IGCSE - Extended Mathematics Transformation

Content:

- Reflection **(M)**: Reflect simple plane figures in horizontal or vertical lines;
- Rotation (**R**)**:** Rotate simple plane figures about the origin, vertices or midpoints of edges of the figures, through multiples of 90°;
- Enlargement **(E)** : Construct given translations and enlargements of simple plane figures;
- Shear (H) &Stretch (S)
- Their **combinations** (if M(a) = b and R(b) = c the notation RM(a) = c will be used;
- **Invariants** under these transformations may be assumed.) Identify and give precise descriptions of transformations connecting given figures.
- **Describe transformations** using co-ordinates and matrices (singular matrices are excluded).

Transformation:

- The word" **transform** "means "**to change**." In geometry, a transformation changes the position of a shape on a coordinate plane. That means a shape is moving from one place to another.
- The **original shape** of the object is called the **pre-image** and the **final shape** and position of the object is the **image** under the transformation.

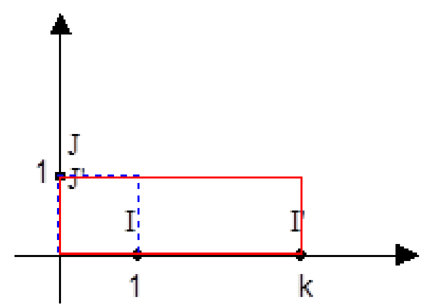
Isometry : Isometric transformation is a transformation that preserves **congruence**. In other words, a transformation in which the image and pre-image have the same side lengths and angle measurements. The following transformations maintain their mathematical congruence Reflection (Flip) ,Translation (Slide), Rotation (Turn).

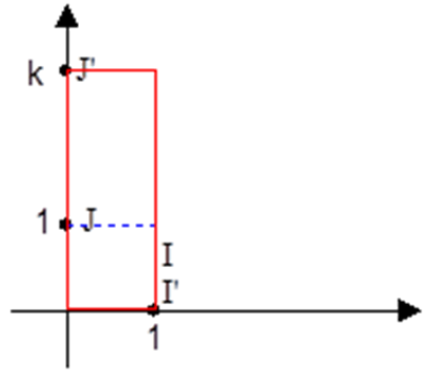
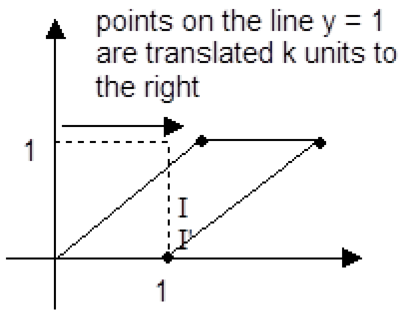
Transformation Matrix	Effect (Image)	Example
1. Identity Matrix	No Effect , Image Remains Same	
$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	This transformation matrix is the identity matrix. When multiplying by this matrix, the point matrix is unaffected and the new matrix is exactly the same as the point matrix.	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4x1)+(3x0) \\ (4x0)+(3x1) \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$
2. Reflection	Reflection in the X axis	
$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	This transformation matrix creates a reflection in the x-axis. When multiplying by this matrix, the x co-ordinate remains unchanged, but the y co-ordinate changes sign.	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4x1)+(3x0) \\ (4x0)+(3x-1) \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$

3. Reflection	Reflection in the Y axis	
$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	This transformation matrix creates a reflection in the y-axis. When multiplying by this matrix, the y co-ordiante remains unchanged, but the x co-ordinate changes sign.	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4x-1)+(3x0) \\ (4x0)+(3x1) \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$
4. Rotation	180 degrees Rotation	
$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	This transformation matrix creates a rotation of 180 degrees. When multiplying by this matrix, the point matrix is rotated 180 degrees around (0,0). This changes the sign of both the x and y co-ordinates.	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4x-1)+(3x0) \\ (4x0)+(3x-1) \end{bmatrix} = \begin{bmatrix} -4 \\ -3 \end{bmatrix}$
5. Reflection	Reflection on y = x line	
$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	This transformation matrix creates a reflection in the line y=x. When multiplying by this matrix, the x co-ordinate becomes the y co-ordinate and the y-ordinate becomes the x co-ordinate.	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4x0)+(3x1) \\ (4x1)+(3x0) \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

6. Rotation	90 degrees clock wise rotation	
$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	This transformation matrix rotates the point matrix 90 degrees clockwise. When multiplying by this matrix, the point matrix is rotated 90 degrees clockwise around (0,0).	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4x0)+(3x1) \\ (4x-1)+(3x0) \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$
7. Rotation	Anti-clockwise rotation 90 degrees	
$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	This transformation matrix rotates the point matrix 90 degrees anti-clockwise. When multiplying by this matrix, the point matrix is rotated 90 degrees anti-clockwise around (0,0).	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} \begin{bmatrix} (4x0)+(3x1) \\ (4x-1)+(3x0) \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$
8. Reflection	Reflection in the line y = -x	
$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$	This [illegible] a refl[illegible] $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ [illegible]ultiplying by t[illegible]eflected in the [illegible] both co-ord[illegible]es.	$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4x0)+(3x1) \\ (4x-1)+(3x0) \end{bmatrix} = \begin{bmatrix} -3 \\ -4 \end{bmatrix}$

9 .Enlargement	Enlargement of scale factor	
$\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$	This transformation matrix is the identity matrix multiplied by the scalar 6. When multiplying by this matrix, the point matrix is enlarged by a factor of 6 in the x and y directions.	$\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4x6) + (3x0) \\ (4x0) + (3x6) \end{bmatrix} = \begin{bmatrix} 24 \\ 18 \end{bmatrix}$
10.Enlargment		
$\begin{bmatrix} 7 & 0 \\ 0 & 0 \end{bmatrix}$	This transformation matrix is the identity matrix but $T_{1,1}$has been enlarged by a factor c 7 and $T_{2,2}$ has been enlarged by a factor of 0. When multiplying by this matrix, the x co-ordinate is enlarged by a factor of 7, whilst the y co-ordinate is enlarged by a factor of 0.	$\begin{bmatrix} 7 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4x7) + (3x0) \\ (4x0) + (3x0) \end{bmatrix} = \begin{bmatrix} 28 \\ 0 \end{bmatrix}$
11. Enlargment		
$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$	This transformation matrix is the identity matrix but $T_{1,1}$has been enlarged by a factor c a and $T_{2,2}$ has been enlarged by a factor of b When multiplying by this matrix, the x co-ordinate is enlarged by a factor of a, whilst the y co-ordinate is enlarged by a factor of b.	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4xa) + (3x0) \\ (4x0) + (3xb) \end{bmatrix} = \begin{bmatrix} 4a \\ 3b \end{bmatrix}$
12. Enlargment		

$\begin{bmatrix} 0 & -5 \\ 7 & 0 \end{bmatrix}$	This transformation matrix creates a rotation and an enlargement. When multiplying by th matrix, the point matrix is rotated 90 degree anticlockwise around (0,0), whilst the x xo-ordinate of the new point matrix is enlarged by a factor of -5 and the y co-ordinate of the new point matrix is enlarged by a factor of 7.	$\begin{bmatrix} 0 & -5 \\ 7 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} (4x0)+(3x-5) \\ (4x7)+(3x0) \end{bmatrix} = \begin{bmatrix} -15 \\ 28 \end{bmatrix}$
13. Streatch	Horizontal axis stretch	
$\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$	Stretch, scale factor k parallel to the x- axis	

13. Streatch	Vertical axis stretch	
$\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$	Stretch, scale factor k parallel to the y-axis	
14. Shear	Shear parallel to the x-axis	
$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$	The matrix corresponds to a shear parallel to the x-axis. Points on the x-axis do not move, whilst points on the line y = 1 are translated k units to the right.	

15. Shear	Shear parallel to y axis	
$\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$	The matrix corresponds to a shear parallel to the y-axis. Points on the y-axis do not move, whilst points on the line x = 1 are translated k units up.