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Handbook for Spoken Mathematics

(Larry's Speakeasy)

Lawrence A. Chang, Ph.D.

With assistance from
Carol M. White
Lila Abrahamson

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Handbook for Spoken Mathematics

SECTION I — INTRODUCTION

This handbook answers some of the needs of the many people who have to deal with spoken mathematics, yet have insufficient background to know the correct verbal expression for the written symbolic one. Mathematical material is primarily presented visually, and when this material is presented orally, it can be ambiguous. While the parsing of a written expression is clear and well-defined, when it is spoken this clarity may disappear. For example, "One plus two over three plus four" can represent the following four numbers, depending on the parsing of the expression: 3/7, 1 2/7, 5, 5 2/3. However, when the corresponding written expression is seen, there is little doubt which of the four numbers it represents. When reading mathematics orally, such problems are frequently encountered. Of course, the written expression may always be read symbol by symbol, but if the expression is long or there are a cluster of expressions, it can be very tedious and hard to understand. Thus, whenever possible, one wishes to have the written expression spoken in a way that is interest retaining and easy to understand.

In an attempt to alleviate problems such as these, this handbook has been compiled to establish some consistent and well-defined ways of uttering mathematical expressions so that listeners will receive clear, unambiguous, and well-pronounced representations of the subject.

Some of the people who will benefit from this handbook are: 1) those who read mathematics orally and have insufficient background in the subject, and their listeners; 2) those interested in voice synthesis for the computer, particularly those who deal with spoken symbolic expressions; and 3) those technical writers and transcribers who may need to verbalize mathematics.

This edition of the handbook is a working one, and it is hoped that the people who use it will add to and refine it. The choice of material and its ordering are my own preferences, and, as such, they reflect my biases. A goal of the handbook is to establish a standard where no standard has existed, so far as I know. However, this standard represents only one of many possibilities. As a blind person, I have learned mathematics by means of others reading the material to me; so my preferences are a result of direct experience.

This handbook is organized as follows: In Section II the various types of alphabets used in mathematics are listed. Section III lists the basic symbols used in mathematics, along with their verbalizations. Sections IV–XI list the expressions used in some of the more common branches of mathematics, along with their verbalizations. Section XII contains some suggestions on how to and how not to describe diagrams.

To use this handbook efficiently, it is suggested that you look over Sections II and III on alphabets and basic symbols. Next, establish which section most closely relates to the subject matter at hand. There may also be material in other sections that you can use if you cannot find what you need in the related section. In many sections, more than one choice for a given expression is offered to the user. Once the choice has been made, the reader should use it consistently throughout the text. If you encounter an expression that is not included in the guide, read the expression literally, that is, read it from left to right, symbol by symbol.

For those who are interested in speech synthesis and speech recognition for the computer, this handbook may provide some basic ideas and suggestions regarding the formulation of spoken mathematics. With speech synthesis, when the computer reads a file containing many mathematical expressions, the speech synthesizer will speak the expressions symbol by symbol. As we have pointed out before, this process can be tedious and hard to understand. A program that could translate the mathematical expressions from the symbol to symbol form into a spoken form that is more intelligible can ease the task for those who use synthetic speech. On the other hand, if one wishes to communicate mathematical expressions to the computer by voice, a program that will translate spoken expressions of mathematics into written expressions with the correct parsing is essential. This handbook provides a basis for writing these programs, both for speech synthesis and voice recognition, by giving examples of written mathematical expressions followed by the word for word spoken form of the same expressions. An example where these ideas are of particular relevance is the voice input and output of computer programs that manipulate symbolic expressions, because both the input and output of the program are mathematical expressions.

I would like to thank the Office of Equal Opportunity of the Lawrence Livermore National Laboratory for their support in bringing this handbook into fruition. Thanks also go to my wife for her untiring help and to my friends and colleagues at the Lab for their assistance.

October, 1983 Lawrence Chang

SECTION II — ALPHABETS

Roman Alphabet

Read capital or upper-case letters as capital *lettername* or cap *lettername*. Read small or lower-case letters as small *lettername*.

Capital or upper-case	**Small or lower-case**
A	a
B	b
C	c
D	d
E	e
F	f
G	g
H	h
I	i
J	j
K	k
L	l
M	m
N	n
O	o
P	p
Q	q
R	r
S	s
T	t
U	u
V	v
W	w
X	x
Y	y
Z	z

Types of Roman Alphabets

Italic

Read capital or upper-case letters as italic capital *lettername*. Read small or lower-case letters as italic *lettername*.

Capital or upper-case	Small or lower-case
A	*a*
B	*b*
C	*c*
D	*d*
E	*e*
F	*f*
G	*g*
H	*h*
I	*i*
J	*j*
K	*k*
L	*l*
M	*m*
N	*n*
O	*o*
P	*p*
Q	*q*
R	*r*
S	*s*
T	*t*
U	*u*
V	*v*
W	*w*
X	*x*
Y	*y*
Z	*z*

Boldface

Read capital or upper-case letters as boldface capital *lettername*. Read small or lower-case letters as boldface *lettername*.

Capital or upper-case	Small or lower-case
A	**a**
B	**b**
C	**c**
D	**d**
E	**e**
F	**f**
G	**g**
H	**h**
I	**i**
J	**j**
K	**k**
L	**l**
M	**m**
N	**n**
O	**o**
P	**p**
Q	**q**
R	**r**
S	**s**
T	**t**
U	**u**
V	**v**
W	**w**
X	**x**
Y	**y**
Z	**z**

Gothic or Old English

Read capital or upper-case letters as Gothic capital *lettername*. Read small or lower-case letters as Gothic *lettername*.

Script

Read capital or upper-case letters as script capital *lettername*. Read small or lower-case letters as script *lettername*.

Gothic: Capital or upper-case	Corresponding Roman-letter	Gothic: Small or lower-case	Script: Capital or upper-case	Script: Small or lower-case
𝔄	A	𝔞	𝒜	𝒶
𝔅	B	𝔟	ℬ	𝒷
ℭ	C	𝔠	𝒞	𝒸
𝔇	D	𝔡	𝒟	𝒹
𝔈	E	𝔢	ℰ	ℯ
𝔉	F	𝔣	ℱ	𝒻
𝔊	G	𝔤	𝒢	ℊ
ℌ	H	𝔥	ℋ	𝒽
ℑ	I	𝔦	ℐ	𝒾
𝔍	J	𝔧	𝒥	𝒿
𝔎	K	𝔨	𝒦	𝓀
𝔏	L	𝔩	ℒ	𝓁
𝔐	M	𝔪	ℳ	𝓂
𝔑	N	𝔫	𝒩	𝓃
𝔒	O	𝔬	𝒪	ℴ
𝔓	P	𝔭	𝒫	𝓅
𝔔	Q	𝔮	𝒬	𝓆
ℜ	R	𝔯	ℛ	𝓇
𝔖	S	𝔰	𝒮	𝓈
𝔗	T	𝔱	𝒯	𝓉
𝔘	U	𝔲	𝒰	𝓊
𝔙	V	𝔳	𝒱	𝓋
𝔚	W	𝔴	𝒲	𝓌
𝔛	X	𝔵	𝒳	𝓍
𝔜	Y	𝔶	𝒴	𝓎
ℨ	Z	𝔷	𝒵	𝓏

Greek Alphabet

Read capital or upper-case letters as capital *lettername* or cap *lettername*. Read small or lower-case letters as small *lettername*.

Capital	Small	Name	Pronunciation	Corresponding Roman letter
Α	α	alpha	aĺ fuh	a
Β	β	beta	baý tuh	b
Γ	γ	gamma	gaḿ muh	g
Δ	δ	delta	deĺ tuh	d
Ε	ϵ	epsilon	eṕ suh lon	e
Ζ	ζ	zeta	zaý tuh	z
Η	η	eta	aý tuh	ē
Θ	θ	theta	thaý tuh	th
Ι	ι	iota	i oh tuh	i
Κ	κ	kappa	kaṕ puh	k
Λ	λ	lambda	laḿ duh	l
Μ	μ	mu	mew	m
Ν	ν	nu	new	n
Ξ	ξ	xi	zigh or ksigh	x
Ο	o	omicron	oḿ uh cron	o
Π	π	pi	pie	p
Ρ	ρ	rho	row (as in rowboat)	r, rh
Σ	σ, s	sigma	siǵ muh	s
Τ	τ	tau	tow (rhymes with cow)	t
Υ	υ	upsilon	uṕ suh lon	y, u
Φ	ϕ	phi	fi (rhymes with hi)	ph
Χ	χ	chi	ki (rhymes with hi)	ch
Ψ	ψ	psi	sigh or psigh	ps
Ω	ω	omega	oh meǵ uh	ō

SECTION III — BASIC SYMBOLS

Symbol		Speak	Notes
$+$	or	plus positive	
$-$	or	minus negative	
$\times$, $\cdot$	or	multiplies times	
$\div$, $/$		divided by	
$\lvert\ \rvert$		absolute value	
$\mid$		divides	
$\pm$		plus or minus	
$\mp$		minus or plus	
$\oplus$		circle plus	
$\otimes$		circle cross	
$=$	or	equals equal to	
$\neq$	or	does not equal not equal to	
$\equiv$		identical to	
$\not\equiv$		not identical to	
$\approx$, $\simeq$		approximately equal to	
$\sim$		equivalent to	
$\lesssim$		approximately equal but less than	
$\leq$		less than or equal to	

Symbol	Speak	Notes
$<$	less than	
$\ll$	much less than	
$\nless$	not less than	
$\gtrsim$	approximately equal but greater than	
$\geq$	greater than or equal to	
$>$	greater than	
$\gg$	much greater than	
$\ngtr$	not greater than	
(	open parenthesis or left parenthesis	
)	closed parenthesis or right parenthesis	
[	open bracket or left bracket	
]	closed bracket or right bracket	
{	open brace or left brace	
}	closed brace or right brace	
——	vinculum	Example: $a-\overline{b-c}$ is read as a minus vinculum b minus c.

In the next examples, the letter a is used with the symbol for clarity — the letter a is a dummy variable.

Symbol		Speak	Notes
$\|a\|$		absolute value of a	In this case a is any real number.
a'		a prime	If a is an angle, a' is read as a minutes.
a''		a double prime	If a is an angle, a'' is read as a seconds.
$a^{[n]}$		a with n primes	
a^n	or	a superscript n a to the n	
$\bar{a}$		a bar	
a^*	or	a star a super asterisk	
a_n	or	a subscript n a sub n	When n = 0, a_n may be read as a naught.
$\sqrt{}$		radical sign	
$\sqrt{a}$		square root of a	
$\sqrt[3]{a}$		cube root of a	
$\sqrt[n]{a}$		nth root of a	
ϕ	or	zero null set	to distinguish from the letter o

Symbol		Speak	Notes
ƶ		the letter z	to distinguish from 2
ℵ		ah́ lef	aleph, the first letter of the Hebrew alphabet
$\prod$		product	Example: $\prod_{i=1}^{n}$ is read product from i=1 to n.
$\sum$		summation	Example: $\sum_{i=1}^{n}$ is read summation from i=1 to n.
$\int$		integral	Example: $\int_{a}^{b}$ is read integral from a to b.
d/dx	or or	d over d x d by d x the derivative with respect to x	
∂/∂x	or	the partial derivative with respect to x partial over partial x	
∇		del	
!		factorial	Example: n! is read n factorial.
*	or	star asterisk	
&	or	ampersand and	
†		dagger	

Symbol	Speak	Notes
††	double dagger	
a°	a degrees	
$a^{(r)}$	a radians	
§	section	
$\parallel$	parallel	
$\perp$	perpendicular	
$<$, ⦣, $\angle$	angle	
∟	right angle	
$\triangle$	triangle	However, Δx is read delta x or increment x.
▱	parallelogram	
$\square$	square	
○	circle	
⬭	ellipse or oval	
⌒	arc	Example: $\overset{\frown}{AB}$ is read arc ab.
$\therefore$	therefore	
$\because$	since or because	
...	dot, dot, dot or ellipsis or etc.	
:	is to or ratio	

Symbol		Speak	Notes
::	or	as proportion	
∧	or	hat circumflex	Example: â is read a hat or a circumflex.
..		oom laut	Example: ä is read a oom laut.
`		accent grave	Example: à is read a accent grave.
´		accent acute	Example: á is read a accent acute.
~		til duh	Example: ñ is read n til duh.
^		caret	
→	or	arrow to the right approaches	
←	or	arrow to the left withdraws	
↑	or	arrow pointing up upward arrow	
↓	or	arrow pointing down downward arrow	
$\vec{a}$		vector a	
∪ V		union	

Symbol		Speak	Notes
$\cap$		intersection	
$\subset$	or	contained in subset of	
$\supset$		contains	
$\Longrightarrow$		implies	
$\Leftrightarrow$		equivalent to	
iff		if and only if	
$\exists$	or	there exists there is	
$\forall$		for every	
$\ni$		such that	
%		percent	
$		dollars	
¢		cents	
@		at	
#	or or	sharp pound sign number sign	
$\flat$		flat	
$\propto$		proportional to	
∞		infinity	

SECTION IV — ALGEBRA

The small letters of the alphabet, a, b, c, d, ..., may be any numbers.

Expression		Speak	Notes
$a + b$		a plus b	
$a + b + c$		a plus b plus c	
$a - b$		a minus b	
$-a - b$		minus a minus b	
$a + b - c$		a plus b minus c	
$a - b - c$		a minus b minus c	
$a - (b + c)$		a minus the sum b plus c	
	or	a minus the quantity b plus c	
	or	a minus open parenthesis b plus c close parenthesis	
$a - (b - c)$		a minus the difference b minus c	
	or	a minus the quantity b minus c	
	or	a minus open parenthesis b minus c close parenthesis	
$a - (-b - c)$		a minus the quantity minus b minus c	
	or	a minus open parenthesis minus b minus c close parenthesis	
$a - (b + c) - d$		a minus the quantity b plus c end of quantity minus d	
	or	a minus open parenthesis b plus c close parenthesis minus d	
$a - b - (c - d)$		a minus b minus the difference c minus d	
	or	a minus b minus the quantity c minus d	
	or	a minus b minus open parenthesis c minus d close parenthesis	

Expression	Speak	Notes
$a \times b$	a times b or a cross b or the product of a and b or a multiplied by b	
$a \cdot b$	a times b or a dot b or the product of a and b or a multiplied by b	
ab	a b or a times b or the product of a and b or a multiplied by b	
$a \cdot -b$	a times minus b	
$ab + c$	a b plus c	
$a (b + c)$	a times the sum b plus c or a times the quantity b plus c or a times open parenthesis b plus c close parenthesis	
$a (b + c) + d$	a times the quantity b plus c end of quantity plus d or a open parenthesis b plus c close parenthesis plus d	
$ab - c$	a b minus c	
$a (b - c)$	a times the difference b minus c or a times the quantity b minus c or a open parenthesis b minus c close parenthesis	
$a (-b - c)$	a times the quantity minus b minus c or a open parenthesis minus b minus c close parenthesis	

Expression		Speak	Notes
$a(b - c + d)$		a times the quantity b minus c plus d	
	or	a open parenthesis b minus c plus d close parenthesis	
$ab + cd$		a b plus c d	
$ad - bc$		a d minus b c	
$a(b + c) - e(f - g)$		a times the quantity b plus c end of quantity minus e times the quantity f minus g	
	or	a open parenthesis b plus c close parenthesis minus e open parenthesis f minus g close parenthesis	
$a[b + c - e(f - g)]$		a times the quantity b plus c minus the product e times the difference f minus g end of quantity	
	or	a open bracket b plus c minus e open parenthesis f minus g close parenthesis, close bracket	
$(a + b)(c + d)$		the sum a plus b times the sum c plus d	
	or	the product of the sum a plus b and the sum c plus d	
	or	open parenthesis a plus b close parenthesis open parenthesis c plus d close parenthesis	
$\frac{1}{2}$		one half	
	or	one over two	
$\frac{1}{3}$		one third	
	or	one over three	
$\frac{1}{n}$		one over n	

Expression	Speak	Notes
$\frac{a}{d}$, a/d, $a \div d$	a over d or a divided by d or the ratio of a to d	
$\frac{a+b}{d}$	the fraction, the numerator is a plus b, the denominator is d or the quantity a plus b divided by d	
$a + \frac{b}{d}$	a plus the fraction b over d	
$a + \frac{b}{c+d}$	a plus the fraction, the numerator is b and the denominator is c plus d or a plus the fraction b divided by the quantity c plus d	
$\frac{a+b}{c} + d$	the quantity a plus b over c, that fraction plus d	
$a + \frac{b}{c} + d$	a plus the fraction b over c, that fraction plus d	
$\frac{a}{b} + \frac{c}{d}$	the fraction a over b plus the fraction c over d	
$\frac{a}{b + \frac{c}{d}}$	the fraction, the numerator is a, the denominator is the sum b plus the fraction c over d	
$\frac{\frac{a}{b}}{d}$	the fraction, the numerator is the fraction a over b, the denominator is d or a over b, that fraction divided by d	

Expression	Speak	Notes
$\dfrac{a}{\dfrac{c}{d}}$	a divided by the fraction c over d	
$\dfrac{\dfrac{a}{b}}{\dfrac{c}{d}}$	the fraction, the numerator is the fraction a over b, the denominator is the fraction c over d or the fraction a over b divided by the fraction c over d	
$\dfrac{\dfrac{a+b}{c}}{d}$	the fraction, the numerator is the quantity a plus b over c, the denominator is d or the quantity a plus b over c, that fraction divided by d	
$\frac{c}{d}(a+b)$	the fraction c over d times the sum a plus b	
$\dfrac{a}{\dfrac{b}{c+d}}$	a divided by the fraction b over the quantity c plus d	
$a\left(b+\frac{c}{d}\right)$	a times the sum b plus the fraction c over d	
$a+\cfrac{b}{a+\cfrac{b}{a+\cfrac{b}{a+\cfrac{b}{\vdots}}}}$	the continued fraction: a plus the fraction b divided by the sum a plus the fraction b divided by the sum a plus the fraction b divided by the sum a plus the fraction b divided by dot dot dot	

Expression	Speak	Notes
$ay + bx + c = 0$	a y plus b x plus c equals zero	linear equation
$y = mx + b$	y equals m x plus b	
$y = ax^2 + bx + c$	y equals a x squared plus b x plus c	quadratic equation
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	x equals minus b plus or minus the square root of the difference b squared minus 4 a c, that whole quantity divided by 2 a or x equals the fraction, the numerator is minus b plus or minus square root of the difference b squared minus 4 a c, the denominator is 2 a	
$x^2 + y^2 = r^2$	x squared plus y squared equals r squared	circle with radius r, center at origin
$y = \pm \sqrt{r^2 - x^2}$	y equals plus or minus square root of the difference r squared minus x squared	respectively, upper or lower semicircle with radius r, center at origin
$(x - h)^2 + (y - k)^2 = r^2$	the difference x minus h squared plus the difference y minus k squared equals r squared or the quantity x minus h squared plus the quantity y minus k squared equals r squared	
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	the fraction x squared over a squared plus the fraction y squared over b squared equals 1	ellipse
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	the fraction x squared over a squared minus the fraction y squared over b squared equals 1	hyperbola

Expression		Speak	Notes
$ax^2 + bxy + cy^2 + dx + ey + f = 0$		a x squared plus b x y plus c y squared plus d x plus e y plus f equals zero	the conics
a^x	or	a to the x a raised to the x power	
e^{x+y}	or	e to the quantity x plus y power e raised to the x plus y power	
$e^x + y$	or	the sum of e to the x and y e to the x power plus y	
$e^x e^y$		the product of e to the x power and e to the y power	
e^{xa^y}	or	e raised to the x times a to the y power e raised to the product of x and a to the y	
$e^x y$	or	the product of e to the x power and y e raised to the x power times y	
$e^{i2\pi z}$	or	e to the quantity i 2 pi z power e raised to the i 2 pi z power	
$\log_b a$		log to the base b of a	
$\log_{10} 3 \cdot 4$		log to the base 10 of the product 3 times 4	
$\log_e \frac{2}{5}$	or	log to the base e of the fraction 2 over 5 log to the base e of the ratio 2 to 5	
$\ln x$	or	the natural log of x l n of x	

Expression		Speak	Notes
$a_1 + a_2 + \cdots + a_n$		a sub 1 plus a sub 2 plus dot dot dot plus a sub n	
	or	a sub 1 plus a sub 2 plus ellipsis plus a sub n	
$a_1 \cdot a_2 \cdot \ldots \cdot a_n$		a sub 1 times a sub 2 times dot dot dot times a sub n	
	or	a sub 1 times a sub 2 times ellipsis times a sub n	
$a_1 \cdot b_1 + a_2 \cdot b_2 + \cdots + a_n \cdot b_n$		a sub 1 times b sub 1 plus a sub 2 times b sub 2 plus dot dot dot plus a sub n times b sub n	
	or	a sub 1 times b sub 1 plus a sub 2 times b sub 2 plus ellipsis plus a sub n times b sub n	
$p(x)$		p of x	in algebra
$p(x) = 3x^2 + 2x - 4$		p of x equals 3 x squared plus 2 x minus 4	
$q(x) = x^3 - 8$		q of x equals x cubed minus 8	
$p(x) = a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n$		p of x equals a sub zero x to the n plus a sub 1 x to the n minus 1 plus dot dot dot plus a sub n minus 1 x plus a sub n	polynomial of n degree
	or	p of x equals a sub zero times x raised to the n power plus a sub 1 times x to the n minus 1 power plus dot dot dot plus a sub the quantity n minus 1 times x plus a sub n.	for extra clarity, when in doubt

SECTION V - TRIGONOMETRIC AND HYPERBOLIC EXPRESSIONS

The Greek letter θ (theta) will be used in this section to denote an angle in degrees or radians.

Expression	Speak	Notes
$\theta°$	theta degrees	
θ'	theta minutes	
θ''	theta seconds	
s.a.s.	side angle side	
s.s.s.	side side side	

The six basic trigonometric functions are:

Function		Speak	Notes
$\sin\theta$	or	sine of theta sine theta	
$\cos\theta$	or	co sine of theta co sine theta	
$\tan\theta$	or	tangent of theta tangent theta	
$\cot\theta$	or	co tangent of theta co tangent theta	
$\sec\theta$	or	see cant of theta see cant theta	
$\csc\theta$	or	co see cant of theta co see cant theta	

Other functions are:

Function		Speak	Notes
$\sin^2 x$		sine squared x	
$\cos^2 x$		co sine squared x	
$\tan^2 x$		tangent squared x	
$\cot^2 x$		co tangent squared x	
$\sec^2 x$		see cant squared x	
$\csc^2 x$		co see cant squared x	
$\sinh \theta$	or	hyperbolic sine theta sinch theta	
$\cosh \theta$	or	hyperbolic co sine theta cosh theta	
$\tanh \theta$	or	hyperbolic tangent theta tange theta	
$\coth \theta$		hyperbolic co tangent theta	
$\text{sech}\ \theta$		hyperbolic see cant theta	
$\text{csch}\ \theta$		hyperbolic co see cant theta	
$\left.\begin{matrix}\text{arc sin } x\\ \sin^{-1} x\end{matrix}\right\}$	or or or	arc sine x inverse sine x anti sine x sine to the minus 1 of x	The negative exponent *does not* mean the reciprocal of the function n nor $\frac{1}{\text{the function}}$
$\left.\begin{matrix}\text{arc cos } x\\ \cos^{-1} x\end{matrix}\right\}$	or or or	arc co sine x inverse co sine x anti co sine x co sine to the minus 1 of x	

Function	Speak	Notes
arc tan x $\tan^{-1}x$	arc tangent x or inverse tangent x or anti tangent x or tangent to the minus 1 of x	
arc cot x $\cot^{-1}x$	arc co tangent x or inverse co tangent x or anti co tangent x or co tangent to the minus 1 of x	
arc sec x $\sec^{-1}x$	arc see cant x or inverse see cant x or anti see cant x or see cant to the minus 1 of x	
arc csc x $\csc^{-1}x$	arc co see cant x or inverse co see cant x or anti co see cant x or co see cant to the minus 1 of x	
arc sinh x $\sinh^{-1}x$	arc hyperbolic sine of x or arc sinch x or inverse hyperbolic sine of x or anti hyperbolic sine of x	
arc cosh x $\cosh^{-1}x$	arc hyperbolic co sine of x or arc cosh x or inverse hyperbolic co sine of x or anti hyperbolic co sine of x	

Function	Speak	Notes
arc tanh x $\tanh^{-1}x$	arc hyperbolic tangent of x or arc tange x or inverse hyperbolic tangent of x or anti hyperbolic tangent of x	
arc coth x $\coth^{-1}x$	arc hyperbolic co tangent of x or inverse hyperbolic co tangent of x or anti hyperbolic co tangent of x	
arc sech x $\text{sech}^{-1}x$	arc hyperbolic see cant of x or inverse hyperbolic see cant of x or anti hyperbolic see cant of x	
arc csch x $\text{csch}^{-1}x$	arc hyperbolic co see cant of x or inverse hyperbolic co see cant of x or anti hyperbolic co see cant of x	

The following expressions can be used for any of the six trigonometric functions: sine, cosine, tangent, cotangent, secant, cosecant. In the examples that follow, sine will be used.

Function	Speak	Notes
$\sin\theta + x$	sine of theta, that quantity plus x	
$\sin(\theta + \omega)$	sine of the sum theta plus omega or sine of the quantity theta plus omega	
$(\sin\theta)\,x$	sine theta times x	
$\sin(\theta\omega)$	sine of the product theta omega	
$(\sin\theta^2)\,x$	sine of theta squared, that quantity times x	
$\sin^2\theta\cos\theta$	sine squared theta times co sine theta	
$\sin\theta\cos\theta$	sine of theta times co sine of theta	
$\sin(\theta\cos\theta)$	sine of the product theta times co sine theta	

SECTION VI — LOGIC AND SET THEORY

Expression	Speak	Notes
	therefore	
∍	such that	
$\sim p$ $-p$ $\bar{p}$ p'	not p	The reader must be careful to differentiate between $\tilde{p}$ (p tilde) and $\sim p$ (not p).
$p \wedge q$ $p \cdot q$ $p \,\&\, q$	both p and q or p and q	
$p \vee q$ $p \text{ v } q$	at least one of p and q or p or q	
$p \mid q$ p / q	not both p and q or not p or not q	
$p \downarrow q$ $p \triangle q$	neither p nor q	
$p \therefore q$ $p < q$	if p, then q or p only if q	
$\Leftrightarrow$ $\cdot\!:\; :\!\cdot$ $\equiv$ $\sim$ iff	if and only if	
V **V** **I** $\mid$	universal class	
ϕ Λ $\boldsymbol{\Lambda}$ 0	null class	

Expression		Speak	Notes
(x) A_x $\forall_x$		for all x	
$A_{x,y,\cdots}$ $\forall_{x,y,\cdots}$	or	for all x, y, ellipsis for all x, y, dot dot dot	
$\exists$		there exists	
$(\exists x)$ (Ex) Σ_x		there is an x such that	
$E_{x,\ y,\ \ldots}$		there exist x, y, dot dot dot such that	
E_x $\mathbf{C}_x$		the class of all objects x that satisfy the condition	Example: $E_x(x - a) < 0$ is read the class of all objects x that satisfy the condition the quantity x minus a is less than zero.
$[x \mid s(x)]$ $[x : s(x)]$		the class of all objects x which satisfy s of x	

Note: In the following expressions the capital letters M, N, and P denote sets.

Expression		Speak
$x \epsilon M$ $x \in M$	or	x is an element of the set capital m the point x belongs to the set capital m
$M \subset N$	or	capital m is a subset of capital n capital m is contained in capital n
$M \subseteq N$		capital m is a subset of or equal to capital n
$M \supset N$		capital m contains capital n
$M \supseteq N$		capital m contains or is equal to capital n
$M \cap N$ $M \cdot N$		intersection of capital m and capital n

Expression		Speak	Notes
$M \cup N$ $M + N$	 or or	union of capital m and capital n join of capital m and capital n sum of capital m and capital n	
$\bigcap_{\alpha\epsilon A} \; M_\alpha$ $\Pi_{\alpha\epsilon A} \; M_\alpha$	or	intersection of all the sets capital m sub alpha with alpha an element of capital a product of all the sets capital m sub alpha with alpha an element of capital a	
$\bigcup_{\alpha\epsilon A} \; M_\alpha$ $\Sigma_{\alpha\epsilon A} \; M_\alpha$	or	union of all the sets capital m sub alpha with alpha an element of capital a sum of all the sets capital m sub alpha with alpha an element of capital a	
$\sim M$ $C(M)$ $\bar{M}$ $\tilde{M}$ M'		complement of the set capital m	
$M - N$ $M \sim N$	or	complement of capital n in capital m relative complement of capital n in capital m	
$M \sim N$	or	the sets capital m and capital n are bijective the sets capital m and capital n can be put into one to one correspondence	
$M \cap (N \cup P)$		intersection of capital m and the set capital n union capital p	
$M \cap N \cup M \cap P$		capital m intersect capital n union capital m intersect capital p	
$M \cup (N \cap P)$		capital m union the set capital n intersect capital p	
$\overline{(M \cup N)}$		complement of the set capital m union capital n	
$\bar{M} \cap \bar{N}$		intersection of the complement of capital m and the complement of capital n	

Expression	Speak	Notes
$\aleph$	ah́ lef	the first letter of the Hebrew alphabet
$\aleph_o$	ah́ lef null	the cardinal number of the set of positive numbers
$M \simeq N$	capital m and capital n are of the same ordinal type	
ω	omega	the ordinal number of the positive integers in their natural order
ω^*	omega superscript star	the ordinal number of the negative integers in their natural order
${}^*\omega$	left superscript star omega	
π	pi	the ordinal number of all integers in their natural order
Q.E.D.	q e d	"Quod erat demonstrandum"—Latin meaning: which was to be demonstrated or which was to be proved

SECTION VII — ELEMENTARY AND ANALYTIC GEOMETRY

Symbol	Speak	Notes
$\angle$	angle	Example: $\angle$ ABC is read the angle ABC.
$\angle$s	angles	
$\perp$	perpendicular	Example: AB $\perp$ CD is read AB is perpendicular to CD.
$\perp$s	perpendiculars	
$\parallel$	parallel	Example: AB $\parallel$ CD is read AB is parallel to CD.
$\parallel$s	parallels	
$\cong$ or $\equiv$	congruent or is congruent to	Example: A $\cong$ B is read A is congruent to B.
$\sim$	similar or is similar to	Example: A $\sim$ B is read A is similar to B.
$\triangle$	triangle	
▱	parallelogram	
□	square	
○	circle	
Ⓢ	circles	
π	pi	See Greek alphabet, Section II.
O	origin	
(a,b)	the point a, b	
P(a,b) / p:(a,b)	the point capital p with coordinates a and b	
(r,θ)	the point r, theta in polar coordinates	See Greek alphabet, Section II.

Symbol		Speak	Notes
(x,y,z)		the point x, y, z in a rectangular coordinate system in space	
(r,θ,z)		the point r, theta, z in a cylindrical coordinate system in space	See Greek alphabet, Section II.
(r,θ,ϕ)	or	the point r, theta, fi	See Greek alphabet, Section II.
(ρ,θ,ϕ)		rho, theta, fi in a spherical coordinate system in space	
$\overline{AB}$	or	the line segment a b	
AB		the line segment between a and b	
$\overrightarrow{AB}$	or	the directed line segment from a to b	
		the ray from a to b	
$\overset{\frown}{AB}$	or	the arc a b	
		the arc between a and b	

SECTION VIII — STATISTICS AND MATHEMATICS OF FINANCE

Greek alphabet–the pronunciation of the Greek letters can be found in Section II.

Symbol		Speak	Notes
χ^2		chi–square	
d.f.		degrees of freedom	
F		capital f	F ratio
i		i	width of a class interval
k		k	coefficient of alienation
P.E.	or	probable error probable deviation	
r	or	r correlation coefficient	Pearson product moment, correlation coefficient between two variables
$r_{12 \cdot 34 \ldots n}$		r sub the quantity one two dot three four dot dot dot n	partial correlation coefficient between variables one and two in a set of n variables
s sd		standard deviation	from a sample
σ_x		sigma sub x	standard deviation of the population of x
σ_{xy}		sigma sub x y	standard error of estimate, standard deviation of an x array for a given value of y
t	or or	t students' t statistic students' t test	
V		capital v	coefficient of variation
$\overline{x}$		x bar	arithmetic average of the variable x from a sample

Symbol	Speak	Notes
μ	mu	arithmetic mean of a population
μ_2	mu sub two	second moment about the mean
μ_r	mu sub r	r^{th} moment about the mean
β_1	beta sub one	coefficient of skewness
β_2	beta sub two	coefficient of kurtosis
$\beta_{12 \cdot 34}$	beta sub the quantity one two dot three four	multiple regression coefficient in terms of standard deviation units
η	eta	correlation ratio
z	z	Fisher's z statistic
Q_1	capital q sub one	first quartile
Q_3	capital q sub three	third quartile
E(x)	capital e of x	expected value of x, expectation of x
$P(x_i)$	capital p of x sub i	probability that x assumes the value x sub i
%	percent	
$	dollar or dollars	
¢	cent or cents	
@	at	Example: three oranges @ $1.00 each is read three oranges at one dollar each.
$j_{(p)}$	j sub p in parentheses	nominal rate (p conversion periods per year)
n	n	number of periods or years

Symbol	Speak	Notes
l_x	l sub x	number of persons living at age x (mortality table)
d_x	d sub x	number of deaths per year of persons of age x (mortality table)
p_x	p sub x	probability of a person of age x living one year
q_x	q sub x	probability of a person of age x dying within one year
${}_nA_x$	left-subscript n capital a sub x	net single premium for $1 of term insurance for n years for a person aged x
${}_nP_x$	left-subscript n capital p sub x	premiums for a limited payment life policy of $1 with a term of n years at age x
$s_{\overline{n}\rceil}$	s sub n right angle	compound amount of $1 per annum for n years at a given interest rate

SECTION IX — CALCULUS AND ANALYSIS

Greek alphabet - the pronunciation of the Greek letters can be found in Section II.

Expression		Speak	Notes
a		a	*usually means* acceleration
I		capital i	usually means inertia
k		k	usually means radius of gyration
s σ		s sigma	usually means length of arc
$\dot{s}$ v		s dot v	usually means velocity
$\bar{x}, \bar{y}, \bar{z}$		x bar, y bar, z bar	
(a,b)	or	open interval a b *point a b*	
[a,b]		closed interval a b	
(a,b]	or	interval a less than x less than or equal to b interval a b, open on the left and closed on the right	
[a,b)	or	interval a less than or equal to x less than b interval a b, closed on the left and open on the right	
[x]	or	greatest integer not greater than x integer part of x	
$\{a_n\}$ $[a_n]$ (a_n)		sequence a sub 1, a sub 2, dot dot dot, a sub n, dot dot dot	
$\sum$		summation	boldface capital sigma

Expression		Speak	Notes
$\sum_{1}^{N}$		summation from one to capital n	
$\sum_{i=1}^{\infty} x_i$		summation from i equals one to infinity of x sub i	
$\prod$		product	boldface capital pi
$\prod_{1}^{n}$		product from one to n	
$\prod_{i=1}^{\infty} y_i$		product from i equals one to inifinity of y sub i	
l.u.b.		least upper bound	
sup	or	supremum soup	
g.l.b.		greatest lower bound	
inf	or	inferior inf	
$\lim_{x \to a} y = b$ $\lim_{x = a} y = b$		limit as x approaches a of y equals b	
$\overline{\lim}_{n \to \infty} t_n$		limit superior as n approaches infinity of t sub n	
$\underline{\lim}_{n \to \infty} t_n$		limit inferior as n approaches inifinity of t sub n	

Expression		Speak	Notes
lim sup $\overline{\lim}$	or	limit superior lim soup	
lim inf $\underline{\lim}$	or	limit inferior lim inf	
$f(x)$		f of x	
$f(g(x))$ $f \circ g(x)$	or	f composed with g of x f of g of x	
$f(a + 0)$		f of the quantity a plus zero	
$f(a+)$		f of the quantity a plus	
$f(a-0)$		f of the quantity a minus zero	
$f(a-)$		f of the quantity a minus	
$\lim_{x \downarrow a} f(x)$		limit as x decreases to a of f of x	
$\lim_{x \to a+} f(x)$		limit as x approaches a plus of f of x	
$\lim_{x \uparrow a} f(x)$		limit as x increases to a of f of x	
$\lim_{x \to a-} f(x)$		limit as x approaches a minus of f of x	
$f'(a+)$		derivative on the right of f at a	
$f'(a-)$		derivative on the left of f at a	
Δy	or	capital delta y an increment of y	
∂y	or or	partial y a variation in y an increment of y	

Expression		Speak	Notes
dy	or	d y differential of y	
$\frac{dx}{dt}$	or or	derivative with respect to t of x derivative of x with respect to t d x over d t	
$\frac{df(x_0)}{dx}$	or	derivative with respect to x of f at x sub zero derivative of f at x sub zero with respect to x	
y'		y prime	
$f'(x)$		f prime of x	
$D_x\,(y)$	or	derivative with respect to x of y capital d sub x of y	
$\frac{d^n y}{dx^n}$		n[th] derivative with respect to x of y	
$y^{(n)}$		y to the n[th] prime	
p' $\dot{p}$	or	p prime first derivative of p	
p'' $\ddot{p}$	or	p double prime second derivative of p	
$f^{(n)}\,(x)$		f to the n[th] prime of x	
$D_x^n\,y$	or	n[th] derivative with respect to x of y capital d sub x super n of y	
$f'\,(g(x))$	or	f prime of g of x f prime at g of x	
$(f(g(x)))'$		the quantity f of g of x, that quantity prime	
$f'(g(x))g'(x)$		the product of f prime of g of x and g prime of x	

Expression		Speak	Notes
$(f(x)g(x))'$		the quantity f of x times g of x, that quantity prime	
$f'(x)g(x)+f(x)g'(x)$		f prime of x times g of x, that product plus f of x times g prime of x	
$\left(\frac{f(x)}{g(x)}\right)'$		the quantity f of x over g of x, that quantity prime	
$\frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$		the fraction, the numerator is f prime of x times g of x, that product minus f of x times g prime of x, the denominator is g squared of x	
$f(x,y)$		f of x, y	
$\frac{\partial u}{\partial x}$		partial derivative of u with respect to x	
	or	partial u over partial x	
	or	partial derivative with respect to x of u	
u_x		partial derivative of u with respect to x	
	or	u sub x	
$f_x(x,y)$		partial derivative with respect to x of f of x, y	
	or	f sub x of x, y	
$f_1(x,y)$		partial derivative with respect to the first variable of f of x, y	
	or	f sub one of x, y	
$\frac{\partial^2 u}{\partial y\, \partial x}$		second partial derivative of u, first with respect to x and then with respect to y	
u_{xy}		second partial derivative of u, first with respect to x and then with respect to y	
	or	u sub x y	

Expression		Speak	Notes
$f_{xy}\ (x,y)$	or	second partial derivative of f of x, y, first with respect to x and then with respect to y f sub x y of x, y	
$f_{12}\ (x,y)$	or	second partial derivative of f of x, y, first with respect to the first variable and then with respect to the second variable f sub one two of x, y	
$D_y\ (D_x\ u)$	or	partial derivative with respect to y of the partial derivative with respect to x of u capital d sub y of capital d sub x u	
D	or	operator d over d x capital d	
$f(x_1, x_2, \ldots, x_n)$		f of x sub one, x sub two, dot dot dot, x sub n	
D_i	or	partial derivative with respect to the i^{th} variable capital d sub i	Example: $D_i f(x_1,x_2,\ldots,x_n) = \frac{\partial}{\partial x_i} f(x_1,x_2,\ldots,x_n)$
D_{ij}	or	second partial derivative first with respect to x sub i then with respect to x sub j capital d sub i j	Example: $D_{ij} f(x_1,x_2,\ldots,x_n) = D_i(D_j f(x_1,x_2,\ldots,x_n))$
$D_s f$	or	directional derivative of f in the direction s capital d sub s of f	
$\Delta f(x)$		delta f of x	
∇		del	
∇f	or	gradient of f del f	
$\vec{u}$		vector u	

Expression		Speak	Notes
grad f		gradient of f	
$\nabla \cdot \vec{u}$	or	divergence of vector u del dot vector u	
div $\vec{u}$	or	divergence of vector u div vector u	
$\nabla \times \mathbf{F}$	or	curl of boldface capital f del cross boldface capital f	
∇^2	or	Laplacian operator del squared	Example: $\nabla^2 u$ is the Laplacian operator on u.
Δ	or	Laplacian operator delta	
δ^i_j		Kronecker delta	
$F(x)]^b_a$	or	capital f of x evaluated from a to b capital f of b minus capital f of a	
$\int f(x)\,dx$	or	integral f of x d x anti derivative of f with respect to x	
$\int_a^b f(x)\,dx$		integral from a to b of f of x d x	
$\overline{\int_a^b}$		upper Darboux integral from a to b	
$\underline{\int_a^b}$		lower Darboux integral from a to b	
$\int_a^b \left[\int_c^d f(x,y)\,dy \right] dx$		iterated integral: integral from a to b of the integral from c to d of f of x,y d y d x	
$\int_\alpha^\beta \left[\int_{r_1(\theta)}^{r_2(\theta)} f(r,\theta) r dr \right] d\theta$		iterated integral: integral from alpha to beta of the integral from r sub one of theta to r sub two of theta of f of r, theta r d r d theta	iterated integral in polar coordinates

Expression	Speak	Notes
$\int_R 1 \, dV$	integral over capital r of one d capital v	
$\int_0^{2\pi}\left(\int_0^a\left(\int_{-\sqrt{a^2-r^2}}^{\sqrt{a^2-r^2}} 1 \cdot r \, dz\right)dr\right)d\theta$	iterated integral: integral from zero to two pi of the integral from zero to a of the integral from minus square root of the quantity a squared minus r squared to square root of the quantity a squared minus r squared of one dot r d z d r d theta	iterated integral with cylindrical coordinates
$\int_0^{2\pi}\left(\int_0^{\pi/2}\left(\int_0^a \rho \cos \phi \, \rho^2 \sin \phi \, d \rho\right) d \phi\right) d\theta$	iterated integral: integral from zero to two pi of the integral from zero to pi over two of the integral from zero to a of rho cosine fi rho squared sine fi d rho d fi d theta	iterated integral with spherical coordinates
$\int_R f \, d \, V$	integral over capital r of f d capital v	
$\int f[x(u)] \frac{dx}{du} \, du$	integral of the product of three factors: f of x of u, and d x over d u, and d u or integral of f of x of u times d x over d u times d u	
$\int_{a_1}^{b_1} \int_{a_2}^{b_2} \cdots \int_{a_n}^{b_n} f(x_1, x_2, \ldots, x_n) \, dx_1 dx_2 \ldots dx_n$	multiple integral: integral from a sub one to b sub one, integral from a sub two to b sub two, dot dot dot, integral from a sub n to b sub n of function f of x sub one, x sub two, dot dot dot, x sub n, end of function, d x sub one d x sub two dot dot dot d x sub n	
$\int_\gamma f(z) \, dz$	integral over gamma of f of z d z	
$\oint_C M(x,y) \, dx$	line integral along capital c in positive direction of function capital m of x, y d x	

Expression	Speak	Notes
$\iint_S g(x,y,z)\, dS$	surface integral over capital s of g of x, y, z d capital s	
$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(t)\, e^{ixt}\, dt$	one divided by the square root of two pi that fraction times integral from minus infinity to infinity of the quantity g of t times e to the i x t power d t	
(f,g)	inner product of the functions f and g	
$\|f\|$	norm of the function f	
$f * g$	convolution of f and g	
$W(u_1,u_2,\ldots,u_n)$	Wronskian of u sub one, u sub two, dot dot dot u sub n	
$\frac{\partial(f_1,f_2,\ldots,f_n)}{\partial(x_1,x_2,\ldots,x_n)}$ $\frac{D(f_1,f_2,\ldots,f_n)}{D(x_1,x_2,\ldots,x_n)}$ $J\left(\frac{f_1,f_2,\ldots,f_n}{x_1,x_2,\ldots,x_n}\right)$	Jacobian of the function f sub one of x sub one, x sub two, dot dot dot x sub n; f sub two of x sub one, x sub two, dot dot dot, x sub n; dot dot dot f sub n of x sub one, x sub two, dot dot dot x sub n	

In the following expressions z is a complex number

$\|z\|$	absolute value of z or modulus of z	
$\bar{z}$	conjugate of z or z bar	
conj z	conjugate of z	
arg z	argument of z	

Expression	Speak	Notes
R (z) $\mathscr{R}(z)$ Re (z)	real part of z	
I (z) $\mathscr{I}(z)$ Im (z)	imaginary part of z	
$\operatorname{Res}_{z=a} f(z)$	residue at z equals a of f of z	

SECTION X — LINEAR ALGEBRA

Note: Matrices are read either by rows or by columns and the number of rows and columns determines the size of the matrix. Hence, a matrix with four rows and three columns is called a four-by-three matrix. (The number of rows is listed first, i.e., 4 by 3.)

Expression		Speak	Notes
$\begin{bmatrix} 2, & 7 \\ 3, & 10 \end{bmatrix}$		two by two matrix first row two seven second row three ten	
$\begin{bmatrix} 2 & 7 \\ 3 & 10 \end{bmatrix}$	or	two by two matrix first column two three second column seven ten	
a_{ij}		a sub i j	
$m \times n$	or	m cross n m by n	
$a_{i+1,j}$		a double subscript i plus one comma j	
$a_{i,\, j-1}$		a double subscript i comma j minus one	
$a_{i+\frac{1}{2},\, j-\frac{1}{2}}$		a double subscript i plus one half, j minus one half	
$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$		m by n matrix: first row a sub one one, a sub one two, dot dot dot, a sub one n second row a sub two one, a sub two two, dot dot dot, a sub two n third row dot dot dot m^{th} (or last) row a sub m one, a sub m two, dot dot dot a sub m n	

Expression		Speak	Notes
$[a_{ij}]\ 1 \leq i \leq m,\ 1 \leq j \leq n$			
		the m by n matrix with elements a sub i j	
	or	open bracket a sub i j close bracket one less than or equal to i less than or equal to m, one less than or equal to j less than or equal to n	
$a\mathbf{A}$		a boldface capital a	
	or	scalar product of a and matrix boldface capital a	
$\mathbf{A}^T$		boldface capital a superscript capital t	
	or	transpose of the matrix boldface capital a	
	or	the matrix boldface capital a transpose	
$\mathbf{A}'$		boldface capital a prime	
	or	transpose of the matrix boldface capital a	
	or	the matrix boldface capital a transpose	
$\mathbf{A}^H$		boldface capital a superscript capital h	
	or	Hermitian transpose of the matrix boldface capital a	
$[\mathbf{AB}]^{-1}$		left bracket boldface capital a boldface capital b right bracket superscript minus one	
	or	inverse of the matrix product boldface capital a boldface capital b	
$[\mathbf{A}+\mathbf{B}]^{-1}$		left bracket boldface capital a plus boldface capital b right bracket superscript minus one	
	or	inverse of the matrix sum boldface capital a plus boldface capital b	
$\mathbf{ABB}^{-1}$		boldface capital a boldface capital b boldface capital b superscript minus one	
	or	the product boldface capital a boldface capital b boldface capital b inverse	

Expression	Speak	Notes
$\mathbf{A}^{-1}$	boldface capital a superscript minus one or inverse of the matrix boldface capital a or matrix boldface capital a inverse	
$\lvert\mathbf{A}\rvert$ $\det \mathbf{A}$	determinant of the square matrix boldface capital a	
$\det \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & \cdots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$	determinant of the matrix: first row a sub one one, a sub one two, dot dot dot, a sub one n second row a sub two one, a sub two two, dot dot dot, a sub two n third row dot dot dot n^{th} (or last) row a sub n one, a sub n two, dot dot dot, a sub n n	
$\sum_{k=1}^{n} a_{ik}b_{kj}$	summation from k equals one to n of the product a sub i k b sub k j	
$\sum_{s=1}^{n} \sum_{t=1}^{p} a_{is}b_{st}c_{tj}$	summation from s equals one to n of summation from t equals one to p of the product a sub i s b sub s t and c sub t j	
$ax + by = e$ $cx + dy = f$	the system of equations first equation: a x plus b y equals e second equation: c x plus d y equals f	

Expression	Speak	Notes
$a_{11}\ x_1 + a_{12}\ x_2 + \ldots + a_{1n}\ x_n = b_1$ $a_{21}\ x_1 + a_{22}\ x_2 + \ldots + a_{2n}\ x_n = b_2$ $\cdots$ $a_{m1}\ x_1 + a_{m2}\ x_2 + \ldots + a_{mn}\ x_n = b_m$	the system of equations first equation: a sub one one x sub one plus a sub one two x sub two plus dot dot dot plus a sub one n x sub n equals b sub one second equation: a sub two one x sub one plus a sub two two x sub two plus dot dot dot plus a sub two n x sub n equals b sub two third line: dot dot dot m[th] (or last) equation: a sub m one x sub one plus a sub m two x sub two plus dot dot dot plus a sub m n x sub n equals b sub m	
$[a_1, a_2, \ldots a_n]$	n row vector a sub one a sub two dot dot dot a sub n	
$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$	n column vector a sub one a sub two dot dot dot a sub n	
$\begin{bmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ & & \ldots & \\ 0 & 0 & \ldots & 1 \end{bmatrix}$	matrix: first row one zero dot dot dot zero second row zero one dot dot dot zero third row dot dot dot last row zero zero dot dot dot one or identity matrix	
I	identity matrix	

Expression	Speak	Notes
$$\begin{bmatrix} d_1 & 0 & 0 & \dots & 0 \\ 0 & d_2 & 0 & \dots & 0 \\ & & \dots & & \\ 0 & 0 & 0 & \dots & d_n \end{bmatrix}$$	matrix: first row d sub one zero zero dot dot dot zero second row zero d sub two zero dot dot dot zero third row dot dot dot n^{th} (or last) row zero zero zero dot dot dot d sub n or n by n diagonal matrix with d sub one to d sub n on the diagonal	
$$\begin{bmatrix} u_{11} & u_{12} & \dots & u_{1n} \\ 0 & u_{22} & \dots & u_{2n} \\ & & \dots & \\ 0 & \dots & 0 & u_{nn} \end{bmatrix}$$	matrix: first row u sub one one u sub one two dot dot dot u sub one n second row zero u sub two two dot dot dot u sub two n third row dot dot dot n^{th} (or last) row zero dot dot dot zero u sub n n or n by n upper triangular matrix	

Expression	Speak	Notes
$\begin{bmatrix} \ell_{11} & 0 & \ldots & 0 \\ \ell_{21} & \ell_{22} & \ldots & 0 \\ & & \ldots & \\ \ell_{n1} & \ell_{n2} & \ldots & \ell_{nn} \end{bmatrix}$	matrix: first row script l sub one one zero dot dot dot zero second row script l sub two one script l sub two two dot dot dot zero third row dot dot dot n^{th} (or last) row script l sub n one script l sub n two dot dot dot script l sub n n or n by n lower triangular matrix	
$\begin{bmatrix} u_{11} & u_{12} & u_{13} & \ldots \\ \ell_{21}u_{11} & \ell_{21}u_{12}+u_{22} & \ell_{21}u_{13}+u_{23} & \ldots \\ \ell_{31}u_{11} & \ell_{31}u_{12}+\ell_{32}u_{22} & \ell_{31}u_{13}+\ell_{32}u_{23}+u_{33} & \ldots \\ & \ldots & & \end{bmatrix}$	matrix: first row first element u sub one one second element u sub one two third element u sub one three, dot dot dot second row first element script l sub two one u sub one one second element script l sub two one u sub one two plus u sub two two third element script l sub two one u sub one three plus u sub two three, dot dot dot third row first element script l sub three one u sub one one second element script l sub three one u sub one two plus script l sub three two u sub two two third element script l sub three one u sub one three plus script l sub three two u sub two three plus u sub three three, dot dot dot fourth row dot dot dot	

SECTION XI — TOPOLOGY AND ABSTRACT SPACES

Note: In the following expressions, the capital letters M and N denote sets.

Expression	Speak	Notes
$\overline{M}$	capital m bar	closure of capital m
M'	capital m prime	derived set of capital m
$d(x,y)$ $\delta(x,y)$ $\rho(x,y)$ (x,y)	d of x, y delta of x,y rho of x,y x, y	distance from x to y
$M \times N$	capital m cross capital n	the Cartesian product of spaces capital m and capital n
M/N	capital m slash capital n	the quotient space of capital m and capital n
E_n E^n R_n R^n	capital e sub n capital e superscript n capital r sub n capital r superscript n	real n-dimensional Euclidean space
Z_n C_n	capital z sub n capital c sub n	complex n-dimensional space
H $\mathfrak{H}$	capital h Gothic capital h	Hilbert space
$(\mathbf{x},\mathbf{y})$	open parenthesis boldface x, boldface y closed parenthesis	inner product of the elements x and y of a vector space
$\Vert\mathbf{x}\Vert$	norm of boldface x	
l_p $l^{(p)}$	italic l sub p space italic l superscript p in parentheses space	

Expression	Speak	Notes
L_p	capital l sub p space	
$L^{(p)}$	capital l superscript p in parentheses space	
$\left[\sum_{i=1}^{\infty} \lvert x_i \rvert^p\right]^{1/p}$	summation i equals one to infinity of the absolute value x sub i, that absolute value raised to the p power, and the whole sum raised to the one over p power	
$\left[\int_s \lvert f(x) \rvert^p ax\right]^{1/p}$	integral over s of the absolute value of f of x, that absolute value raised to the p power a x and the whole integral raised to the one over p power	
∂S	partial capital s	boundary of the set capital s
ΔS	capital delta capital s	
$d(S)$	d of capital s	

SECTION XII — DIAGRAMS AND GRAPHS

In this section the approach changes from previous sections. Here suggestions are merely offered to alleviate the very complicated problem of diagram description.

Diagrams are visual aids and are very useful to illustrate qualitative information. Because of their visual nature, it is somewhat clumsy and sometimes even impossible to describe them verbally. The old saying, "A picture is worth a thousand words", sums up the difficulty faced when trying to describe a picture with words. The degree of complexity of the diagram should determine whether "reading" the diagram is worth the effort. Some illustrations require so many words from the reader that it can render the listener in a state of depressed confusion from which there is no reasonable hope of bringing him out clear-headed again.

This section deals mainly with suggestions for describing diagrams in general. These suggestions should help the interpreter convey the information in the illustration to the listener in as clear a manner as possible. It is most important that diagrams be described clearly. A poorly read diagram is worse than one not read at all, because it can confuse and frustrate the listener and even give misleading information. When taping, if the reader finds that the material to be described is not clear or comprehensible to himself, the reader should consult the listener in person. Specific questions from the listener will likely elicit the desired information. If the listener is blind, there are other ways to facilitate understanding of the diagram, such as tracing the diagram using the blind person's hand, or using raised line drawing paper to duplicate the essential parts.

The following are some specific suggestions that I have personally found helpful when having diagrams read to me. First, read the caption, for it may contain a very good description of the diagram itself. Next, describe the shapes either contained in the diagram or comprising the entire diagram. An example of the former case is a flow chart, a chart consisting of circles, squares, triangles, etc., with connecting arrows. An example of the latter case would be a pie diagram, where a circle is cut into pie-shaped sections or wedges. Besides stating the basic geometric shapes, use words for the shapes of any familiar objects, such as crescent, football, piece of bread, sausage, tear drop, etc. Describe the relative sizes of the shapes and any labels, markings, or shading on them. In addition, describe the orientation of the various figures in the diagram, i.e., how the various figures are related to one another. Describe the basic layout, if there is one.

An important subcategory of the diagram is the graph. Particularly in mathematics, graphs are widely used. Often they are hard to describe, for they can depict complicated figures, such as the projection of a three-dimensional object on a plane. Nonetheless, from my experience, having certain key features of a graph described facilitates the listener's understanding of whatever the graph is depicting.

First, a framework upon which the graph is constructed is needed. In a graph, the horizontal and vertical lines form the axes of a coordinate system. The horizontal line in general is known as the x-axis and the vertical line as the y-axis. (Any letters may be used to label the axes.) If there is a scale marked on the axes, for the horizontal axis it increases from left to right; for the vertical axis it increases from down to up. The point where the axes meet is the origin. The axes divide the plane into four quadrants: the upper right is the first, upper left is the second, lower left is the third, and lower right is the fourth. This is the basic framework upon which the graph is constructed.

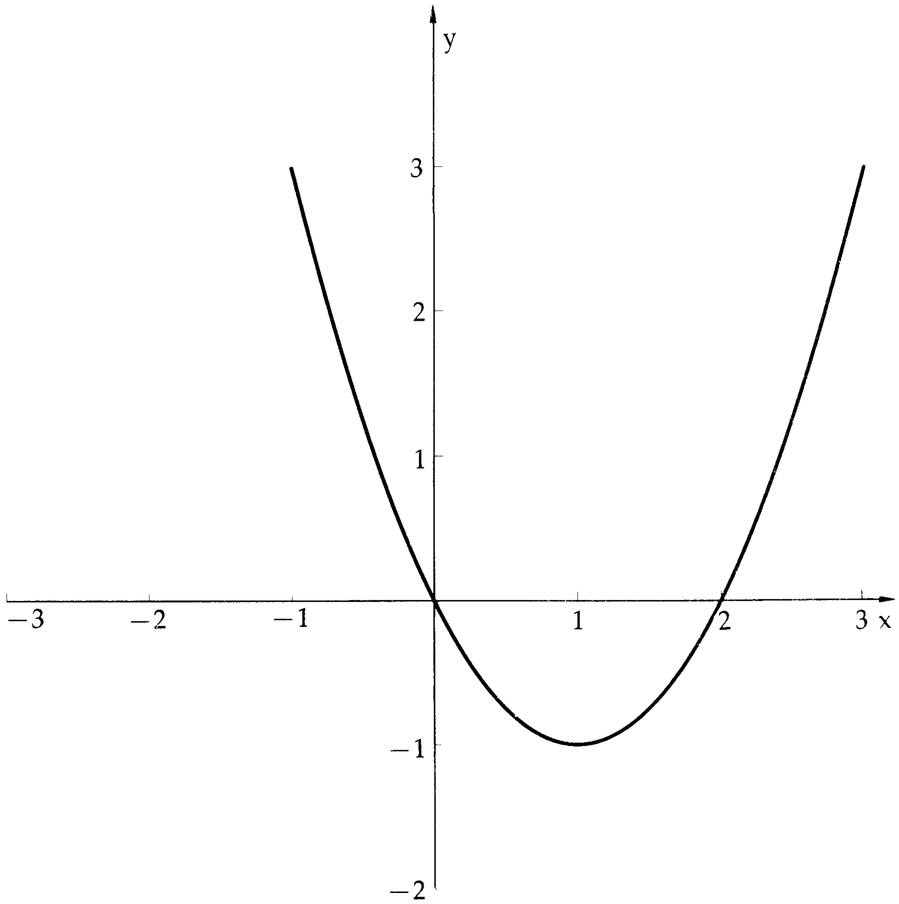
The following is a list of some of the key features of a graph that should be described:

- Read the labels on the axes and any marking or scale on the axes.
- If possible, read from left to right, and state in which quadrant the graph begins and in which it ends.
- As the graph traverses from left to right, state where it goes up or down and over what point on the x-axis it changes direction.
- Describe how steeply each portion of the graph goes up or down. Compare that portion to a line which forms a particular angle with the x-axis, such as 15°, 30°, 45°, etc., if desired.
- State at what points the graph crosses the axes, and where it reaches its local minima or maxima.
- Describe the shapes of the various portions of the graph. Examples of shapes are: straight line, semicircle, parabola, sinusoid, etc.
- Describe the concavity of the various portions of the graph; specify which portion is concave up (a curve that opens up or a dip) and which portion of the graph is concave down or convex (a curve that opens down or a hump).
- Describe the point of inflection, i.e., the point on the graph at which the graph changes concavity.
- Specify any points of discontinuity (breaks in the graph) and any cusps (sharp points on the graph).
- Describe the symmetry of the graph, i.e., on which line one half of the graph is the mirror image of the other.
- If there is more than one graph in the figure, describe each graph individually, and describe where they intersect or how they are related to each other.

The types of diagrams and graphs are so varied that these few pages cannot help specifically in every case. These suggestions are limited, but it is hoped that not only will they be useful in themselves, but also will inspire the interpreter to develop his own ideas to describe diagrams clearly.

This section concludes with a few examples of graphs, each (except the last) accompanied by a suggested verbal description. The last one cannot be reasonably described.

EXAMPLES

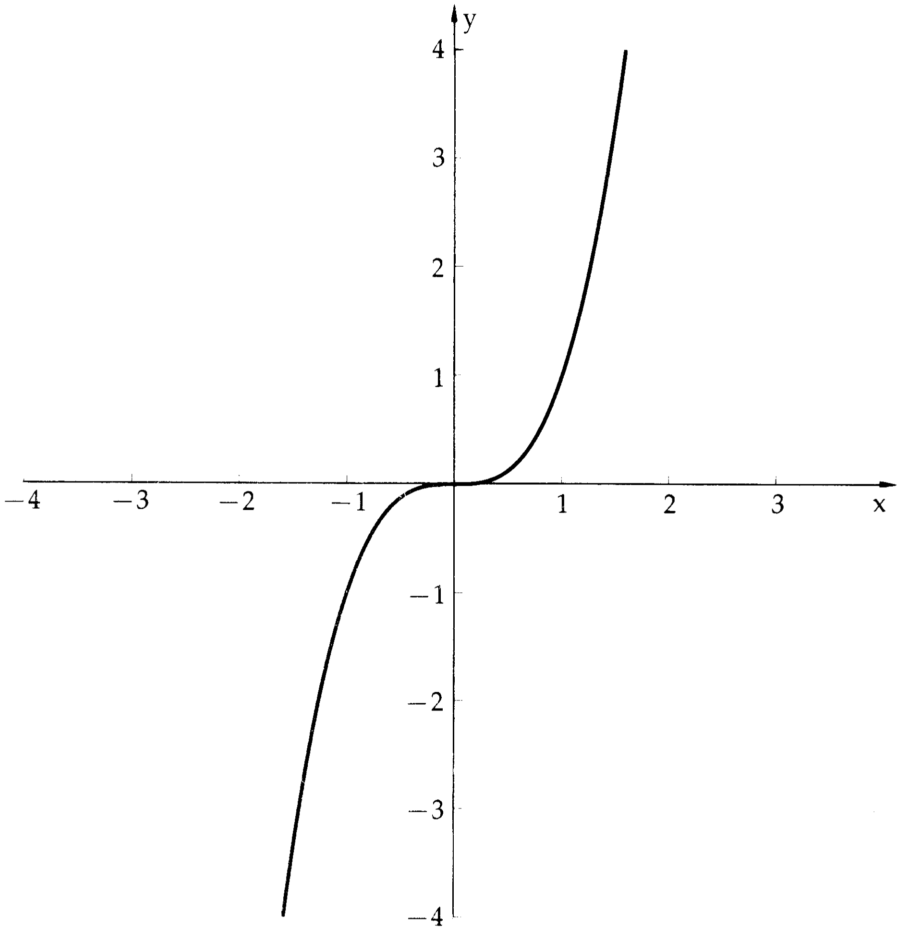


$y = x^2 - 2x$, a parabola

Speak

The graph is captioned: y equals x squared minus two x, a parabola. The graph has x- and y-axes and the scale for both axes is in units of one, labeled from minus three to plus three. The shape of the graph is a parabola, concave up. It is symmetric about the vertical line x equals one. The graph begins in the second quadrant and decreases steeply, almost vertically, from the upper left as it moves to the right. It crosses the origin and continues to go down into the fourth quadrant and reaches the minimum at the point one, minus one. The graph then changes direction to go up and crosses the x-axis again at the point two, zero, moves into the first quadrant and continues to go up steeply.

2)

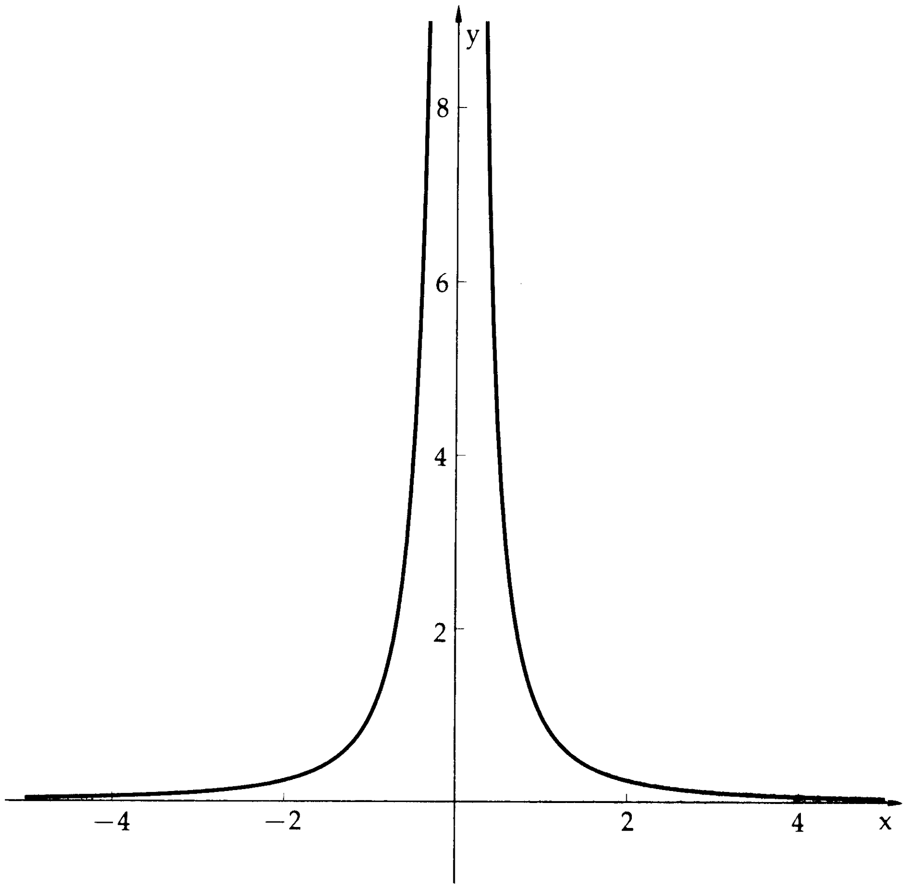


$y = x^3$, the cubic

Speak

The graph is captioned: y equals x cubed, the cubic. The graph has x- and y-axes, and the scale for both axes is in units of one, labeled from minus four to plus four. The graph is antisymmetric about the vertical line x equals zero, the y-axis. The graph begins in the third quadrant and increases steeply as it moves to the right. As it nears the origin it flattens out somewhat, crosses the axes at the origin, remains somewhat flat close to the origin, after which it increases steeply again in the first quadrant. It is concave down for x less than zero and concave up for x greater than zero.

3)

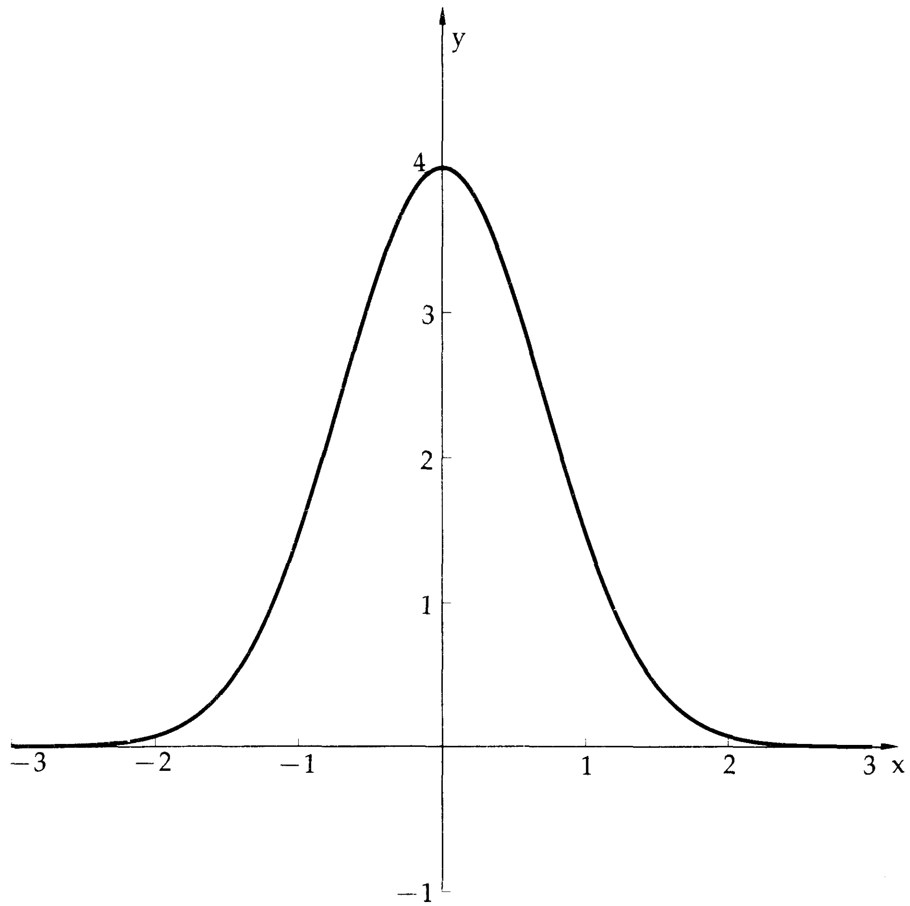


$$y = \frac{1}{x^2}$$

Speak

The graph is captioned: y equals the fraction one over x squared. The graph has x- and y-axes and the scale for both axes is in units of two, labeled from minus four to plus eight. The graph is symmetric about the vertical line x equals zero, the y-axis. The graph consists of two separate branches. The first begins in the second quadrant very close to the x-axis. As it moves to the right, the graph increases very slowly until it reaches the point minus one, one. As it continues to approach zero from the left, the graph increases steeply and nears but never touches the y-axis. That is the end of the first branch of the graph, which is entirely contained in the second quadrant. The graph has a discontinuity at x equals zero. The second branch of the graph is entirely contained in the first quadrant. It begins very close to the y-axis. As it moves to the right, the graph decreases steeply, until it reaches the point one, one, where it begins to flatten out, and slowly approaches the x-axis but never touches it. That is the end of the second branch of the graph.

4)

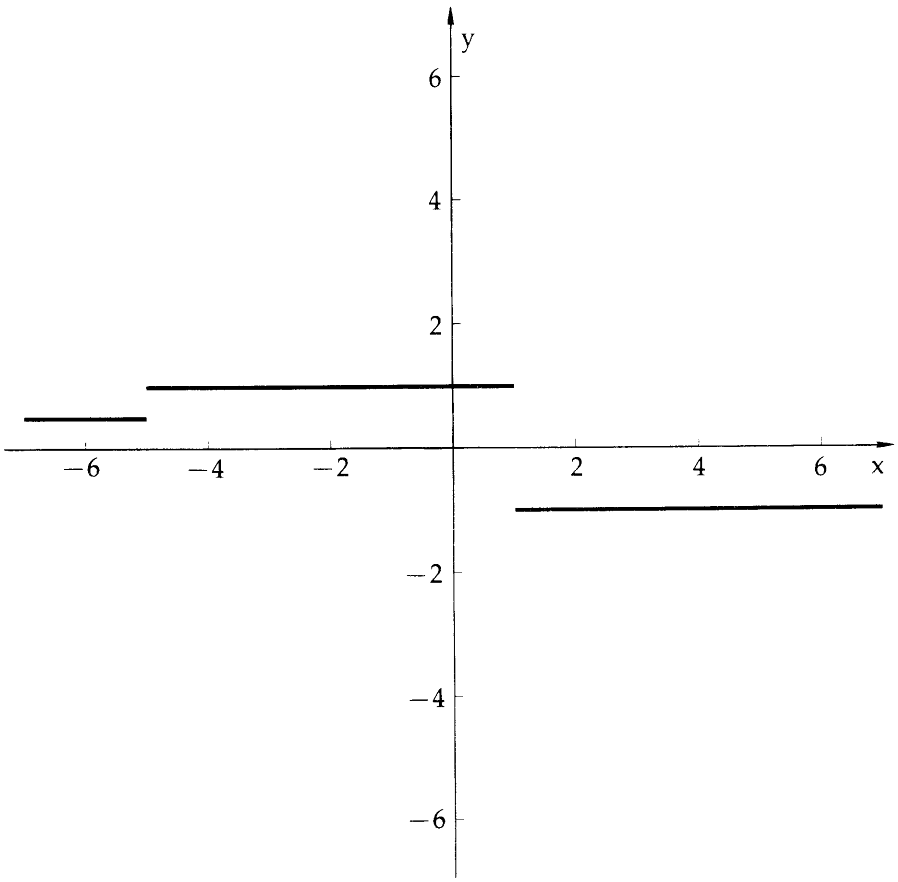


$$y = 4e^{-x^2}$$

Speak

The graph is captioned: y equals four times e raised to the quantity minus x squared. The graph has x- and y-axes and the scale for both axes is in units of one, labeled from minus three to plus four. The graph is a bell-shaped curve symmetric about the y-axis and concave down. The graph begins in the second quadrant near the x-axis. When x is less than minus two, the graph increases slowly. When x is greater than minus two and less than zero, the graph increases sharply and crosses the y-axis at the point zero, four. The graph then decreases rapidly for x greater than zero and less than two. For x greater than two, it decreases slowly as it approaches the x-axis but never touches it.

5)



$$\text{The step function: } y = \begin{cases} \frac{1}{2}, & x < -5 \\ 1, & -5 \leq x < 1 \\ -1, & x \geq 1 \end{cases}$$

Note: This graph is an example where the caption obstensively describes the graph.

Speak

The graph is captioned: the step function: y equals one half when x is less than minus five, y equals one, when minus five is less than or equal to x is less than one, and y equals minus one, when x is greater than or equal to one. The graph has x- and y-axes and the scale for both axes is in units of two, labeled from minus six to plus six. The graph consists of three disjoint horizontal line segments parallel to the x-axis. The first line segment is located at y equals one half when x is less than minus five. It is entirely contained in the second quadrant. The second line segment is located at y equals one for x greater than or equal to minus five and less than one. It begins in the second quadrant, crosses the y-axis and ends near the point x equals one in the first quadrant. The third line segment is located at y equals minus one when x is greater than one. It is entirely contained in the fourth quadrant.

6)

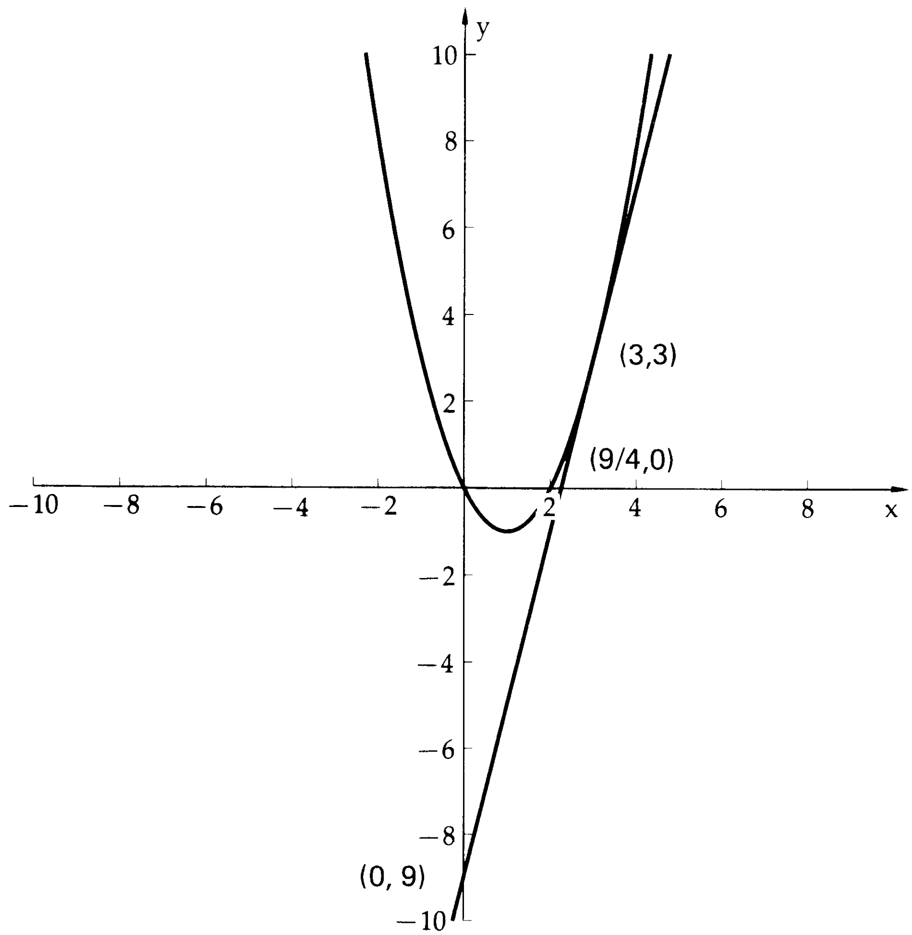
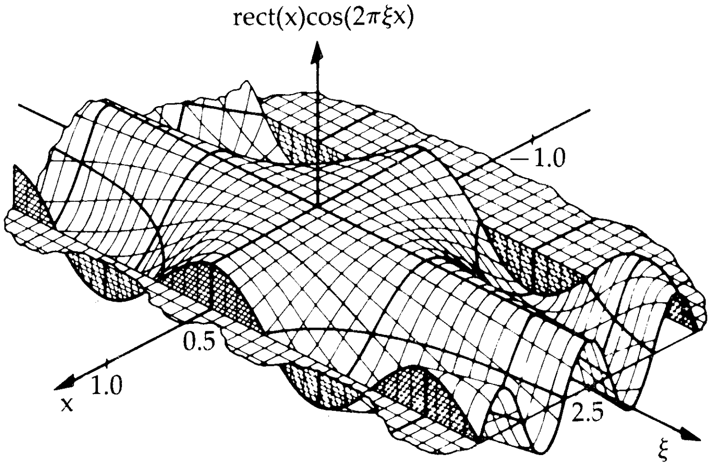


Diagram containing 2 graphs: $y = x^2 - 2x$ and $y = 4x - 9$

Speak

The graph is captioned: diagram containing two graphs: y equals x squared minus two x and y equals four x minus nine. The scale for the x- and y-axes is in units of two and is labeled from minus ten to plus ten. The parabola is described as in Example 1. The second graph is a straight line which starts in the third quadrant, intersects the y-axis at the point zero, minus nine, and continues through the fourth quadrant. It intersects the x-axis at the point nine fourths, zero, and continues up into the first quadrant. The angle between the graph and the x-axis is fairly close to ninety degrees. The two graphs, the parabola and the straight line intersect at the point three, three; or the straight line is tangent to the parabola at the point three, three.

7)



Two-dimensional representation of the integrand of the Fourier integral of the rectangular function of x.

Speak

The diagram is captioned: two-dimensional representation of the integrand of the Fourier integral of the rectangular function of x.

Comments

This picture is worth more than a thousand words. This diagram is so complicated that one should probably not consider describing it verbally other than reading the caption. Use of raised line drawing paper or a discussion between reader and listener of some of its main points could be useful.