Guidance for Writing Lab Reports

Written by Adam Beagles, Stephen Beck, Lizzy Cross, Andrew Garrard and Jen Rowson

1 Writing lab reports

To write a successful scientific report you need to be clear about what you are trying to achieve. The main purpose of a scientific report is to communicate the finding from the work and to help the reader to understand them. The report should include a record of the process used to establish the findings, so they can be reproduced at a later stage for validation. It should be written as an independent record that can be read without further input from the author.

Initially focus on the audience for your report, as this will assist you in getting the level of complexity and explanation right. You need to think about who you are writing, how much they will already understand and what they want to know?

A typical technical report should document what has been done, how it was done, what the findings were, and the author's interpretation of those findings. A story should be told through a logical delivery of information. A technical engineering report should be presented in logical sections. The structure of these sections and style of presentation has evolved to convey essential information as concisely and effectively as possible. Each report will vary depending on what is being documented. However, there are typical sections that will be relevant to the majority of reports you write.

1.1 The start of the report

At the start of a technical engineering report, there is a certain amount of preliminary material. This may include a title page, contents page (with page numbers), list of tables, list of figures, list of equations, acknowledgements, and nomenclature. The type and amount of material provided should be based on what is appropriate for the document. For example, it is unnecessary to have a contents page for a 3 page document.

1.2 Abstract

The first item to appear after the title of the document is the abstract (sometimes called the summary or executive summary). It is a very concise summary of all the salient aspects of the entire document. An abstract is written so that a reader interested in the work, can gather an impression of the contents of the report and decide if investigating the details further is worthwhile.

It should include:

- the aim of the experiment,
- the background context,
- the procedures followed and equipment used,
- the results that were obtained,

- any observations made,
- the findings drawn and the impact those findings have towards fulfilling the original aim.

Compressing all this information it a very short piece of text makes writing an abstract a difficult task to perform and one that is often done badly by undergraduate students. Practice writing abstracts is one of the best methods for improving technique. There are some rules that should be followed when writing an abstract:

- the structure of an abstract should follow the structure of the report
- only the critically important "headlines" from the report should be included
- it shouldn't include tables, graphs, pictures or equations
- it should be self-contained, i.e. can be read and understood without needing to refer to other documents
- it should not include abbreviations, acronyms or jargon
- it is the first thing to appear after the title of the document, but should be the last part of the document to be written

1.3 Introduction

The introduction provides the reader with the background to the work documented in the report. This section should set the scene for what is to follow. It should contain the aims or objectives of the proposed work. If an aim of the experiment is to investigate a hypothesis, then this should be stated in the introduction. The aims, objectives and/or hypothesis should be given in the context of the real world application outside the experiment.

There should be a broad introduction to the background of the science, the reasons for doing the work, and who will benefit from the results. For example: if the subject of the lab report is discussing an experiment conducted on a photovoltaic solar panel, the introduction should mention the fundamentals of collecting solar energy from the sun and conversion of solar energy to electrical energy by photons operating on photodiodes. It isn't necessary to derive equations from first principles or exhaustively describe theory, as reference to alternative material can point the reader towards where to find this information. Given that the reader may not be familiar with the specifics of the discipline it may be necessary to explain acronyms or technical terms. This should be done in the introduction.

To place the purpose of this example experiment in context, the introduction could include a discussion of the benefits of increasing the efficiency of solar cells, or of reducing the manufacturing cost of solar panels, compared with reducing the carbon emissions of alternative methods of energy production. In addition, there should be a summary of previously conducted work in the same field and a description of how the contents of the report furthers the advancement of knowledge.

In summary, the introduction should include:

- a background to the subject
- previously conduced work in the same subject
- aims and objectives for the work that will be presented in the lab report
- reasons why the work is being conducted

1.4 Procedure

The procedure section is a record of what was done, a chronological description of the steps followed and the equipment used. It should not be a list of instructions but should be written as prose, in the third person and past tense (as should the rest of the report). Details of what variables were recorded, what observations were made, and what types of instrumentation were used should be included.

The procedure should contain sufficient detail to allow the experiment to be repeated by another person at a later date. It is necessary to give a detailed record of any important conditions of the experiment (e.g. operating temperatures, atmospheric pressure, humidity), any specific techniques that were used (e.g. equipment calibration) and any materials involved (e.g. 10M hydrochloric acid, cast iron). It is critical to include all relevant information but to ensure the report is sufficiently concise and excludes extraneous detail. For example, in detailing equipment, it may be useful to record the manufacturer and model number, the precision of the instrument, the zero-offset, any calibration that was performed, and the accuracy at different recording ranges. A record of the colour of the equipment will probably be of no consequence to the results and should not be included in the report.

A well written procedure should include not only a description of what was performed, but also the reasoning behind the experimental design. Why was the experiment set up in the way it was and how does it conform to the scientific method? What special measures have been put in place to ensure accuracy and repeatability of the results?

To describe equipment, labelled diagrams and photographs can be included. Photographs are usually not sufficient to replace explanatory diagrams, and should only be used if they enhance readers' understanding of the experiment. Any safety precautions or procedures that were observed, or any PPE (personal protective equipment) that was used can be discussed if appropriate.

1.5 Results

The results section of a lab report contains an impartial description of the results obtained from the experiment, typically presented as tables or graphs, and observations that were made. At this point in the report, interpretation of the results should not be performed. To convey the main findings of the experiment, processed, rather than raw data, should be shown. A brief description of the method used to covert the raw data to the results could be included, possibly using an illustrative sample calculation. However, large datasets and numerous intermediate calculations should not be shown in the results section. These can be included for reference in an appendix if useful for the reader. Large quantities of raw data can be stored electronically and an explanation of how to access it given in the report.

In addition to the measurements taken during the experiment, the results section should include any observations that were made during the experiment. Unexpected phenomena may affect the results in ways that are not known by the author of the lab report but may be of significance to the reader. For example, if work is conducted on a water flow system and a large number of bubbles are observed in the supply or there is a large oscillation in the values reported from measurement equipment, record this in the results section.

A well written results section of a lab report highlights the trends observed rather than giving details of exact results. The data presented in the results section should demonstrate how the experiment's objectives have been met. For example, if the aim of an experiment was to optimize the level of fuel consumption in a petrol car by varying travelling speed, then the results section could show a plot of kilometres per litre against meters per second. The details of the amount of fuel used, distance travelled by the car, the variation of lengths of journeys, the elimination of effects of acceleration and deceleration on the results, and other processing techniques should only be described briefly.

1.6 Discussion

The purposed of a discussion section is to answer the questions:

- What do the results mean?
- Do they answer the questions the experiment was to investigate?
- What is the relevance to engineering problems?
- Where are errors introduced?"

The discussion section is used to analyse and interpret the information presented in the results section. Mention should be made of whether or not the results achieve the aims or prove/disprove the hypothesis previously set out, within the context of the background science. In doing so, the discussion should refer to the introduction section so that the document is a coherent piece of work.

The interpretation of the results should discuss the physical principles for the trends or phenomena that were observed. If unexpected results are produced, that were not suggested from the background theory presented in the introduction, the discussion allows possible reasons for these findings to be proposed.

The discussion section should attempt to report on the errors and uncertainties in the experiment. Errors may include the limited precision of instruments, the result of ignoring wind resistance, or human error/reaction time. Where possible these errors should be quantified, even approximately, and ranked. Further details on handling and manipulating errors are given in this document. There are two reasons to quantify errors and uncertainties: firstly, it allows a degree of confidence to be placed on the results presented; secondly, it allows efforts to reduce error in future experiments to be focused correctly.

The potential impact of the results on the real world applications to which the experiment was designed to apply should be discussed in this section. For example, by:

- comparison between field scale and lab scale results
- proposing design changes to existing products based on new knowledge
- quantifying the impact of the results on beneficiaries

1.7 Conclusion

The conclusion is a short review of that which has been deduced from the work conducted. It is an opportunity to restate the aims or key questions and to summarise the key points raised in the results and discussion sections. <u>No new information</u> should be given in the conclusion that hasn't been stated previously in the document.

Proposals for further work or potential improvements identified during the experiment can be suggested in the conclusion, or this can be placed in a separate "further work" section following the conclusion.

1.8 The end of the report

Following the conclusion should be additional, non-essential information. Any previously published work cited in the body of the document should be referenced in a dedicated "references" section. If previously published work has been used but not explicitly cited, this should be placed in a "bibliography" section.

Other information, such as raw data, manufacture's user manuals, complex numerical tables of results…etc. can be placed in an appendix, to which the reader can refer for detail. The appendix is not a substitute for the results section and important information must be in the body of the report.

2 Presenting lab reports

In addition to the content, there are a set of professional standards that should be observed when created a technical engineering report. Ensure the documents you produce conform to these standards

2.1 Layout and Typesetting

There are a number of aspects of a technical document that should ALWAYS be present. These are:

- your name, student ID, institution name (The University of Sheffield) and your department name
- page numbers
- a title
- the date the document was written
- any other pertinent information for the report, such as the collaborators in the experiment or personal tutor's name.

The decision to include other layout features of the document is, to a certain extent, based on common sense. If a document is made up of several pages, a dedicated title page and contents page (indicating the page number that starts each section) may be appropriate. If the document contains a large number of tables, figures or equations, a list of these may appear at the start of the document.

Technical documents should have the content divided into logical pieces in order to make the information manageable for the reader. Sections can be further broken down into subsections and maybe into sub-subsections. The degree to which the document is sectioned should be appropriate for the size of the document. All sections and subsections should be numbered. This document is an example of how to use section and subsection numbers.

The key to creating a professionally presented document layout is consistency. If a certain font is used for sections or subsections, this should be the same throughout the document. If certain standards for how page numbers, line spacing or text justifications are adopted, this should not vary. Finally, and this almost goes without saying, all lab reports should be word processed and the spelling and grammar checked.

2.2 Figures and Tables

A technical engineering report will, mostly likely, contain pictures, diagrams, graphs and tables, in order to help convey information to the reader. All pictures, diagrams and graphs are considered "Figures" and any tabulated data is considered a "Table", Figures and Tables must be numbered sequentially in the order they appear and have titles, e.g.

Figure 1. A graph showing the relationship between stress and strain for a mild steel tensile specimen

or

Table 3. Dimensions of the specimens

If a table or figure is included in a document, it must be referred to in the body text. The reader will only know to look at a table or figure if instructed to do so while reading the document. Indication to the reader to look at a figure or table should be done using the figure number rather than the location on the page. For example, **do not** write:

the readings from the oscilloscope are shown in the table below

Instead write

the readings from the oscilloscope, as shown in Table 4.2.

When producing tables, ensure that the column and row headings are distinct from the other contents, the precision of the numbers is appropriate, and that units of the numerical values are clear. All graph axes should have labels and appropriate units. Common errors when producing graphs, which can occur using the default settings in software, are to

- produce series of data that are indistinguishable from one another (especially if a colour graph is printed in black and white)
- add an unnecessary legend when there is only one series of data
- include a title on the plot instead of (or as well as) using the numbered title described above
- have a range on the axis that produces too much whitespace
- have inappropriate precision for the axis numbering
- have a graph type that is inappropriate for the data (for example using bar graph for continuous, rather than discrete, data)

2.3 Equations, numbers and nomenclature

As with figures and tables, all equations should be numbered sequentially and then referred to in the body text using those numbers. Any nomenclature used in an equation needs to be defined. For example

Newton's second law of motion dictates that the force experienced by a solid body is the product of its mass and acceleration, as shown in equation 4.1,

$$F = m_b \times a \quad [4.1]$$

where F is the force on the body, m_b is the mass of the body and a is the body's acceleration.

In this case, the nomenclature has been defined with the equation. A symbol should be defined the first time it appears, but need not be defined with subsequent use. If a document contains a large number of equations, with regularly repeated symbols to denote physical parameters, it can be more appropriate to define all the symbols used in a nomenclature section at the start of the document.

When numbers are presented in the text of a document they should always be accompanied by an appropriate unit and quoted to the correct precision.

2.4 Language and style

The report should be grammatically sound, with correct spelling, and generally free of errors. The use of jargon, slang or colloquial terms should be avoided. The style of writing should be formal and precise, so that the meaning of sentences is clear and unambiguous with no unnecessary information. Define acronyms and any abbreviations not used as standard measurement units. The use of contractions (such as can't, isn't) and personal pronouns (subjective: I, you, he, she, we, they; or objective: me, you, him, her, us, them) are not appropriate for technical engineering documents.

Scientific and technical reports should be written in the third person and usually in the past tense, unless you are specifically referring to a prediction about the future. For example,

During the experiment, it was found that buckling occurs at loads greater than 15 kN

Or

It was found that efficiency could be increased by 17% by using the new material

2.5 Referencing & Plagiarism

If you copy pictures or information from a published source it must be referenced, otherwise you have plagiarised (i.e. stolen!) them. Therefore it is important to properly reference the primary source (i.e. the originator of the material, not someone who has referenced them; so Wikipedia is almost never a valid source). There are many styles of referencing. The library has a useful guide to referencing on their Information Skills Resource pages.

http://www.librarydevelopment.group.shef.ac.uk/referencing.html

As with all standards of a technical report, if a referencing style is adopted it should be consistent throughout the document.

3 Presenting results and data

Each lab will require different ways of looking at the data you collect; there is not one single 'data analysis' technique to learn, but it is important to know the range of methods available. This section summarises some of the approaches you should know and apply.

3.1 What is data?

It can be almost anything, but in terms of engineering it is normally numerical, and can be some measurement, experimental result, dependency, or event.

Table 3: Table of numbers used as an example of data

	1	2	3	4	5	6	7	8	9	10	11
a	4.82364	4.83259	3.57005	1.43533	4.66193	1.78892	3.01846	4.82364	4.83259	3.57005	1.43533
b	4.27232	6.2184	4.51902	2.91401	6.48955	3.80554	4.40747	3.3546	5.04946	2.51684	0.44293
c	5.24213	8.26946	9.84735	3.42236	7.91179	4.95712	6.15483	7.705	5.60667	7.6175	8.96083
d	3.3546	5.04946	2.51684	0.44293	3.97267	1.11522	2.61395	1.85115	3.05947	1.7842	0.993583
e	3.27397	5.25909	4.32896	1.86461	5.1291	2.44637	4.01061	1.54094	4.2876	5.10179	0.466502
f	4.98103	5.66918	4.66601	2.17074	5.56137	3.71159	5.19948	1.66021	2.62715	2.24575	1.48672
g	7.705	5.60667	7.6175	8.96083	4.78917	9.31167	8.4	4.14705	7.19573	6.58351	2.87798
h	2.78659	3.81197	3.34776	1.48663	4.45638	1.88005	2.44895	0.750786	0.778602	1.15623	0.209091
i	2.53497	3.81894	3.73222	1.93026	4.24599	2.20449	2.84811	0.648482	1.33915	1.1468	0.295838
j	1.85115	3.05947	1.7842	0.993583	2.59132	1.05959	2.24717	5.51351	4.18695	6.81261	2.93632
a	3.11308	3.47604	3.61033	2.24816	3.45288	2.27261	2.33611	0.954924	1.31511	0.983211	0.514119
b	4.24693	4.45319	4.1288	2.50872	4.6046	3.79175	2.56685	2.39961	2.17394	3.28255	1.7492
c	4.51085	3.45892	4.7743	2.80733	7.31364	2.75202	6.38688	0.631981	0.639996	1.49568	0.376927
d	1.54094	4.2876	5.10179	0.466502	6.79288	1.76346	1.4622	1.42354	1.97844	2.57294	1.28682
e	4.14705	7.19573	6.58351	2.87798	9.15683	3.99217	3.43829	1.7149	1.54046	1.19442	0.29018
f	4.26474	3.21329	10.5594	3.17532	7.78569	4.03504	4.44756	1.63051	2.00861	2.93171	-0.04549
g	1.66021	2.62715	2.24575	1.48672	2.27262	0.828575	2.73842	4.27232	6.2184	4.51902	2.91401
h	5.51351	4.18695	6.81261	2.93632	5.37107	3.61462	5.63621	3.27397	5.25909	4.32896	1.86461
i	6.24765	5.37011	6.67001	3.89066	6.69345	4.64867	5.83701	2.78659	3.81197	3.34776	1.48663
j	0.750786	0.778602	1.15623	0.209091	1.1501	0.209091	1.22412	3.11308	3.47604	3.61033	2.24816

It is almost always important to plot the data to reveal any dependencies or anomalies. Considering which plots will be of most use requires an understanding of the context in which the data arose.

We then manipulate this data, often using statistical methods, to look at, examine, and predict: numbers, populations, distributions, variations, models, and/or errors. It is beneficial to understand what kind of data is being gathered:

Univariate data is data that can be described by a single distribution such as heights, weights, speeds, times. This data records how one thing varies independently of anything else, and as such can only be described relative to itself, looking at trends and patterns within the data set. It is possible with such data to look at probabilities of an event e.g. runners in a marathon running sub 2½ hrs.

Bivariate data is more common in experimental tests where we see how one thing varies with another. This could be a simple experiment to observe how stress varies with strain, or load with deflection. Or could the result of ordering data against another factor, such as marathon times against body mass index (BMI).

A few definitions to start:

> **Mean** – the sum of the values divided by the number of samples.
>
> **Median** – the number where the value of half the samples have a value greater and half less.

Mode – the most common value.

Quartile – one quarter of the data, the first quartile is the top quartile of the data, the last quartile is the bottom quarter of the data.

Deciles and **centiles** are the equivalent of quartiles but for tenths and hundredths.

3.2 Sampling

Choosing the number of readings to take, or how many repeat tests to be carried out is called sampling:. It is vital to get a representative sample to review, therefore, deciding how many times to carry out a test or how many incremental measurements to take and in which situations is a key part of data gathering.

The target population is every situation that a researcher is interested in and wishes to draw conclusions about. There are different techniques for sampling (i.e. choosing which specific situations to study) from this population:

Random sampling: is where each situation in the sample is chosen entirely by chance and each member of the target population has a known, but possibly non-equal, chance of being included in the sample.

Cluster sampling: is where the target population is divided into groups, or clusters, and a random sample of these clusters are selected. This is typically used when the researcher cannot get a complete list of the members of a population they wish to study.

The number of situations to include will often depend on the cost or time associated with each test. The study of how best to design experiments has resulted in guidelines that need to be followed when testing medicines, and for many established mechanical tests there are guidelines in relevant international standards (often requiring at least 5 repeats of each situation of interest).

Bias and Precision: It is generally impossible to know how data is distributed relative to the true value. **Precision** is a measure of how close an estimator (e.g. the result of an experiment) is expected to be to the true value of a parameter; the larger the difference the less precise the measurement. **Bias** is a term which refers to how far the average statistic lies from the parameter it is estimating. Errors from chance will be seen in the precision of the value and will cancel each other out in the long run, those from bias will not. To illustrate this, you can represent the target value as the centre of a bulls eye, as shown in Figure 2, with different combinations of bias and precision.

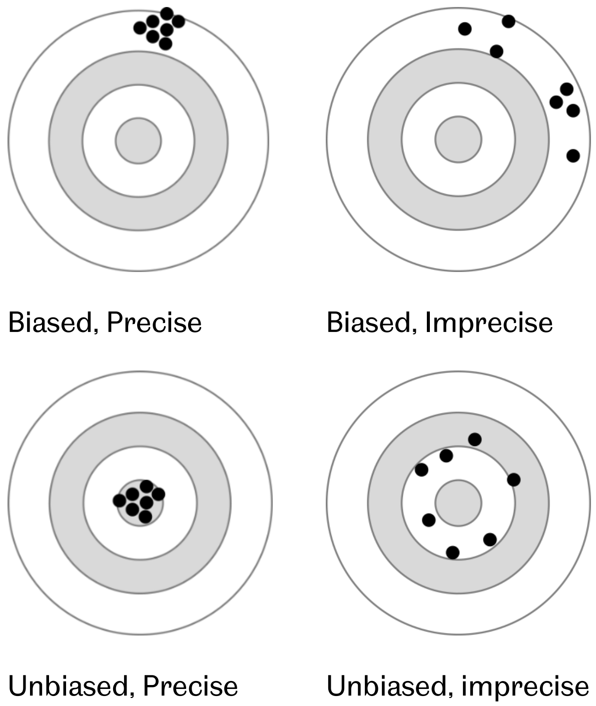

Figure 2: Analogy of a target used to demonstrate the principles of accuracy and precision.

3.3 Linear regression

On suitable plots, experimental data can often appear to lie along or close to a straight line. This indicates that linear regression through the data points, as shown in Figure 3, may be appropriate. There are several steps in creating the linear regression and then assessing its appropriateness to the data. The steps below give the equations that underpin the “Insert trendline” feature of Microsoft Excel and are also built into MATLAB.

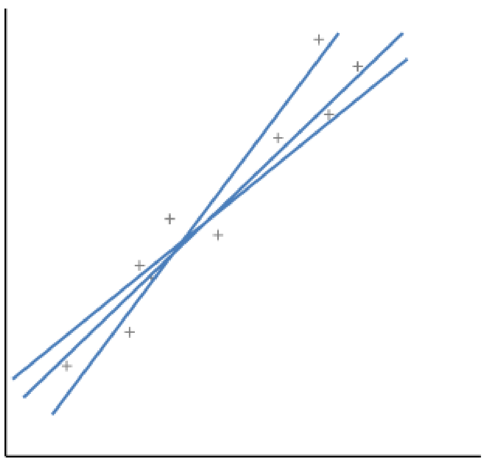

Figure 3: Example of the result of linear regression applied to a data set

Firstly, the slope is calculated using the least squares method and assuming a straight line equation of the form: $y = mx + c$:

$$m = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(x_i - \bar{x})^2}$$

where x_i is the ith sample and $\bar{x}$ denotes the mean value of x, $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$. This can be rearranged as follows for ease of use:

$$m = \frac{n(\sum x_i y_i) - (\sum x_i)(\sum y_i)}{n(\sum x_i^2) - (\sum x_i)^2}$$

Then the intercept:

$$c = \bar{y} - m\bar{x}$$

$$c = \frac{(\sum y_i)(\sum x_i^2) - (\sum x_i)(\sum x_i y_i)}{n(\sum x_i^2) - (\sum x_i)^2}$$

If a data set has columns containing values for x and y, it is possible to form extra columns containing calculated x^2, y^2 and xy, then sum each column to give the terms in these equations. See the example in Table 4 and Figure 4.

Example:

	x	y	x²	y²	xy
	0.369704	-2.38803	0.136681	5.702687	-0.88286
	1.588133	0.402435	2.522166	0.161954	0.63912
	2.829449	4.956806	8.005782	24.56993	14.02503
	3.594139	7.415136	12.91784	54.98424	26.65103
	4.592764	10.31141	21.09348	106.3252	47.35787
	5.526358	12.64368	30.54063	159.8626	69.8735
	6.709325	16.45262	45.01504	270.6887	110.386
	7.687919	18.93695	59.1041	358.6081	145.5857
	8.761176	22.72286	76.7582	516.3284	199.079
	9.683235	25.73131	93.76504	662.1003	249.1623
	10.73824	27.8915	115.3098	777.9358	299.5056
Sums	**62.08044**	**145.0767**	**465.1688**	**2937.268**	**1161.382**

Table 4: Example data used to demonstrate the process of linear regression

$$m = \frac{(11 \times 1161.4) - (62.1 \times 145.1)}{(11 \times 465.2) - (62.1)^2} = 2.99$$

$$c = \frac{(145.1 \times 465.2) - (62.1 \times 1161.4)}{(11 \times 465.2) - (62.1)^2} = -3.67$$

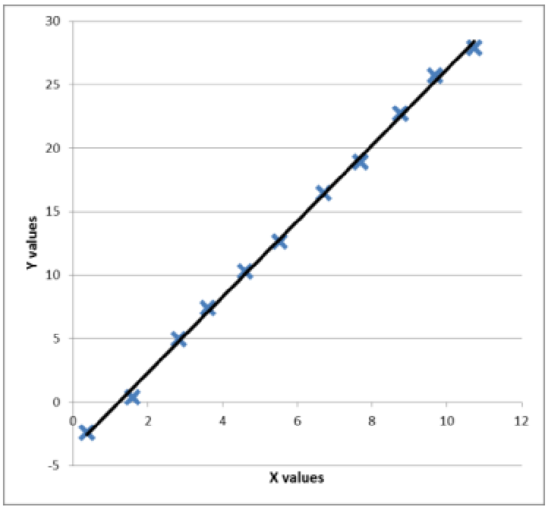

Figure 4: Example of linear regression line plotted against original data

3.4 Goodness of fit

Goodness of fit is a measure of correlation - how well a relationship represents the data. For linear relationships the most common measure is Pearson's correlation coefficient which is denoted by r (or sometimes by R in Excel). This coefficient measures the strength of a linear relationship and is always between -1 and +1

-1 means there is a perfect negative linear correlation (all points lie on a line of negative slope)

+1 means there is a perfect positive linear correlation (all points lie on a line of positive slope)

The correlation coefficient always has the same sign as the slope of the regression line. It does not change if the independent (x) and dependent (y) variables are interchanged. Neither does it change if the scale on either variable is changed (i.e. you may multiply, divide, add or subtract from the entire x, y variables without changing the value of r).

Pearson's correlation coefficient:

$$r = \frac{n(\sum x_i y_i) - (\sum x_i)(\sum y_i)}{\sqrt{(n\sum x_i^2 - (\sum x_i)^2)(n\sum y_i^2 - (\sum y_i)^2)}}$$

For the previous example:

$$r = \frac{(11 \times 1161.4) - (62.1 \times 145.1)}{\sqrt{(11 \times 465.2 - 62.1^2)(11 \times 2937.3 - 145.1^2)}} = 0.99$$

In order to state the magnitude of r, typically r^2 is quoted.

3.5 Variance

The mean average of a set of data can be written in the form:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

The variance of a set of data is a non-negative number which indicates how widely the values of the data are spread. The larger the variance the further that individual values of the data tend to be from the mean. Stating the variance gives an impression of how closely concentrated around the expected value the distribution is; it is a measure of the 'spread' of a distribution about its average value. Variance is symbolised by *Var*(x) or σ^2.

Taking the positive square root of the variance gives the standard deviation, $\sigma(x)$,i.e.

$$Var(x) = \sigma(x)^2$$

The variance and standard deviation of a set of data are always positive. Variance is the sum of the square of the distance away from the mean that each value is, divided by the number of values, see Table 5.

$$Var(x) = \frac{1}{n}\sum_{i=1}^{n}(\bar{x} - x_i)^2$$

	x	$(\bar{x} - x_i)$	$(\bar{x} - x_i)^2$	
	2.62	-0.08	0.01	
	2.57	-0.03	0.00	
	1.15	1.39	1.93	
	2.28	0.26	0.07	
	2.90	-0.36	0.13	
	3.72	-1.18	1.39	
Sum	**15.24**		**3.53**	
Sum/n	**2.54**	Average	**0.59**	Variance

Table 5: Example data used to demonstrate variance

The variance equation can be rearranged to give:

$$Var(x) = \frac{1}{n}\sum_{i=1}^{n}(\bar{x} - x_i)^2 = \frac{1}{n}\sum_{i=1}^{n} x^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n} x\right)^2$$

So that variance can be worked out from x and x^2 terms.

3.6 Distributions

If the numbers of occurrences of values in univariate data are plotted (it is necessary to choose a set of ranges of values and count the number in each range) it will reveal the distribution trend. The most common is called a **Normal/Gaussian distribution**, shown in Figure 5.

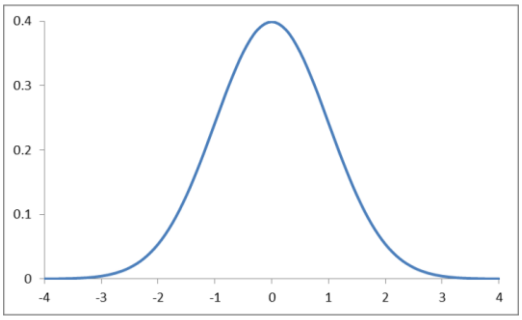

Figure 5: Shape of a normal distribution

It has the equation:

$$y = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

where μ is the mean and σ the variance

The area under the curve is 1, with the mean halfway along the distribution, and the standard deviations are evenly spread out from the mean, 68% of the data is within one standard deviation and 95% within two standard deviations.

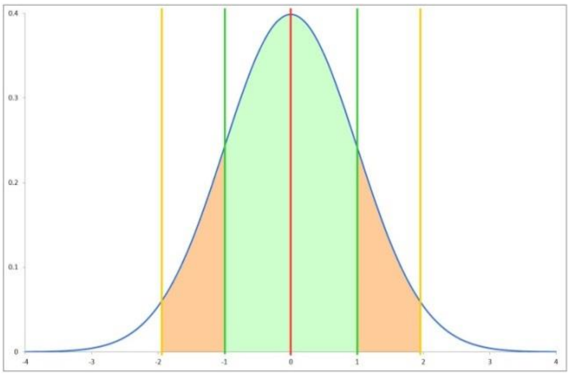

Figure 6: A normal distribution showing standard deviations

If your results can be normalised it is possible to use a standard normal distribution look up table to produce values. This standard score (z-score) allows you to calculate the probability of it occurring within the normal distribution. Table 6 is part of a standard normal distribution look up table, which states the probability for each z-score (the value of z is rounded to 2 decimal places, the first gives the column and the second, the row in which the probability is found).

Table 6: Standard Normal Distribution z-score table

z	**0**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0	0.5	0.504	0.508	0.512	0.516	0.519	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.591	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.648	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.67	0.6736	0.6772	0.6808	0.6844	6879
0.5	0.6915	0.695	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.719	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7157	0.7549
0.7	0.758	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.791	0.7939	0.7969	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.834	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8513	0.8554	0.8577	0.8529	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.877	0.879	0.881	0.883
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.898	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9215	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.937	0.9382	0.9394	0.9406	0.9418	0.9492	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.975	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.983	0.9834	0.9838	0.9842	0.9846	0.985	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.989
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.992	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.994	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.996	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.997	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.998	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.999	0.999
3.1	0.999	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Using the central limit theorem, the z-score is given by:

$$z = \frac{\bar{x} - \mu}{\sigma}$$

Example: The average height for UK women is 1.637 m with a standard deviation of 0.065. Assuming a normal distribution what percentage of the population is larger than 1.59m?

Calculate the z-score

$$z = \frac{1.637 - 1.59}{0.065} = 0.72 \text{ to 2 d.p.}$$

Look up value in table

z	**0**	**0.01**	**0.02**	**0.03**
0	0.5	0.504	0.508	0.512
0.1	0.5398	0.5438	0.5478	0.5517
0.2	0.5793	0.5832	0.5871	0.591
0.3	0.6179	0.6217	0.6255	0.6293
0.4	0.6554	0.6591	0.6628	0.6664
0.5	0.6915	0.695	0.6985	0.7019
0.6	0.7257	0.7291	0.7324	0.7357
0.7	0.758	[illegible]	0.7642	0.7673
0.8	0.7881	0.791	0.7939	0.7969
0.9	0.8159	0.8186	0.8212	0.8238

Figure 7: Example of how to use the Standard Normal Distribution z-score table

Value = 0.7642, therefore 76% of the female UK population is taller than Dr Rowson. The Excel function "NORM.S.DIST(0.72,TRUE)" and the MATLAB command "normcdf(0.72)" also return this value.

Poisson Distribution: is a discrete distribution which takes on the values x = 0, 1, 2, 3, ... It is often used as a model for the number of events (such as the number of telephone calls at a business) in a specific time period. It is determined by one parameter, λ. The distribution function for a Poisson distribution is

$$f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

Where $x!$ denotes factorial (e.g. $4! = 4 \times 3 \times 2 \times 1$). It can be used to determine the probability of a rare event.

Example: Assuming the probability of carrying a particular genetic mutation is described by a Poisson distribution with a mean of 2.3 occurrences per 10,000, as shown in Figure 8. What is the probability of observing a local occurrence rate of 6 per 10,000 (assuming that nothing is acting to increase the rate)?

$$P(6) = \frac{e^{-2.3}2.3^{6}}{6!} = \frac{0.1003 \times 148.04}{720} = 2.1\%$$

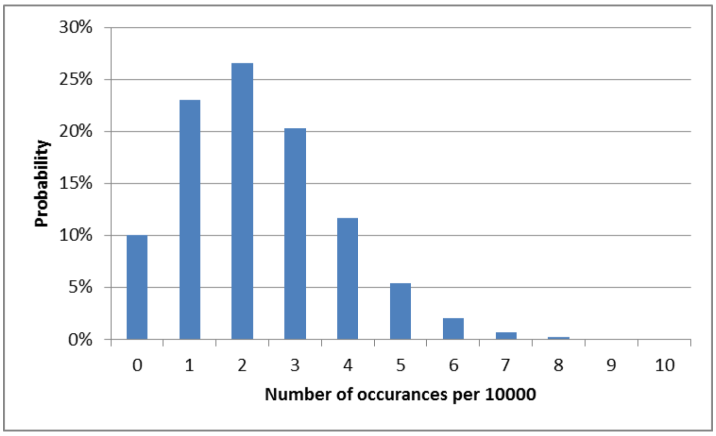

Figure 8: Analogy of a target used to demonstrate the principles of accuracy and precision.

Note: Even if the main population behaves in a normal way the extremes may not.

4 Understanding and manipulating error

This section is to help you understand measurements and their significance. It is always important to think about where possible sources of error may occur in your experiment. This will allow you to estimate the cumulative error and estimate the reliability of your measurements. To introduce the ideas some dictionary definitions are helpful:

4.1 Errors and Uncertainty

Error: a measure of the estimated difference between the observed or calculated value of a quantity and its true value.

Uncertainty: the lack of certainty, the estimated amount or percentage by which an observed or calculated value may differ from the true value.

It is unfortunate that 'error' has the alternative definition that something is 'wrong', and it is helpful to remember that 'errors' in your measurements refer to uncertainties, and not necessarily that your measurements are wrong.

In order to tell if a measurement is significant there needs to be an awareness of the level of error.

e.g. two measurements of body temperature, before and after a drug is administered were measure as 38.2 °C and 38.4 °C

Is the temperature rise significant? – depends on the associated errors

(38.2±0.01) °C and (38.4±0.01) °C *a significant change*

(38.2±0.5) °C and (38.4±0.5) °C *not significant*

Common practice is to assume that a value is quoted to the precision to which it is known so that this example would be

(38.2±0.05) °C and (38.4±0.05) °C *a significant change*

Random errors (also known as reading errors): an error that varies between successive measurements, it is equally likely to be positive or negative. They are always present in an experiment and are obvious from the distribution of values obtained. Random errors can be minimised by performing multiple measurements of the same quantity or by measuring one quantity as a function of a second then performing a straight line fit of the data.

Systematic errors: an error that is constant throughout a set of readings, it may result from equipment that has not been correctly calibrated or due to how the measurements are performed. These errors cause the average (mean) of measured values to depart from the correct value. It is often difficult to spot the presence of systematic errors in an experiment.

Epistemic errors: errors due to lack of knowledge, e.g. not knowing that the equipment wasn't calibrated.

Artifacts: a variation in the measured quantities that occurs as a result of the measurement procedure.

Human errors: these are a subset of random errors and are dependent on a personal reaction or style; they can be an error due to line-of-sight readings, or reaction time in starting a stop watch.

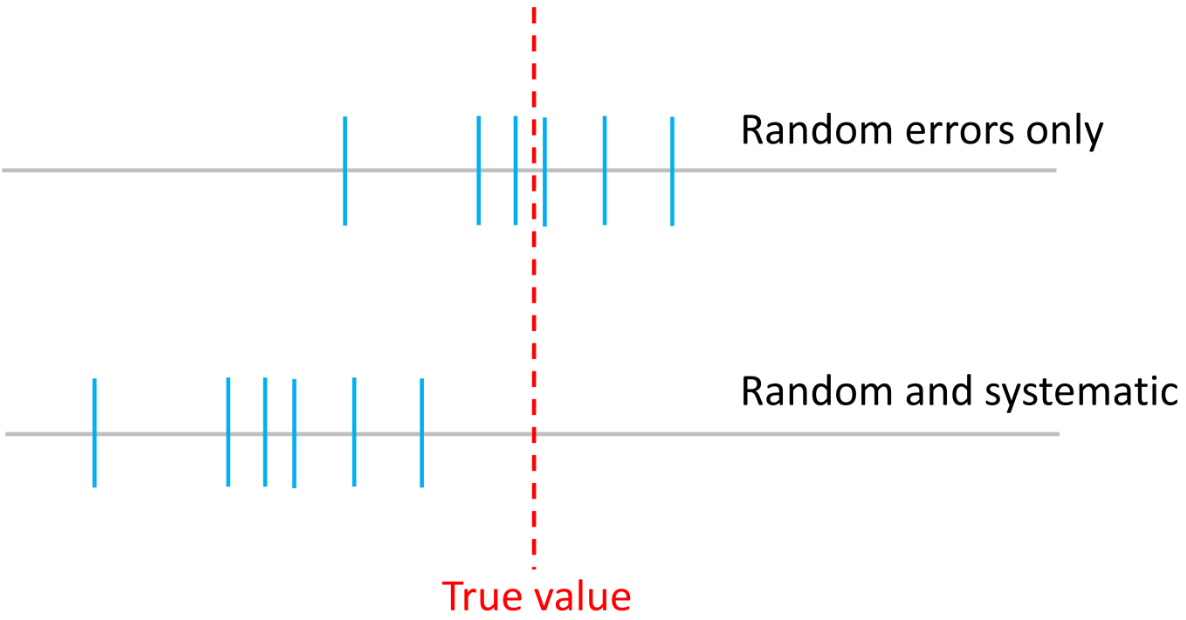

Figure 9: Demonstration of random vs systematic error

A result is said to be accurate if it is relatively free from systematic error, and said to be precise if the random error is small.

When quoting results and errors it is generally accepted that you state the error to one significant figure, although in a small number of cases this may be extended to two significant figures. It is important that the result is quoted to the same significance as the error and when using scientific notation, quote the value and error with the same exponent.

Value 44, error 5 → 44±5

Value 128, error 32 → 130±30

Value $4.8x10^{-3}$, error $7x10^{-4}$ → $(4.8±0.7)x10^{-3}$

Value 1092, error 56 → 1090±60

Value 12.345, error 0.35 → 12.3±0.4

Note that some errors can cancel out.

The convention is that, unless indicated otherwise, a number can be assumed to be accurate to the precision quoted. So that values of 31.1 or 31.10 would be 31.1±0.05 or 31.1±0.005, respectively.

If the error is not quoted explicitly, then always choose the units of the quantities that you quote to ensure the reader knows the precision. E.g. 31.10 MPa would be assumed to be 31.10 ± 0.005 MPa, whereas if this were quoted as 31,100,000 Pa the reader would probably not guess the correct error bounds.

4.2 Examples of estimating reading errors

Oscilloscope – shown in Figure 10, the reading error is related to the width of the trace (~0.2 division), scale is 169.2 mV/division. The reading would be stated as (16.9±0.2)mV.

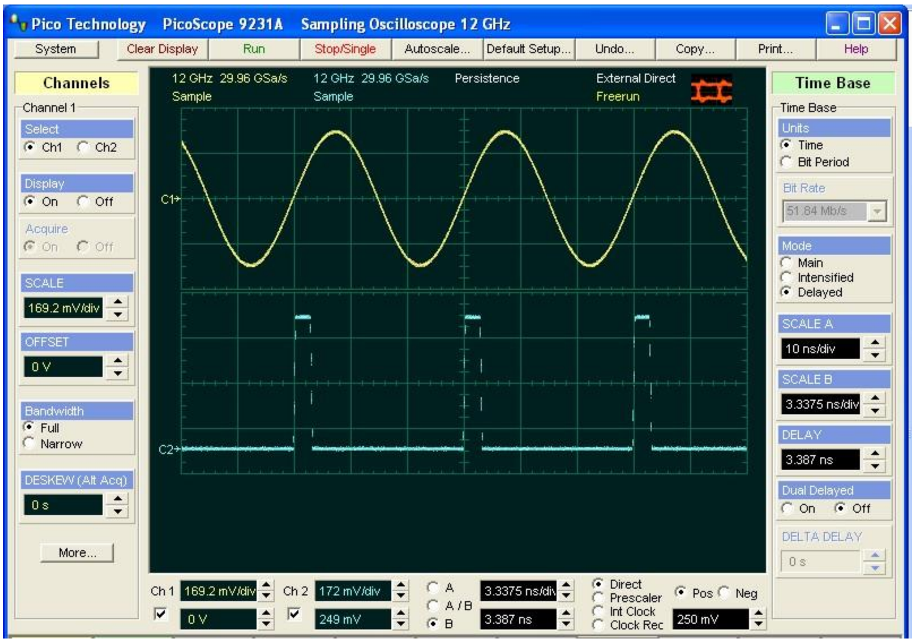

Figure 10: Typical image displayed on oscilloscope with 169.2mV/division

Analogue meter, as shown in Figure 11 : the error is related to width of pointer e.g. pointer has a width of 0.1V so reading would be stated as (0.0±0.1) V

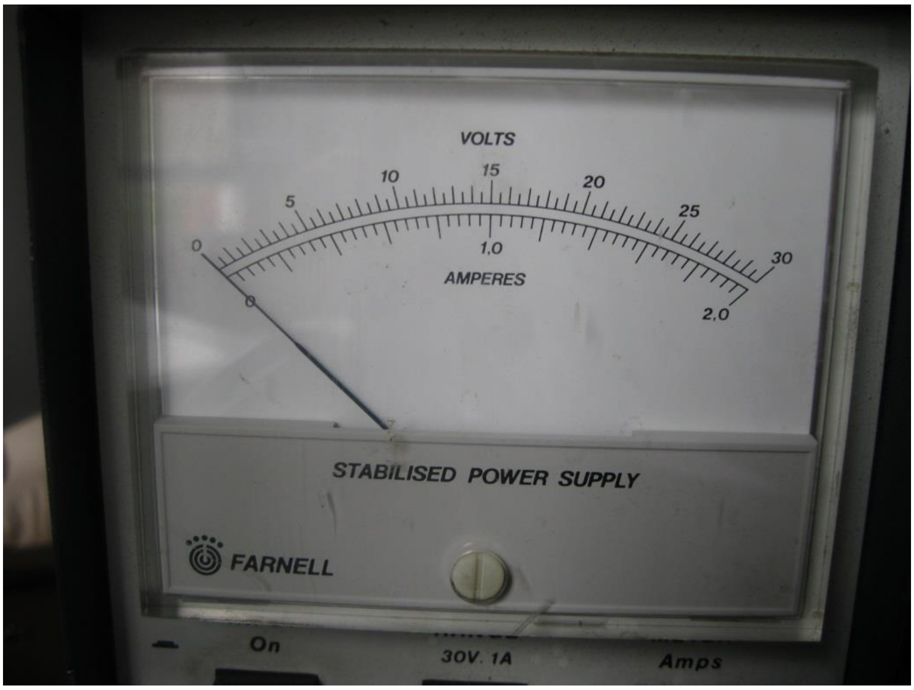

Figure 11: Analogue meter for measuring current and voltage.

Digital meter, as shown in Figure 12: error is taken as ±5 in the next significant figure e.g. (32.5480±0.0005) Hz.

Figure 12: Digital meter showing frequency of waveform to nearest 0.0005 Hz

Linear scale, as shown in Figure 13: need to estimate the precision with each measurement made, it may be a subjective choice (38.42±0.02) cm

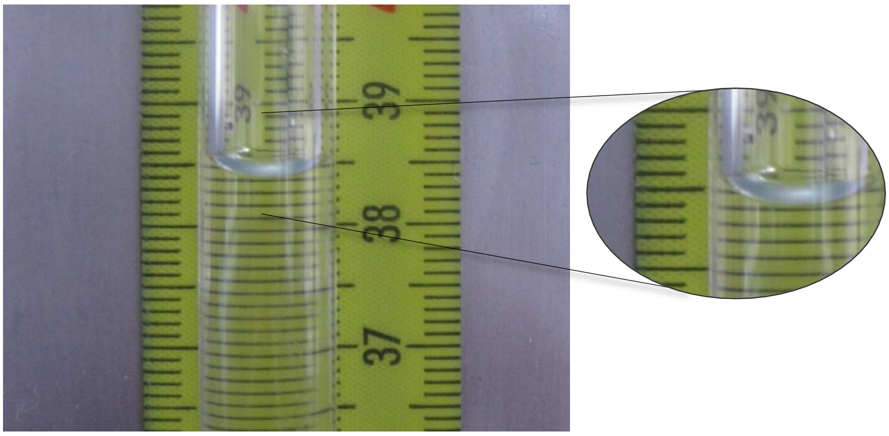

Figure 13: Linear scale of length, to nearest 0.02 cm

4.3 Examples of *corrections* as opposed to *errors*

- Corrections may allow for a false **instrumental reading**. For example, a rotameter calibrated for air may be used for another gas if a density correction is made to the observed readings.
- Correction to data to bring an experimental result to standard conditions. For example, correction of a measured gas volume at a particular temperature and pressure to the standard condition of 0 °C and 1 bar.

You should try to eliminate as many different sources of error as possible. This way, it will give you the most accurate measure of the desired parameters, and a trustworthy value. Of course, it is impossible to make any experimentally determined value completely error free.

4.4 Combining errors

The degree of error in two (or more) variables will combine to create an overall error. The way they combine depends upon how the variables are related.

In general we will calculate a result using a formula which has as an input one or more measured values. For example: volume of a cylinder$A = \pi r^2 h$. How errors in the measured values feed through to the final results is an important part of understanding the significance of your result.

If $A, B, C\ and\ Z$ are absolute values, and $\Delta A, \Delta B, \Delta C\ and\ \Delta Z$ are their respective absolute errors then $\Delta A/A$ is the fractional error in A, and $100 \cdot \left(\frac{\Delta A}{A}\right)$% is the percentage error. A method of treating errors in formulae is give in Figure 14. This assumes that the errors represent spreads of the distributions of values and gives expected spreads of the distributions of the results. Often this is smaller than the range that would be predicted by directly using the limits of the values (see examples below).

$$Z = A \pm B \pm C \qquad (\Delta Z)^2 = (\Delta A)^2 + (\Delta B)^2 + (\Delta C)^2$$

$$Z = ABC \text{ or } AB/C \text{ etc} \qquad \left(\frac{\Delta Z}{Z}\right)^2 = \left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2 + \left(\frac{\Delta C}{C}\right)^2$$

$$Z = A^n B^m C^l \qquad \left(\frac{\Delta Z}{Z}\right)^2 = \left(n\frac{\Delta A}{A}\right)^2 + \left(m\frac{\Delta B}{B}\right)^2 + \left(l\frac{\Delta C}{C}\right)^2$$

$$Z = \ln A \qquad \Delta Z = \frac{\Delta A}{A}$$

$$Z = \exp A \qquad \frac{\Delta Z}{Z} = \Delta A$$

Figure 14: Treatment of errors in formulae

Example 1:

$$A = \pi r^2 \quad where\ r = (5 \pm 0.5)m$$

$$A = 78.5398\ m^2 \text{ (raw theoretical value)}$$

$$\frac{\Delta r}{r} = \frac{0.5}{5} = 0.1$$

$$\left(\frac{\Delta A}{A}\right)^2 = \left(2\frac{\Delta r}{r}\right)^2 = (2 \times 0.1)^2 = 0.04$$

$$\Rightarrow \frac{\Delta A}{A} = 0.2 \quad hence \quad \Delta A = 0.2A \quad \Delta A = 0.2(78.5398\ m^2) = 16\ m^2$$

Hence the final result is $A = (79 \pm 16)\ m^2$

Example 2:

$$P = 2L + 2W \quad where\ L = (4 \pm 0.2)m\ and\ W = (5 \pm 0.2)m$$

$$P = 18\ m$$

$$(\Delta P)^2 = (0.2)^2 + (0.2)^2 = 0.08$$

$$\Rightarrow \Delta P = 0.28\ m$$

Hence the final result is $P = (18 \pm 0.3)m$, but note that using the extreme values for L and W would give $P = (18 \pm 0.8)\ m$

Example 3:

$$\tau = 2\pi\sqrt{\frac{l}{g}} \qquad where\ l = (2.5 \pm 0.1)m\ and\ g = (9.8 \pm 0.2)m/s^2$$

$$\tau = 3.1735\ s$$

$$\left(\frac{\Delta\tau}{\tau}\right)^2 = \left(\frac{1}{2}\frac{\Delta l}{l}\right)^2 + \left(-\frac{1}{2}\frac{\Delta g}{g}\right)^2 = \left(\frac{1}{2}\times\frac{0.1}{2.5}\right)^2 + \left(-\frac{1}{2}\times\frac{0.2}{9.8}\right)^2 = 5.04\times10^{-4}$$

$$\frac{\Delta\tau}{\tau} = 0.022 \quad hence\ \Delta\tau = 0.022\times3.1735 = 0.070$$

Hence the final result is $\tau = (3.17 \pm 0.07\,)s$, whereas using 2.4 with 10 and 2.6 with 9.6 gives $\tau = (3.17 \pm 0.10)\ s$

To combine random and systematic errors (if known) add the squares of the separate errors. An example of which is: A length is measured with a reading (random error) given by $(89 \pm 2)cm$ using a rule of calibration accuracy 2 %.

$$\left(\begin{matrix}Total\ fractional\\ error\end{matrix}\right)^2 = (fractional\ reading\ error)^2 + (systematic\ error)^2$$

$$\left(\begin{matrix}Total\ fractional\\ error\end{matrix}\right)^2 = \left(\frac{2}{89}\right)^2 + (0.02)^2 = 0.000905$$

$$Total\ fractional\ error = 0.03$$

$$Absolute\ error = 0.03\times89 = 2.7\ cm$$

$$Value = (89 \pm 3)\ cm$$

4.5 The statistical nature of errors

When stating the total error associated with a value, this is not the maximum possible range of values. Instead the total error provides information concerning the probability that the value falls within certain limits.

If measurements of a quantity σ have a normal distribution with a standard deviation of Δσ then there is a 67% chance that the true value lies within the range (σ-Δσ) to (σ+Δσ). There is a 95% chance that the true value lies within the range (σ-2Δσ) to (σ+2Δσ), see Figure 15. It is often more reasonable to assume that the observed error is related to the standard deviation of a normal distribution than that it provides an absolute limit to all possible measurements.

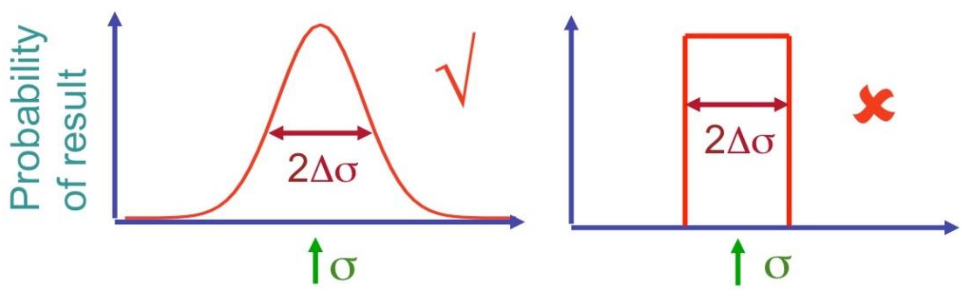

Figure 15: Distribution of possible results for a value with an associated error

When comparing values, it is therefore important to look at the overlap of the distributions. For example, consider two quantities σ_A and σ_B normally distributed with standard deviations $\Delta\sigma_A$ and $\Delta\sigma_B$. that differ by $\Delta\sigma_A + \Delta\sigma_B$:

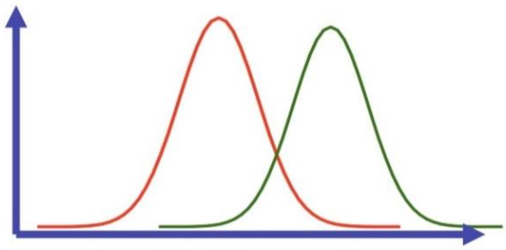

Figure 16: Overlap of potential errors in two different readings

The probability of agreement $\sim 2 \times \frac{1}{36} = 6\ \%$

4.6 Reducing errors

One way to reduce error and increase confidence in the measurements made is to repeat tests many times and calculate an average value. Repeating experiments many times will give you a scatter of values, which will lie within a boundary of the true value given by the single sample error.

For instance, consider a data set that can be expected to be described by $a=Pb$, where a and b are variables and P is a proportionality constant. Each of the values of a and b could be used to arrive at a value of P, with a similar degree of error, e_i. However, if the data set was plotted and a line of best fit applied, the gradient of the line could be used to obtain P. This now uses all of the data points and so is effectively a form of averaging. The total standard error, e_N, now becomes:

$$e_N = \frac{e_i}{\sqrt{N}}$$

where N is the number of data points. So, **the overall error in the value of P is reduced by $\sqrt{N}$ compared to the single data point case**.

This is similar to if data were measured at the same point repeatedly, i.e. for fixed a measure b many (N) times. However, it is often more convenient to obtain a series of changing data values in an experiment.

5 An example lab report

Determination of the Spring Constant and Natural Frequency of Bungee Cord for Varying Applied Load

Ben G. Cord

12th September 2010

Summary

An experiment was carried out to measure the spring constant of a piece of bungee cord for a range of applied loads. The results were used to calculate the variation in natural frequency. The natural frequency for a particular load was measured experimentally and compared with the predicted natural frequency, calculated from the spring constant. The two values were found to be in good agreement.

Nomenclature

k	Spring constant	N m^{-1}
m	Mass	N
x	Displacement	m
g	Gravitational constant	9.81 m s^{-2}
ω_n	Natural resonant frequency (Hz)	s^{-1}

Introduction

Bungee cord has a variety of uses, including strapping down loads during transit and in the recreational activities of 'Bungee jumping' and sailing. The elastic behaviour of bungee cord is quite complex, as it exhibits creep and visco-elastic properties.

The aim of this experiment was to determine how the elastic behaviour of a piece of bungee cord varied with applied load.

The objectives of the experiment were:

1. To apply increasing load to a piece of bungee cord and measure the deflection.
2. To examine the relationship between spring constant and applied load.
3. To calculate the natural frequency from spring constant values, at various loads.
4. To compare an experimental value of natural frequency with a predicted value.

Theory

Hooke's law [1] for an ideal spring states that the spring constant (or stiffness) k (Nm^{-1}) is the gradient of the force-displacement graph (where m is the applied mass).

$$k = \frac{\text{Force (Newtons)}}{\text{Extension (metres)}} = \frac{mg}{x} \qquad \textbf{(1)}$$

For a non-linear spring, the constant k at any point is the differential of this.

$$k = g\frac{dm}{dx} \qquad \textbf{(2)}$$

In terms of experimental points,

$$k = g\frac{\Delta m}{\Delta x} = g \times \left(\frac{m_i - m_{i-1}}{x_i - x_{i-1}}\right) \qquad \textbf{(3)}$$

For a system that exhibits simple harmonic motion, the natural frequency, ω_n (in Hz), is given by [2] and was originally described by Rayleigh, [3],:

$$\omega_n = \left(\frac{1}{2\pi}\right)\sqrt{\frac{k}{m}} \qquad \textbf{(4)}$$

Procedure

The apparatus was set up as shown in Figure 1. A piece of bungee cord was hung from a loading frame and a 50 g mass carrier was attached to the bottom end of the cord. A measuring scale was then attached to the loading frame, and adjusted so that the bottom edge of the mass was aligned with a zero reading on the scale.

Masses were then added to the carrier in increments of 50 g and 100 g until a total of 1100 g was attached to the cord. As each mass was applied, the position of the bottom edge of the mass carrier was measured on the scale and recorded.

The mass was then reduced to 250 g and the cord stretched slightly downwards and released. This allowed the cord and mass to behave as a spring-mass system, exhibiting simple harmonic motion. The time taken for four oscillations to occur was measured using a stop watch and recorded.

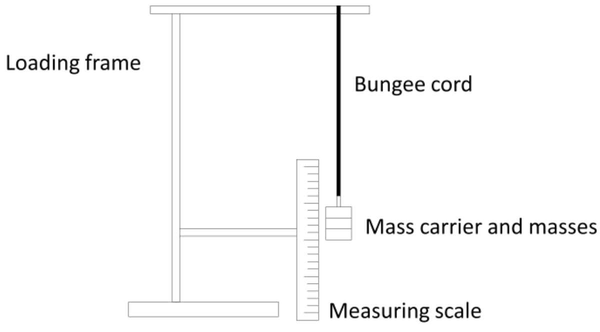

Figure 1. Apparatus for applying load to bungee cord and measuring deflection.

Results

Figure 2 shows the deflection of the bungee cord relative to the applied load.

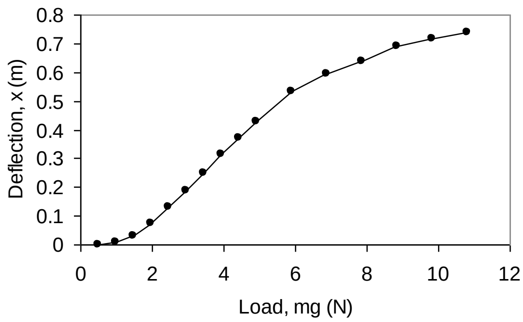

Figure 2. Deflection of bungee cord vs. applied load.

In Table 1, one can see the data collected; as each mass was applied the deflection was measured and recorded. The load and deflection were then used to calculate the spring constant and natural frequency, see following for sample calculations.

Table 1. Data from experiment and calculated values of spring constant and natural frequency.

Applied Mass g	Measured Deflection mm	Applied Mass kg	Applied Load N	Measured Deflection m	Spring Constant, k N/m	Natural Frequency, ω_n Hz
50	0	0.05	0.49	0	nda	nda
100	10	0.10	0.98	0.010	49.1	3.52
150	32	0.15	1.47	0.032	22.3	1.94
200	75	0.20	1.96	0.075	11.4	1.20
250	129	0.25	2.45	0.129	9.08	0.96
300	189	0.30	2.94	0.189	8.18	0.83
350	250	0.35	3.43	0.250	8.04	0.76
400	316	0.40	3.92	0.316	7.43	0.69
450	372	0.45	4.41	0.372	8.76	0.70
500	429	0.50	4.91	0.429	8.61	0.66
600	535	0.60	5.89	0.535	9.25	0.63
700	595	0.70	6.87	0.595	16.3	0.77
800	640	0.80	7.85	0.640	21.8	0.83
900	690	0.90	8.83	0.690	19.6	0.74
1000	715	1.00	9.81	0.715	39.2	1.00
1100	740	1.10	10.79	0.740	39.2	0.95

Sample calculations at applied mass of 250g

Calculating spring constant, *k*, from Equation 3,

$$k = g \times \left(\frac{m_i - m_{i-1}}{x_i - x_{i-1}} \right) = 9.81 \times \left(\frac{0.25 - 0.20}{0.129 - 0.075} \right) = 9.08 \text{ N/m}$$

Calculating natural frequency, ω_n,from Equation 4,

$$\omega_n = \left(\frac{1}{2\pi} \right) \sqrt{\frac{k}{m}} = \left(\frac{1}{2\pi} \right) \times \sqrt{\left(\frac{9.08}{0.25} \right)} = 0.96 \text{ Hz}$$

Figure 3 shows the calculated spring constant with repect to the load applied.

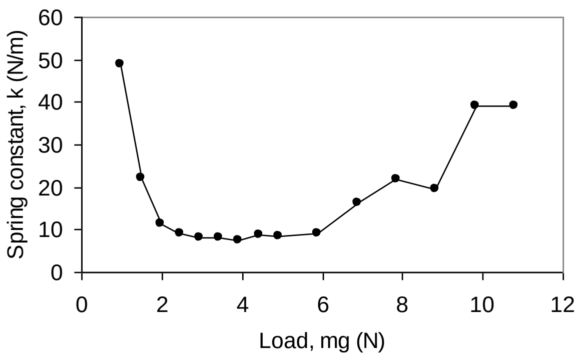

Figure 3. Spring constant of bungee cord vs. applied load.

The time taken for four oscillations of the system, with an applied mass of 250 g was 4.11s. Therefore, the measured natural frequency was 4/4.11 = 0.97 Hz

Discussion

Figure 2 shows the relationship between the deflection of the bungee cord and the applied load. This is initially non-linear, but for loads in the range of 2 to 6 N, the relationship can be seen to be linear. Above 6N, the gradient of the curve in Figure 2 begins to decrease, indicating that the spring constant increases.

The spring constant (or stiffness) for each applied load was calculated by dividing the increase of each applied load, by the corresponding increase in deflection (see sample calculation in Results section). This data is shown in Table 1. Figure 3 shows the relationship between the spring constant of the bungee cord and the applied load. At low loads (0 to 2 N) a relatively high spring constant was measured, indicating that the spring-mass system was initially stiff and became less stiff as increased load was applied. At loads ranging from 2 to 6 N, the spring constant was found to be fairly constant, at approximately 9 N/m. As the applied load was increased above 6 N, the system became increasingly stiff again and the spring constant increased.

If the load had been increased further still, is thought that the stiffness of the system would gradually level off to a constant value, until the point at which the cord snapped.

The natural frequency of the bungee cord for each applied load was calculated from the spring constant values using Equation 4 (see sample calculation in Results section). This data can be seen in Table 1. The predicted value of natural frequency for an applied mass of 250 g (applied load of 2.45 N) was 0.96 Hz. The natural frequency measured by experiment was 0.97 Hz, showing excellent agreement (approximately 1% different).

There were a number of sources of error in this experiment. The deflection of the cord could only be measured to ± 1 mm and the scale could only be originally placed to the same degree of accuracy. This led to an inaccuracy of up to 20% for the smaller deflection measurements (around 10 mm). For larger deflections (up to 740 mm), this inaccuracy reduced to 0.3%. Further error could have been introduced by deflection of the loading frame and slippage of the attachments at each end of the cord.

Another error arose from the assumption shown in Equation 3, that the gradient of the force-deflection graph can be approximated using the finite difference between consecutive data points. This error may be reduced by using more data points and a more accurate method of gradient approximation.

The main error in the measurement of the natural frequency was caused by the human reaction time of operating the stop watch and assessing the point at which four oscillations had occurred. These sources of error may be reduced by using a longer piece of cord, which would oscillate more slowly and by averaging over a large number of measurements.

Errors

For the spring stiffness, the main error arises from the measurement of the displacement. It is possible to read the scale to an accuracy of 1 mm. For a mean reading of 300 mm, this gives an error of

$$1/300 = 0.3\%$$

which is negligible. When calculating the resonant frequency, as

$$\omega_n = \left(\frac{1}{2\pi}\right)\sqrt{\frac{k}{m}}$$

the error is going to be the square root of this.

$$1 - \sqrt{1.003} = 0.15\%$$

When working with small numbers like this, it is possible to multiply the error by the exponent (this come out of the Taylor's expansion).

For the resonant frequency, however, the error is made up of two components. Firstly the time, the error on a stopwatch can be estimated at about a quarter of a second. For a time 0f 4.11 seconds, this gives an error of

$$^{0.25}/_{4.11} = 6\%$$

It is also possible to judge the bottom of the final swing to within about 1/10th of a swing as it is small by this time. For four swings, this gives an additional error of

$$^{0.1}/_{4} = 2.5\%$$

So the total error for the resonant frequency is

$$1 - (1.06 * 1.025) = 1.087 = 8.7\%$$

Which is $\pm$0.084 Hz. Once again, for small errors, you can get away with adding the percentages.

Conclusions

The elastic behaviour of bungee cord was found to be non-linear for varying applied loads apart from in the range of 2 to 6 N.

At applied loads ranging from 2 to 6 N, the spring constant was found to remain constant at approximately 908 $\pm$0.027 N/m. At loads above and below this range, the spring constant was higher, and the spring-mass system was therefore stiffer.

For an applied mass of 250 g, the measured natural frequency (0.97 $\pm$0.084 Hz) was found to be in good agreement (and within the predicted errors) when compared with the natural frequency predicted using the equation for a spring-mass system with the calculated spring constant (0.96 Hz).

References

1. Gere, JM and Timoshenko, SP, Mechanics of materials, 3rd edition, page 22, Chapman and Hall, London, 1991
2. Meriam, JL, and Kraige, LG, Engineering Mechanics Volume 2 Dynamics, 3rd Edition, page 595John Wiley and Sons inc, New York, 1993
3. Strutt, JW, 3rd Baron Rayleigh, The theory of sound, Chapter 1, London, 1877.