Fundamentals of Nuclear Engineering

Module 9: *Core Heat Transfer*

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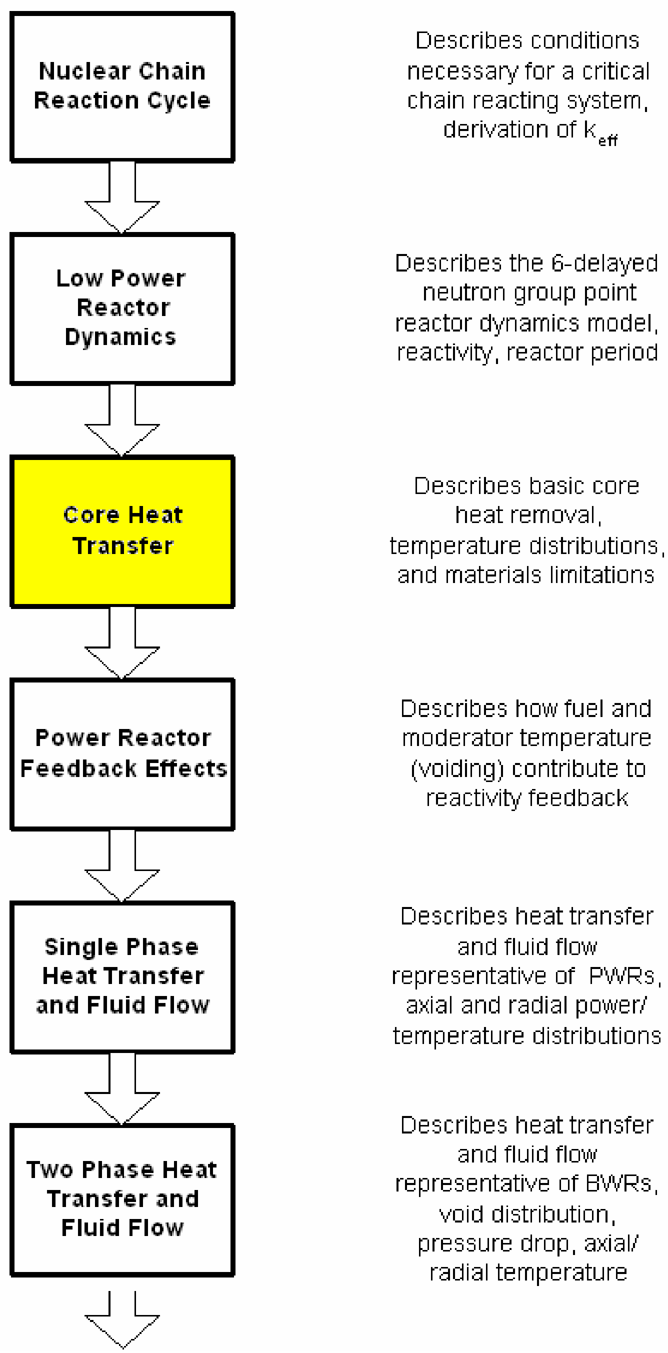

Nuclear Chain Reaction Cycle
Describes conditions necessary for a critical chain reacting system, derivation of k_{eff}
Low Power Reactor Dynamics
Describes the 6-delayed neutron group point reactor dynamics model, reactivity, reactor period
Core Heat Transfer
Describes basic core heat removal, temperature distributions, and materials limitations
Power Reactor Feedback Effects
Describes how fuel and moderator temperature (voiding) contribute to reactivity feedback
Single Phase Heat Transfer and Fluid Flow
Describes heat transfer and fluid flow representative of PWRs, axial and radial power/ temperature distributions
Two Phase Heat Transfer and Fluid Flow
Describes heat transfer and fluid flow representative of BWRs, void distribution, pressure drop, axial/ radial temperature

Objectives

This lecture will:

1. Review basic laws of heat transfer
2. Describe fuel/cladding thermal and mechanical limitations
3. Describe basic heat sources considered
4. Describe temperature dependent fuel/cladding material properties
5. Describe simplified steady state core heat transfer from fuel to coolant and fuel/clad temperature distributions
6. Describe lumped parameter transient heat transfer model for dynamic behavior of fuel temperature, heat flux

Unit Systems:

- Historically all Nuclear Engineering texts used English units
- USNRC Technical Specifications are in English Units
- A few obvious *"problem children"*: power density in kW/ft
- Reactor Physics historically used CGS units because:

  macroscopic cross sections Σ_f naturally arise in cm^{-1}

  fuel pellet diameters are on order of cm.
- Expressing parameters consistently in SI units is cumbersome - particularly when describing length.
- For remainder of this course CGS units will be used for describing power and heat transfer within fuel pellets
- A unit conversion tool (free software) will be distributed with this course

Unit Systems:

	English	**CGS**	**SI**
Mass	lb_m	gram	kilogram
Density	lb_m/ft^3	gr/cm^3	Kg/m^3
Power	BTU/hr.	Watts	Watts
Linear Power Density	kW/ft	Watt/cm	Watt/m
Heat Flux	BTU/hr ft^2	$Watt/cm^2$	$Watt/m^2$
Pressure	psi (lb_f/in^2)		Pascals
Temperature	°F	°C	°K
Length	ft	cm.	m.

Basic Laws of Heat Transfer

Basic Laws of Heat Transfer

- Energy balance equation describing heat input, temperature, heat transfer is similar to neutron transport equation:
- $$\frac{\partial}{\partial t}\left(\rho C_p T(r,t)\right) = P(r,t) + \nabla \bullet k \nabla T(r,t) \cong P(r,t) + k \nabla^2 T(r,t)$$
- $P(r,t)$ is volumetric heat input source in Watts/cm^3
- $k = k(T)$ is thermal conductivity in Watts/cm.°K
- $\rho = \rho(T)$ is mass density in grams/cm^3
- $C_p = C_p(T)$ is Heat Capacity in Joules/gram°K or alternately: Watt-sec/gram°K
- Units of this overall Heat Transfer equation work out to:
- (grams/cm^3)(Watt-sec/gram°K)(°K/sec) = Watts/cm^3

Basic Laws of Heat Transfer

- Flow of heat is governed by Fourier's law of thermal conduction:

  $$Q = - k\ A\ dT/dx = - k\ A\ Grad\ T$$

  - where: Q is heat expressed in Watts
- Frequently equation is rearranged to units of $Q/A = q$

  $$q = - k\ dT/dx = - k\ Grad\ T$$

  - where: q is expressed in Watts/cm^2

  $k = k(T)$ is thermal conductivity in Watts/cm.°K

Basic Laws of Heat Transfer

- Heat transfer between two adjacent surfaces is governed by Newton's Law of cooling:

  $$Q/A = q = h \, \Delta T$$

  \- where: h is conductance in units of Watts/cm^2.°K
- Newton's law of cooling is particularly useful when evaluating temperature drops across:
- Fuel pellet – clad Characterized via "h_{gap}"
- Clad – Coolant film Characterized via "h_{film}"

- Using these three basic relationships with appropriate property values all temperature, heat transfer effects can be evaluated.

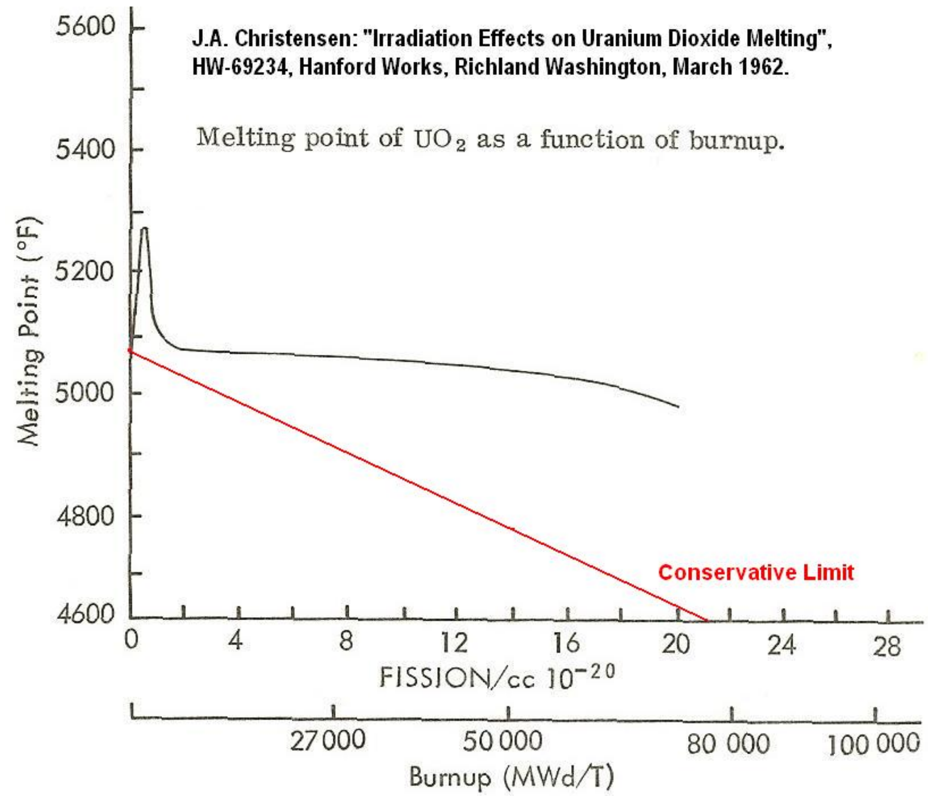
Fuel, Cladding Thermal and Mechanical Limitations

Ceramic UO_2 is Preferred Fuel Material

- Pure Uranium metal has $T_{melt} \sim 2070°F \;\; (\sim 1405°K)$
- Pure Uranium metal undergoes 3 separate crystalline phase changes before reaching 2070 °F
- Ceramic UO_2 does not have such phase changes
- Ceramic UO_2 has significantly higher melting point implying higher operating temperature capability

UO_2 Temperature Limits

- Observed UO_2 melting temperature depends on extent of burnup
- Epstein (1967) measured $T_{melt} = 5144 +/- 68\ °F$
- NRC has used Peak Fuel Centerline Temperature (PFCT) limit of 5080 °F (3077 °K)
- Decreasing 58 °F (288 °K) every 10,000 MWD/MTU as appropriate



UO_2 Enthalpy Rise Limitations

- Rapid enthalpy rise in fuel causes fracturing of ceramic UO_2
- Experimental studies (SPERT-IV, PBF) indicate:
- *ΔH < 260 Cal/gram (1.088 Joule/kg):* no apparent effects
- *ΔH between 260-300 Cal/gram (1.088-1.25 Joule/kg)* fuel element fractures into large chunks
- *ΔH ~ 350 Cal/gram (~1.46 Joule/kg)* in PBF reactivity insertion tests: local fuel melt, 0.3% conversion to mechanical energy destructive pressure pulse from: 870 → 5076 psia
- *ΔH > 600 Cal/gram (2.51 Joule/kg):* fracturing of fuel to less than 0.15mm particles, significantly larger mechanical energy conversion efficiencies
- 1986 Chornobyl-4 reactivity accident exceeded ΔH limitations
- Worst US BWR Rod Drop analyses: *ΔH < 110 Cal/gram*

Zircaloy Replaced Stainless Steel Clad

- Stainless Steel $T_{melt} \sim 2550\text{-}2600°F \ \ (< 1700°K)$
- Zircaloy $T_{melt} \sim 3360\ °F \ \ (\sim 2122°K)$
- Zircaloy-2 in BWRs
- Zircaloy-4 used in PWRs
- Key thermal limitation is related to *metal-water reaction*
- *Zr-Water exothermic reaction* accelerates rapidly at surface temperatures $T_c > 2200\ °F$ and *generates* H_2 *gas*
- NRC currently uses Peak Clad Temperature limit of
  $PCT < 2200\ °F\ (1200°C)$ as safety limit for LWRs

Zircaloy Clad Strain Limitations

- Because of metallurgical differences between UO_2 and Zircaloy – expansion/contraction vs. temperature is different
- UO_2 thermal expansion rate is ~2x that of Zircaloy
- Rapid local power increase in fuel pellet can cause high local strain on Zircaloy cladding leading to local failure
- *Limit of < 1% Plastic Strain* is conservative limit below which fuel-clad failure due to excessive strain not expected
- Strain limit can be translated back to specific power density limits.

Heat Sources Considered in Core Thermal Analysis

Three Core Heat Sources are Considered

- *Steady state power operation*, characterized by: $\Phi(r,z)$

  $P(r,z)\ (Watts/cm^{3)} \approx$

  $E_f(Watts/fission)\ \Phi(r,z)(neutrons/cm^2)\ \sum_f(fission/cm)$
- *Transient power operation*, described by neutron kinetics models (with feedback), characterized by: $\Phi(r,z,t)$
- *Long term fission product decay heat*, best characterized by ANS 5.1-2005 Decay Heat Standard
- If Clad temperature > 2200°F (1200°C) Zircaloy-Water chemical reaction heat would need consideration.
- Distribution of heat throughout reactor is roughly:
- 88% within fuel pellets (fission product recoil, α,β)
- 2.5% in moderator (neutrons slowing down)
- 9.5% in metals, structural materials (γ-*ray* absorption)

Core Heat Sources

- Scenarios involving reactor trip from high power operation must consider when fission/activation product *decay heat* exceeds *neutron flux power*.
- Decay heat sources include: *α,β,γ*-decay modes of fission products and activation products such as: Pu^{239}, Np^{239}
- Example problem
- CE System 80 NPP generating 3800 MWt trips from full power after running for more than one year
- Compare heat generated by post-trip neutron fission power vs. NRC Branch Technical Position ASB-9 (Rev2) Decay Heat Model
- Modern ANSI ANS 5.1-2005 Decay Heat Standard considers equivalent heat sources

CE System 80 NPP Trips from 3800 MWt

Comparison of ASB 9-2 and Simplified Core Decay Heat Models for Long Term Cooling Analysis

The following fission product and activation product decay energy correlations are taken from NRC Branch Technical Position ASB 9-2 Rev. 02 (July 1981). This decay heat model is used for evaluation of long term core cooling capability.

USER Input: Reactor Operating Thermal Power in MWt $Po := 3800$

USER Input: Core Operating time in seconds: (16000 hours considered to be equilibrium value) $to := 5.76 \cdot 10^{7}$

Fission Product Decay Term:

$$k := \begin{pmatrix} 0.5980 \\ 1.6500 \\ 3.1000 \\ 3.8700 \\ 2.3300 \\ 1.2900 \\ 0.4620 \\ 0.3280 \\ 0.1700 \\ 0.0865 \\ 0.1140 \end{pmatrix} \qquad \lambda := \begin{pmatrix} 1.77 \cdot 10^{0} \\ 5.774 \cdot 10^{-1} \\ 6.743 \cdot 10^{-2} \\ 6.214 \cdot 10^{-3} \\ 4.739 \cdot 10^{-4} \\ 4.810 \cdot 10^{-5} \\ 5.344 \cdot 10^{-6} \\ 5.716 \cdot 10^{-7} \\ 1.036 \cdot 10^{-7} \\ 2.959 \cdot 10^{-8} \\ 7.585 \cdot 10^{10} \end{pmatrix}$$

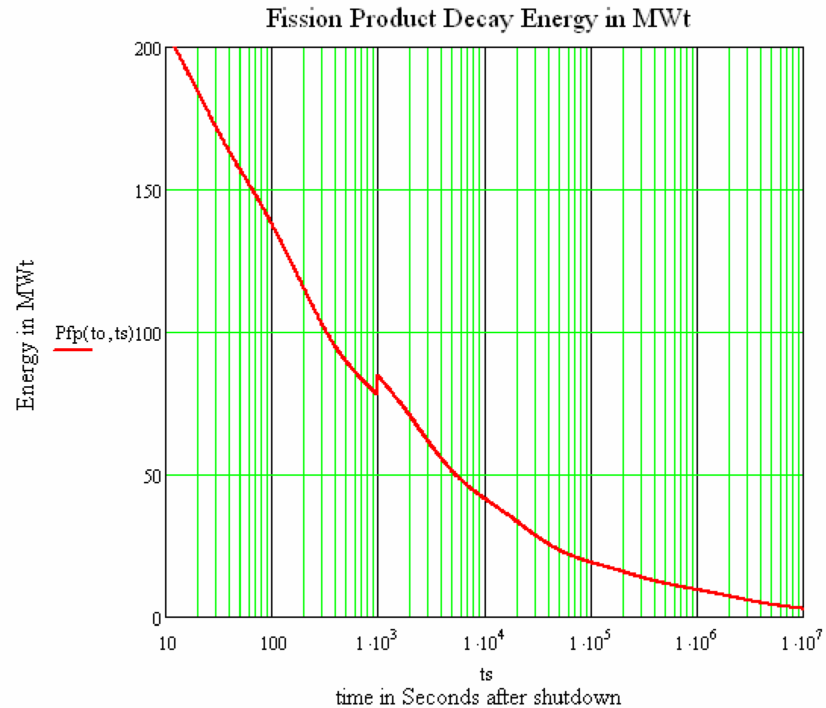
$$P(ts) := \frac{Po}{200} \cdot \sum_{i=0}^{10} \left(k_i \cdot e^{-\lambda_i \cdot ts} \right)$$

$$K(ts) := \begin{cases} 0.1 & \text{if } 0 \le ts < 10^{3} \\ 0.2 & \text{if } 10^{3} \le ts \le 10^{7} \\ 0.0 & \text{otherwise} \end{cases}$$

Cooling time uncertainty factor adjustment per NRC Branch Technical Position ASB 9-2 Rev. 02 (July 1981)

$$Pfp(to, ts) := (1 + K(ts)) \cdot P(ts) - P(to + ts)$$

Adjusted fission product energy source which accounts for uncertainties.



Heavy Metal Decay Heat:

Uranium-239 term:

Conversion ratio, atoms of Pu-239 produced per atom U-235 consumed multiplied by ratio: Σ_aU-235 / Σ_fU-235 $C := 0.7$

Value suggested in NRC Branch Technical Position ASB 9-2 Rev. 02 (July 1981)

$$PU239(to, ts) := Po \cdot \left[2.28 \cdot 10^{-3} \cdot C \cdot \left(1 - e^{-4.91 \cdot 10^{-4} \cdot to} \right) \cdot \left(e^{-4.91 \cdot 10^{-4} \cdot ts} \right) \right]$$

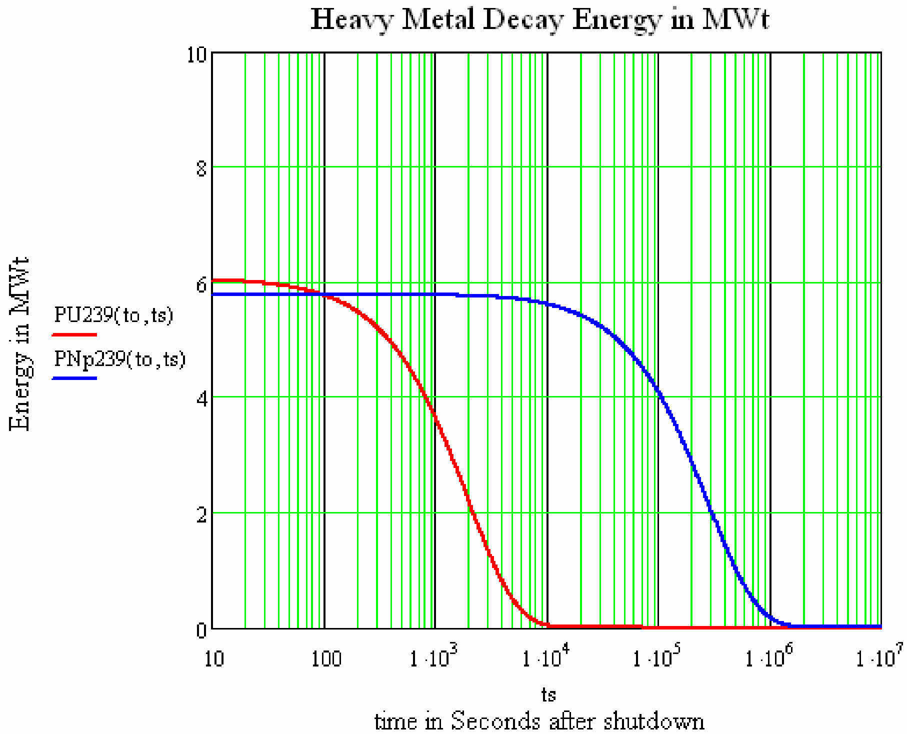
CE System 80 NPP Trips from 3800 MWt

Neptunium-239 term:

$$F(to,ts) := 1.007\cdot\left(1 - e^{-3.41\cdot10^{-6}\cdot to}\right)\cdot\left(e^{-3.41\cdot10^{-6}\cdot ts}\right)$$

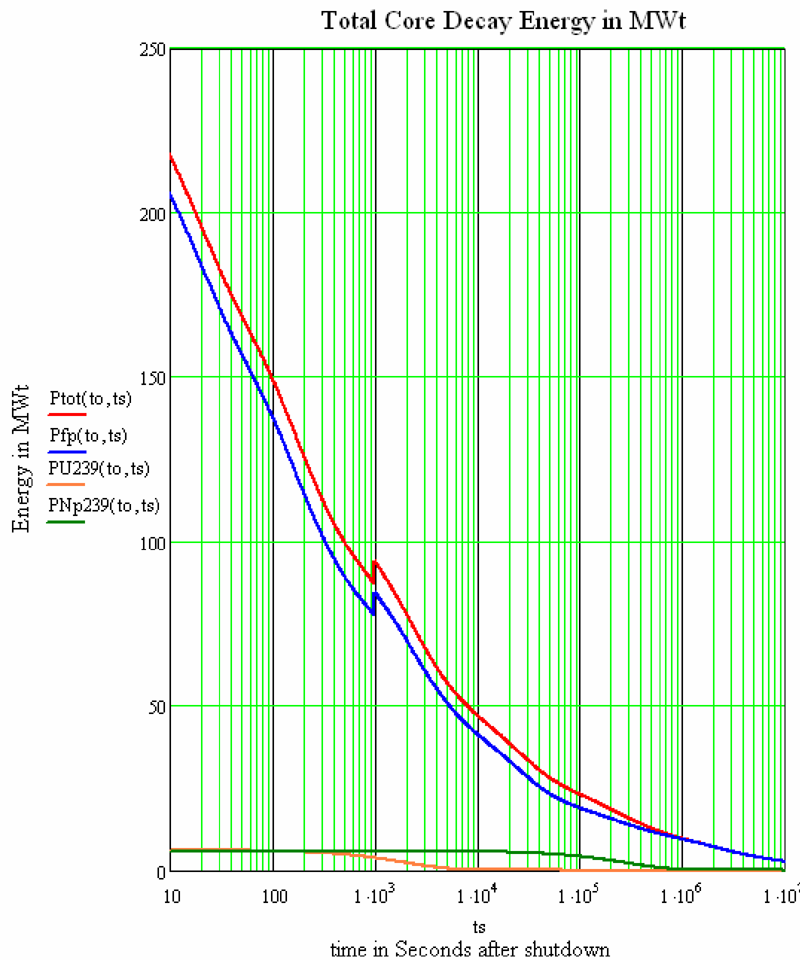
$$G(to,ts) := 0.007\cdot\left(1 - e^{-4.91\cdot10^{-4}\cdot to}\right)\cdot\left(e^{-4.91\cdot10^{-4}\cdot ts}\right)$$

$$PNp239(to,ts) := Po\cdot\left|2.17\cdot10^{-3}\cdot C\right|\cdot(F(to,ts) - G(to,ts))$$

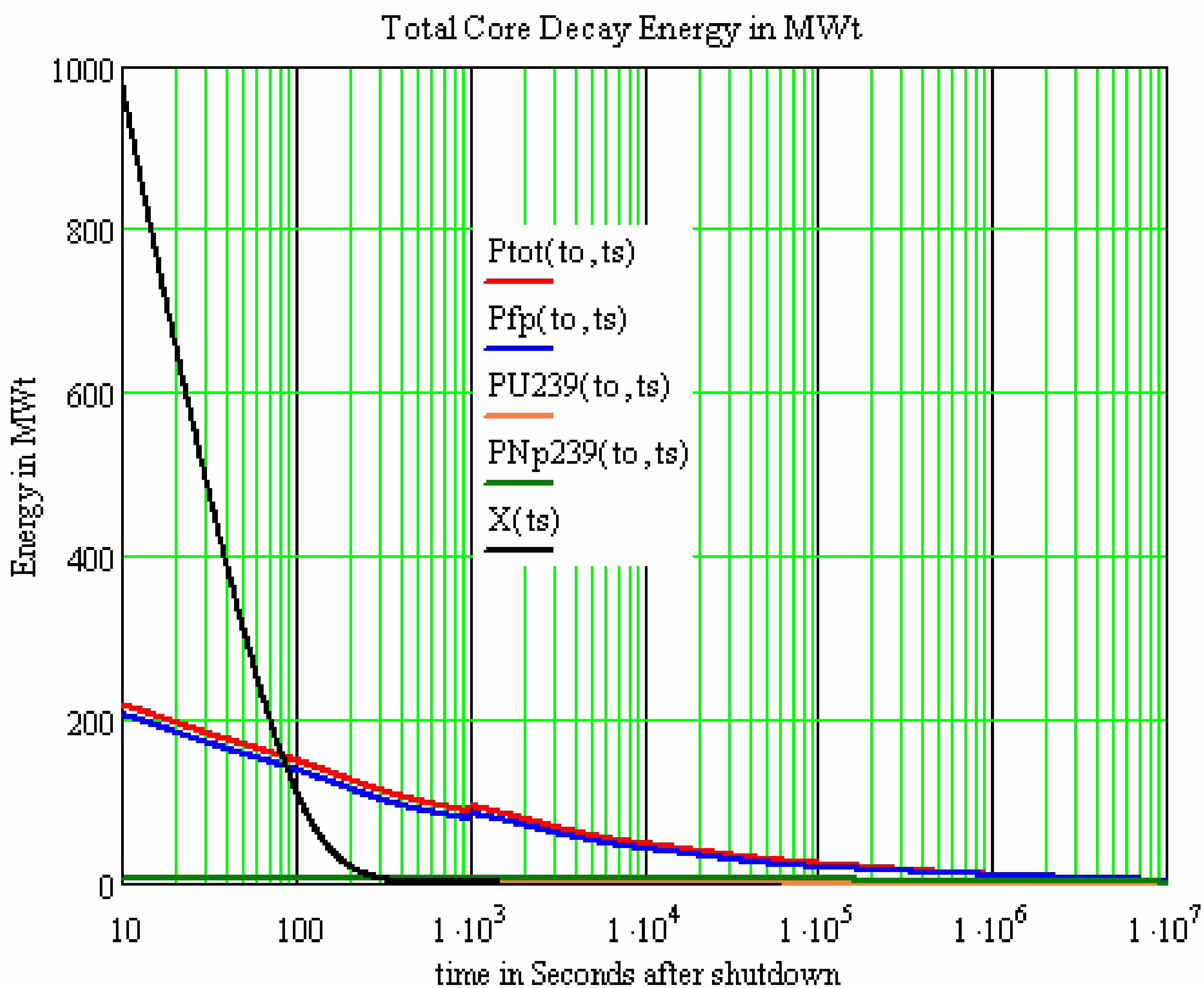


Calculation of the total energy sums the fission product and heavy metal decay energy tersm as follows:

$$Ptot(to,ts) := Pfp(to,ts) + PU239(to,ts) + PNp239(to,ts)$$



Comparison of Neutron Flux Power to NRC BTP ASB-9 Rev2 Decay Heat



Material Properties Which Impact Core Heat Transfer

Heat Transfer from Fuel to Coolant

- General equation for heat transfer:

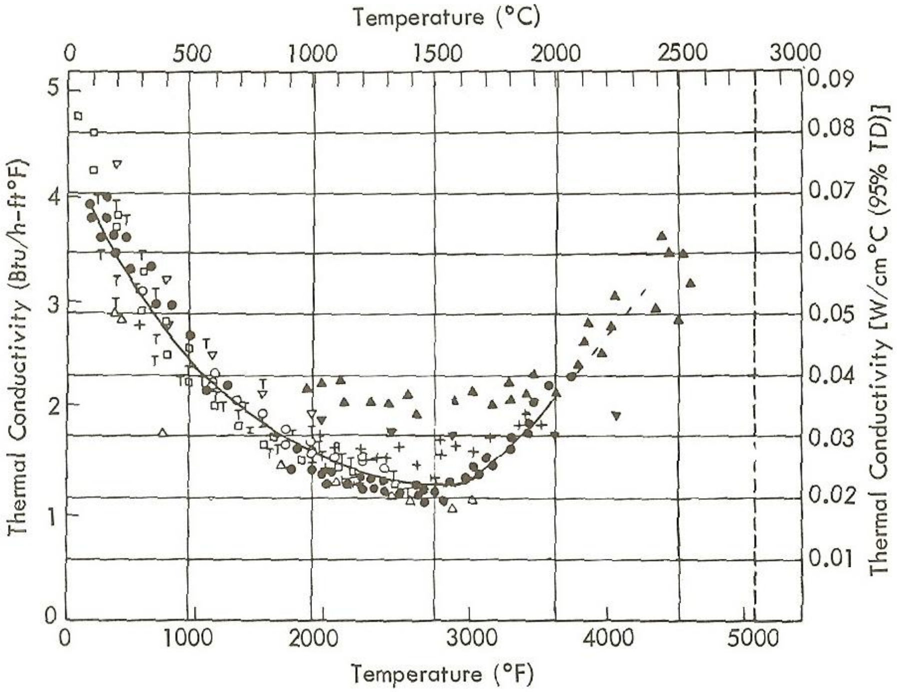
$$\rho(T_f(r,t))c_p(T_f(r,t))\frac{\partial T_f(r,t)}{\partial t} = P(r,t) + \nabla(k_f(T_f(r,t))\nabla T_f(r,t))$$

- Equation is written this way because material parameters are *strongly dependent on temperature*

- Simplified calculations can be performed in narrow temperature ranges assuming relatively constant, temperature averaged $k(T)$, $C_p(T)$ parameters

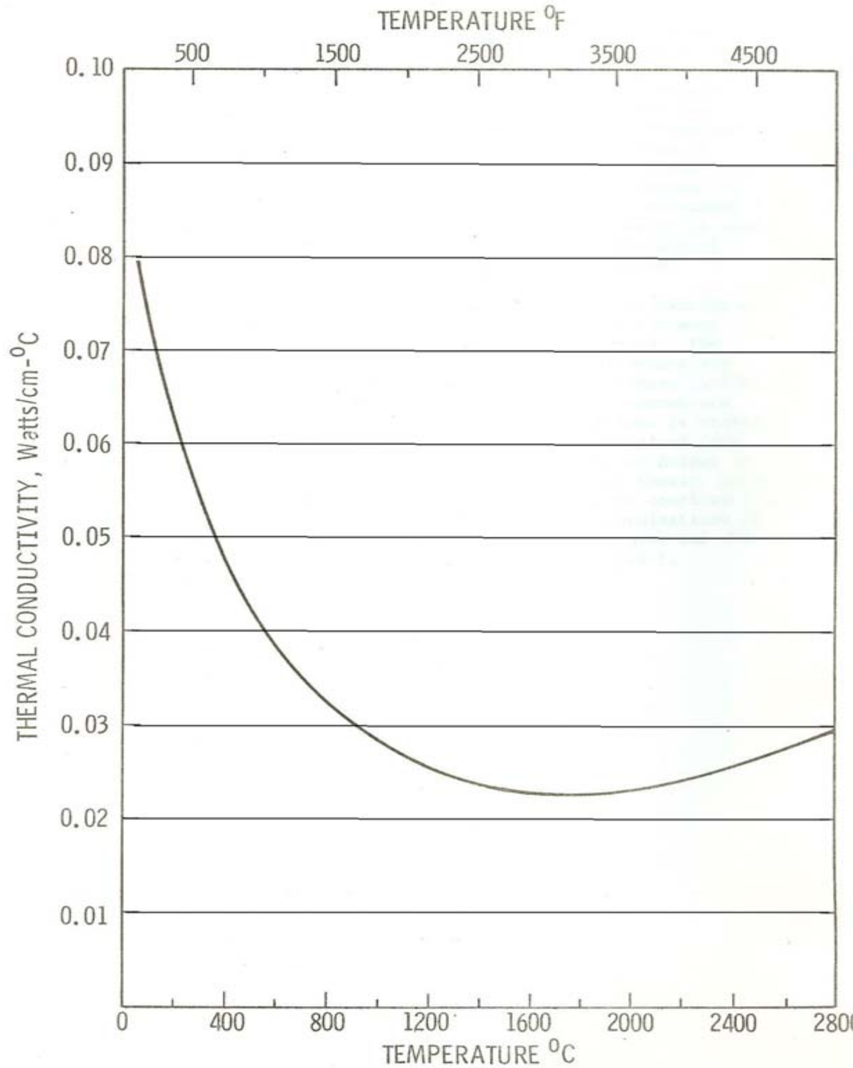
- Doing accident analysis generally requires more sophisticated analysis

Data for UO_2 Thermal Conductivity



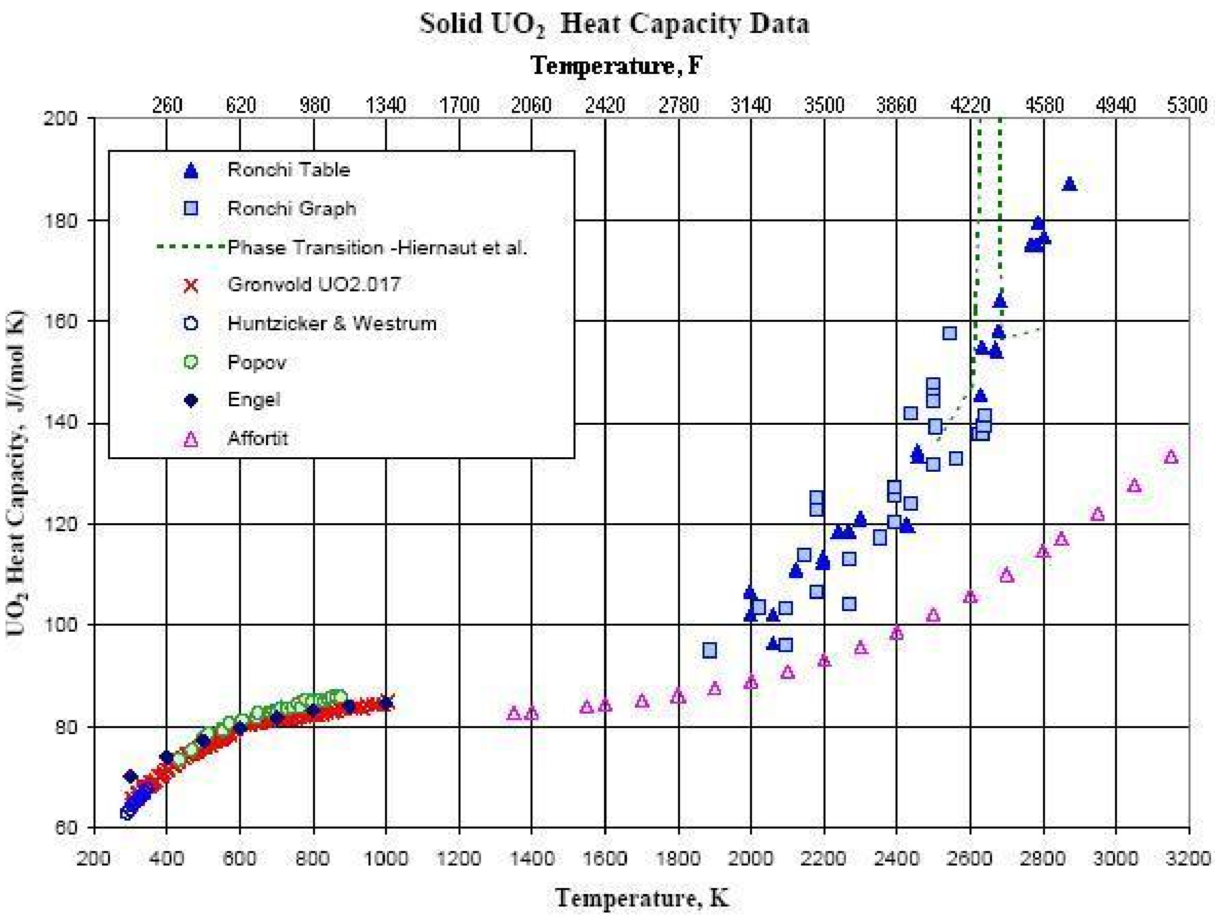
Thermal conductivity of unirradiated UO_2.
Notation ○ Godfrey et al., Ref. 27, 1964.
□ Dayton & Tipton, Battelle Memorial Institute Report BMI-1448, 1960.
▽ Kingery et al., *J. Am. Ceram. Soc.*, **37**, 107, 1954.
T Howard & Galvin, UKAEA IG Report 51, 1960.
⊢ Reiswig, *J. Am. Ceram. Soc.*, **44**, 48, 1961.
● Nishijima, Ref. 31, 1965.
+ Bush et al., *Trans. Am. Nucl. Soc.*, **7**, 392, 1964.
▼ Feith, Ref. 32, 1963.
▲ Feith unpublished data.

Taken from L.S. Tong & J. Weisman, "Thermal Analysis of Pressurized Water Reactors", p. 60



Taken from "CE Fuel Evaluation Model Topical Report", CENPD-139, July 1974, p.5-21

Data for UO_2 Specific Heat

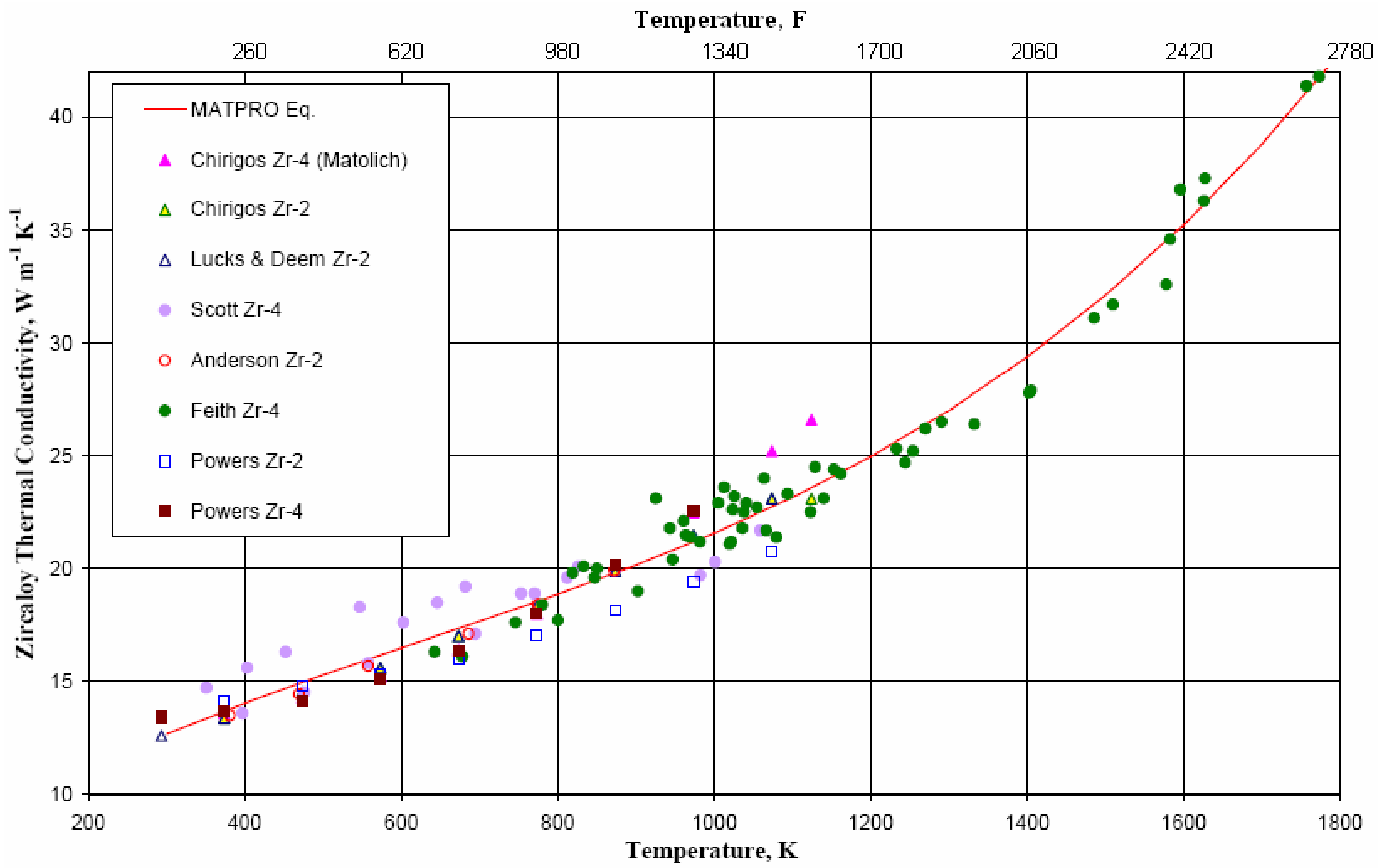


for 298.15 K ≤ T ≤ 3120 K

$$C_p(T) = +\ 52.1743 + 87.951\tau - 84.2411\tau^2 + 31.542\tau^3 - 2.6334\tau^4 - 0.71391\tau^{-2}$$

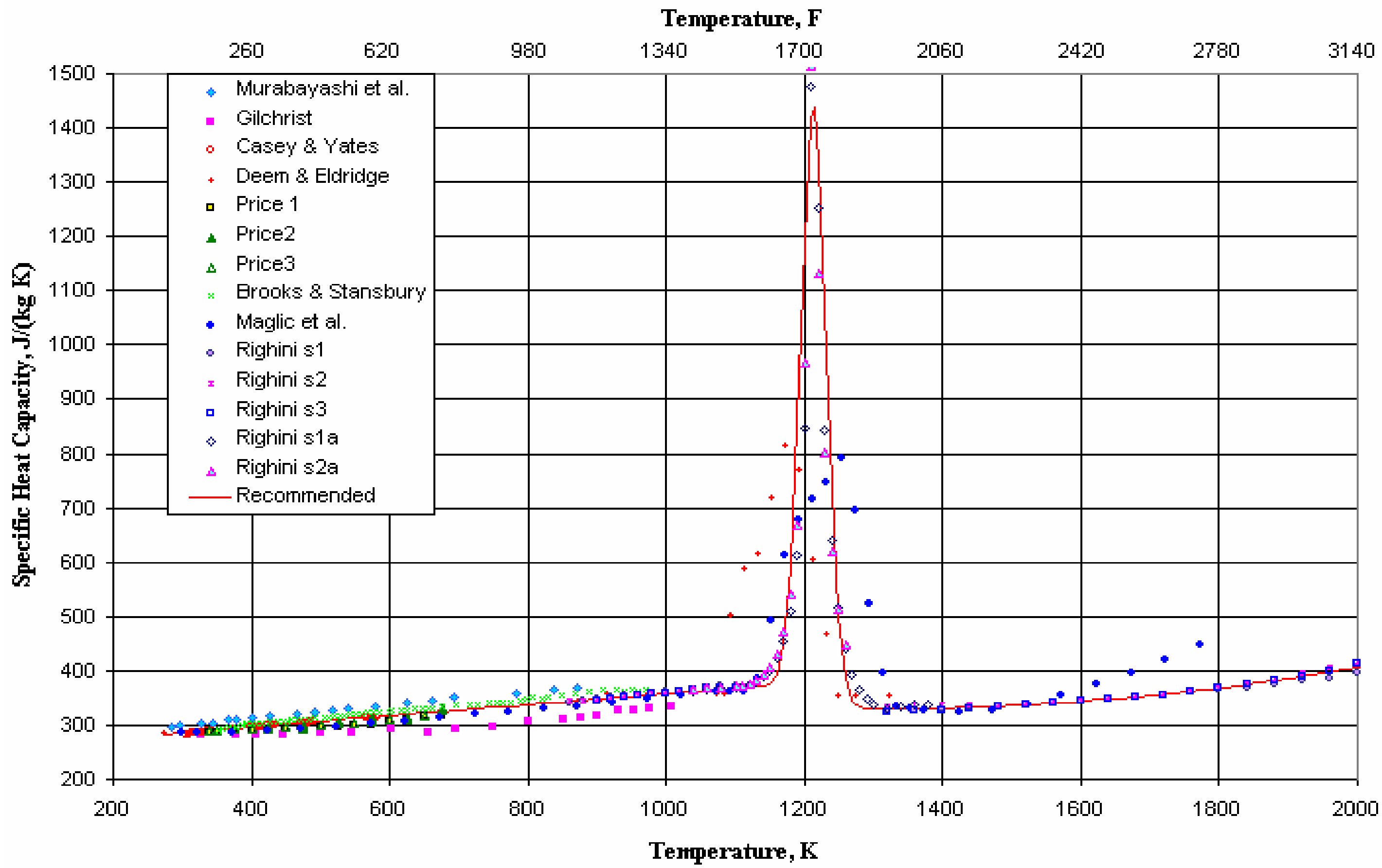
where τ =T/1000,
T is the temperature in K,
and the heat capacity, C_p, is in $J \cdot mol^{-1} \cdot K^{-1}$.

Data for Zircaloy Thermal Conductivity



- $k_{Zr}(T)$ data in *Watts/m.°K vs. T°K* in fitted as:
- $k_{Zr}(T)=12.767 - 5.4348x10^{-4}T + 8.9818x10^{-6}T^2$

Data for Zircaloy Specific Heat



- Zircaloy-2 undergoes phase change >*1200°K*
- $C_p(T) = 255.66 + 0.1024T$ for: $T<1100°K$
- $C_p(T) = 1058.4\ exp[(T-1213.8)^2/719.61]$ $1100°K< T< 1214°K$
- $C_p(T) = 597.1 – 0.4088T + 1.565x10^{-4}T^2$ $1214°K< T< 2000°K$

Steady State Core Heat Transfer

Assumptions:

- Steady state: $dT_f/dt = 0, dT_c/dt = 0$
- Power derived from fission is uniformly deposited within fuel pin (no flux depression, no power within clad)
- Fuel pin radius is: R_o in cm.
- Clad outer radius is: R_c in cm.
- Gap is of negligible dimension
- Assume volumetric power density: P in units of Watts/cm^3
- Linear power density is: $q = \pi R_o^2 P$ in units of Watts/cm
- Alternately: $P = q / \pi R_o^2$
- Thermal conductivity values: k, in units of Watts/cm.°K
- Conductance values: h, in units of Watts/cm^2.°K

Heat Transfer Assuming Constant k-values

- Assume averaged heat transfer coefficients – heat conduction equation in fuel pellet becomes:

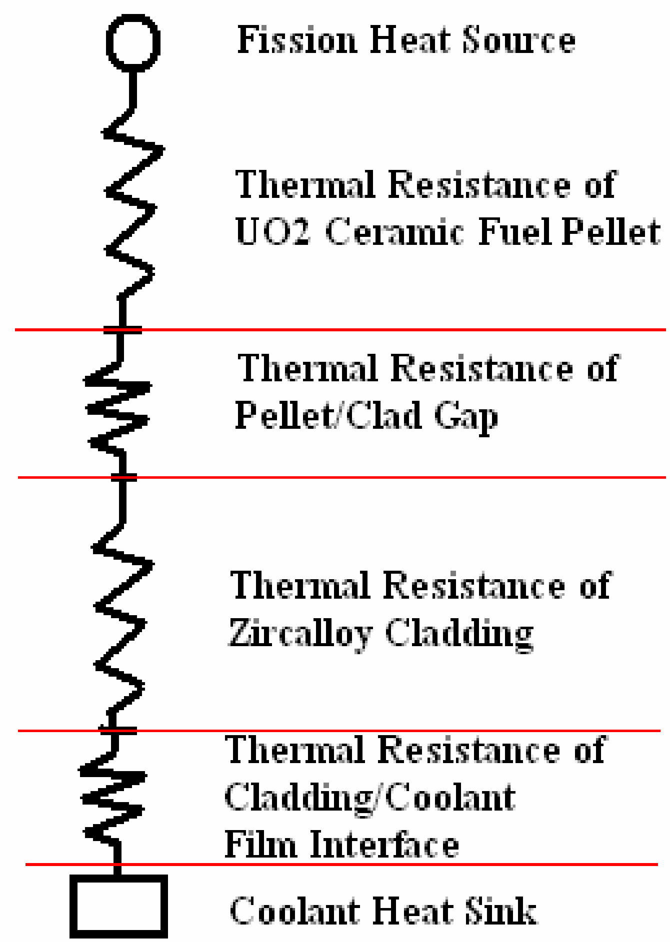
$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT_f}{dr}\right)+\frac{P}{k_f}=0$$

$$with: \left.\frac{dT_f}{dr}\right|_{r=0}=0;$$

$$T_f(r)\big|_{r=Ro}=T_f(Ro)$$

- A series of temperature drops are calculated proportional to *"q"*
- Temperature drop across gap is:

$$T_f(R_o)\text{-}T_c(R_o)= q/2\pi R_o h_{gap}$$



Thermal Resistances

Using thermal resistance analogy:

$$\Delta T = q \cdot Resistance$$

Resistance across gap:

$$1/2\pi R_o h_{gap}$$

Resistance across cladding:

$$ln(R_c/R_o)/4\pi k_c$$

Resistance across cladding/coolant film:

$$1/2\pi R_c h_{film}$$

Heat Transfer across fuel pellet assuming constant k-values

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT_f}{dr}\right)+\frac{P}{k_f}=0$$

$$\int\frac{d}{dr}\left(r\frac{dT_f}{dr}\right)dr=-\int\frac{rP}{k_f}dr+A$$

$$r\frac{dT_f}{dr}=-\frac{P}{2k_f}r^2+A$$

$$\frac{dT_f}{dr}=-\frac{P}{2k_f}r+\frac{A}{r}$$

$$Applying:\left.\frac{dT_f}{dr}\right|_{r=0}=0$$

$$yields:A=0$$

Heat Transfer across fuel pellet assuming constant k-values

$$\int \frac{dT_f}{dr} dr = -\int \frac{P}{2k_f} r dr$$

$$T_f(r) = -\frac{P}{4k_f} r^2 + B$$

$$If : T_f(r)\Big|_{r=Ro} = T_f(R_o)$$

$$T_f(R_o) = -\frac{P}{4k_f} R_o^{\ 2} + B$$

$$Thus : B = T_f(R_o) + \frac{P}{4k_f} R_o^{\ 2}$$

$$Substituting : P = \frac{q}{\pi R_o^{\ 2}}$$

$$T_f(r) = T_f(R_o) + \frac{P}{4k_f}(R_o^{\ 2} - r^2) = T_f(R_o) + \frac{q}{4\pi k_f}(1 - \frac{r^2}{R_o^{\ 2}})$$

Heat Transfer across fuel pellet assuming constant k-values

- Substituting in temperature drop across gap using thermal resistance model yields:

$$T_f(R_o) - T_c(R_o) = q \times \text{Resistance}$$

$$\text{Resistance} = \frac{1}{2\pi R_o h_{gap}}$$

$$T_f(R_o) - T_c(R_o) = \frac{q}{2\pi R_o h_{gap}}$$

$$T_f(r) = T_c(R_o) + \frac{q}{2\pi}\left(\frac{\left(1 - \frac{r^2}{R_o^{\ 2}}\right)}{2k_f} + \frac{1}{R_o h_{gap}}\right)$$

Heat Transfer across cladding assuming constant k-values

- Additional heat transfer equation used for clad
- *No internal heat* assumed

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT_c}{dr}\right)=0$$

$$with\ :T_c(r)\big|_{r=R_o}=T_c(R_o)$$

$$and\ :T_c(r)\big|_{r=R_c}=T_c(R_c)$$

$$\int\frac{d}{dr}\left(r\frac{dT_c}{dr}\right)dr=C$$

$$\int\frac{dT_c}{dr}dr=\int\frac{C}{r}dr+D$$

$$T_c(r)=C\ln(r)+D$$

$$T_c(R_o)=C\ln(R_o)+D$$

$$T_c(R_C)=C\ln(R_C)+D$$

Heat Transfer across cladding assuming constant k-values

- To solve for coefficients substitute thermal resistance across cladding:

$$T_c(R_o) - T_c(R_c) = q \times \text{Resistance}$$

$$\text{Resistance} = \ln\left[\frac{R_c}{R_o}\right] / 4\pi \mathrm{k_c}$$

$$T_c(R_o) - T_c(R_c) = C \ln\left[\frac{R_c}{R_o}\right]$$

$$C = \ln\left[\frac{R_c}{R_o}\right] / 4\pi \mathrm{k_c}$$

$$T_c(R_o) = T_c(R_c) + \frac{qR_o{}^2}{4\pi k_c} \ln\left[\frac{R_c}{R_o}\right]$$

Heat Transfer Assuming Constant k-values

- Substituting in yields following for $T_f(r)$:

$$T_f(r) = T_c(R_c) + \frac{q}{2\pi}\left(\frac{\left(1 - \frac{r^2}{R_o{}^2}\right)}{2k_f} + \frac{1}{R_o h_{gap}} + \frac{\ln\left(\frac{R_c}{R_o}\right)}{2k_c} \right)$$

- A final substitution (similar to expression for gap) is temperature drop across film layer from clad to coolant:

$$T_f(R_c) - T_{coolant}(R_c) = q/2\pi R_c h_{film}$$

- Film conductance depends on coolant flow, geometry, temperature, pressure, voids, etc. *(more to come later!)*

Heat Transfer Assuming Constant k-values

- Overall solution for fuel pellet temperature becomes:

$$T_f(r) = T_{coolant}(R_c) + \frac{q}{2\pi}\left(\frac{\left(1 - \frac{r^2}{R_o^{\ 2}}\right)}{2k_f} + \frac{1}{R_o h_{gap}} + \frac{\ln\left(\frac{R_c}{R_o}\right)}{2k_c} + \frac{1}{R_c h_{film}} \right)$$

- Overall solution for clad temperature becomes:

$$T_c(r) = T_{coolant}(R_c) + \frac{q}{2\pi}\left(\frac{\ln\left(\frac{R_c}{r}\right)}{2k_c} + \frac{1}{R_c h_{film}} \right)$$

Example Applications

- Westinghouse 17 x 17 fuel bundle design at 14 kW/ft
- GE 8 x 8 fuel bundle design at 13.4 kW/ft
- Effective (constant) thermal conductivity assumed
- All thermal energy assumed to originate in fuel pin
- No thermal neutron flux depression considered
- Constant h_{film} = 4.5 Watts/cm°K in both cases
- Objective is to calculate:
- Peak centerline temperature (compare to melting point)
- Fuel temperature distribution
- Clad temperature distribution

Example: W 17x17 Fuel at 14kW/ft.

Fuel Pin Temperature Distribution Assuming Constant: q, kfuel, kc

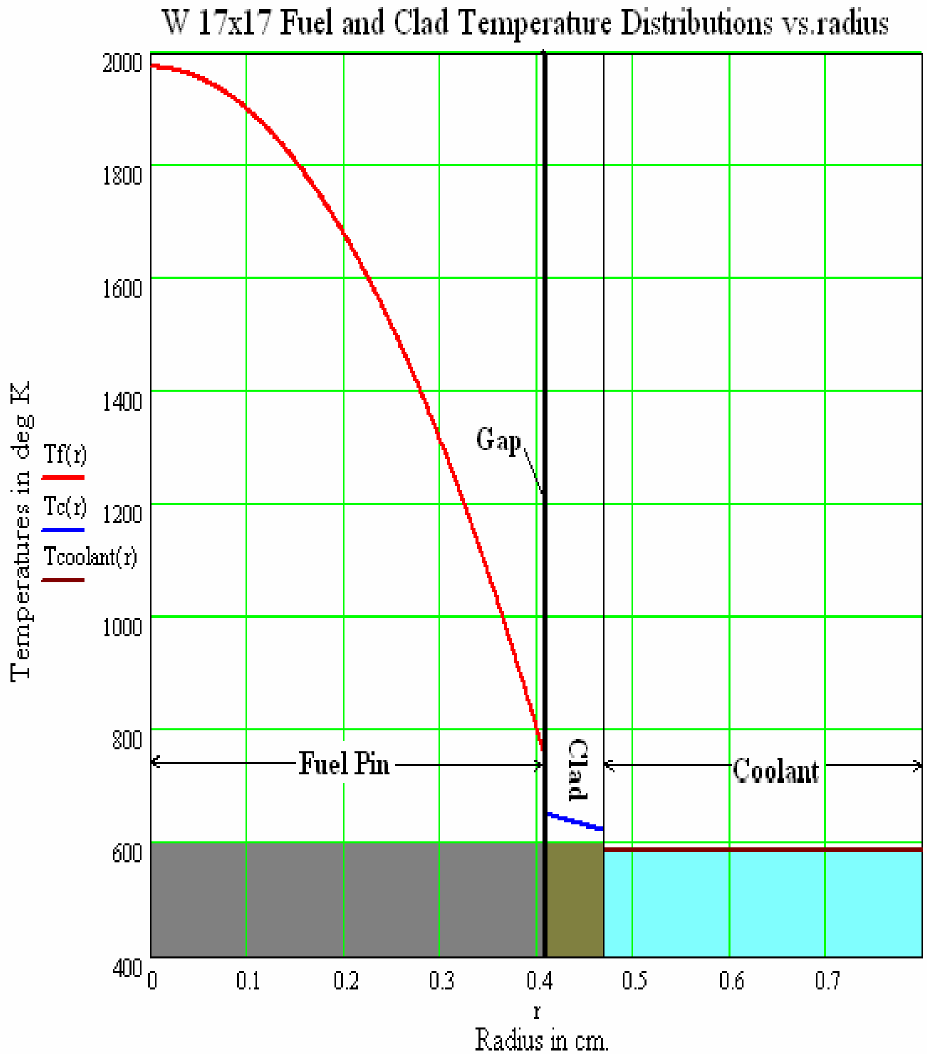
Fuel pin radius in cm:	$Ro := \frac{0.819}{2}$	Source:Westinghouse Duderstadt/Hamilton p.634
Clad radius in cm.:	$Rc := \frac{0.94}{2}$ $Rc = 0.47$	Source:Westinghouse Duderstadt/Hamilton p.634
Maximum linear Power Density in Watts/cm.:	$q := 459$	Source:Westinghouse WCAP-8720, p.A-17 14kW/ft = 459.32 Watts/cm
Effective fuel thermal conductivity in Watts/cm.K:	$kfuel := 0.03$	Source:MATPRO
Effective clad thermal conductivity in Watts/cm.K:	$kc := 0.17$	Source:MATPRO
Gap conductance in Watts/cm.K:	$hgap := 1.7$	Source:Westinghouse WCAP-8720, p.2-7 notes: 3000 BTU / hr.ft2 degF
Film conductance in Watts/cm.K	$hfilm := 4.5$	Source:Westinghouse Duderstadt/Hamilton p.481 notes: 4.5 Watts/cm K
Local Coolant temperature in K:	$Tcool := 588.15$	Source:Westinghouse Duderstadt/Hamilton p.635 notes: 300-332 C range. 315 C is assumed.

Predicted steady state fuel pin and clad temperature distributions:

$$Tf(r) := \begin{cases} Tcool + \frac{q}{2\cdot\pi}\cdot\left(\frac{1-\frac{r^2}{Ro^2}}{2\cdot kfuel} + \frac{1}{Ro\cdot hgap} + \frac{\ln\left(\frac{Rc}{Ro}\right)}{2\cdot kc} + \frac{1}{Rc\cdot hfilm}\right) & \text{if } r \le Ro \\ \text{break} & \text{otherwise} \end{cases}$$

$$Tc(r) := \begin{cases} Tcool + \frac{q}{2\cdot\pi}\cdot\left(\frac{\ln\left(\frac{Rc}{r}\right)}{2\cdot kc} + \frac{1}{Rc\cdot hfilm}\right) & \text{if } Ro < r \le Rc \\ \text{break} & \text{otherwise} \end{cases}$$

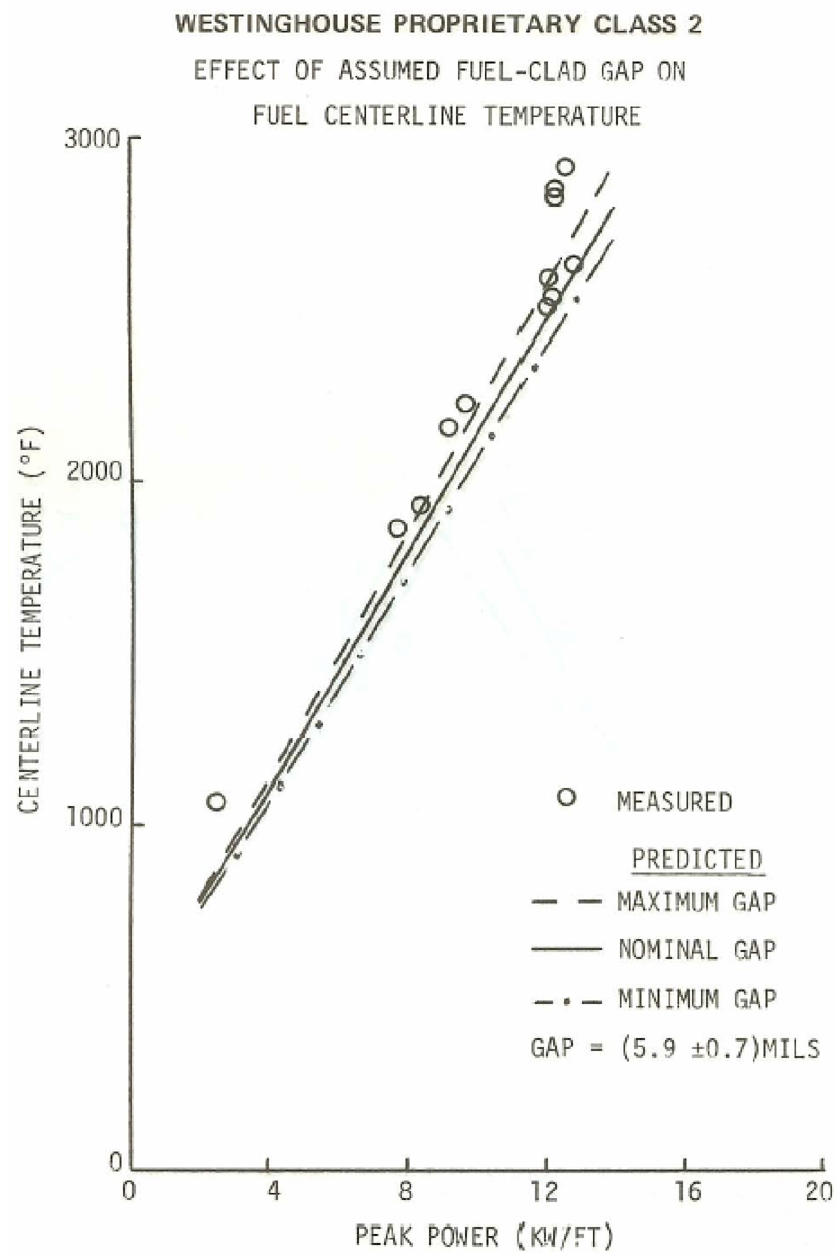
$$Tcoolant(r) := \begin{cases} Tcool & \text{if } Rc < r \\ \text{break} & \text{otherwise} \end{cases}$$



Peak Centerline Temperature in deg K: $Tf(0) = 1.975 \times 10^{3}$

Example: W 17x17 fuel at 14kW/ft.

- Predicted Peak Centerline Temperature ~1975°K
- This would be ~3095°F
- This is comparable to data described in WCAP-8720
- Actual reactor vendor fuel temperature evaluation models consider: burnup and temperature effects on density, heat transfer processes



Example: GE BWR-6 8x8 Fuel at 13.4kW/ft

Fuel Pin Temperature Distribution Assuming Constant: q, kfuel, kc

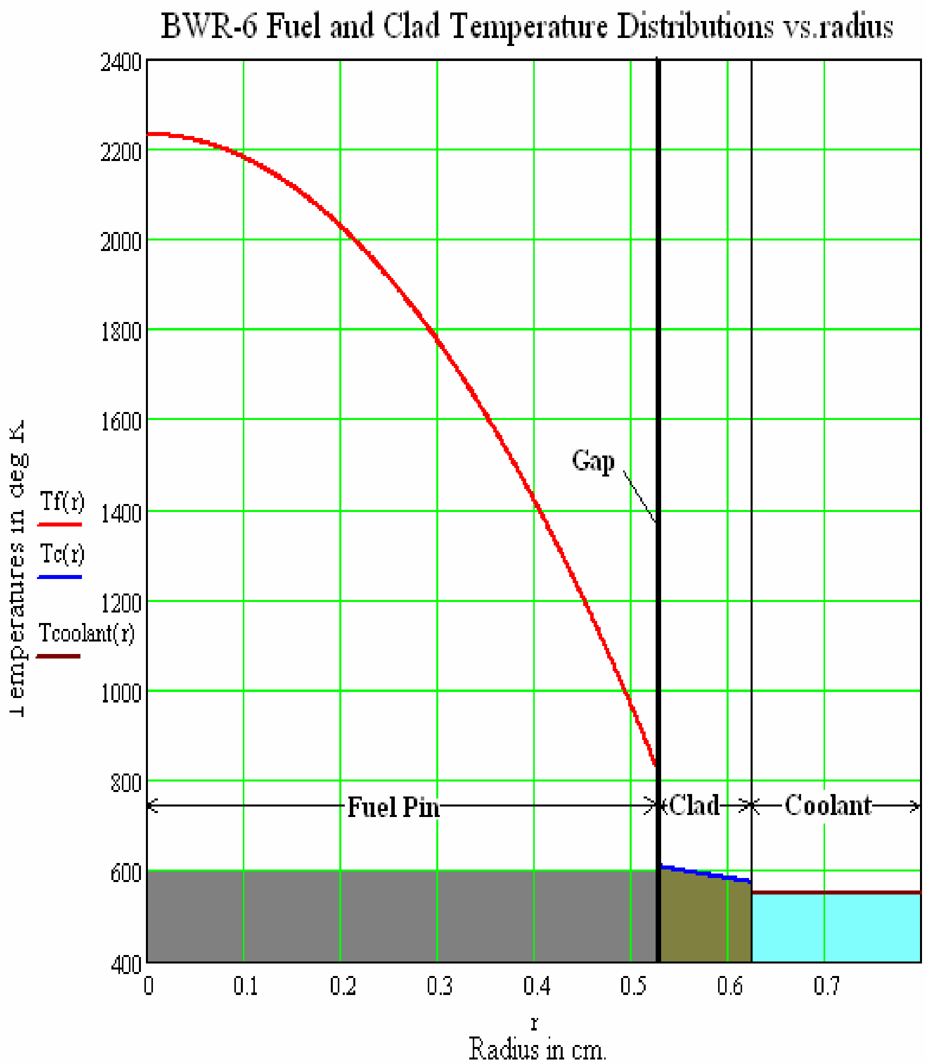
Fuel pin radius in cm:	$Ro := \frac{1.056}{2}$	$Ro = 0.528$	Source: GE BWR-6 Duderstadt/Hamilton p.634
Clad radius in cm.:	$Rc := \frac{1.25}{2}$	$Rc = 0.625$	Source: GE BWR-6 Duderstadt/Hamilton p.634
Maximum linear Power Density in Watts/cm.:	$q := 440$		Source: GE BWR-6 Duderstadt/Hamilton p.634
Effective fuel thermal conductivity in Watts/cm.K:	$kfuel := 0.025$		Source: MATPRO
Effective clad thermal conductivity in Watts/cm.K:	$kc := 0.17$		Source: MATPRO
Gap conductance in Watts/cm.K:	$hgap := 0.6$		Source: Duderstadt/Hamilton p.480 notes: 0.5 - 1.1 Watts/cm2K
Film conductance in Watts/cm.K	$hfilm := 4.5$		Source: Duderstadt/Hamilton p.481 notes: 4.5 Watts/cm2K
Local Coolant temperature in K:	$Tcool := 550.65$		Source: BWR-6 Duderstadt/Hamilton p.635 notes: 269-286 C range. 277.5 C is assumed.

Predicted steady state fuel pin and clad temperature distributions:

$$Tf(r) := \begin{cases} Tcool + \frac{q}{2\cdot\pi}\cdot\left(\frac{1-\frac{r^2}{Ro^2}}{2\cdot kfuel} + \frac{1}{Ro\cdot hgap} + \frac{\ln\left(\frac{Rc}{Ro}\right)}{2\cdot kc} + \frac{1}{Rc\cdot hfilm}\right) & \text{if } r \le Ro \\ \text{break} & \text{otherwise} \end{cases}$$

$$Tc(r) := \begin{cases} Tcool + \frac{q}{2\cdot\pi}\cdot\left(\frac{\ln\left(\frac{Rc}{r}\right)}{2\cdot kc} + \frac{1}{Rc\cdot hfilm}\right) & \text{if } Ro < r \le Rc \\ \text{break} & \text{otherwise} \end{cases}$$

$$Tcoolant(r) := \begin{cases} Tcool & \text{if } Rc < r \\ \text{break} & \text{otherwise} \end{cases}$$



Peak Centerline Temperature in deg. K: $Tf(0) = 2.232 \times 10^3$

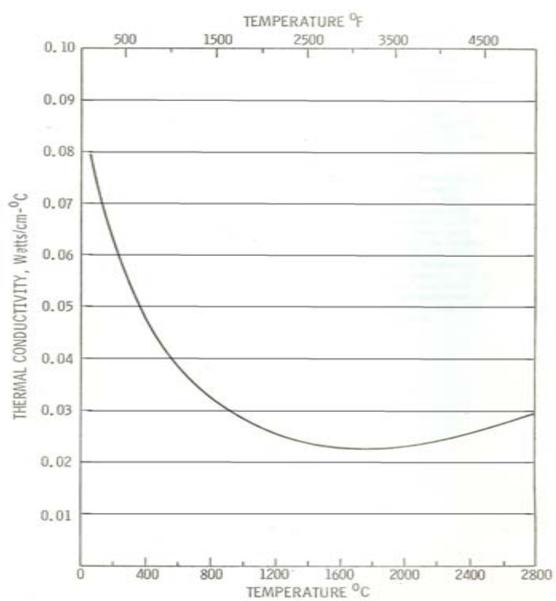
How Significant are Nonlinearities in $k_f(T)$?

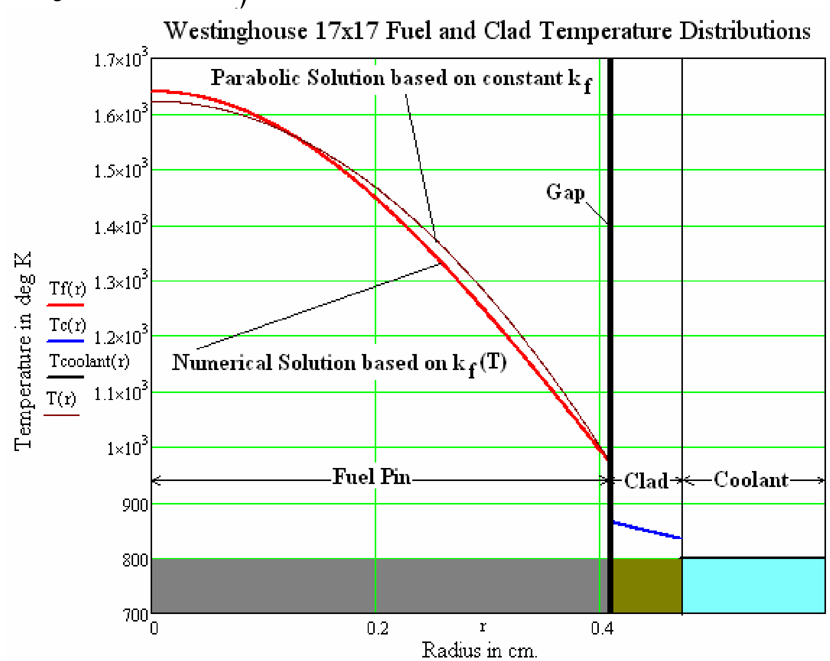
- Recall that: $k_f(T)$ temperature is parabolic in range of interest
- Previous calculations used constant $k_f \sim 0.025$ *Watts / cm °K*
- Actual equation is:

$$q + \nabla (k_f (T_f (r)) \nabla T_f (r)) = 0$$

$$q + k_f (T_f (r)) \frac{d^2 T_f (r)}{dr^2} + \frac{dk_f (T_f (r))}{dT_f (r)} \left(\frac{dT_f (r)}{dr} \right)^2 + \frac{k_f (T_f (r))}{r} \frac{dT_f (r)}{dr} = 0$$

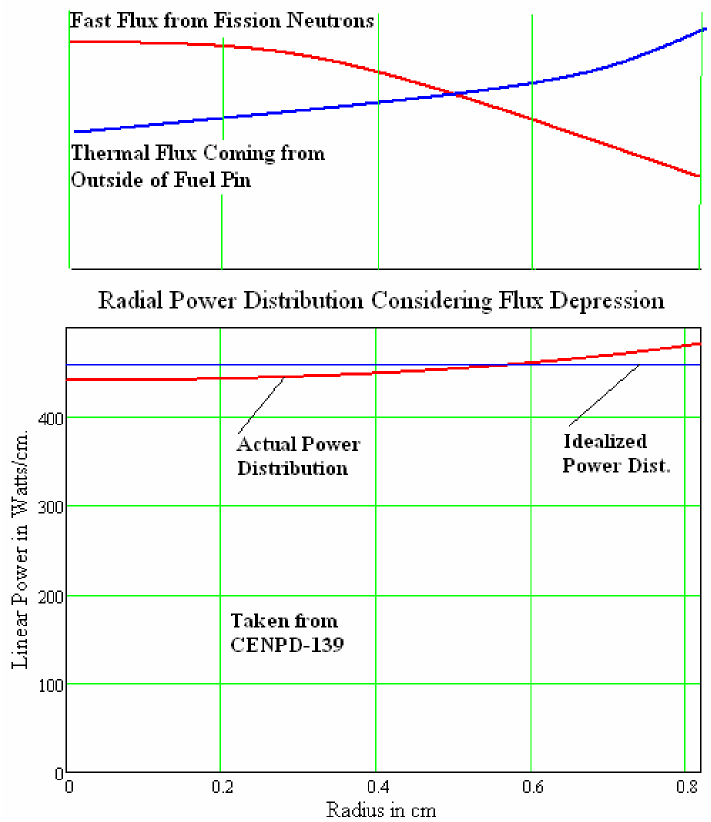
- Fitting k_f to "detailed calculation" yields: $k_f \sim 0.056$ *Watts / cm °K*





Other Nonlinearities:

- Thermal/fast neutrons *distribute* differently within fuel pin
- Fast neutrons peak in center
- Thermal neutrons peak in water
- Actual power distribution is *depressed* in center of pin
- Steady state diffusion equation reflecting flux depression:



$$P(r) + \nabla (k_f(T_f(r)) \nabla T_f(r)) = 0$$

$$P(r) + k_f(T_f(r)) \frac{d^2 T_f(r)}{dr^2} + \frac{dk_f(T_f(r))}{dT_f(r)} \left(\frac{dT_f(r)}{dr} \right)^2 + \frac{k_f(T_f(r))}{r} \frac{dT_f(r)}{dr} = 0$$

Transient Core Heat Transfer

Transient Heat Transfer via Lumped Parameter Method

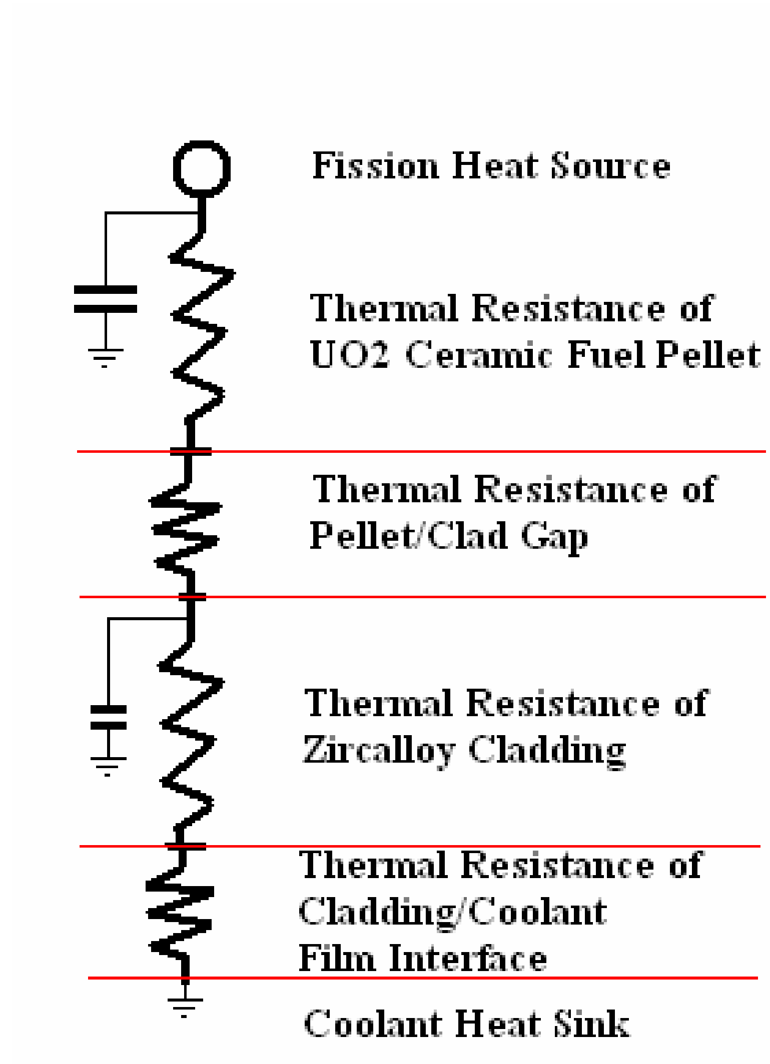
- Heat transfer equation for fuel previously noted as:

$$\rho(T_f(r,t))c_p(T_f(r,t))\frac{\partial T_f(r,t)}{\partial t} = P(r,t) + \nabla(k_f(T_f(r,t))\nabla T_f(r,t))$$

- Licensing calculations require considerable detail
- Majority of dynamic core heat transfer can be understood based on *Lumped Parameter Heat Transfer Model*
- Lumped Parameter approach recognizes that fuel and clad behave differently due to different “time constants”
- Method involves computing “effective” temperatures and rates of heat transfer between regions.
- Lumped Parameter approach can be directly used with fuel temperature (Doppler) reactivity feedback simulations

Transient Heat Transfer via Lumped Parameter Method

- Steady state heat transfer network modeled as resistance network
- Transient heat transfer network modeled as having "heat storage"
- During steady state heat stored in fuel, clad
- During transients temperature response is delayed via thermal capacitance



Transient Heat Transfer via Lumped Parameter Method

- Using RC electrical circuit analogy:
- Temperature in fuel pellet is like *voltage source*
- Heat flux is like *electrical current*
- Temperature drops are equivalent to *voltage drops*
- There is *thermal resistance* in fuel, gap, clad, film
- Thermal resistance if fuel pellet is: $R_f = 1/(4\pi k_f)$
- Thermal resistance of gap is: $R_{gap} = 1/(2\pi R_o h_{gap})$
- Thermal capacitance in fuel is: $C_f = \pi R_o^2 C_{pf} \rho_f$
- C_{pf} is Heat Capacity in Watt-sec/gram°K
  ($C_{pf} = 0.377$ *Watt-sec/gram°K*)
- ρ_f is fuel pellet density in gram/cm^3 ($\rho_f = 10.97$ *gram/cm*^{3})

Lumped Parameter Fuel Heat Transfer

- Dynamics of (spatially averaged) Fuel temperature:

$$C_f \frac{d\langle T_f \rangle}{dt} = \pi R_o \langle P(t) \rangle - \frac{\langle T_f \rangle - \langle T_c \rangle}{R_{fc}}$$

$$C_f = \pi R_o^{\ 2} C_{pf} \rho_f$$

- R_{fc} is effective thermal resistance between $<T_f>, <T_c>$
- $<T_f>, <T_c>$ are respectively spatial averaged fuel and clad temperatures in °K
- $<P(t)>$ is volumetric heat source, $\pi R_o <P(t)> = <q(t)>$ is linear power density in Watts/cm.

Lumped Parameter Clad Heat Transfer

- Dynamics of effective (spatially averaged) clad temperature are similarly given by:

$$C_c \frac{d\langle T_c \rangle}{dt} = \frac{\langle T_f \rangle - \langle T_c \rangle}{R_{fc}} - \frac{\langle T_c \rangle - \langle T_{AVG} \rangle}{R_{cc}}$$

$$C_c = \pi (R_c^{\ 2} - R_o^{\ 2}) C_{pc} \rho_c$$

- R_{cc} is effective thermal resistance between: $<T_c(t)>, <T_{AVG}(t)>$
- Where: $<T_{AVG}(t)>$ is average coolant temperature in °K
- C_{pc} is Heat Capacity in Watt-sec/gram°K
  (C_{pc} = 0.331 Watt-sec/gram°K)
- ρ_c is clad density in gram/cm^3 ($\rho_c = 6.57$ gram/cm^3)

Lumped Parameter Heat Transfer Model

- Effective fuel and clad temperatures are related to previously calculated temperatures via following averages:

$$\left\langle T_f(t) \right\rangle = \frac{1}{\pi R_o^{\ 2}} \int_0^{R_o} T_f(r,t) 2\pi r dr$$

$$\left\langle T_c(t) \right\rangle = \frac{1}{\pi (R_c^{\ 2} - R_o^{\ 2})} \int_{R_o}^{R_c} T_c(r,t) 2\pi r dr$$

- Heat flux from clad to coolant, which lags power, can be expressed:

$$q_{c\text{-}c}(t) = [\ \langle T_c(t) \rangle - T_{avg}(t)\ ] / R_{cc}$$

Effective Temperatures

Computation of Effective Fuel Temperature $<T_f>$:

$$<T_f> = \frac{1}{\pi \cdot Ro^2} \cdot \int_0^{Ro} \left[Tcool + \frac{q}{2 \cdot \pi} \cdot \left(\frac{1 - \frac{r^2}{Ro^2}}{2 \cdot kfuel} + \frac{1}{Ro \cdot hgap} + \frac{\ln\left(\frac{Rc}{Ro}\right)}{2 \cdot kc} + \frac{1}{Rc \cdot hfilm} \right) \right] \cdot 2 \cdot \pi \cdot r \, dr$$

$$<T_f> = Tcool + \frac{q}{2 \cdot \pi} \cdot \left(\frac{1}{4 \cdot kfuel} + \frac{1}{Ro \cdot hgap} + \frac{\ln\left(\frac{Rc}{Ro}\right)}{2kc} + \frac{1}{Rc \cdot hfilm} \right) = 1.366 \times 10^3 \ {}^oK$$

Computation of Effective Clad Temperature $<Tc>$:

$$<Tc> = \frac{1}{\pi \cdot \left(Rc^2 - Ro^2\right)} \cdot \int_{Ro}^{Rc} \left[Tcool + \frac{q}{2 \cdot \pi} \cdot \left(\frac{\ln\left(\frac{Rc}{r}\right)}{2 \cdot kc} + \frac{1}{Rc \cdot hfilm} \right) \right] \cdot 2 \cdot \pi \cdot r \, dr$$

$$<Tc> = Tcool + \frac{q}{2 \cdot \pi} \cdot \left[\frac{1}{Rc \cdot hfilm} + \frac{1}{4 \cdot kc} + \frac{Ro^2 \cdot \ln\left(\frac{Ro}{Rc}\right)}{2\left(Rc^2 - Ro^2\right) \cdot kc} \right] = 636.814 \ {}^oK$$

Lumped Parameter Time Constants

Time constant for fuel:

Fuel Heat Capacity in Watt-sec/gram K: $Cpf := 0.377$ Based upon 0.09 BTU/lb.F Tong & Weissman, p.64

Fuel Density in gr/cm[3]: $\rho f := 10.97$ Based upon Tong & Weissman, p.64

$$Cf := \pi \cdot Ro^2 \cdot Cpf \cdot \rho f$$

$$Rfc := \frac{1}{2\cdot\pi}\left[\frac{1}{4\cdot kfuel} + \frac{1}{Ro\cdot hgap} - \frac{1}{4\cdot kc} + \frac{\ln\left(\frac{Rc}{Ro}\right)}{2\cdot kc} - \frac{Ro^2\cdot\ln\left(\frac{Ro}{Rc}\right)}{\left[2\left(Rc^2 - Ro^2\right)\cdot kc\right]}\right]$$

$\tau f := Cf\cdot Rfc$ Time constant in seconds: $\tau f = 3.461$

Time constant for clad:

Zircaloy Heat Capacity in Watt-sec/gram K: $Cpc := 0.3307$ Based upon 0.0789 BTU/lb.F Tong & Weissman, p.67

Zircaloy Density in gr/cm[3]: $\rho c := 6.57$ Based upon Tong & Weissman, p.67

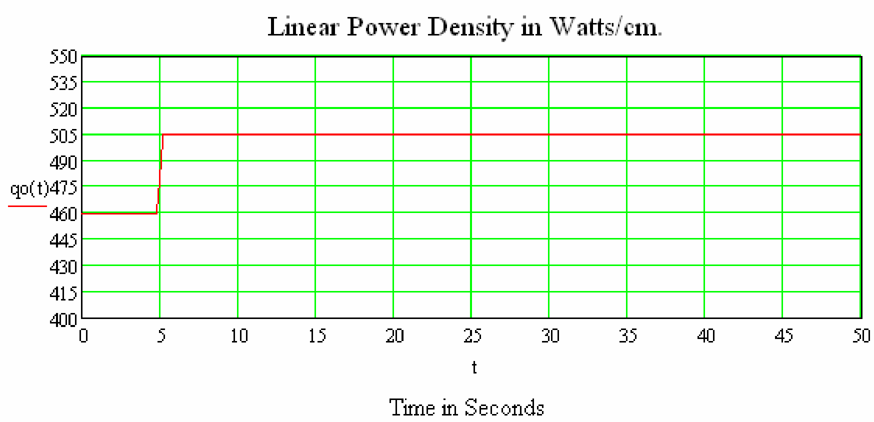
$$Rcc := \frac{1}{2\cdot\pi}\cdot\left(\frac{1}{4\cdot kfuel} + \frac{1}{Ro\cdot hgap} + \frac{\ln\left(\frac{Rc}{Ro}\right)}{2kc} + \frac{1}{Rc\cdot hfilm}\right)$$

$$Cc := \pi\cdot\left(Rc^2 - Ro^2\right)\cdot Cpc\cdot\rho c$$

$\tau c := Cc\cdot Rcc$ Time constant in seconds: $\tau c = 0.615$

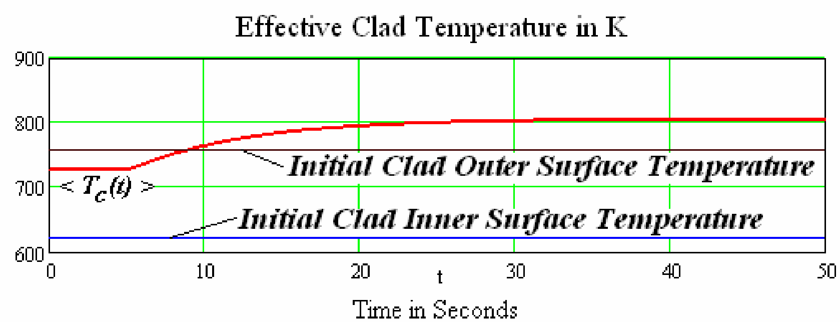
$Req := \frac{Rfc\cdot Rcc}{Rfc + Rcc}$ $\tau eq := Cc\cdot Req$ $\tau eq = 0.298$

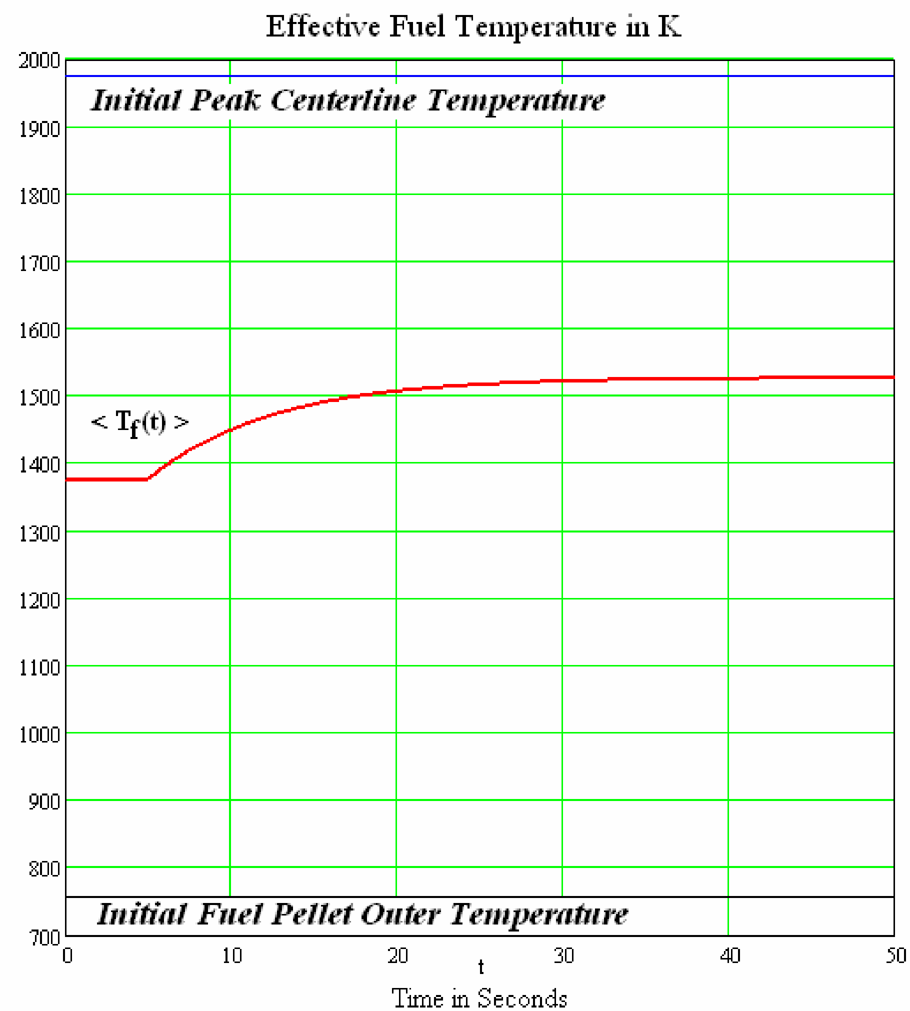
MATHCAD Simulation of +10% Rise in q



Linear Power Density from 459 Watts/cm. by +10% at t = 5 seconds

Coolant Temperature Assumed to Remain constant



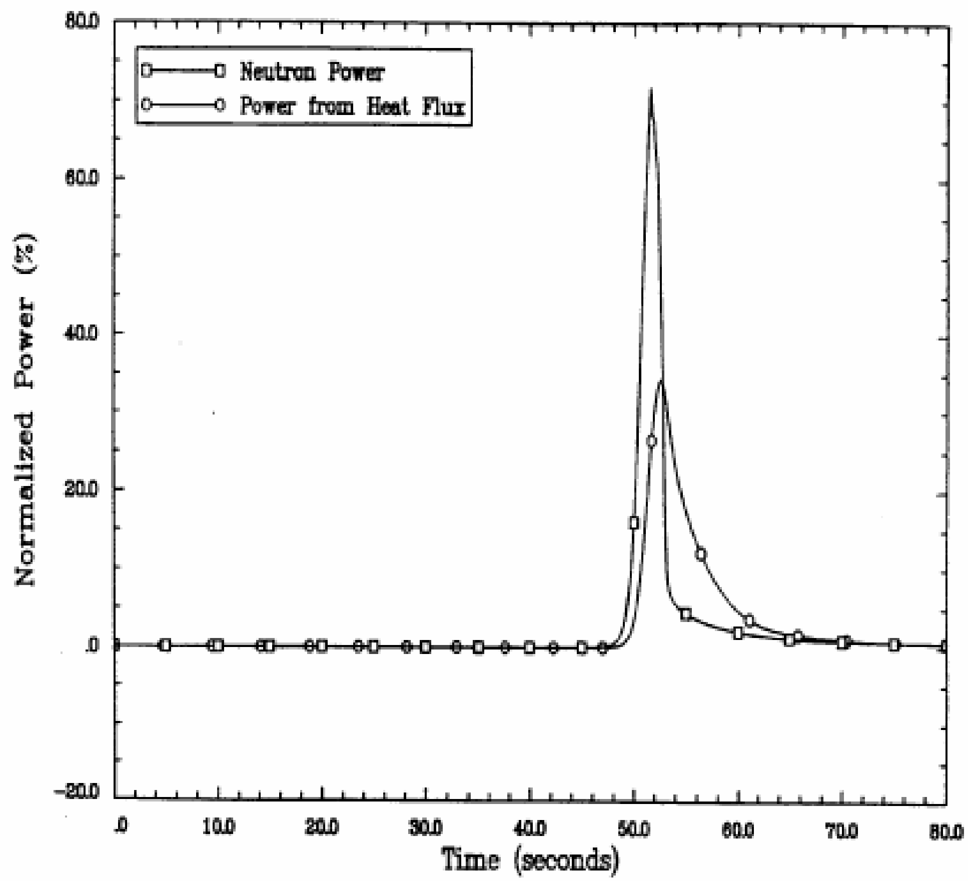


Example: PWR Surface Heat Flux vs Power Response During Startup Transient

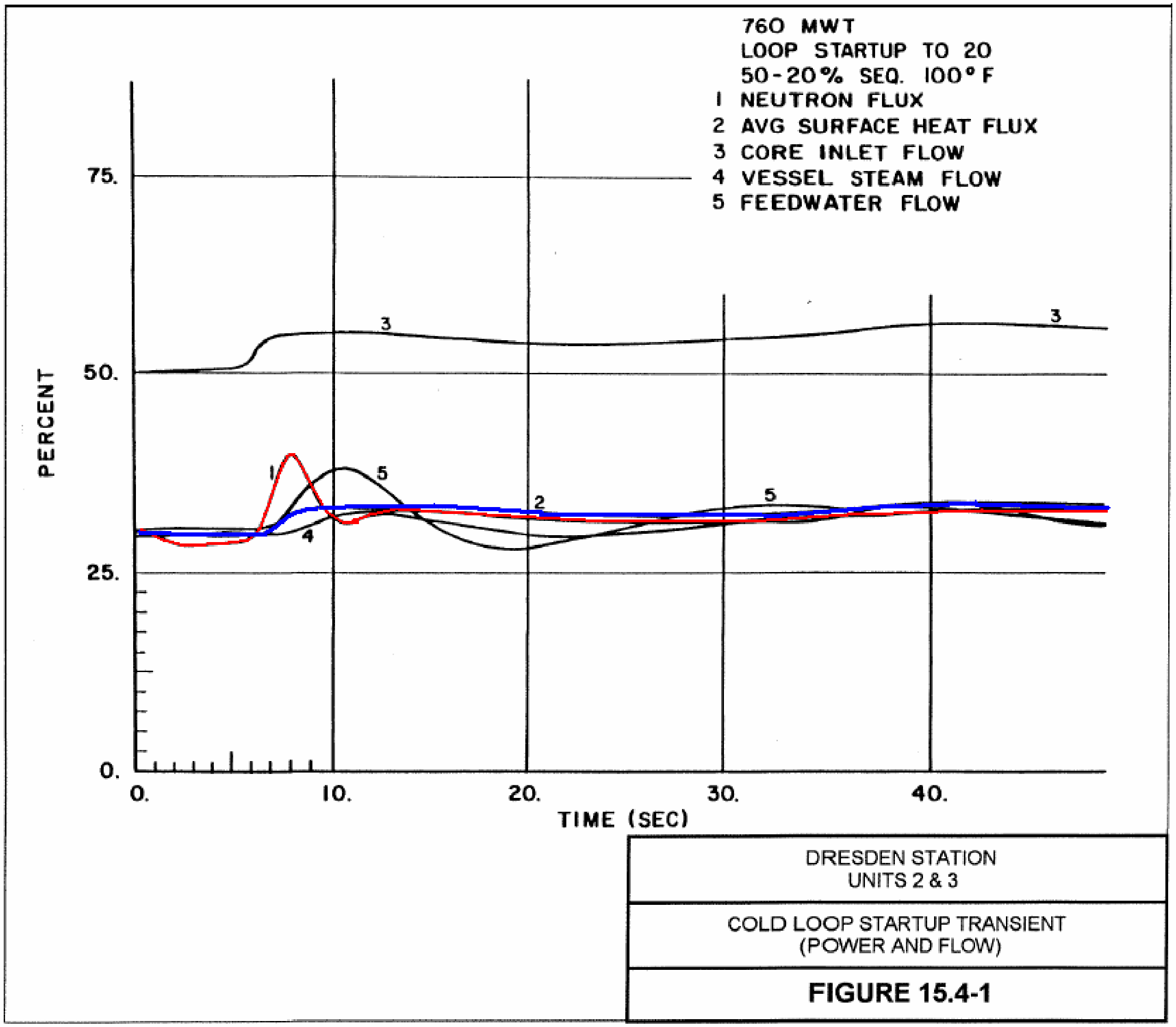
FSAR CHAPTER 14 - SAFETY ANALYSIS

FIGURE 14.2.1-4
Revision 21

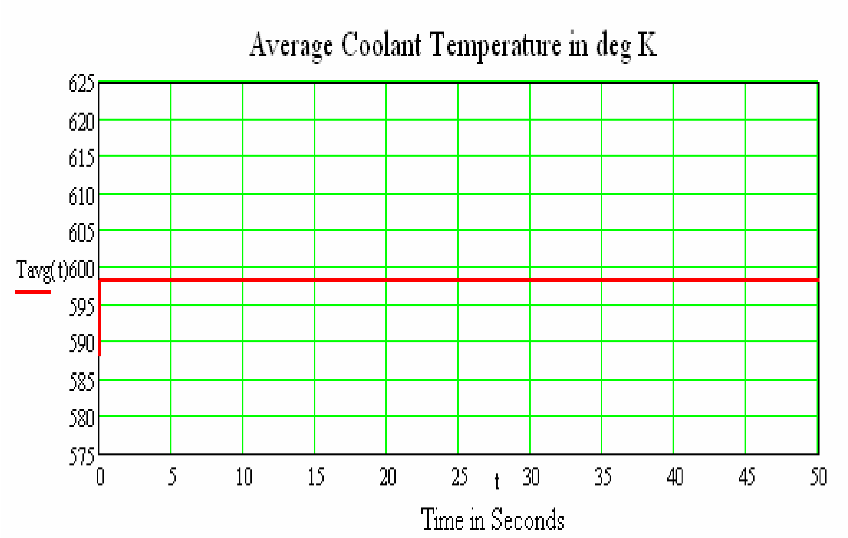
CONTROL ROD WITHDRAWAL INCIDENT HZP POWER AND HEAT FLUX

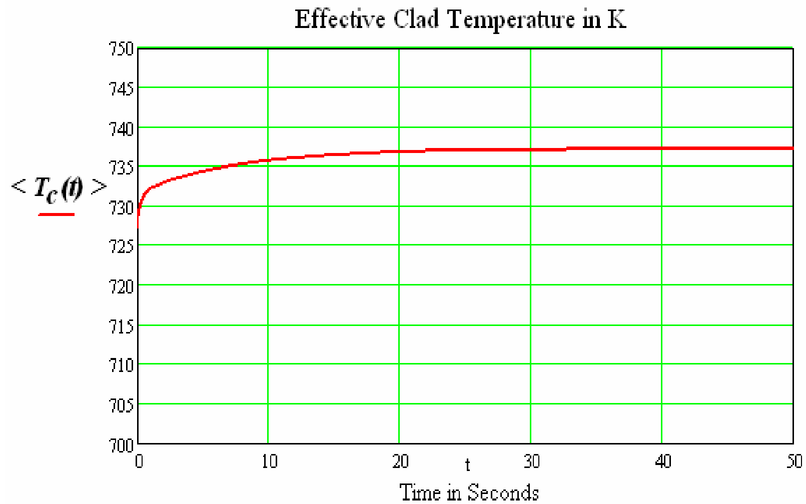


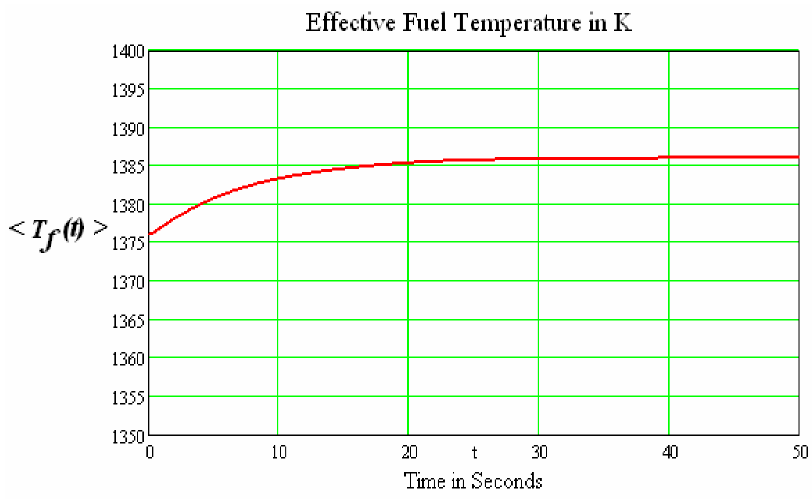
Example: BWR Surface Heat Flux vs Power Response During Loop Startup Transient



*MATHCAD Simulation of +10°K Rise in **Tavg***







Summary

- Laws governing heat transfer can be used to define temperature and heat flow rates in fuel rods
- Key material fuel/clad properties (thermal conductivity, conductance, heat capacity) are *temperature dependent*
- *Simple heat transfer calculation* assuming "flat neutron power distribution" and constant material properties shows:
    - Peak centerline temperature is linear function of *coolant temperature* and *linear power density*
    - Fuel temperature distribution is *roughly parabolic*
- *Detailed fuel models* consider:
    - Thermal neutron flux depression in fuel pin
    - Temperature dependencies of material properties, densification..
    - Gap distance, fill gas composition……