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Illustrative Mathematics

Access as:

Unit 6 Practice Problems

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Lesson 1

Problem 1

Here is data on the number of cases of whooping cough from 1939 to 1955.

year	number of cases
1941	222,202
1950	120,718
1945	133,792
1942	191,383
1953	37,129
1939	103,188
1951	68,687
1948	74,715
1955	62,786
1952	45,030
1940	183,866
1954	60,866
1944	109,873
1946	109,860
1943	191,890
1949	69,479
1947	156,517

- 1. Make a new table that orders the data by year.
- 2. Which years in this period of time had fewer than 100,000 cases of whooping cough?

3. Based on this data, would you expect 1956 to have closer to 50,000 cases or closer to 100,000 cases?

Solution

1.

year	number of cases
1939	103,188
1940	183,866
1941	222,202
1942	191,383
1943	191,890
1944	109,873
1945	133,792
1946	109,860
1947	156,517
1948	74,715
1949	69,479
1950	120,718
1951	68,687
1952	45,030
1953	37,129
1954	60,886
1955	62,786

- 2. The years 1948, 1949, 1951, 1952, 1953, 1954, and 1955 had fewer than 100,000 cases of whooping cough.
- This data seems to show the number of cases decreasing over time, so I would expect 1956 to have closer to 50,000 cases than 100,000.

Problem 2

In volleyball statistics, a block is recorded when a player deflects the ball hit from the opposing team. Additionally, scorekeepers often keep track of the average number of blocks a player records in a game. Here is part of a table that records the number of blocks and blocks per game for each player in a women's volleyball tournament. A scatter plot that goes with the table follows.

blocks per game
1.18
0.17
0.42
0
0
0.64

2					
1.5		•		•	
1	•	•			
0.5	•••	•			
0	7.	.5	15	22.5	30

Label the axes of the scatter plot with the necessary information.

Solution

The horizontal axis should be labeled "blocks," and the vertical axis should be labeled "blocks per game."

Problem 3

(from Unit 5, Lesson 18)

A cylinder has a radius of 4 cm and a height of 5 cm.

- 1. What is the volume of the cylinder?
- 2. What is the volume of the cylinder when its radius is tripled?
- 3. What is the volume of the cylinder when its radius is halved?

Solution

- 1. $80\pi \text{ cm}^3$
- 2. $720\pi \text{ cm}^3$
- 3. $20\pi \text{ cm}^3$

Lesson 2

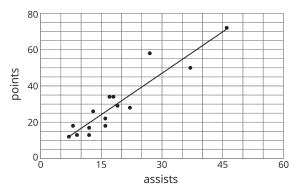
Problem 1

In hockey, a player gets credited with a "point" in their statistics when they get an assist or goal. The table shows the number of assists and number of points for 15 hockey players after a season.

assists	points
22	28
16	18
46	72
19	29
13	26
9	13
16	22
8	18
12	13
12	17
37	50
7	12
17	34
27	58
18	34

Make a scatter plot of this data. Make sure to scale and label the axes.

Solution



Problem 2

Select **all** the representations that are appropriate for comparing bite strength to weight for different carnivores.

- A. Histogram
- B. Scatter plot
- C. Dot plot
- D. Table
- E. Box plot

B, D

Problem 3

When is it better to use a table? When is it better to use a scatter plot?

Solution

Answers vary. Sample response: Scatter plots are best when looking for an overall pattern (or lack of one). Tables are best when looking for the precise details of the data.

Problem 4

(from Unit 5, Lesson 17)

There are many cylinders with radius 6 meters. Let h represent the height in meters and V represent the volume in cubic meters.

- 1. Write an equation that represents the volume V as a function of the height h.
- 2. Sketch the graph of the function, using 3.14 as an approximation for π .
- 3. If you double the height of a cylinder, what happens to the volume? Explain this using the equation.
- 4. If you multiply the height of a cylinder by $\frac{1}{3}$, what happens to the volume? Explain this using the graph.

Solution

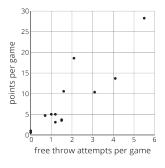
- 1. $V = 36\pi h$
- 2. The graph is a line starting from (0,0) then through about (1,113) and (2,226).
- 3. If you double the height, the volume doubles. Replacing h with 2h in the equation gives $V=36\pi \cdot 2h=2(36\pi h)$, double the original volume.
- 4. If you multiply the height by $\frac{1}{3}$, the volume is also multiplied by $\frac{1}{3}$. On the graph this can be seen using similar triangles, or by noting the relationship is proportional.

Lesson 3

Problem 1

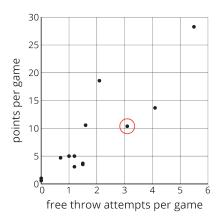
Here is a table and a scatter plot that compares points per game to free throw attempts for a basketball team during a tournament.

player	free throw attempts	points
player A	5.5	28.3
player B	2.1	18.6
player C	4.1	13.7
player D	1.6	10.6
player E	3.1	10.4
player F	1	5
player G	1.2	5
player H	0.7	4.7
player I	1.5	3.7
player J	1.5	3.5
player K	1.2	3.1
player L	0	1
player M	0	0.8
player N	0	0.6

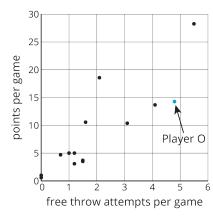


- Circle the point that represents the data for Player E.
- 2. What does the point (2.1, 18.6) represent?
- 3. In that same tournament, Player O on another team scored 14.3 points per game with 4.8 free throw attempts per game. Plot a point on the graph that shows this information.

1.



- 2. The point (2.1,18.6) represents the free throw attempts and points per game for Player B.
- 3.



(from Unit 6, Lesson 2)

Select **all** the representations that are appropriate for comparing exam score to number of hours of sleep the night before the exam.

- A. Histogram
- B. Scatter plot
- C. Dot plot
- D. Table
- E. Box plot

Solution

B, D

Problem 3

(from Unit 5, Lesson 17)

A cone has a volume of 36π cm³ and height h. Complete this table for volume of cylinders with the same radius but different heights.

height (cm)	volume (cm ³)
h	36π
2 <i>h</i>	
5h	
<u>h</u> 2	
<u>h</u> 5	

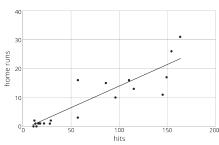
height (cm)	volume (cm ³)
h	36π
2 <i>h</i>	72π
5h	180π
<u>h</u> 2	16π
<u>h</u> 5	$\frac{36}{5}\pi$

Lesson 4

Problem 1

The scatter plot shows the number of hits and home runs for 20 baseball players who had at least 10 hits last season. The table shows the values for 15 of those players.

The model, represented by y = 0.15x - 1.5, is graphed with a scatter plot.



Use the graph and the table to answer the questions.

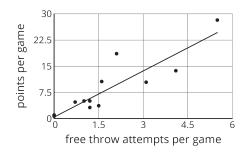
- Player A had 154 hits in 2015. How many home runs did he have? How many was he predicted to have?
- 2. Player B was the player who most outperformed the prediction. How many hits did Player B have last season?
- 3. What would you expect to see in the graph for a player who hit many fewer home runs than the model predicted?

hits	home runs	predicted home runs
12	2	0.3
22	1	1.8
154	26	21.6
145	11	20.3
110	16	15
57	3	7.1
149	17	20.9
29	2	2.9
13	1	0.5
18	1	1.2
86	15	11.4
163	31	23
115	13	15.8
57	16	7.1
96	10	12.9

Solution

- 1. Home runs: 26. Predicted home runs: 21.6
- 2. 57
- 3. The point should be much lower on the graph than the line.

Here is a scatter plot that compares points per game to free throw attempts per game for basketball players in a tournament. The model, represented by y=4.413x+0.377, is graphed with the scatter plot. Here, x represents free throw attempts per game, and y represents points per game.



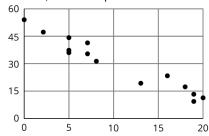
- 1. Circle any data points that appear to be outliers.
- 2. What does it mean for a point to be far above the line in this situation?
- 3. Based on the model, how many points per game would you expect a player who attempts 4.5 free throws per game to have? Round your answer to the nearest tenth of a point per game.
- 4. One of the players scored 13.7 points per game with 4.1 free throw attempts per game. How does this compare to what the model predicts for this player?

Solution

- 1. Circle the point at (2.1, 18.6).
- A point above the line represents a player who scores more points per game than predicted by their number of free throw attempts.
- 3. 20.2 points per game, because 4.413(4.5) + 0.377 is roughly equal to 20.2.
- 4. The model predicts that with 4.1 free throw attempts per game, the player should score 4.413(4.1) + 0.377, or about 18.5 points per game. That means the player is scoring less than the model predicts they should.

Lesson 5

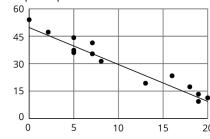
1. Draw a line that you think is a good fit for this data. For this data, the inputs are the horizontal values, and the outputs are the vertical values.



2. Use your line of fit to estimate what you would expect the output value to be when the input is 10.

Solution

1. Answers vary. Sample response:

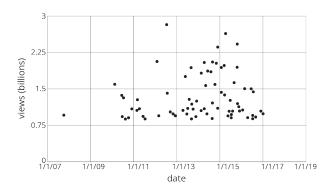


2. Answers vary. Sample response: The output would be close to 30.

Problem 2

(from Unit 6, Lesson 3)

Here is a scatter plot that shows the most popular videos in a 10-year span.



1. Use the scatter plot to estimate the number of views for the most popular

video in this 10-year span.

2. Estimate when the 4th most popular video was released.

Solution

- 1. The most popular video has roughly 2.8 billion views.
- 2. Late 2014

Problem 3

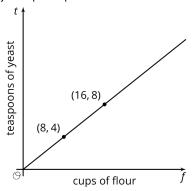
(from Unit 5, Lesson 8)

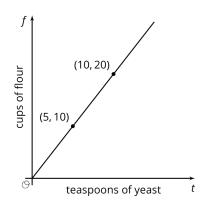
A recipe for bread calls for 1 teaspoon of yeast for every 2 cups of flour.

- 1. Name two quantities in this situation that are in a functional relationship.
- 2. Write an equation that represents the function.
- Draw the graph of the function. Label at least two points with input-output pairs.

Solution

- 1. The amount of yeast and the amount of flour are in a functional relationship.
- 2. Let t represent the number of teaspoons of yeast and f represent the number of cups of flour. If the amount of flour is treated as a function of the amount of yeast, then the equation is f=2t. If it's the other way around, the equation is $t=\frac{1}{2}f$.
- 3. Points plotted vary. Sample responses:

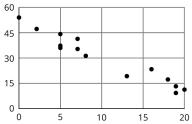




Lesson 6

Problem 1

Which of these statements is true about the data in the scatter plot?



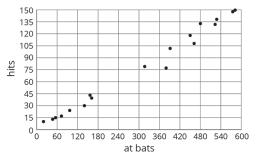
- A. As *x* increases, *y* tends to increase.
- B. As *x* increases, *y* tends to decrease.
- C. As x increases, y tends to stay unchanged.
- D. x and y are unrelated.

Solution

В

Problem 2

Here is a scatter plot that compares hits to at bats for players on a baseball team.



Describe the relationship between the number of at bats and the number of hits using the data in the scatter plot.

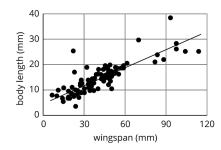
Solution

As the number of at bats increases, the number of hits also increases.

Problem 3

The linear model for some butterfly data is given by the equation y=0.238x+4.642. Which of the following best describes the slope of the model?





- A. For every 1 mm the wingspan increases, the length of the butterfly increases 0.238 mm.
- B. For every 1 mm the wingspan increases, the length of the butterfly increases 4.642 mm.
- C. For every 1 mm the length of the butterfly increases, the wingspan increases 0.238 mm.
- D. For every 1 mm the length of the butterfly increases, the wingspan increases 4.642 mm.

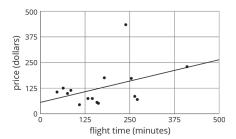
Solution

Α

Problem 4

(from Unit 6, Lesson 4)

Nonstop, one-way flight times from O'Hare Airport in Chicago and prices of a one-way ticket are shown in the scatter plot.



- Circle any data that appear to be outliers.
- Use the graph to estimate the difference between any outliers and their predicted values.

- 1. The point at (239, 436) appears to be an outlier.
- This point represents a destination that costs around \$250 more than the model predicts for its flight time.

Problem 5

(from Unit 4, Lesson 14)
Solve:
$$\begin{cases} y = -3x + 13 \\ y = -2x + 1 \end{cases}$$

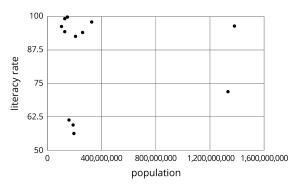
Solution

(12, -23)

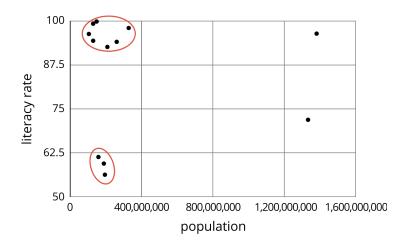
Lesson 7

Problem 1

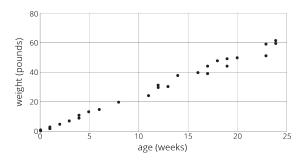
Literacy rate and population for the 12 countries with more than 100 million people are shown in the scatter plot. Circle any clusters in the data.



Solution



Here is a scatter plot:



Select all the following that describe the association in the scatter plot:

- A. Linear association
- B. Non-linear association
- C. Positive association
- D. Negative association
- E. No association

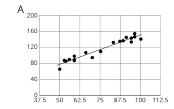
Solution

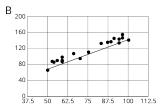
A, C

Problem 3

(from Unit 6, Lesson 5)

For the same data, two different models are graphed. Which model more closely matches the data? Explain your reasoning.





Model A more closely matches the data. In Model B, most of the points are above the line in the graph. In Model A, the points are more evenly arranged around the line.

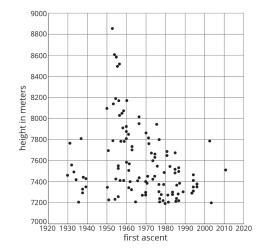
Problem 4

(from Unit 6, Lesson 3)

Here is a scatter plot of data for some of the tallest mountains on Earth.

The heights in meters and year of first recorded ascent is shown. Mount Everest is the tallest mountain in this set of data.

- Estimate the height of Mount Everest.
- Estimate the year
 of the first
 recorded ascent of
 Mount Everest.



Solution

- Approximately 8,848 meters (The vertical coordinate of the data point with the greatest vertical value is closer to 8800 than 9000.)
- Approximately 1953 (The horizontal coordinate of the same point is slightly to the right of 1950.)

Problem 5

(from Unit 5, Lesson 18)

A cone has a volume V, radius r, and a height of 12 cm.

- 1. A cone has the same height and $\frac{1}{3}$ of the radius of the original cone. Write an expression for its volume.
- 2. A cone has the same height and 3 times the radius of the original

cone. Write an expression for its volume.

Solution

- 1. $\frac{V}{9}$
- 2. 9V

Lesson 8

Problem 1

Different stores across the country sell a book for different prices. The table shows the price of the book in dollars and the number of books sold at that price.

price in dollars	number sold
11.25	53
10.50	60
12.10	30
8.45	81
9.25	70
9.75	80
7.25	120
12	37
9.99	130
7.99	100
8.75	90

- 1. Draw a scatter plot of this data. Label the axes.
- 2. Are there any outliers? Explain your reasoning.
- 3. If there is a relationship between the variables, explain what it is.
- Remove any outliers, and draw a line that you think is a good fit for the data.

Solution

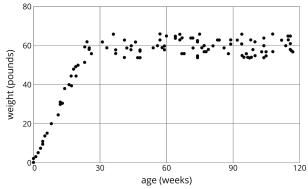
1.



- 2. Yes, at (9.99, 130). This point is much higher than expected on the scatter plot.
- 3. There is a negative linear relationship between the variables. When the price increases, the number of books sold decreases.
- 4. Answers vary. Sample response:



(from Unit 6, Lesson 7) Here is a scatter plot:



Select all the following that describe the association in the scatter plot:

- A. Linear association
- B. Non-linear association
- C. Positive association

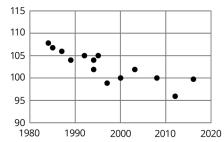
- D. Negative association
- E. No association

B, C

Problem 3

(from Unit 6, Lesson 6)

Using the data in the scatter plot, what can you tell about the slope of a good model?



- A. The slope is positive.
- B. The slope is zero.
- C. The slope is negative.
- D. There is no association.

Solution

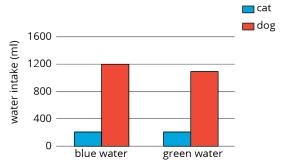
С

Lesson 9

Problem 1

A scientist wants to know if the color of the water affects how much animals drink. The average amount of water each animal drinks was recorded in milliliters for a week and then graphed. Is there evidence to suggest an association between water color and animal?

	cat intake (ml)	dog intake (ml)	total (ml)
blue water	210	1200	1410
green water	200	1100	1300
total	410	2300	2710



No, the relative frequencies of the animals drinking each color of water are about the same, so there is no evidence of association.

Problem 2

A farmer brings his produce to the farmer's market and records whether people buy lettuce, apples, both, or something else.

	bought apples	did not buy apples
bought lettuce	14	58
did not buy lettuce	8	29

Make a table that shows the relative frequencies for each row. Use this table to decide if there is an association between buying lettuce and buying apples.

Solution

	bought apples	did not buy apples	total
bought lettuce	19% since $14 + 58 = 72$ and $14 \div 72 = 0.19\overline{4}$	$81\% \text{ since}$ $58 \div 72 = 0.80\overline{5}$	100%
did not buy lettuce	22% since $8 + 29 = 37$ and $8 \div 37 = 0.\overline{216}$	$78\% \text{ since}$ $29 \div 37 = 0.\overline{783}$	100%

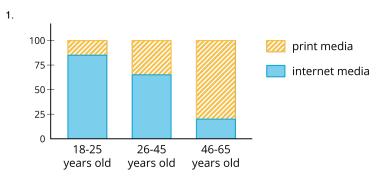
Since the percentages in each column are close, there is not enough evidence to suggest an association between buying apples and lettuce.

Problem 3

Researchers at a media company want to study news-reading habits among different age groups. They tracked print and online subscription data and made a 2-way table.

	internet articles	print articles
18–25 year olds	151	28
26–45 year olds	132	72
45–65 year olds	48	165

- 1. Create a segmented bar graph using one bar for each row of the table.
- 2. Is there an association between age groups and the method they use to read articles? Explain your reasoning.

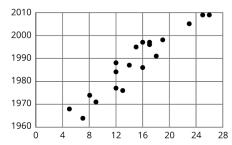


2. Yes. Explanations vary. Sample explanation: The segments of the bars are not very close to being the same size. Younger age groups use the internet articles much more than print articles, while the oldest age group reverses that pattern.

Problem 4

(from Unit 6, Lesson 6)

Using the data in the scatter plot, what is a reasonable slope of a model that fits this data?



- A. -2.5
- B. -1
- C. 1
- D. 2.5

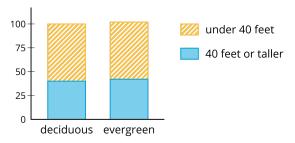
D

Lesson 10

Problem 1

An ecologist is studying a forest with a mixture of tree types. Since the average tree height in the area is 40 feet, he measures the height of the tree against that. He also records the type of tree. The results are shown in the table and segmented bar graph. Is there evidence of an association between tree height and tree type? Explain your reasoning.

	under 40 feet	40 feet or taller	total
deciduous	45	30	75
evergreen	14	10	24
total	59	40	99



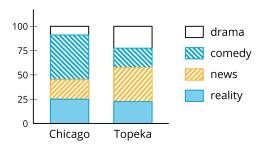
Solution

No, 60% of the deciduous trees are under 40 feet, and 40% are at least 40 feet. Similarly, 58% of evergreens are under 40 feet, and 42% are at least 40 feet. From the data recorded, there is not a clear association.

Problem 2

Workers at an advertising agency are interested in people's TV viewing habits. They take a survey of people in two cities to try to find patterns in the types of shows they watch. The results are recorded in a table and shown in a segmented bar graph. Is there evidence of different viewing habits? If so, explain.

	reality	news	comedy	drama
Chicago	50	40	90	20
Topeka	45	70	40	45



Yes, there are differences. Topekans watch news and dramas much more than Chicagoans. Chicagoans watch comedies much more. They watch about the same amount of reality TV.

Problem 3

A scientist is interested in whether certain species of butterflies like certain types of local flowers. The scientist captures butterflies in two zones with different flower types and records the number caught. Do these data show an association between butterfly type and zone? Explain your reasoning.

	zone 1	zone 2
eastern tiger swallowtail	16	34
monarch	24	46

Solution

No, there is not an association. 32% of eastern tiger swallowtails and 34% of monarchs were found in zone 1, so there is not a large difference in type of butterfly.