Lecture 10

University of Massachusetts
UMASS Lowell
Physics I
10.09.2013

Chapter 6

Gravitation and Newton's Synthesis

Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsI

Lecture Capture:
http://echo360.uml.edu/danylov2013/physics1fall.html

Outline

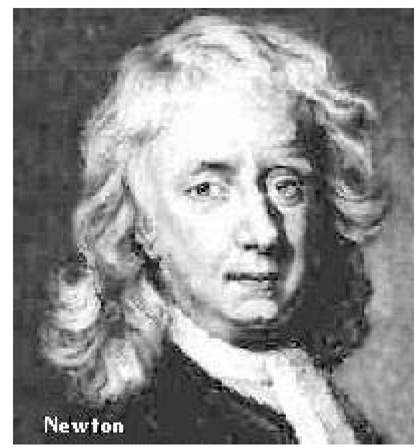
Chapter 6

- Newton's Law of Universal Gravitation
- Kepler's Laws

Heaven and Earth, Apple and Moon

A classic example of scientific advancement

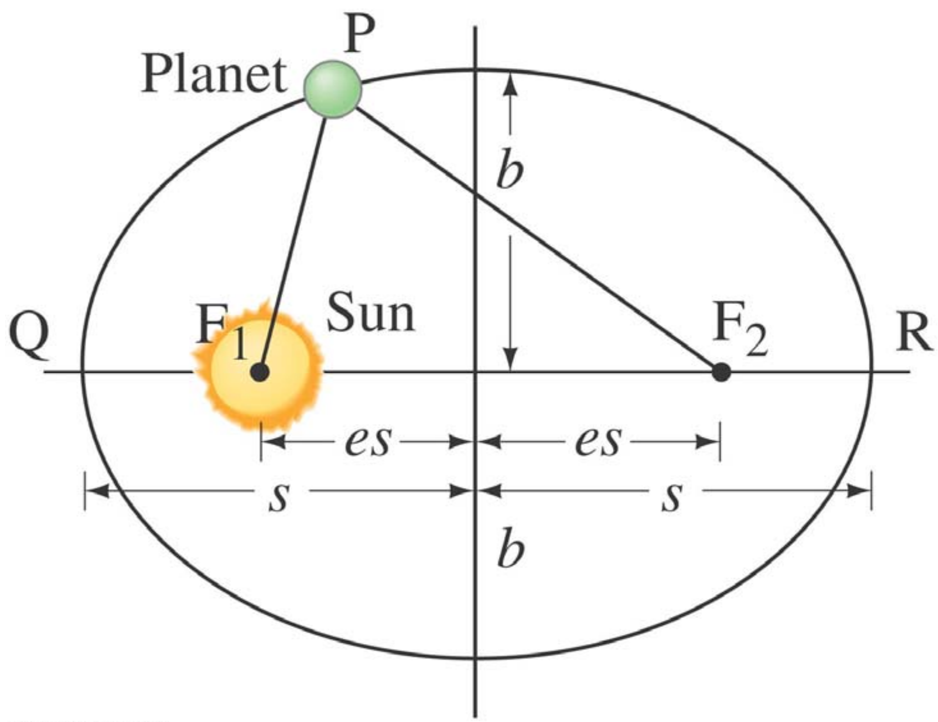
Observations → pattern → model

Tycho Brahe	Johannes Kepler	Isaac Newton
		
1546-1601	1571-1630	1642-1727

Kepler's Laws: 1

http://phys23p.sl.psu.edu/phys_anim/astro/embederQ4.40100.html

Planets move in planar elliptical paths with the Sun at one focus of the ellipse.

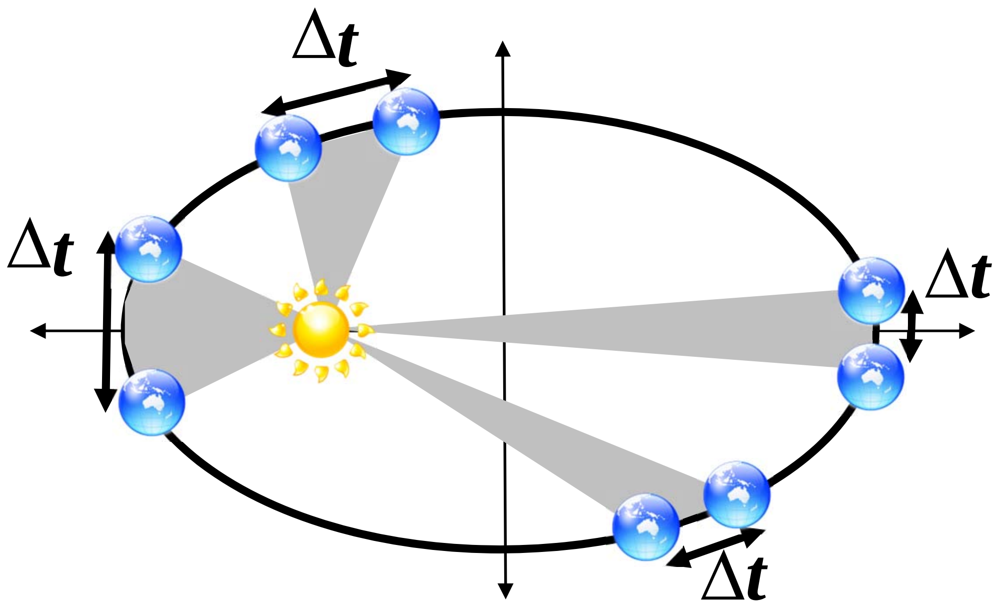


s → semi-major axis
b → semi-minor axis

Kepler's Laws: 2

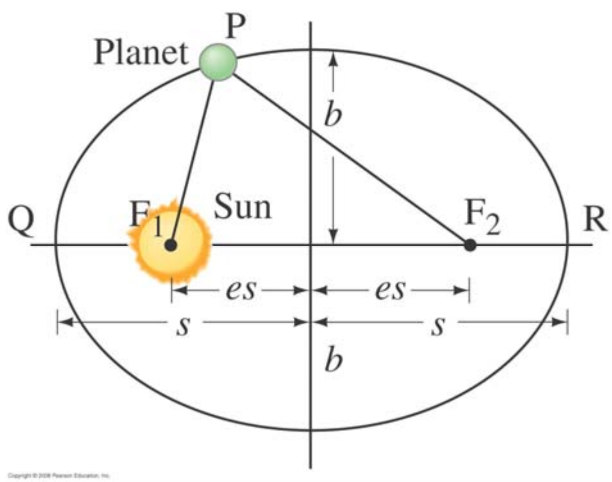
http://phys23p.sl.psu.edu/phys_anim/astro/kepler_2.avi

During equal time intervals the radius vector from the Sun to a planet sweeps out ***equal areas***.



Kepler's Laws: 3

If a planet makes one revolution around the Sun in time T, and if R is the semi-major axis of the ellipse (R is radius if orbit circular):



$$T^2 \propto R^3$$

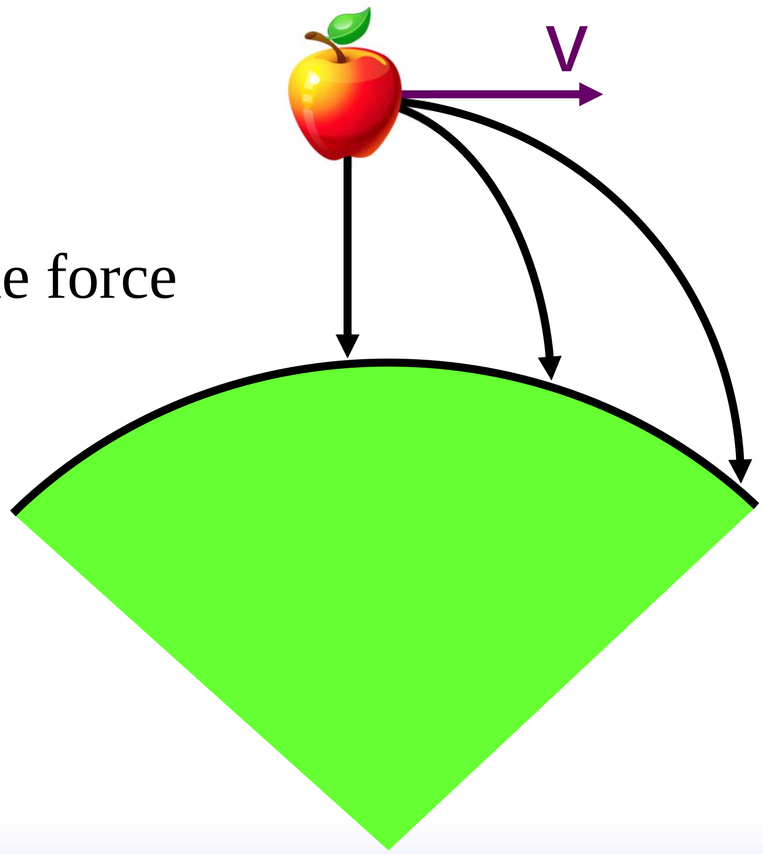
or

$$T^2 / R^3 = C$$

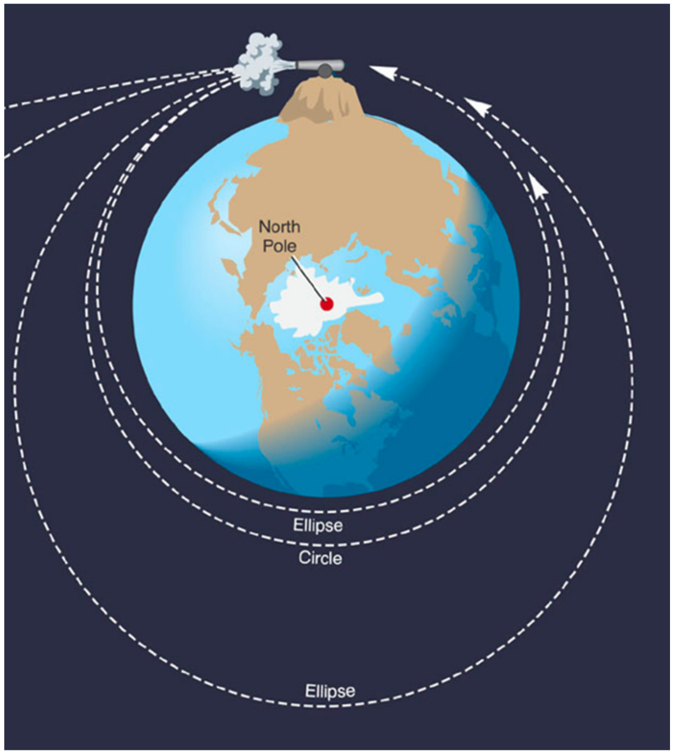
where the constant *C* is *the same for all planets*.

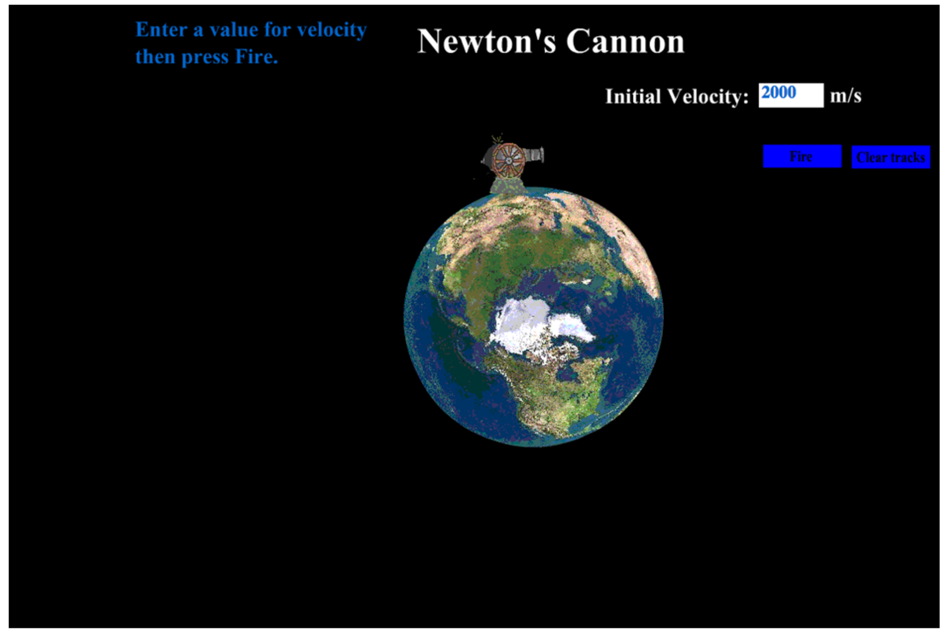
Newton and Gravitation

- Newton asked why?
- The apple does not go sideways or upwards, but always falls towards the earth's center. The apple accelerates towards the center of the earth
- So a force must be directed along a line joining the centers
- Newton's postulate: the Earth exerts the force and give it a name: GRAVITATION



Newton's Cannon on a Mountain



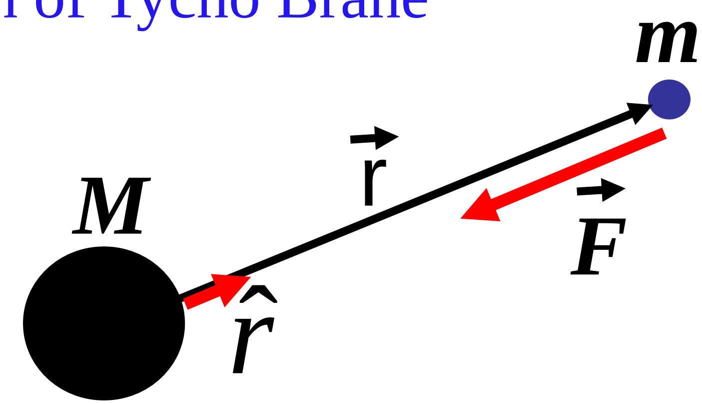


http://waowen.screaming.net/revision/force&motion/ncananim.htm

Newton's Law of Universal Gravitation

One force equation between any two point masses, can explain all three laws of Kepler, which explains all the detailed observations of planetary motion of Tycho Brahe

$$\vec{F} = -\left(\frac{GmM}{r^2}\right)\hat{r}$$



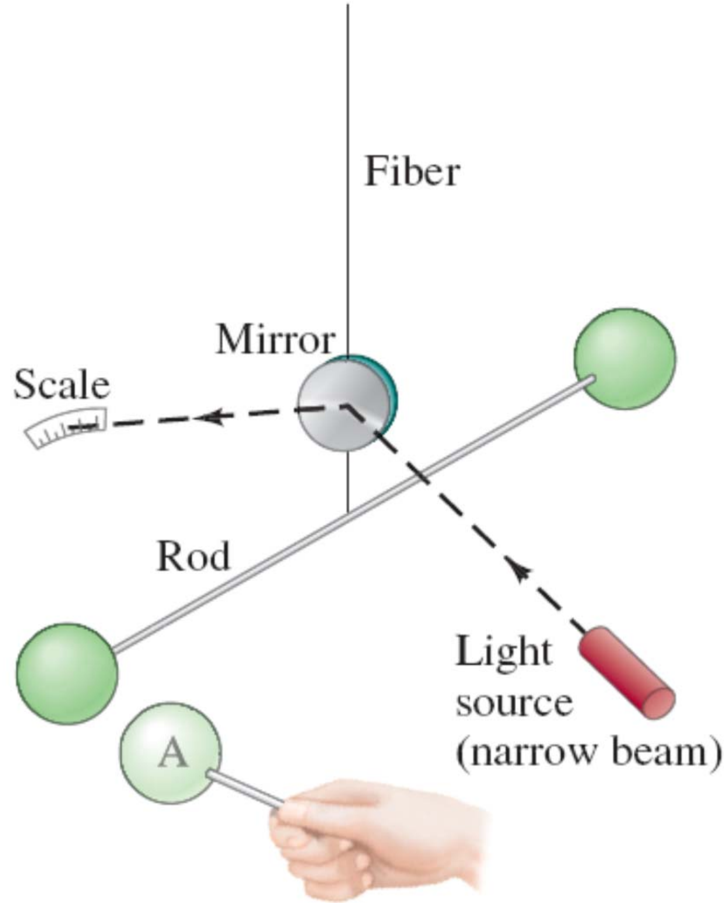
Force is central, attractive, proportional to masses, and inversely proportional to the square of the distance

$$G = 6.67 \times 10^{-11} N \cdot m^2 / kg^2$$

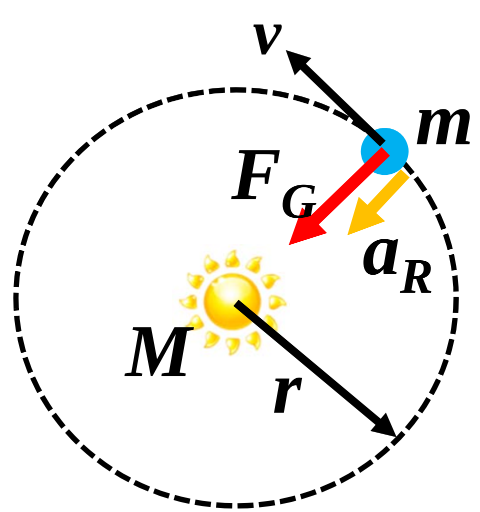
universal gravitational constant

Cavendish experiment.

The ***magnitude*** *of the gravitational constant G can be measured in the laboratory.*



Kepler's Third Law



Look at circular orbit for simplicity

Gravitational force provides centripetal acceleration ⇒ $$\frac{GMm}{r^2} = \frac{mv^2}{r}$$ ⇒ $$v^2 r = GM$$

Period of rotation ⇒ $$T = \frac{2\pi r}{v}$$ ⇒ $$T^2 = \frac{4\pi^2 r^2}{v^2}$$

$$\frac{T^2}{r^3} = \frac{4\pi^2}{v^2 r}$$ ⇒ $$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

Constant for all planets

Gravity at Earth's Surface

$$F = \frac{GM_E m}{r^2}$$

With Newton's Law of Universal Gravitation, what should we get for the acceleration due to gravity at the Earth's surface?

$$M_{EARTH} = 5.98\times10^{24}\,kg \qquad R_{EARTH} = 6.38\times10^{6}\,m \qquad G = 6.67\times10^{-11}\,{}^{Nm^2}\!/_{kg^2}$$

$$F = \frac{GM_E m}{R_E{}^2} = mg$$

$$g = \frac{GM_E}{R_E^2} = 9.80\ \text{m/s}^2$$

The local acceleration of gravity on the surface of the Earth

Gravity Near the Earth's Surface

TABLE 6–1
Acceleration Due to Gravity at Various Locations on Earth

Location	Elevation (m)	g (m/s^2)
New York	0	9.803
San Francisco	0	9.800
Denver	1650	9.796
Pikes Peak	4300	9.789
Sydney, Australia	0	9.798
Equator	0	9.780
North Pole (calculated)	0	9.832

The acceleration due to gravity varies over the Earth's surface due to altitude, local geology, and the shape of the Earth, which is not quite spherical.

ConcepTest 1 Mt. Everest

Two objects are dropped simultaneously from 2m above the ground at the top of Mount Everest and at sea level. Which hits the ground first?

A) Object at sea level

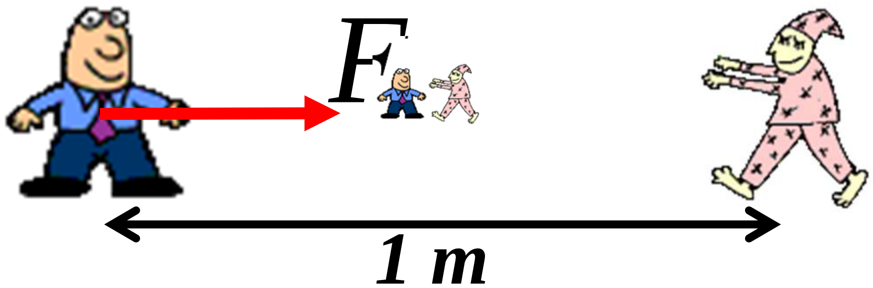
B) Object on Mt. Everest

C) They hit simultaneously

The acceleration due to gravity varies over the Earth's surface due to altitude and it is smaller on Mt. Everest.

May The Force Be With You

Estimate the attractive Force of Gravity between two average persons 1 m apart.



$$F = \frac{Gm_1m_2}{R^2} = \frac{6.67\times10^{-11}\frac{Nm^2}{kg^2}\times70kg\times70kg}{1m\times1m} = 3.3\times10^{-7}N$$

Compare this to the weight of each person.

700 N

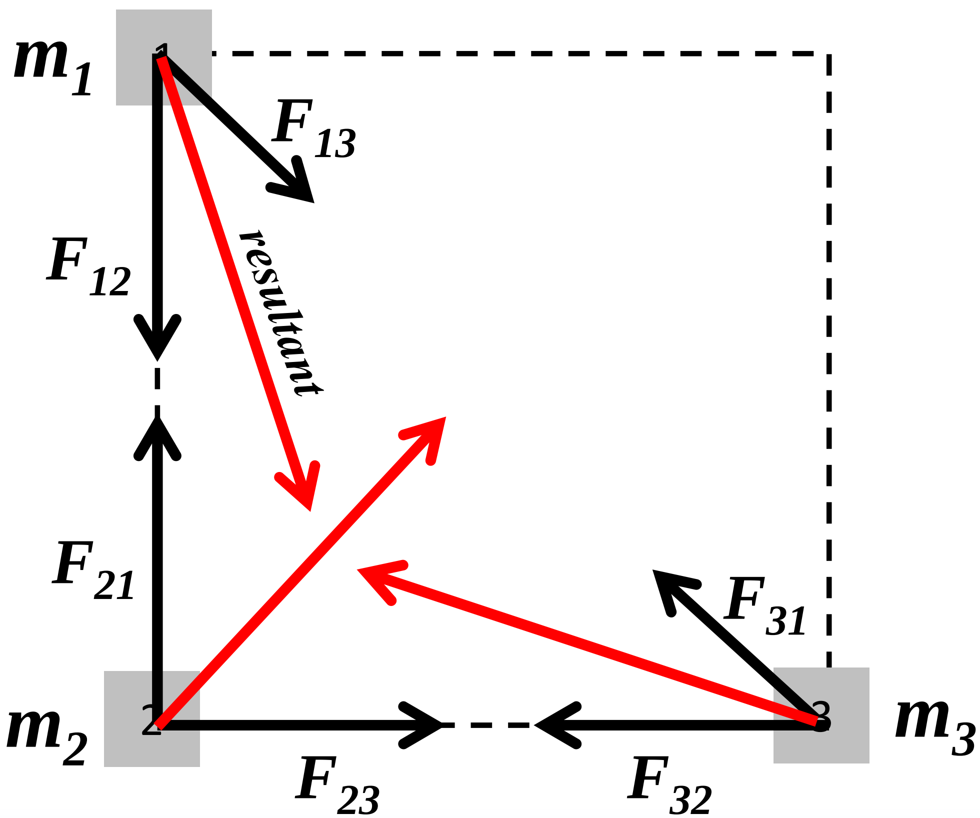
Vector Form of Newton's Universal Gravitation

If there are many particles, the total force is the vector sum of the individual forces:

$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{13} + \vec{\mathbf{F}}_{14} + \cdots + \vec{\mathbf{F}}_{1n} = \sum_{i=2}^{n} \vec{\mathbf{F}}_{1i}.$$

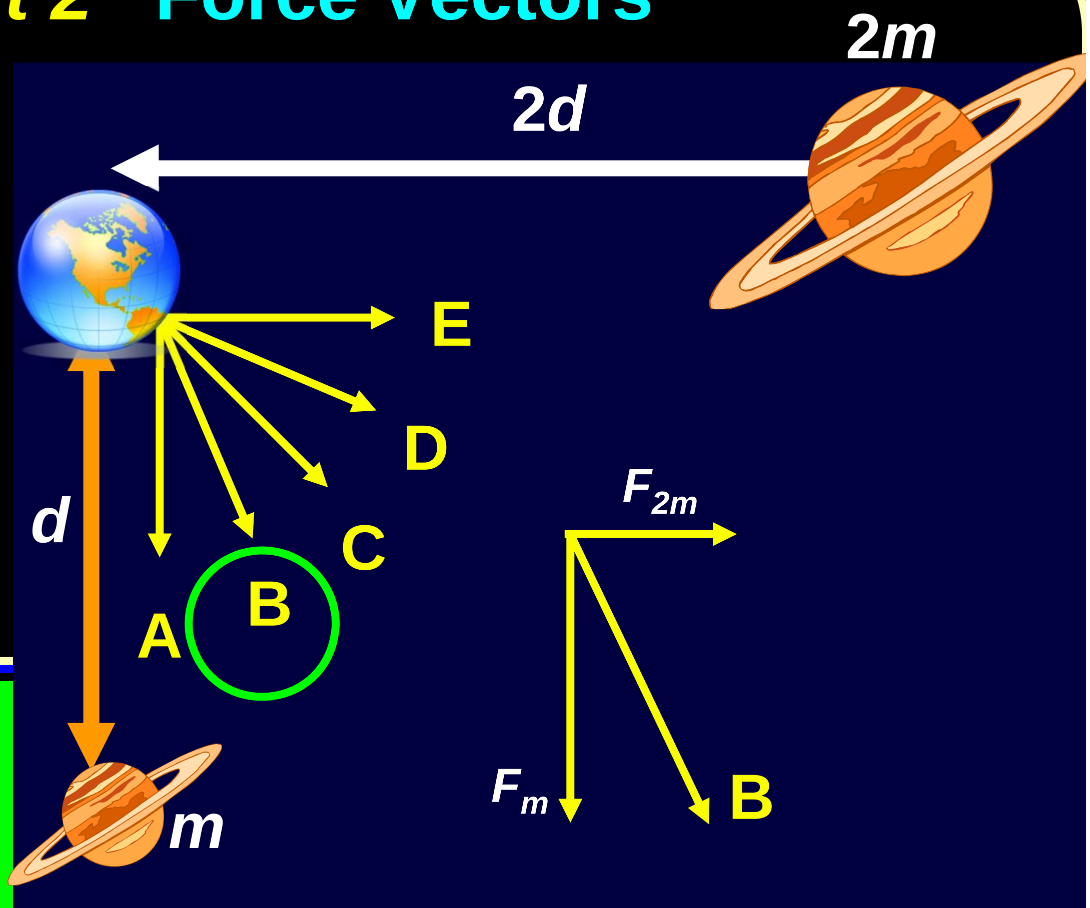
Example Problem

Three objects, of equal mass, are at three corners of a square. Draw the gravitational force vectors acting on each block.



ConcepTest 2 Force Vectors

A planet of **mass *m*** is a **distance d** from Earth. Another planet of **mass 2*m*** is a **distance 2*d*** from Earth. Which force vector best represents the direction of the **total gravitation force** on Earth?



The force of gravity on the Earth due to *m* is greater than the force due to 2*m*, which means that the force component pointing down in the figure is greater than the component pointing to the right.

$$\vec{F} = -\left(\frac{GmM}{r^2}\right)\hat{r}$$

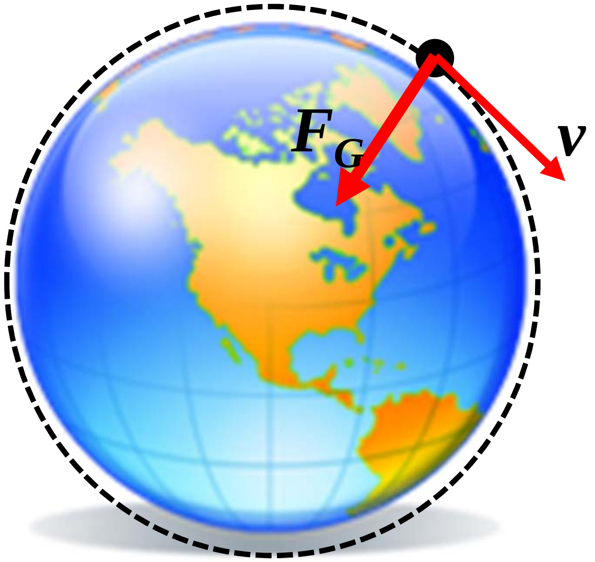
$$F_{2m} = GM_E(2m) / (2d)^2 = \frac{1}{2}\, GMm / d^2$$

$$F_m = GM_E\, m / d^2 = GMm / d^2$$

Satellite in an orbit just above ground level

If a cannonball is launched horizontally just above ground level, what speed would make it travel in a circular orbit at the surface of the Earth?

(Radius of earth: 6400 km, acceleration due to gravity at earth's surface: 10 m/s²)



Gravitational force provides centripetal acceleration

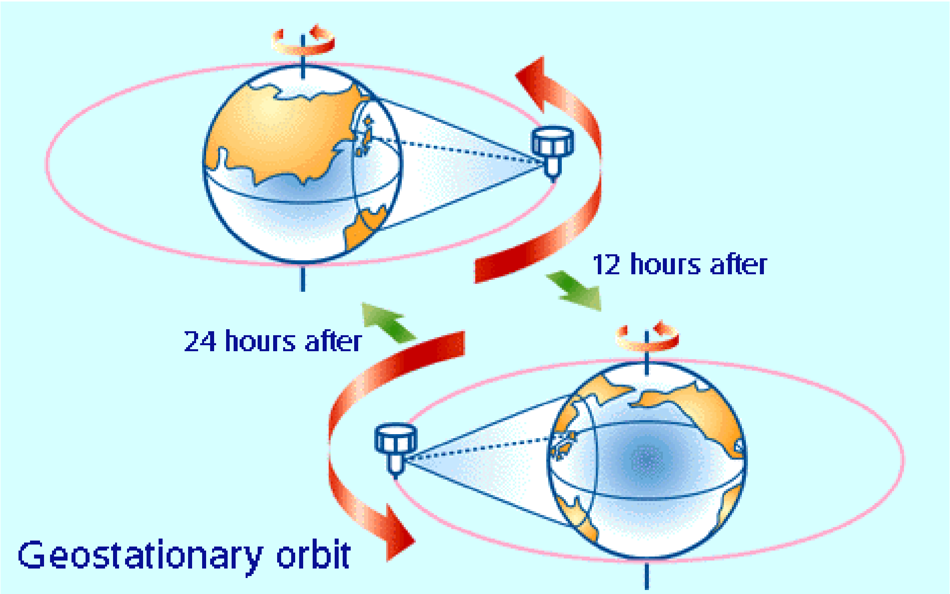
$$F_G = mg = m\frac{v^2}{R_E}$$

$$g = \frac{v^2}{R_E} \quad \Rightarrow \quad v = \sqrt{gR_E} \quad \Rightarrow \quad v = \sqrt{10 \times 6400 \times 10^3} = 8000 \text{ m/s}$$

Geosynchronous Orbit

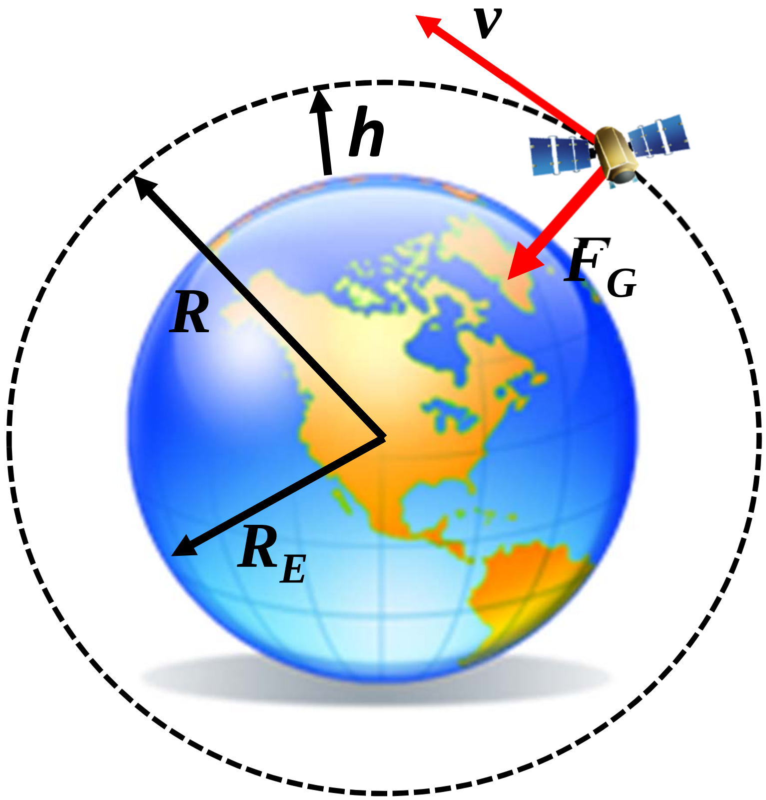
- A satellite in a *geosynchronous* orbit stays at the same position with respect to Earth as it orbits.
- Such satellites are used for TV and radio transmission, for weather forecasting, and as communication relays.

http://sphere.ssec.wisc.edu/images/geo_orbit.animated.gif



Geosynchronous Orbit

To calculate the height of a satellite of mass ***m*** in geosynchronous orbit:



Gravitational force provides centripetal acceleration

$$F = \frac{GM_E m}{R^2}$$

Satellite's velocity $v = \frac{2\pi R}{T}$

$$F = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$$

$$F = \frac{4\pi^2 Rm}{T^2} = \frac{GM_E m}{R^2}$$

$$R = \sqrt[3]{\frac{GM_E T^2}{4\pi^2}}$$

Distance from the Earth's center

$$T = 1\ day = 24\ hrs = 86,400s$$

Final caution: Height is often given by $h = R - R_E$

ConcepTest 3 Averting Disaster

The Moon does not crash into Earth because:

A) it's in Earth's gravitational field

B) the net force on it is zero

C) it is beyond the main pull of Earth's gravity

D) it's being pulled by the Sun as well as by Earth

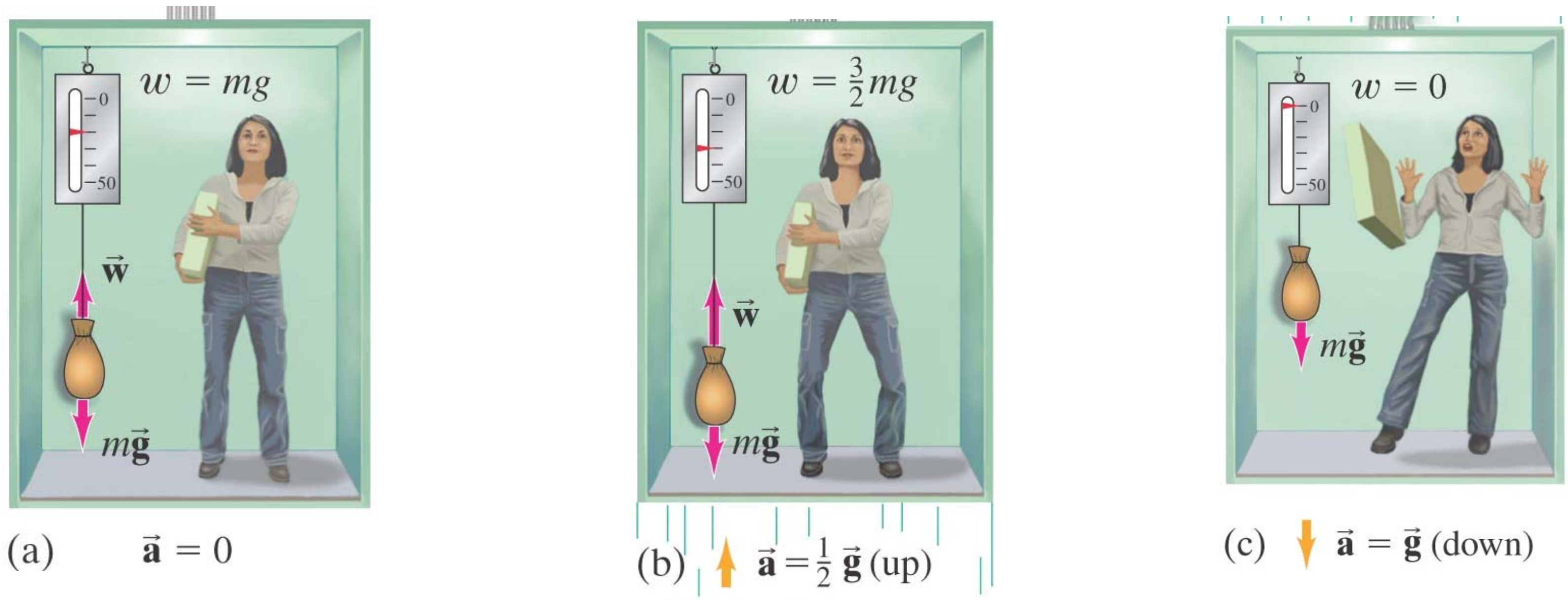
E) none of the above

The Moon does not crash into Earth because of its high speed. If it stopped moving, it would, of course, fall directly into Earth. With its high speed, the Moon would fly off into space if it weren't for gravity providing the centripetal force.

"Weightlessness"

Weight is defined as the magnitude of the force of gravity on an object.
At the surface of Earth this is *mg*.

But we measure weight, by the force a mass exerts on, say, a spring scale.



The ***"weightlessness"*** in a freely falling elevator is because everything is accelerating equally independent of mass. So, there is no normal force.

She does have a gravitational force acting on her, though!

"Weightlessness"

- The "weightlessness" in orbit is exactly the same, everything is accelerating equally independent of mass.

- The satellite and all its contents are in free fall, except with a huge tangential velocity. So, ***there is no normal force***. This is what leads to the experience of weightlessness.

They do have a gravitational force acting on them, though!

“Weightlessness”

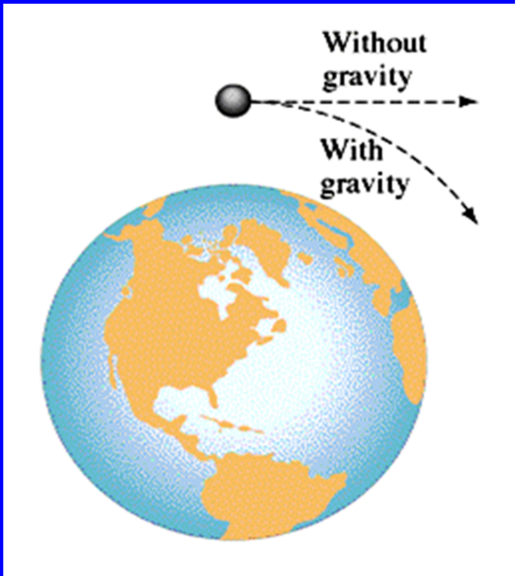
More properly, this effect is called **apparent** *weightlessness, because the gravitational force still exists. It can be experienced on Earth as well, but only briefly:*

ConcepTest 4 In the Space Shuttle

Astronauts in the space shuttle float because:

1) they are so far from Earth that Earth's gravity doesn't act any more
2) gravity's force pulling them inward is cancelled by the centripetal force pushing them outward
3) while gravity is trying to pull them inward, they are trying to continue on a straight-line path
4) their weight is reduced in space so the force of gravity is much weaker

Astronauts in the space shuttle float because they are in "free fall" around Earth, just like a satellite or the Moon. Again, it is gravity that provides the centripetal force that keeps them in circular motion.



Thank you
See you on Monday