the planning of differential gears.

References:

1.Theory of Machines, Rattan, Tata McGraw-Hill Education, 2009.

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